Brief historical overview from LEP

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EWWG Precision Subgroup meeting, CERN, 13/03/2019

main references

- ADLO-SLD-LEPEWWG, hep-ex/0509008
- D.Y. Bardin, M. Grünewald, G. Passarino, hep-ph/9902452

- **Realistic observables**: cross sections and A_{FB} within experiment dipendent event selection
- $e^+e^- \to f\bar{f}$ amplitude

$$A_{SM} = A_\gamma + A_Z + \mathrm{non-factorizable}$$

- non-factorizable = terms that don't factorize into the Born-like amplitude
  - e.g.: weak boxes, exclusive hard QED radiation
- $\sigma_{e^+e^-\to f\bar{f}}$ is convoluted with ISR and FSR QED and FSR QCD

$$\sigma_{\mathrm{T}}(s) = \int_{z_0}^{1} dz H(z;s)\hat{\sigma}_{\mathrm{T}}(zs)$$

$$A_{FB}(s) = \frac{\pi\alpha^2 Q_e^2 Q_f^2}{\sigma_{\mathrm{tot}}} \int_{z_0}^{1} dz \frac{1}{(1+z)^2} H_{\mathrm{FB}}(z;s)\,\hat{\sigma}_{\mathrm{FB}}(zs)$$

- ISR radiator function known up to $\mathcal{O}(\alpha^3)$
  1. additive form

     G. Montagna, O. Nicrosini, F.P., PLB 406, (1997) 243
  2. factorized form

     S. Jadach, M. Skrzypek, B.F.L. Ward, PLB257 (1991) 173, M. Skrzypek, APPB23 (1992) 135
- H_{FB} known up to $\mathcal{O}(\alpha^2)$

$$
\begin{aligned}
\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\hat{\sigma}}{d\cos\theta}(e^+e^- \to f\bar{f}) \;=\;& \\
& \underbrace{|\alpha(s)Q_f|^2\,(1+\cos^2\theta)}_{\sigma^\gamma} \\
& \underbrace{-8\Re\left\{\alpha^*(s)Q_f\chi(s)\left[G_V^eG_V^f(1+\cos^2\theta)+2G_A^eG_A^f\cos\theta\right]\right\}}_{\gamma-Z\ \text{interference}} \\
& \underbrace{\begin{array}{l}+16|\chi(s)|^2\left[(|G_V^e|^2+|G_A^e|^2)(|G_V^f|^2+|G_A^f|^2)(1+\cos^2\theta)\right. \\ \qquad\qquad\left.+8\mathcal{R}\left\{G_V^eG_A^{e\,*}\right\}\mathcal{R}\left\{G_V^fG_A^{f\,*}\right\}\cos\theta\right]\end{array}}_{\sigma^{\mathrm{Z}}}
\end{aligned}
$$

with:

$$
\chi(s) \;=\; \frac{G_\mu M_Z^2}{8\pi\sqrt{2}}\frac{s}{s-M_Z^2+is\Gamma_Z/M_Z}
$$

- idea: characterize the Z resonance in a **model independent** way as a spin 1 resonance with general g_V^f, g_A^f couplings

A. Borrelli et al., Nucl.Phys. B333 (1990) 357

- **without γ exchange and rad. corr. except for pure QED/QCD**

$$\sigma^{\rm Z}_{\rm f\bar{f}} = \sigma^{\rm peak}_{\rm f\bar{f}} \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\sigma^{\rm peak}_{\rm f\bar{f}} = \frac{1}{R_{\rm QED}}\sigma^0_{\rm f\bar{f}} \qquad \sigma^0_{\rm f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}$$

$$\Gamma_f = c_f(=1,3)\frac{G_\mu M_Z^3}{6\sqrt{2}\pi}\left((g_V^f)^2 R_V^f + (g_A^f)^2 R_A^f\right)$$

$$R_V^f, R_A^f \Longrightarrow \text{FSR QED/QCD radiation and finite mass effects}$$

$$\frac{d\sigma_{\rm f\bar{f}}}{d\cos\theta} = \frac{3}{8}\sigma^{\rm tot}_{\rm f\bar{f}}\left[1+\cos^2\theta + 2\mathcal{A}_e\mathcal{A}_f\cos\theta\right]$$

$$\mathcal{A}_f = 2\frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2} \qquad A_{FB}^{0f} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

Actually there are γ exchange and rad. corrections

- G_V^f and G_A^f become complex quantities

$$
\begin{aligned}
\Gamma_f &= c_f(=1,3)\frac{G_\mu M_Z^3}{6\sqrt{2}\pi}\left(|G_V^f|^2 R_V^f + |G_A^f|^2 R_A^f\right) + \Delta_{\mathrm{EW/QCD}} \\
\mathcal{A}_f &= 2\frac{\mathrm{Re}[G_V^f (G_A^f)^*}{|G_V^f|^2 + |G_A^f|^2} \sim 2\frac{\mathrm{Re}(G_V^f)\mathrm{Re}(G_A^f)}{\mathrm{Re}(G_V^f)^2 + \mathrm{Re}(G_A^f)^2} \\
\sin^2\theta_{\mathrm{eff}}^f &\equiv \frac{1}{4|Q_f|}\left(1 - \frac{\mathrm{Re}(G_V^f)}{\mathrm{Re}(G_A^f)}\right) = \mathrm{Re}(k_f)\left(1 - \frac{M_W^2}{M_Z^2}\right)
\end{aligned}
$$

- in order to determine the MI PO, SM remnants ($Z-\gamma$ interference, non-factorizing rad. corrections, imaginary parts) need to be calculated and subtracted from data $\Longrightarrow$ level of dependence on SM parameters has to be checked

semi-analytical codes

- data fitting with the help of semianalitical codes
  - TOPAZ0

    G. Montagna et al., NPB401 (1993) 3; CPC 76 (1993) 328, CPC 93 (1996) 120; CPC 117 (1999) 278
  - ZFITTER

    D. Bardin et al., NPB351 (1991) 1l Z. Phys. C44 (1989) 493; PLB255 (1991) 290; CERN-TH.6442/1992;

    hep-ph/9412201; CPC 133 (2001) 229
  - realistic observables with analytical/one-dim numerical integration with kinematical cuts
    - ISR described with structure functions / radiator functions
    - IFI added with $\mathcal{O}(\alpha)$
    - different renormalization schemes, with G_μ, α, M_Z as input
    - exact electroweak one-loop plus available higher orders on top of Z peak (and other factorized higher orders like $\Delta\alpha(s)$)
  - Pseudo Observables
    - implementing all available higher order corrections on top of NLO

Two independent codes crucial to study the remaining intrinsic theoretical uncertainties

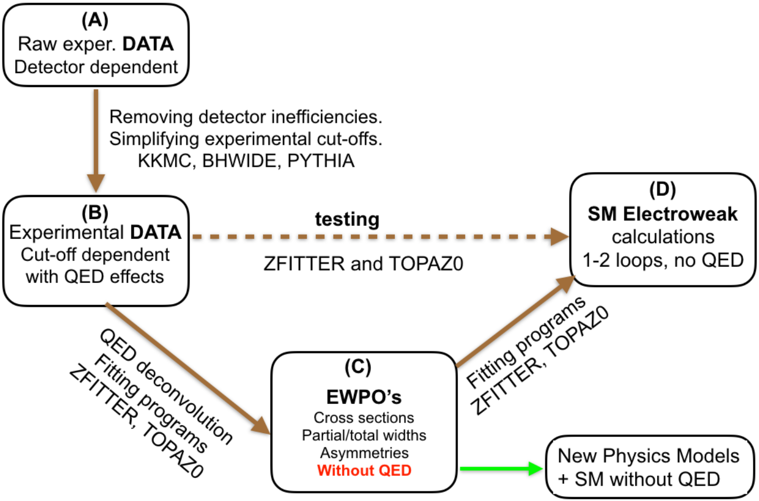
Schematic flow for data analysis

- row data extrapolated by means of Monte Carlo (KORALZ) to
  - idealized/simplified event selection (different for each experiment)
  - fully inclusive setup
- deconvolution of ISR/FSR QED/QCD effects
- subtraction of QED γ exchange and $Z-\gamma$ interference (this step depends on SM assumptions (eg.: m_t, m_H, $\alpha(M_Z)$, $\alpha_s(M_Z)$)
- calculation of the relevant SM remnants (for certain SM Lagrangian param values)

$$R_V^f, \qquad R_A^f, \qquad \Delta_{EW/QCD}, \qquad \mathrm{Im} G_V^f, \qquad \mathrm{Im} G_A^f$$

- extrapolated "Z-exchange data" can be used to make a 9- or 5-parameter fit
  - M_Z, Γ_Z, $\sigma^0_{\rm had}$, R^0_ℓ, $A^{0,\ell}_{FB}$, assuming lepton universality
  - M_Z, Γ_Z, $\sigma^0_{\rm had}$, R^0_e, R^0_μ, R^0_τ, $A^{0,e}_{FB}$, $A^{0,\mu}_{FB}$, $A^{0,\tau}_{FB}$

$$R_\ell = \frac{\Gamma_{\rm had}}{\Gamma_\ell}$$



A. Freitas, J. Gluza, , S. Jadach, in arXiv:1809.01830

- several calculations of higher order effects for PO $\Longrightarrow$ e.g.

$\sin^2\vartheta^\ell_{\rm eff}$ calculated in the SM: 0.231596 ± 0.000035

- at one loop $\mathcal{O}(\alpha)$

A. Sirlin, PRD22, (1980) 971, W.J. Marciano, A. Sirlin, PRD22 (1980) 2695

G. Degrassi, A. Sirlin, NPB352 (1991) 352, P. Gambino and A. Sirlin, PRD49 (1994) 1160

- at higher orders:
  - $\mathcal{O}(\alpha\alpha_s)$

A. Djouadi, C. Verzegnassi, PLB195 (1987) 265
B. Kiehl, NPB353 (1991) 567; B. Kniehl, A. Sirlin, NPB371 (1992) 141, PRD47 (1993) 883

A. Djouadi, P. Gambino, PRD49 (1994) 3499

  - $\mathcal{O}(\alpha\alpha_s^2)$

L. Avdeev et al., PLB336 (1994) 560;

K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394; NPB482 (1996) 213

  - $\mathcal{O}(\alpha\alpha_s^3)$

Y. Schröder, M. Steinhauser, PLB622 (2005) 124;

K.G. Chetyrkin et al., hep=ph/0605201; R. Boughezal, M. Czakon, hep-ph/0606232

  - $\mathcal{O}(\alpha^2)$ for large Higgs / top mass

G. Degrassi, P. Gambino, A. Sirlin, PLB394 (1997) 188

  - exact $\mathcal{O}(\alpha^2)$

M. Awramik, M. Czakon, A. Freitas, JHEP0611 (2006) 048

$\mathrm{M_W}$ calculated in the SM: $80.373 \pm 0.003\,\mathrm{GeV}$

- one loop $\mathcal{O}(\alpha)$ calculation

A. Sirlin, PRD22 (1980) 971

- two loop $\mathcal{O}(\alpha\alpha_s)$

A. Djouadi, C. Verzegnassi, PLB195 (1987) 265

- three loop $\mathcal{O}(\alpha\alpha_s^2)$

L. Avdeev et al., PLB336 (1994) 560;

K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394

- $\mathcal{O}(\alpha^2)$ for large top / Higgs mass

R. Barbieri et al., PLB288 (1992) 95; NPB409 (1993) 105

G. Degrassi, P. Gambino, A. Vicini, PLB383 (1996) 219

- exact $\mathcal{O}(\alpha^2)$

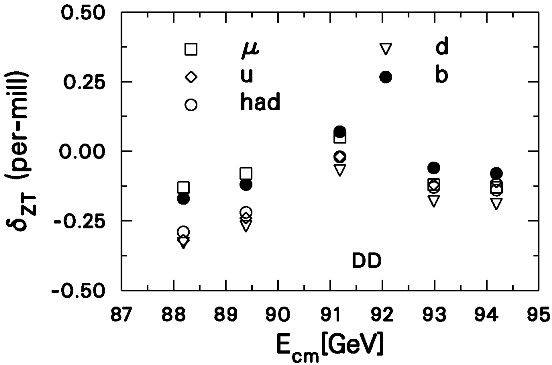
A. Freitas et al., PLB495 (2000) 338; NPB632 (2002) 189
M. Awramik, M. Czakon, PLB568 (2003) 48; PRL89 (2002) 241801

A. Onishchenko, O. Veretin, PLB551 (2003) 111; M. Awramik et al., PRD68 (2003) 053004

TH uncertainties on realistic observables

- final detailed estimate by TOPAZ0 and ZFITTER
  - within 0.01% at the Z peak
  - within 0.03∼0.05% at $\sqrt{s} = M_Z \pm 3$ GeV
  - some differences in estimates of IFI but ascribed to approximations in the analytical integrations

recalculated and clarified in P. Christova et al., hep-ph/9908289



Summary

- Th. predictions for LEP (Z peak ± 3 GeV) affected by uncertainties within $\sim$ 0.03% in the wings and 0.01% at peak
- main ingredients: one loop ew, resummed photon radiation, selected classes of two-loop corrections (e.g. fermionic corrections to $\Delta\alpha$, $\Delta\rho$, $Z \to f\bar{f}$), $\alpha^3_{\rm top}$ mixed $\mathcal{O}(\alpha\alpha_s^n)$, $\mathcal{O}(\alpha^m_{\rm top}\alpha_s^n)$
- "contamination" from SM in the MI parameters extraction $\sim 10^{-4}$
- $\sin^2\theta^\ell_{eff}$
  - measured through a model independent analysis of Z lineshape, with subtraction of the SM non-factorizing terms
  - calculated in the SM in the G_μ, α, M_Z scheme
  - consistency of the model independent results cross-checked with direct determination of Lagrangian parameters through realistic observables
- complete ew two-loop corrections to PO completed very recently