Hadron PDFs in the valence region

A DSE viewpoint

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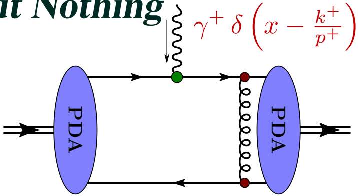
Pion PDF Puzzle – Much Ado About Nothing

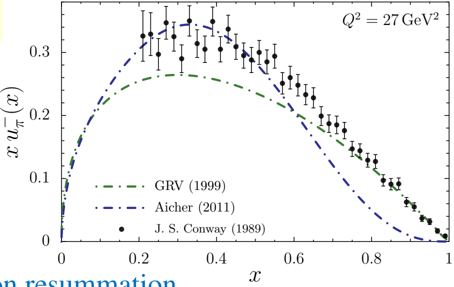


- Longstanding perturbative QCD prediction that pion PDF near $x = 1$ behaves as

$$q_\pi(x) \simeq (1-x)^{2+\gamma_q(\mu^2)}$$

- Since large-x quarks are the source of large-x gluons, near $x = 1$ expect

$$g_\pi(x) \simeq (1-x)^{3+\gamma_g(\mu^2)}$$



- However, pion-induced DY data, and a recent re-analysis that also included leading-neutron data, seem to prefer $q(x) \sim (1-x)^1$ near $x = 1$

- Potential resolution to this "puzzle" provided by Aicher *et al.* $\Longrightarrow$ soft-gluon resummation

- *However, pQCD predictions need only set in very near $x = 1$, the observed $q(x) \simeq (1-x)^1$ behavior could be real where data exists*

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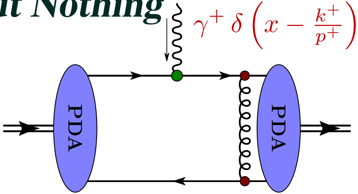

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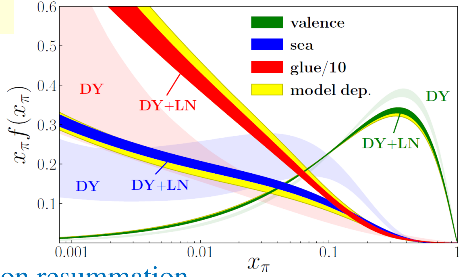
$$g_\pi(x) \simeq (1-x)^{3+\gamma_g(\mu^2)}$$

- However, pion-induced DY data, and a recent re-analysis that also included leading-neutron data, seem to prefer $q(x) \sim (1-x)^1$ near $x = 1$

[Barry, Sato, Melnitchouk, and Ji, Phys. Rev. Lett., 152001 (2018)]



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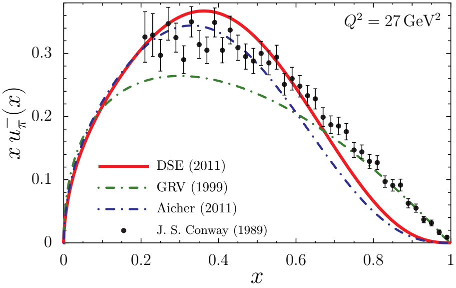
DSEs + Pion PDFs

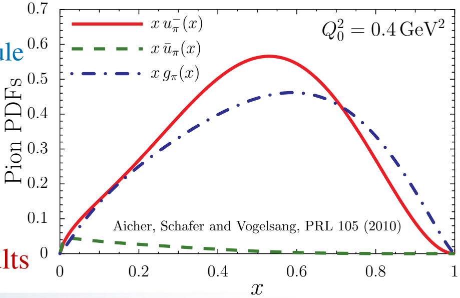
- DSE prediction [Hecht (2001); etc] that $q_\pi(x) \simeq (1-x)^2$ as $x \to 1$
  - related to $1/k^2$ dependence of BSE kernel at large relative momentum



- Previous DSE PDF calculations have used Ward-identity ansatz (WIA)

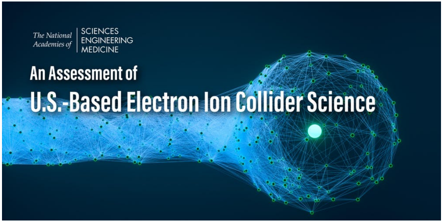
$$\Lambda_q(z,p,n) \to \Lambda_q^{\text{WIA}}(z,p,n) = \delta\,(1-z)\;\, n^\mu\, \frac{\partial}{\partial p^\mu}\, S_q^{-1}(p)$$

  - WIA respects baryon sum rule but not higher moments e.g. momentum sum rule
  - momentum is not distributed correctly between quarks and gluons
- Aicher: $g_\pi(x) \sim (1-x)^{1.3}$ as $x \to 1$
  - pQCD predicts: $g_\pi(x) \sim (1-x)^3$
- Inconsistencies in Aicher & DSE results

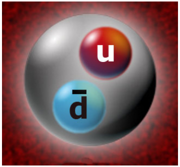


Pion / Kaon + High-priority science questions

- The NAS *Assessment of a U.S. based Electron-Ion Collider* identified three high-priority science questions
  - How does the mass of the nucleon arise?
  - How does the spin of the nucleon arise?
  - What are the emergent properties of dense systems of gluons?



- The pion and kaon can be understood as a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT, but are also Goldstone modes associated with DCSB in QCD



- The dynamical breaking of chiral symmetry (DCSB) in QCD gives rise to $\sim 500\,$MeV mass splittings in hadron spectrum & massless Goldstone bosons in chiral limit (π, K, η)
- Therefore, understanding the nucleon mass is not sufficient
  - must also understand the mass of the pion ($u\bar{d}, \ldots$) and kaon ($u\bar{s}, \ldots$)

Hadron Masses in QCD

- Quark/gluon contributions to masses (& angular momentum) are accessed via matrix elements of QCD's (symmetric) energy-momentum tensor

$$T^{\mu\nu} = T^{\nu\mu}, \qquad \partial_\mu T^{\mu\nu} = \partial_\mu T_q^{\mu\nu} + \partial_\mu T_g^{\mu\nu} = 0, \qquad T^{\mu\nu} = \underset{\text{[traceless]}}{\overline{T}^{\mu\nu}} + \underset{\text{[trace]}}{\widehat{T}^{\mu\nu}}$$

- The trace piece of $T^{\mu\nu}$ takes the form (un-renormalized)

$$T^\mu_\mu = \sum_{q=u,d,s} \underbrace{m_q\,(1+\gamma_m)\,\overline{\psi}_q\psi_q}_{\text{quark mass term}} + \underbrace{\frac{\tilde{\beta}(g)}{2\,g}\,F^{\mu\nu,a}F^a_{\mu\nu}}_{\text{trace anomaly}}$$

- At zero momentum transfer

$$\langle p\,|T^{\mu\nu}|\,p\rangle = 2\,p^\mu p^\nu \quad \Longrightarrow \quad \left\langle p\,\middle|T^\mu_\mu\middle|\,p\right\rangle = 2\,m^2$$

  - in chiral limit entire hadron mass from gluons!
  - e.g., Dmitri Kharzeev – Proton Mass workshops at Temple University and ECT*

- Understanding difference in pion/kaon and proton is key to hadron masses:

$$\left\langle \pi\,\middle|T^\mu_\mu\middle|\,\pi\right\rangle = 2\,m_\pi^2 \overset{\text{chiral limit}}{\to} 0, \qquad \left\langle N\,\middle|T^\mu_\mu\middle|\,N\right\rangle = 2\,m_N^2$$

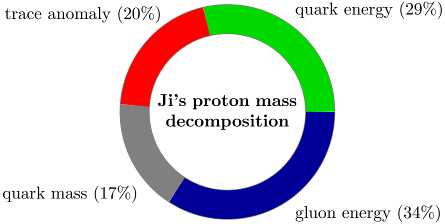
Rest Frame Hadron Mass Decompositions

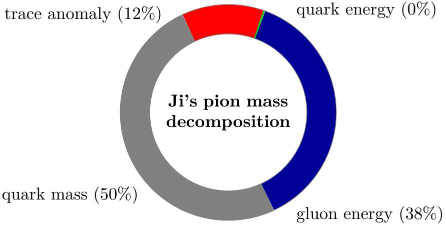
- Xiangdong Ji proposed hadron mass decomposition [PRL **74**, 1071 (1995); PRD **52**, 271 (1995)]

$$m_p = \frac{\langle p \,|\int d^3x\, T^{00}(0,\vec{x})|\, p\rangle}{\langle p|p\rangle}\Bigg|_{\text{at rest}} = \underbrace{M_q \;+\; M_g}_{\text{quark and gluon energies}} + \underbrace{M_m}_{\text{quark mass}} + \underbrace{M_a}_{\text{trace anomaly}}$$

$$M_q = \tfrac{3}{4}\,(a - b)\, m_p, \quad M_g = \tfrac{3}{4}\,(1 - a)\, m_p, \quad M_m = b\, m_p, \quad M_a = \tfrac{1}{4}\,(1 - b)\, m_p,$$

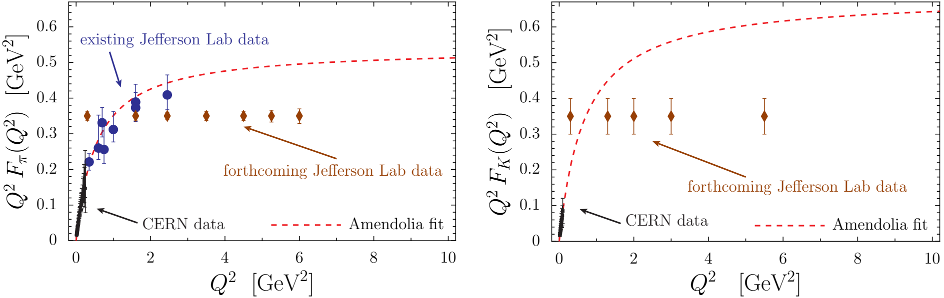
  - a = quark momentum fraction, b related to sigma-term or anomaly contribution
  - [See Cédric Lorcé, EPJC **78**, (2018) for decomposition with pressure effects]





- In chiral limit ($m_q \to 0$) pion has no rest frame ($m_\pi = 0$) – how to interpret Ji's pion mass decomposition? Limit as $m_q \to 0$ is likely well behaved.

What we know about the Pion and Kaon



- Pion and kaon structure is slowly being revealed using: π^-/K^- beams at CERN; Sullivan type experiments at Jefferson Lab; π^- beam at Fermilab; and $e^+e^- \to \pi^+\pi^-, \ K^+K^-$ in the time-like region
- 40 years of experiments has revealed, e.g.
  - $r_{\pi^+} = 0.672 \pm 0.008,$ $r_{K^+} = 0.560 \pm 0.031,$ $r_{K^0} = -0.277 \pm 0.018$
- Still a lot more to learn about pion and kaon structure:
  - quark and gluon PDFs; TMDs including Boer-Mulders function; $q, g \to \pi/K$ fragmentation functions, quark and gluon GPDs; gravitational form factors

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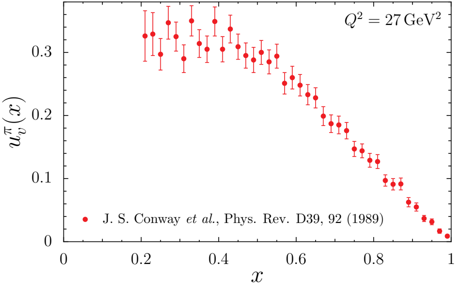

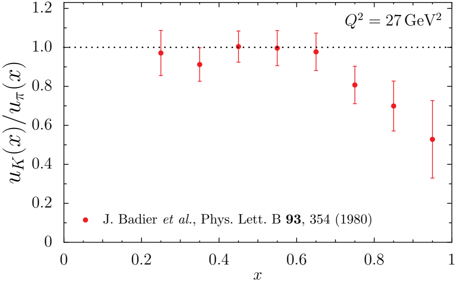

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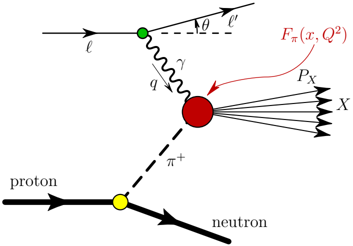
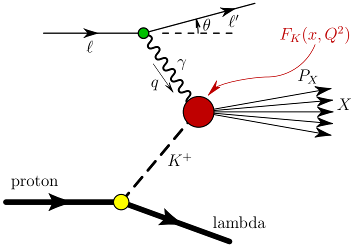
Pion & Kaon Structure at JLab and an EIC

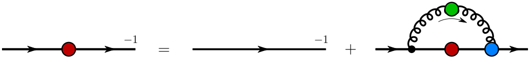




- At Jefferson Lab and an EIC pion and kaon structure can be accessed via the so-called *Sullivan* processes
  - initial pion/kaon is off mass-shell – need extrapolation to pole
  - existing results for form factors – what about quark and gluon PDFs, TMDs, GPDs, *etc*, at an EIC?
- Explored this ideal at a series of workshops on "*Pion and Kaon Structure at an Electron–Ion Collider*" (PIEIC)
  - 1−2 June 2017, Argonne National Laboratory www.phy.anl.gov/theory/pieic2017/
  - 24−25 May 2018, The Catholic University of America www.jlab.org/conferences/pieic18/
- Drell-Yan also very nice way to measure pion/kaon structure

QCD's Dyson-Schwinger Equations

- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator

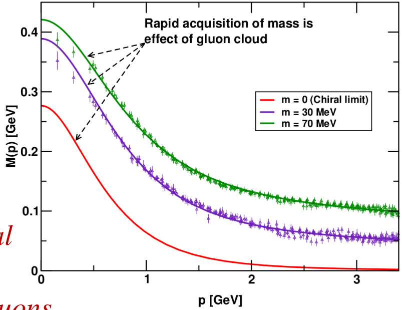


  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

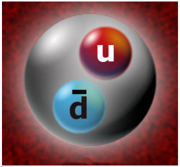
$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

- Mass function, $M(p^2)$, exhibits dynamical mass generation, even in chiral limit
  - *mass function is gauge dependent and therefore NOT an observable!*
- *Hadron masses are generated by dynamical chiral symmetry breaking – caused by a cloud of gluons dressing the quarks and gluons*

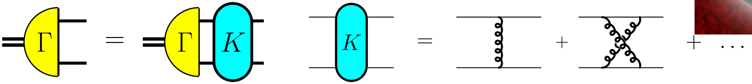
[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



Calculating and Predicting Pion Structure



- In QFT a two-body bound state (e.g., a pion, kaon, etc) is described by the Bethe-Salpeter equation (BSE):

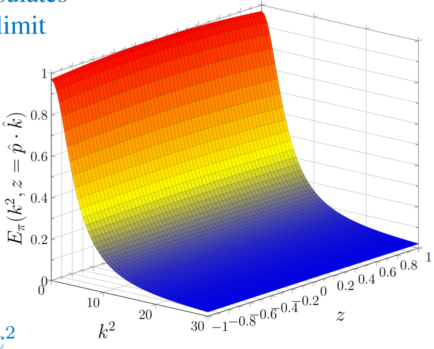


  - the kernel must yield a solution that encapsulates the consequences of DCSB, e.g., in chiral limit $m_\pi = 0$ & $m_\pi^2 \propto m_u + m_d$

- Pion Bethe-Salpeter vertex

$$\Gamma_\pi(p,k) = \gamma_5\Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, G_\pi(p,k) + i\sigma^{\mu\nu}k_\mu p_\nu\, H_\pi(p,k)\Big]$$

  - $\chi_{\rm BSE} = S(k+\frac{1}{2}p)\,\Gamma_\pi(p,k)\,S(k-\frac{1}{2}p)$
  - large relative momentum: $E_\pi \sim F_\pi \sim 1/k^2$



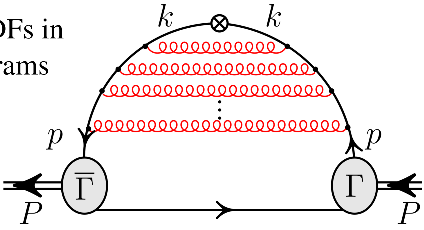
- Challenging to go beyond rainbow-ladder trunction and maintain symmetries

Pion & Kaon PDFs

Pion PDFs – Self-Consistent DSE Calculations

- To self-consistently determine hadron PDFs in rainbow-ladder must sum all planar diagrams

$$q(x) \propto \mathrm{Tr} \int \frac{d^4p}{(2\pi)^4}\, \overline{\Gamma}_M(p,P)\, S(p) \times \Gamma_q(x,p,n)\, S(p)\, \Gamma_M(p,P)\, S(p-P)$$



- DSEs are formulated in Euclidean space – evaluate $q(x)$ by taking moments
- The *hadron dependent* vertex $\Gamma_q(x,p,n)$ satisfies an inhomogeneous BSE
- However can define a *hadron independent* vertex $\Lambda_q(x,p,n)$

$$\Gamma_q(x,p,n) = \iint dy\, dz\, \delta(x-yz)\, \delta\left(y - \frac{p\cdot n}{P\cdot n}\right) \Lambda_q(z,p,n)$$

- $\Lambda_q(x,p,n)$ satisfies the inhomogeneous BSE

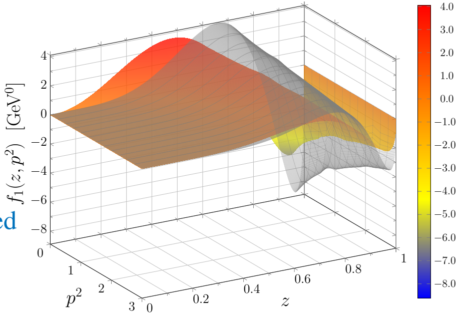
$$\Lambda_q(z,p,n) = iZ_2\, \not{n}\, \delta(1-z) - \iint du\, dw\, \delta(z-uw) \int \frac{d^4\ell}{(2\pi)^4} \delta\left(w - \frac{\ell\cdot n}{p\cdot n}\right) \times \gamma_\mu\, \mathcal{K}_{\mu\nu}(p-\ell)\, S(\ell)\, \Lambda_q(u,\ell,n)\, S(\ell)\, \gamma_\nu$$

PDFs of a Dressed Quark

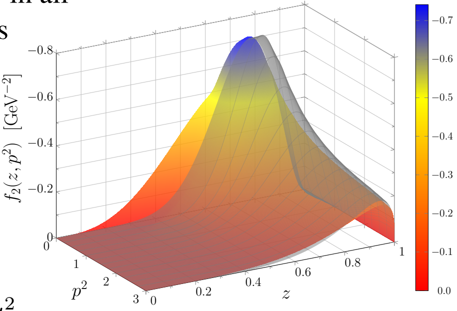
- *Hadron independent* vertex has form

$$\Lambda_q(z,p,n) = i\not{n}\,\delta(1-z) + i\not{n}\,f_1^q(z,p^2) + n\cdot p\left[i\not{p}\,f_2^q(z,p^2) + f_3^q(z,p^2)\right]$$

  - the functions $f_i^q(z,p^2)$ can be interpreted as unpolarized PDFs in a dressed quark of virtuality p^2



- These functions are universal – appear in all RL-DSE unpolarzied PDF calculations



- Distributed support in z is immediate indication gluons carry significant momentum
  - heavier s quark support nearer $z = 1$
  - WIA $\Longrightarrow$ $\Lambda_q(z,p,n) \propto \delta(1-z)$
- Renormalization condition means dressing functions vanish when $p^2 = \mu^2$

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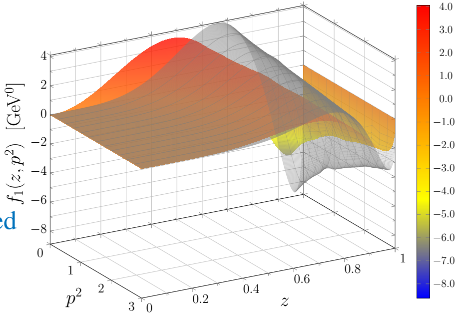

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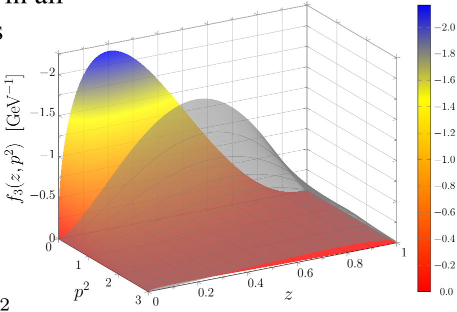

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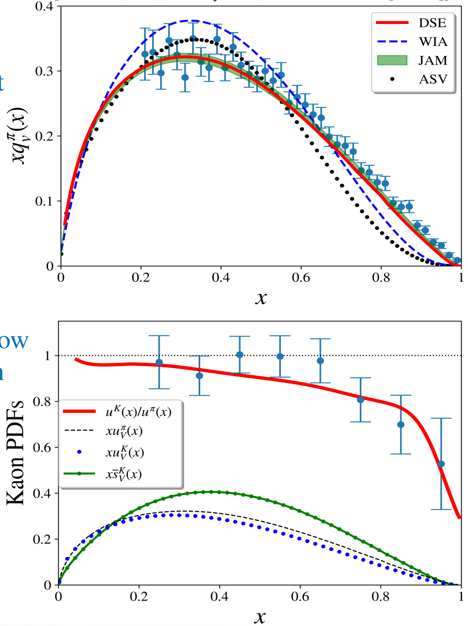
Self-Consistent DSE Results

- For pion and kaon PDFs included for first time gluons self-consistently
  - correct RL-DSE pion PDFs in excellent agreement with Conway *et al.* data and recent JAM analysis
  - agrees with $x \to 1$ pQCD prediction
- Treating non-perturbative gluon contributions correctly pushes support of $q_\pi(x)$ to larger x
  - gluons remove strength from $q_\pi(x)$ at low to intermediate x – baryon number then demands increased support at large x
  - cannot be replicated by DGLAP – DSE splitting functions are dressed
- *Immediate consequence of gluon dressing is that gluons carry 35% of pion's and 30% of kaon's momentum*

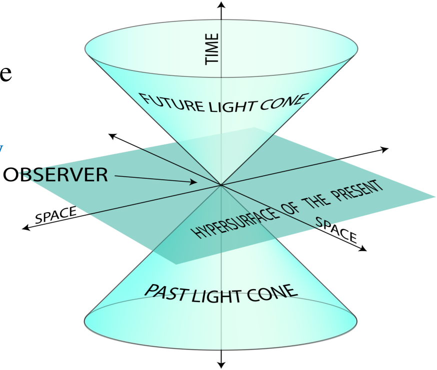
[K. Bednar, ICC, P. Tandy, *et al.*, arXiv:1811.12310 [nucl-th]]



Pion & Kaon
LF Wave Functions

Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame-dependent concept
  - boost operators are dynamical, that is, they are interaction dependent
- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$



- Light-front quantization $\Longrightarrow$ light-front wave functions, which have some interesting properties
  - frame dependence is trivial, and yield a probability interpretation
  - boosts are kinematical – *not dynamical*
- BSE wave function $\Longrightarrow$ light-front wave functions (LFWFs) $[\psi(x, \boldsymbol{k}_T)]$
  $\Longrightarrow$ parton distribution amplitudes (PDAs) $[\varphi(x)]$

$$\psi(x, \boldsymbol{k}_T) = \int dk^- \, \chi_{\text{BSE}}(p, k), \qquad \varphi(x) = \int d^2\boldsymbol{k}_T \, \psi(x, \boldsymbol{k}_T)$$

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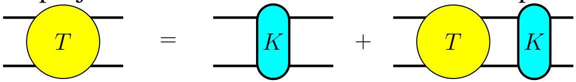
DSEs + Light-Front Wave Functions

- On light-front hadron states can be represented by a Fock-state expansion

$$|\pi^+\rangle = |u\bar{d}\,\rangle + |u\bar{d}\,g\rangle + |u\bar{d}\,gg\rangle + \ldots + |u\bar{d}\,q\bar{q}\rangle + |u\bar{d}\,q\bar{q}\,g\rangle + \ldots$$

- Associated with each Fock-state is a number of LFWFs
  - diagonalizing the light-cone QCD Hamiltonian operator $\Longrightarrow$ LFWFs
  - *methods include*: discretized lightcone quantization, basis light-front quantization, and holographic QCD
- LFWFs can be projected from solutions to the Bethe-Salpeter equation



  - BSE self-consistently sums an infinite number of Fock states
  - in rainbow-ladder, e.g, $|\pi^+\rangle = |u\bar{d}\,\rangle + |u\bar{d}\,g\rangle + |u\bar{d}\,gg\rangle + \ldots$
- Obtaining LFWFs from DSE solutions of the BSE has several key features
  - in the DSEs emergent pheonmena, such as confinement and DCSB, arise through the infinite sum of diagrams
  - these effects are encoded in DSE dressed propagators and BS amplitudes, and therefore the projected LFWFs

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$$\left|\pi^+\right\rangle = \left|u\bar{d}\right\rangle + \left|u\bar{d}\,g\right\rangle + \left|u\bar{d}\,gg\right\rangle + \ldots + \left|u\bar{d}\,q\bar{q}\right\rangle + \left|u\bar{d}\,q\bar{q}\,g\right\rangle + \ldots$$

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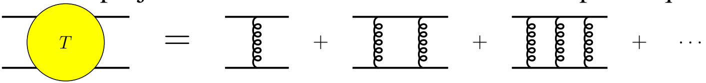


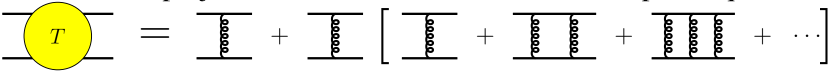
  - BSE self-consistently sums an infinite number of Fock states
  - in rainbow-ladder, e.g, $\left|\pi^+\right\rangle = \left|u\bar{d}\right\rangle + \left|u\bar{d}\,g\right\rangle + \left|u\bar{d}\,gg\right\rangle + \ldots$
- Obtaining LFWFs from DSE solutions of the BSE has several key features
  - in the DSEs emergent pheonmena, such as confinement and DCSB, arise through the infinite sum of diagrams
  - these effects are encoded in DSE dressed propagators and BS amplitudes, and therefore the projected LFWFs

DSEs + Light-Front Wave Functions

- On light-front hadron states can be represented by a Fock-state expansion

$$|\pi^+\rangle = |u\bar{d}\,\rangle + |u\bar{d}\,g\rangle + |u\bar{d}\,gg\rangle + \ldots + |u\bar{d}\,q\bar{q}\rangle + |u\bar{d}\,q\bar{q}\,g\rangle + \ldots$$

- Associated with each Fock-state is a number of LFWFs
  - diagonalizing the light-cone QCD Hamiltonian operator $\Longrightarrow$ LFWFs
  - *methods include*: discretized lightcone quantization, basis light-front quantization, and holographic QCD
- LFWFs can be projected from solutions to the Bethe-Salpeter equation



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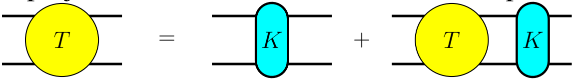

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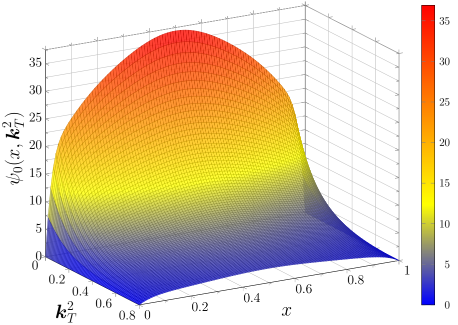
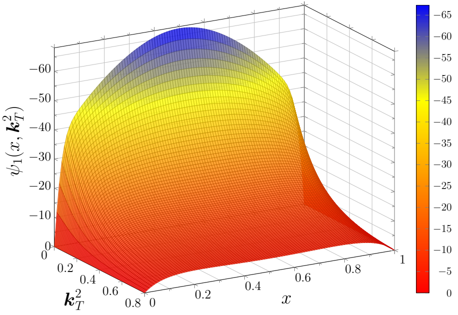
Pion and Kaon LFWFs

[Chao Shi and ICC, Phys. Rev. Lett. **122**, no. 8, 082301 (2019)]





- Pion has two leading Fock-state LFWFs: $\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)$ & $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)$

$$\psi_0(x,\boldsymbol{k}_T^2)=\sqrt{3}\,i\int \frac{dk^+ dk^-}{2\,\pi}\mathrm{Tr}_D\big[\gamma^+\gamma_5\,\chi(k,p)\big]\delta\big(k^+ - x\,p^+\big); \qquad \psi_1(x,\boldsymbol{k}_T^2)=\ldots$$

- DSE result finds broad (almost) concave functions at hadronic scales, with features at small $\boldsymbol{k}_T^2$ driven by DCSB
  - large $\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)$ indicates significant orbital angular momentum and relativistic effects in pion and kaon
  - at large $\boldsymbol{k}_T^2$ find same power-law behavior as predicted by perturbative QCD

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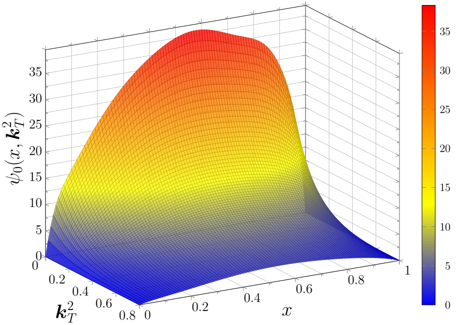

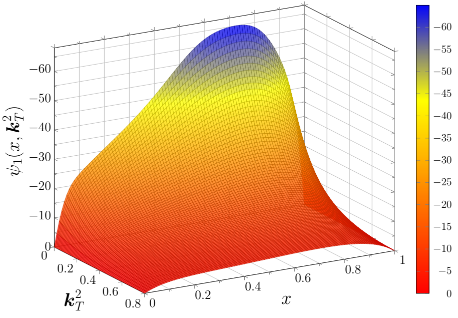

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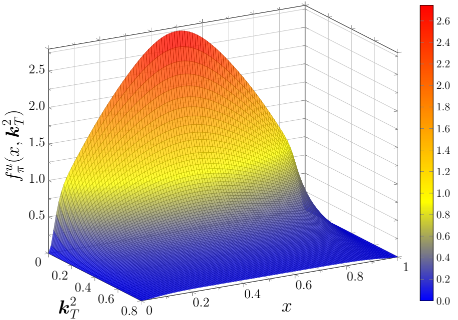
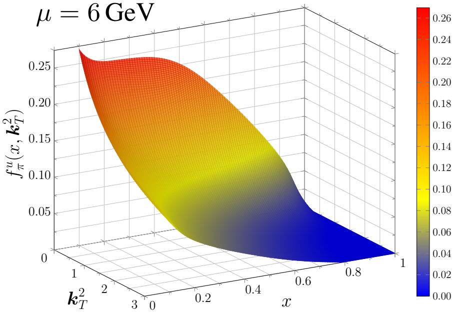
Pion's T-even TMD

[Chao Shi and ICC, Phys. Rev. Lett. **122**, no. 8, 082301 (2019)]



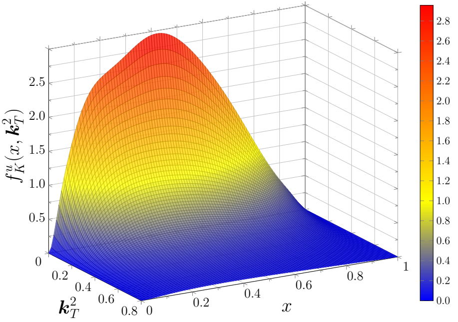


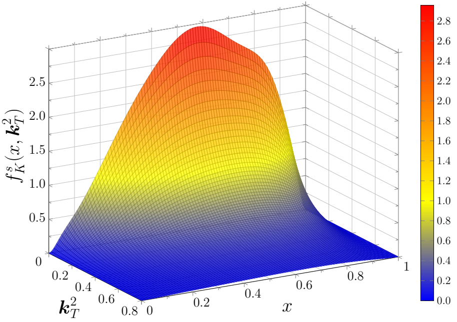
- Using pion's LFWFs straightforward to make predictions for pion TMDs

$$f(x, \boldsymbol{k}_T^2) \propto \left|\psi_{\uparrow\downarrow}(x, \boldsymbol{k}_T^2)\right|^2 + \boldsymbol{k}_T^2 \left|\psi_{\uparrow\uparrow}(x, \boldsymbol{k}_T^2)\right|^2$$

  - numerous features inherited from LFWFs: TMDs are broad functions as a result of DCSB and peak at zero relative momentum ($x = 1/2$)
  - evolution from model scale ($\mu = 0.52\,\mathrm{GeV}$) to $\mu = 6\,\mathrm{GeV}$ results in significant broadening in $\langle \boldsymbol{k}_T^2 \rangle$, from $0.16\,\mathrm{GeV}^2$ to $0.69\,\mathrm{GeV}^2$
- Need careful treatment of gauge link to study pion Boer-Mulders function

Kaon's T-even TMD





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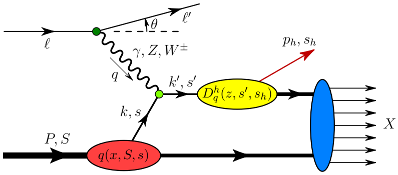
  - numerous features inherited from LFWFs
  - TMDs are broad functions as a result of DCSB and with significant flavor breaking effects
- TMDs satisfy: $f_K^s(x, \boldsymbol{k}_T^2) = f_K^u(1-x, \boldsymbol{k}_T^2)$; $f(x, \boldsymbol{k}_T^2) \to x^2(1-x)^2/\boldsymbol{k}_T^4$
- In general both pion and kaon LFWFs do not factorize in x and $\boldsymbol{k}_T^2$

Probing Transverse Momentum

leading twist		quark polarization		
		unpolarized [U]	longitudinal [L]	transverse [T]
nucleon polarization	U	$f_1 =$ unpolarized		$h_1^\perp =$ − Boer-Mulders
	L		$g_1 =$ − helicity	$h_{1L}^\perp =$ − worm gear 1
	T	$f_{1T}^\perp =$ − Sivers	$g_{1T}^\perp =$ − worm gear 2	$h_1 =$ − transversity $h_{1T}^\perp =$ − pretzelosity



- Measuring the pion/kaon TMDs will be a challenge, however progress can be made now by studing the $q \to \pi/K$ TMD fragmentation functions
- Fragmentation functions are particularly important and interesting
  - *potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark or gluon becomes a tower of hadrons*
- Also interesting tool with which to probe color entanglement at an EIC
  - over what length scales can colored correlations be observed?

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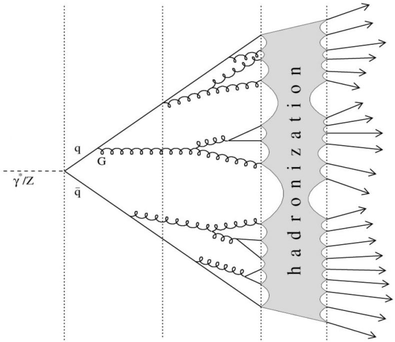

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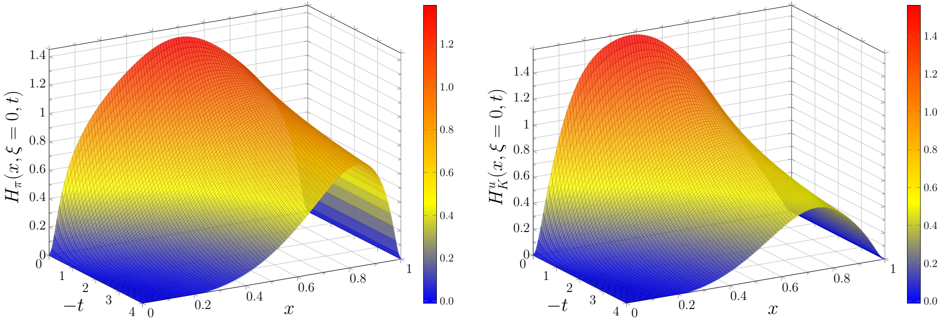
Pion and Kaon GPDs

[Chao Shi and ICC, forthcoming publication]



- Straightforward to make predictions for pion and kaon GPDs from overlaps of LFWFs – only one type of GPD at leading twist

$$H_\pi(x,0,t) = \int d\boldsymbol{k}_T \left[\psi_0(x,\hat{\boldsymbol{k}}_T)\psi_0(x,\boldsymbol{k}_T) + (\hat{k}_1 + i\hat{k}_2)(k_1 - ik_2)\,\psi_1(x,\hat{\boldsymbol{k}}_T)\psi_1(x,\boldsymbol{k}_T)\right]$$

  - access to DGLAP region $[x > \xi]$ only with leading Fock state
  - impossible to self-consistently respect polynomiality with truncated Fock space
- Our Fock-state expansion is in terms of dressed quarks and gluons
  - as momentum transfer t increases dressing of quarks and gluons stripped away

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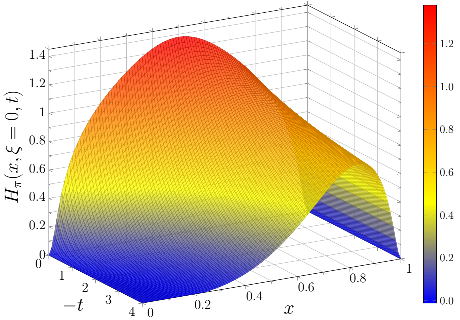

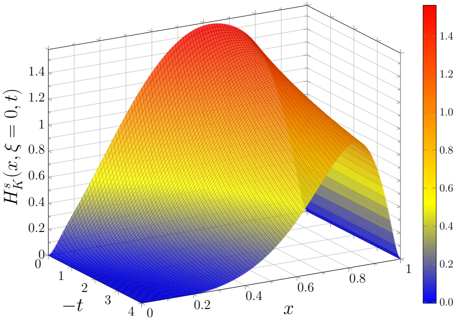

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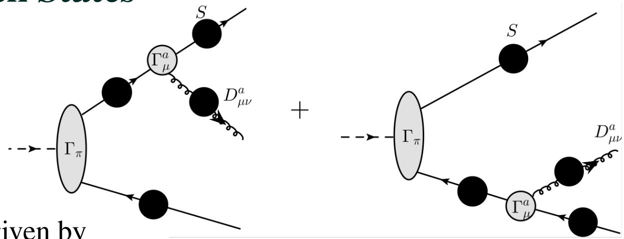
DSEs + Higher Fock States

- From existing DSE ingredients can project out higher Fock states



- For example, the $|q\bar{q}\,g\rangle$ Fock state is given by

$$\psi_{\lambda_1\lambda_2\lambda_3}(x_1,x_2,\boldsymbol{k}_{1T},\boldsymbol{k}_{2T})\sim\int\frac{dk_1^- \, dk_2^-}{(2\pi)^2}\;\bar{u}(x_1P^+,\boldsymbol{k}_{1T},\lambda_1)\gamma^+\chi^\mu(k_1,k_2;P)\gamma^+v(x_2P^+,\boldsymbol{k}_{2T},\lambda_2)\,\varepsilon^*_\mu(\lambda_3)$$

  - for a pion there are nine 5-dimensional LFWFs associated with $|q\bar{q}\,g\rangle$ Fock state

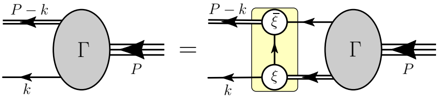
- Key question: *When is a leading Fock-state approximation reliable?*
  - leading Fock state dominates at (very) large x and (very) large Q^2
  - can generate numerous higher Fock states using, e.g., DGLAP evolution – however non-perturbative content is missing

- Increasing difficult to calculate these higher Fock-state LFWFs and their impact on observables – *need to use full BSE solutions*

Nucleon PDFs

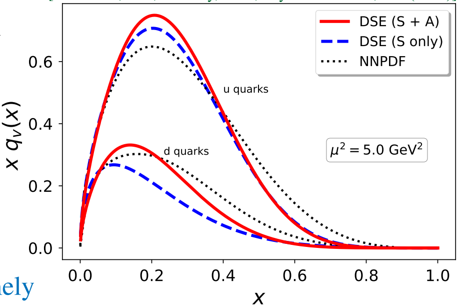
Spin-Independent PDFs

- Solve Poincaré covariant Faddeev eqn for nucleon bound state:

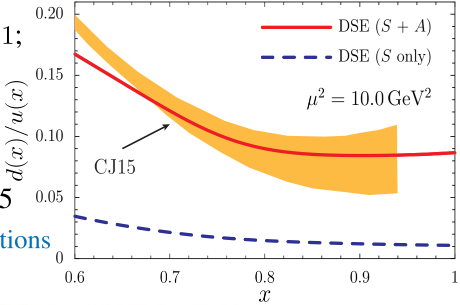


  - key approximation is that the nucleon consists of quark + dynamical diquark
  - approximation is known to work extremely well, e.g., masses, form factors, etc

[K. Bednar, ICC, P. Tandy, *et al.*, Phys. Lett. B **782**, 675 (2018)]



- QCD predicts $q(x) \sim (1-x)^3$ as $x \to 1$; our result is $q(x) \sim (1-x)^5$
  - quark-diquark approximation breaks down at (very) large x
- Find d/u in good agreement with CJ15
  - ratio is very sensitive to diquark correlations
- *At what x does $q + qq$ break down?*



Conclusions

- For pion and kaon PDFs included for first time gluons self-consistently
  - correct RL-DSE pion PDFs in excellent argeement with Conway *et al.* data and recent JAM analysis
  - agrees with $x \to 1$ pQCD prediction
- Using DSE solutions to the BSE we determined the leading Fock-state LFWFs for the pion and kaon
  - using these LFWFs straightforward to determine FFs, PDFs, TMDs, GPDs, etc
  - key advantage of DSE method is BSE sums an infinite number of Fock states $\Longrightarrow$ LFWFs encapsulate effects from emergent phenomena: confinement & DCSB
- Much work remains in experiment and theory to understand the pion and kaon

[K. Bednar, ICC, P. Tandy, arXiv:1811.12310 [nucl-th]]