# Multi-loop $Z$-boson calculations for future lepton colliders

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## Deriving mass and width of the $Z$-boson

- **LEP:** Collected $\simeq 17 \times \mathbf{10^6}$ decays (a few years of collecting data) [PDG 2017]

  $\Gamma_Z = 2495.2 \pm \Delta\Gamma_Z^{LEP}$ $\quad \Delta\Gamma_Z^{LEP} = 2.3$ MeV
- **FCC-ee:** Expected $\mathbf{10^{12}}$ $Z$-boson decays. [1] $\quad \Delta\Gamma_Z^{FCC} \simeq 0.1$ MeV
- Other EWPOs are $R_l$, $R_b$, $\sin^2\theta^l_{\rm eff}$,$\sin^2\theta^b_{\rm eff}$, e.g.: $\quad \Delta R_l^{LEP} = 250 \cdot 10^{-4}$
- Projects of new, more accurate lepton colliders: $\quad \Delta R_l^{FCC} \simeq 2 \div 20 \cdot 10^{-4}$

  **ILC, CLIC, CEPC, FCC-ee**
- **FCC-ee** is the most demanding with huge statistics and precise systematics and beam energy
- **Need for precise higher orders SM radiative corrections at 3 and 4 loops.**

## Electroweak pseudo-observables [EWPOs]

The n-loop corrections $z^\mu_{f(n)}$ to the effective $Zf\bar{f}$ vertex is given by:

$$z^\mu_{f(n)} = v_{f(n)}\gamma^\mu + a_{f(n)}\gamma^\mu\gamma^5,$$

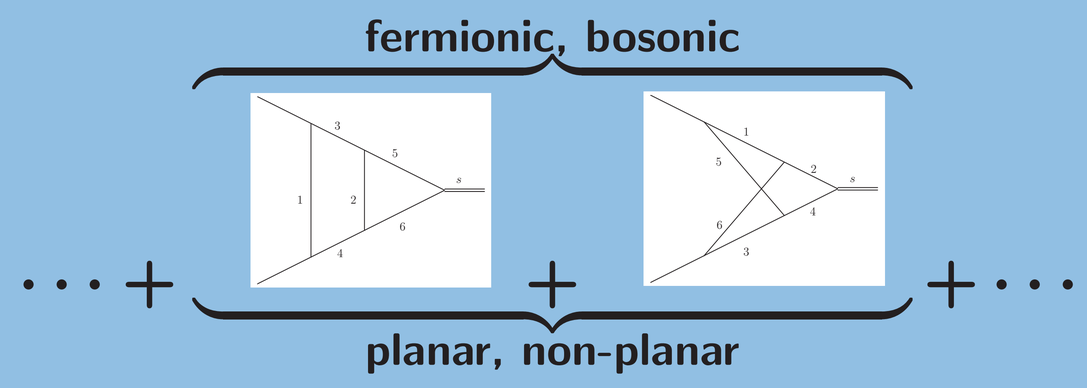
Calculated quantities of interest:

$$v_f(k^2) = \frac{1}{2(2-d)k^2} Tr[\gamma_\mu \not{p}_1 z_f^\mu(k^2) \not{p}_2],$$

$$a_f(k^2) = \frac{1}{2(2-d)k^2} Tr[\gamma_5\gamma_\mu \not{p}_1 z_f^\mu(k^2) \not{p}_2]$$

**Typical examples of 2-loop vertex diagrams [2,3]:**

$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b^{th} - a_b^{th}\gamma_5] =$

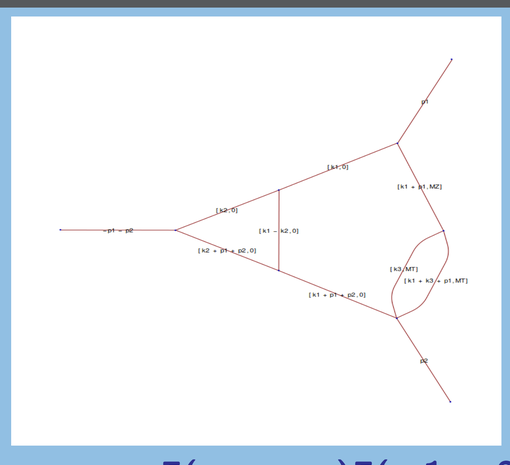



## An example of 3-loop calculation

Kinematical point:

$x = -\frac{M_t^2}{s} = \mathbf{3.6}$

$y = -\frac{M_Z^2}{s} = \mathbf{1}$



Similar accuracy can be obtained for other diagrams.

$$I_{4dim} = \int dz_1 dz_2 dz_3 dz_4 \; x^{z_3}\, y^{z_4} \; \frac{\Gamma(-\epsilon - z_1)\Gamma(-1 - 2\epsilon + z_1 - z_3 - z_4)\cdots}{\Gamma(1-2\epsilon)\Gamma(1-z_2)\cdots}$$

$$\longrightarrow I_{MB} = -14.8845 \pm 1.572 \times 10^{-8} - \frac{3.6449 \pm 8.975 \times 10^{-12}}{\epsilon}$$

$$\longrightarrow I_{FIESTA} = -14.8848 \pm 3.316 \times 10^{-3} - \frac{3.6453 \pm 2.09 \times 10^{-4}}{\epsilon}$$

$$\longrightarrow I_{pySecDec} = -14.8845 \pm 4.244 \times 10^{-7} - \frac{3.6449 \pm 5.42 \times 10^{-8}}{\epsilon}$$

## Completing 2-loops: bosonic corrections [2,3]

| $\Gamma_i$ [MeV] | $\Gamma_e, \Gamma_\mu, \Gamma_\tau$ | $\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$ | $\Gamma_d, \Gamma_s$ | $\Gamma_u, \Gamma_c$ | $\Gamma_b$ | $\Gamma_Z$ |
|---|---|---|---|---|---|---|
| $\mathcal{O}(\alpha)$ | 2.273 | 6.174 | 9.717 | 5.799 | 3.857 | 60.22 |
| $\mathcal{O}(\alpha\alpha_s)$ | 0.288 | 0.458 | 1.276 | 1.156 | 2.006 | 9.11 |
| $\mathcal{O}(N_f^2\alpha^2)$ | 0.244 | 0.416 | 0.698 | 0.528 | 0.694 | 5.13 |
| $\mathcal{O}(N_f\alpha^2)$ | 0.120 | 0.185 | 0.493 | 0.494 | 0.144 | 3.04 |
| $\mathcal{O}(\alpha^2_{\rm bos})$ | **0.017** | **0.019** | **0.058** | **0.057** | **0.167** | **0.505** |
| $\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,$ $\alpha_t^2\alpha_s,\ \alpha_t^3)$ | 0.038 | 0.059 | 0.191 | 0.170 | **0.190** | 1.20 |

**Table 1:** Contributions of different orders in perturbation theory to the partial and total $Z$ widths [3]. $N_f$ and $N_f^2$ refer to corrections with one and two closed fermion loops respectively

## Three-loop corrections needed: theory estimations [4]

| | $\delta\Gamma_Z$[MeV] | $\delta R_l[10^{-4}]$ | $\delta R_b[10^{-5}]$ | $\delta sin^{2,l}_{eff}\theta[10^{-6}]$ |
|---|---|---|---|---|
| **Present EWPO theoretical uncertainties** | | | | |
| EXP-2018 | 2.3 | 250 | 66 | 160 |
| TH-2018 | 0.4 | 60 | 10 | 45 |
| **EWPO theoretical uncertainties when FCC-ee will start** | | | | |
| EXP-FCC-ee | 0.1 | 10 | $2 \div 6$ | 6 |
| TH-FCC-ee | 0.07 | 7 | 3 | 7 |

**Table 2:** Comparison for selected precision observables of present experimental measurements (EXP-2018), current theory errors (TH-2018), FCC-ee precision goals at the end of the Tera-Z run (EXP-FCC-ee) and rough estimates of the theory errors assuming that electroweak 3-loop corrections and the dominant 4-loop EW-QCD corrections are available at the start of FCC-ee (TH-FCC-ee). Based on discussion in [5].

## Next decade: complete 3-loop calculations [3]

| $Z \to b\bar{b}$ | | | |
|---|---|---|---|
| Number of topologies | 1 loop | 2 loops | 3 loops |
| | 1 | 5 | 51 |
| Number of diagrams | 15 | 1114 | 120187 |
| Fermionic loops | 0 | 150 | **17580** |
| Bosonic loops | 15 | 964 | 102607 |
| QCD / EW | 1 / 14 | 98 / 1016 | 10405 / 110067 |

**Table 3:** The number of Z decay Feynman diagrams needed to be calculated for TH-FCC-ee of Table 2. Tadpoles, products of lower loop diagrams and symmetrical diagrams are not included.

**A first tackle might concentrate on the 17580 electroweak 3-loop diagrams with closed internal fermionic loops, to be determined with a net accuracy of two relevant digits.**

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I. Dubovyk, J. Usovitsch

## References

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**[2]** I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Phys. Lett. B762 (2016) 184.
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**[4]** A. Blondel, et al., Theory Requirements and Possibilities for the FCC-ee and other Future High Energy and Precision Frontier Lepton Colliders, arXiv:1901.02648[hep-ph]
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