
Isospin dependence in DIS from $A=3$ mirror nuclei

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Motivation

- DIS from helium-3 and tritium proposed as a unique and independent means of determining neutron/proton, and hence d/u PDF, ratio at large x

→ MARATHON experiment at JLab Hall A



G. Petratos
Mon. 11:30

- Independent determination of nuclear EMC effect in deuterium and $A=3$ nuclei

→ never before been experimentally determined

- Understanding structure of helium-3 also vital for determination of polarized neutron structure

Basic idea (ca. 2001)

PHYSICAL REVIEW C **68**, 035201 (2003)

Deep inelastic scattering from $A=3$ nuclei and the neutron structure function

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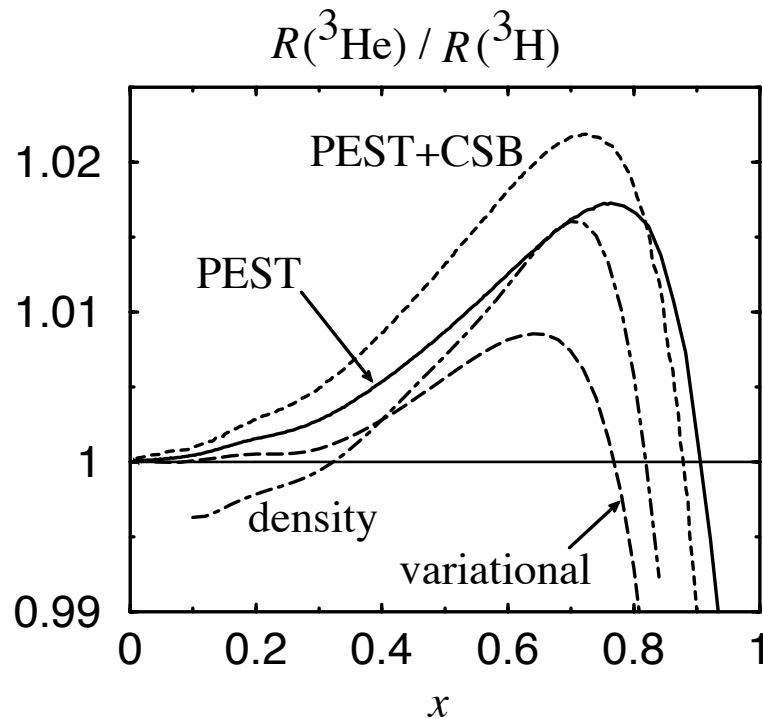
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We present a comprehensive analysis of deep inelastic scattering from ^3He and ^3H , focusing in particular on the extraction of the free neutron structure function F_2^n . Nuclear corrections are shown to cancel to within 1–2% for the isospin-weighted ratio of ^3He to ^3H structure functions, which leads to more than an order of magnitude improvement in the current uncertainty in the neutron to proton ratio F_2^n/F_2^p at large x . Theoretical uncertainties originating from the nuclear wave function, including possible non-nucleonic components, are evaluated. Measurements of the ^3He and ^3H structure functions will, in addition, determine the magnitude of the EMC effect in all $A \leq 3$ nuclei.



$$R(^3\text{He}) = \frac{F_2^{^3\text{He}}}{2F_2^p + F_2^n}$$

$$R(^3\text{H}) = \frac{F_2^{^3\text{H}}}{F_2^p + 2F_2^n}$$

The ratio of these,

$$\mathcal{R} = \frac{R(^3\text{He})}{R(^3\text{H})}$$

can be inverted to yield the ratio of free neutron to proton structure functions,

$$\frac{F_2^n}{F_2^p} = \frac{2\mathcal{R} - F_2^{^3\text{He}}/F_2^{^3\text{H}}}{2F_2^{^3\text{He}}/F_2^{^3\text{H}} - \mathcal{R}}$$

→ **most calculations gave ~2% variation in super-ratio**

Pace, Salme, Scopetta (2001)

Sargsian, Simula, Strikman (2002)

Update (2019)

- Revisit problem including latest theoretical developments and recent data (JLab E03-103) on helium-3 / deuterium ratios

→ generalized convolution in “weak binding approximation” (WBA)

$$F_2^A(x, Q^2) = \sum_N \int \frac{d^4 p}{(2\pi)^4} \mathcal{F}_0^N(\varepsilon, \mathbf{p}) \left(1 + \frac{\gamma p_z}{M}\right) C_{22} \tilde{F}_2^N(x/y, Q^2, p^2)$$

nuclear spectral function

bound nucleon momentum $p = (p_0; \mathbf{p}) = (M + \varepsilon; \mathbf{p}_\perp, p_z)$

kinematic factor $C_{22} = \frac{1}{\gamma^2} \left[1 + \frac{(\gamma^2 - 1)}{2y^2 M^2} (2p^2 + 3\mathbf{p}_\perp^2) \right]$

$$\gamma^2 = 1 + \frac{4M^2 x^2}{Q^2}$$

nuclear momentum fraction $y = \frac{M_A}{M} \frac{p \cdot q}{P \cdot q} = \frac{p_0 + \gamma p_z}{M}$

→ factorized formula valid up to $\mathcal{O}(\mathbf{p}^2/M^2)$ corrections

WM, Schreiber, Thomas (1994)
Kulagin et al. (1994)

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off-shell nucleon structure function

→ expand to lowest order in nucleon virtuality ($p^2 - M^2$)

$$\tilde{F}_2^N(x, Q^2, p^2) = F_2^N(x, Q^2) \left(1 + \frac{p^2 - M^2}{M^2} \delta f^N(x)\right)$$

on-shell structure function

off-shell correction

$$\delta f^N = \left. \frac{\partial \log \tilde{F}_2^N}{\partial \log(p^2/M^2)} \right|_{p^2=M^2}$$

Update (2019)

- Write total nuclear structure function as a sum of nucleon on-shell and off-shell contributions

$$F_2^A(x, Q^2) = F_2^{A(\text{on})}(x, Q^2) + F_2^{A(\text{off})}(x, Q^2)$$

where

$$F_2^{A(\text{on})}(x, Q^2) = \sum_N \int dy f^{N/A}(y, \gamma) F_2^N(x/y, Q^2)$$

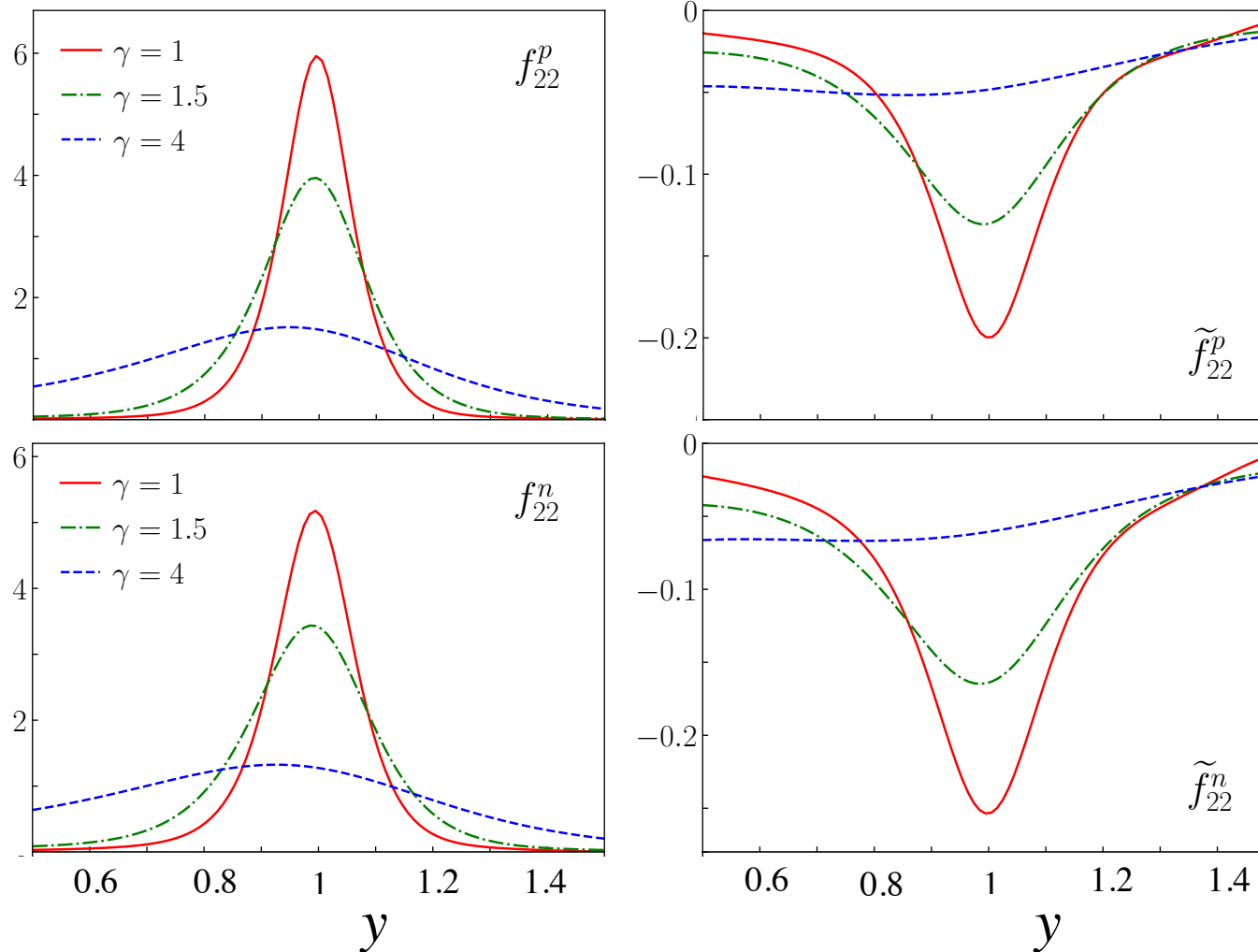
$$F_2^{A(\text{off})}(x, Q^2) = \sum_N \int dy \left[\tilde{f}^{N/A}(y, \gamma) F_2^N(x/y, Q^2) \right] \delta f^N(x/y)$$

- Nucleon “smearing functions” (light-cone momentum distributions)

on-shell $f^{N/A}(y, \gamma) = \int \frac{d^4 p}{(2\pi)^4} \mathcal{F}_0^N(\varepsilon, \mathbf{p}) \left(1 + \frac{\gamma p_z}{M} \right) C_{22} \delta \left(y - 1 - \frac{\varepsilon + \gamma p_z}{M} \right)$

off-shell $\tilde{f}^{N/A}(y, \gamma) = \int \frac{d^4 p}{(2\pi)^4} \mathcal{F}_0^N(\varepsilon, \mathbf{p}) \left(1 + \frac{\gamma p_z}{M} \right) C_{22} \frac{(p^2 - M^2)}{M^2} \delta \left(y - 1 - \frac{\varepsilon + \gamma p_z}{M} \right)$

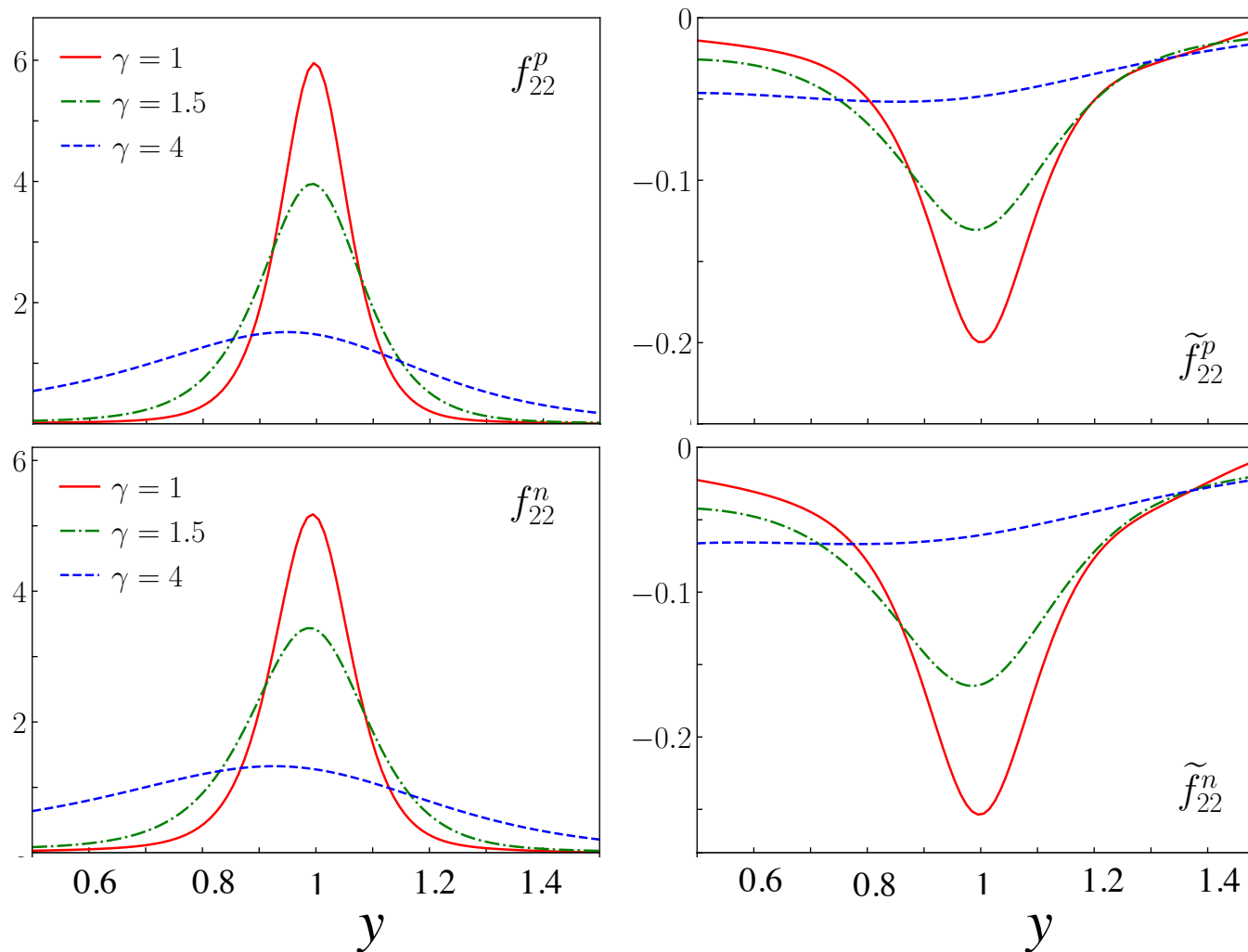
Smearing functions



*Tropiano et al.,
PRC 99, 035201 (2019)*

→ off-shell smearing functions \ll on-shell smearing functions
for most kinematics of interest

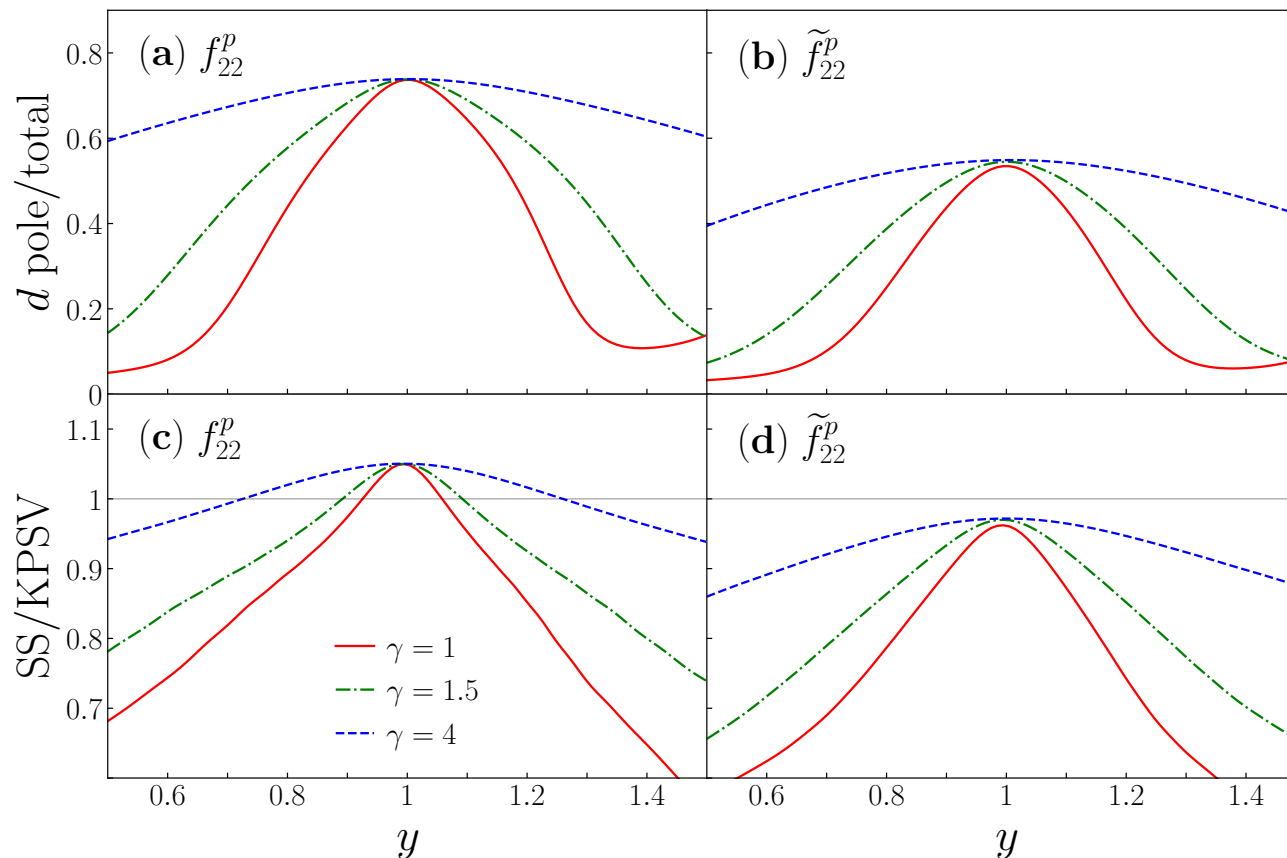
Smearing functions



*Tropiano et al.,
PRC 99, 035201 (2019)*

- for tritium, assume isospin symmetry $p/{}^3\text{H} = n/{}^3\text{He}$, $n/{}^3\text{H} = p/{}^3\text{He}$
- but isospin symmetry does not imply $p/{}^3\text{He} = n/{}^3\text{He}$!

Smearing functions



→ spectral function models similar at peak

KPSV = Kievsky, Pace, Salme, Viviani
SS = Schulze, Sauer

→ large d pole contribution to proton

$$\mathcal{F}^p(\varepsilon, \mathbf{p}) = \mathcal{F}_{d \text{ pole}}^p(\mathbf{p}) \delta(\varepsilon + \varepsilon_{\text{He}} - \varepsilon_d) + \mathcal{F}_{\text{cont}}^p(\varepsilon, \mathbf{p})$$

$$\mathcal{F}^n(\varepsilon, \mathbf{p}) = \mathcal{F}_{\text{cont}}^n(\varepsilon, \mathbf{p})$$

Quasielastic scattering

■ Check smearing functions against quasielastic ${}^3\text{He}$ data

→ nucleon elastic structure function given by elastic form factors

$$F_2^{N(\text{el})}(x, Q^2) = \left[\frac{G_{EN}^2 + \tau G_{MN}^2}{1 + \tau} \right] \delta(1 - x) \quad \tau = \frac{Q^2}{4M^2}$$

→ off-shell generalization minimally accounted for by kinematics

$$Q^2 \delta((p + q)^2 - M^2) = \frac{x}{y} \delta\left(1 - \kappa(p^2) \frac{x}{y}\right)$$

where

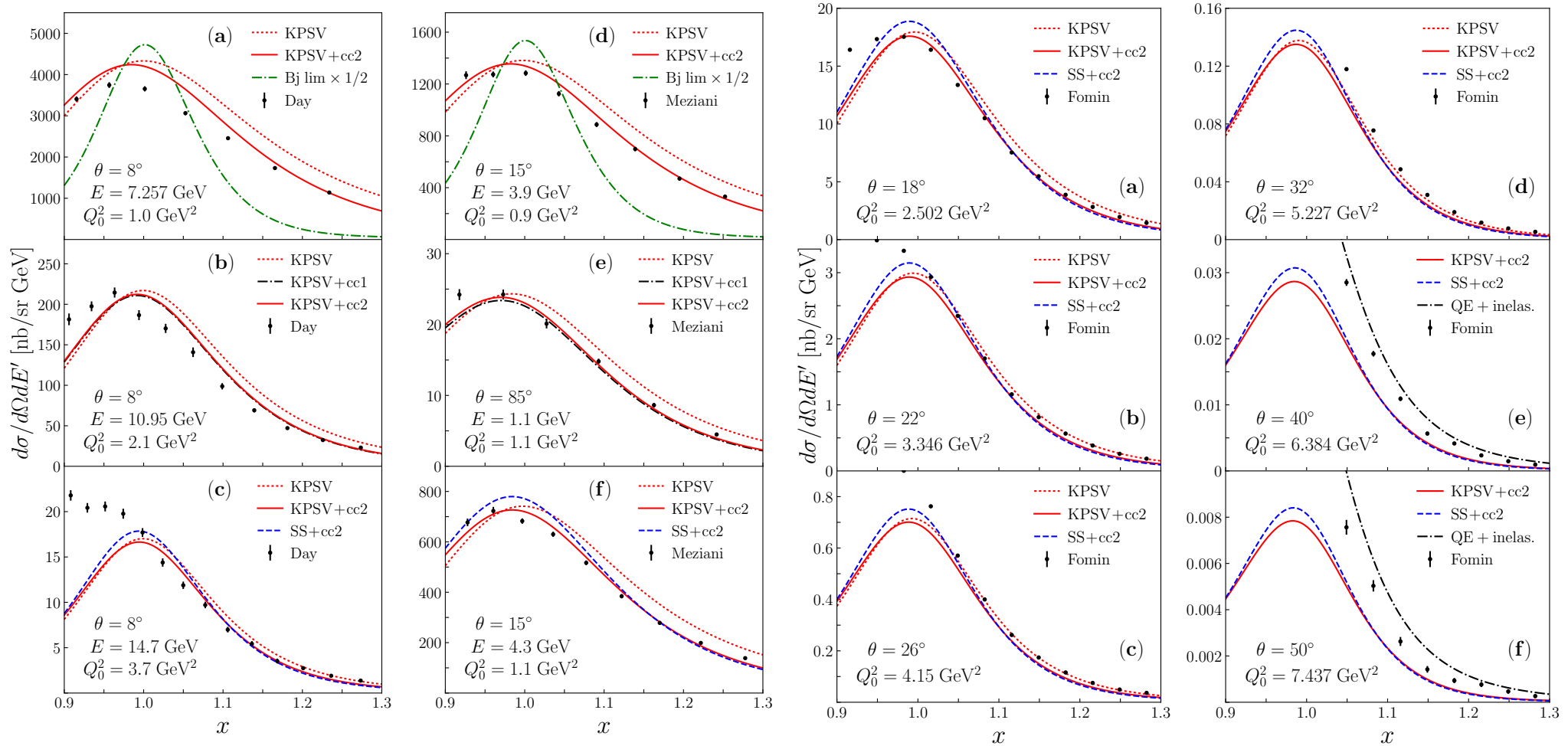
$$\kappa(p^2) = 1 - (p^2 - M^2)/Q^2$$

and choice of “cc1” or “cc2” prescription for off-shell nucleon current

DeForest (1983)

Quasielastic scattering

Check smearing functions against quasielastic ^3He data



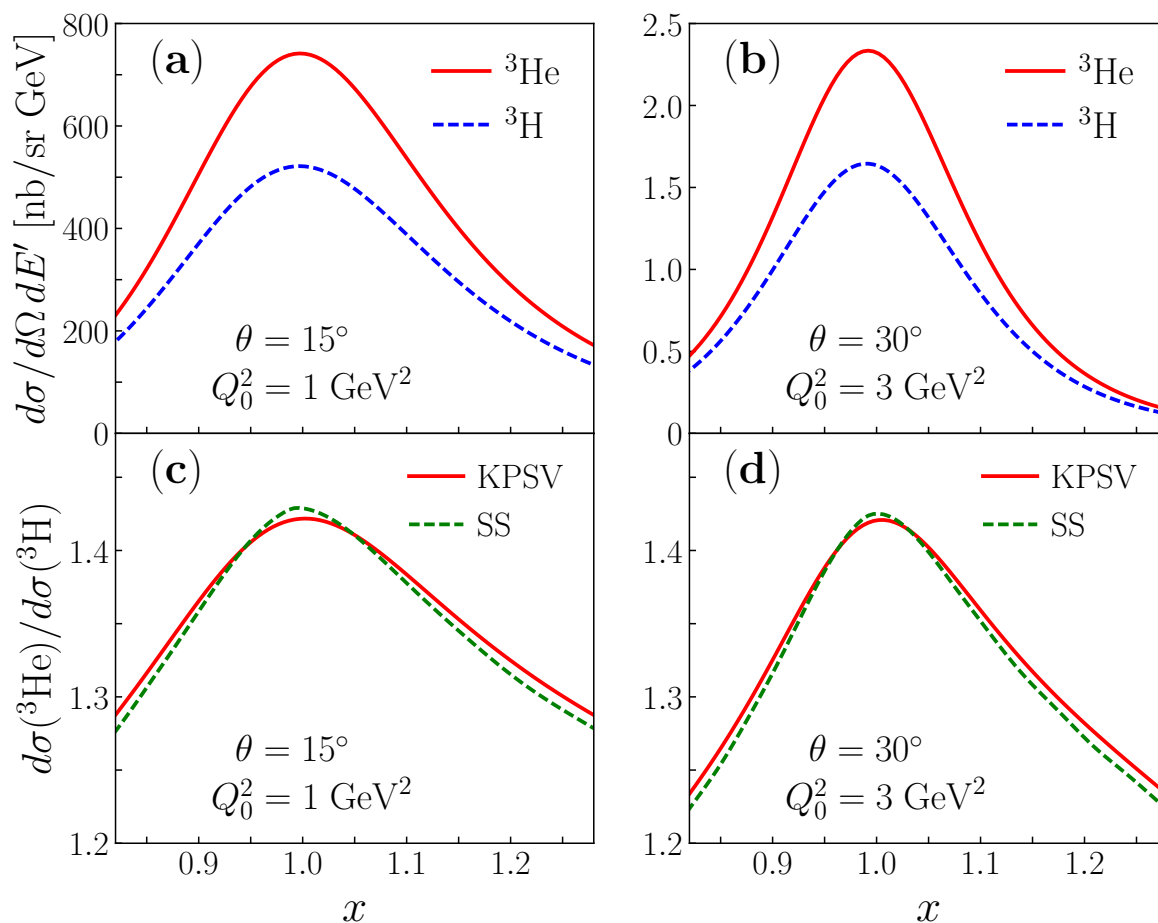
→ impulse approximation gives reasonable description of QE data at $x \gtrsim 1$ for $Q^2 \gtrsim 1$ GeV 2

*Tropiano et al.,
PRC 99, 035201 (2019)*

Quasielastic scattering

- If smearing functions well constrained at $y \approx 1$, can one use QE ^3He and ^3H data to extract nucleon's e.m. form factors?

→ *e.g.*, at kinematics of JLab E12-11-112 experiment



*Tropiano et al.,
PRC 99, 035201 (2019)*

Quasielastic scattering

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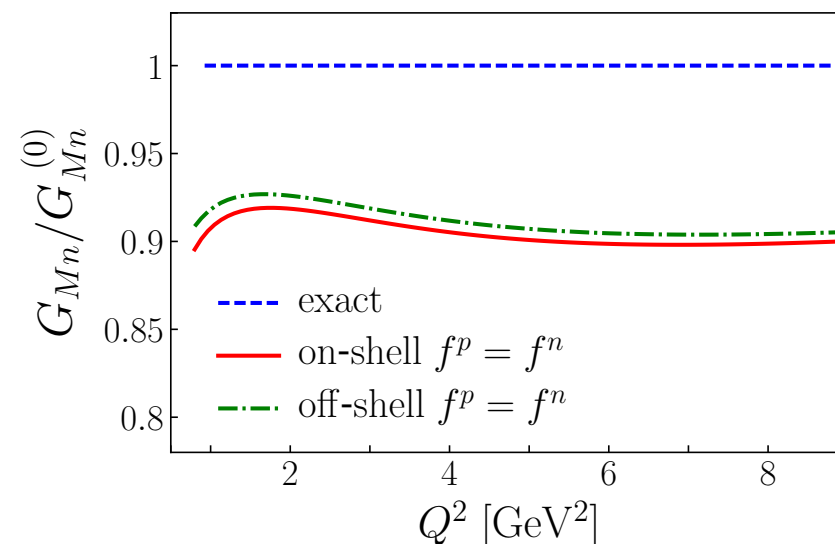
$$R^{(\text{QE})} \equiv \frac{F_2^{3\text{He}(\text{QE})}}{F_2^{3\text{H}(\text{QE})}} = \frac{2 + (f^n/f^p)R_{np}}{(f^n/f^p) + 2R_{np}}$$

$$f^N \equiv f_{22}^N(x=1)$$

$$R_{np} = \frac{G_{En}^2 + \tau G_{Mn}^2}{G_{Ep}^2 + \tau G_{Mp}^2}$$

→ if 3 of the form factors known,
can extract the remaining one,
e.g., for neutron magnetic

→ $f^n/f^p \approx 0.87$ at QE peak
 $\approx 10\%$ error if $f^n = f^p$ assumed

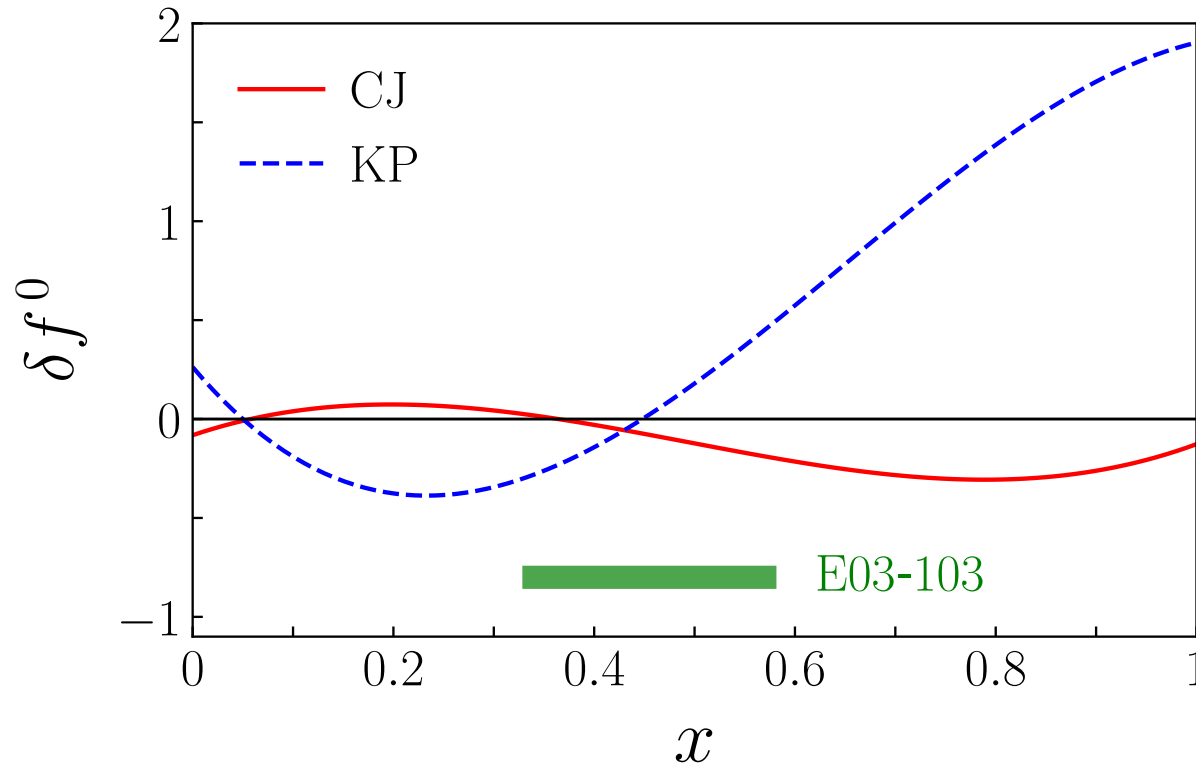


Nucleon off-shell corrections

- Off-shell effects unphysical (of course), but can be discussed within a given theoretical framework
- Within WBA approach, Kulagin & Petti suggested to fit them to nuclear structure function data
Kulagin, Petti, NPA 765, 126 (2006)
- Similar approach adopted in CJ15 global QCD analysis of *proton and deuteron* data
Accardi et al., PRD 93, 114017 (2016)
 - parametrize (isoscalar) off-shell function as 3rd order polynomial, with parameters C and x_0 , with x_1 determined from normalization condition (off-shell effects do not modify valence quark number)

$$\delta f^0 = C(x - x_0)(x - x_1)(1 + x_0 - x)$$

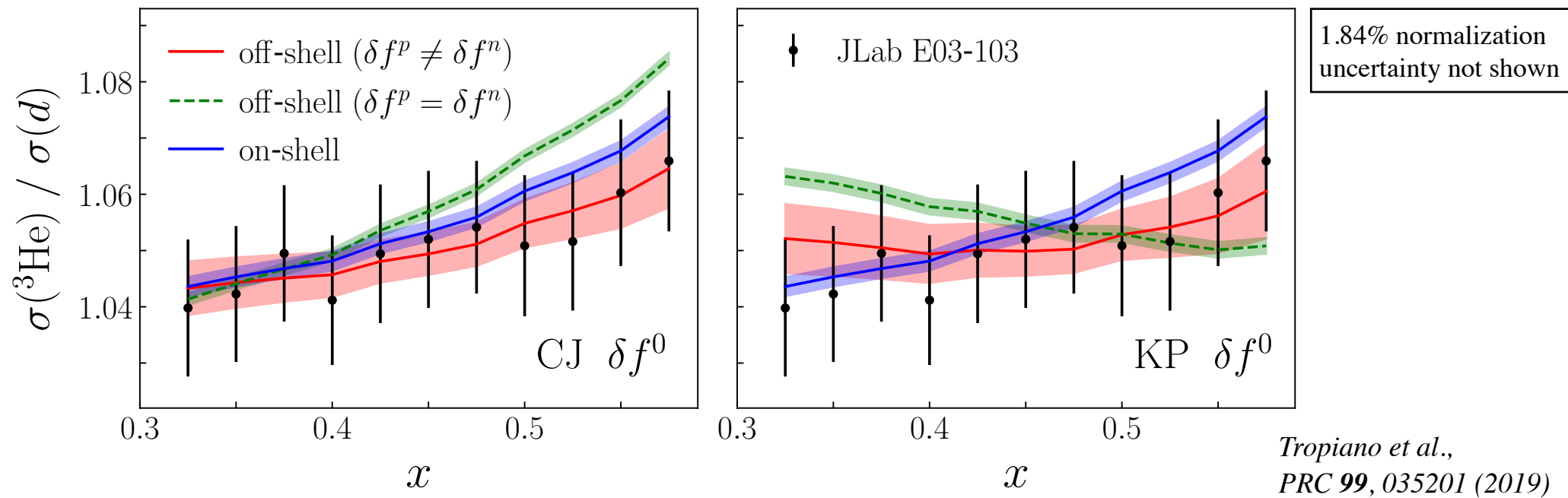
Nucleon off-shell corrections



- some difference in shape for isoscalar off-shell function from KP and CJ analyses
- CJ sensitive only to $\delta f^p + \delta f^n$;
KP assume $\delta f^p = \delta f^n$ for heavier nuclei also

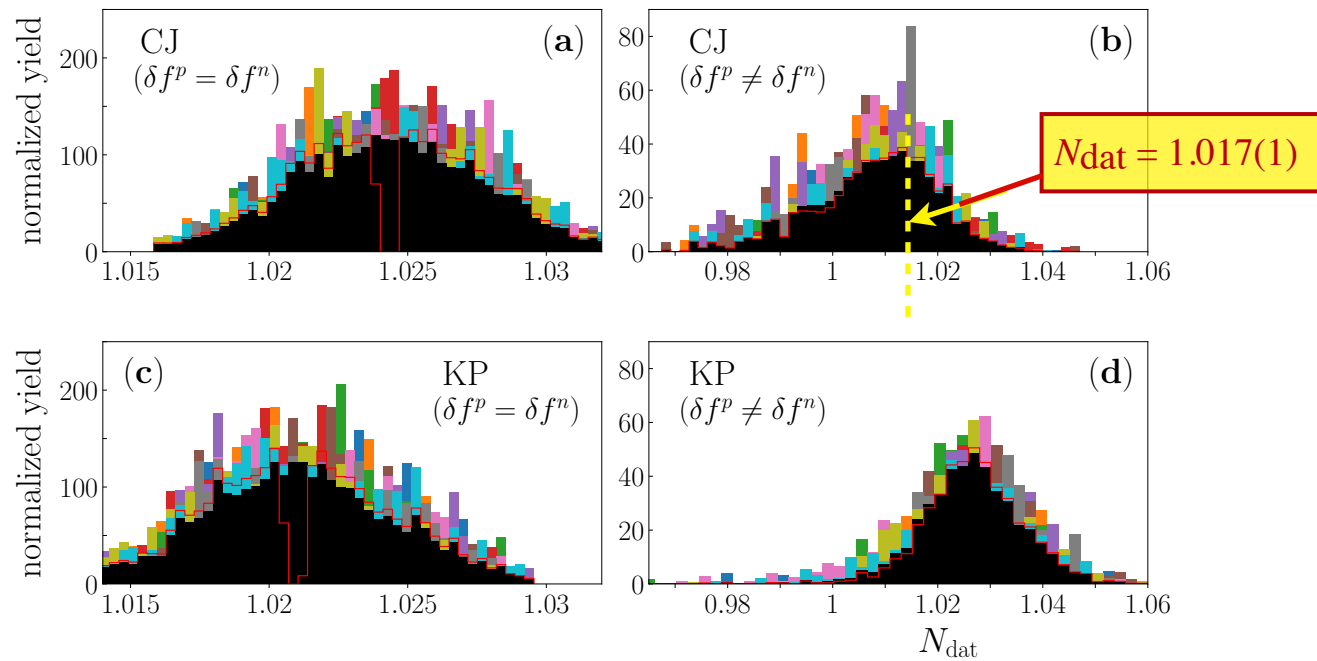
Nucleon off-shell corrections

- JLab E03-103 experiment measured ratios of cross sections for light nuclei, including helium-3 / deuterium



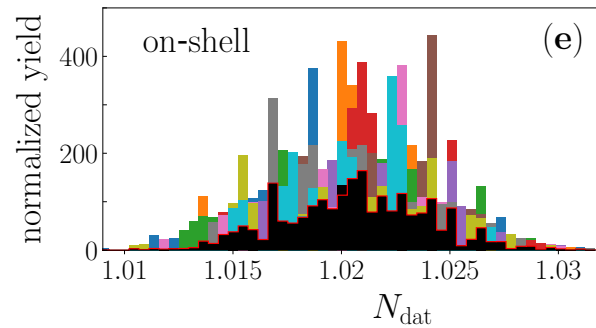
→ since helium-3 is more sensitive to proton than neutron, fit proton off-shell function and extract neutron from

$$\begin{aligned}\delta f^n &= \frac{1}{F_2^n} [(F_2^p + F_2^n)\delta f^0 - F_2^p \delta f^p] \\ &= \delta f^0 - \frac{F_2^p}{F_2^n} (\delta f^p - \delta f^0)\end{aligned}$$



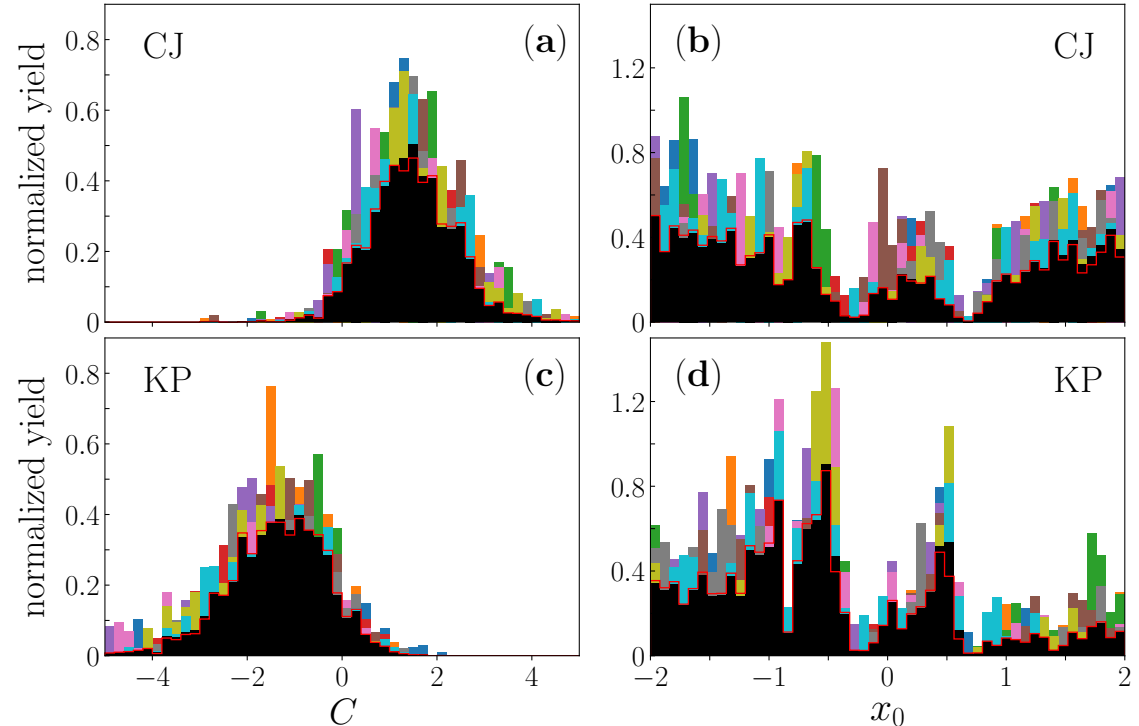
Monte Carlo
parameter
distributions
(using JAM technology)

→ **JAM talk**
Tue. 12:00

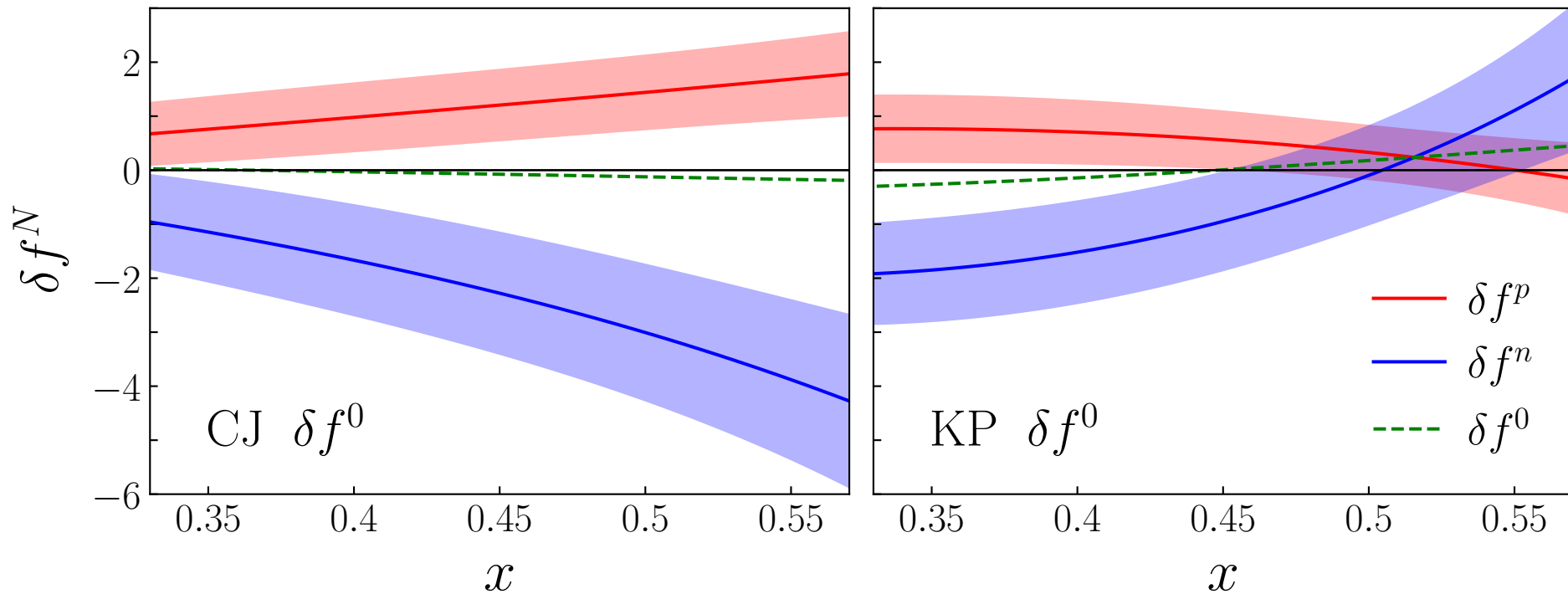


→ MC fits disfavor zero
off-shell correction

— easier for fit to vary one
of the params. than keep
same shape & compensate
by normalization shift

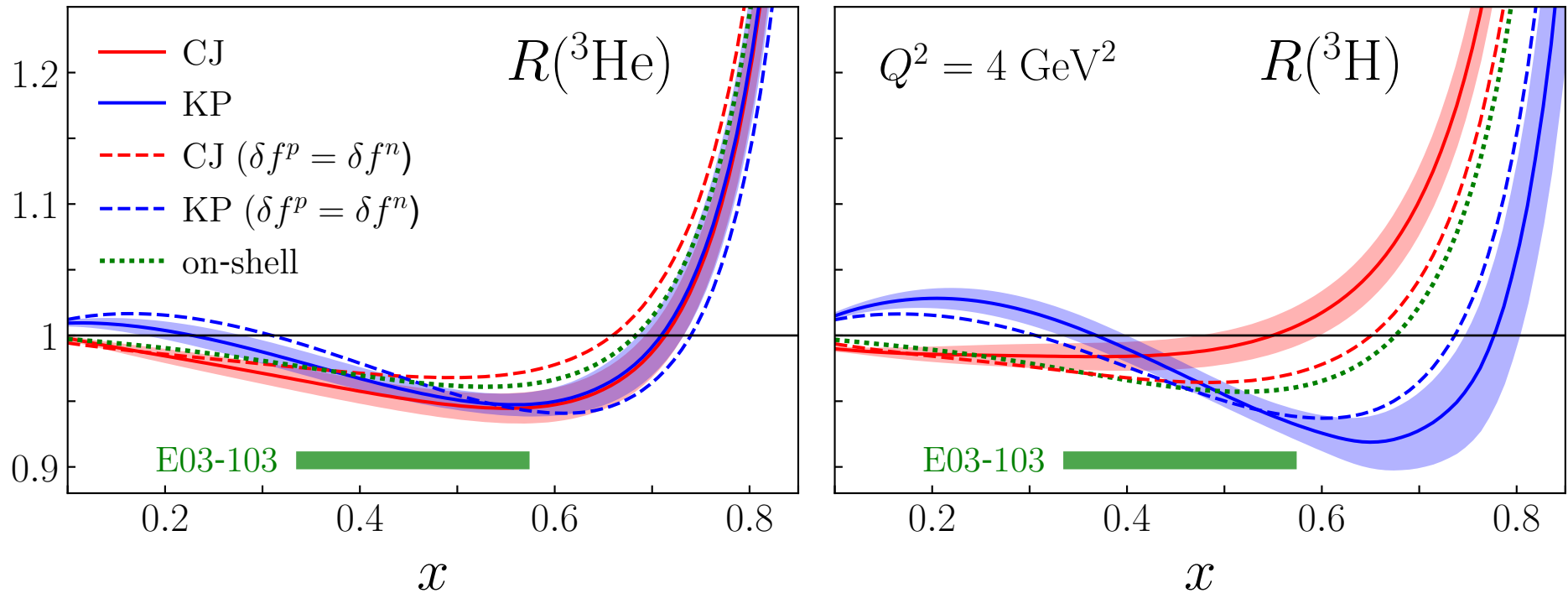


Isospin dependent off-shell functions



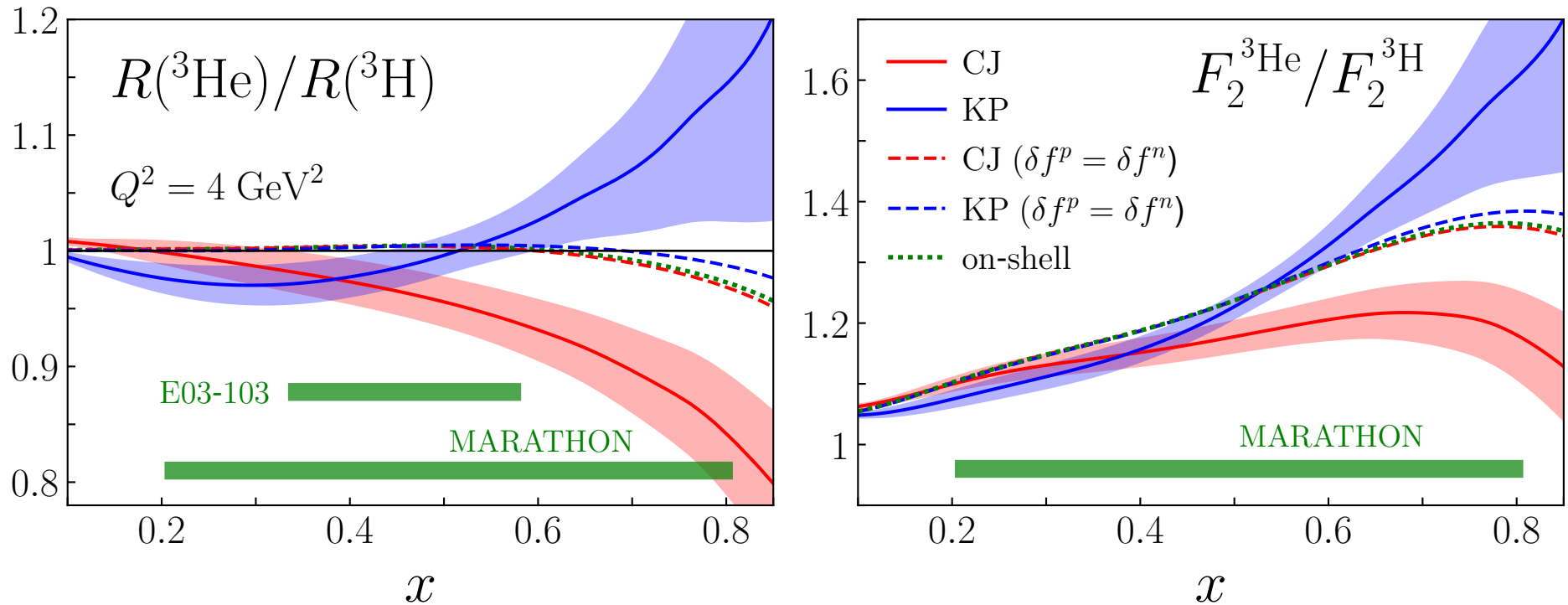
- fits favor large cancellations between proton and neutron off-shell effects: isovector off-shell \gg isoscalar off-shell
- off-shell functions weighted by nucleon virtuality $|v| \ll 1$, where $v^2 = (p^2 - M^2)/M^2$
 - corrections to structure functions range from $\sim 10\%$ for p at $x \sim 0.3$ to $\sim 30\%$ for n at $x \sim 0.6$

Impact on $A=3$ EMC ratios



- limited model variation for helium-3 EMC ratio due to stronger sensitivity of helium-3 to proton structure
- larger variation for tritium EMC ratio due to stronger dependence of tritium on (less well known) neutron structure

Impact on super-ratios



→ potential for sizeable isospin dependent* off-shell effects suggested (not ruled out) by E03-103 data

* note: this is not violation of any isospin/charge symmetry

Synopsis

■ Assumptions made in the analysis:

- E03-103 data & uncertainties are correct as given
- Theoretical WBA framework valid for $A = 2$ & 3
 - total structure function = on-shell part + off-shell part
 - expand off-shell function to lowest order in v
 - same off-shell functions δf^p , δf^n in $A = 2$ & 3
- Isospin symmetry for smearing functions in ${}^3\text{He}$ & ${}^3\text{H}$
- δf^0 off-shell functions from CJ and KP analyses

■ Some of these can be improved:

- should perform combined fit of all p , d and $A=3$ data to self-consistently determine on-shell neutron and off-shell p and n functions under same set of conditions

Strategy for analyzing MARATHON data

- Least model-dependent ways to extract d/u ratio from MARATHON data, without any assumption about super-ratio:
 - with ≥ 3 observables — ${}^3\text{He}/d$, ${}^3\text{H}/d$, d (or d/p) + p — perform global fit at structure function level to extract 3 unknowns — F_2^n , δf^p , δf^n — to be used as input into global QCD analysis (at parton level)

or

- perform global QCD fit directly on all p , d , ${}^3\text{He}$, ${}^3\text{H}$ data to extract PDFs, δf^p , δf^n — planned by CJ, JAM, ... collaborations

Το τέλος.

Ευχαριστώ!