

# Theory issues for $\sin^2\theta_W$ measurement: QED and EW part

**E. Richter-Was, IF UJ, Kraków**

- **QED/EW corrections in data analysis**
  - EWPOs at LEP: theory meeting the data
  - $A_i$  and  $\sin^2\theta_{\text{eff}}$  measurement with ATLAS
  - EWPOs at LEP: QED ISR/FSR/IFI
- **Discussion on theory and parametric uncertainties**
  - Few comments on the EW schemes

# QED/EW corrections in data analysis: how it evolved with time

## LEP data analysis

- Libraries in semi-analytical fitting programs
  - ZFITTER ( DIZET ), TOPAZO
- Code of ZFITTER (DIZET) used in sophisticated MCs: KORALZ, BHLUMI, BHWIDE, KORALW, KKMC

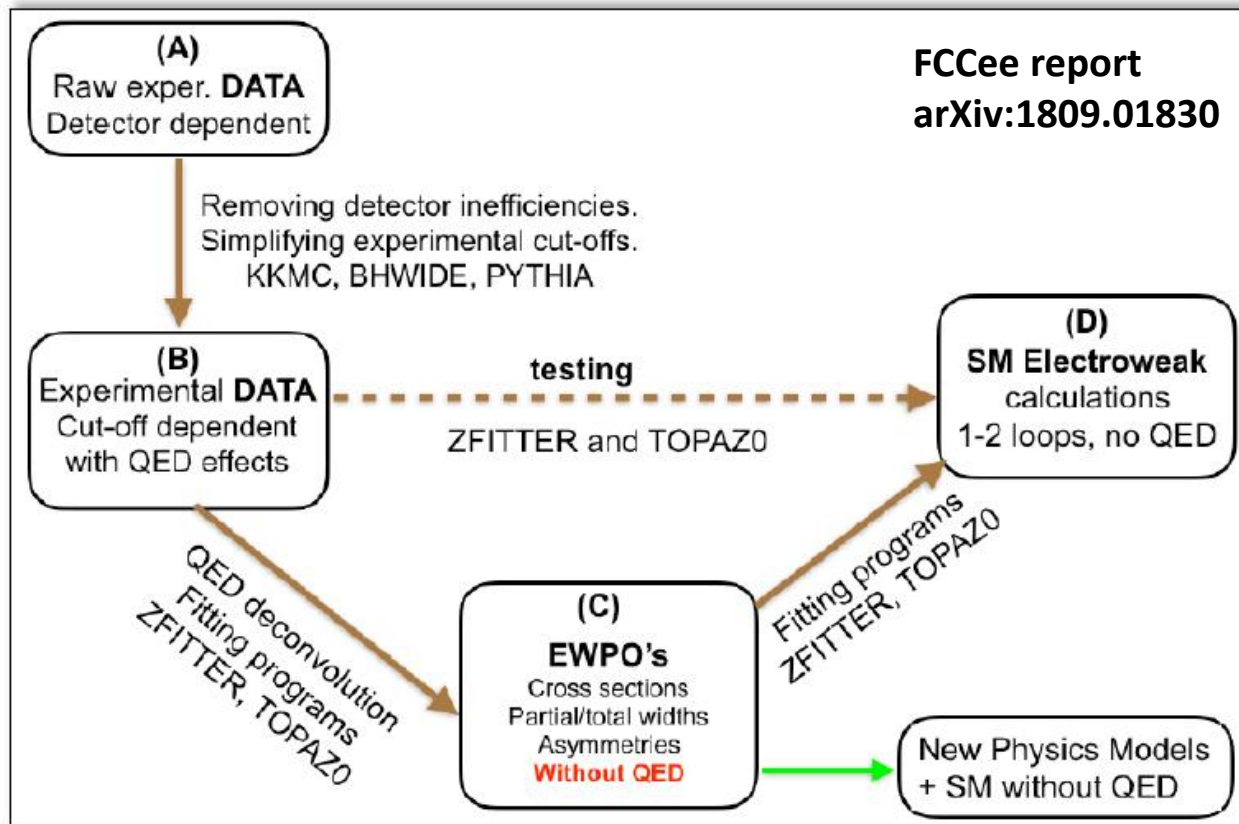
## Year 2010 and later

- ZFITTER → GFITTER, same/rewritten code for EW libraries, now being improved with more complete two-loop calculations. But the scheme of calculations has not changed
- PDG reviews: since several years based on  $\overline{\text{MS}}$  scheme for defining EW observables, GAPP program
- Tevatron analyses: based on LEP codes for QED/EW corrections
- FCCee: for now back to LEP codes as a starting point, 10-100 better precision needed

## LHC data analysis

- Changed approach for QED/EW calculations, motivated by easier? handling of QCD, QED and EW corrections simultaneously?
- Used widely in LHC MC's but precision not established to the LEP standards.

# Electroweak Pseudo-Observables at LEP: the meeting point between data and theory

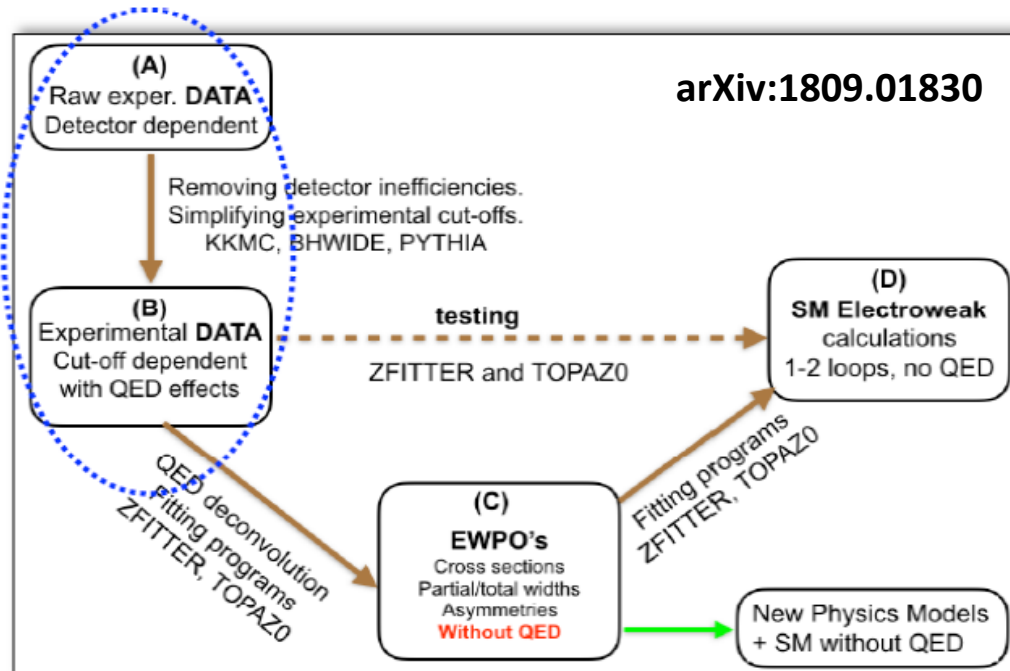


## EWPOs at LEP1 (OPAL )

$\chi^2/\text{dof} = 155/194$	
$m_Z$ [GeV]	$91.1858 \pm 0.0030$
$\Gamma_Z$ [GeV]	$2.4948 \pm 0.0041$
$\sigma_{\text{had}}^0$ [nb]	$41.501 \pm 0.055$
$R_e^0$	$20.901 \pm 0.084$
$R_\mu^0$	$20.811 \pm 0.058$
$R_\tau^0$	$20.832 \pm 0.091$
$A_{\text{FB}}^{0,e}$	$0.0089 \pm 0.0045$
$A_{\text{FB}}^{0,\mu}$	$0.0159 \pm 0.0023$
$A_{\text{FB}}^{0,\tau}$	$0.0145 \pm 0.0030$

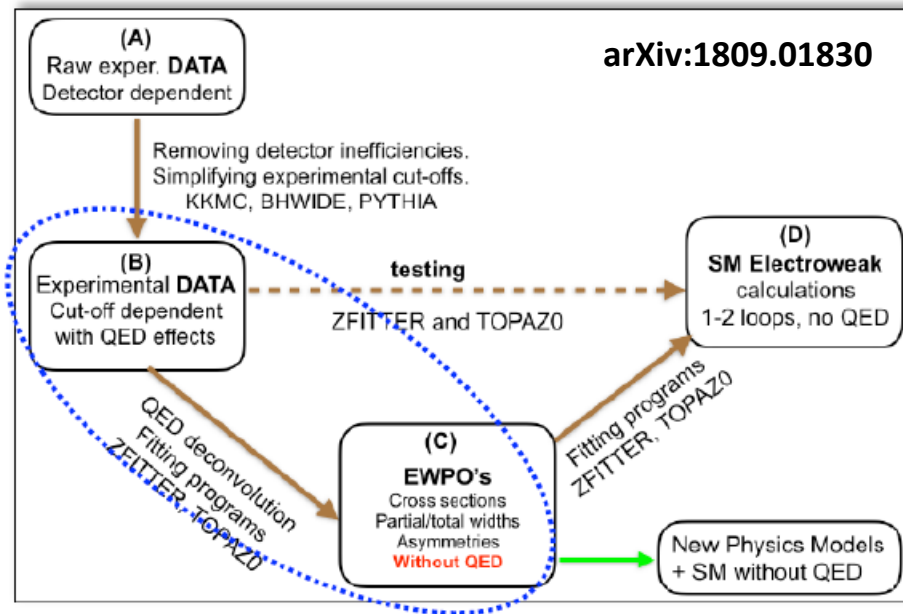
EWPOs are in the centre of the ADLO+SLD scheme in analysing/fitting data from LEP1  
arXiv:hep-ex/0509008

# Electroweak Pseudo-Observables at LEP



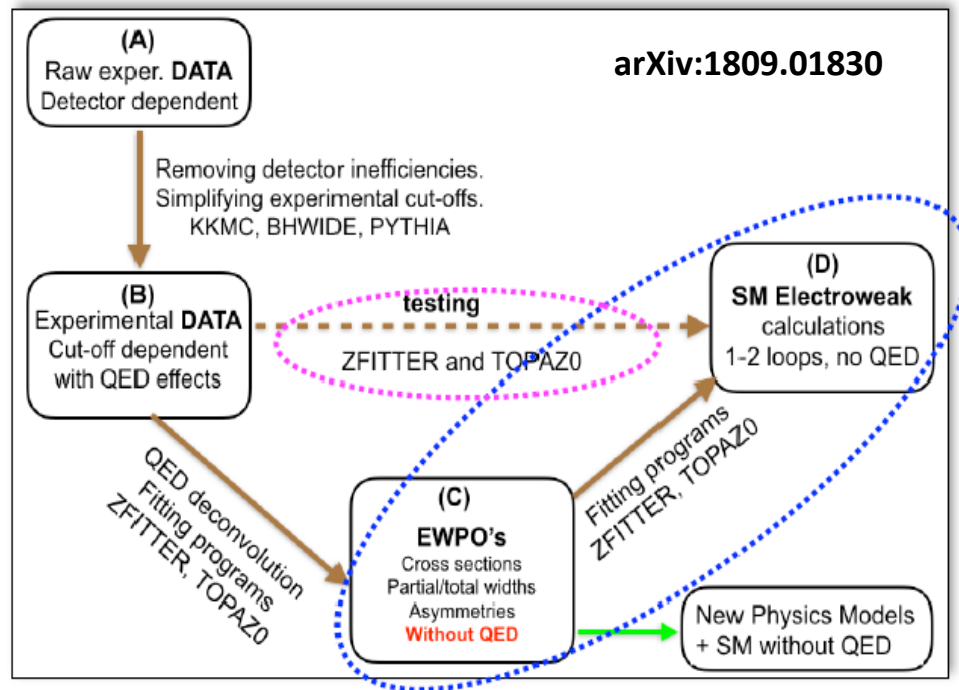
- From (A) to (B) raw data are corrected for inefficiencies of the detector and kinematics cut-offs and rounded to a simpler shape, which could be treated by non-MC fitter programs ZFITTER and TOPAZ0
- The transition (A) -> (B) was done with sophisticated MC event generators
- Data at stage (B) were obtained separately for each LEP experiment (cut-offs might be different)

# Electroweak Pseudo-Observables at LEP



- From (B) to (C) non-MC fitter programs ZFITTER and TOPAZO used to remove QED effects and cut-off dependence.
- This step introduced certain loss of precision, acceptable at LEP.
- An effective Born spin amplitudes used in the fitter programs.
- Combining data from all LEP experiments and SLD done at stage (C).

# Electroweak Pseudo-Observables at LEP



- In the (C) -> (D) step the fitting of the SM lagrangian parameters was done at LEP, with QED eliminated. The EWPOs at stage (C) represent data and in principle know nothing about parameters in the SM lagrangian.
- For a given LEP experiment important cross-check of (B)->(D) step directly was done.

# Electroweak Pseudo-Observables at LEP

## EWPO's: cross-sections, asymmetries

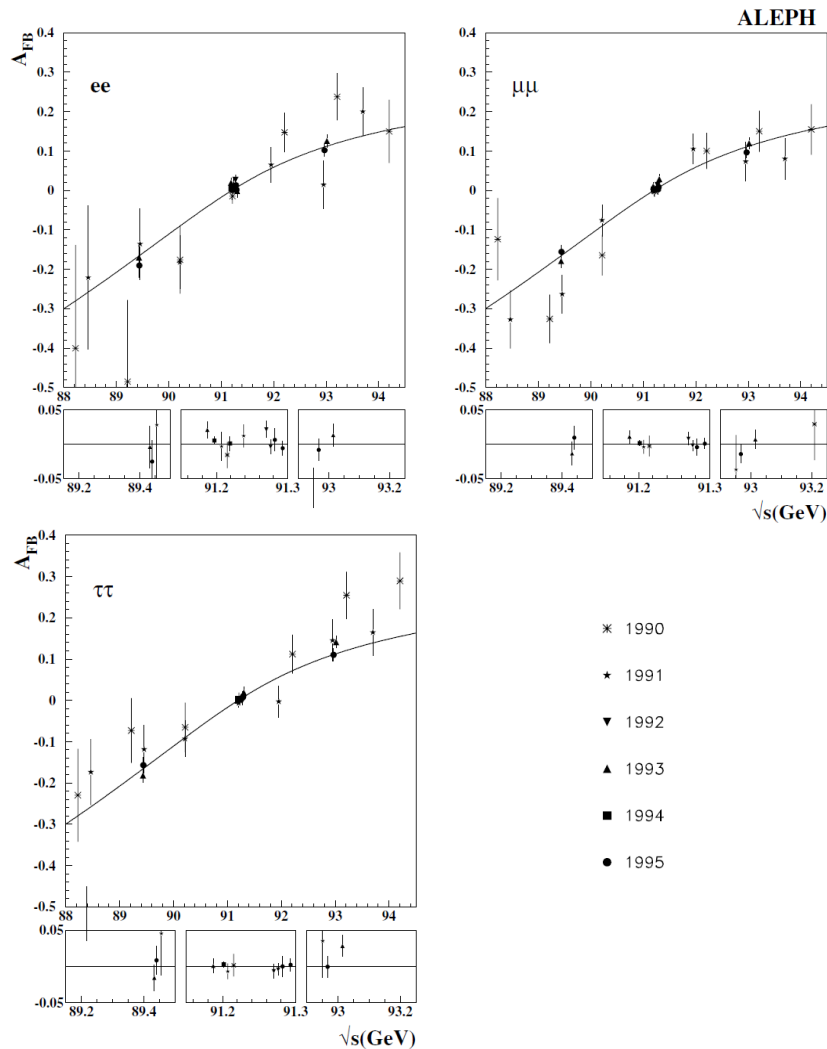
- Measurements at different energies,
- Extrapolated to the Z-peak
- Corrected for: QED ISR, imaginary part of the couplings, pure photon exchange, presence of box diagrams, etc.

$\chi^2/\text{dof} = 155/194$	
$m_Z$ [GeV]	$91.1858 \pm 0.0030$
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$A_{\text{FB}}^{0,\tau}$	$0.0145 \pm 0.0030$

## Example of functional dependence

$$A_{FB}^f(s) \simeq A_{FB}^f(m_Z^2) + \frac{(s - m_Z^2)}{s} \frac{3\pi\alpha(s)}{\sqrt{2}G_F m_Z^2} \frac{2Q_e Q_f g_{Ae} g_{Af}}{(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)}$$

# Electroweak Pseudo-Observables at LEP



E. Richter-Was, IF JU

## At the Z-pole

$$A_{FB}^0(e) = 0.0145 \pm 0.0025,$$

$$A_{FB}^0(\mu) = 0.0169 \pm 0.0013,$$

$$A_{FB}^0(\tau) = 0.0188 \pm 0.0017.$$

## Combination from four LEP experiments

$$A_{FB} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

$$\mathcal{A}_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} = \frac{2 g_{Ve}/g_{Ae}}{1 + (g_{Ve}/g_{Ae})^2}$$

## Then combined assuming lepton universality.

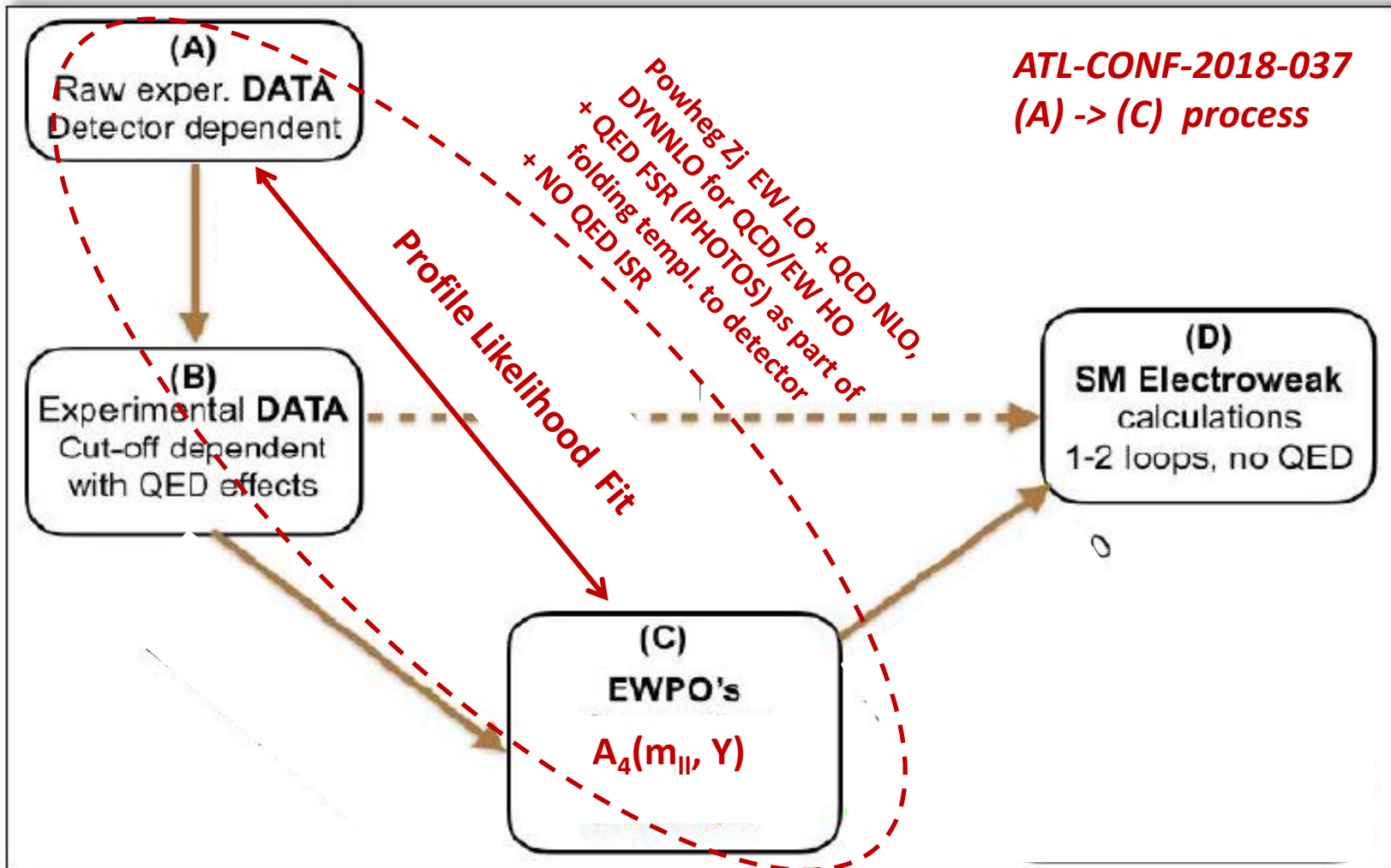
$$A_{FB}^0(\ell) = 0.0171 \pm 0.0010,$$

$$\sin^2 \theta_{\text{eff}}^\ell = 0.23099 \pm 0.00053$$

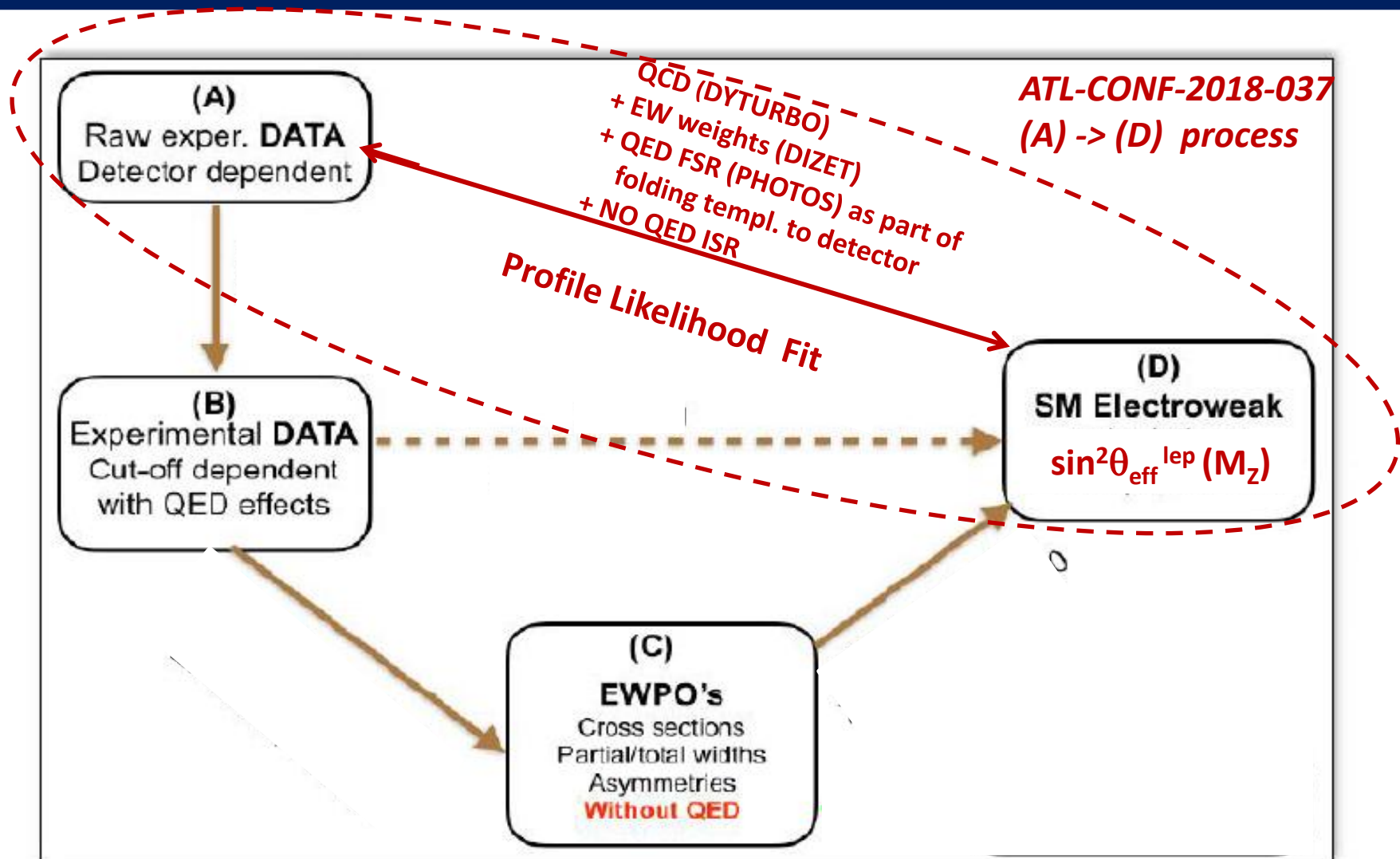
It is not a result of a single measurement  
at fixed energy!!

$$\sin^2 \theta_{\text{eff}}^\ell \equiv \frac{1}{4} \left( 1 - \frac{g_{V\ell}}{g_{A\ell}} \right)$$

# ATLAS ZAi analysis: $A_4$ measurement



# ATLAS ZAi analysis: $\sin^2\theta_w$ measurement



# ATLAS ZAi analysis: $\sin^2\theta_w$ measurement

$$\mathcal{L}(A, \sigma, \theta | N_{\text{obs}}) = \prod_n^{N_{\text{bins}}} \left\{ P(N_{\text{obs}}^n | N_{\text{exp}}^n(A, \sigma, \theta)) P(N_{\text{eff}}^n | \gamma^n N_{\text{eff}}^n) \right\} \times \prod_m^M G(0 | \beta^m, 1)$$

N exp. in data  
phase-space  
(uncorrected  
data)

Likelihood fit

Templates  
t<sub>ij</sub> folded with  
MC to data  
phase-space

## Fit model

$$N_{\text{exp}}^n(A, \sigma, \theta) = \left\{ \sum_{j=0}^{N_{\text{bins}}} \sigma_j \times L \times \left[ t_{8j}^n(\beta) + \sum_{i=0}^7 A_{ij} \times t_{ij}^n(\beta) \right] \right\} \times \gamma^n + \sum_B^{\text{bkgs}} T_B^n(\beta),$$

where:

- $N_{\text{bins}} = 1280$  is the total number of measurement bins in  $(\cos\theta, \phi, m^{\ell\ell}, y^{\ell\ell})$  space
- $A$  is the set of all angular coefficients,  $A_{ij}$
- $\sigma$  is the set of all polarised cross sections,  $\sigma_j$
- $\theta$  is the set of all nuisance parameters representing the systematic uncertainties,  $\{\beta^m, \gamma^n\}$
- $t_{ij}$  is the set of all signal  $P_i$  templates
- $T_B$  is the set of background templates, where the sum runs over all background sources
- $L$  is the total integrated luminosity.

single value at  $M_z^2$

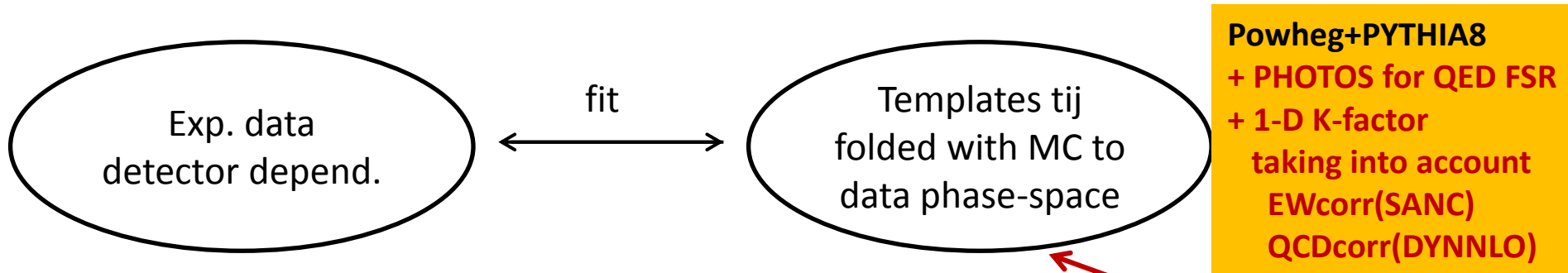


Instead of fitting  $A_{4j}$ , we fit  $\sin^2\theta_{\text{eff}}$ ,  
given predicted  $A_{4j}(\theta)$

## EW fit model

$$A_{4,j}(\sin^2\theta_{\text{eff}}^\ell, \theta) = a_j(\theta) \times \sin^2\theta_{\text{eff}}^\ell + b_j(\theta)$$

# ATLAS ZAi analysis: $\sin^2\theta_w$ measurement



QED FSR: treated as part of folding into data phase-space

Template shapes:

-> no QED IFI

-> no EW box correction ( $s, \cos\theta$ )

$$A_{4,j}(\sin^2\theta_{\text{eff}}^\ell, \theta) = a_j(\theta) \times \sin^2\theta_{\text{eff}}^\ell + b_j(\theta)$$

EW corrections to  $A_{4j}$ :

- EW genuine and lineshape, estimated with  $wt^{\text{EW}}$  using Improved Born Approximation and Dizet form-factors
- QED ISR (not yet)
- QED IFI (not yet, negligible?)

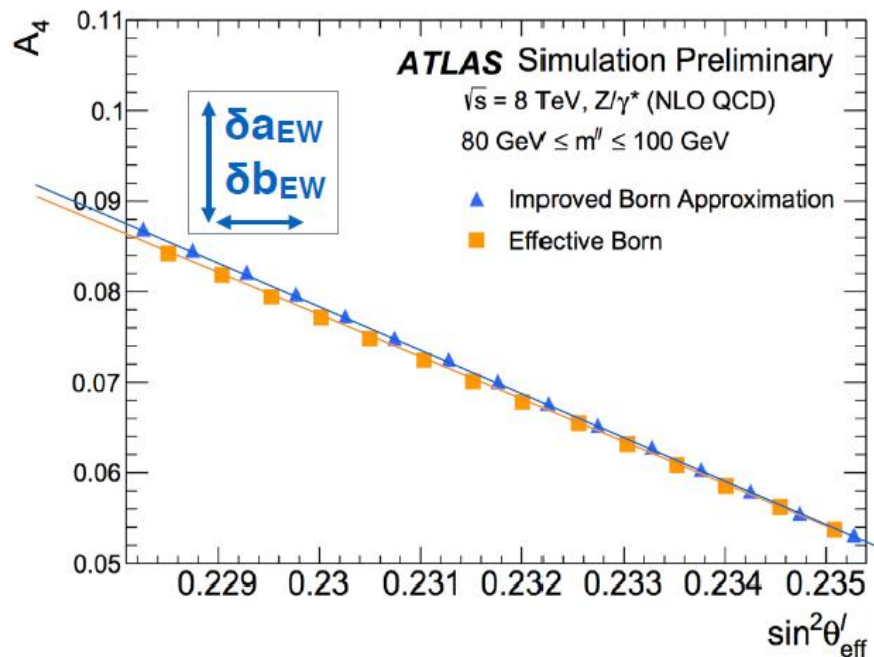
**DYTURBO**  
+ EWcorr: form-factors (DIZET)

# ATLAS ZAi: EW corrections to A4

LO EW

NLO+HO EW

$$A_4 = a \cdot \sin^2\theta_W + b \rightarrow (a + \delta a_{EW}) \cdot \sin^2\theta_{eff} + (b + \delta b_{EW})$$



$$v_\ell = (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 \cdot K_\ell(s, t) + \delta_V)) / \Delta$$

↑  
 nominal  $\sin^2\theta_{eff}$

## Step A:

A4 calculated using PowhegZj MC. Adding term  $\delta V$  to vector couplings allows to derive  $A_4(\sin^2\theta_{eff})$  dependency.

1) EW weights with form-factors from Dizet library

2) EW weights with effective Born-like couplings

—  $\delta a_{EW}, \delta b_{EW}$  calculated assuming linear relation for  $\sin^2\theta_{eff}^{LEP} \pm 100 \cdot 10^{-5}$

## Step B:

A4, **a**, **b** calculated using DYTURBO with  $\sin^2\theta_W = 0.23152 \pm 100 \cdot 10^{-5}$  and Born ME

—  $\delta a_{EW}, \delta b_{EW}$  derived in step (A) applied as shift to **a**, **b** in step (B)

# QED corrections: LEP legacy

## In nutshell

- KMC : Monte Carlo, complete  $O(\alpha^2)$ , soft resummation for QED ISR, FSR, IFI
- ZFITTER, TOPAZO: integrators, complete  $O(\alpha)$ , including IFI, combined with „QED radiator” functions for ISR, FSR
- Why rather crude QED treatment in ZFITTER/TOPAZO worked for LEP 1?:
  - Suppressed hard photons in ISR due to resonance structure of the Z boson. The „QED-radiator” function was sufficient.
  - Suppressed IFI because of  $\Gamma_Z/M_Z$  ratio around the Z-peak. Kinoshita-Lee-Nauenberg theorem for FSR.

# QED corrections: LEP legacy

**Precision**: calculations are not done order-by-order

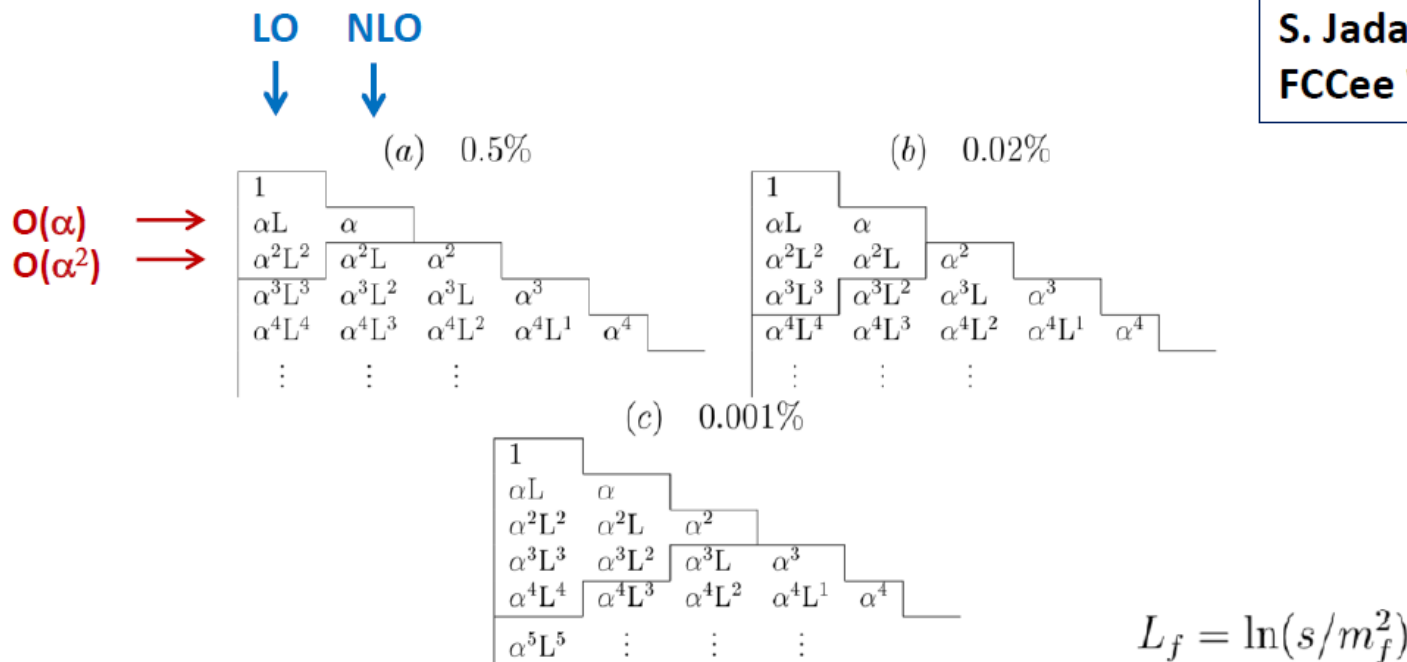


Figure 2: QED perturbative leading and subleading corrections. Rows represent corrections in consecutive perturbative orders – the first row is the Born contribution. The first column represents the leading logarithmic (LO) approximation and the second column depicts the next-to-leading (NLO) approximation. In the figure, terms selected for the same precision level (a)  $5 \cdot 10^{-2}$  (b)  $2 \cdot 10^{-3}$  and (c)  $1 \cdot 10^{-5}$  are limited with the help of an additional line.

# Residual QED corrections: LEP EWPO's

After deconvoluting for QED effects, small residual dependence remains and added to experimental error.

Observable	Where from	Present
$M_Z$ [MeV]	Z linesh. [2]	$91187.5 \pm 2.1\{0.3\}$
$\Gamma_Z$ [MeV]	Z linesh. [2]	$2495.2 \pm 2.1\{0.2\}$
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [3]	$20.767 \pm 0.025\{0.012\}$
$\sigma_{\text{had}}^0$	$\sigma_{\text{had}}^0$ [2]	$41.541 \pm 0.037\{25\}\text{nb}$
$N_\nu$	$\sigma(M_Z)$ [2]	$2.984 \pm 0.008\{0.006\}$
$N_\nu$	$Z\gamma$ [4]	$2.69 \pm 0.15\{0.06\}$
$\sin^2 \theta_W^{\text{eff}}$	$A_{FB}^{\text{lept.}}$ [3]	$0.23099 \pm 0.00053\{06\}$
$\sin^2 \theta_W^{\text{eff}}$	$A_{\text{pol.}}^\tau$ [2, 3]	$0.23159 \pm 0.00041\{12\}$
$M_W$ [MeV]	ADLO [5]	$80376 \pm 33\{7\}$
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [2]	$\pm 0.020\{0.001\}$

S. Jadach  
FCCee WS 2019

Babha scattering  
luminosity measurement

Table 1: Table of present experimental precision of electroweak observables, which are most sensitive to QED effects. The numbers in the braces {...} in the 3-rd column are component of the systematic error components due to QED calculation uncertainty.

# QED corrections: LEP legacy

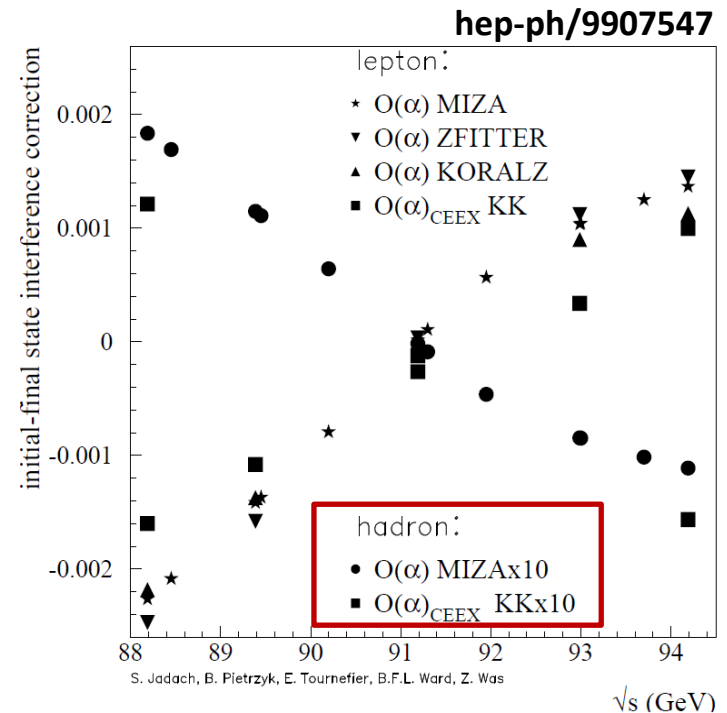
## QED Initial-final state Interference (IFI)

- Suppressed by the  $\Gamma_Z/M_Z$  factor, does not contain mass logarithms. Additional factor 10 of suppression comes from partial cancellation of contributions from different quark flavours
- Codes used at LHC should reproduce LEP benchmarks around Z-pole

$$\frac{\sigma^{IFI}}{\sigma} \sim Q_e Q_f \frac{\alpha}{\pi} \text{Max} \left\{ \frac{\Gamma_Z}{M_Z} ; \frac{s - M_Z^2}{M_Z^2} \right\}$$

## QED IFI: (arXiv:hep-ph/9907547)

- 0.02% corrections to cross-section  $\sigma_{\text{had}}$  at the Z peak
- 0.15 MeV for the Z mass shift at  $M_Z$



# Theoretical and parametric uncertainties

## ALDO+ SLD + Tevatron, arXiv:1012.2367

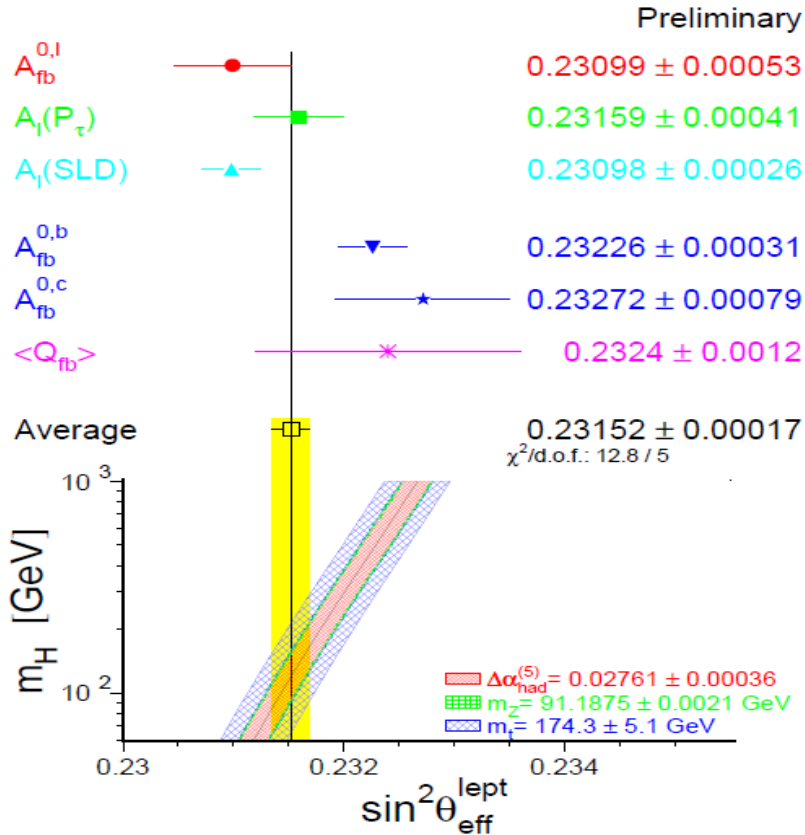
- The remaining theoretical uncertainties were estimated to be 4 MeV on  $m_W$  and **0.000049 on  $\sin^2\theta_{\text{eff}}^{\text{lep}}$** . We can use this estimate @ Z-pole.
- The parametric uncertainties were dominated by  $\Delta\alpha_{\text{had}}(M_Z^2)$ . The uncertainty of  **$\pm 0.00035$  caused an error of  $\pm 0.00013$  on  $\sin^2\theta_{\text{eff}}^{\text{lep}}$** .

## ATLAS:

- We are using the same code for calculating EW genuine corrections. We can use this estimate.  
**Theoretical uncertainties (EW, @Z-pole):  $5 \cdot 10^{-5}$**
- The parametric uncertainties should be updated for better precision on  $\Delta\alpha_{\text{had}}(M_Z^2)$  and  $m_t$ .

# Re-discovered impact of $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

arXiv:hep-ex/0112021



## Recent measurements:

arXiv: 1706.09436

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02753 \pm 0.00009$$

arXiv: 1711.06089

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02774 \pm 0.00016$$

$$= 0.02752 \pm 0.00012$$

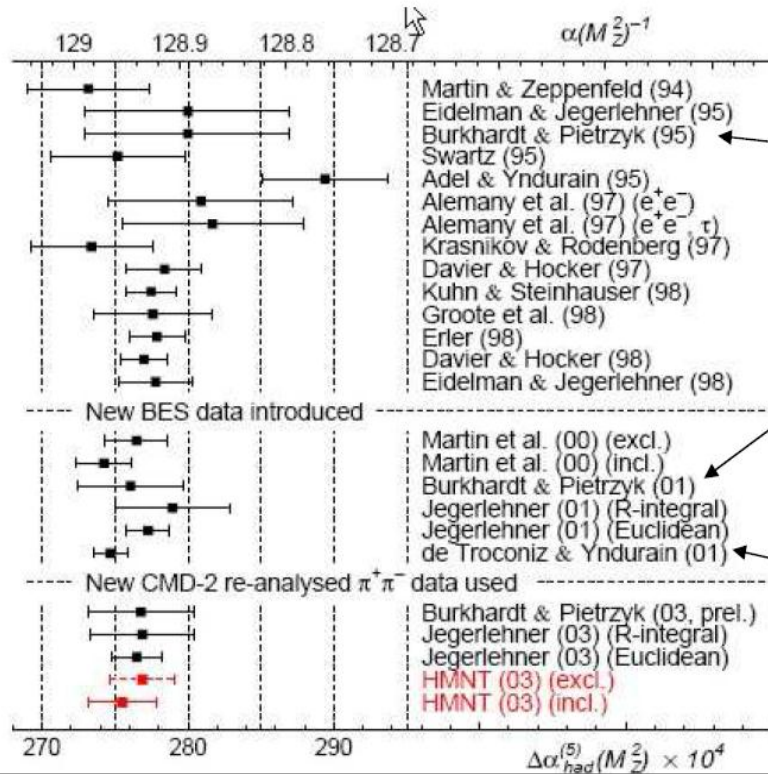
arXiv: 1802.02995

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02761 \pm 0.00011$$

$\pm 0.00036$  on  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$   $\rightarrow$   $\pm 0.00013$  on  $\sin^2\theta_{\text{eff}}^{\text{lept}}$

# Re-discovered impact of $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

Dominant uncertainty for LEP  $\sin^2\theta_{\text{eff}}$  measurements



At end of LEP  $\Delta\alpha_{\text{had}}$  became limiting uncertainty in SM fits.

$$\Delta\alpha_{\text{had}}(M_Z) = 0.02804 \pm 0.00065$$

Post LEP measurements from BES and CMD-2 improvement.

$$\Delta\alpha_{\text{had}}(M_Z) = 0.02758 \pm 0.00035$$

*used by LEP EW WG*

Using perturbative QCD

$$\Delta\alpha_{\text{had}}(M_Z) = 0.02749 \pm 0.00012$$

Dizet 6.21, 6.44, both version have implemented old parametrisations:

Eidelman&Jegerlehner (95)

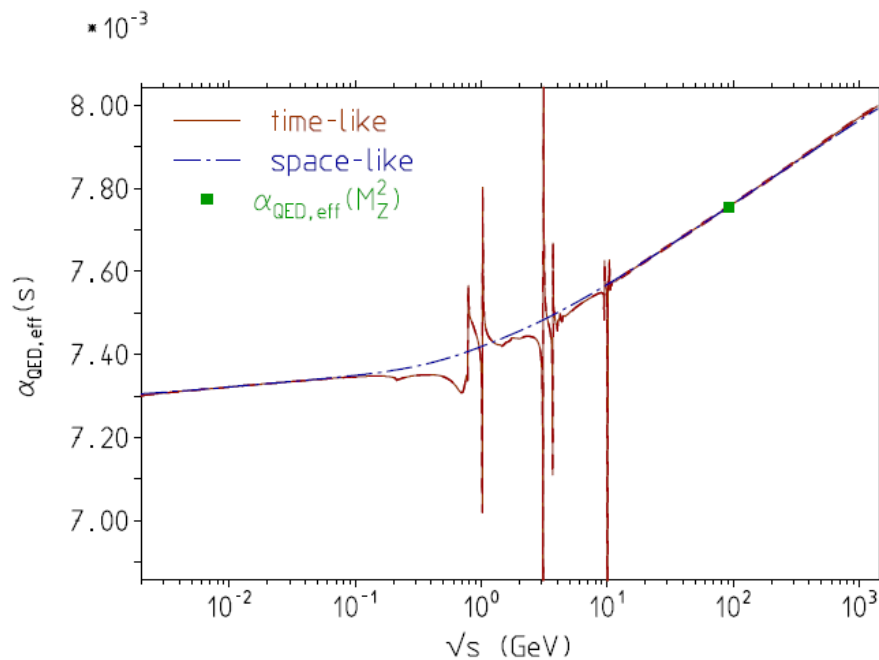
Burkhardt&Pietrzyk (95)

$$\Delta\alpha_{\text{had}}(M_Z) = 0.02804 \pm 0.00065$$

# $\alpha_{\text{QED}}(s)$ and $\sin^2\theta_W(s)$

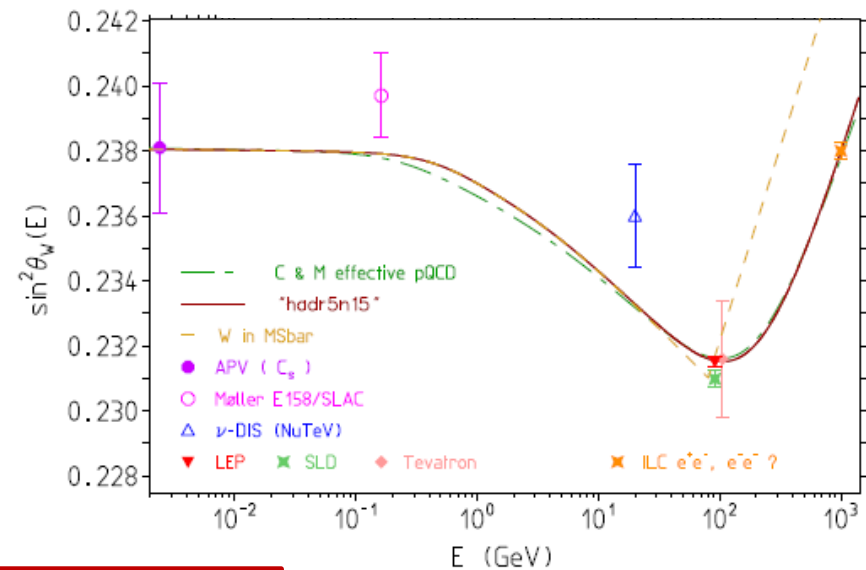
F. Jegerlehner  
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$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}; \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)$$



$$\sin^2\theta_i \cos^2\theta_i = \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2} \frac{1}{1-\Delta r_i}$$

$$\Delta r_i = \Delta r_i(\alpha, G_\mu, M_Z, m_H, m_{f \neq t}, m_t)$$



$$\Delta\alpha_{\text{hadrons}}^{(5)}(M_Z^2) = 0.027738 \pm 0.000158 [0.027523 \pm 0.000119]$$

# Impact of $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

Dizet 6.21, 6.42

NEW parametrisation

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0280398$ (param. Jegerlehner 1995)	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0275762$ (param. Jegerlehner 2017)	$\Delta$
$\alpha(M_Z^2)$	0.00775884	0.00775492	
$1/\alpha(M_Z^2)$	128.885224	128.9503292	
$s_W^2$	0.22351946	0.22332758	- 0.00019
$\sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23175990	0.23158294	- 0.00018
$\sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23164930	0.23147645	- 0.00017
$\sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23152214	0.23134945	- 0.00017
$M_W$	80.35281 GeV	80.36274 GeV	+10 MeV
$\Delta r$	0.03694272	0.036609	
$\Delta r_{\text{rem}}$	0.01169749	0.01170287	

$$M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1 - \Delta r)}}}$$

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

# Impact of AMT=4 → AMT=6

Updated to Dizet 6.42, which comes with more complete two-loop corrections

NEW

Parameter	AMT4= 4	AMT4 = 6	$\Delta$
$\alpha(M_Z^2)$	0.00775492	0.00775492	
$1/\alpha(M_Z^2)$	128.9503239	128.9503239	
$s_W^2$	0.22332758	0.22343647	+ 0.00011
$\sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23158294	0.23153917	-0.000044
$\sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23147645	0.23143261	-0.000044
$\sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23134945	0.23130551	-0.000044
$M_W$	80.36274 GeV	80.35710 GeV	- 5.6 MeV
$\Delta r$	0.036609	0.03636609	
$\Delta r_{rem}$	0.01170287	0.01170287	

$$M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1 - \Delta r)}}}$$

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

- **AMT4=4:** Subleading two-loop corrections and re-summation recipe (best option Dizet 6.21)
- **AMT4=6:** Complete two-loop corrections to  $M_W$  and fermionic two-loop corrections to  $\sin^2\theta_{eff}^{lep}$  (best option in Dizet 6.42)

# Parametric uncertainties

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2) - 0.0001$	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0275762$	$\Delta\alpha_h^{(5)}(M_Z^2) + 0.0001$	$\Delta/2$
$\alpha(M_Z^2)$	0.0077540999	0.0077549240	0.0077557482	
$1/\alpha(M_Z^2)$	128.9640328306	128.9503292550	128.9366256793	
$s_W^2$	0.22340146	0.22343647	0.22347148	0.000035
$\sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23150412	0.23153917	0.23157421	0.000035
$\sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23139759	0.23143261	0.23146763	0.000035
$\sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23127052	0.23130551	0.23134049	0.000035
$M_W$	80.35892 GeV	80.35710 GeV	80.35529 GeV	1.8 MeV
$\Delta r$	0.03625683	0.036609	0.03647535	
$\Delta r_{rem}$	0.01170310	0.01170287	0.01170264	

Parameter	$m_t - 0.5$ GeV	$m_t = 173.2$ GeV	$m_t + 0.5$ GeV	$\Delta/2$
$\alpha(M_Z^2)$	0.0077549205	0.0077549240	0.0077549274	
$1/\alpha(M_Z^2)$	128.9503873792	128.9503292550	128.9502716590	
$s_W^2$	0.22349450	0.22343647	0.22337836	0.000058
$\sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23155486	0.23153917	0.23152344	0.000016
$\sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23144830	0.23143261	0.23141688	0.000016
$\sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23132119	0.23130551	0.23128979	0.000016
$M_W$	80.354102 GeV	80.35710 GeV	80.360111 GeV	3 MeV
$\Delta r$	0.03654697	0.036609	0.03618477	
$\Delta r_{rem}$	0.01169343	0.01170287	0.01171229	

## At Z-pole:

**Parametric uncertainties: 4 10<sup>-5</sup> [35 10<sup>-6</sup> (from  $\Delta\alpha_{had}$ ) and 16 10<sup>-6</sup> ( from  $m_t$ )]**

# EW schemes: LEP and LHC paradigms

## LEP

- EW scheme: „on-mass-schell” regularisation; input:  $(\alpha(0), G_F, M_Z)$ , the most precisely known quantities:  
 $\Delta\alpha/\alpha \sim 3.6 \cdot 10^{-9}$ ;  $\Delta G_F/G_F \sim 8.6 \cdot 10^{-6}$ ;  $\Delta M_Z/M_Z \sim 2.4 \cdot 10^{-5}$
- TH precision @Z-pole: 4 MeV on  $m_W$ ;  $5 \cdot 10^{-5}$  on  $\sin^2\theta_{\text{eff}}$
- Parametric precision on  $\sin^2\theta_{\text{eff}}$  :  $4 \cdot 10^{-5}$  (dominated by  $\Delta\alpha_{\text{had}}$  )
- Used and developed still today: GFitter, FCCee preparation.

## LHC

- EW scheme: „pole-mass” regularisation; input:  $(G_F, M_Z, M_W)$   
 $\Delta G_F/G_F \sim 8.6 \cdot 10^{-6}$ ;  $\Delta M_Z/M_Z \sim 2.4 \cdot 10^{-5}$ ;  $\Delta M_W/M_W \sim 1.9 \cdot 10^{-4}$
- Relations to EWPO's: not established.
- EW corrections complete at EW NLO, only some at EW NLO+HO
- TH precision @Z-pole: not established.
- Parametric precision:  $\pm 15$  MeV on  $M_W$  ->  $30 \cdot 10^{-5}$  on  $\sin^2\theta_w$

# My „shopping list”

- **Conclude on question asked at some meetings if „ $\sin^2\theta_{\text{eff}}^{\text{lep}}$ ” measured at LEP and LHC are the same physics quantities.**
- **Explore more LEP legacy benchmarks @Z-pole.**
  - Eg. to confirm consistency of LHC estimates for IFI.
- **Concise and conclude on theory and parametric uncertainties between „LEP EW scheme” and „LHC EW scheme”**

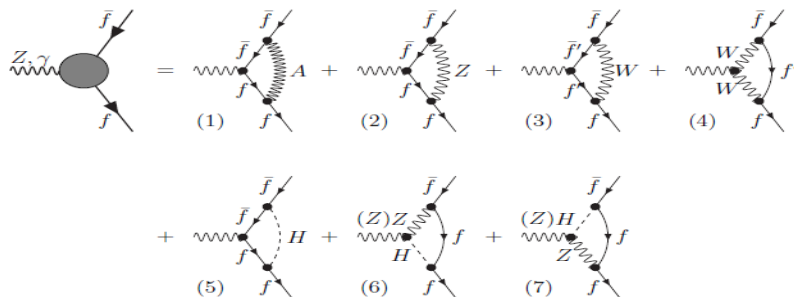
# Extra slides

# Genuine EW and lineshape corrections

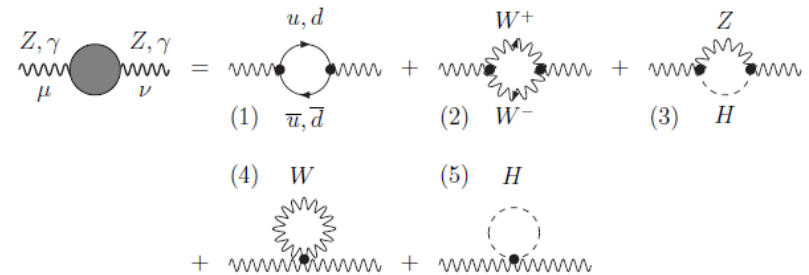
Gauge-invariant set of diagrams.

For IBA approach calculated form-factor corrections to couplings, propagators and masses.

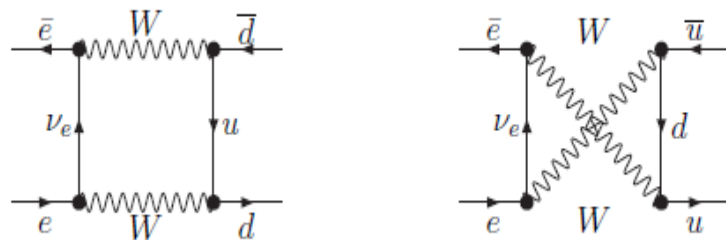
## Zff and $\gamma$ ff vertices



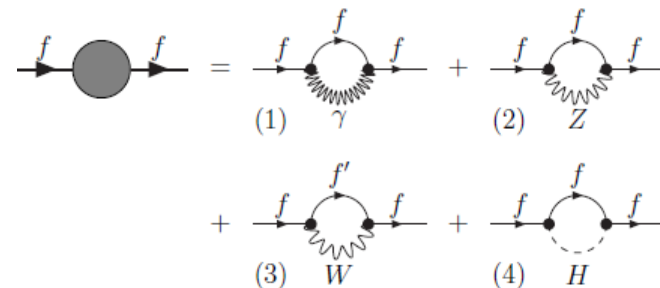
## Bosonic self-energies



## WW, ZZ boxes (shown only WW diagrams)



## Fermionic self-energies



# From Zfitter/Dizet documentation

D. Bardin et al.  
arXiv:9908433

**Zfitter** is a **semi-analytical program** for calculating total cross-sections and pseudo-observables (eg.  $A_{fb}$ ,  $\sin^2\theta_w^{\text{eff}}$ ), used by LEP1, and to a lesser degree by LEP2.

**DIZET** is a library for calculating form-factors and some other corrections. Provides complete EW  $O(\alpha)$  weak-loop corrections supplemented with selected higher order terms (eg. vacuum polarisation,  $\alpha_{\text{QED}}(Q^2)$  ).

For analyses at LEP1, LEP2 used always in parallel with **MC generators (KoralZ, KoralW)** eg. to evaluate systematics of simplified cuts used in analysis integration.

$$\begin{aligned} \mathcal{A}_Z^{OLA}(s, t) = & i\sqrt{2}G_\mu I_e^{(3)} I_f^{(3)} M_Z^2 \chi_Z(s) \boxed{\rho_{ef}(s, t)} \left\{ \gamma_\mu(1 + \gamma_5) \otimes \gamma_\mu(1 + \gamma_5) \right. \\ & - 4|Q_e|s_W^2 \boxed{\kappa_e(s, t)} \gamma_\mu \otimes \gamma_\mu(1 + \gamma_5) - 4|Q_f|s_W^2 \boxed{\kappa_f(s, t)} \gamma_\mu(1 + \gamma_5) \otimes \gamma_\mu \\ & \left. + 16|Q_e Q_f|s_W^4 \boxed{\kappa_{e,f}(s, t)} \gamma_\mu \otimes \gamma_\mu \right\}. \end{aligned} \quad (\text{A.4.75})$$

one loop  
amplitude

$$A_\gamma^{OLA} = i\chi_\gamma(s) \boxed{\alpha(s)} \gamma_\mu \otimes \gamma_\mu. \quad (2.2.36)$$

Dyson summation leads to the change of  $\alpha$  into  $\alpha(s)$ :

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha^{\text{fer}}(s)} = \frac{\alpha(0)}{1 - \boxed{\Delta\alpha^{(5)}(s) - \Delta\alpha^t(s) - \Delta\alpha^{\alpha\alpha_s}(s)}}. \quad (2.2.37)$$

Vacuum polarisation  
corrections

# LEP legacy: from Zfitter/Dizet documentation

After some trivial algebra one derives the final expressions:

$$\boxed{\rho_{ef}} = 1 + \frac{g^2}{16\pi^2} \left\{ -\Delta\rho_z^F + \mathcal{D}_z^F(s) + \frac{5}{3}B_0^F(-s; M_W, M_W) - \frac{9}{4}\frac{c_W^2}{s_W^2} \ln c_W^2 - 6 \right. \\ \left. + \frac{5}{8}c_W^2(1 + c_W^2) + \frac{1}{4c_W^2}(3v_e^2 + a_e^2 + 3v_f^2 + a_f^2)\mathcal{F}_z(s) + \hat{\mathcal{F}}_w^0(s) + \hat{\mathcal{F}}_w(s) \right. \\ \left. - \frac{r_t}{4}[B_0^F(-s; M_W, M_W) + 1] - c_W^2(R_z - 1)s\hat{\mathcal{B}}_{WW}^d(s, t) \right\}, \quad (\text{A.4.80})$$

$$\boxed{\kappa_e} = 1 + \frac{g^2}{16\pi^2} \left\{ -\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi_{Z\gamma}^F(s) - \frac{1}{6}B_0^F(-s; M_W, M_W) - \frac{1}{9} - \frac{v_e\sigma_e}{2c_W^2}\mathcal{F}_z(s) \right. \\ \left. - \hat{\mathcal{F}}_w^0(s) + (R_z - 1)\left[\frac{|Q_f|}{2}(1 - 4|Q_f|s_W^2)\mathcal{F}_z(s) + c_W^2[\hat{\mathcal{F}}_{wn}(s) \right. \right. \\ \left. \left. - |Q_f|\mathcal{F}_{wa}(s) + s\hat{\mathcal{B}}_{WW}^d(s, t)\right] \right\}, \quad (\text{A.4.81})$$

$$\boxed{\kappa_f} = 1 + \frac{g^2}{16\pi^2} \left\{ -\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi_{Z\gamma}^F(s) - \frac{1}{6}B_0^F(-s; M_W, M_W) - \frac{1}{9} - \frac{v_f\sigma_f}{2c_W^2}\mathcal{F}_z(s) \right. \\ \left. - \hat{\mathcal{F}}_w(s) + (R_z - 1)\left[\frac{|Q_e|}{2}(1 - 4|Q_e|s_W^2)\mathcal{F}_z(s) + c_W^2[\hat{\mathcal{F}}_{wn}^0(s) \right. \right. \\ \left. \left. - |Q_e|\mathcal{F}_{wa}(s) + s\hat{\mathcal{B}}_{WW}^d(s, t)\right] - \frac{r_t}{4}[B_0^F(-s; M_W, M_W) + 1] \right\}, \quad (\text{A.4.82})$$

interference

$$\boxed{\kappa_{ef}} = 1 + \frac{g^2}{16\pi^2} \left\{ -2\frac{c_W^2}{s_W^2}\Delta\rho^F - 2\Pi_{Z\gamma}^F(s) - \frac{1}{3}B_0^F(-s; M_W, M_W) - \frac{2}{9} \right. \\ \left. - \frac{1}{4c_W^2}\left[\frac{\delta_e^2 + \delta_f^2}{s_W^2}(R_W - 1) + 3v_e^2 + a_e^2 + 3v_f^2 + a_f^2\right]\mathcal{F}_z(s) \right. \\ \left. - \hat{\mathcal{F}}_w^0(s) - \hat{\mathcal{F}}_w(s) - \frac{r_t}{4}[B_0^F(-s; M_W, M_W) + 1] \right. \\ \left. + c_W^2(R_z - 1)\left[\frac{2}{3} - \hat{\Pi}_{\gamma\gamma}^{\text{bos}, F}(s) + s\hat{\mathcal{B}}_{WW}^d(s, t)\right] \right\}. \quad (\text{A.4.83})$$

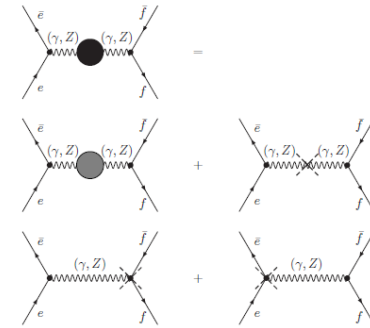


Figure A.11. Bosonic self-energies and bosonic counter-terms for  $e\bar{e} \rightarrow (Z, \gamma) \rightarrow f\bar{f}$

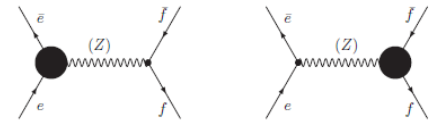


Figure A.10. Electron (a) and final fermion (b) vertices in  $e\bar{e} \rightarrow (Z) \rightarrow f\bar{f}$

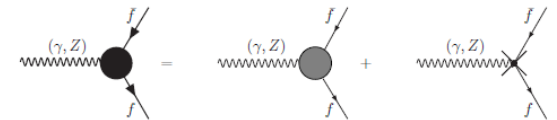


Figure A.6. Off-shell  $Z f\bar{f}$  and  $\gamma f\bar{f}$  vertices

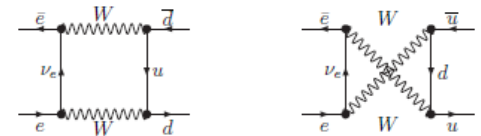
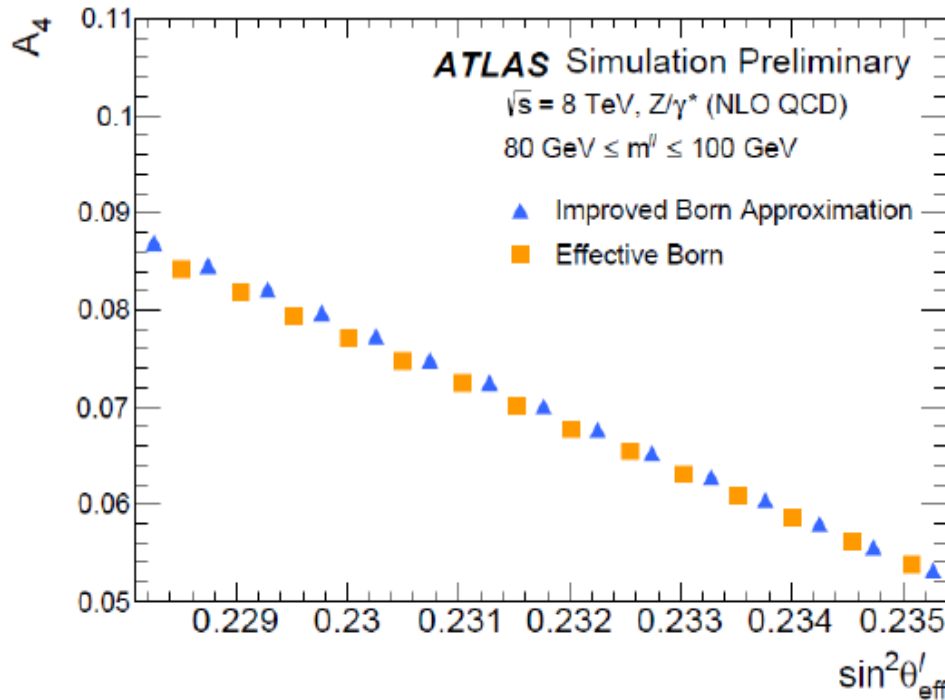


Figure A.7. The  $WW$  boxes

etc. etc.

# Predicting $\Delta A_4(\sin^2\theta_{\text{eff}})$



## Formulas used for this plot, varied $\delta V$

$$v_\ell = (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 \cdot K_\ell(s, t) + \delta_V)) / \Delta$$

$$v_f = (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 \cdot K_f(s, t) + \delta_V)) / \Delta$$

$$vv_{\ell f} = \frac{1}{v_\ell \cdot v_f} [(2 \cdot T_3^\ell)(2 \cdot T_3^f) - 4 \cdot q_\ell \cdot (s_W^2 + K_f(s, t) + \delta_V)(2 \cdot T_3^\ell) - 4 \cdot q_f \cdot (s_W^2 \cdot K_\ell(s, t) + \delta_V)(2 \cdot T_3^f) + (4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2)K_{\ell f}(s, t) + 2 \cdot (4 \cdot q_\ell)(4 \cdot q_f) \cdot s_W^2 \cdot K_{\ell f}(s, t) \cdot \delta_V] \frac{1}{\Delta^2}$$

Figure 2: Predicted variation of  $A_4$  as a function of  $\sin^2\theta_{\text{eff}}^\ell$ , integrated over  $y^{\ell\ell}$ ,  $p_T^Z$  and over the range  $80 < m^{\ell\ell} < 100 \text{ GeV}$ , where  $\sin^2\theta_{\text{eff}}^\ell$  is varied as described in the text. The orange squares show the prediction using the effective Born approximation with  $\sin^2\theta_W = 0.23152$ , while the blue triangles show the prediction from the improved Born approximation.

# EW schemes

- **LEP legacy: input (  $\alpha(0)$ ,  $G_\mu$ ,  $M_Z$  )**

D. Bardin et al.  
arXiv:9908433

- Inputs are very precisely measured physics quantities
- $M_Z$ ,  $M_W$  are on-shell masses
- Genuine EW and lineshape corrections in form of (multiplicative) form-factors to LO couplings

- **LHC paradigm: input (  $G_\mu$ ,  $M_Z$ ,  $M_W$  ).**

S. Dittmaier, M. Huber  
arXiv:0911.2329

- $M_Z$ ,  $M_W$  are pole-masses or complex masses.
- Most of universal corrections absorbed into lowest-order couplings
- Higher-order corrections redefine couplings in non-multiplicative manner