Mainz Laboratory Highlights

proton radius, polarizabilities, …, muon g—2, dark photons

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Institute for Nuclear Physics
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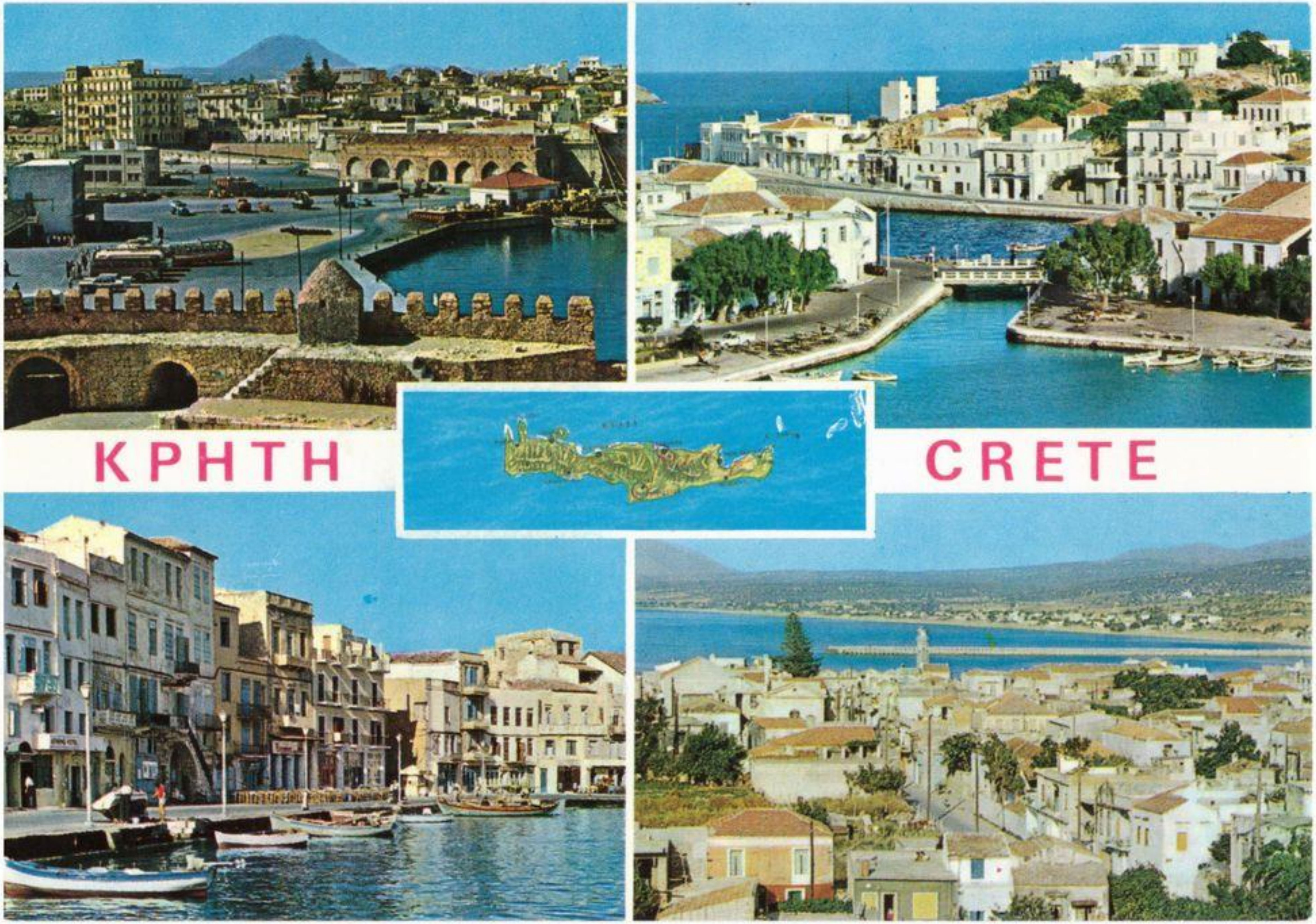
@ HiX Workshop, KPHTH,
Greece
Aug 17, 2019

KernPHhysik Inst. — THeorie (KPHTH)



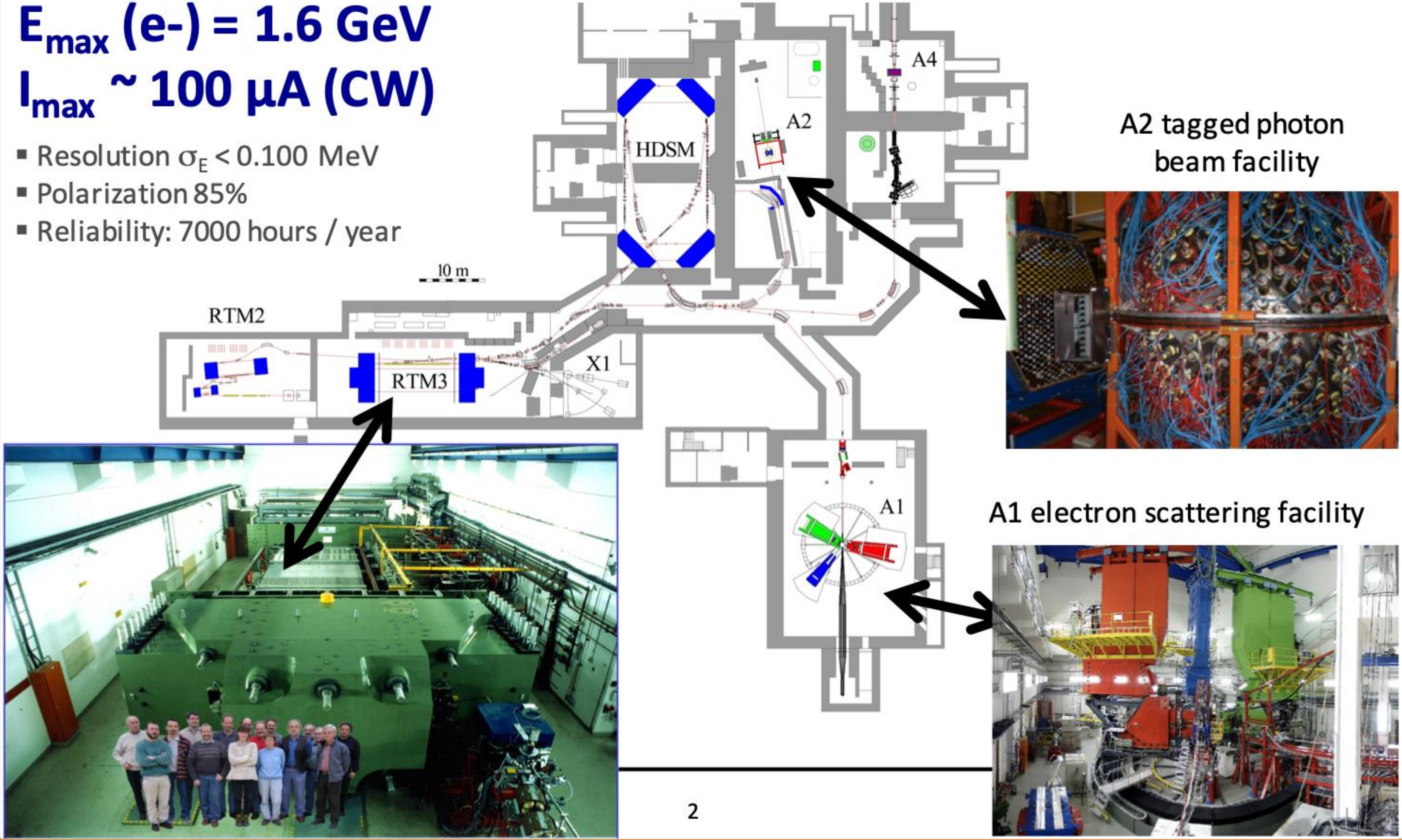
The Mainz Microtron MAMI

Electron Accelerator for Fixed Target Experiments

E_{max} (e-) = 1.6 GeV

I_{max} ~ 100 µA (CW)

- Resolution $\sigma_E < 0.100$ MeV
- Polarization 85%
- Reliability: 7000 hours / year



A2 tagged photon beam facility

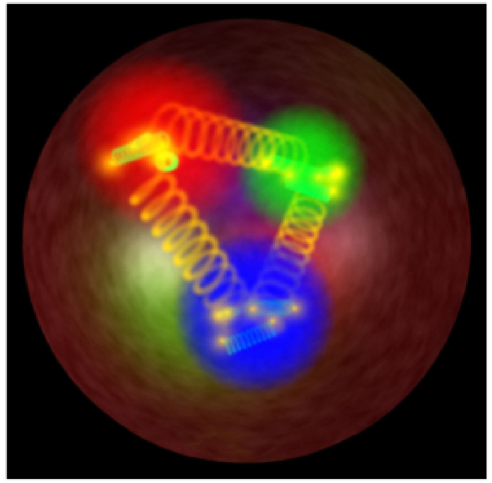
A1 electron scattering facility

Collaborative Research Centre 1044

coordinators: Achim Denig, Marc Vanderhaeghen

Hadron physics (= The Low-Energy Frontier of the Standard Model)

plays a central and connecting role in interpretation of measurements at the precision frontier of the Standard Model



SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

$\sin^2\theta_W$

$(g-2)_\mu$

R_E
Proton charge radius

Dark Photon

Nuclear EOS

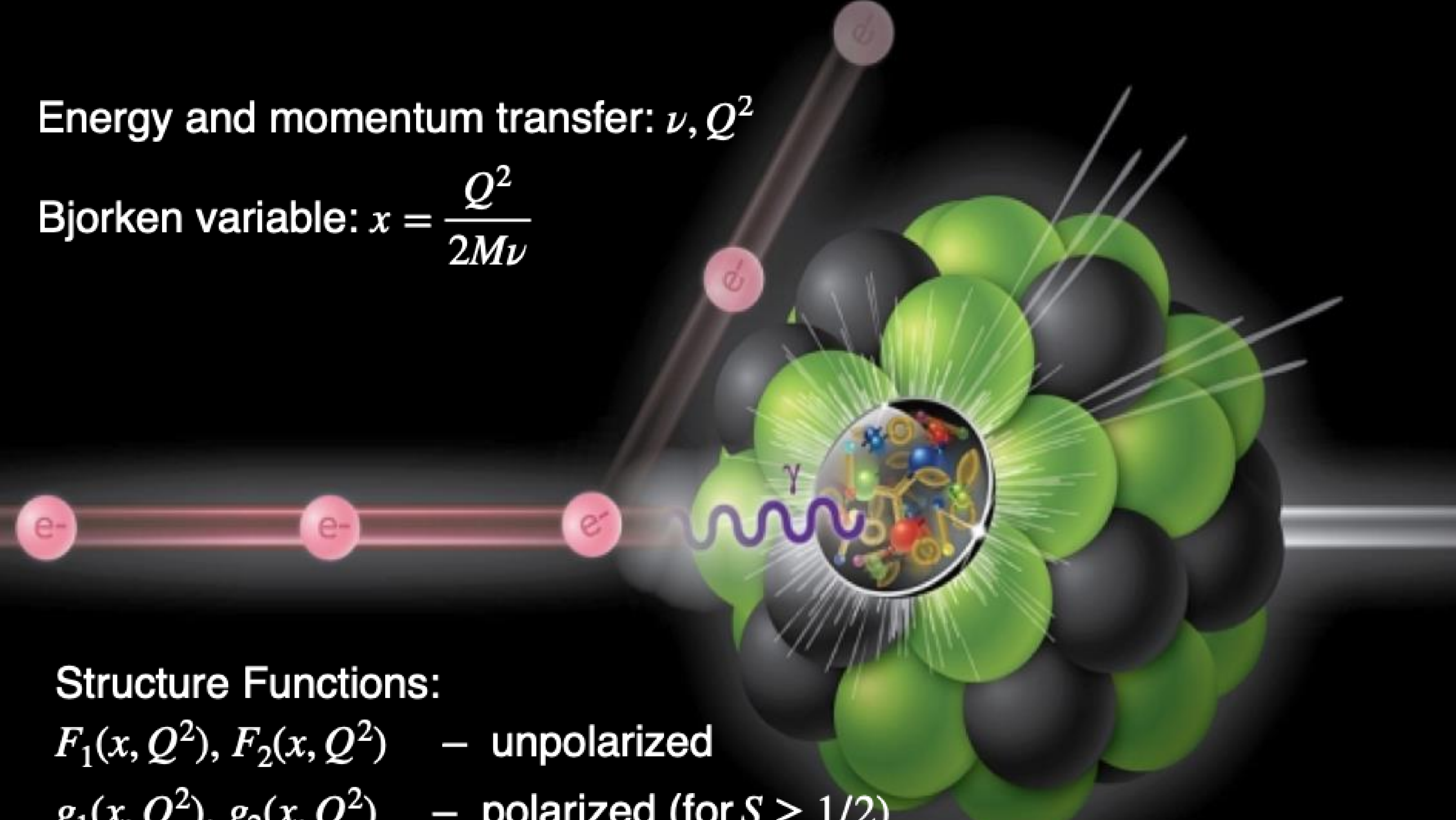
Strong interactions
Hadron structure
Hadron spectroscopy

↔

Particle physics
Atomic physics
Astro(particle) physics

Energy and momentum transfer: ν, Q^2

Bjorken variable: $x = \dfrac{Q^2}{2M\nu}$



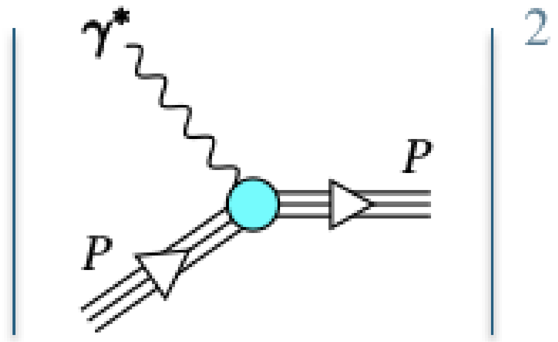
Structure Functions:

$F_1(x, Q^2)$, $F_2(x, Q^2)$ – unpolarized

$g_1(x, Q^2)$, $g_2(x, Q^2)$ – polarized (for $S \geq 1/2$)

$b_{1,2,3,4}(x, Q^2)$ – tensor (for $S \geq 1$)

Elastic $(x = 1)$



$$F_1^{\rm el}(x, Q^2) = \frac{1}{2} G_M^2(Q^2)\ \delta(1 - x)$$

$$F_2^{\rm el}(x, Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau}\ \delta(1 - x)$$

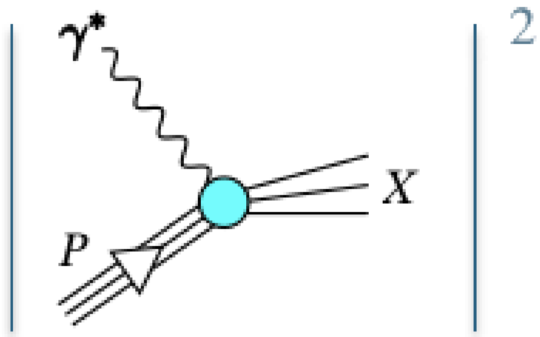
$$g_1^{\rm el}(x, Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1 + \tau)} G_M(Q^2)\ \delta(1 - x)$$

$$g_2^{\rm el}(x, Q^2) = \frac{G_E(Q^2) - G_M(Q^2)}{2(1 + \tau)} \tau G_M(Q^2)\ \delta(1 - x)$$

with G_E electric and G_M magnetic Form Factors,

$\tau = Q^2/4M^2$

Inelastic $(0 < x \leq x_0 < 1)$



$$F_1(x, Q^2) \sim \sigma_T(\nu, Q^2)$$

$$F_2(x, Q^2) \sim (\sigma_T + \sigma_L)(\nu, Q^2)$$

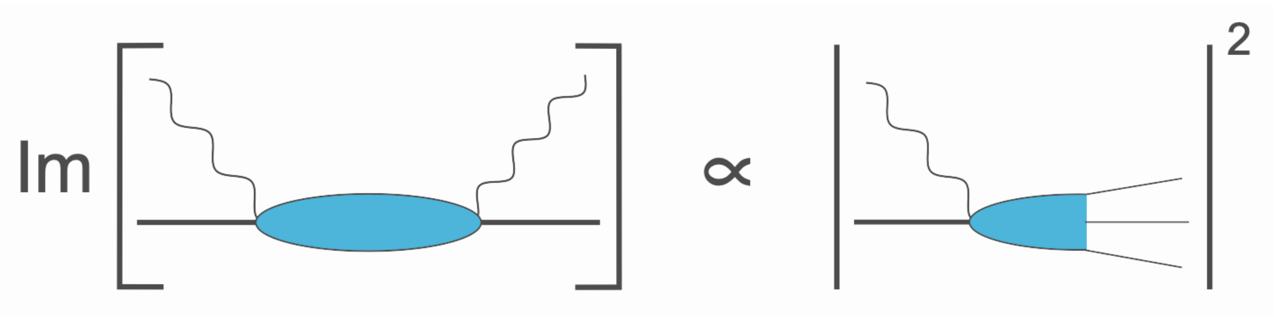
$$g_1(x, Q^2) \sim [(Q/\nu)\sigma_{LT} + \sigma_{TT}](\nu, Q^2)$$

$$g_2(x, Q^2) \sim [(\nu/Q)\sigma_{LT} - \sigma_{TT}](\nu, Q^2)$$

with total photoabsorption cross sections $\sigma(\nu, Q^2)$

Relation to forward Compton scattering

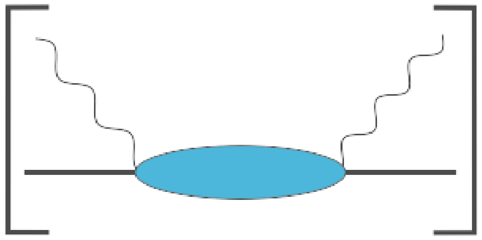
- Optical theorem:



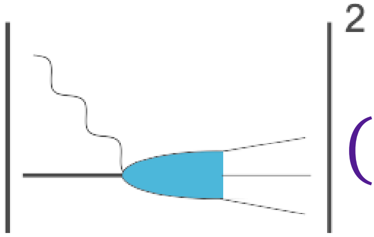
- Causality (analyticity, Cauchi formula):

$$f(w) = \frac{1}{2\pi i}\oint_C \frac{f(z)\,dz}{z-w}$$

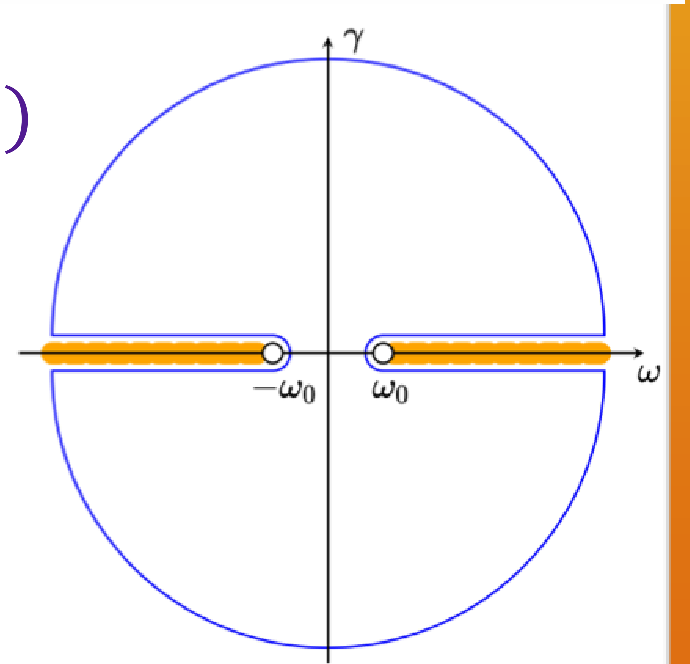
for any interior pt. w of C



$$(\nu, Q^2) = \frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu' \,\frac{\nu'}{\nu'^2-\nu^2}$$



$$(\nu', Q^2)$$

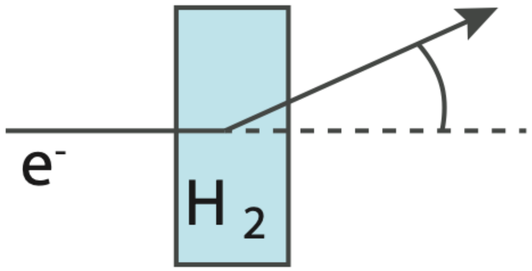


Polarizabilities

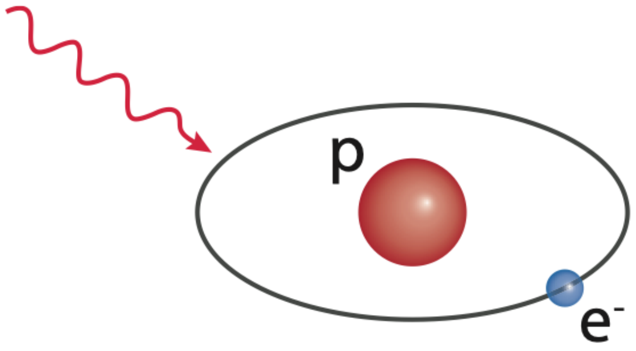
Sum Rules

eg: GDH,
Baldin,
Schwinger

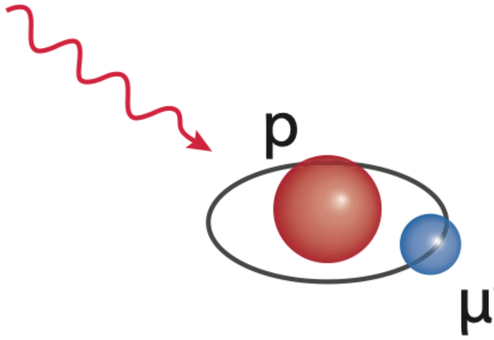
Structure functions



e^--p scattering



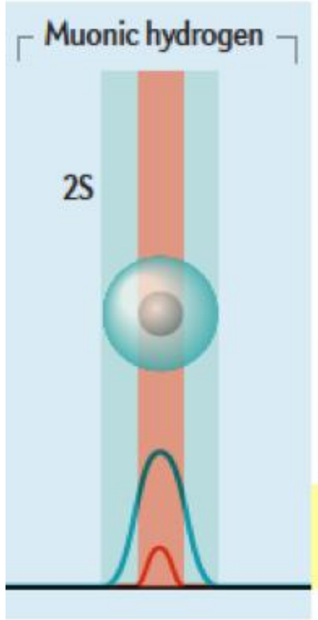
H spectroscopy

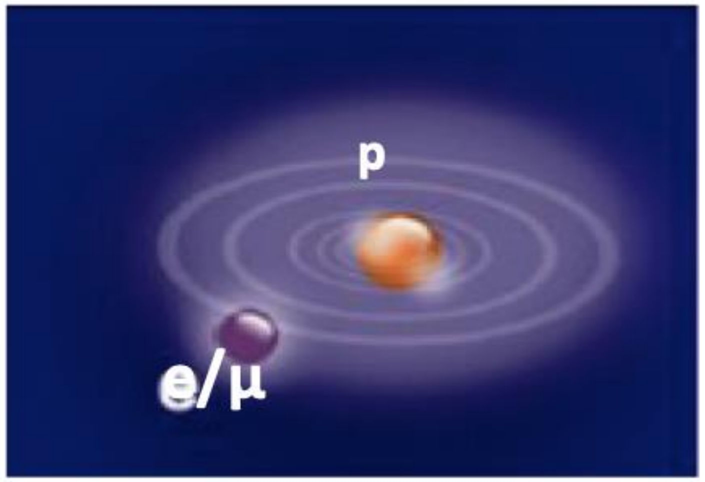


μp spectroscopy

Proton radius puzzle

Atomic Spectroscopy
(PSI: Lamb Shift in muonic hydrogen)

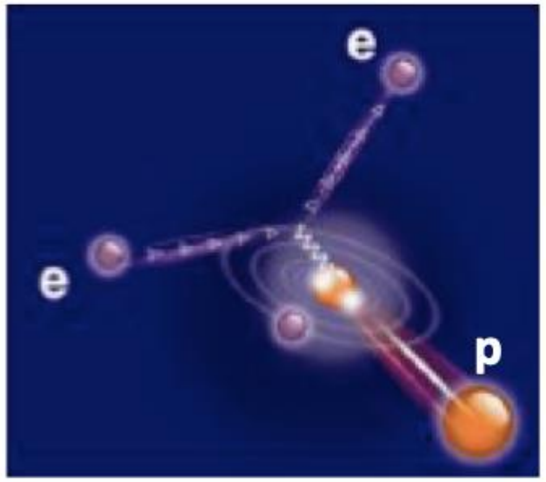




$R_E = 0.8409 \pm 0.0004$ fm

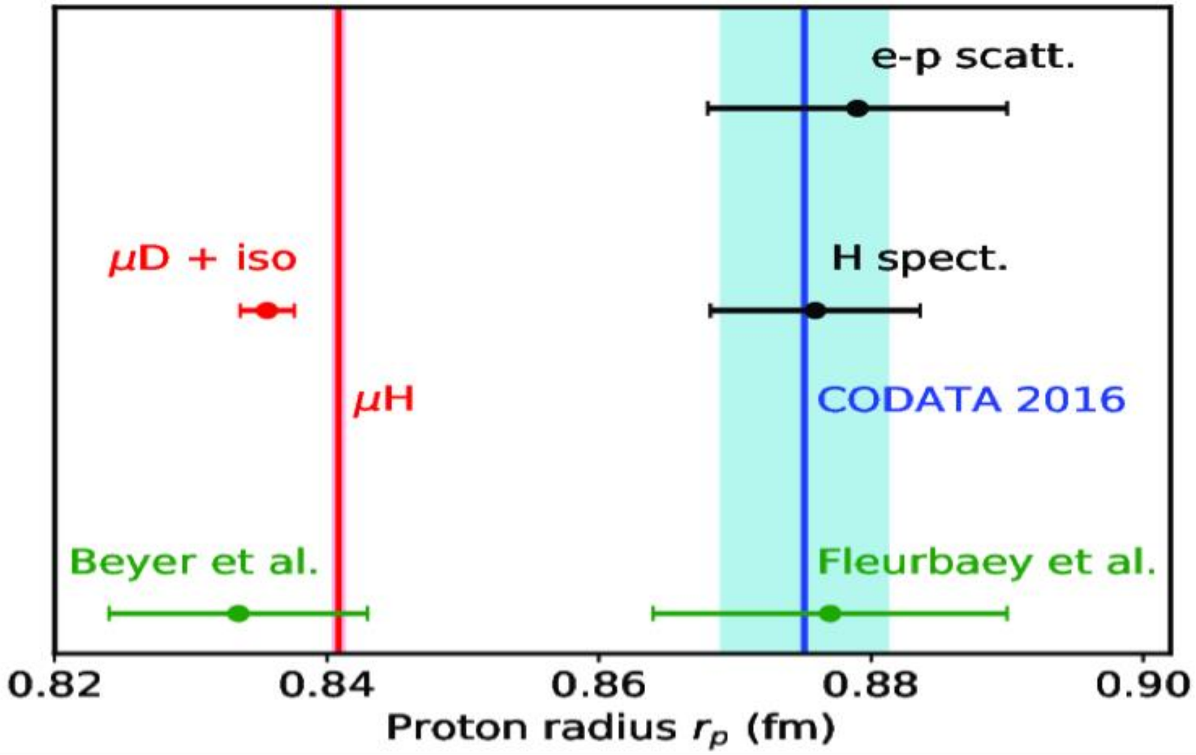
Nature (2012)
Science (2013)

Electron Scattering on proton
(EM form factor measurements)



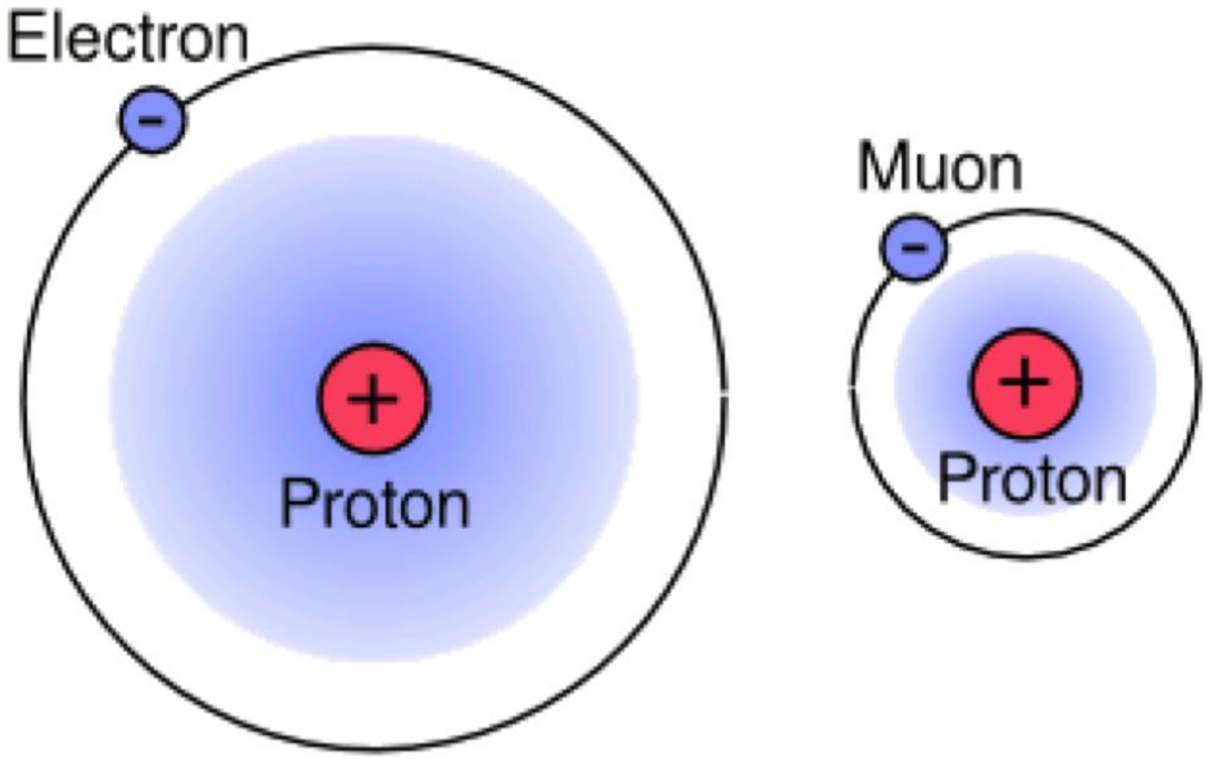
$R_E = 0.879 \pm 0.008$ fm

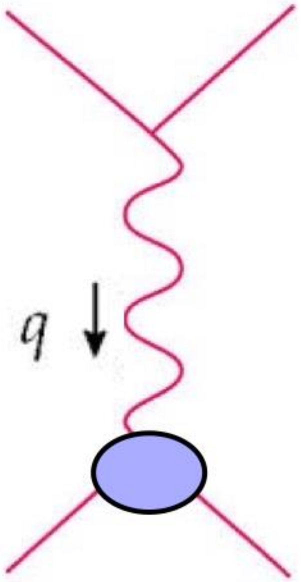
PRL (2010)
PRD(2014)



The New York Times

Hydrogens sensitive to proton structure





$$\delta V^{(1\gamma)} = -\frac{4\pi\alpha}{\vec{q}^{\,2}}\left[G_E(-\vec{q}^{\,2}) - 1\right] = \frac{2}{3}\pi\alpha\, r_E^2 + O(\vec{q}^{\,2})$$

$$\Delta E_{nl}^{(\mathrm{FS})} = \left\langle nlm \right|\, \delta V^{(1\gamma)} \left| nlm \right\rangle = \delta_{l0}\frac{2}{3}\pi\alpha\, r_E^2\, \frac{\alpha^3 m_r^3}{\pi n^3} + O(\alpha^5)$$

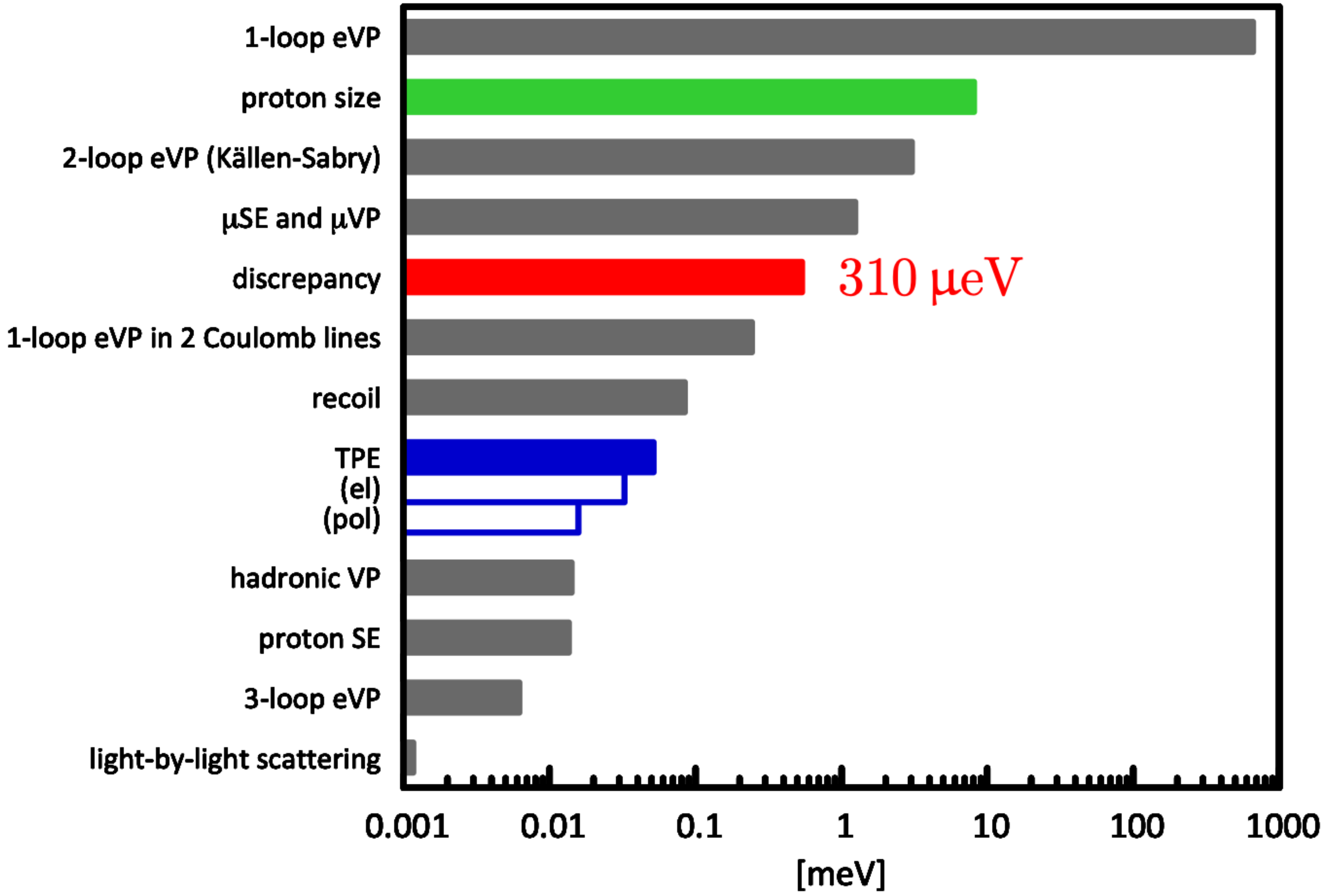
wave function at origin

Muonic Hydrogen Lamb shift

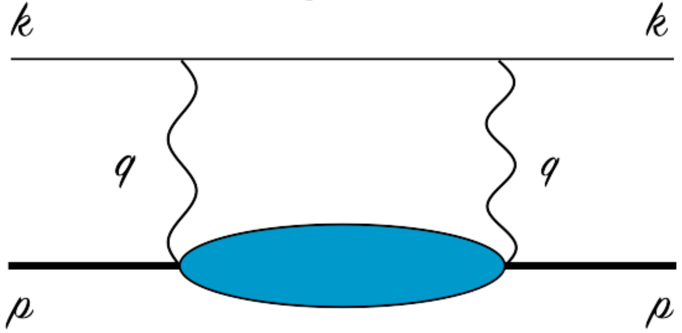
$$\Delta E^{\rm th}_{\rm LS} = 206.0668(25) - 5.2275(10)\,(R_E/{\rm fm})^2$$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).

theory uncertainty:
$2.5\,\mu eV$



subleading effects of proton structure proposed to resolve the puzzle



$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\rm elastic} + \delta V^{(2\gamma)}_{\rm polariz.}$$

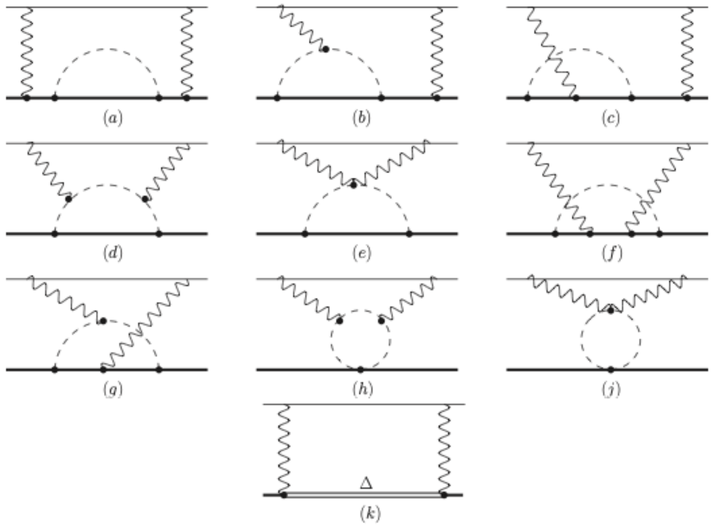
A. De Rujula, Phys. Lett. B693 (2010)
G. A. Miller, Phys. Lett. B718 (2013)

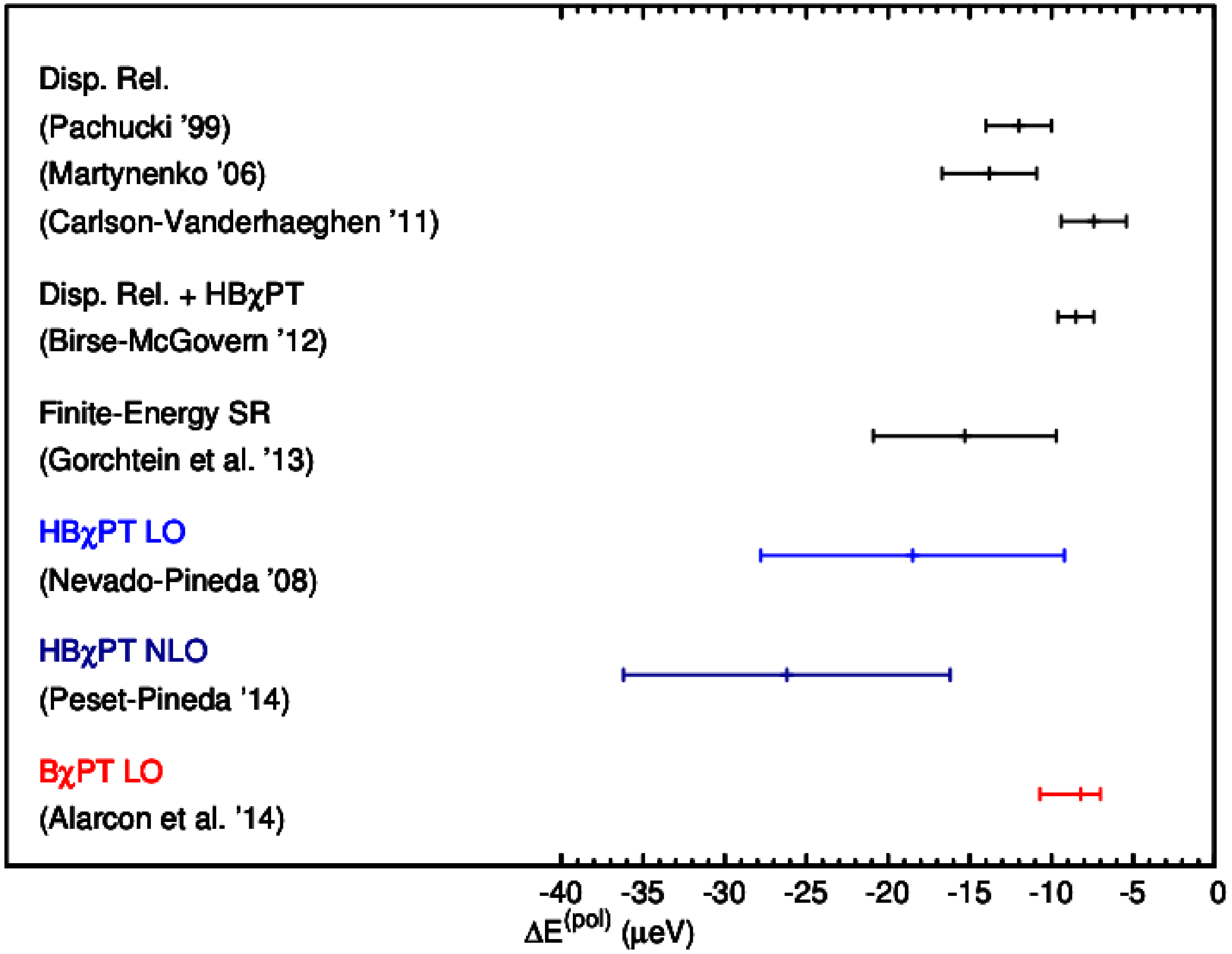
Proton polarizability in muonic-H Lamb shift

Can be computed with dispersion th. + data

But subtraction term is needed — model dependent

vs.
Chiral perturbation theory predictive at LO





Compiled by: Hagelstein, Miskimen & VP, *Prog. Part. Nucl. Phys. (2016)*

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = \left[1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}\right] E_F(nS)$$

with $\Delta_{\mathrm{structure}} = \boxed{\Delta_Z} + \Delta_{\mathrm{recoil}} + \boxed{\Delta_{\mathrm{pol}}}$

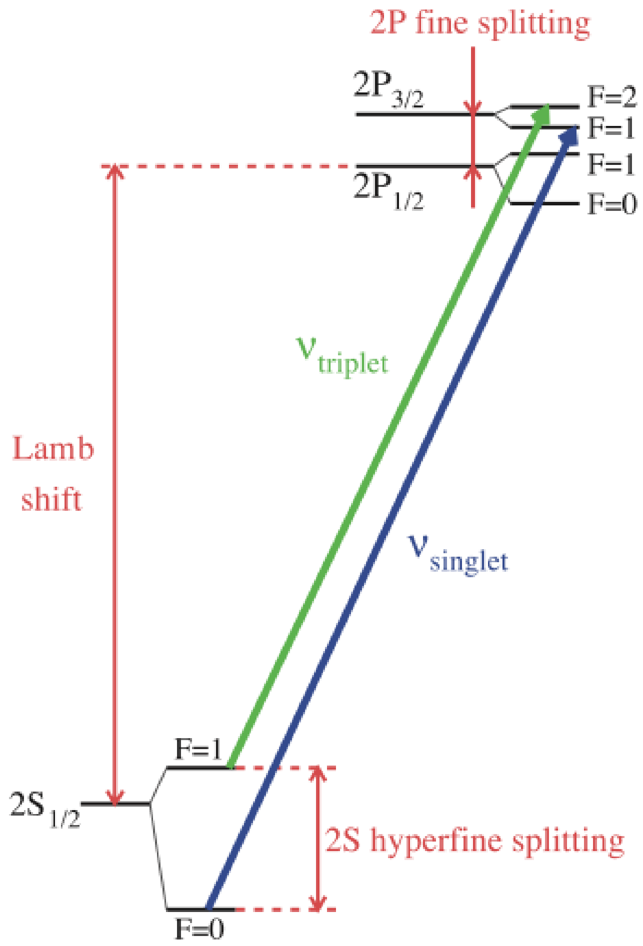
↓

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\,\mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



Measurements of the μH ground-state HFS planned by CREMA, FAMU and J-PARC / Riken-RAL collaborations

- Very precise input for the 2γ polarizability effect needed to find the μH ground-state HFS transition in experiment
- Zemach radius involves magnetic properties of the proton

HFS theory status

$$\Delta E_{\mathrm{HFS}}(1S) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak+hVP}} + \underbrace{\Delta_{\mathrm{Zemach}} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}}_{\Delta_{\mathrm{TPE}}}]\Delta E_0^{\mathrm{HFS}}$$

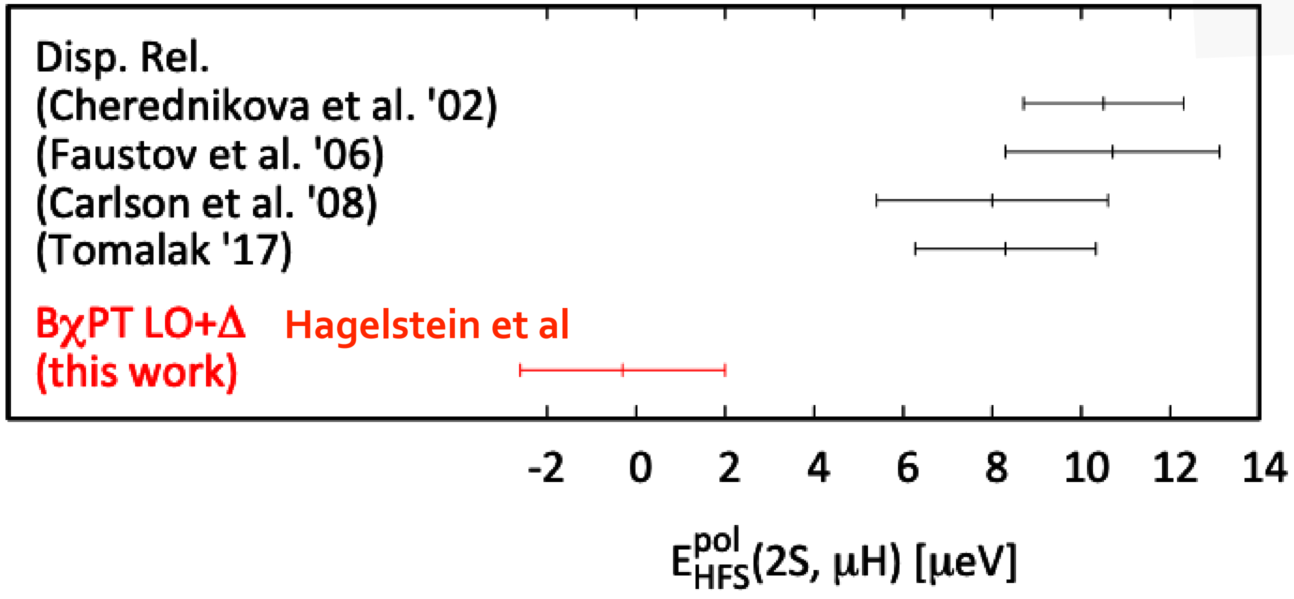
Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

	μp	
	Magnitude	Uncertainty
$\Delta E_0^{\mathrm{HFS}}$	182.443 meV	0.1×10^{-6}
Δ_{QED}	1.1×10^{-3}	1×10^{-6}
$\Delta_{\mathrm{weak+hVP}}$	2×10^{-5}	2×10^{-6}
Δ_{Zemach}	7.5×10^{-3}	7.5×10^{-5}
Δ_{recoil}	1.7×10^{-3}	10^{-6}
Δ_{pol}	4.6×10^{-4}	8×10^{-5}

$\leftarrow G_E(Q^2), G_M(Q^2)$

$\leftarrow G_E, G_M, F_1, F_2$

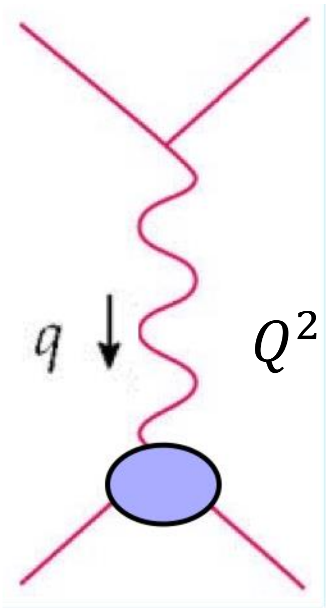
$\leftarrow g_1(x,Q^2), g_2(x,Q^2)$



Polarizability correction is fully expressed in terms of spin structure functions (no subtractions),

yet their poor knowledge leads disagreement with ChPT !

Radius from elastic e-p scattering

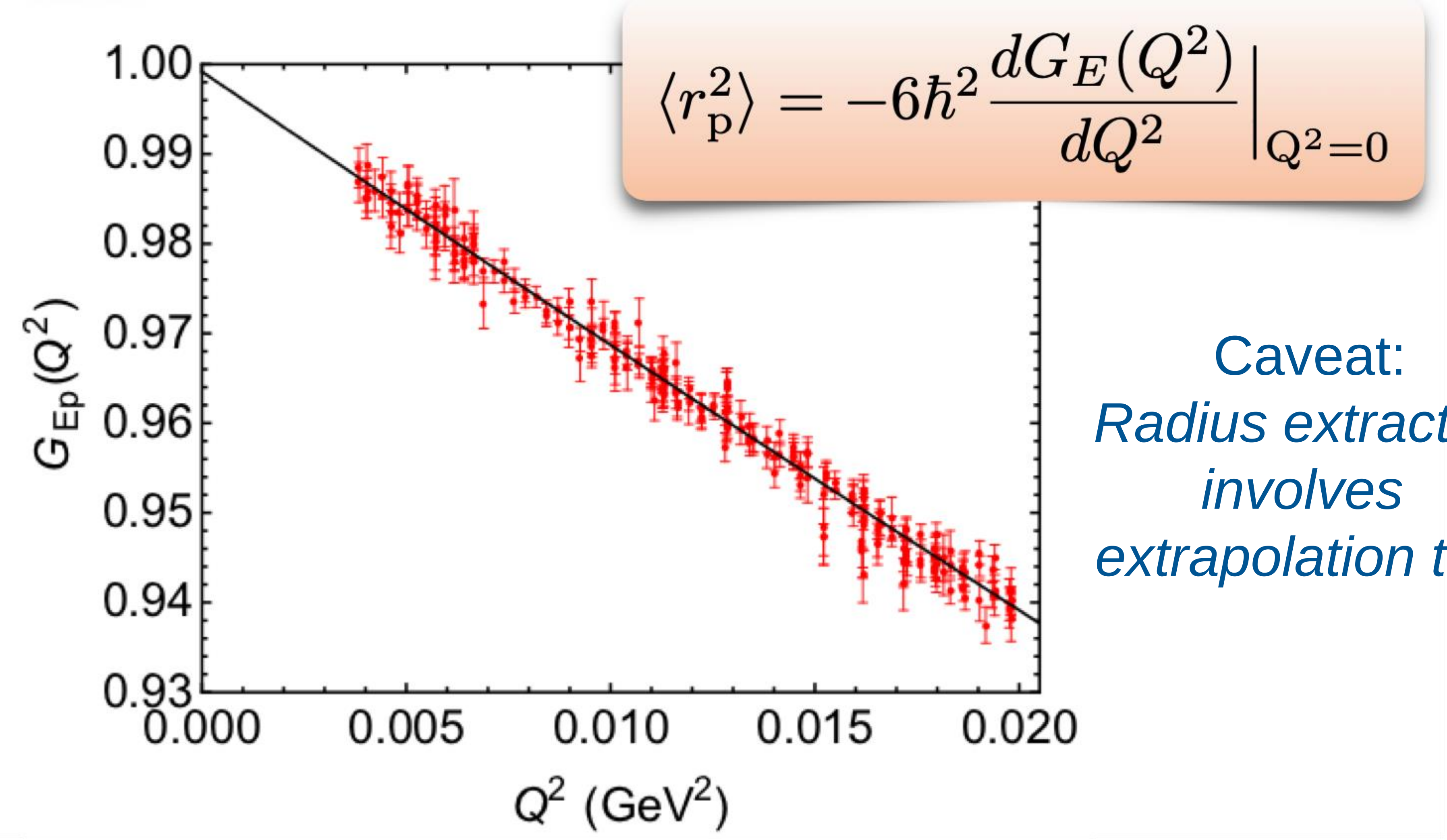


$$Q^2 = -q^2 > 0$$

$$G_E(Q^2), G_M(Q^2)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{(1+\tau)}\left(\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right)$$

with$\tau = Q^2/4M_p^2, \varepsilon \lesssim 1$



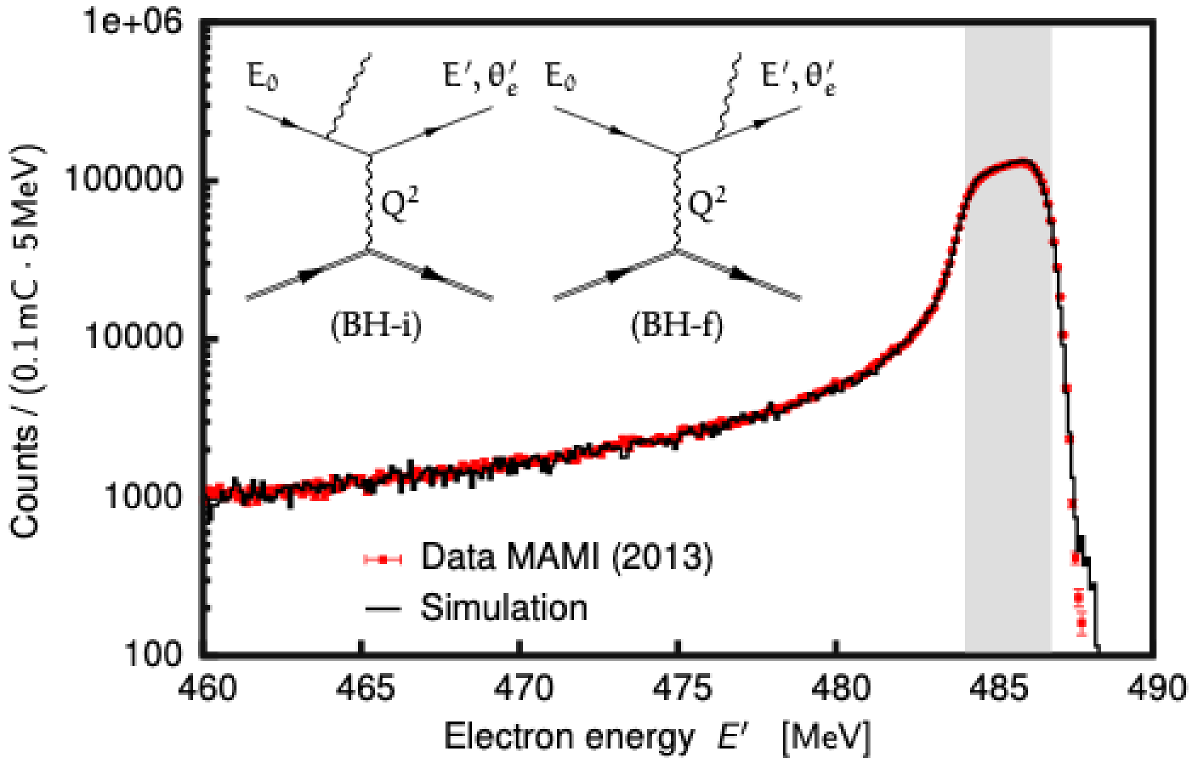
$$\langle r_{\text{p}}^2\rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{\text{Q}^2=0}$$

Caveat:
Radius extraction involves extrapolation to 0

data points: J. C. Bernauer *et al.*, Phys. Rev. C90,015206 (2014).

Initial State Radiation (ISR) Expt @ MAMI

M. Mihovilovic *et al.*, Phys. Lett. B, 771 (2017)
M. Mihovilovič *et al.*., arXiv:1905.11182 [nucl-ex] |

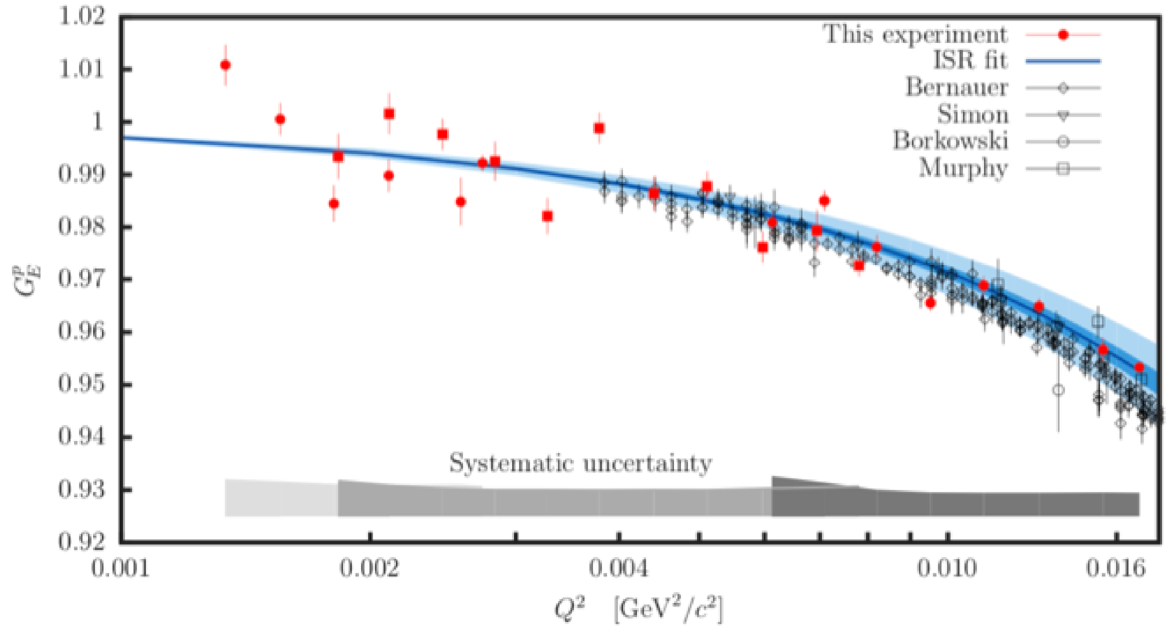


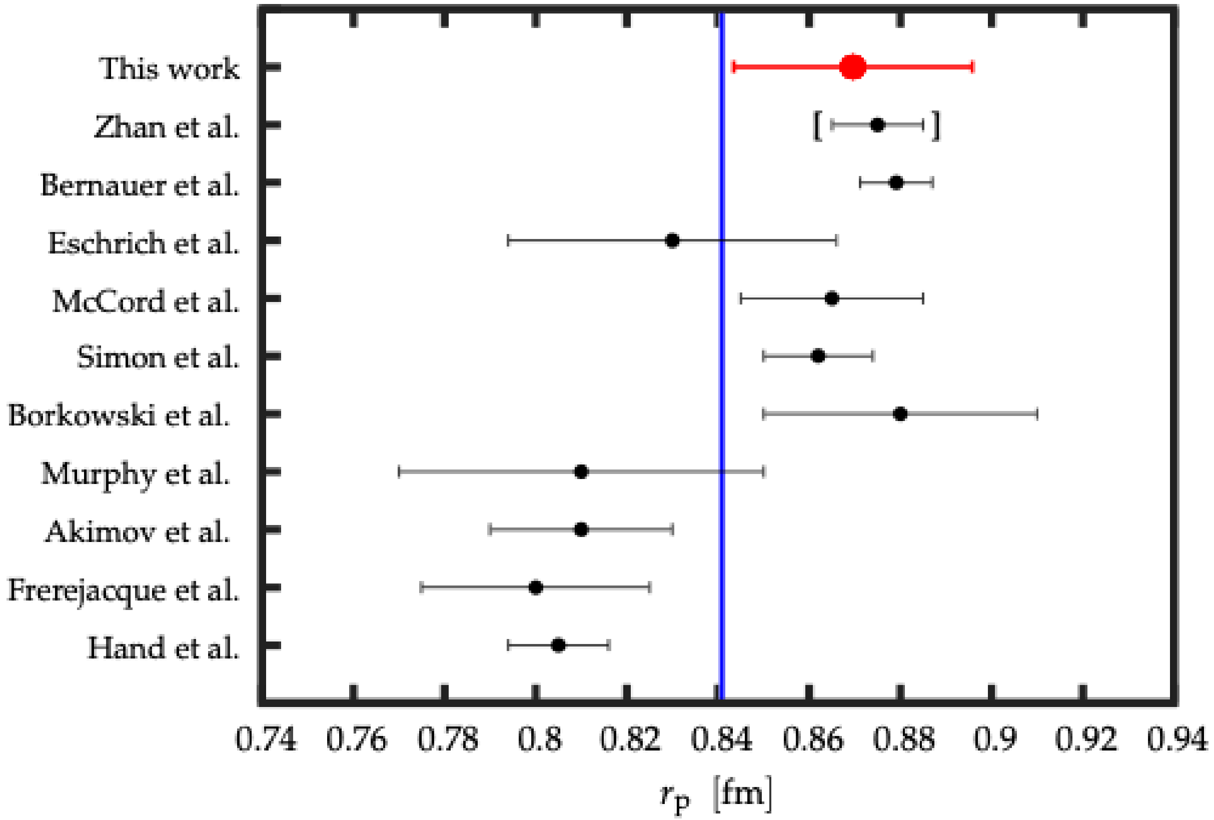
Results on Initial State Radiation

- **Data at $E_0 = 495\,$MeV, $330\,$MeV, $195\,$MeV**
- **Published results: method works**

Next steps

- **Reducing experimental background**
- **Theoretical description of radiative tail**
- **Jet Target without walls**





Lower bound directly from e-p data

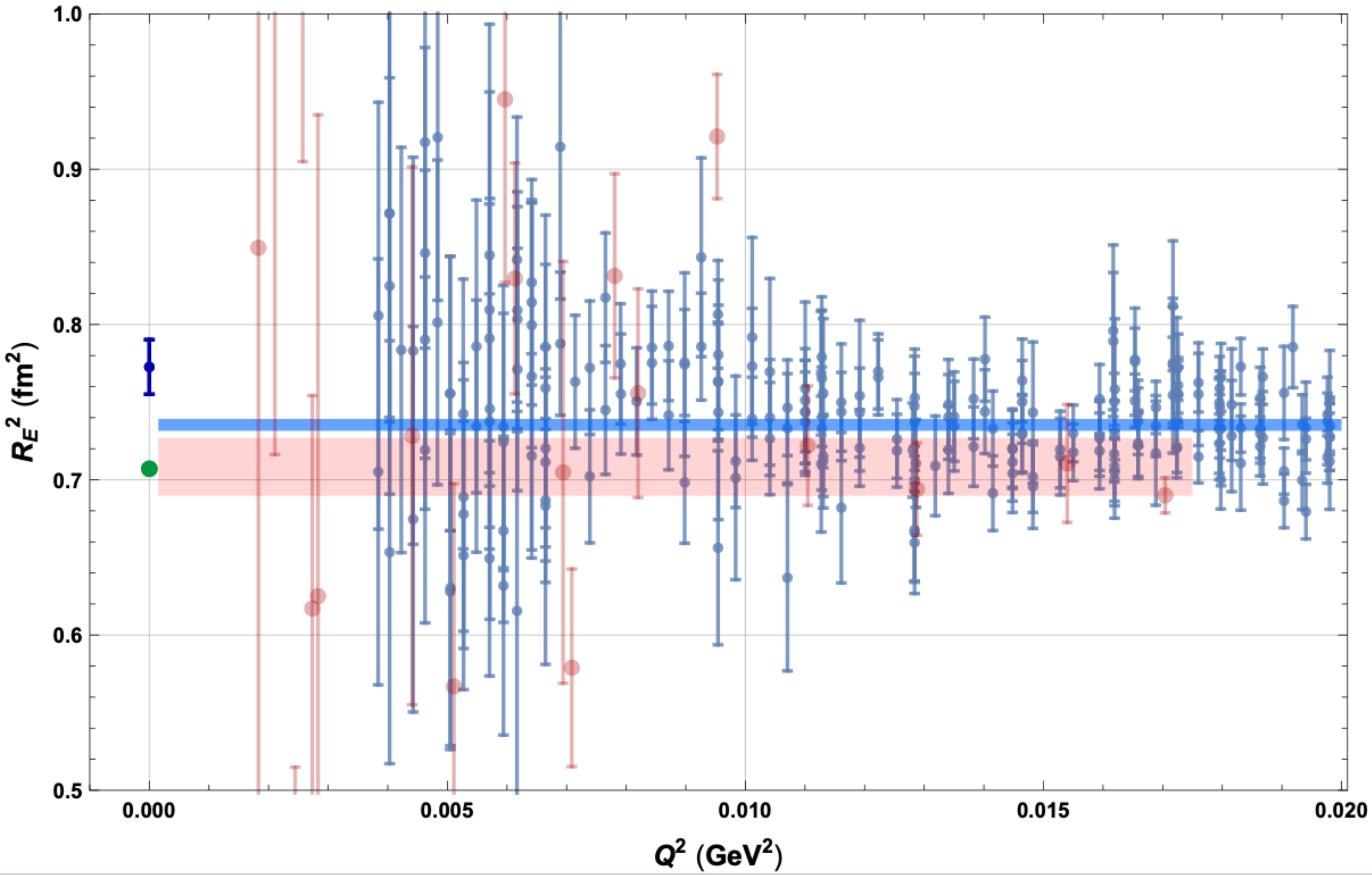
Hagelstein & VP,

Phys. Lett. B 797 (2019).

$$R_E^2(Q^2) = -\frac{6}{Q^2}\log G_E(Q^2) \xrightarrow{Q^2=0} R_E^2$$

This function sets a lower bound:

$$\boxed{R_E^2(Q^2) \leq R_E^2, for Q^2 \geq 0}$$



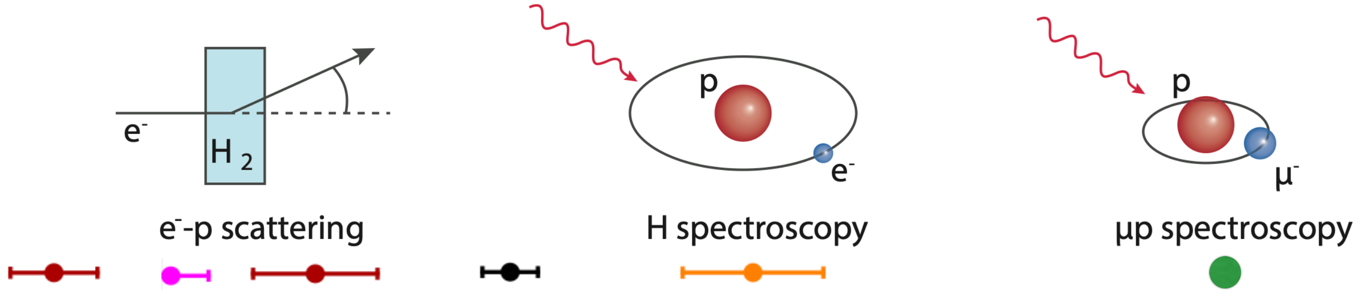
Data points from A1 Coll.:

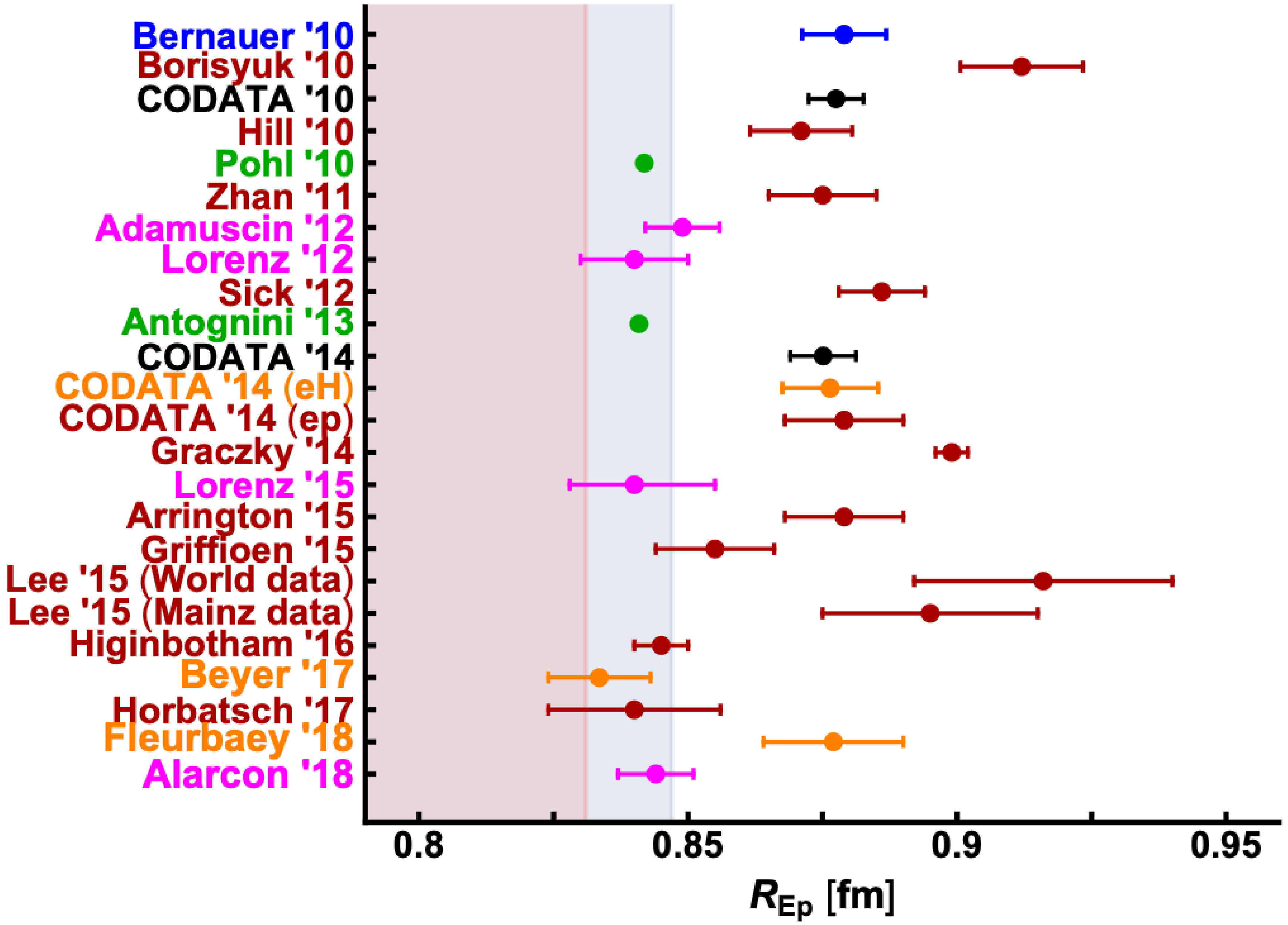
Bernauer et al (2010), (2014)

Mihovilovic et al (2017), (2019)

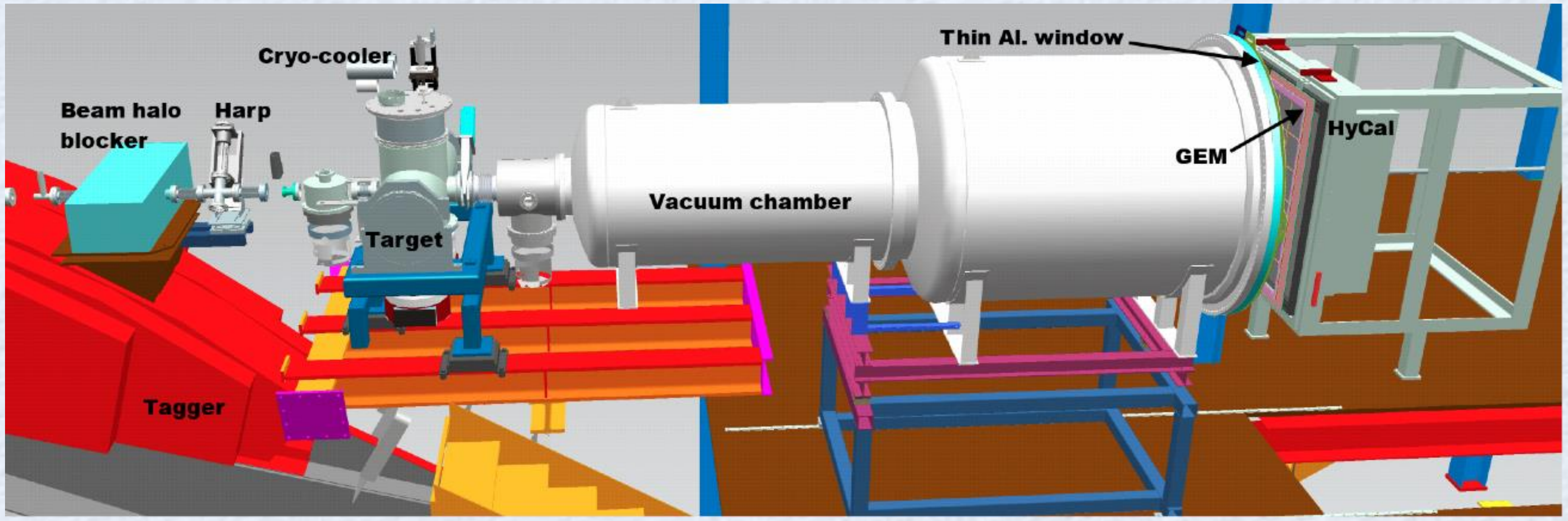
No extrapolation required

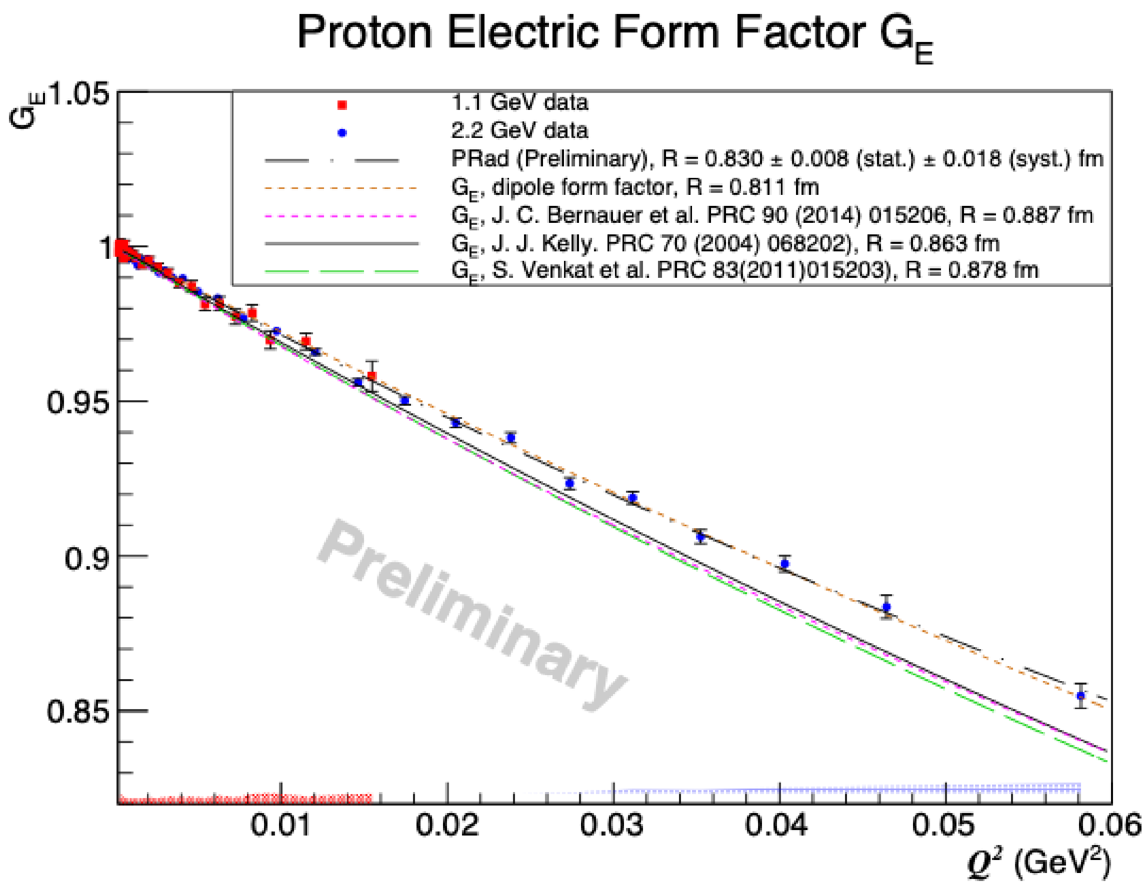
Various extractions





Low-Q^2 proton FF: PRAD@JLab





Preliminary PRAD@JLab

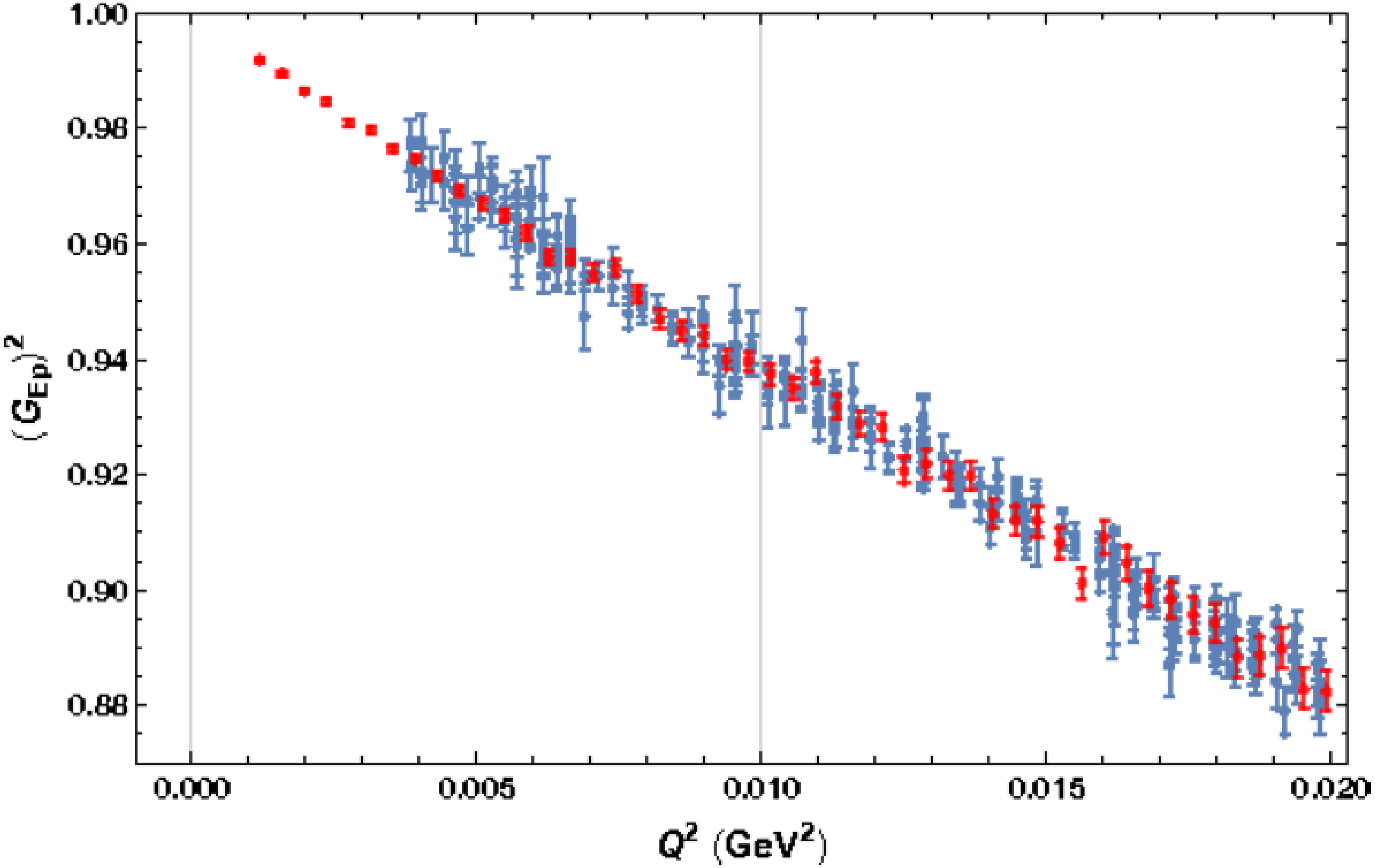
[H. Gao, A. Gasparyan, et al.]

proton charge radius value

$$R_p = 0.831 \pm 0.007(\text{stat.}) \pm 0.012(\text{syst.}) \quad \text{fm}$$

Planned ep-scattering experiment by A2@MAMI [V. Sokhoyan et al.]

- **Newly built active target: Time Projection Chamber (TPC)**
- **New beam line in A2 Hall**
- **Overcomplete kinematics: both scattered *e*- and recoiled *p***
- **Projected data vs Bernauer's [A1 Coll.] data**



New facility *MESA*

Mainz Energy-Recovering Superconducting Accelerator

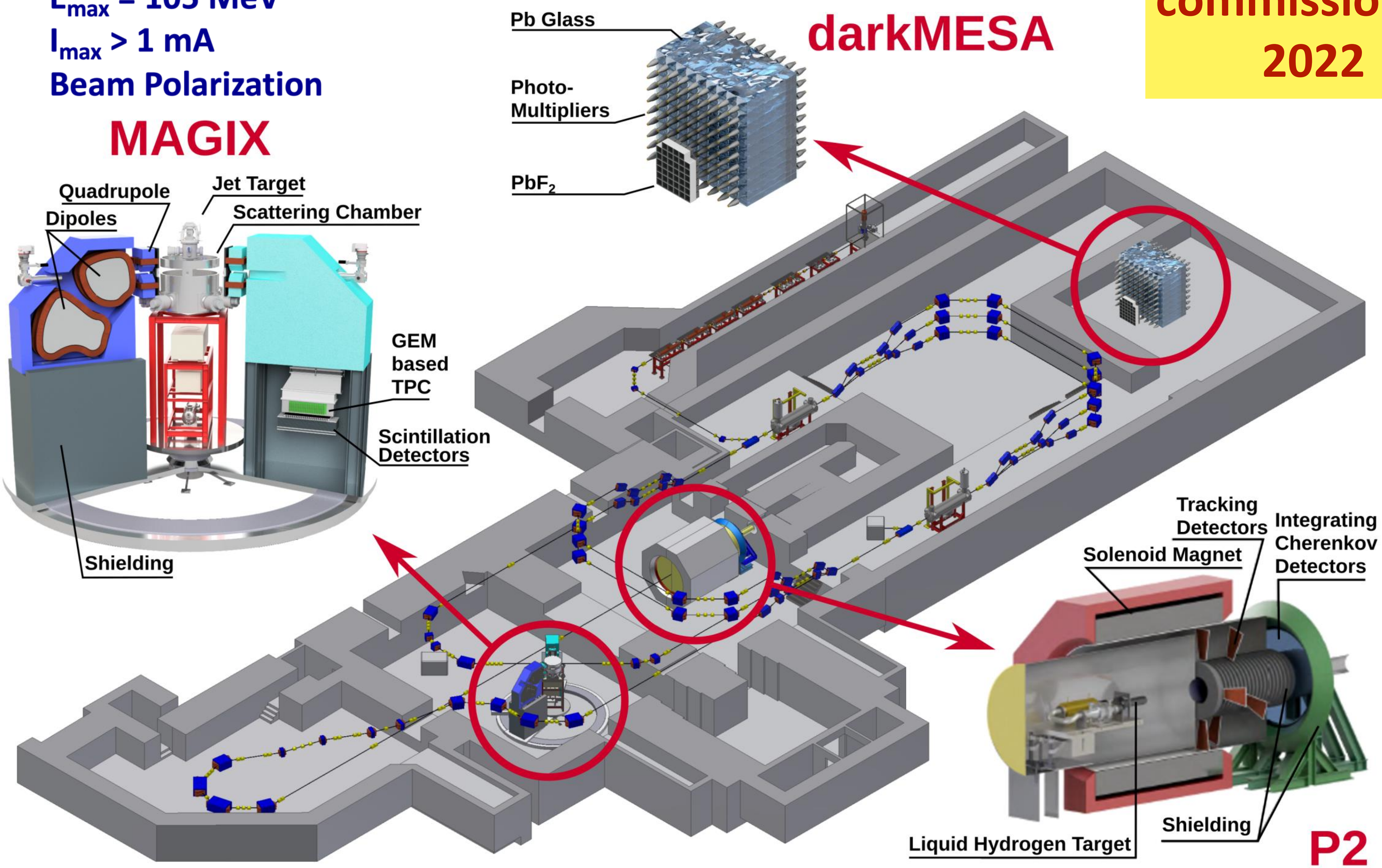
Recirculating ERL Mode

E_{max} = 105 MeV

I_{max} > 1 mA

Beam Polarization

commissioning 2022

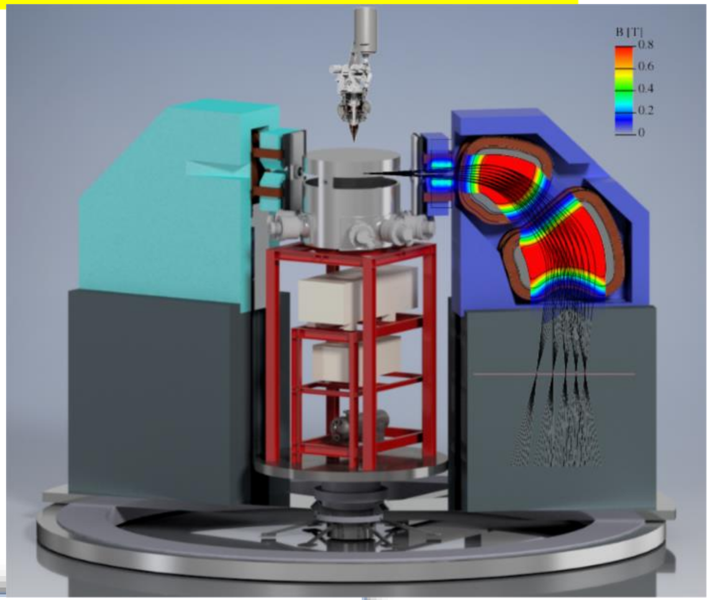


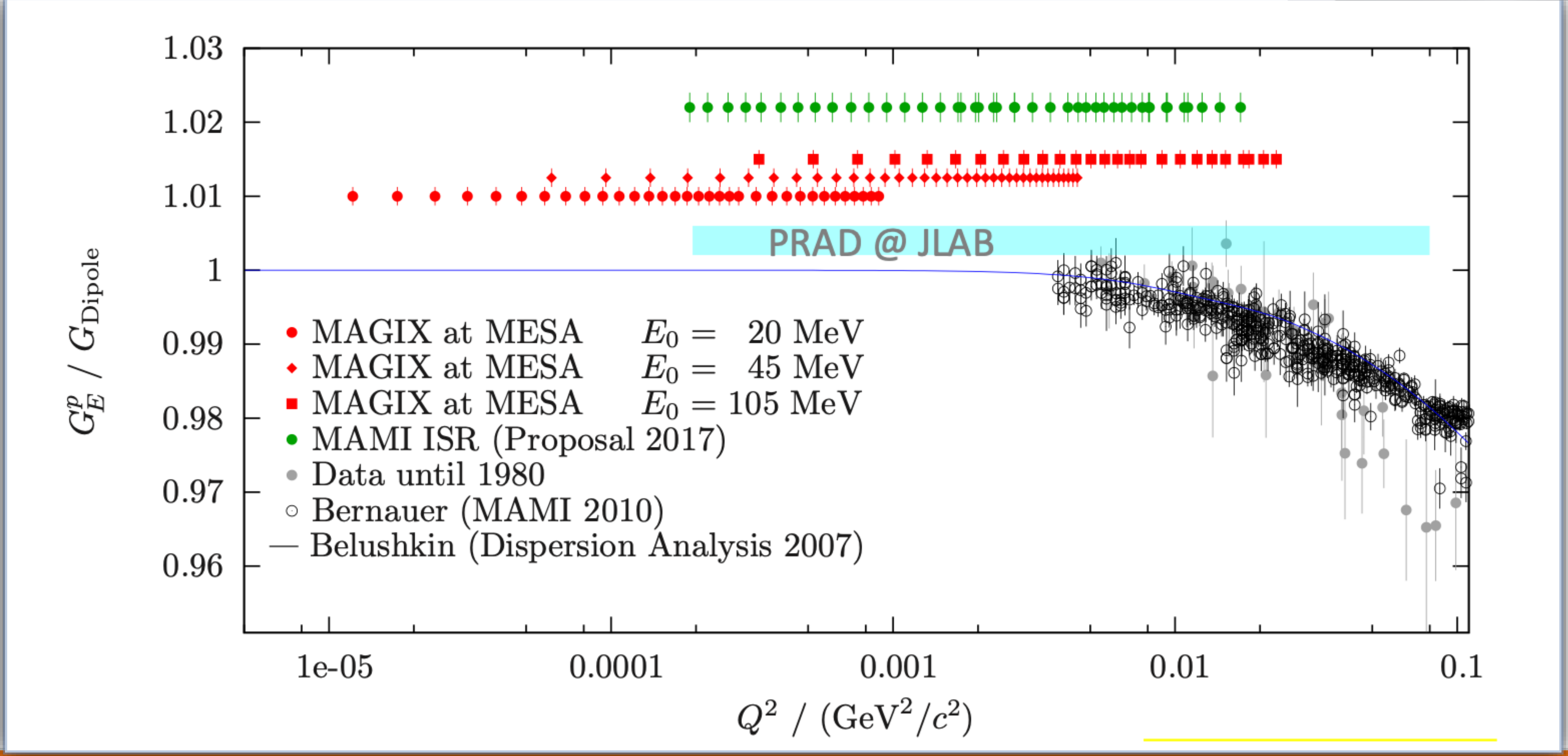
Low-Q^2 proton FF: MAGIX@MESA

Operation of a high-intensity (polarized) ERL beam in conjunction with light internal target
→ a novel technique in nuclear and particle physics

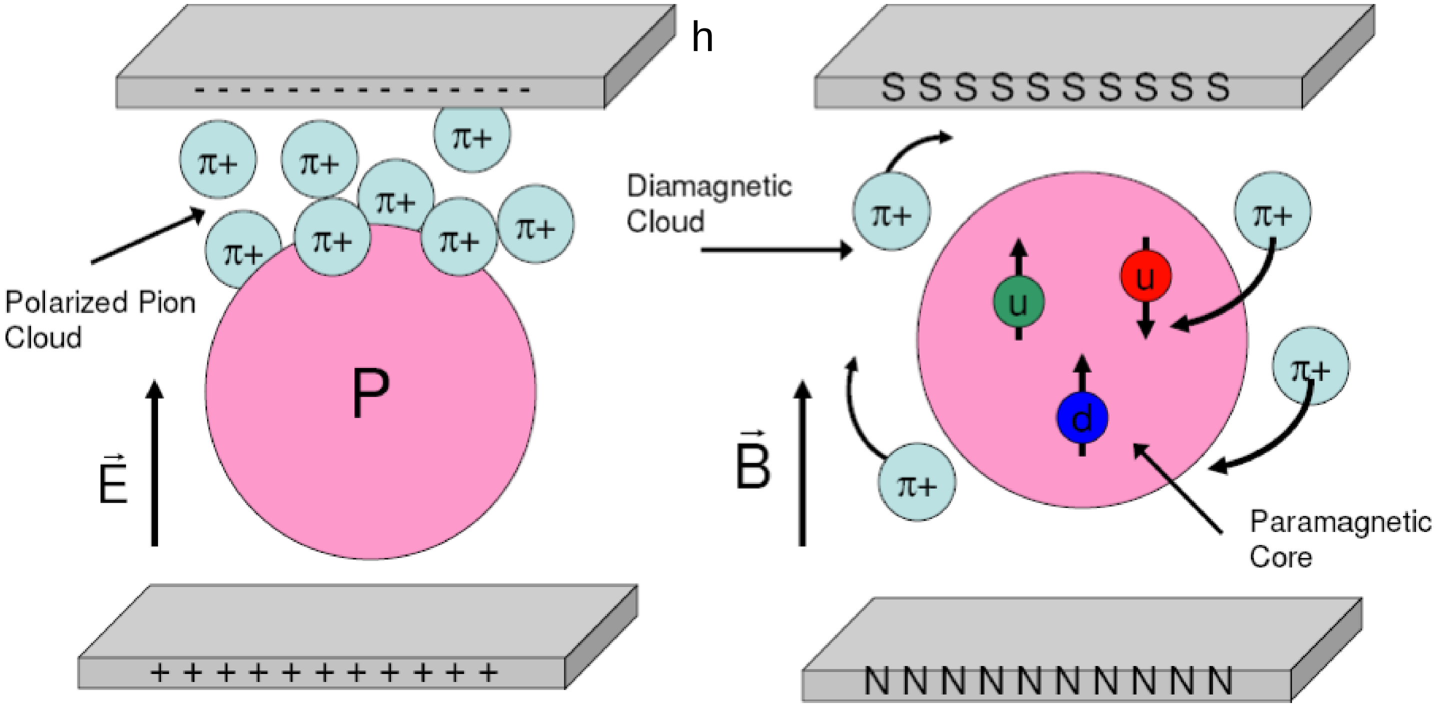
High resolution spectrometers MAGIX:

- double arm, compact design
- momentum resolution: $\Delta p/p < 10^{-4}$
- acceptance: ±50 mrad
- GEM-based focal plane detectors
- Gas Jet or polarized T-shaped target



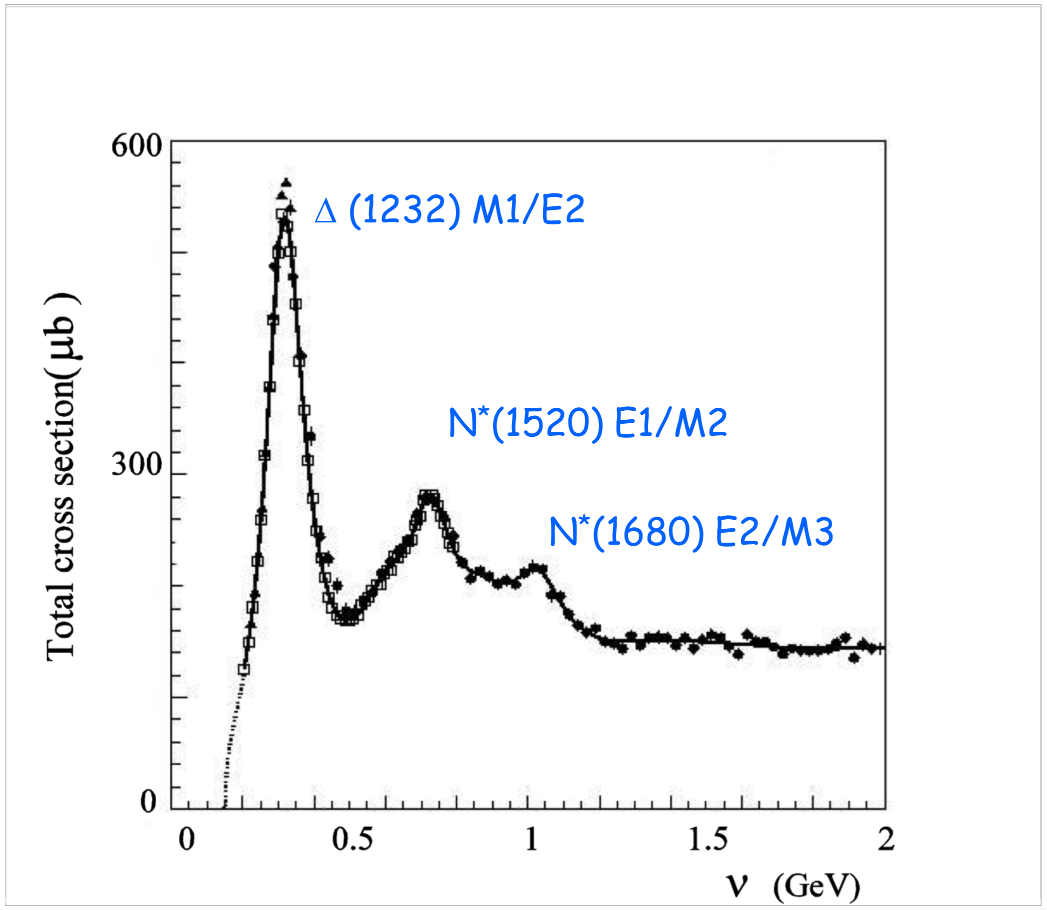


Nucleon Polarizabilities
and Compton scattering
by A2 @ MAMI



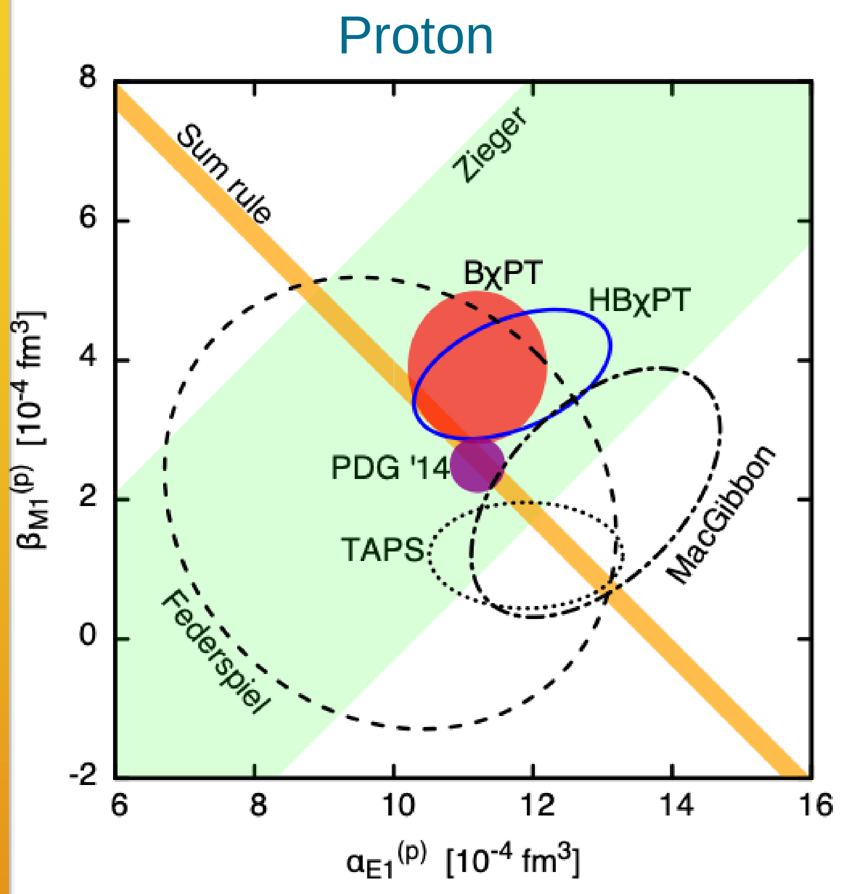
$$\alpha_{E1} + \beta_{M1} = \frac{1}{4\pi^2} \int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4} \mathrm{fm}^3$$

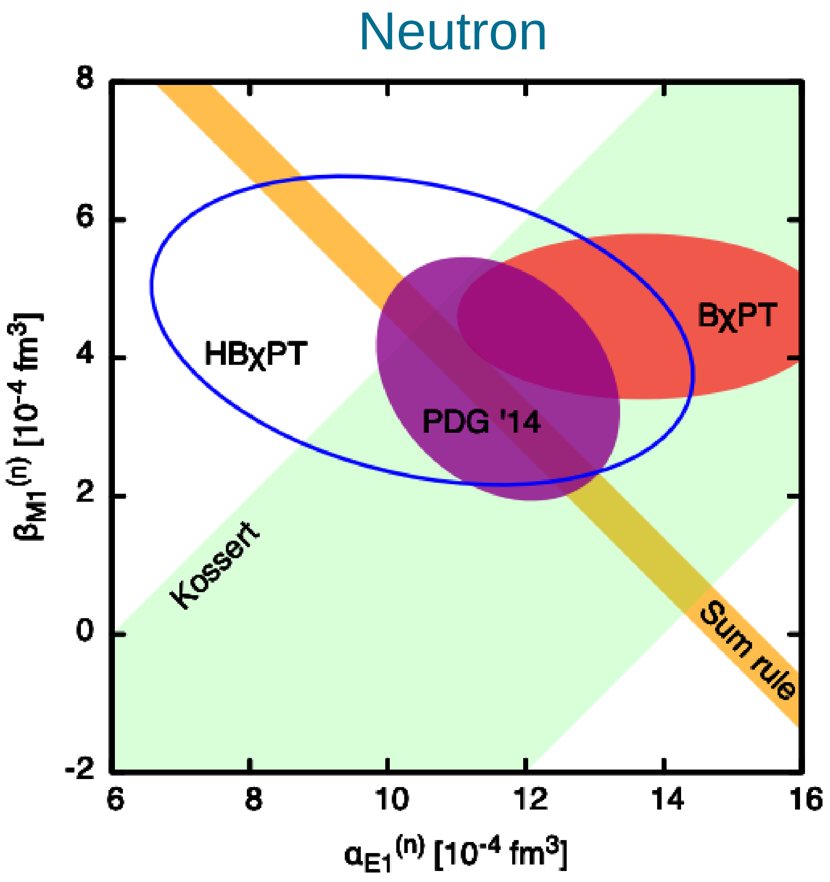
[Baldin sum rule (1960)]



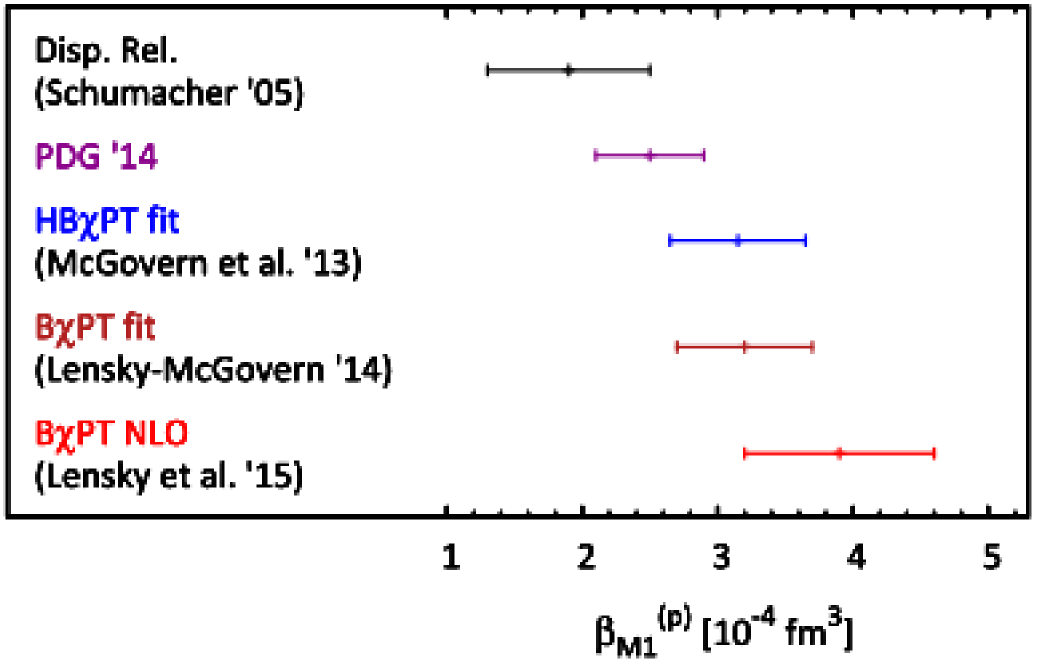
diamagnetic: $\beta_{M1} < 0$
paramagnetic: $\beta_{M1} > 0$

Magnetic vs Electric polarizability





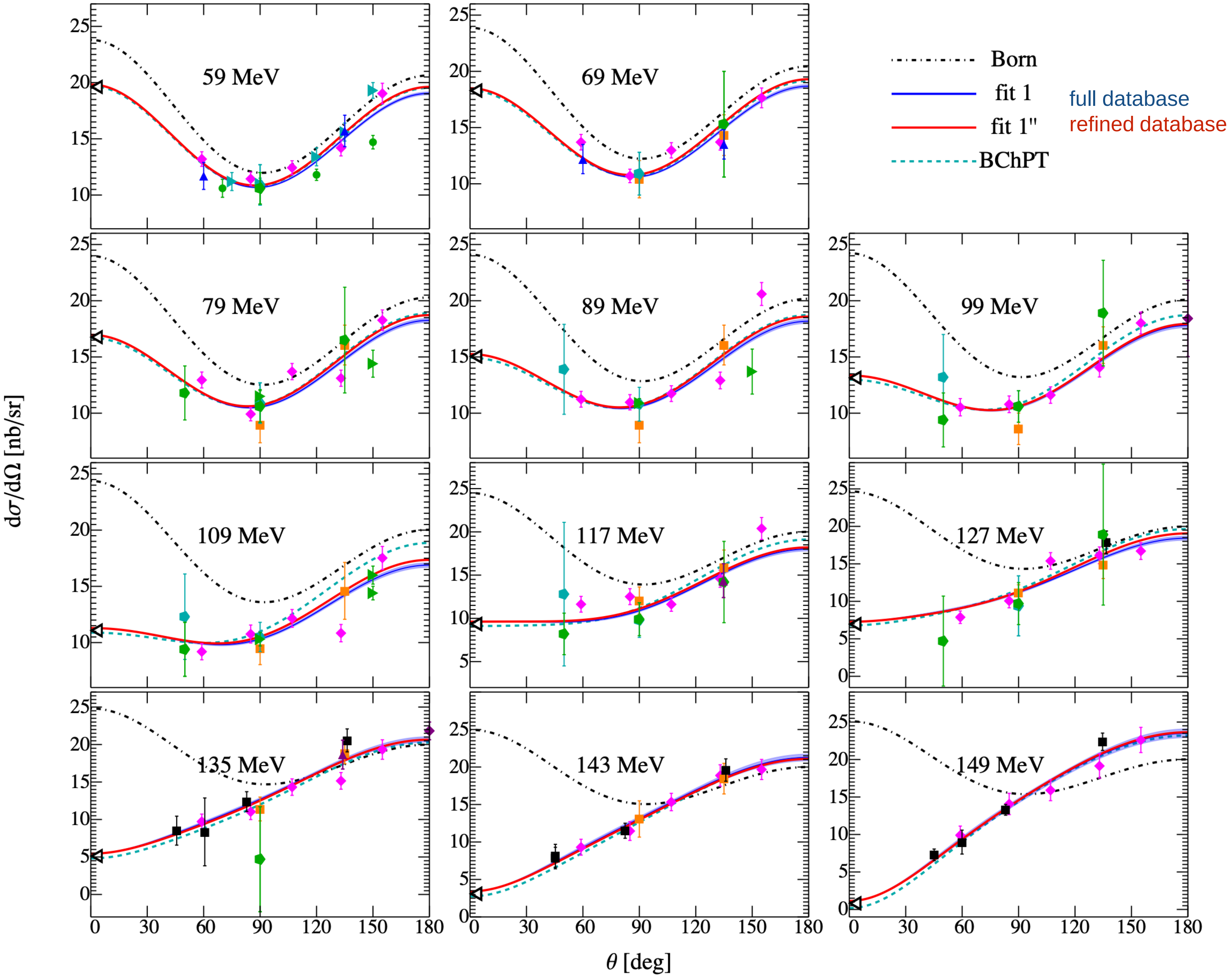
- **TAPS:** fit to TAPS/MAMI data based on fixed-*t* DRs of L'vov et al. Olmos de Leon et al., *EPJA (2001)*



- **BChPT:** "postdiction" Lensky & VP, *EPJC (2010)* Lensky, McGovern & VP, *EPJC (2015)*
- **HBChPT:** fit to world data Grieβhammer, McGovern & Phillips, *EPJA (2013)*

systematic discrepancies between DR and ChPT extractions/predictions

World database of Compton scattering

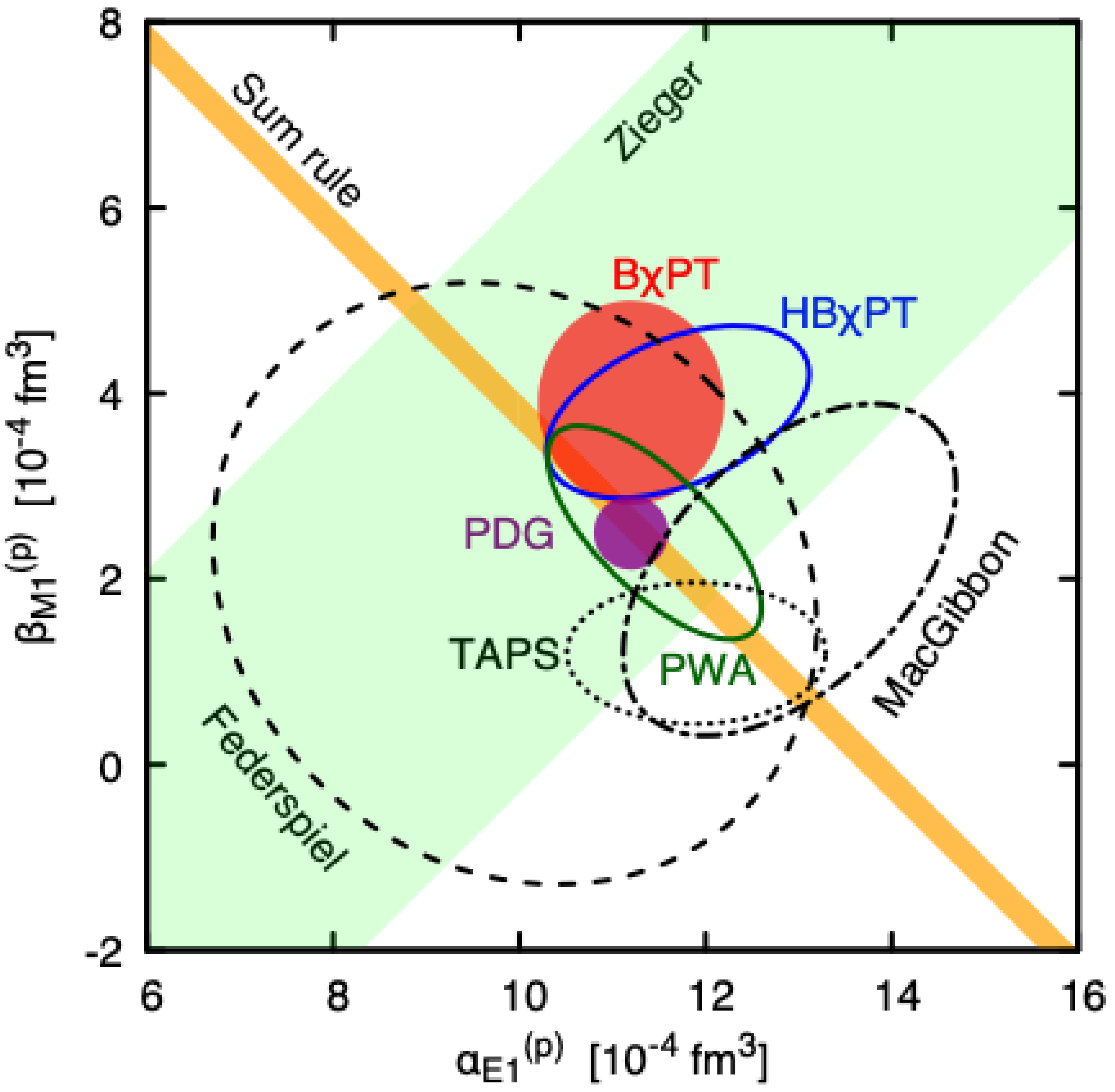


Database of Compton scattering (below pion production)

Author	Ref.	E_γ [MeV]	θ [deg]	$\mathrm{N}_{\mathrm{data}}$	Symbol
Oxley et al.	[27]	60	70-150	4	●
Hyman et al.	[28]	60-128	50, 90	12	⬟
Goldansky et al.	[29]	55	75-150	5	▶
Bernardini et al.	[30]	120, 139	133	2	▲
Pugh et al.	[31]	59-135	50, 90, 135	16	⬢
Baranov et al.	[32]	79, 89, 109	90, 150	7	▶
Federspiel et al.	[33]	59, 70	60, 135	4	▲
Zieger et al.	[34]	98, 132	180	2	◆
Hallin et al.	[35]	130-150	45, 60, 82, 135	13	■
MacGibbon et al.	[36]	73-145	90-135	18	🟧
Olmos de Leon et al.	[20]	59-149	59-155	55	◆

[27] C. L. Oxley, "Scattering of 25-87 Mev Photons by Protons," Phys. Rev. **110** (1958) 733.
[28] L. G. Hyman, R. Ely, D. H. Frisch and M. A. Wahlig, "Scattering of 50- to 140-Mev Photons by Protons and Deuterons," Phys. Rev. Lett. **3** (1959) 93.
[29] V. I. Goldansky, O. A. Karpukhin, A. V. Kutsenko and V. V. Pavlovskaya, " Elastic -p scattering at 40 to 70 MeV and polarizability of the proton," Nuclear Phys. **18** (1960) 473.
[30] G. Bernardini, A. O. Hanson, A. C. Odian, T. Yamagata, L. B. Auerbach and I. Filosofo, "Proton compton effect," Nuovo cim. **18** (1960) 1203.
[31] G. E. Pugh, R. Gomez, D. H. Frisch and G. S. Janes, "Nuclear Scattering of 50- to 130-Mev gamma Rays," Phys. Rev. **105** (1957) 982.
[32] P. Baranov *et al.*, "New experimental data on the proton electromagnetic polarizabilities," Phys. Lett. B **52** (1974) 122.
[33] F. J. Federspiel, R. A. Eisenstein, M. A. Lucas, B. E. MacGibbon, K. Mellendorf, A. M. Nathan, A. O'Neill and D. P. Wells, "The Proton Compton effect: A Measurement of the electric and magnetic polarizabilities of the proton," Phys. Rev. Lett. **67** (1991) 1511.
[34] A. Zieger, R. Van de Vyver, D. Christmann, A. De Graeve, C. Van den Abeele and B. Ziegler, "180-degrees Compton scattering by the proton below the pion threshold," Phys. Lett. B **278** (1992) 34.
[35] E. L. Hallin *et al.*, "Compton scattering from the proton," Phys. Rev. C **48** (1993) 1497.
[36] B. E. MacGibbon, G. Garino, M. A. Lucas, A. M. Nathan, G. Feldman and B. Dolbilkin, "Measurement of the electric and magnetic polarizabilities of the proton," Phys. Rev. C **52** (1995) 2097 [nucl-ex/9507001].

Electric vs magnetic polarizability of the proton



- **TAPS:** fit to TAPS/MAMI data based on fixed-t DRs of L'vov et al.
  Olmos de Leon et al., *EPJA (2001)*
- **BChPT:** "postdiction"
  Lensky & VP, *EPJC (2010)*
  Lensky, McGovern & VP, *EPJC (2015)*
- **HBChPT:** fit to world data
  Grieβhammer, McGovern & Phillips, *EPJA (2013)*
- **PWA:** fit to world data
  Krupina, Lensky & VP, *PLB (2018)*

Partial-Wave Analysis (PWA):
differences between DR and ChPT extractions are due to database inconsistencies, improvements — new experiments — are needed

New (preliminary) data from A2@MAMI

[E. Mornacchi et al.]

Higher statistics,

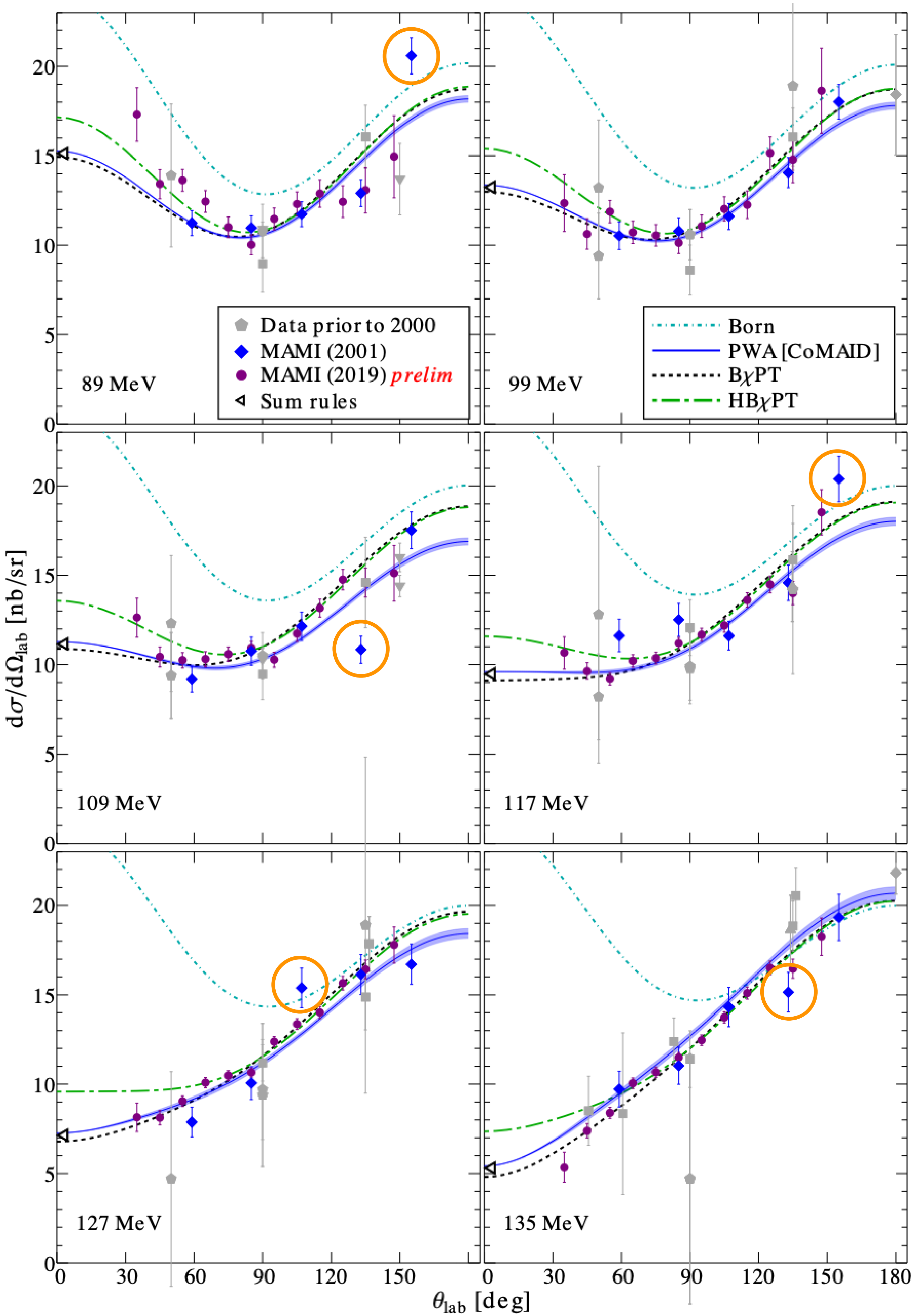
better systematics

than any other dataset

No outliers seen

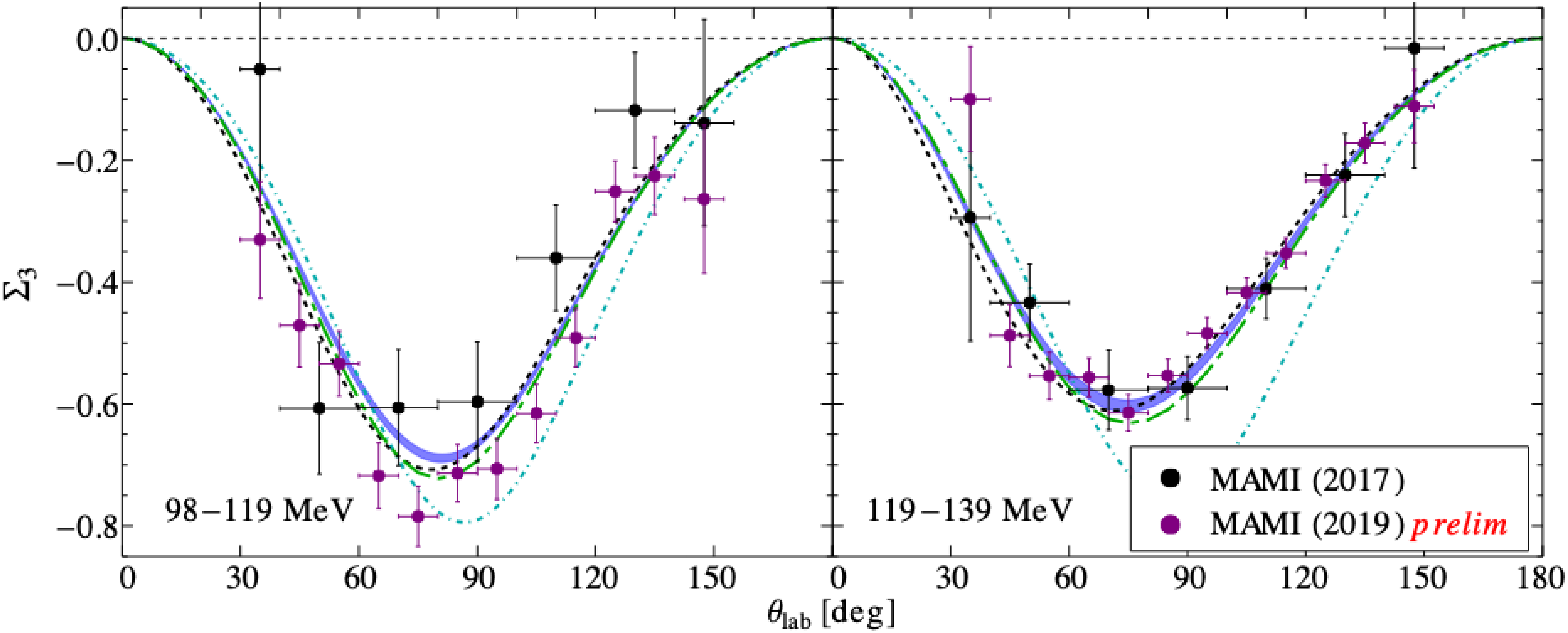
previous MAMI data!

Impact on proton polarizabilities yet to be analyzed…



Beam asymmetry (polarized photon beam)

cleaner separation of magnetic from electric [N. Krupina & VP, PRL (2013)]



Pilot expt: V. Sokhoyan et al., EPJA (2017)

New expt.: E. Mornacchi et al.

Generalzied polarizabilities from Virtual Compton Scattering (VCS)— new results from A1

J. Beričič *et al.*. Jul 23, 2019. 5 pp.
e-Print: **arXiv:1907.09954 [nucl-ex]**

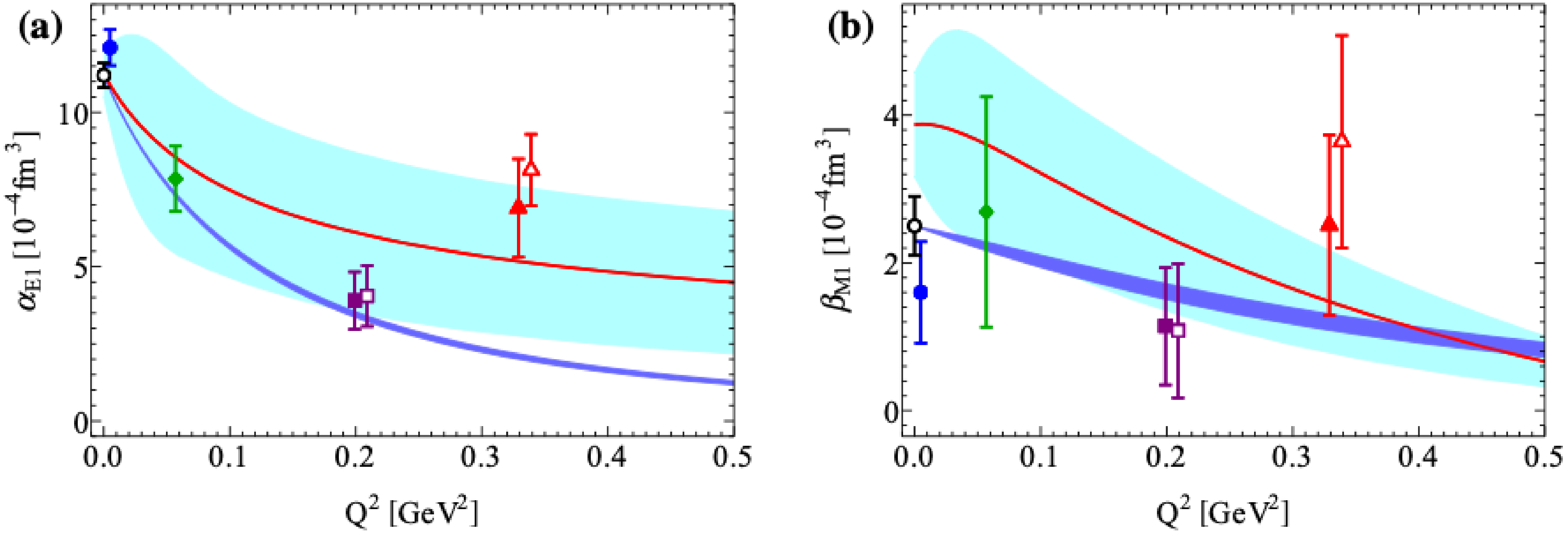


Fig. 11 Generalized scalar polarizabilities: **a** $\alpha_{E1}(Q^2)$, **b** $\beta_{M1}(Q^2)$.

[from V. Lensky, VP & M. Vanderhaeghen, EPJC (2017)]

Spin polarizabilities — new results from A2

(doubly)-polarized Compton scattering

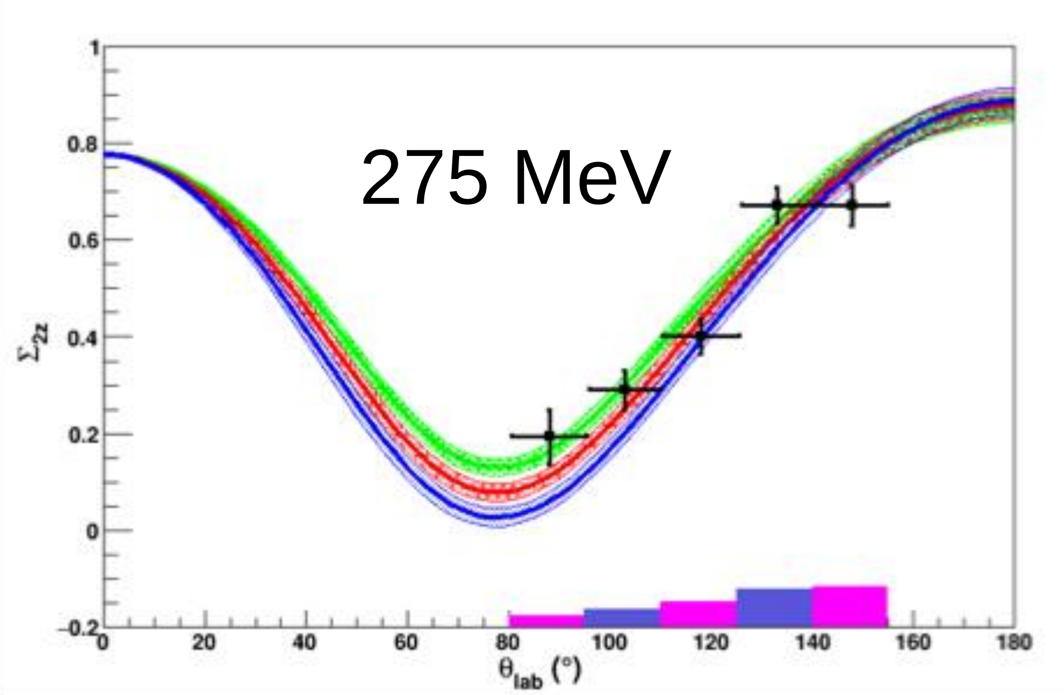
$\gamma E1E1, \gamma M1M1, \gamma M1E2, \gamma E1M2;$

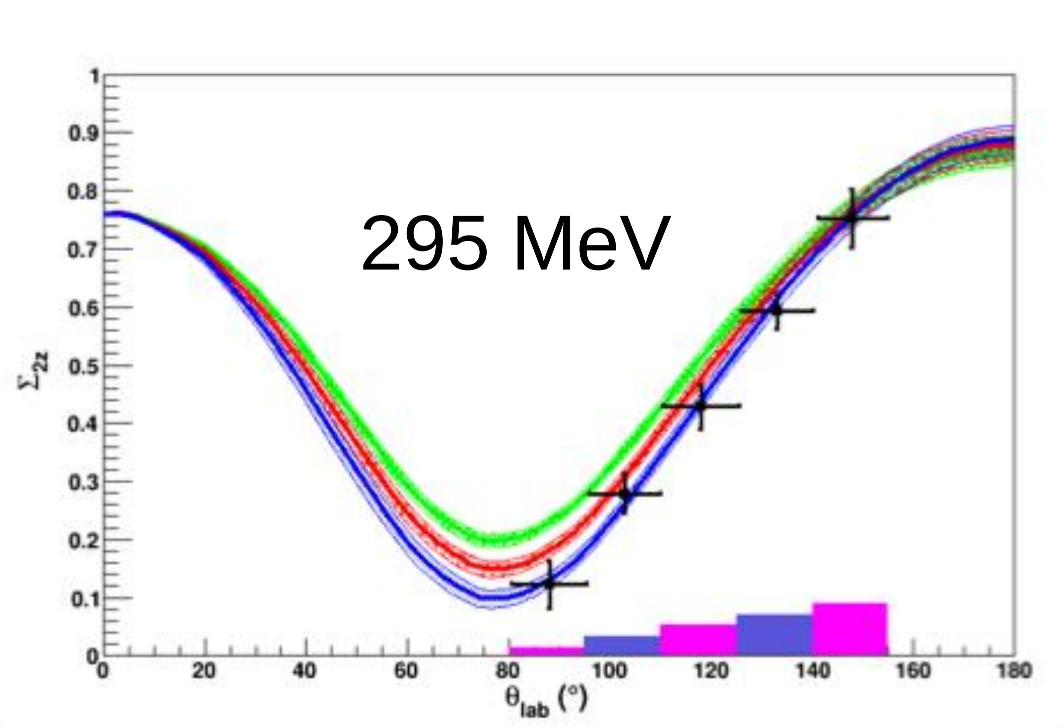
PhD work of P. Martel: measurement of **Σ_{2x}**

[P.Martel et al, PRL (2015)]

PhD work of C. Collicott: measurement of **Σ_3**

PhD work of D. Paudyal: measurement of **Σ_{2z}**





Summary and outlook

MAMI is up and running, with recent results on

elastic *ep*
ISR@A1,
TPC@A2

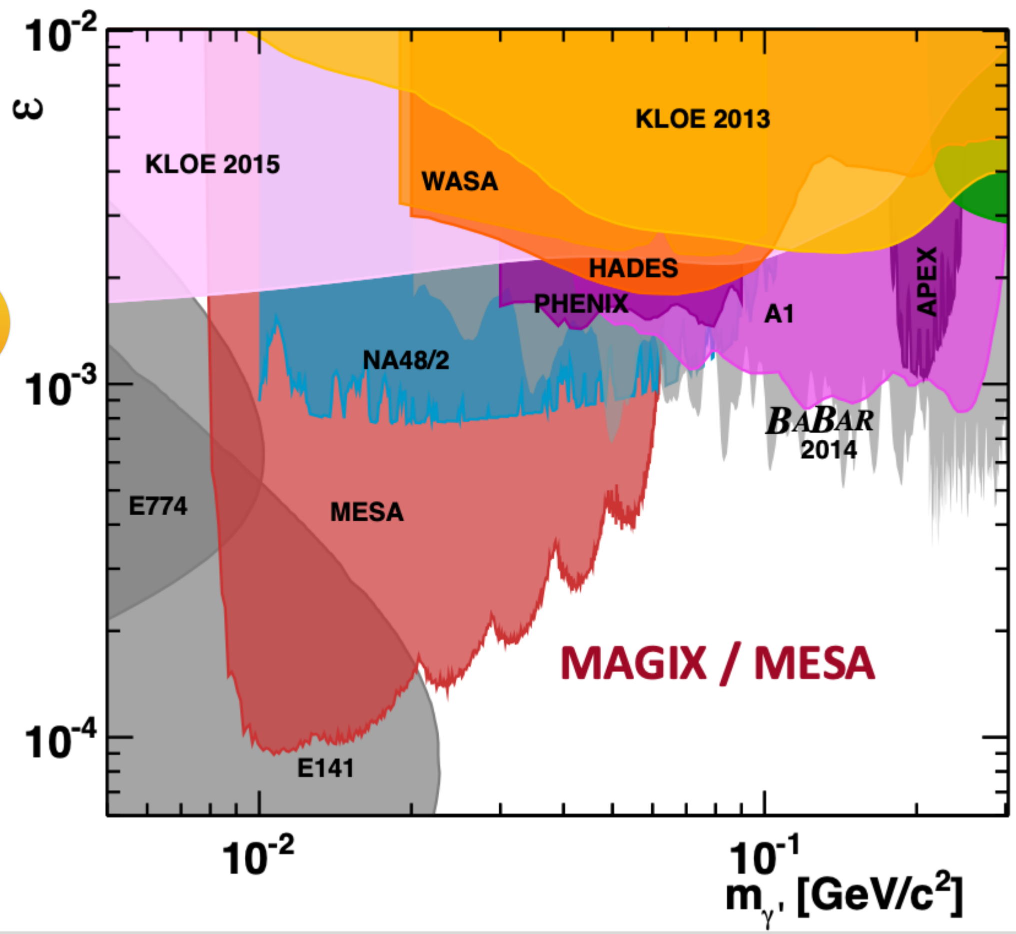
Form factors, radius
(lower bound)

(muonic)
Hydrogen

Compton
Real @ A2
Virtual @ A1

Polarizabilities,
spin, generalized

Dark photon searches



MESA under construction..

MESA Physics Programme

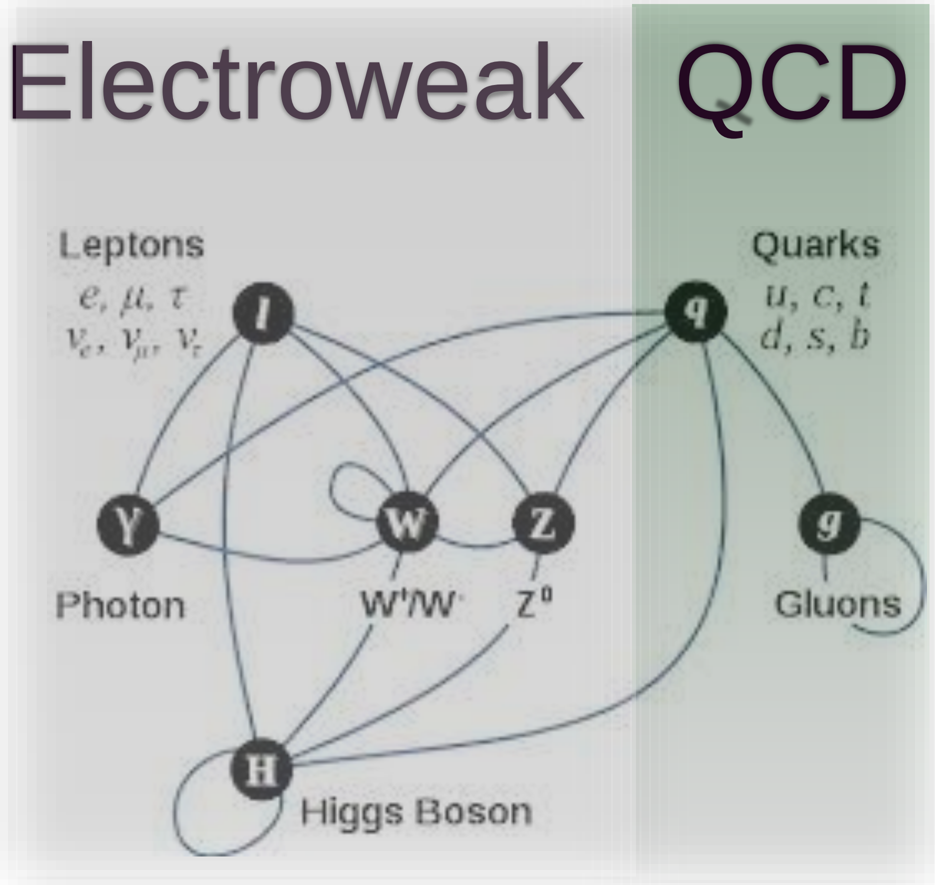
	ERL Mode **MAGIX expt.**	**Extracted Beam Mode** **P2 expt.**	**Extracted Beam Mode** **BDX expt.**
Nucleon From Factors	✓		
EW Mixing Angle		✓	
Nuclear Astrophysics	✓ $^{12}C\ (\alpha,\gamma)\ ^{16}O$	neutron skin of nuclei ✓	
Few Body Physics	✓		
Light Dark Matter Search	✓		✓

Start running in 2022 ?

Thank you!

Backup

Standard Model

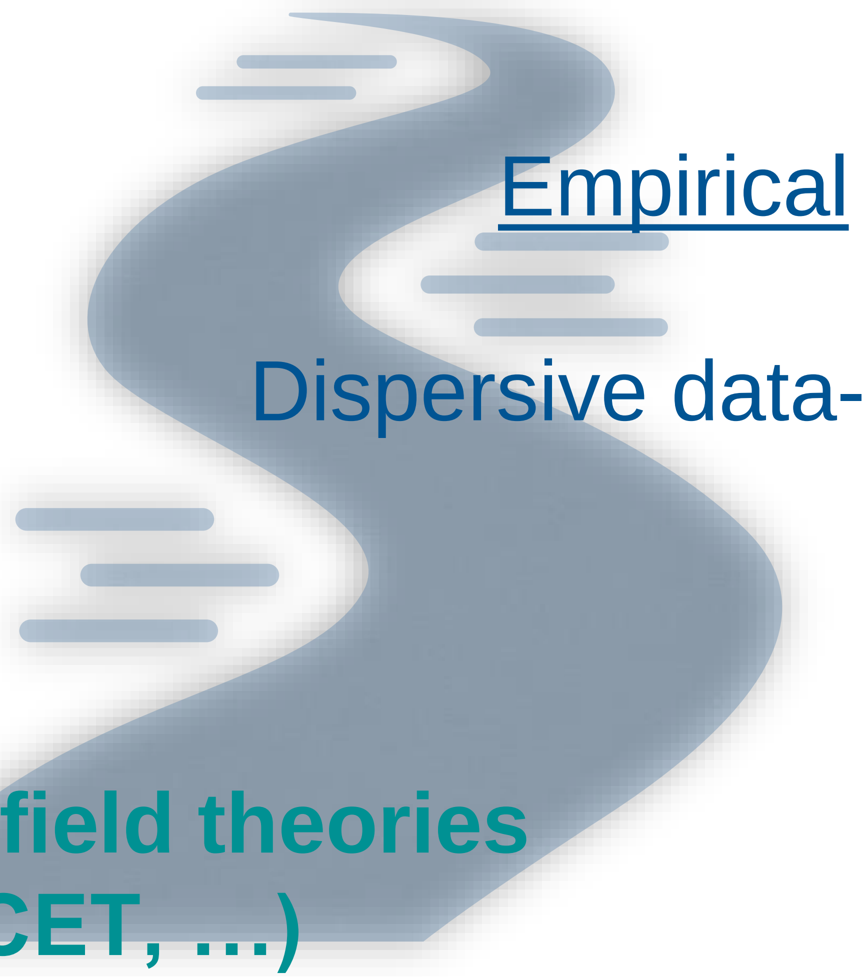


is presently the best theory of (nearly) everything

Approaches to hadronic effects in the SM

QCD Based

Lattice QCD,
Dyson-Schwinger eqs



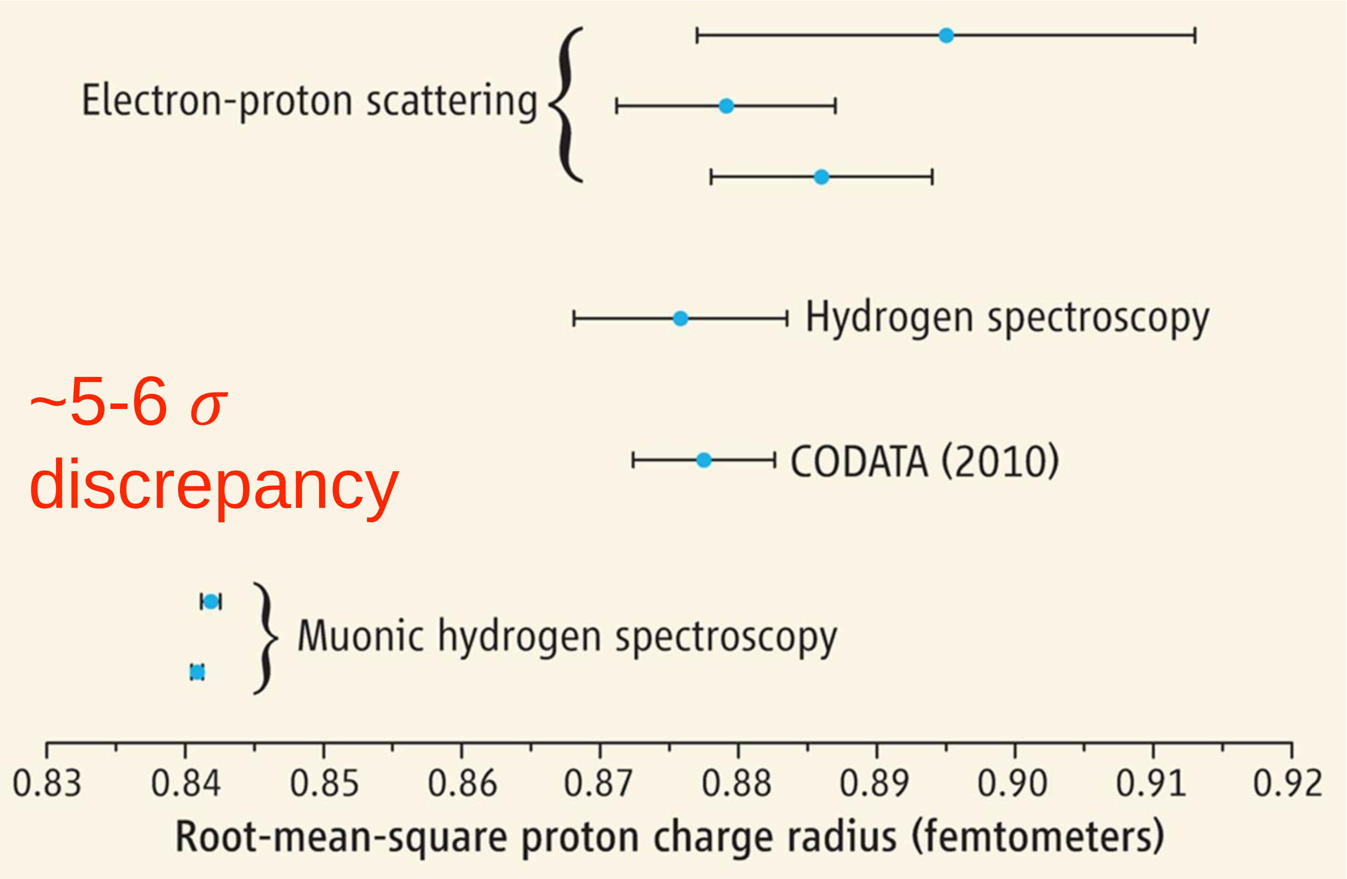
Empirical

Dispersive data-driven

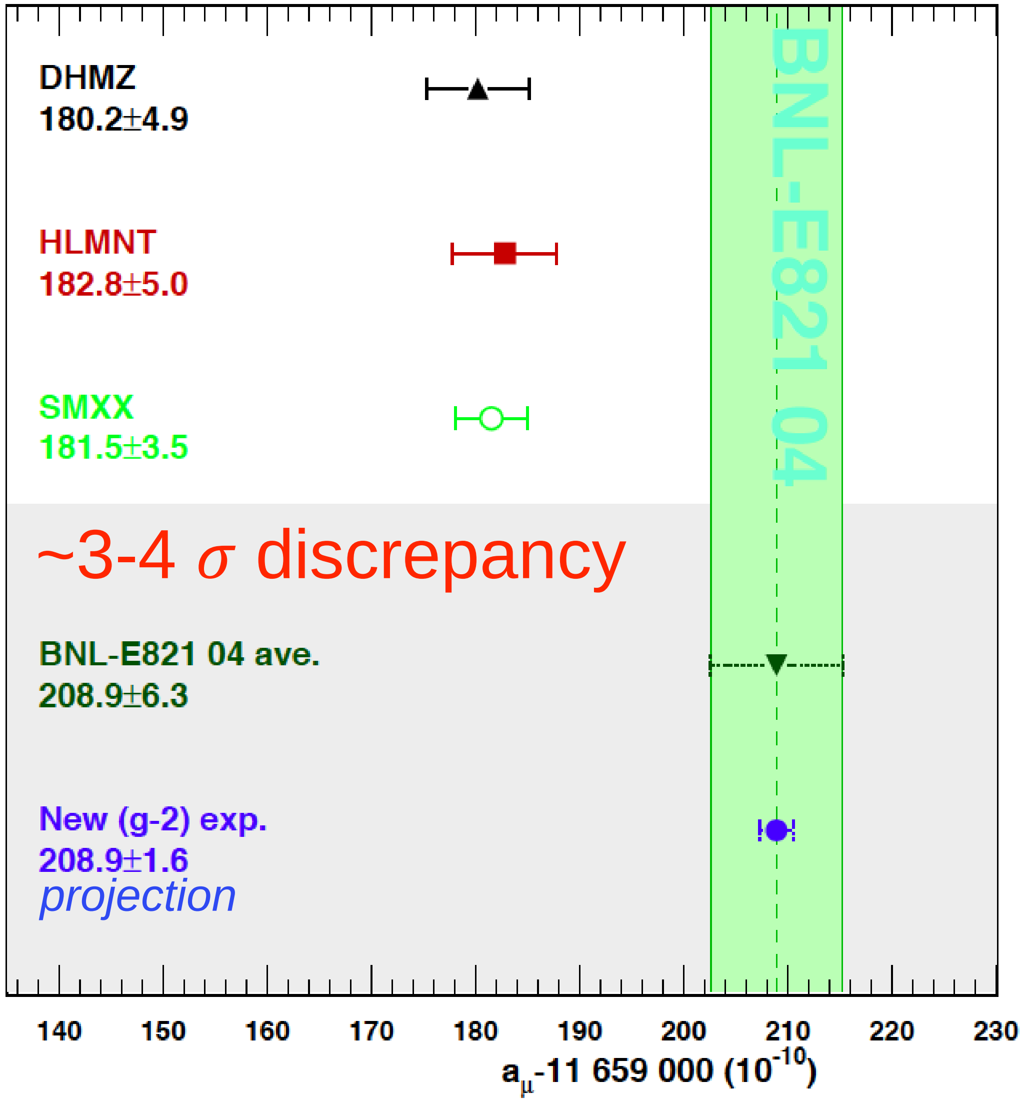
**Effective-field theories
(ChPT, SCET, …)**

Constituent quark models
Other “QCD-inspired” models

Muon anomalies today



~5-6 σ discrepancy



~3-4 σ discrepancy

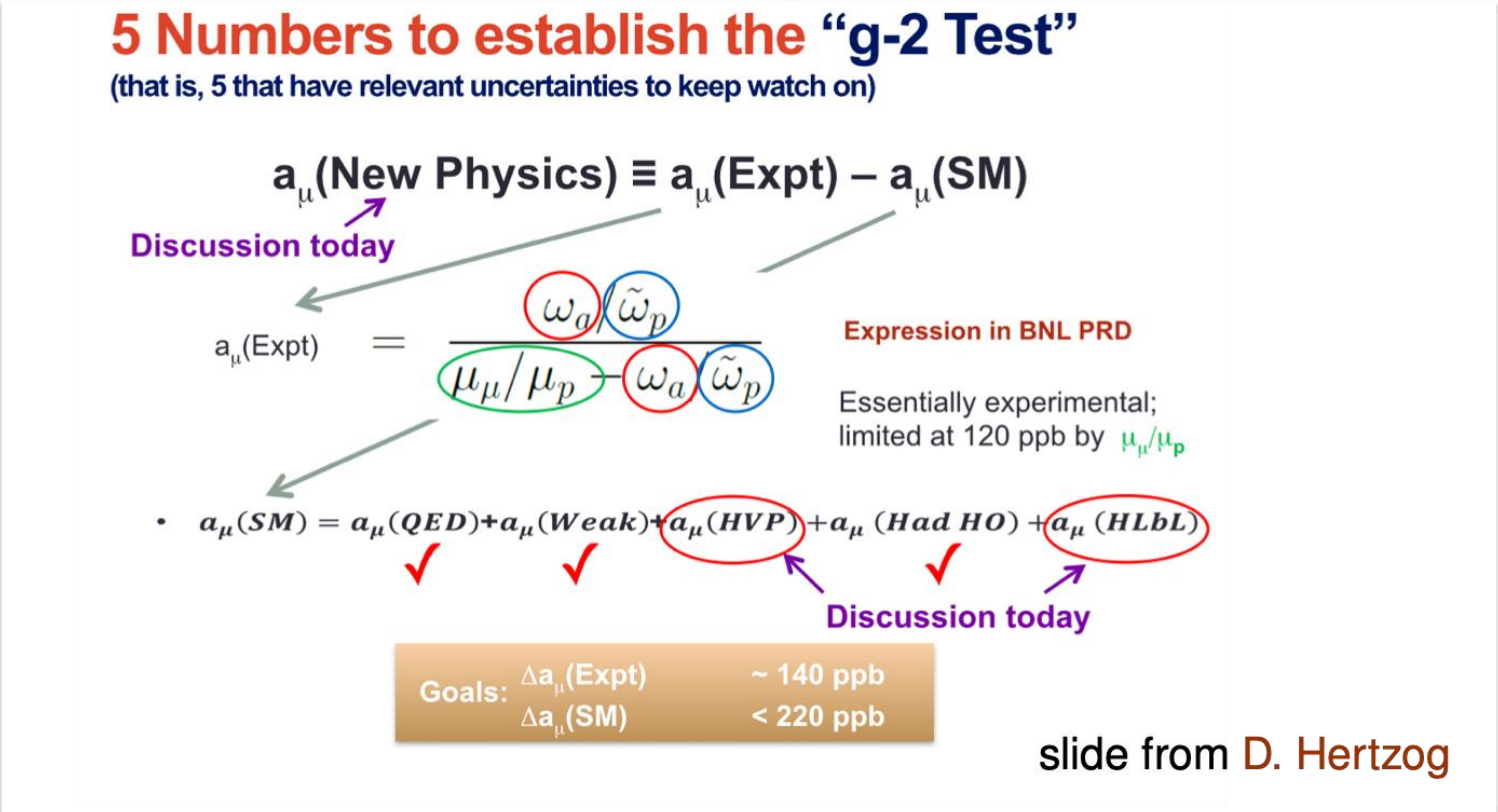
Proton radius puzzle

Muon g-2

Hadronic effects are the main sources of uncertainty

Muon *g*—2

or, anomalous magnetic moment: $a_\mu \equiv (g-2)_\mu/2$



5 Numbers to establish the "g-2 Test"

(that is, 5 that have relevant uncertainties to keep watch on)

$$a_\mu(\text{New Physics}) \equiv a_\mu(\text{Expt}) - a_\mu(\text{SM})$$

Discussion today

$$a_\mu(\text{Expt}) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p}$$

Expression in BNL PRD

Essentially experimental;
limited at 120 ppb by μ_μ/μ_p

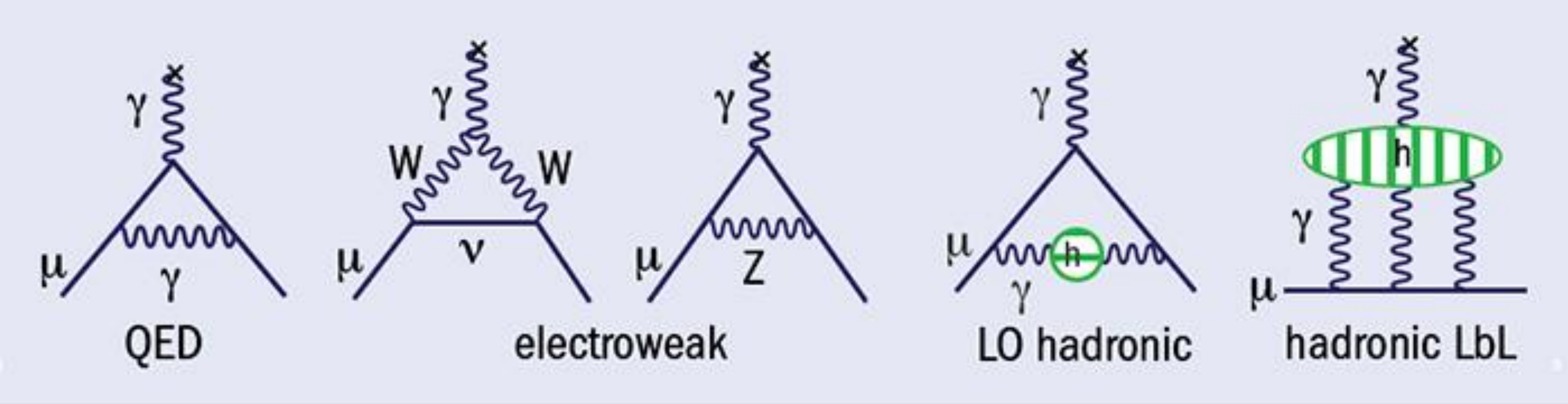
- $a_\mu(SM) = a_\mu(QED) + a_\mu(Weak) + a_\mu(HVP) + a_\mu\,(Had\ HO) + a_\mu\,(HLbL)$

✓ ✓ ✓

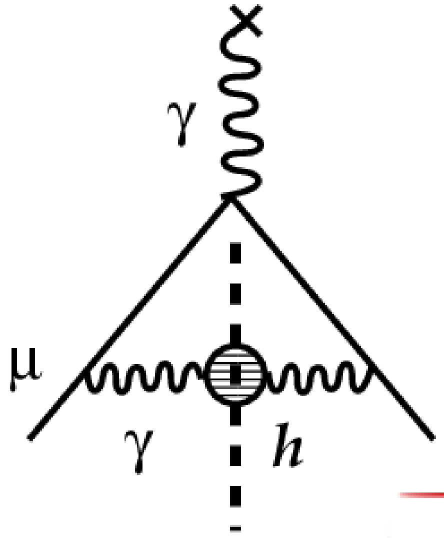
Discussion today

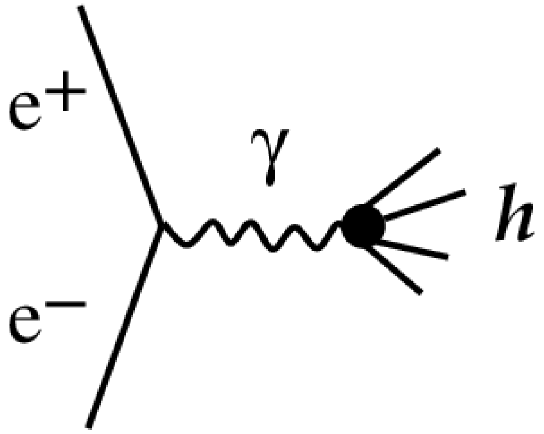
Goals:	$\Delta a_\mu(\text{Expt})$	~ 140 ppb
	$\Delta a_\mu(\text{SM})$	< 220 ppb

slide from D. Hertzog



Dispersive “data-driven” evaluation of HVP





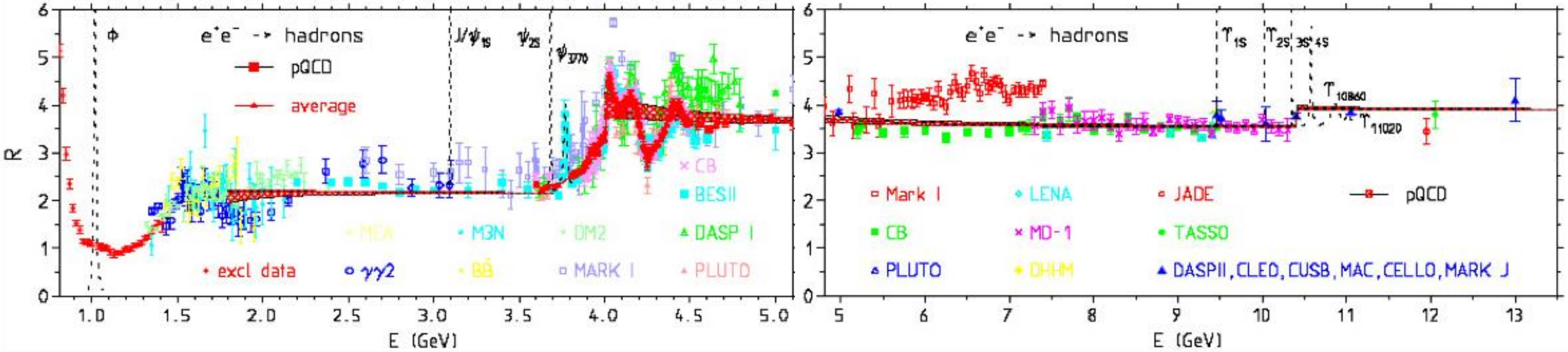
Reviews:

F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017).

M. Davier, Nucl. Part. Phys. Proc. 287-288, 70 (2017).

$$\boxed{a^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{Im}\,\Pi^{\mathrm{had}}(s)\,K(s/m^2)}$$

$$\mathrm{Im}\Pi^{\mathrm{had}} \equiv \frac{s}{4\pi\alpha}R(s) = \frac{s}{4\pi\alpha}\frac{\sigma(e^+e^-\to\mathrm{had})}{\sigma(e^+e^-\to\mu^+\mu^-)}$$

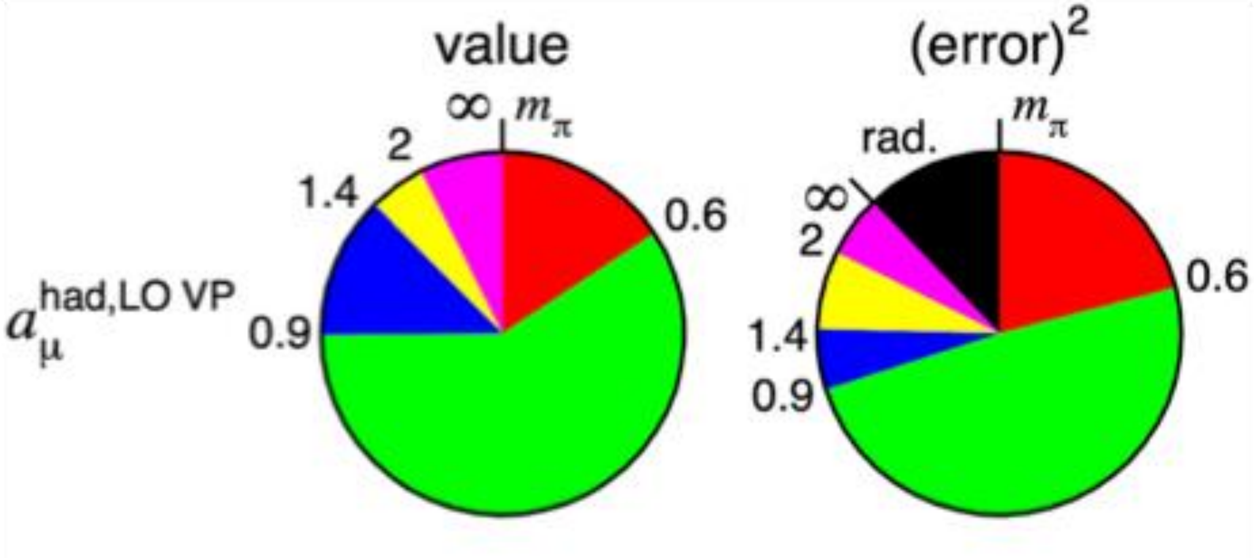


And the result, for HVP contribution

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$ **situation today** (from M. Knecht's talk in Feb'18):

$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.27(2.46)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]
$688.07(4.14)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

$\sim 0.4\%$



(a) Fractional contributions to $a_\mu^{\text{had, LO VP}}$.

What about hadronic light-by-light (HLbL), etc.?

Universal dispersive formula:
Schwinger Sum Rule

J. S. Schwinger, Proc. Nat. Acad. Sci. 72, 1 (1975); ibid. 72, 1559 (1975) [Acta Phys. Austriaca Suppl. 14, 471 (1975)].

A. M. Harun ar-Rashid, Nuovo Cim. A 33, 447 (1976).

photon lab-frame energy ν
and virtuality $Q^2 = -q^2$

muon mass m

anomalous
magnetic moment
a=½(g−2)

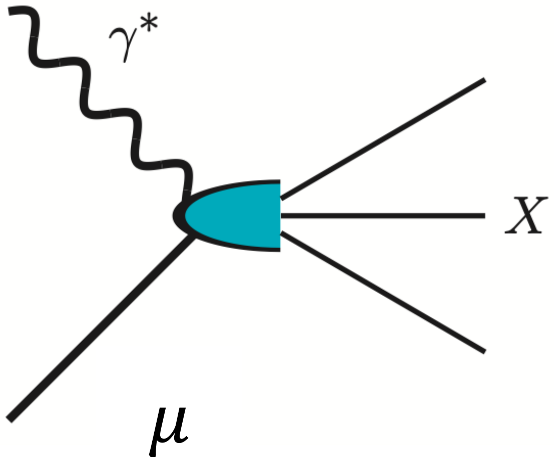
$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q}\right]_{Q^2=0}$$

fine-structure
constant $\alpha \approx 1/137$

photo-absorption threshold ν_0

longitudinal-transverse
photo-absorption
cross section σ_{LT}

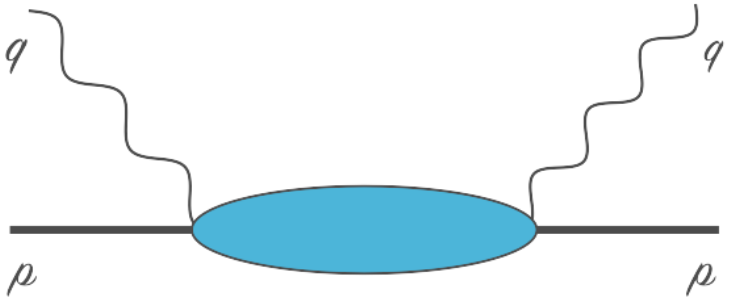
- σ_{LT} is inclusive cross section of polarized photo-absorption on muon:



$=\gamma\mu, \gamma\gamma\mu, \pi^0\mu, \gamma\pi^0\mu, \dots$

Origin of sum rules

- Sum rules are model-independent relations based on general principles of:
  - Analyticity/causality (dispersion relations),
  - unitarity (optical theorem)
  - crossing symmetry



- Examples of sum rules include:

$$(1+\mathrm{a})\,\mathrm{a} = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}}{Q} - \frac{\sigma_{TT}}{\nu}\right]_{Q^2=0}$$

Burkhardt—Cottingham sum rule (1970) $\int_0^1 \mathrm{d}x\, g_2(x,Q^2) = 0$

⊖ $$\mathrm{a}^2 = -\frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu\, \frac{\sigma_{TT}(\nu)}{\nu}$$

Gerasimov—Drell—Hearn sum rule (1966)

$$\mathrm{a} = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$

Schwinger sum rule (1975)

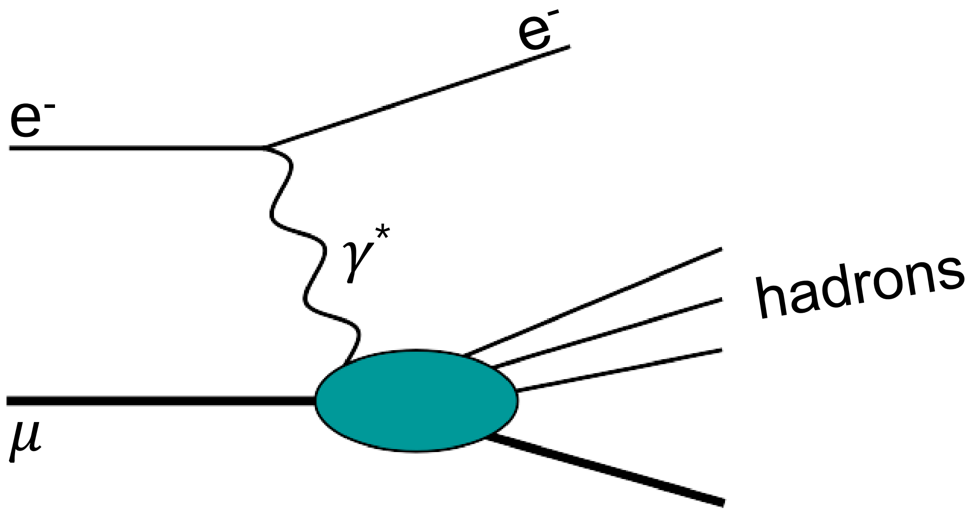
Expression in terms of spin structure functions

$$a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$$

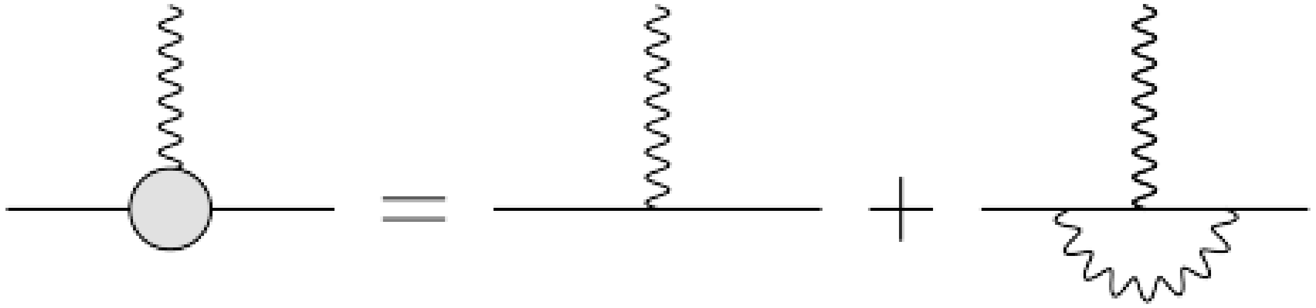
$$= \lim_{Q^2 \to 0} \frac{8m^2}{Q^2} \int_0^{x_0} \mathrm{d}x \, [\bar{g}_1 + \bar{g}_2](x, Q^2)$$

muon spin structure functions

g_1 and g_2



Verifying the Schwinger sum rule in QED



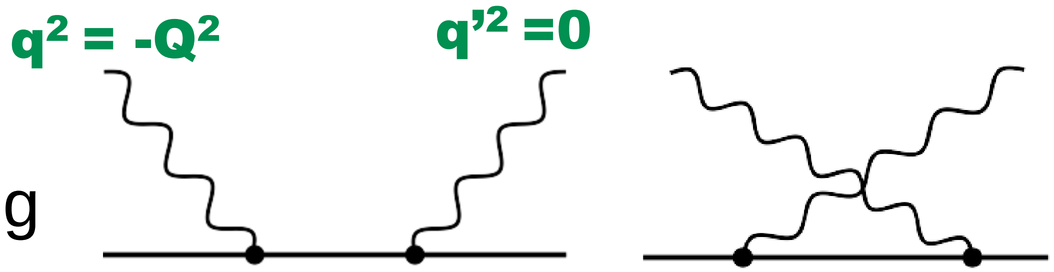
$F_2(0)=a$ $a^{(0)}=0$ $a^{(1)} = \alpha/2\pi$

Schwinger term — the leading QED result

- Schwinger sum rule: $a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$

- Input:

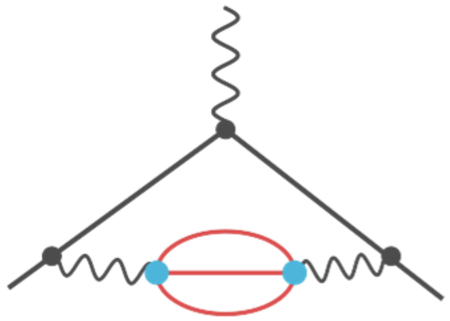
tree-level QED
Compton scattering

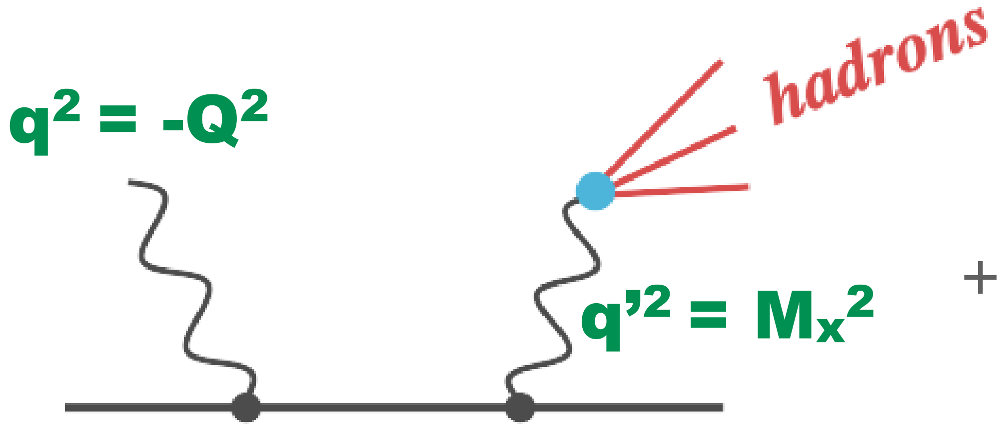


$$\sigma_{LT}^{\gamma^*\mu\to\gamma\mu}(\nu, Q^2) = \frac{\pi\alpha^2 Q\,(s-m^2)^2}{4m^3\nu^2\,(\nu^2+Q^2)} \left(-2 - \frac{m(m+\nu)}{s} + \frac{3m+2\nu}{\sqrt{\nu^2+Q^2}} \operatorname{arccoth} \frac{m+\nu}{\sqrt{\nu^2+Q^2}} \right)$$

$withs = m^2 + 2mv - Q^2$

HVP from Schwinger sum rule





+ crossed diagram

- Cross section of hadron production through timelike Compton scattering:

factorizes as: $$\sigma(\gamma\mu \to \mu X) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}M_X^2}{M_X^2}\, \underset{\substack{\uparrow\\ \text{timelike}\\ \text{Compton scattering}}}{\sigma(\gamma\mu \to \gamma^*\mu)}\, \underset{\substack{\uparrow\\ \text{virtual-photon}\\ \text{decay into hadrons}}}{\mathrm{Im}\,\Pi_X(M_X^2)}$$

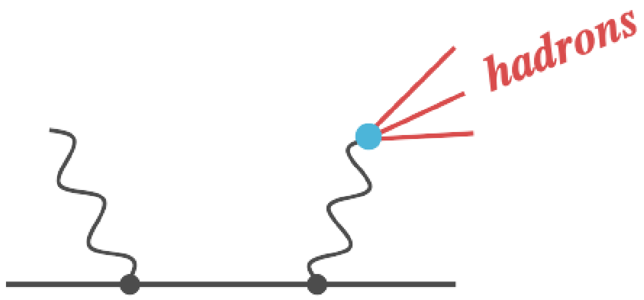
- Timelike Compton scattering cross section:

$$\boxed{\left[\frac{\sigma_{LT}^{\gamma\mu\to\gamma^*\mu}(\nu, Q^2)}{Q}\right]_{Q^2=0} = \frac{\pi\alpha^2}{2m^2\nu^3}\left[-(5s+m^2+M_X^2)\lambda + (s+2m^2-2M_X^2)\log\frac{\beta+\lambda}{\beta-\lambda}\right]}$$

$$\beta = (s+m^2-M_X^2)/2s \qquad s = m^2 + 2m\nu$$
$$\lambda = (1/2s)\sqrt{[s-(m+M_X)^2]\,[s-(m-M_X)^2]}$$

HVP from Schwinger sum rule

Hagelstein & VP, Phys. Rev. Lett. 120, 072002 (2018).



$$\boxed{a = \frac{m^2}{\pi^2\alpha}\int\limits_{4m_\pi^2}^{\infty} \mathrm{d}M_X^2 \int\limits_{\nu_0}^{\infty}\mathrm{d}\nu \left[\frac{1}{Q}\frac{\mathrm{d}\sigma_{LT}^{\gamma\mu\to\mu X}(\nu,Q^2)}{\mathrm{d}M_X^2}\right]_{Q^2=0}}$$

$$= \frac{1}{\pi}\int\limits_{4m_\pi^2}^{\infty}\mathrm{d}M_X^2\,\frac{\mathrm{Im}\,\Pi^{\mathrm{had}}(M_X^2)}{M_X^2}\,\frac{m^2}{\pi^2\alpha}\int\limits_{\nu_0}^{\infty}\mathrm{d}\nu\left[\frac{\sigma_{LT}^{\gamma\mu\to\gamma^*\mu}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$

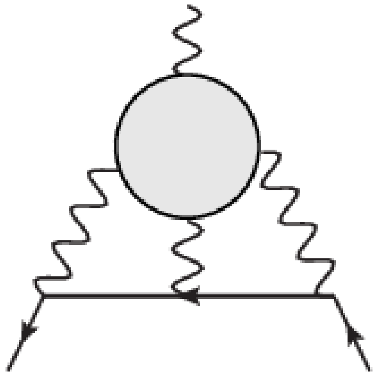
kernel function: $= \frac{\alpha}{\pi}K(M_X^2/m^2) \equiv \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x\,\frac{x^2(1-x)}{x^2+(1-x)(M_X^2/m^2)}$

for $M_X=0$, we find $K(0)=1/2$, and therefore the Schwinger term: $a^{(1)} = \alpha/2\pi$

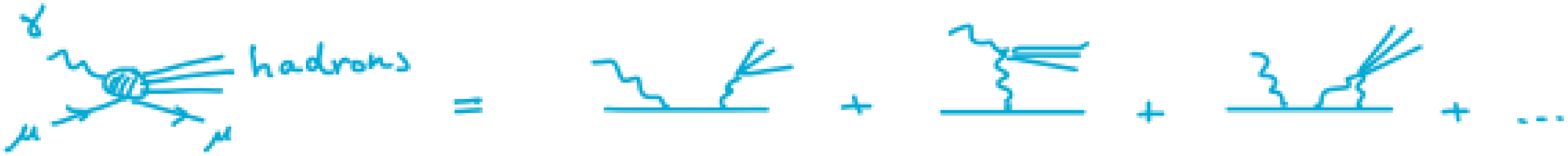
- reproduces the HVP standard dispersion formula

$$\boxed{a^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{Im}\,\Pi^{\mathrm{had}}(s)\int_0^1 \mathrm{d}x\,\frac{x^2(1-x)}{x^2+(1-x)(s/m^2)}}$$

Hadronic Contributions beyond HVP:
4 channels to order α^3



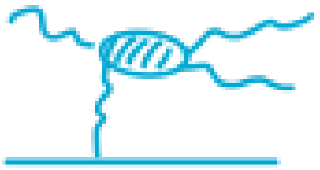
$\gamma\mu \to \mu + \text{hadrons}$



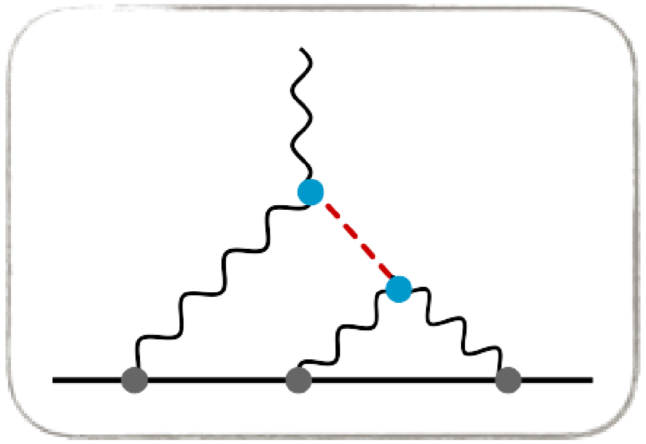
$\gamma\mu \to \gamma\mu + \text{hadrons}$

$\gamma\mu \to \gamma\mu$

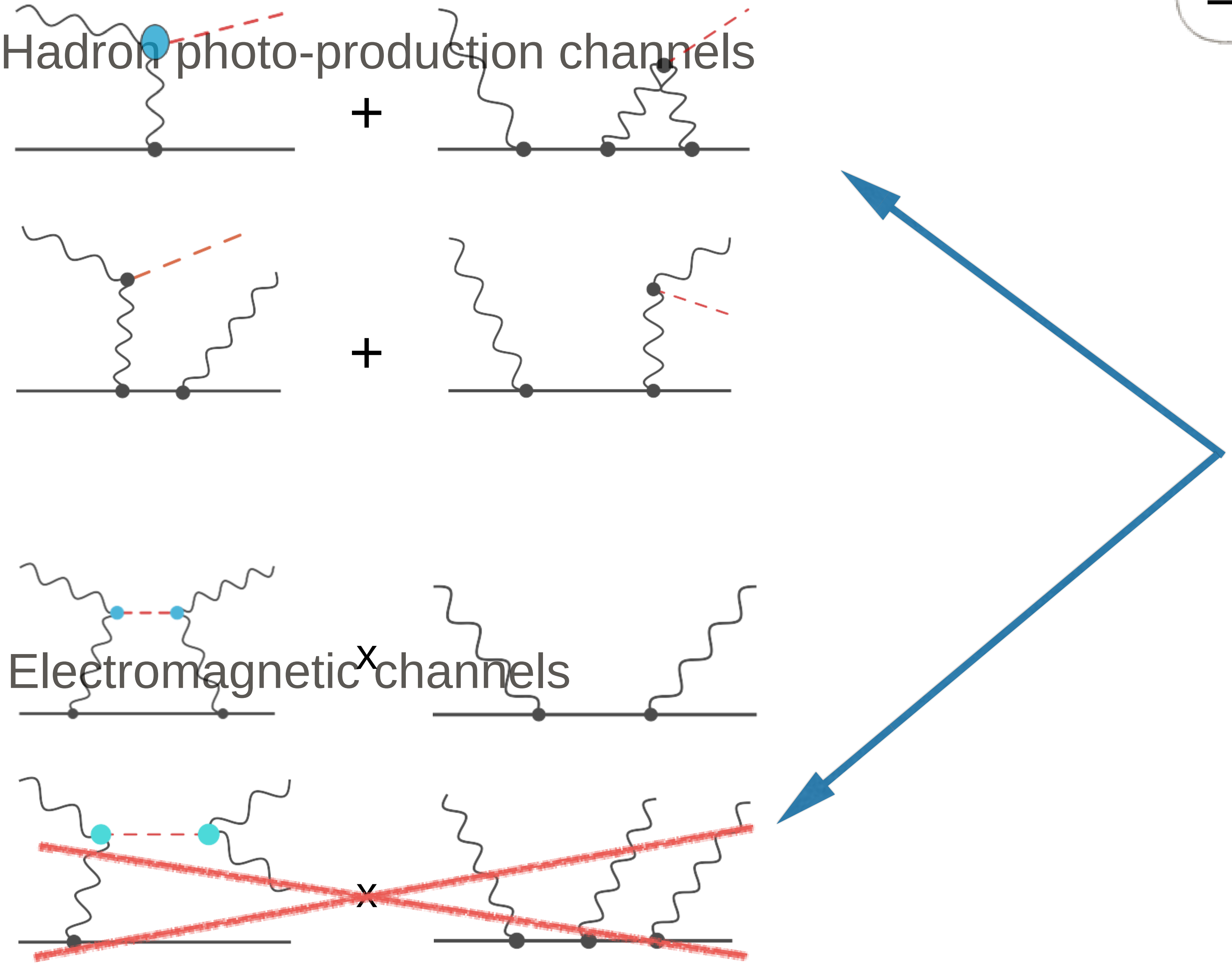
$\gamma\mu \to \gamma\gamma\mu$



Light-by-Light meson-exchange contributions



(pseudo-)scalar contribution



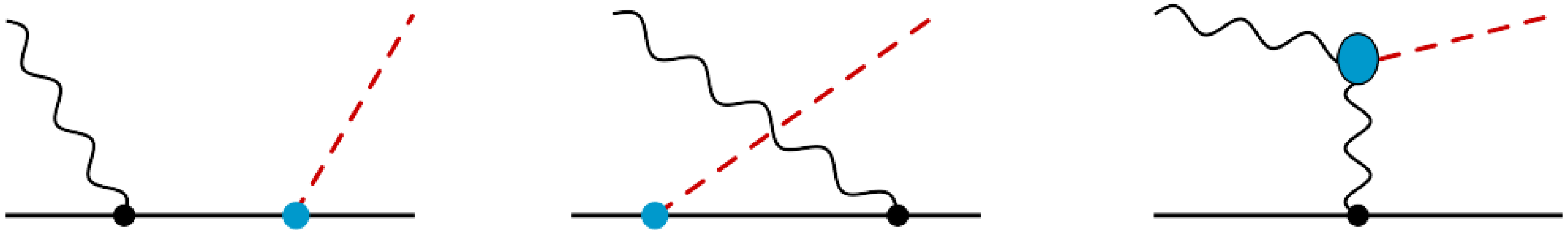
I. Hadron photo-production channels

II. Electromagnetic channels

Double-counting as $\mu\pi^0$ is contained in $\mu\gamma\gamma$

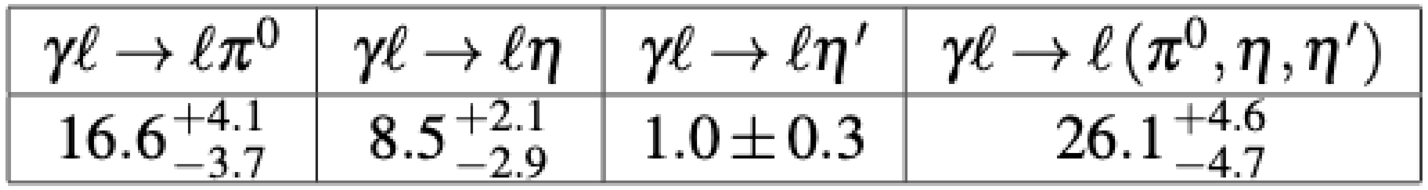
Pseudoscalar meson-photoproduction channel

- No data yet. We use chiral EFT to compute:



1 out of 4 channels at $\mathcal{O}(\alpha^3)$: $\mu + \gamma \to \mu + (\pi^0, \eta, \eta')$

Contribution of the pseudoscalar-meson production channel to a_μ in units of 10^{-10}



$\gamma\ell \to \ell\pi^0$	$\gamma\ell \to \ell\eta$	$\gamma\ell \to \ell\eta'$	$\gamma\ell \to \ell(\pi^0, \eta, \eta')$
$16.6^{+4.1}_{-3.7}$	$8.5^{+2.1}_{-2.9}$	1.0 ± 0.3	$26.1^{+4.6}_{-4.7}$

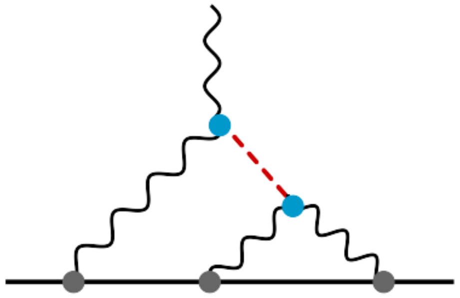
Hagelstein & VP,

arXiv:1907.06927 (2019).

- Compare with the conventional (π^0, η, η')-pole contributions:

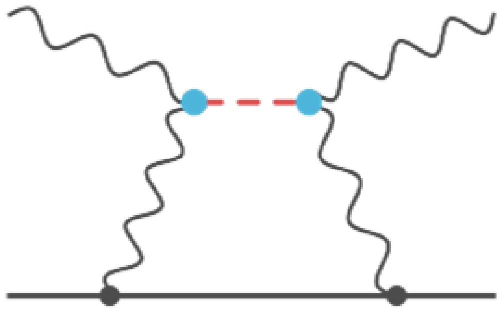
$$a_\mu^{\text{PS-pole}} = 8.3(1.2) \times 10^{-10} \qquad \text{Knecht \& Nyffeler 2002}$$

$$a_\mu^{\text{PS-pole}} = 11.4(10) \times 10^{-10} \qquad \text{Melnikov \& Vainshtein 2004}$$



- Pseudoscalar production gives a contribution to a_μ which is a factor of 2 to 3 larger than the conventional pseudoscalar-pole calculations

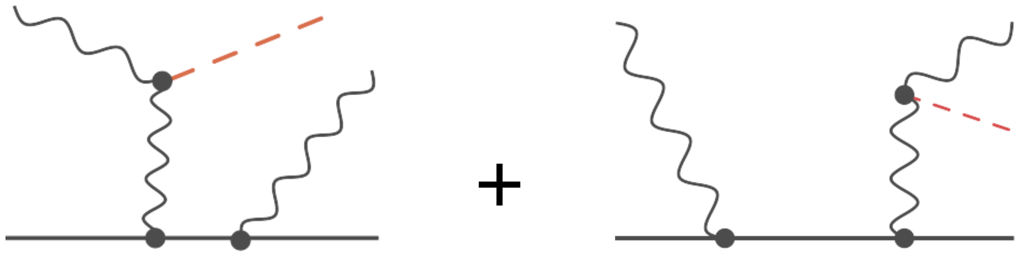
Other channels ?



$\sim 0.5 \times 10^{-10}$

Biloshytskyi, Hagelstein & VP, in preparation.

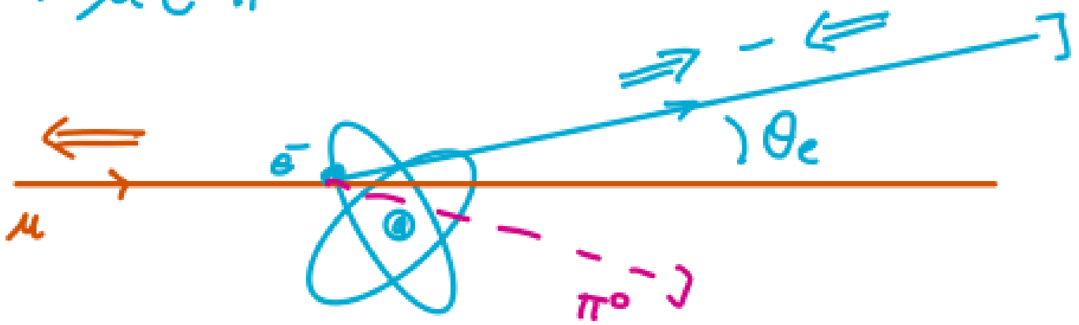
- Compton scattering channel contribution is nearly two orders of magnitude smaller than the pseudoscalar production contribution.



- Radiative pseudoscalar-production channel not calculated yet, but is expected to increase the discrepancy.

Prospects for neutral-pion production measurement at COMPASS
(as part of MUonE experiment)

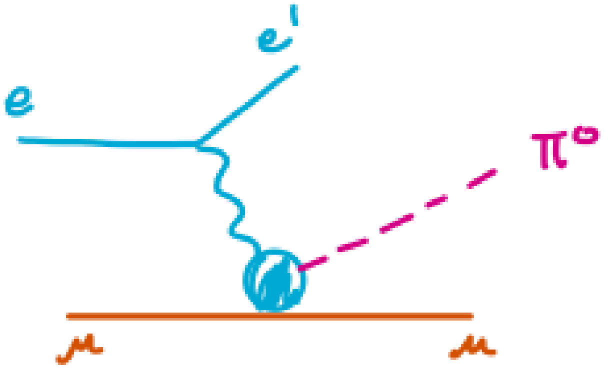
$\mu e \to \mu e \pi^0$



$E_\mu = 150, 200$ GeV

$E_e' \simeq 1$ GeV

$\theta_e \simeq 10$ mrad



$$Q^2 \simeq 2 m_e E_e' \simeq 10^{-3}\,\text{GeV}^2$$

$$\nu \simeq \frac{m_e E_\mu}{m_\mu}\left(1 - 2\frac{E_e'}{m_e}\sin^2\frac{\theta}{2}\right) = (\nu_{\pi^0}, 1\,\text{GeV})$$

$$\nu_{\pi^0} = \frac{m_{\pi^0}}{m_\mu}\left(\frac{1}{2}m_{\pi^0} + m_\mu\right) \simeq 230\,\text{MeV}$$

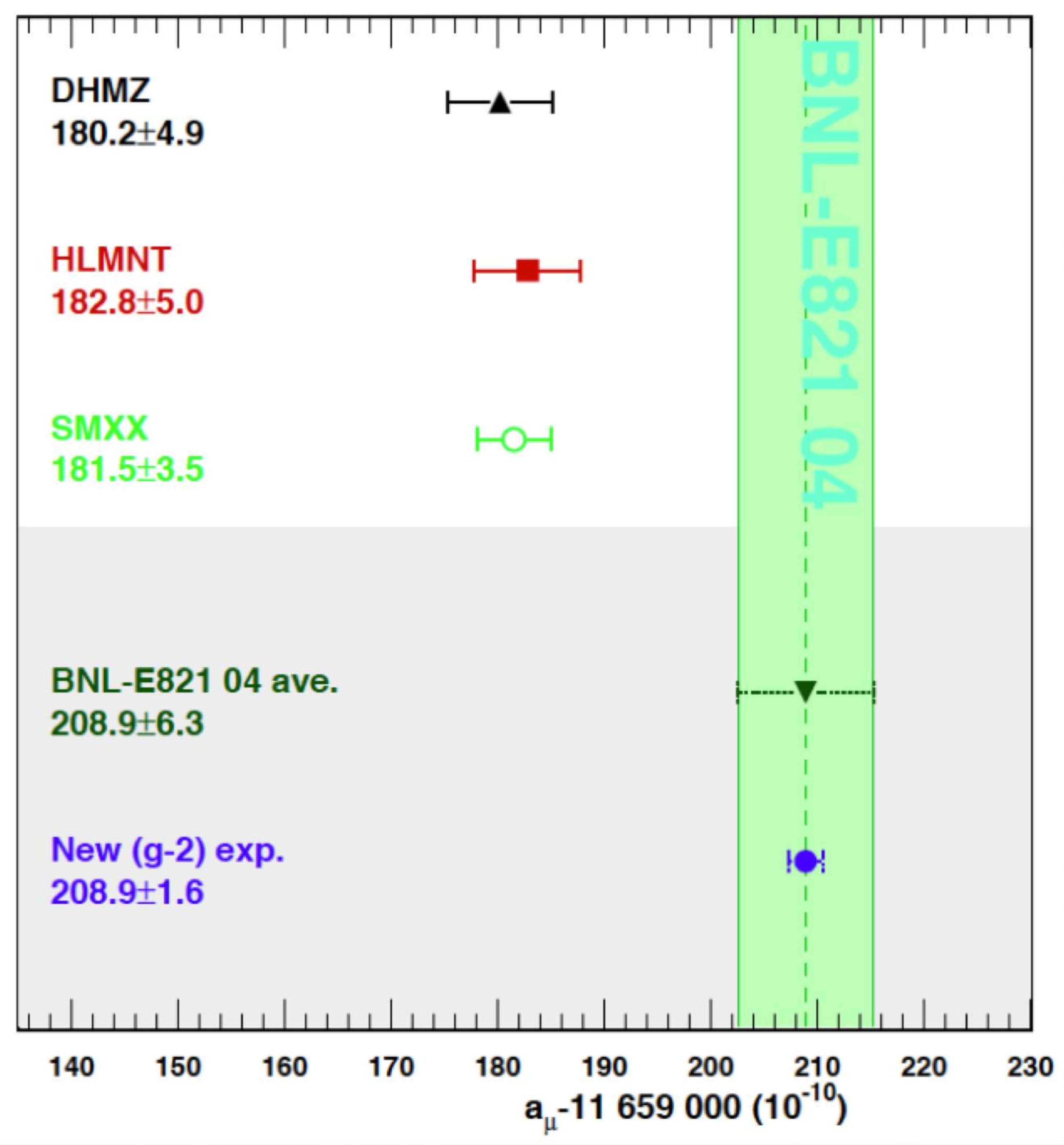
Present status of muon g—2

Standard model theory and experiment comparison F. Jegerlehner

Contribution	Value×10^{10}	Error×10^{10}	Reference
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	Aoyama et al 12,Laporta 17
Hadronic LO vacuum polarization	689.46	3.25	
Hadronic light–by–light	10.34	2.88	
Hadronic HO vacuum polarization	-8.70	0.06	
Weak to 2-loops	15.36	0.11	Gnendiger et al 13
Theory	11 659 178.3	3.5	–
Experiment	11 659 209.1	6.3	BNL 04
The. - Exp. 4.3 standard deviations	-30.6	7.2	–

Muon ***g***—2

or, anomalous magnetic moment: $a_\mu \equiv (g-2)_\mu/2$



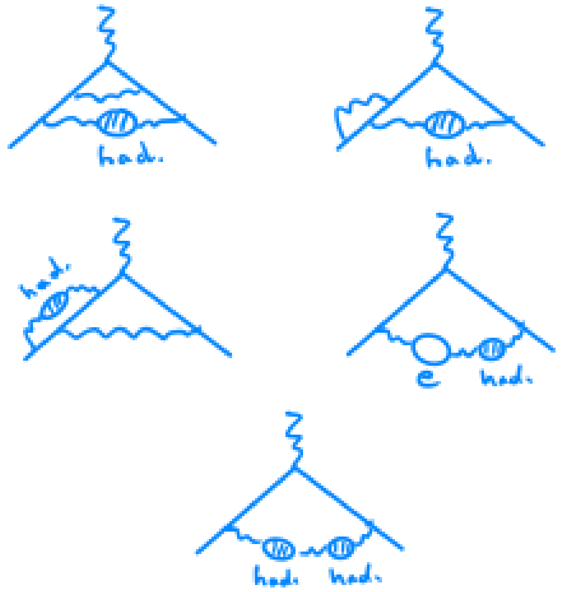
Hadronic Contributions to g-2

$O(\alpha^2)$

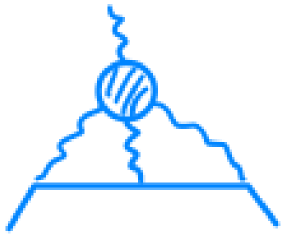
$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$	**situation today** (from M. Knecht's talk in Feb'18):
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.27(2.46)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]
$688.07(4.14)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

$\sim 0.4\%$

$O(\alpha^3)$



VP or LbL



had LbL

“had VP NLO”

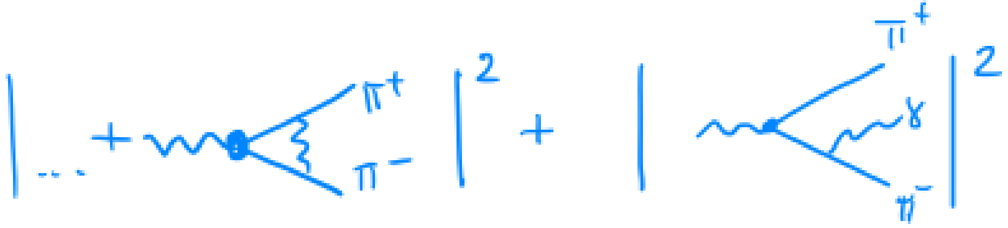
$\sim -10(1) \cdot 10^{-10}$

empirically in LO HVP

$\sim 10 \cdot 10^{-10}$

$a_\mu^{\text{HLxL}} = +(10.3 \pm 2.9) \cdot 10^{-10}$

F. Jegerlehner, arXiv:1705.00263 [hep-ph]



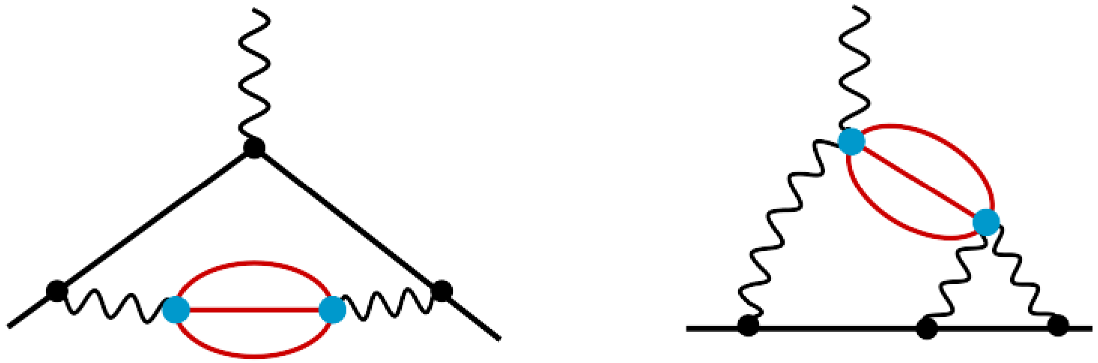
$\sim 4 \cdot 10^{-10}$ (sQED)

$\sim 4 \cdot 10^{-10}$ $(\pi^0\gamma)$

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$	
$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]
$-9.82(4)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]

Motivation

- Uncertainty of the SM prediction for the muon anomaly $(g-2)_\mu$ is dominated by hadronic contributions (HVP and HLbL)



- HVP is calculated with a data-driven dispersive approach:

$$\boxed{a^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{Im}\,\Pi^{\mathrm{had}}(s)\,K(s/m^2)}$$

$$\mathrm{Im}\,\Pi^{\mathrm{had}}(s) = \frac{s}{4\pi\alpha}\,\sigma(\gamma^* \to \text{anything})$$

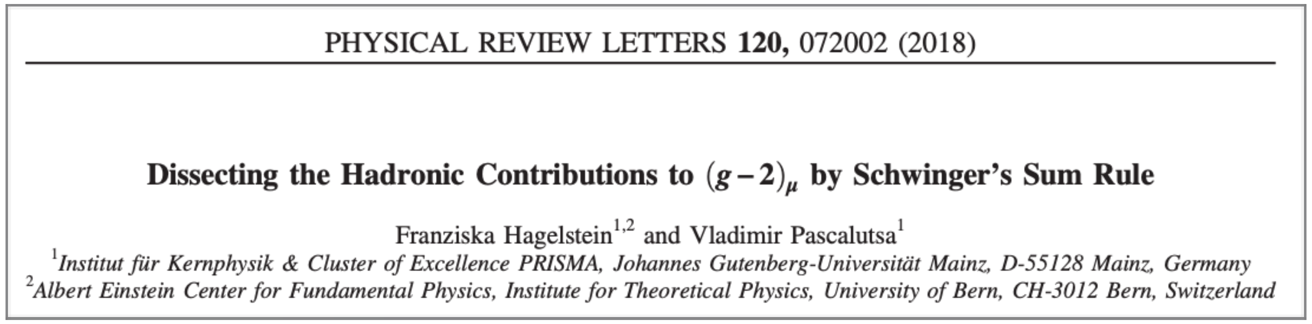
F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017).

M. Davier, Nucl. Part. Phys. Proc. 287-288, 70 (2017)

- HLbL is not as simple, data-driven, systematic

- Is there an exact dispersive formula which treats HVP and HLbL (and everything else) in the same way?

Outline of this talk


PHYSICAL REVIEW LETTERS **120,** 072002 (2018)

Dissecting the Hadronic Contributions to $(g-2)_\mu$ by Schwinger's Sum Rule

Franziska Hagelstein[1,2] and Vladimir Pascalutsa[1]

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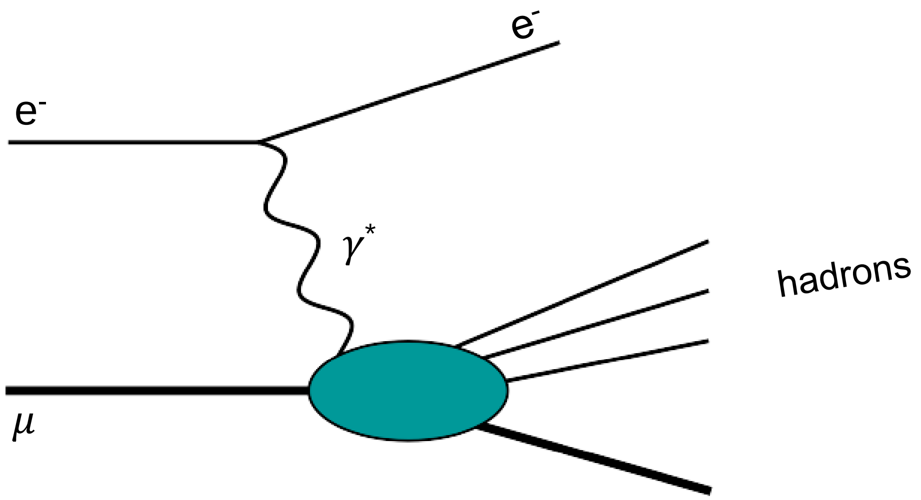
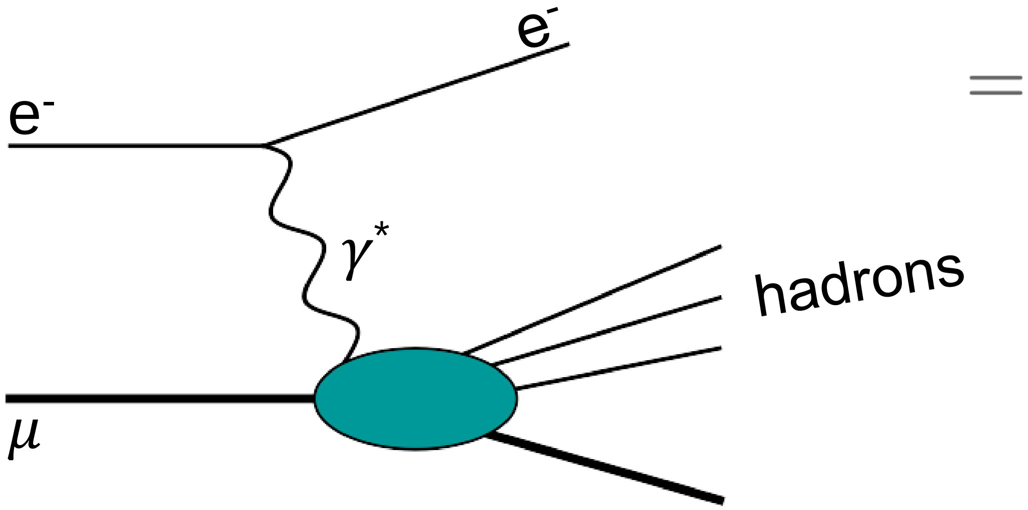
$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$
$$= \lim_{Q^2\to 0}\frac{8m^2}{Q^2}\int_0^{x_0} dx\,[\bar g_1+\bar g_2](x,Q^2)$$

- THE SCHWINGER SUM RULE:
- Reproducing $\alpha/2\pi$
- HADRONIC *VACUUM POLARIZATION* AND *LIGHT-BY-LIGHT* CONTRIBUTIONS ON THE SAME FOOTING
- PSEUDOSCALAR-MESON CONTRIBUTION
- *MUON STRUCTURE FUNCTIONS* FROM INELASTIC MUON-ELECTRON SCATTERING



Spin structure functions



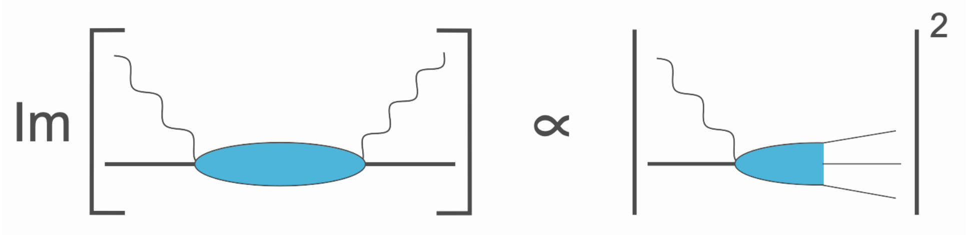
$$a = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu,Q^2)}{Q}\right]_{Q^2=0}$$

$$= \lim_{Q^2\to 0} \frac{8m^2}{Q^2}\int_0^{x_0} \mathrm{d}x \, [\bar{g}_1 + \bar{g}_2](x,Q^2)$$

muon spin structure functions
g_1 and g_2

- Spin-dependent forward doubly-virtual Compton scattering:

$$T_A^{\mu\nu}(q,p) = -\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha \, S_1(\nu,Q^2) + \frac{Q^2}{M^2}\gamma^{\mu\nu}S_2(\nu,Q^2)$$



- Optical theorem:

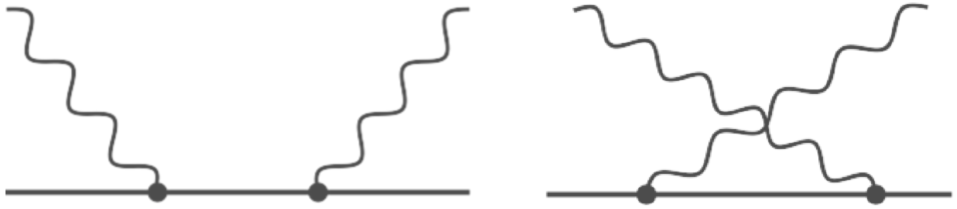
$$\mathrm{Im}\, S_1(\nu,Q^2) = \frac{4\pi^2\alpha}{\nu}\, g_1(x,Q^2) = \frac{M\nu^2}{\nu^2+Q^2}\left[\frac{Q}{\nu}\sigma_{LT} + \sigma_{TT}\right](\nu,Q^2)$$

$$\mathrm{Im}\, S_2(\nu,Q^2) = \frac{4\pi^2\alpha M}{\nu^2}\, g_2(x,Q^2) = \frac{M^2\nu}{\nu^2+Q^2}\left[\frac{\nu}{Q}\sigma_{LT} - \sigma_{TT}\right](\nu,Q^2)$$

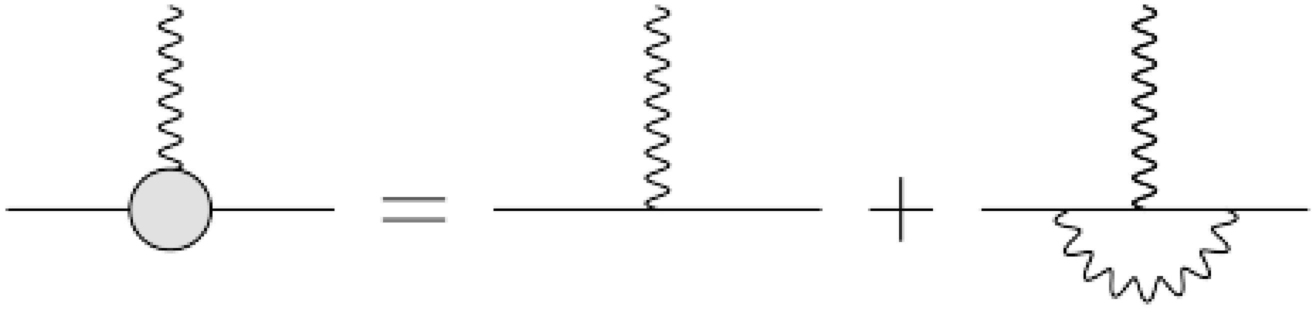
Reproducing the leading QED result

- Schwinger sum rule $a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$
- Input: longitudinal-transverse photo-absorption cross section

  tree-level QED
  Compton scattering

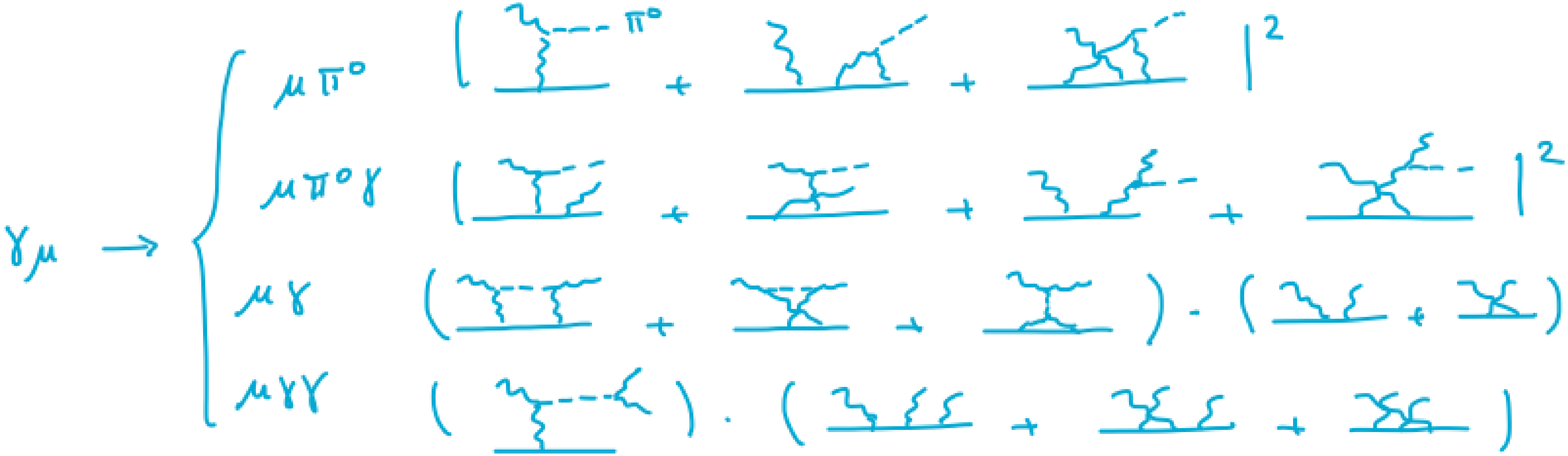


$$\sigma_{LT}^{\gamma^*\mu\to\gamma\mu}(\nu, Q^2) = \frac{\pi\alpha^2 Q\,(s-m^2)^2}{4m^3\nu^2\left(\nu^2+Q^2\right)} \left(-2 - \frac{m(m+\nu)}{s} + \frac{3m+2\nu}{\sqrt{\nu^2+Q^2}}\,\mathrm{arccoth}\,\frac{m+\nu}{\sqrt{\nu^2+Q^2}} \right)$$

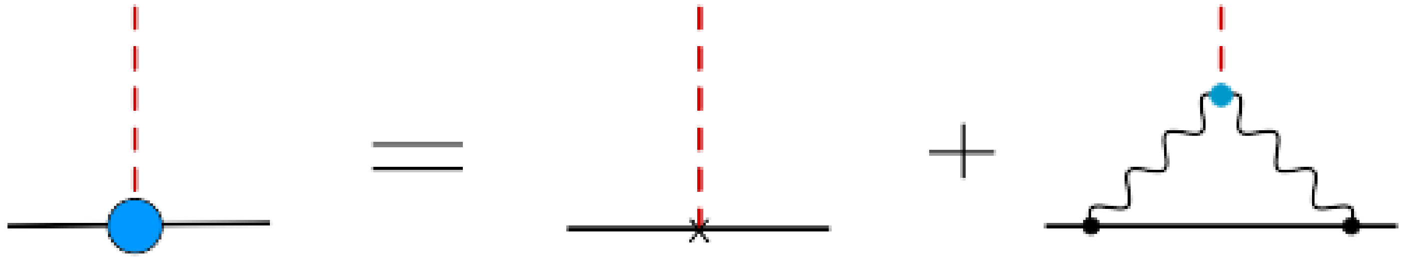


$F_2(0)=a$ $\quad$ $a^{(0)}=0$ $\quad$ $a^{(1)} = \alpha/2\pi$

Pseudo-scalar contribution in full glory

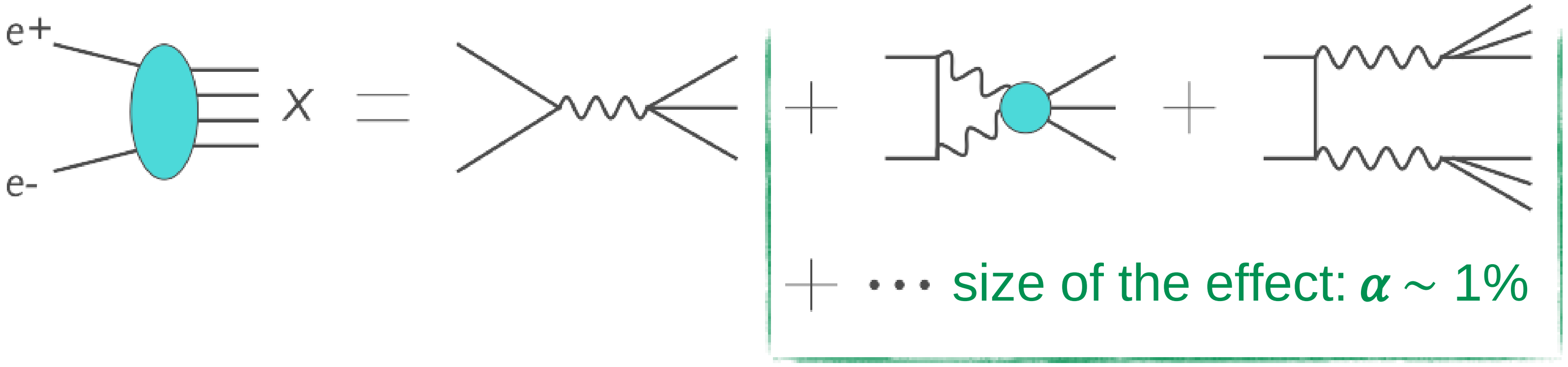


- **No doubly-virtual transition form factors needed, if hadronic channels are measured**



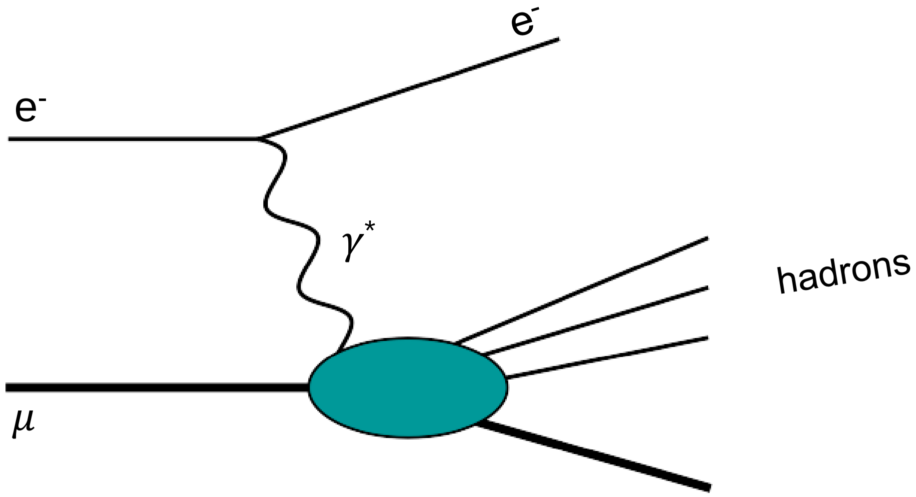
Pseudoscalar meson coupling to leptons.

Alternative way of accessing the HVP



+ • • • size of the effect: $\alpha \sim 1\%$

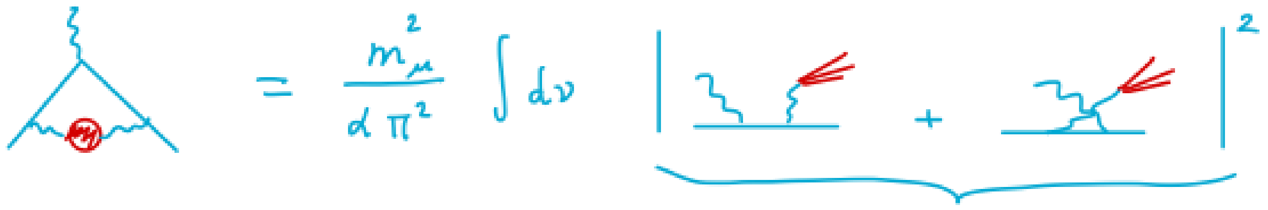
vs.



Summary and Conclusions

1. Schwinger sum rule — dispersive formula applying equally to HVP and HLbL

2. Reproduces α/2π and HVP formula:



$$a_\mu^{HVP} = \frac{m_\mu^2}{\alpha\pi^2}\int d\nu \int dM_x^2 \, \sigma_{LT}(\gamma\mu \to \gamma_x^* \mu)\, \Gamma(\gamma_x^* \to \text{hadrons})$$

3. Splits contributions into hadron production and e.m. (LbL) channels

$\gamma\mu \to \mu + \text{hadrons}$

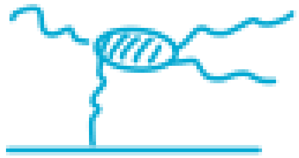
measurable

spin structure functions

$\gamma\mu \to \gamma\mu$

LQCD ?

$\gamma\mu \to \gamma\gamma\mu$



direct LbL scattering (ATLAS)

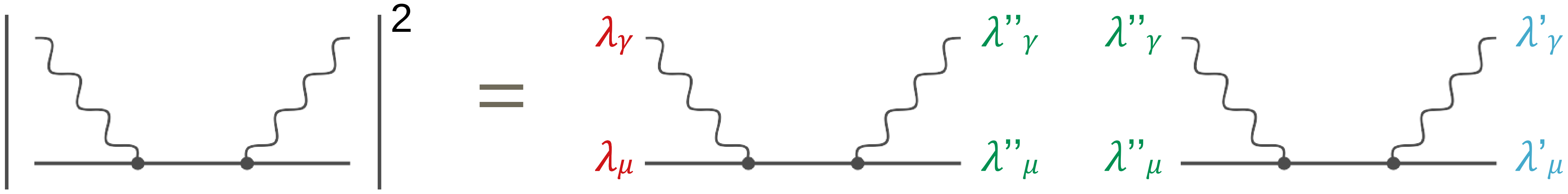
4. Partial calculation of PS-meson contributions: a factor of 2 to 3 larger than the conventional model calculations.

The Cross section σ_{LT}

- Example: tree-level QED Compton scattering cross section

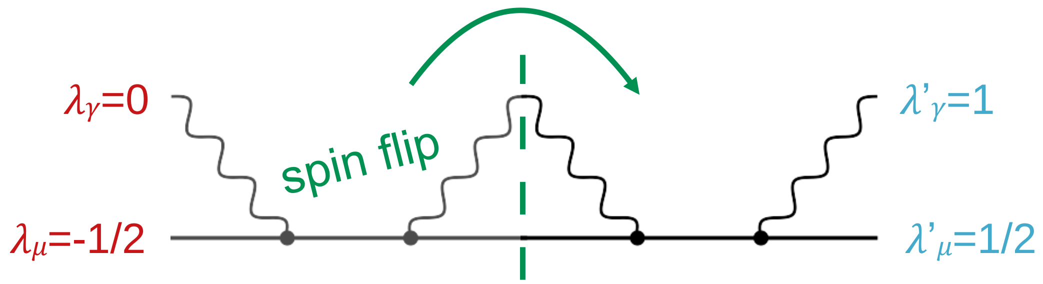
$$d\sigma_{\lambda'_\gamma\lambda'_\mu\lambda_\gamma\lambda_\mu} = (2\pi)^4\delta^{(4)}(p_f - p_i)\sum_{\lambda''_\gamma,\lambda''_\mu}\frac{\mathcal{M}^\dagger_{\lambda'_\gamma\lambda'_\mu\lambda''_\gamma\lambda''_\mu}\mathcal{M}_{\lambda''_\gamma\lambda''_\mu\lambda_\gamma\lambda_\mu}}{4I}\prod_a\frac{d^3p'_a}{(2\pi)^3 2E'_a},$$

with conserved helicity: H=λ'_γ-λ'_μ=λ_γ-λ_μ

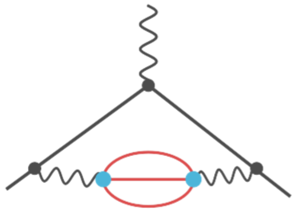


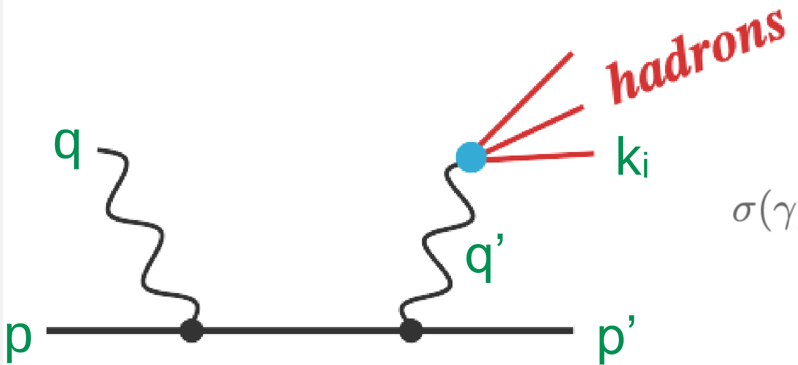
- helicity difference photo-absorption cross section: $\boxed{\sigma_{TT} = 1/2\,(\sigma_{1/2} - \sigma_{3/2})}$
- longitudinal-transverse photo-absorption cross section:

$\gamma^*(\lambda_\gamma=0) + \mu(\lambda_\mu=-1/2) \rightarrow \gamma(\lambda'_\gamma=1) + \mu(\lambda'_\mu=1/2)$



Timelike CS mechanism





$q^2 = -Q^2$

$q'^2 = M_X^2$

+ crossed diagram

Λ^ν: virtual-photon decay vertex

$$\sigma(\gamma\mu \to \mu X) = \frac{(2\pi)^4}{4I} \int \mathrm{d}^4 q' \underbrace{\int \prod_i \frac{\mathrm{d}^3 \boldsymbol{k}_i}{2E_{k_i}(2\pi)^3} \int \frac{\mathrm{d}^3 \boldsymbol{p}'}{2E_{p'}(2\pi)^3}} \left[\frac{\Lambda^{\dagger\mu}\Lambda^\nu \rho_{\mu\nu}}{(-q'^2)^2}\right] \delta^4(q' - \sum_i k_i)\, \delta^4(p + q - p' - q')$$

initial flux factor

$I^2 = (p\cdot q)^2 - p^2 q^2$

phase space of the final state

$\rho_{\mu\nu}$: squared matrix element of timelike CS

- Virtual-photon decay width into hadronic state X:

$$[\Gamma(\gamma^* \to X)]^{\mu\nu} = \int \prod_i \frac{\mathrm{d}^3 \boldsymbol{k}_i}{2E_{k_i}(2\pi)^3} \frac{\Lambda^{\dagger\mu}\Lambda^\nu}{2E_{q'}} (2\pi)^4 \delta^4(q' - \sum_i k_i)$$

$$= -\frac{1}{\sqrt{q'^2}} (q'^2 g^{\mu\nu} - q'^\mu q'^\nu) \mathrm{Im}\, \Pi_X(q'^2)$$

Im Π_X: contribution of state X to the VP

- Combine into:

$$\sigma(\gamma\mu \to \mu X) = -\frac{1}{2I} \int \mathrm{d}^4 q' \int \frac{\mathrm{d}^3 \boldsymbol{p}'}{2E_{p'}(2\pi)^3}\, \rho^\mu_\mu \frac{\mathrm{Im}\, \Pi_X(q'^2)}{q'^2} \delta^4(p + q - p' - q')$$

- Final factorized cross section:

$$\boxed{\sigma(\gamma\mu \to \mu X) = \frac{1}{\pi} \int_{4m_\pi^2}^\infty \frac{\mathrm{d}M_X^2}{M_X^2} \sigma(\gamma\mu \to \gamma^*\mu)\, \mathrm{Im}\, \Pi_X(M_X^2)}$$