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IR singularities, factorization and effective field theory

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Precision Physics at the LHC

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Outline

- structure of infrared singularities of four-loop amplitudes TB, Neubert, 1908.11379

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- Infrared singularities and low-energy effective field theory
    - Factorization constraints
    - Renormalization and resummation of large logarithms in cross sections
- structure of infrared singularities of four-loop amplitudes TB, Neubert, 1908.11379

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- Infrared singularities and low-energy effective field theory
  - Factorization constraints
  - Renormalization and resummation of large logarithms in cross sections
- structure of infrared singularities of four-loop amplitudes TB, Neubert, 1908.11379
- Application: resummation at N^{3}LL
  - Event shapes, transverse momentum spectra, …

Infrared singularities

Scattering amplitudes in theories with massless particles, such as QED or QCD suffer from infrared divergences.

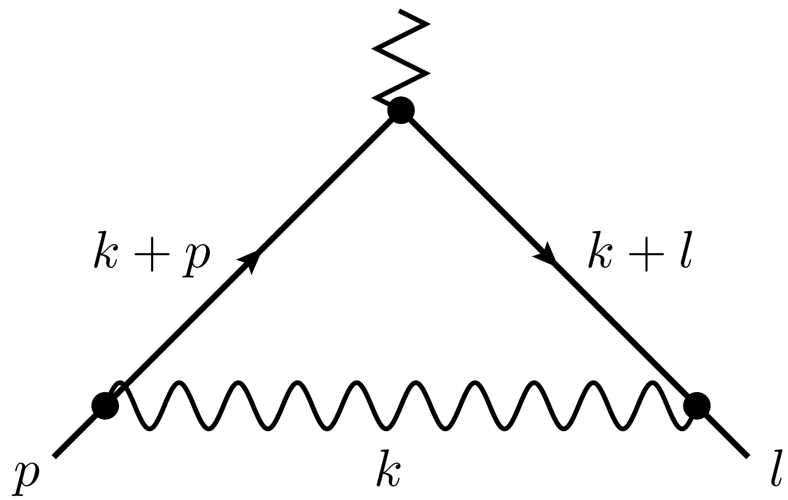
Bloch, Nordsieck 1937
Kinoshita 1962; Lee, Nauenberg 1964

A nuisance for cross section calculations.

- Regularize scattering amplitudes and phase-space integrals.
- Isolate and cancel divergences before obtaining numerical predictions.

→ talks by Sandro Uccirati and Zoltan Trocsanyi

Example: form factor integral



$$p^2 = l^2 = m^2 = 0$$

$$Q^2 = (p-l)^2$$

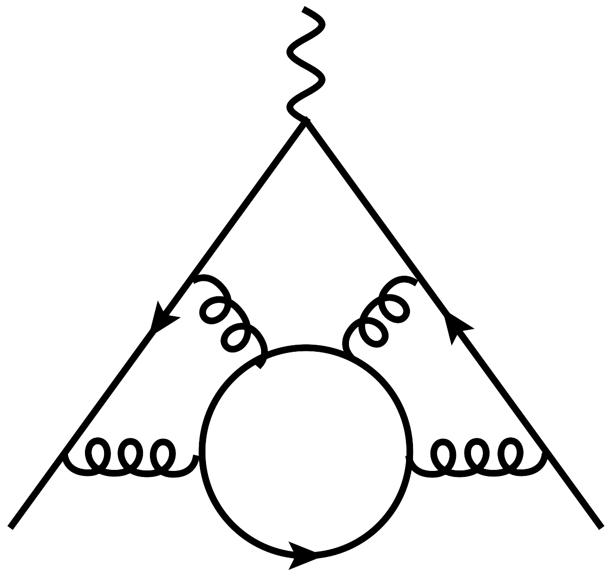
$$\boldsymbol{T}^a \boldsymbol{T}^a = C_F$$

$$F(Q^2) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \frac{\pi^2}{6} - 8 + \mathcal{O}(\varepsilon) \right) \left(\frac{Q^2}{\mu^2} \right)^{-\varepsilon}$$

Use dimensional regularization d=4-2ε

- Two divergent integrations: energy and angle. Soft and collinear divergences.
- Massive case: only single, soft divergence.

Form factor at 4 loops



Interesting color structures!

$$d_R^{a_1 \ldots a_n} = \mathrm{Tr}_R\big(\boldsymbol{T}^{a_1} \ldots \boldsymbol{T}^{a_n}\big)_+$$

symm.

$\rightarrow$ higher Casimir invariants.

Two powers of 1/ε per loop. At four loops

$$\Delta F(Q^2) = \left(\frac{\alpha_s(\mu)}{4\pi}\right)^4 \left[\frac{c_8}{\epsilon^8} + \frac{c_7}{\epsilon^7} + \ldots \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon} + c_0\right] \left(\frac{Q^2}{\mu^2}\right)^{4\epsilon}$$

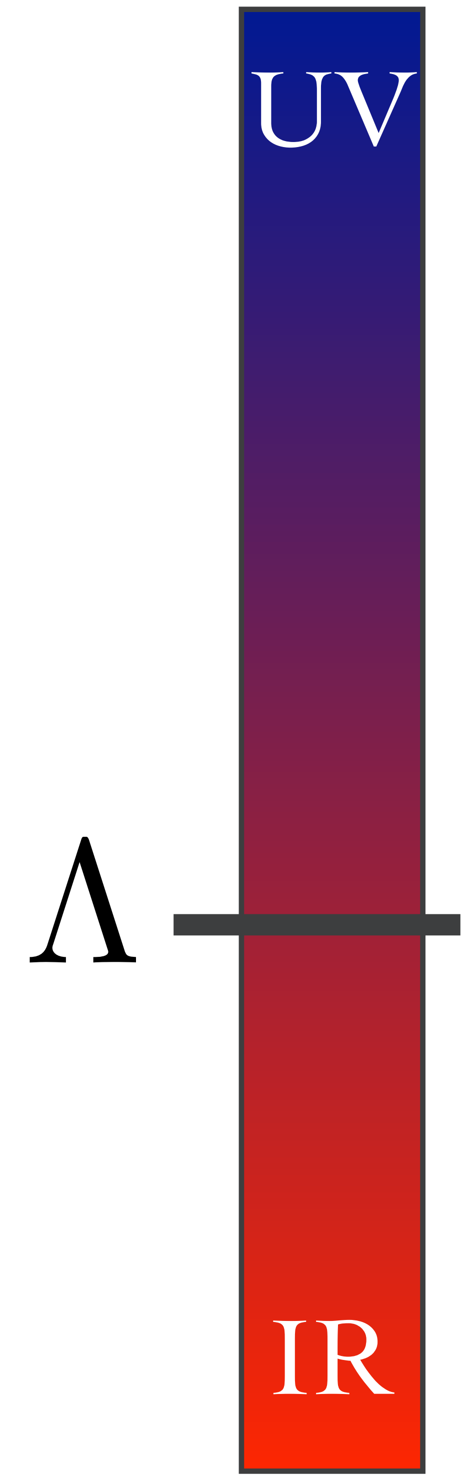
The analytical calculation of the coefficient c_2 of the 1/ε^2 pole ("cusp anomalous dimension") was finished this week: Henn, Korchemsky and Mistlberger 1911.10174. Numerical result Moch, Ruijl, Ueda, Vermaseren and Vogt '18 and many color structures were known earlier.

Misconception

Conventional thinking is that UV and IR divergences are of totally different nature:

- UV divergences are absorbed into renormalization of parameters of theory; structure constrained by RG equations
- IR divergences arise in unphysical calculations; cancel between virtual corrections and real emissions

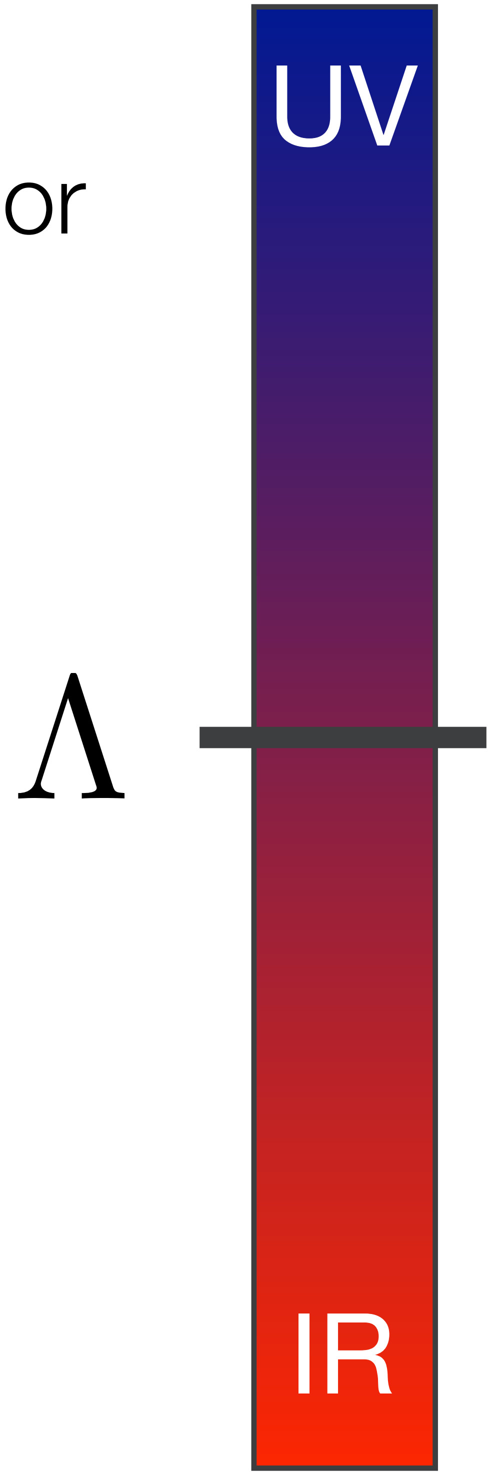
In fact, IR divergences can be mapped onto UV divergences of operators in effective field theory!



High-energy perspective: Λ is infrared regulator

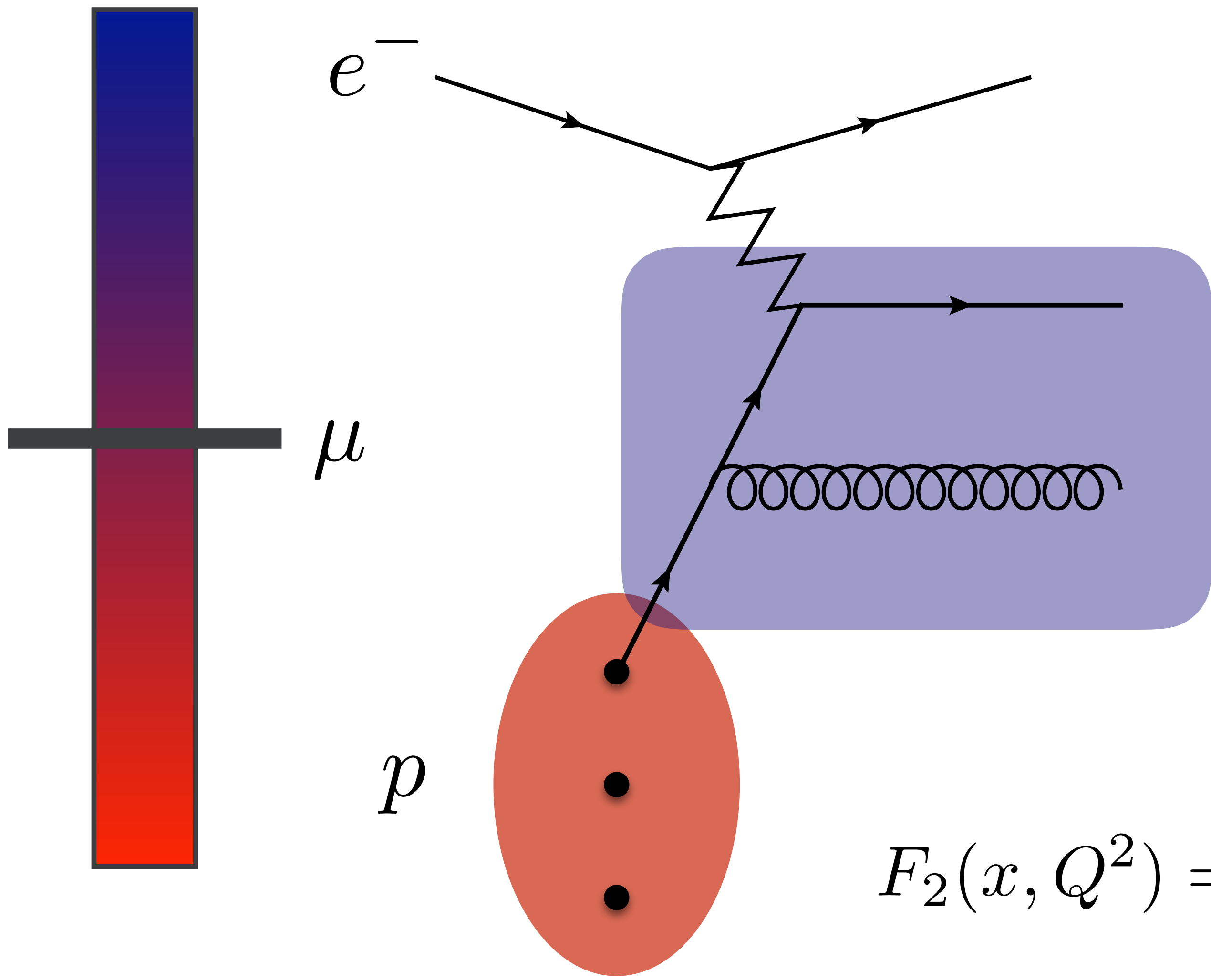
Low-energy perspective: Λ is ultraviolet regulator

- Effective Field Theory (EFT)
- Renormalization, RG evolution



Physics example: DIS

$$e^- + p \to e^- + X$$



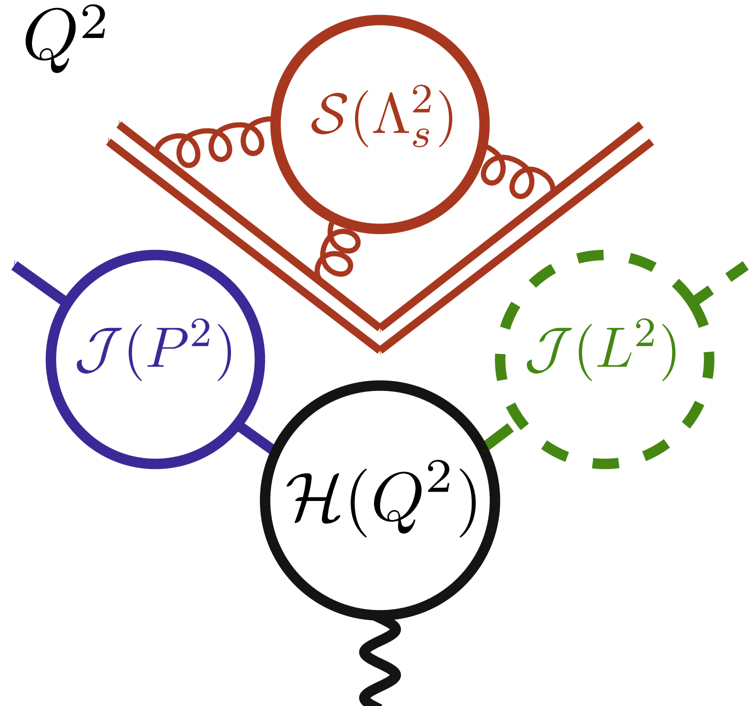
PDF
operator matrix element
needs renormalization

$$F_2(x, Q^2) = \sum_i \int_x^1 d\xi \, H_i(\frac{x}{\xi}, Q, \mu) \, f_i(\xi, \mu)$$

One-to-one correspondence between UV divergences in PDFs and IR-div's in H_i !

Unphysical example: off-shell form factor

$$\Lambda_s^2 = \frac{P^2 L^2}{Q^2}$$



*

$$I_h = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\ln\frac{\mu^2}{Q^2} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{6} + \mathcal{O}(\lambda)\right)$$

$$I_c = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(-\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\ln\frac{\mu^2}{P^2} - \frac{1}{2}\ln^2\frac{\mu^2}{P^2} + \frac{\pi^2}{6} + \mathcal{O}(\lambda)\right)$$

$$I_{\bar{c}} = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(-\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\ln\frac{\mu^2}{L^2} - \frac{1}{2}\ln^2\frac{\mu^2}{L^2} + \frac{\pi^2}{6} + \mathcal{O}(\lambda)\right)$$

$$I_s = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\ln\frac{\mu^2 Q^2}{L^2 P^2} + \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{L^2 P^2} + \frac{\pi^2}{6} + \mathcal{O}(\lambda)\right)$$

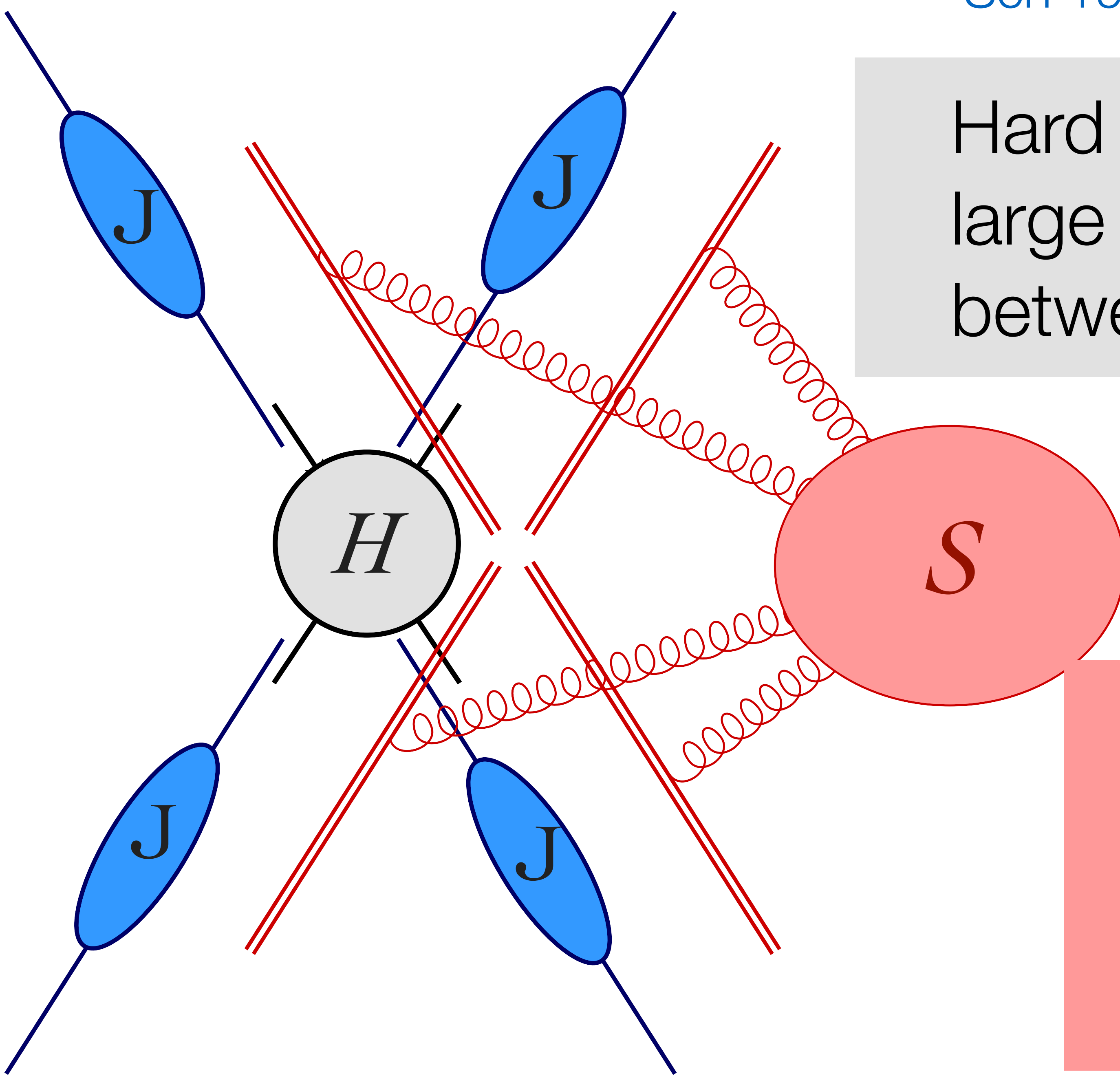
$$I \equiv I_h + I_c + I_{\bar{c}} + I_s = \frac{1}{Q^2}\left(\ln\frac{Q^2}{L^2}\ln\frac{Q^2}{P^2} + \frac{\pi^2}{3} + \mathcal{O}(\lambda)\right).$$

- Cancellations of divergences implies remarkable relations among H, J and S
- Factorization can be obtained in Soft-Collinear Effective Theory (SCET)
- Soft function is given by Wilson line matrix element

* result is for scalar loop integral instead of form factor

Soft-collinear factorization: *n* jet case

Sen 1983; Kidonakis, Oderda, Sterman 1998



Hard function H depends on large momentum transfers s_{ij} between jets

Soft function S depends on scales $\Lambda_{ij}^2 = \dfrac{p_i^2 p_j^2}{s_{ij}}$

Jet functions $J_i = J_i\,(p_i{}^2)$

Factorization

Off-shell Green's function factorize as

on-shell amplitude

$$\boldsymbol{S}(\{\underline{\beta}\},\epsilon)\prod_i J(L_i^2,\epsilon)\,|\mathcal{M}(\{\underline{s}\},\epsilon\rangle = \text{finite}$$

$$L_i \equiv \ln\frac{\mu^2}{-p_i^2}$$

$$s_{ij} \equiv \pm 2 p_i \cdot p_j$$

$$\beta_{ij} = \ln\frac{(-s_{ij})\,\mu^2}{(-p_i^2)(-p_j^2)}$$

Soft function $\boldsymbol{S}$ and on-shell amplitude $\mathcal{M}$ depend on colors of all particles!

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Soft function $\boldsymbol{S}$ and on-shell amplitude $\mathcal{M}$ depend on colors of all particles!

Renormalization

Soft and jet functions are operators in SCET. Renormalize:

$$\boldsymbol{S}(\{\underline{\beta}\},\mu)\prod_i J(L_i^2,\mu)\,|\mathcal{M}(\{\underline{s}\},\mu)\rangle = \text{finite}$$

Renormalized, finite amplitude

$$|\mathcal{M}_n(\{\underline{s}\},\mu)\rangle = \lim_{\epsilon\to 0}\,\boldsymbol{Z}^{-1}(\epsilon,\{\underline{s}\},\mu)\,|\mathcal{M}_n(\epsilon,\{\underline{s}\})\rangle$$

TB, Neubert ‘09

This renormalized amplitude defines a finite *S*-matrix for massless theories. Corresponds to subtracting asymptotic soft+collinear int’s.

Hannesdottir and Schwartz ‘19

Renormalization

Renormalization Group (RG) equation

$$\frac{d}{d\ln\mu}\,|\mathcal{M}_n(\{\underline{s}\},\mu)\rangle = \mathbf{\Gamma}(\{\underline{s}\},\mu)\,|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle$$

Anomalous dimension $\mathbf{\Gamma}$ determines IR singularities.
Independence of μ imposes constraint

$$\mathbf{\Gamma}(\{\underline{s}\},\mu) = \mathbf{\Gamma}_s(\{\underline{\beta}\},\mu) + \sum_{i=1}^{n} \Gamma_c^i(L_i,\mu)\,\mathbf{1}\,,$$

Note:

TB, Neubert '09; Gardi, Magnea '09

- $\boldsymbol{\Gamma}_x$ contains logarithms of associated scales
- $\boldsymbol{\Gamma}$ and $\boldsymbol{\Gamma}_s$ are matrices in color space

Dipole form

The following form is consistent with factorization

$$\boldsymbol{\Gamma}(\{\underline{s}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i^a \boldsymbol{T}_j^a}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\, \mathbf{1}$$

Using color conservation

$$\sum_j \boldsymbol{T}_j^a = 0 \quad \to \quad \sum_{(ij)} \boldsymbol{T}_i^a \boldsymbol{T}_j^a = -\sum_i \boldsymbol{T}_i^a \boldsymbol{T}_i^a = -\sum_i C_i$$

one can rewrite the hard logarithms as soft+jet using

$$\beta_{ij} = \ln\frac{(-s_{ij})\,\mu^2}{(-p_i^2)(-p_j^2)} = L_i + L_j - \ln\frac{\mu^2}{-s_{ij}}$$

Up to 2 loops above dipole form is correct. IR singularities agree with Catani '98 and gives $H^{(2)}{}_{\rm RS}$.

Additional terms beyond 2 loops?

1.) Extra terms must be the same when expressed in $\ln(s_{ij})$ or β_{ij} to be compatible with factorization.

$\longrightarrow$ functions of conformal cross ratios

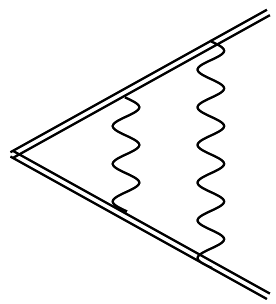
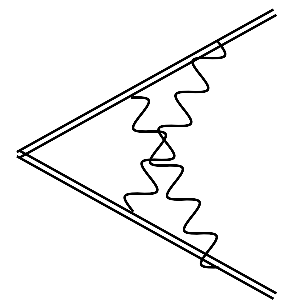
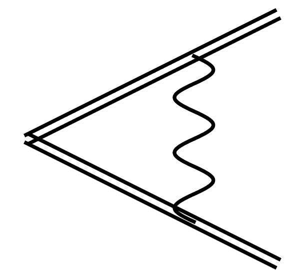
$$\beta_{ijkl} = \beta_{ij} + \beta_{kl} - \beta_{ik} - \beta_{jl} = \ln \frac{(-s_{ij})(-s_{kl})}{(-s_{ik})(-s_{jl})}$$

Gardi, Magnea '09

independent of collinear scales.

2.) Non-abelian exponentiation: only connected color structures.

Non-abelian exponentiation

 +  $= \frac{1}{2}\Bigg($  $\Bigg)^2$

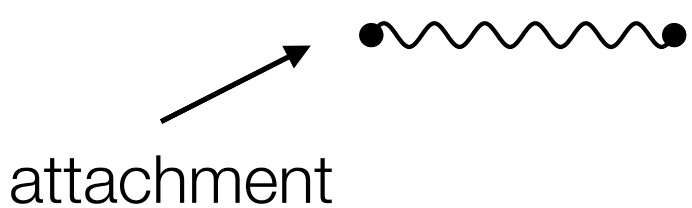
In massive QED, the soft function exponentiates

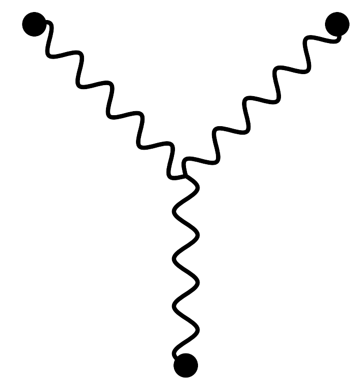
$$S = \exp\left(\frac{\alpha}{4\pi} S^{(1)}\right)$$

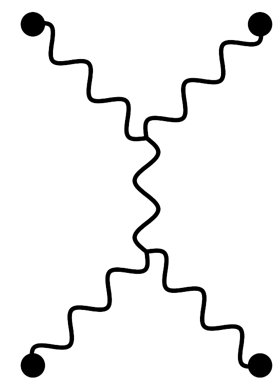
In QCD, simple exponentiation does not hold, but only connected webs contribute to the anomalous dimension. (2 legs: Gatheral ’83, Frenkel and Taylor ’84. *n* legs: Gardi, Smillie and White ’11, ’13)

Connected webs

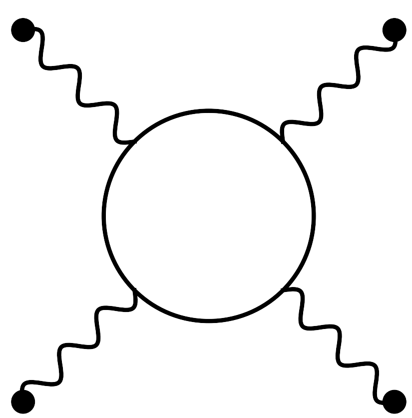
Show that we only need color connected webs that are *symmetrized* in their attachments to legs i,j,k…

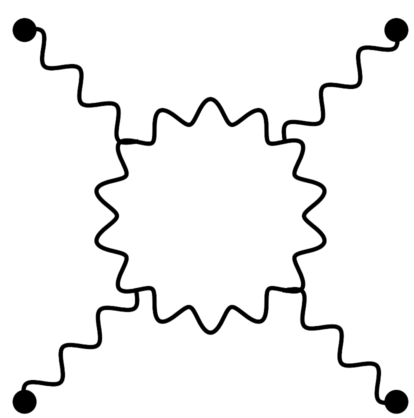


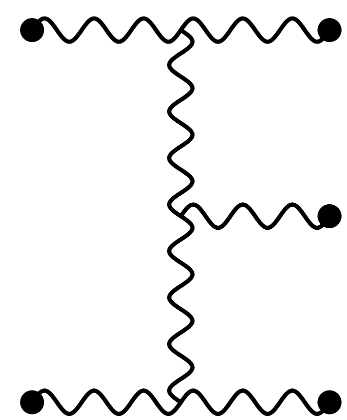




$\mathcal{D}_{ij} = \boldsymbol{T}_i^a \boldsymbol{T}_j^a$ $\qquad$ $\mathcal{T}_{ijk} = i f^{abc} \left(\boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \right)_+$ $\qquad$ $\mathcal{T}_{ijkl} = f^{ade} f^{bce} \left(\boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d \right)_+$







$\mathcal{D}^R_{ijkl} = d_R^{abcd}\, \boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d$ $\qquad$ $\mathcal{T}_{ijklm} = i f^{adf} f^{bcg} f^{efg} \left(\boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d \boldsymbol{T}_m^e \right)_+$

4-loop anomalous dimension

$$
\begin{aligned}
\mathbf{\Gamma}(\{\underline{s}\},\mu) = {} & \sum_{(i,j)} \frac{\boldsymbol{T}_i^a \boldsymbol{T}_j^a}{2}\, \gamma_{\rm cusp}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\, \mathbf{1} \\
& + f(\alpha_s) \sum_{(i,j,k)} \mathcal{T}_{iijk} + \sum_{(i,j,k,l)} \mathcal{T}_{ijkl}\, F(\beta_{ijlk}, \beta_{iklj}; \alpha_s) \\
& + \sum_R g^R(\alpha_s) \Big[\sum_{(i,j)} (\mathcal{D}^R_{iijj} + 2\mathcal{D}^R_{iiij}) \ln \frac{\mu^2}{-s_{ij}} + \sum_{(i,j,k)} \mathcal{D}^R_{ijkk} \ln \frac{\mu^2}{-s_{ij}} \Big] \\
& + \sum_R \sum_{(i,j,k,l)} \mathcal{D}^R_{ijkl}\, G^R(\beta_{ijlk}, \beta_{iklj}; \alpha_s) + \sum_{(i,j,k,l)} \mathcal{T}_{ijkli}\, H_1(\beta_{ijlk}, \beta_{iklj}; \alpha_s) \\
& + \sum_{(i,j,k,l,m)} \mathcal{T}_{ijklm}\, H_2(\beta_{ijkl}, \beta_{ijmk}, \beta_{ikmj}, \beta_{jiml}, \beta_{jlmi}; \alpha_s) + \mathcal{O}(\alpha_s^5)\,.
\end{aligned}
$$

Simplified compared to TB Neubert '09, Ahrens, Neubert and Vernazza '12. Earlier papers concluded that higher Casimir terms were excluded by factorization in collinear limit — true individually, but certain linear combinations are allowed!

Ingredients

$$
\begin{aligned}
\mathbf{\Gamma}(\{\underline{s}\},\mu) = & \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\, \mathbf{1} \\
& + f(\alpha_s) \sum_{(i,j,k)} \mathcal{T}_{iijk} + \sum_{(i,j,k,l)} \mathcal{T}_{ijkl}\, F(\beta_{ijlk}, \beta_{iklj}; \alpha_s) \\
& + \sum_R g^R(\alpha_s) \Bigg[\sum_{(i,j)} \left(\mathcal{D}^R_{iijj} + 2\mathcal{D}^R_{iiij}\right) \ln \frac{\mu^2}{-s_{ij}} + \sum_{(i,j,k)} \mathcal{D}^R_{ijkk} \ln \frac{\mu^2}{-s_{ij}} \Bigg] \\
& + \sum_R \sum_{(i,j,k,l)} \mathcal{D}^R_{ijkl}\, G^R(\beta_{ijlk}, \beta_{iklj}; \alpha_s) + \sum_{(i,j,k,l)} \mathcal{T}_{ijkli}\, H_1(\beta_{ijlk}, \beta_{iklj}; \alpha_s) \\
& + \sum_{(i,j,k,l,m)} \mathcal{T}_{ijklm}\, H_2(\beta_{ijkl}, \beta_{ijmk}, \beta_{ikmj}, \beta_{jiml}, \beta_{jlmi}; \alpha_s) + \mathcal{O}(\alpha_s^5)\,.
\end{aligned}
$$

known to 4 loops

Henn, Smirnov, Smirnov, Steinhauser '16, Henn, Smirnov, Smirnov, Steinhauser, Lee '16, Davies, Vogt, B. Ruijl, T. Ueda and Vermaseren '16; Moch, Ruijl, Ueda, Vermaseren and Vogt '17 '18; Grozin '18; Lee, Smirnov, Smirnov and Steinhauser '17 '19; Henn, Peraro, Stahlhofen and Wasser '19, von Manteuffel and Schabinger '19; Brüser, Grozin, Henn and Stahlhofen '19, Henn, Korchemsky and Mistlberger 19

known to 3 loops

f, F : Almelid, Duhr and Gardi '16

unknown, 4 loops

Vladimirov '17 claims only even structures should arise: H_1 and H_2 zero?

Ingredients

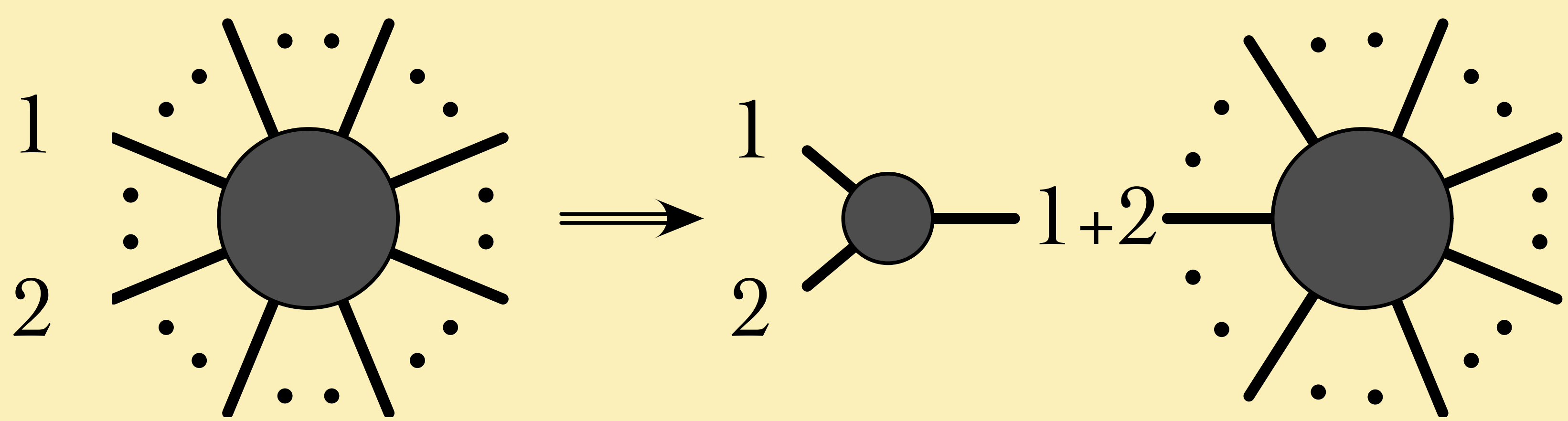
$$\begin{aligned}\boldsymbol{\Gamma}(\{\underline{s}\},\mu) = & \sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\,\mathbf{1} \\ & + f(\alpha_s)\sum_{(i,j,k)} \mathcal{T}_{iijk} + \sum_{(i,j,k,l)} \mathcal{T}_{ijkl}\,F(\beta_{ijlk},\beta_{iklj};\alpha_s) \\ & + \sum_R g^R(\alpha_s)\left[\sum_{(i,j)}\left(\mathcal{D}^R_{iijj}+2\mathcal{D}^R_{iiij}\right)\ln\frac{\mu^2}{-s_{ij}} + \sum_{(i,j,k)}\mathcal{D}^R_{ijkk}\,\ln\frac{\mu^2}{-s_{ij}}\right] \\ & + \sum_R\sum_{(i,j,k,l)}\mathcal{D}^R_{ijkl}\,G^R(\beta_{ijlk},\beta_{iklj};\alpha_s) + \sum_{(i,j,k,l)}\mathcal{T}_{ijkli}\,H_1(\beta_{ijlk},\beta_{iklj};\alpha_s) \\ & + \sum_{(i,j,k,l,m)}\mathcal{T}_{ijklm}\,H_2(\beta_{ijkl},\beta_{ijmk},\beta_{ikmj},\beta_{jiml},\beta_{jlmi};\alpha_s) + \mathcal{O}(\alpha_s^5)\,.\end{aligned}$$

- The full three-loop result is known
  - IR singularities of all 3-loop amplitudes are known
- All logarithmic pieces are known to four loops
  - All IR singularities at 4-loops, except 1/ε are known
  - Resummation to N³LL for *n*-jet processes

Consistency with collinear limits

- When two partons become collinear, an n-point amplitude M_{n} reduces to an (n-1)-parton amplitude times a splitting function: Berends, Giele '89; Mangano, Parke '91 Kosower '99; Catani, de Florian, Rodrigo '03

$$|\mathcal{M}_n(\{p_1,p_2,p_3,\dots,p_n\})\rangle = \mathbf{Sp}(\{p_1,p_2\})\,|\mathcal{M}_{n-1}(\{P,p_3,\dots,p_n\})\rangle + \dots$$



$$\mathbf{\Gamma}_{\mathrm{Sp}}(\{p_1,p_2\},\mu) = \mathbf{\Gamma}(\{p_1,\dots,p_n\},\mu) - \mathbf{\Gamma}(\{P,p_3\dots,p_n\},\mu)\Big|_{\boldsymbol{T}_P\to\boldsymbol{T}_1+\boldsymbol{T}_2}$$

TB, Neubert '09

- $\mathbf{\Gamma}_{\mathrm{Sp}}$ must be independent of momenta and colors of partons 3, ..., n

Consistency with collinear limits

- The fact that $\mathbf{\Gamma}_{\text{Sp}}$ must be independent of the colors and momenta of the remaining particles imposes strong constraint on $\mathbf{\Gamma}$.
- ’09, ‘12 papers concluded that the coefficients of the higher-multiplicity terms should vanish in the collinear limit.
- Deriving the 3-loop result Almelid, Duhr and Gardi ‘16 realized that this is not true: different terms can conspire in the limit to be compatible!

$$\lim_{\omega \to -\infty} F(\omega, 0; \alpha_s) = \frac{f(\alpha_s)}{2}$$

- Similarly, the higher Casimir coefficients must obey

$$\lim_{\omega \to -\infty} G^R(\omega, 0; \alpha_s) = -\frac{g^R(\alpha_s)}{6}\,\omega$$

Result for $\mathbf{\Gamma}_{\text{Sp}}$

Evaluating *

$$\mathbf{\Gamma}_{\text{Sp}}(\{p_1,p_2\},\mu) = \mathbf{\Gamma}(\{p_1,\dots,p_n\},\mu) - \mathbf{\Gamma}(\{P,p_3\dots,p_n\},\mu)\big|_{\boldsymbol{T}_P\to\boldsymbol{T}_1+\boldsymbol{T}_2}$$

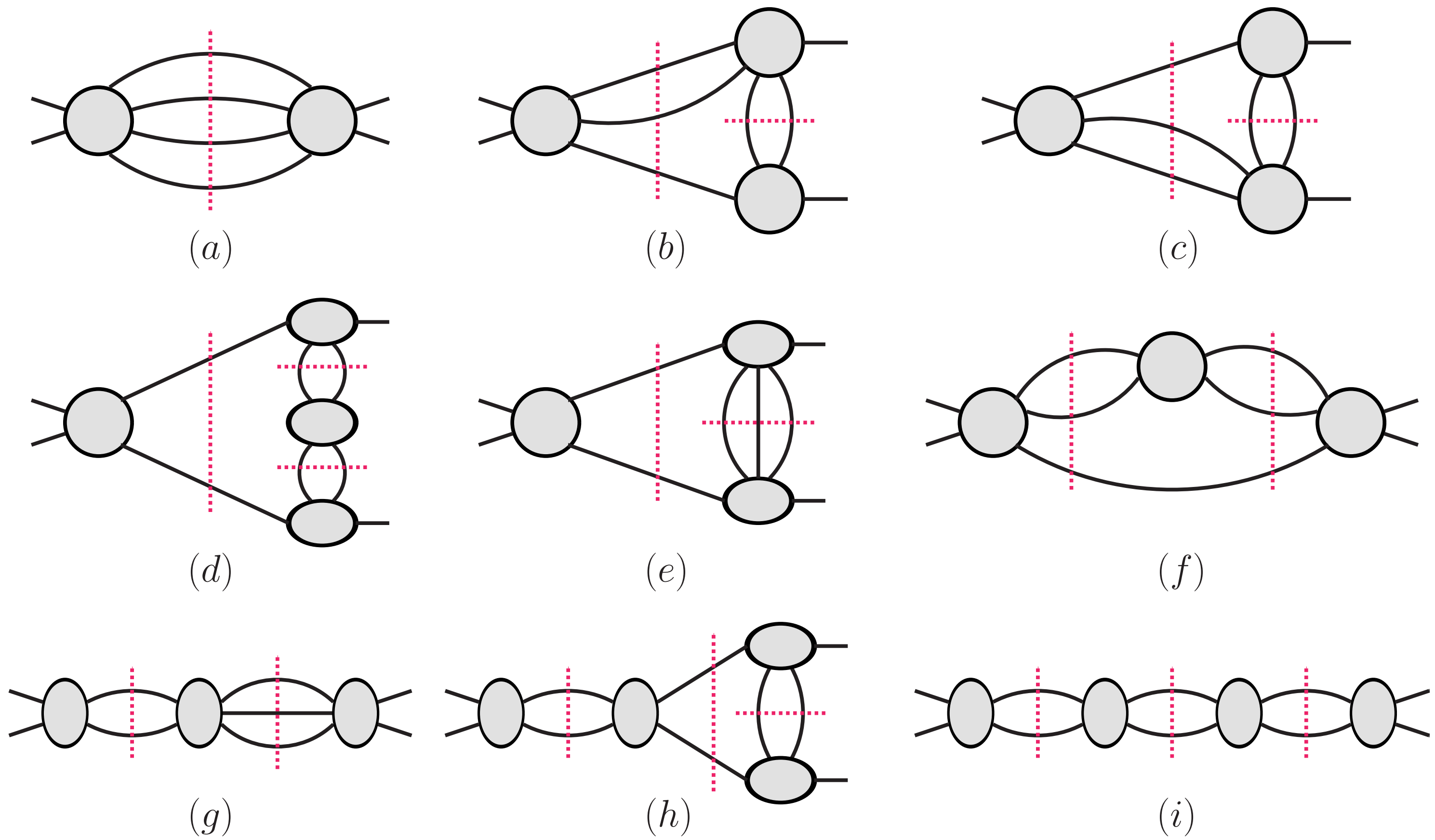
in the collinear limit, one obtains

$$\begin{aligned}
&\mathbf{\Gamma}_{\text{Sp}}(\{p_1,p_2\},\mu)\\
&= \left\{\gamma_{\text{cusp}}(\alpha_s)\,\boldsymbol{T}_1\cdot\boldsymbol{T}_2 + \sum_R 2g^R(\alpha_s)\left[3\mathcal{D}^R_{1122} + 2\big(\mathcal{D}^R_{1112}+\mathcal{D}^R_{1222}\big)\right]\right\}\left[\ln\frac{\mu^2}{-s_{12}} + \ln z(1-z)\right]\\
&\quad + \gamma_{\text{cusp}}(\alpha_s)\left[C_{R_1}\ln z + C_{R_2}\ln(1-z)\right] + \gamma^1(\alpha_s) + \gamma^2(\alpha_s) - \gamma^P(\alpha_s)\\
&\quad - 6f(\alpha_s)\left(\mathcal{T}_{1122} + \frac{C_A^2}{8}\boldsymbol{T}_1\cdot\boldsymbol{T}_2\right) + \sum_R 2g^R(\alpha_s)\left[\mathcal{D}^R_{1111}\ln z + \mathcal{D}^R_{2222}\ln(1-z)\right] + \mathcal{O}(\alpha_s^5)\,.
\end{aligned}$$

Log terms known to 4 loops! (f, γ^i only to 3 loops)

* a painful exercise in color algebra!!

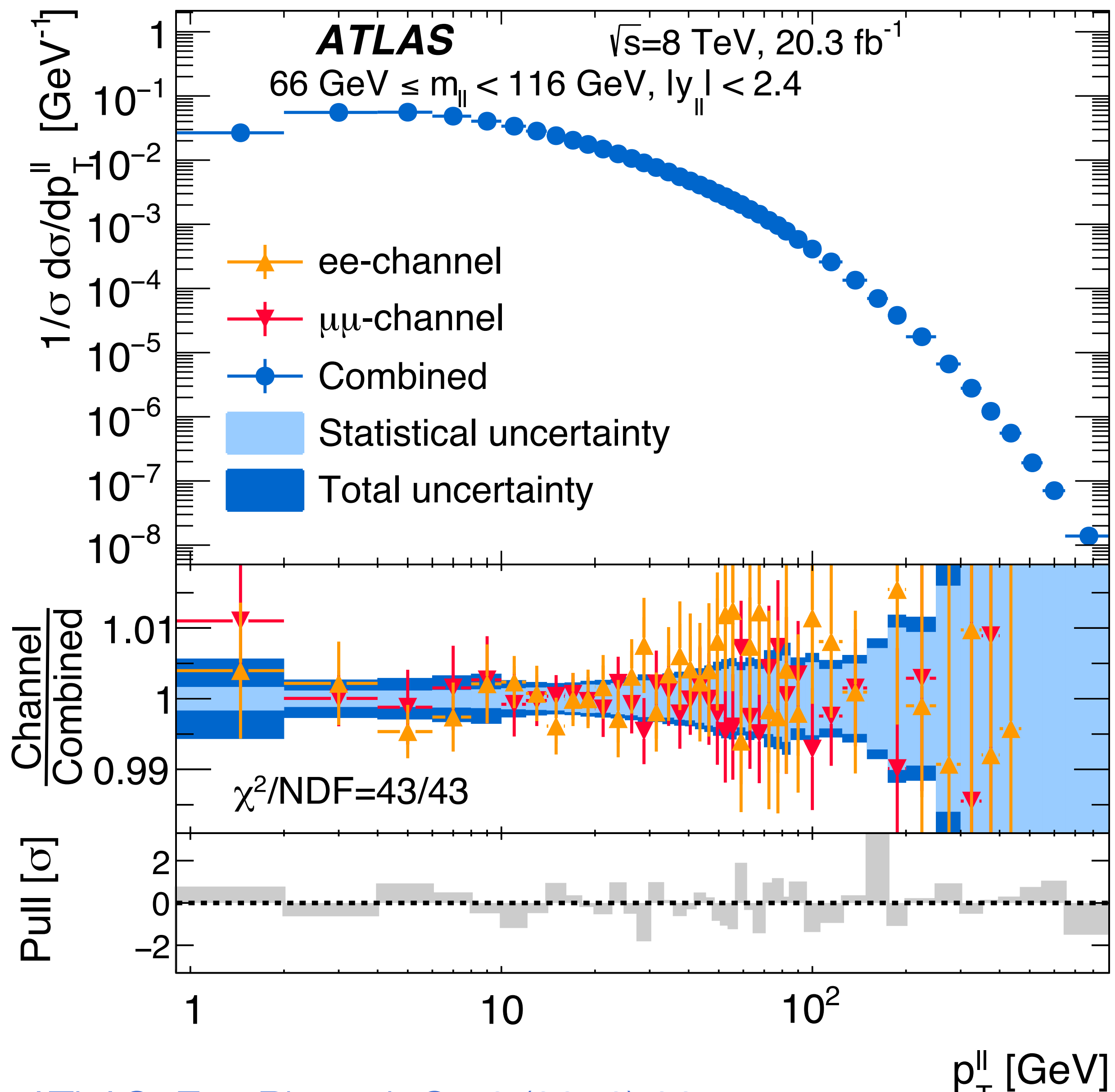
Does it work?



Yes! Recent computation of 3-loop four-gluon amplitude in pure YM theory verified that IR singularities agree with general result. Jin, Luo '19

Resummation at N^3LL

Precision measurements at the LHC



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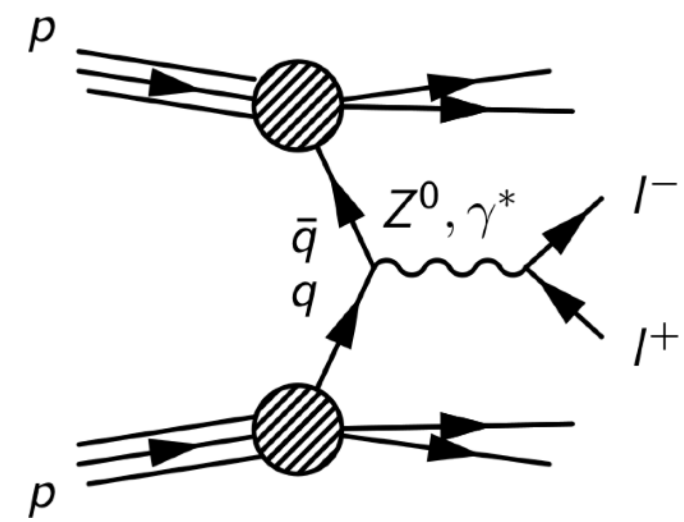


Sub-percent accuracy
over large range of energies and many orders of cross section!

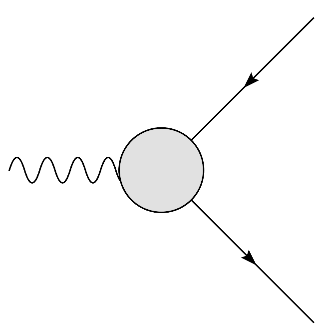
transverse momentum of the lepton pair

A huge challenge for theory!

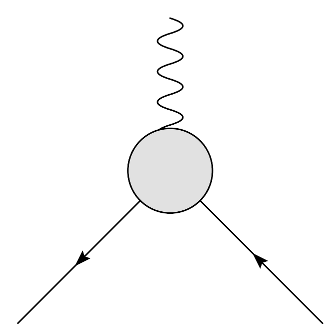
We have derived our factorization formula using off-shell Green’s functions, but the factorization

$$d\sigma = \mathrm{tr}\left[\boldsymbol{H}_n \cdot \prod_{i=1}^{n} J \otimes \boldsymbol{S}_n\right]$$

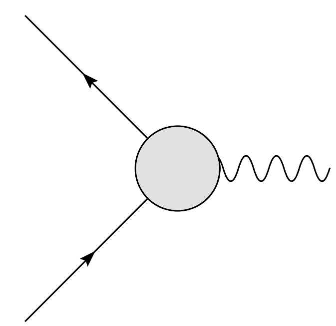
arises for many physical cross sections. $\boldsymbol{J}$ and $\boldsymbol{S}$ are observable dependent, but H is square of on-shell amplitudes.



$e^+e^- \longrightarrow 2$ jets



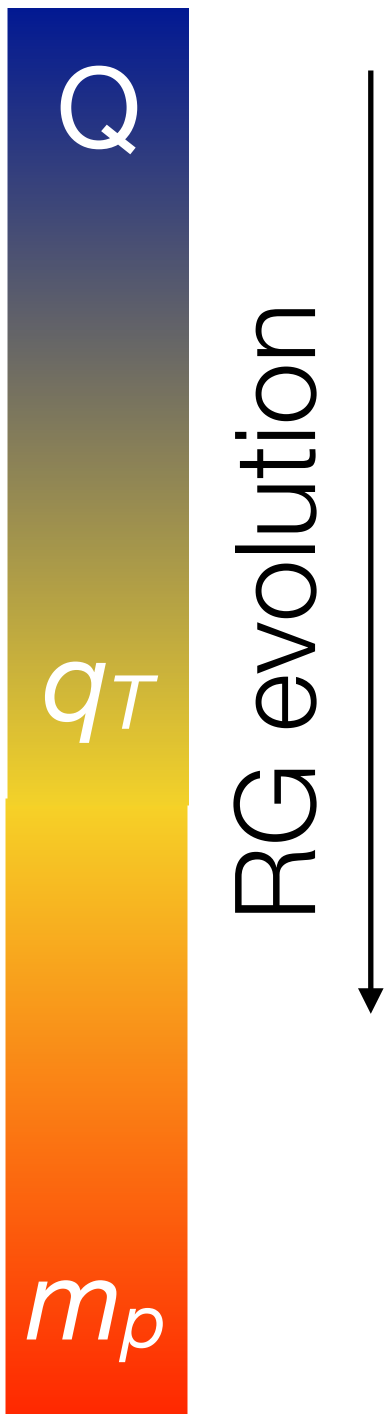
DIS

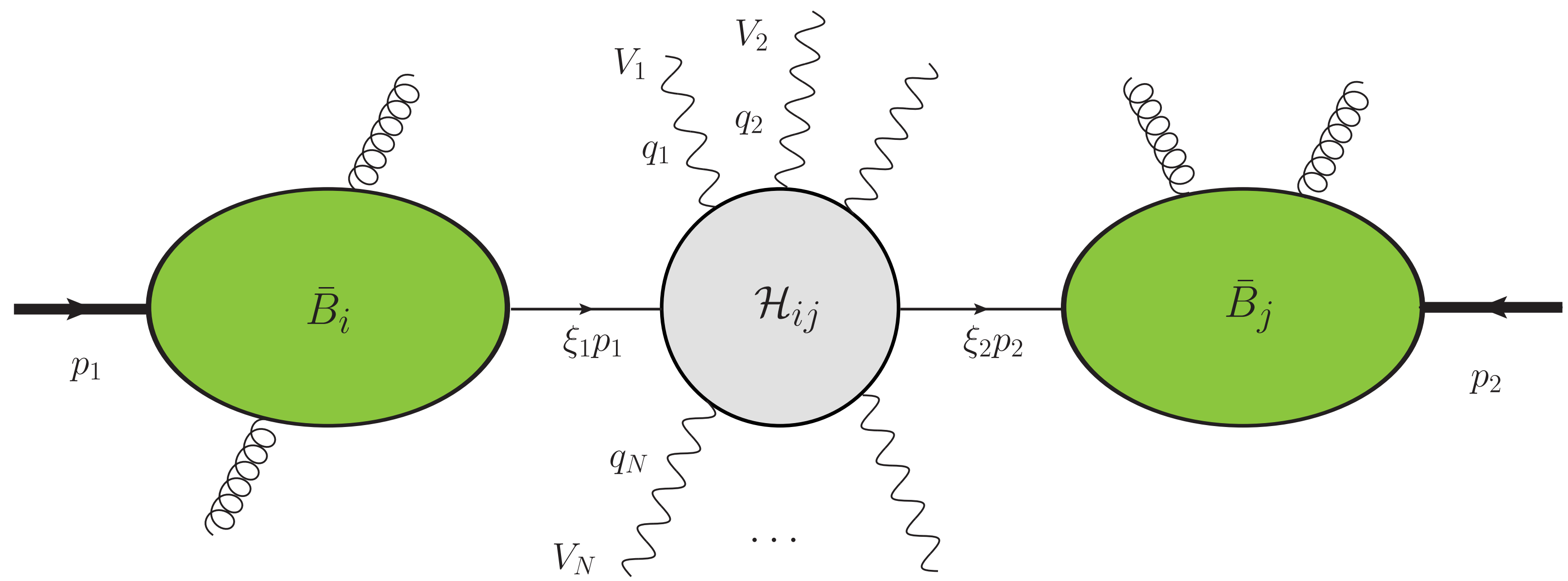


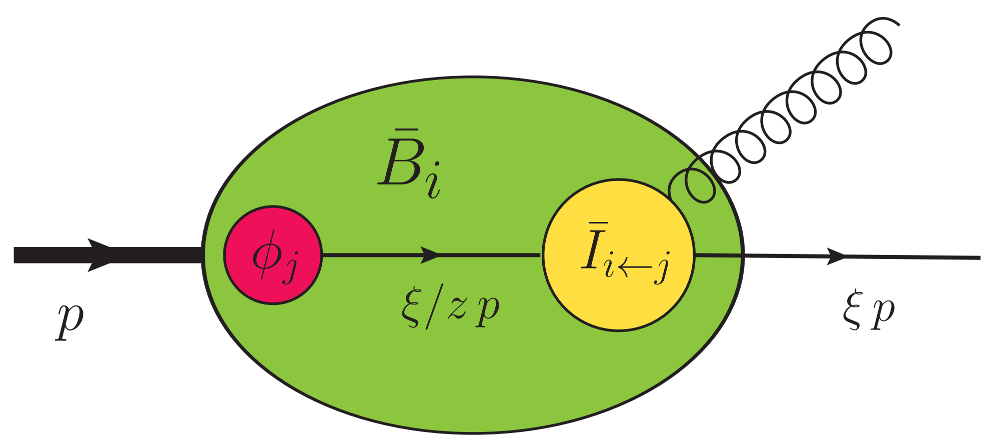
Z,W,H production

→ Zoltan Trocsanyi’s talk

EW boson production at small q_T







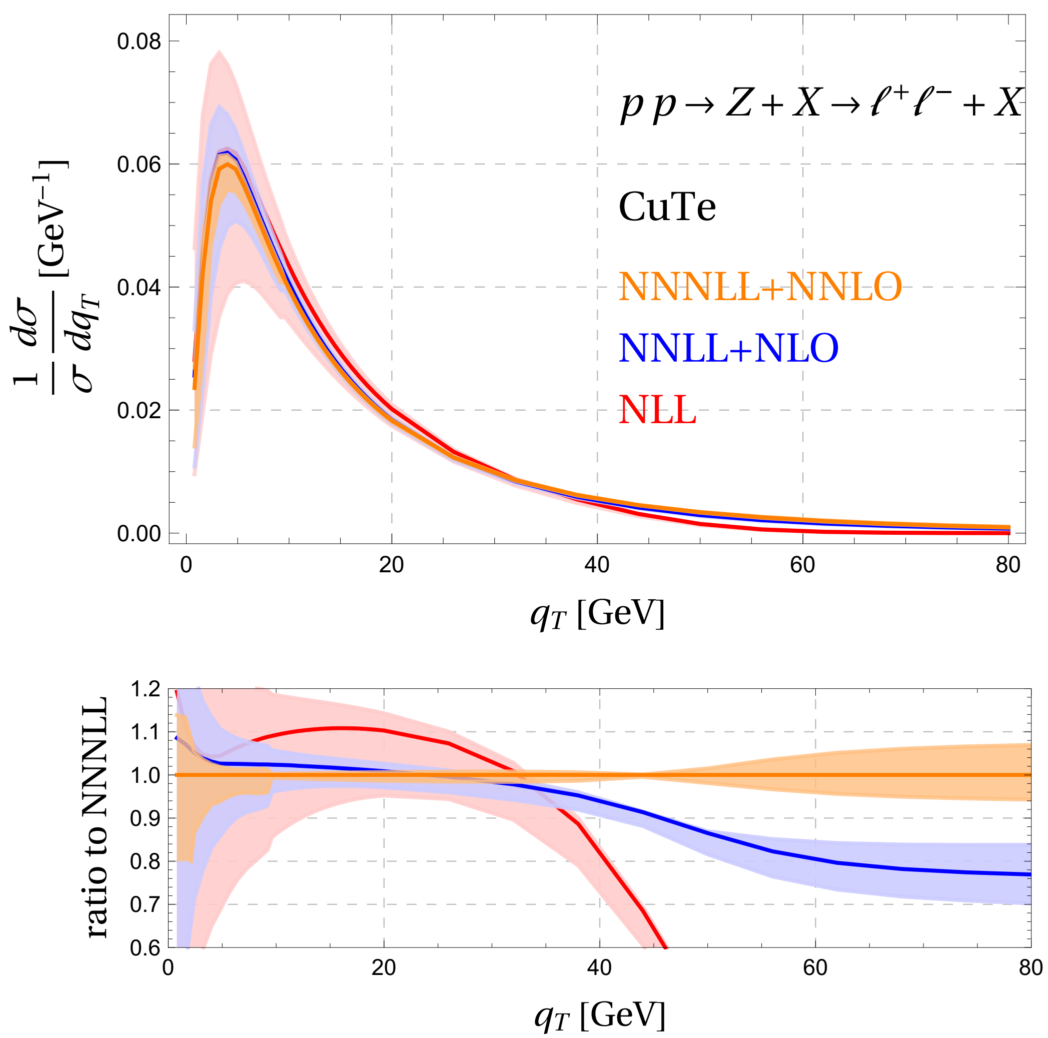
RG can be used to resum large logarithms $\alpha_s{}^n \ln^{\mathrm{m}}(q_T/Q)$.

Ingredients for resummation

Log. approx.	γ_{cusp}	γ^i	H, J, S
LL	1-loop	tree-level	tree-level
NLL	2-loop	1-loop	tree-level
NNLL	3-loop	2-loop	1-loop
NNNLL	4-loop	3-loop	2-loop

- NNNLL has parametrically the same accuracy as NNLO fixed order!
- NNNLL resummations have been performed in the past, but were missing 4-loop γ_{cusp}.
  - now in place, also for *n*-jet processes

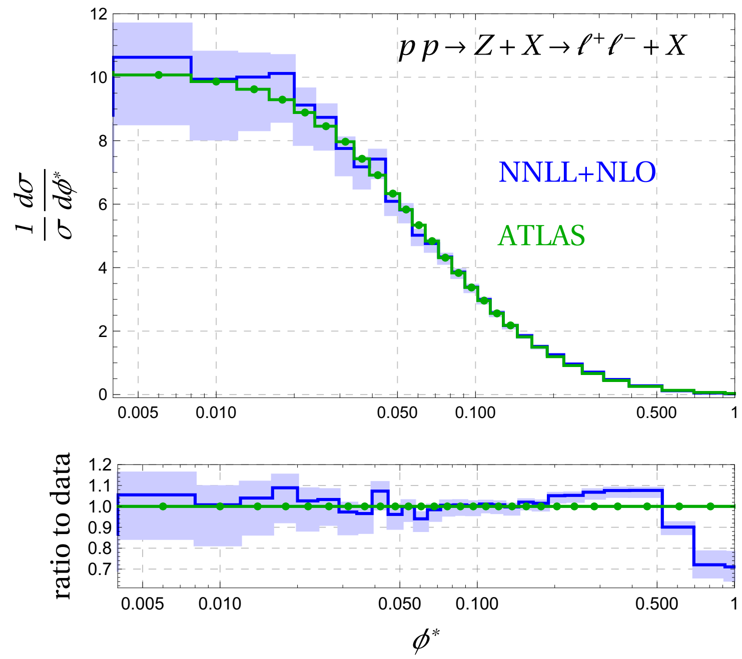
Transverse momentum spectrum

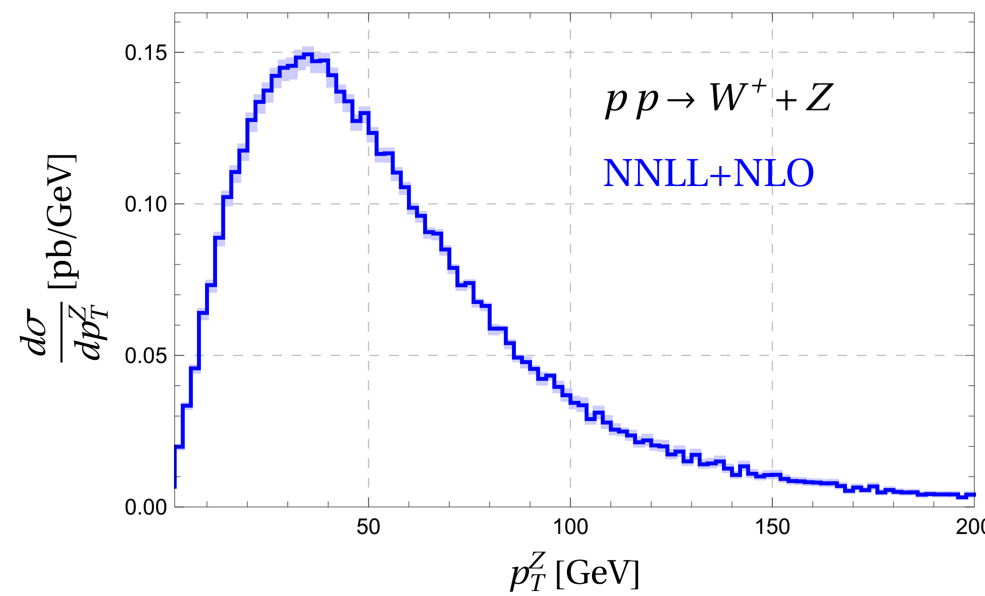


CuTe
TB, Neubert,Wilhelm '12,
+ Lübbert, '16

- At NNNLL, one reaches an accuracy of a few per cent
- 4-loop cusp has numerically only very small effect
- At higher q_T one matches to fixed-order result.
- Here: NNLO = $O(\alpha_s{}^2)$, but $O(\alpha_s{}^3)$ is known.
- CuTe only produces inclusive spectrum.

CuTeR

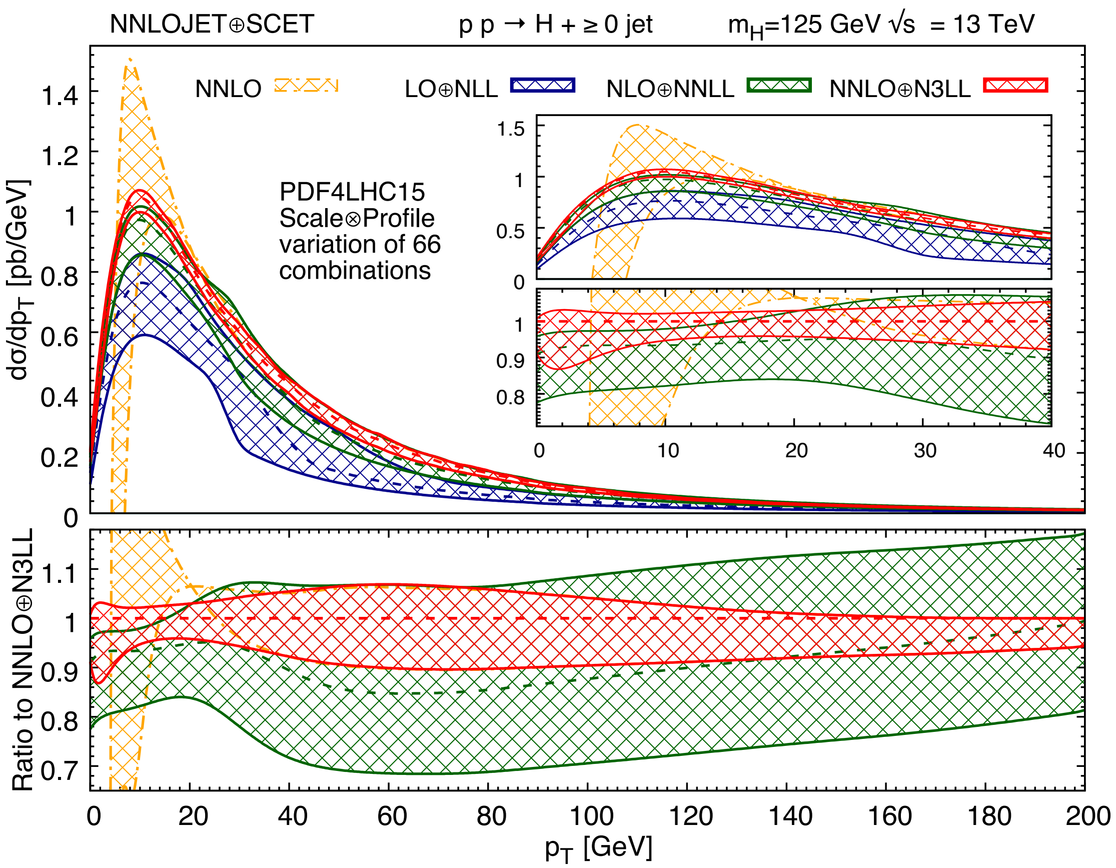


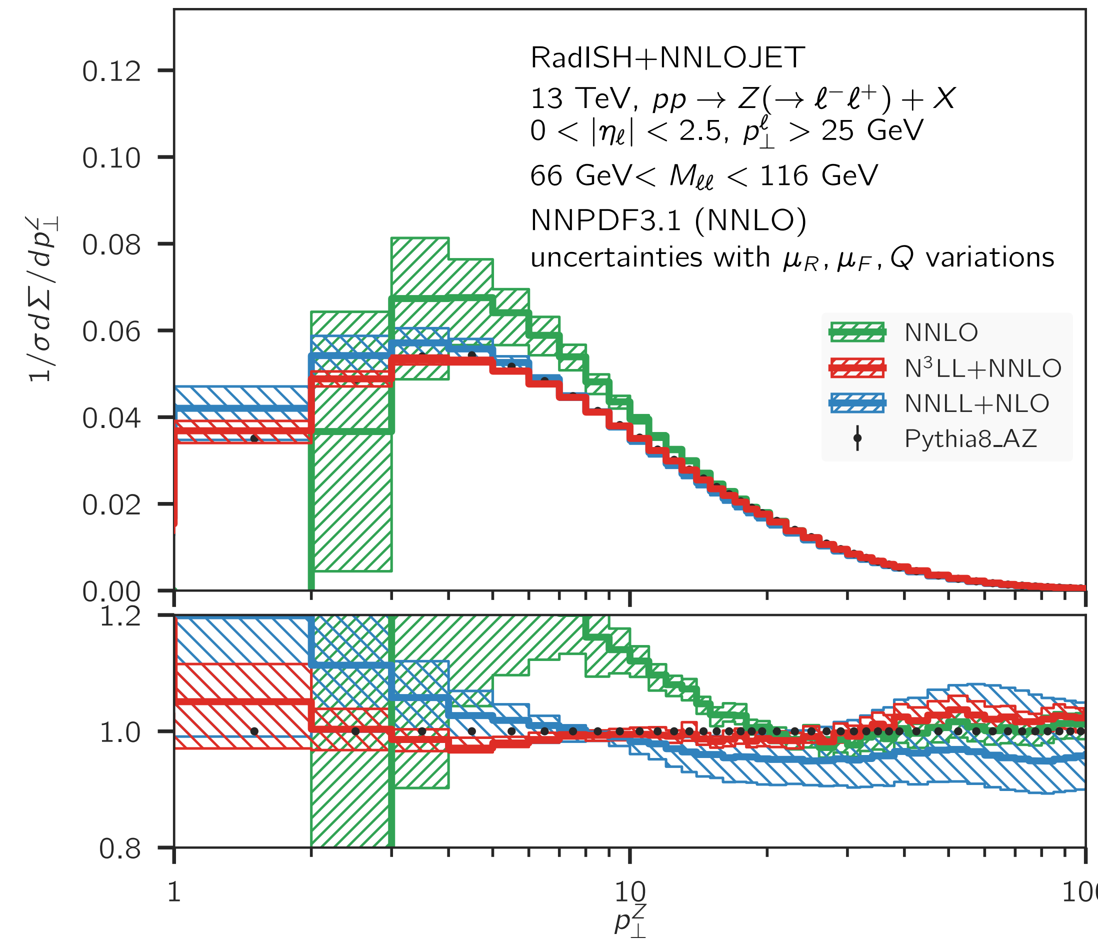


Have implemented q_T resummation in an event-based framework TB, Hager 1904.08325.

- Reweight tree-level event files from MG5_aMC@NLO
- Arbitrary (quark-induced) electroweak boson processes (W,Z, WZ, ZZ, …) at NNLL + O(α_s)
- Can impose experimental cuts on leptonic final states and compute related variables such as ϕ^*

1805.00736 1905.05171

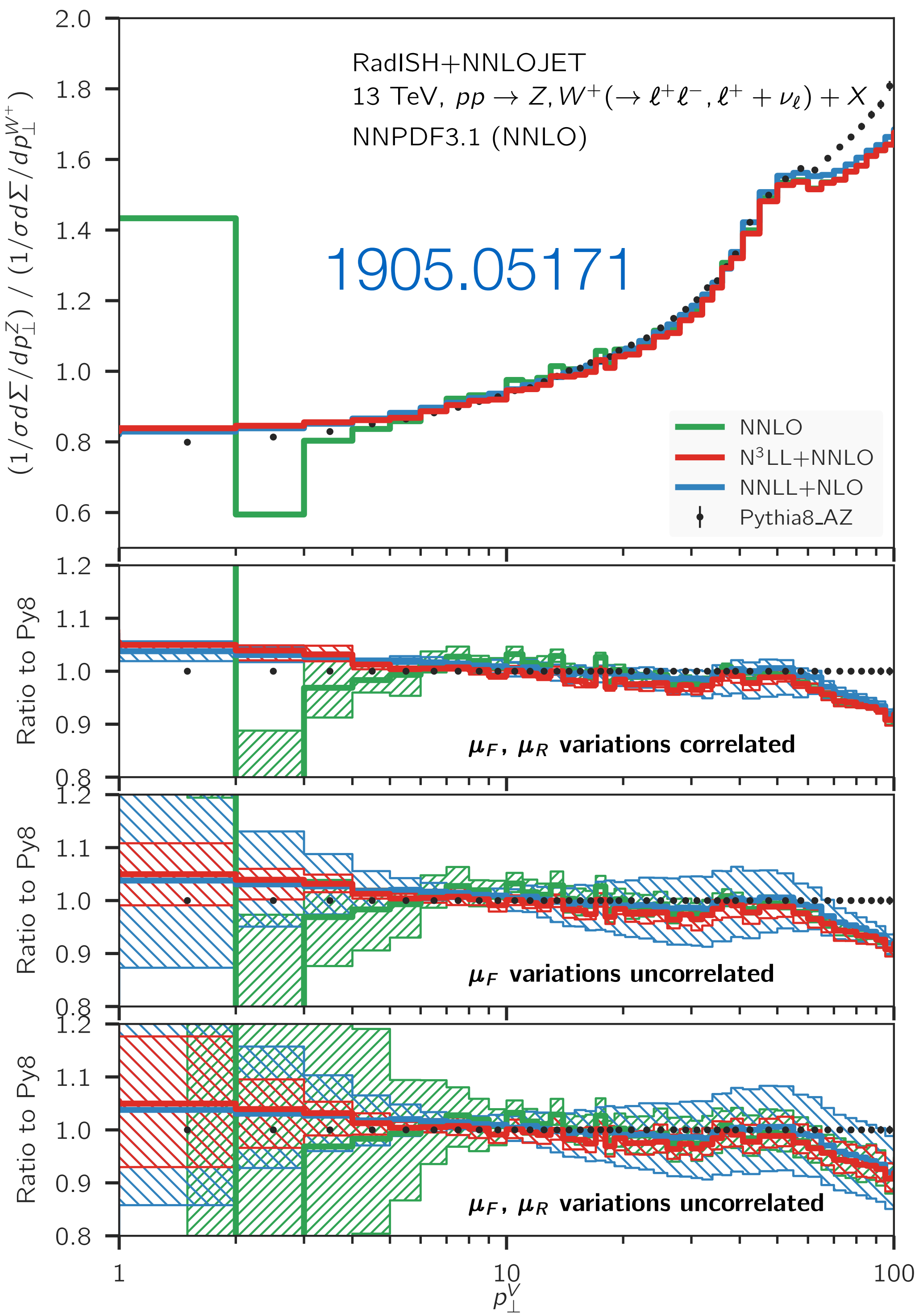




State of the art is now N³LL + $O(\alpha_s^3)$ (here called NNLO) matching

- *W*, *Z*, *H* using RadISH Bizon, Chen, Gehrmann-De Ridder, Glover, Huss, Monni, Re, Rottoli, Torrielli Walker '18 '19
- *H* using SCET Chen, Gehrmann, Glover, Alexander Huss, Li, Neill, Schulze, Stewart, Zhu '18

Ratio of Z and W spectrum



W-spectrum is important for M_W measurement. Analysis needs extremely precise predictions

- Experiments use measured *Z*-spectrum to tune Pythia
- Pythia is then used to predict *W/Z* ratio

A better understanding of the uncertainties would be important

- Ongoing effort to compare and benchmark results of different resummation codes.

Towards NNNNLL

By now even some ingredients for resummation beyond N^3LL have become available

- 3-loop dijet hard functions Baikov, Chetyrkin, Smirnov, Smirnov Steinhauser ’10, Lee, Smirnov, Smirnov ’10, Gehrmann, Glover, Huber, Ikizlerli, Studerus ’10, …
- 3-loop jet functions: quark Brüser, Liu, Stahlhofen ’18; gluon Banerjee, Dhani, Ravindran ’18
- 3-loop soft function for q_T Li and Zhu for EEC, Moult, Zhu ’18, for heavy-to-light decays Brüser, Liu, Stahlhofen ‘19
- double-real for 3-loop quark beam function Melnikov, Rietkerk, Tancredi, Wever ‘18

Summary

Have discussed the structure of IR singularities of amplitudes with massless particles

- heavily constrained by
  - soft-collinear factorization, collinear limits,non-abelian exponentiation
  - regge limit Del Duca, Claude Duhr, Einan Gardi, Lorenzo Magnea, White '11; Caron-Huot, Gardi, Reichel, Vernazza '17
- determined by an anomalous dimension $\mathbf{\Gamma}$
  - known to three loops, logarithmic part to 4 loops
- $\mathbf{\Gamma}$ is an important ingredient to resummation of n-jet processes
  - N^3LL + NNLO for weak boson q_T spectra!

$$
\begin{aligned}
\mathbf{\Gamma}(\{\underline{s}\},\mu) &= \sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} \\
&\quad + \sum_R g^R(\alpha_s)\Bigg[\sum_{(i,j)}\big(\mathcal{D}^R_{iijj}+2\mathcal{D}^R_{iiij}\big)\ln\frac{\mu^2}{-s_{ij}} + \sum_{(i,j,k)}\mathcal{D}^R_{ijkk}\,\ln\frac{\mu^2}{-s_{ij}}\Bigg] \\
&\quad + \sum_i \gamma^i(\alpha_s) + f(\alpha_s)\sum_{(i,j,k)}\mathcal{T}_{iijk} + \sum_{(i,j,k,l)}\mathcal{T}_{ijkl}\,F(\beta_{ijlk},\beta_{iklj};\alpha_s) \\
&\quad + \mathcal{O}\bigg(\alpha_s^4,\alpha_s^5\ln\frac{\mu^2}{-s_{ij}}\bigg).
\end{aligned}
$$

Thank you!

Extra slides

4-loop Z-factor

$$\ln \mathbf{Z} = \frac{\alpha_s}{4\pi}\left(\frac{\Gamma'_0}{4\epsilon^2} + \frac{\mathbf{\Gamma}_0}{2\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_0\Gamma'_0}{16\epsilon^3} + \frac{\Gamma'_1 - 4\beta_0\mathbf{\Gamma}_0}{16\epsilon^2} + \frac{\mathbf{\Gamma}_1}{4\epsilon}\right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left(\frac{11\beta_0^2\Gamma'_0}{72\epsilon^4} - \frac{5\beta_0\Gamma'_1 + 8\beta_1\Gamma'_0 - 12\beta_0^2\mathbf{\Gamma}_0}{72\epsilon^3} + \frac{\Gamma'_2 - 6\beta_0\mathbf{\Gamma}_1 - 6\beta_1\mathbf{\Gamma}_0}{36\epsilon^2} + \frac{\mathbf{\Gamma}_2}{6\epsilon}\right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^4 \left(-\frac{25\beta_0^3\Gamma'_0}{192\epsilon^5} + \frac{13\beta_0^2\Gamma'_1 + 40\beta_0\beta_1\Gamma'_0 - 24\beta_0^3\mathbf{\Gamma}_0}{192\epsilon^4}\right.$$
$$- \frac{7\beta_0\Gamma'_2 + 9\beta_1\Gamma'_1 + 15\beta_2\Gamma'_0 - 24\beta_0^2\mathbf{\Gamma}_1 - 48\beta_0\beta_1\mathbf{\Gamma}_0}{192\epsilon^3}$$
$$\left. + \frac{\Gamma'_3 - 8\beta_0\mathbf{\Gamma}_2 - 8\beta_1\mathbf{\Gamma}_1 - 8\beta_2\mathbf{\Gamma}_0}{64\epsilon^2} + \frac{\mathbf{\Gamma}_3}{8\epsilon}\right) + \mathcal{O}(\alpha_s^5)\,,$$

$$\mathbf{\Gamma}(\alpha_s) = \sum_{n=0}^{\infty} \mathbf{\Gamma}_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

$$\Gamma'(\alpha_s) = \frac{\partial}{\partial \ln\mu}\mathbf{\Gamma}(\{\underline{s}\},\mu) = -\sum_i \Gamma^i_{\text{cusp}}(\alpha_s)$$

Three-loop coefficients

$$F(x_1, x_2; \alpha_s) = 2\,\mathcal{F}(e^{x_1}, e^{x_2})\left(\frac{\alpha_s}{4\pi}\right)^3 + \mathcal{O}(\alpha_s^4)\,,$$

$$f(\alpha_s) = 16\,(\zeta_5 + 2\zeta_2\zeta_3)\left(\frac{\alpha_s}{4\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

$$\mathcal{L}(z) = \mathcal{L}_{10101}(z) + 2\zeta_2\left[\mathcal{L}_{001}(z) + \mathcal{L}_{100}(z)\right]$$