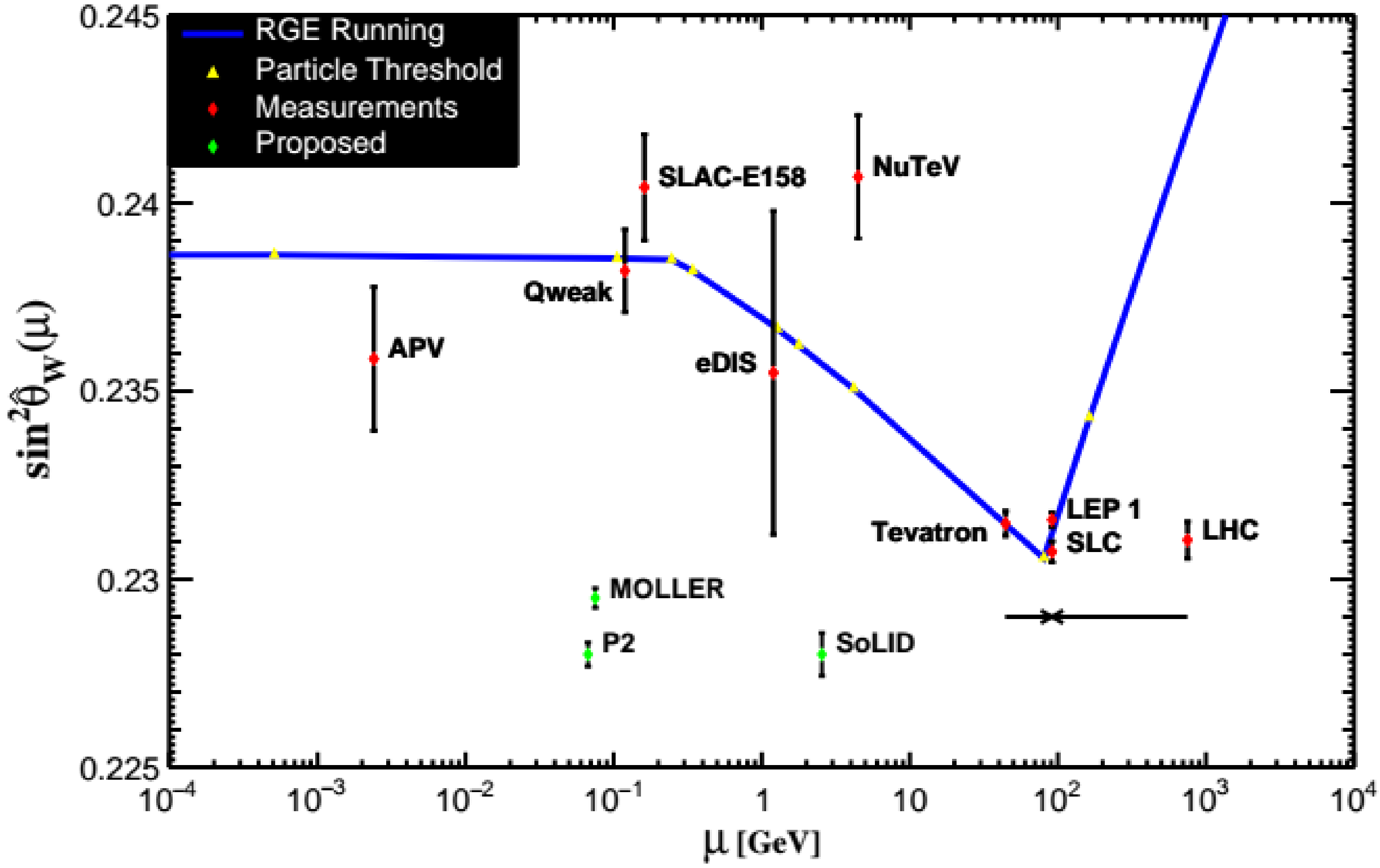
Weak Mixing Angle in the Thomson Limit


RGE Running
Particle Threshold
Measurements
Proposed
SLAC-E158
NuTeV
Qweak
APV
eDIS
Tevatron
LEP 1
SLC
LHC
MOLLER
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0.245
0.24
0.235
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0.225
$\sin^2\hat{\theta}_W(\mu)$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^2
10^3
10^4
μ [GeV]

Uncertainties

source	$\delta \sin^2 \hat{\theta}_W(0) \times 10^5$
$\Delta\hat{\alpha}^{(3)}(2\ \text{GeV})$	1.2
flavor separation	1.0
isospin breaking	0.7
singlet contribution	0.3
PQCD	0.6
Total	1.8

J. Erler and R. Ferro Hernández JHEP 1803 (2018) 196.

There are different schemes

- The on shell scheme, preserves the mass relation to all orders.

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

- The effective angle, used by LEP collaboration.

$$1 - 4\sin^2\theta_{eff}^{\mathrm{lep}} = \frac{g_V^l}{g_A^l}$$

- The $\overline{\mathrm{MS}}$ scheme preserves the dependence on the couplings. We will use this scheme.

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}$$

Effective and $\overline{\text{MS}}$ weak angle

From a paper written by Sirlin and Gambiono in 1993, we know that the relation between this and the $\overline{\text{MS}}$ mixing angle is

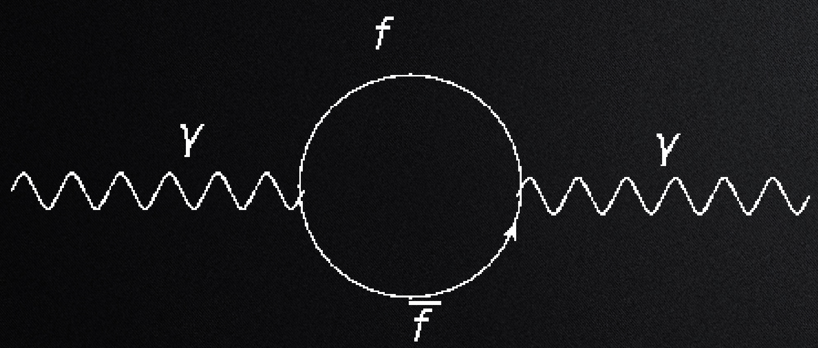
$$\sin^2\theta_{eff}^{\text{lep}} = \hat{s}^2 \text{Re}\,\hat{k}_l(m_Z^2)$$

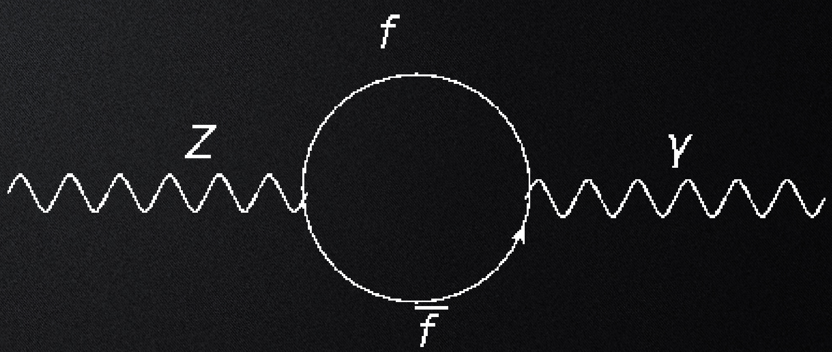
where

$$\text{Re}\,\hat{k}_l(m_Z^2) = 1.0012 + O(10^{-4})$$

Renormalization group evolution

Main idea: Relate the running of α to the running of the weak mixing angle (based on J. Erler et al (2005))
Due to the same Lorentz structure, we can obtain the running of the vector coupling from the running of α. Then from here the running of the WMA.

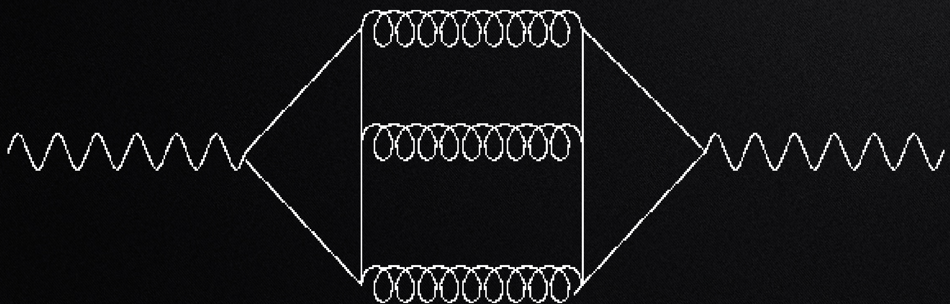


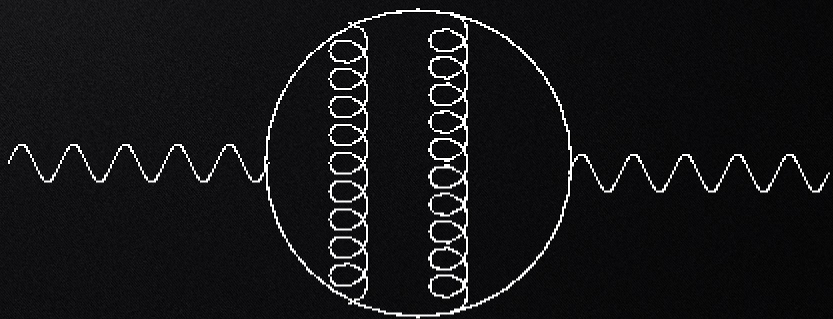


Renormalization group evolution

- α is the electromagnetic Running coupling constant.
- K_i contains information about higher order loops.
- γ_i is a numerical factor that depends on the type of particle.
- Q_i is the charge of the particle.
- σ are the disconnected contributions.

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$





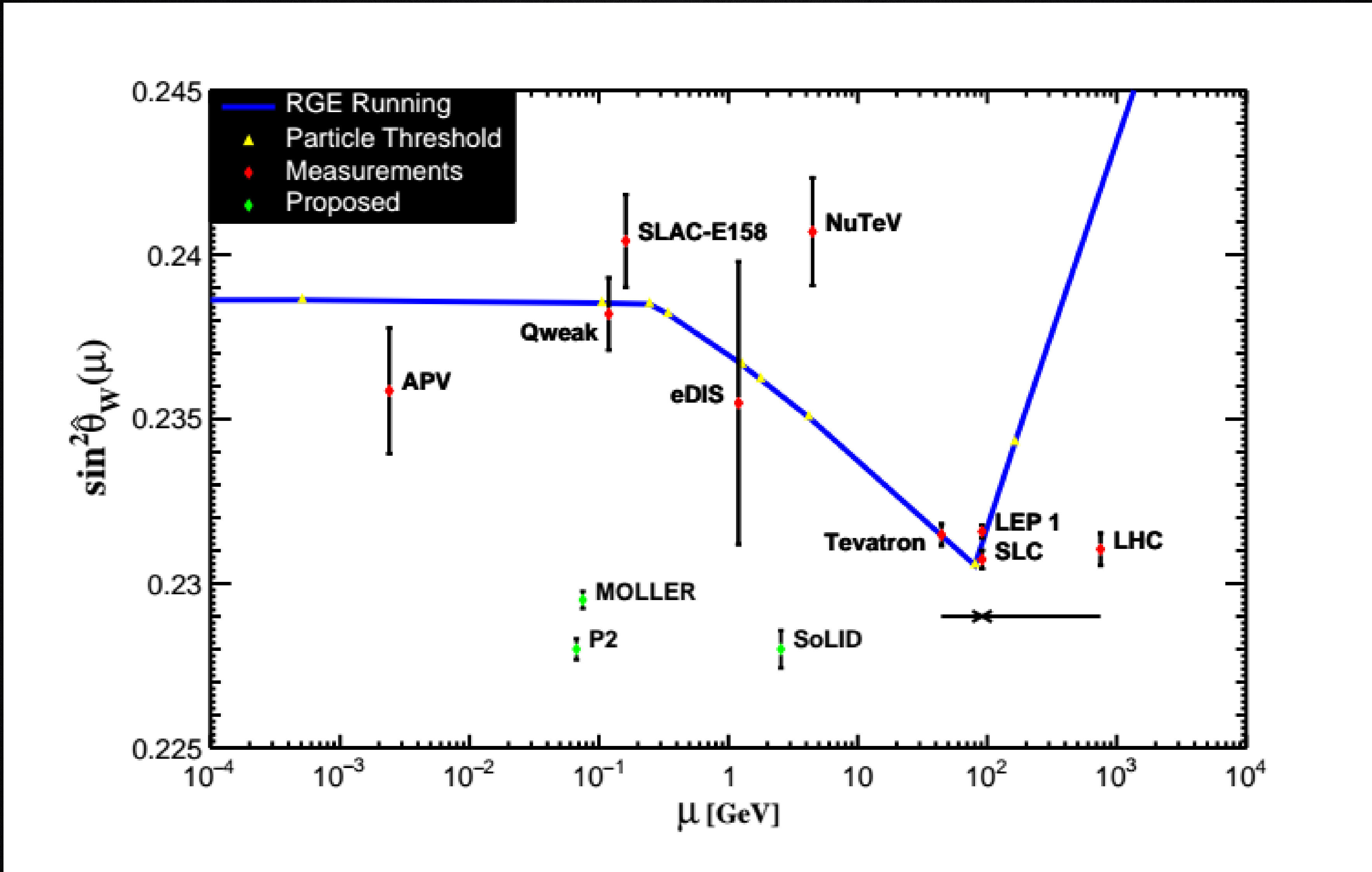
Renormalization group evolution

$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\hat{s}^2(\mu_0) + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

- Weak mixing angle at a scale μ.
- Fine structure constant at some scale μ.
- Numerical constants that depend on the number of particles in the EFT.
- Disconnected contributions.

Where are the uncertainties?

- Hadronic data.
- How much does the strange contributes? different λ in this case!!
- Size of the explicit α_s dependent OZI.
- Perturbative error,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Δα from hadronic data

The running of the fine structure constant can be written as

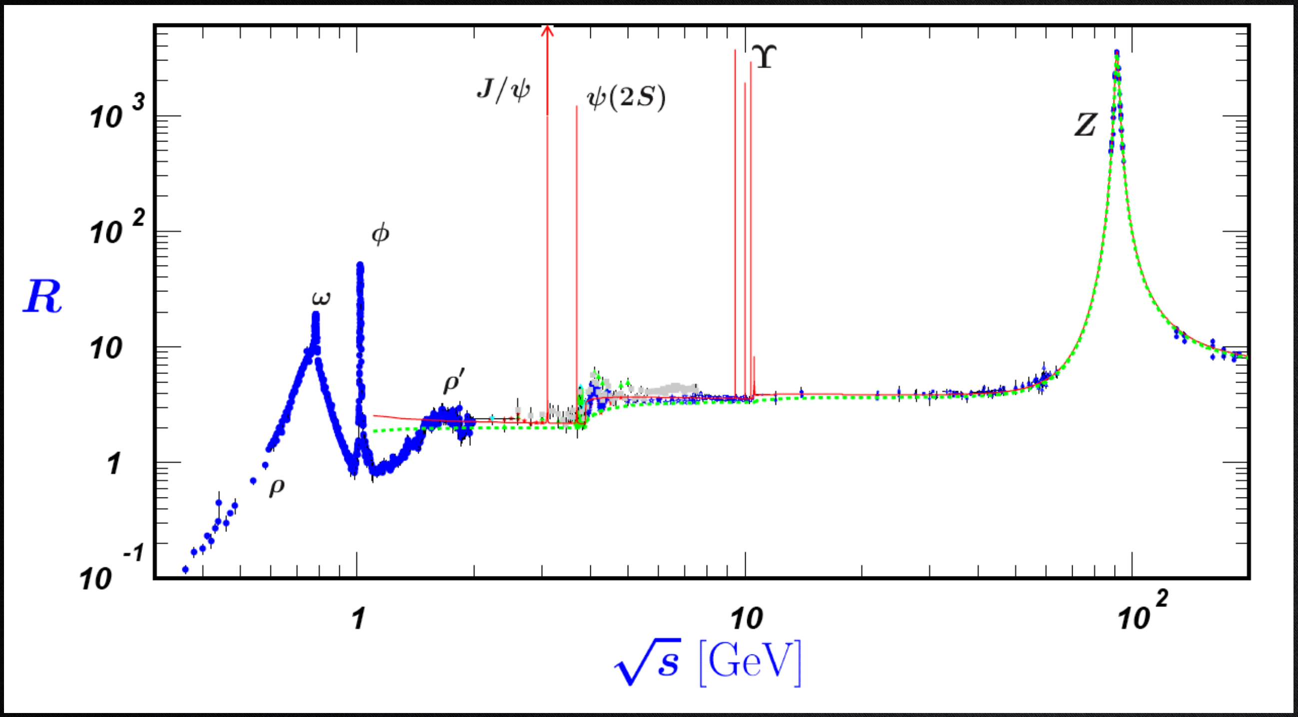
$$\hat{\alpha}\left(\mu\right) = \frac{\alpha}{1 - \Delta\alpha\left(\mu\right)}$$

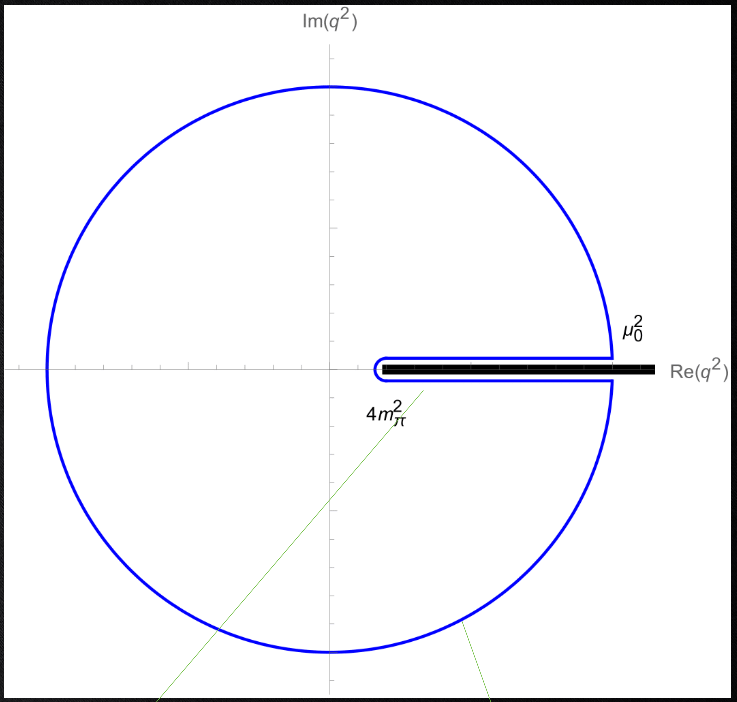
where

$$\Delta\alpha(\mu) = 4\pi\alpha\hat{\Pi}(0, \mu)$$

Where ∏ is the vacuum polarization function, to compute this for the three light quarks we use a contour in the complex q^2 plane.

Δα from hadronic data





$$\Delta\hat{\alpha}^{(3)}(\mu_0) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\mu_0^2} ds\,\frac{R(s)}{s-i\epsilon} + 4\pi I^{(3)},$$

Δα from hadronic data

Including the integration of the cross section ratio R we get

$$\Delta^{(3)}\hat{\alpha}(2.0\text{GeV}) = (83.56 \pm 0.45 \pm 0.18) \times 10^{-4}$$

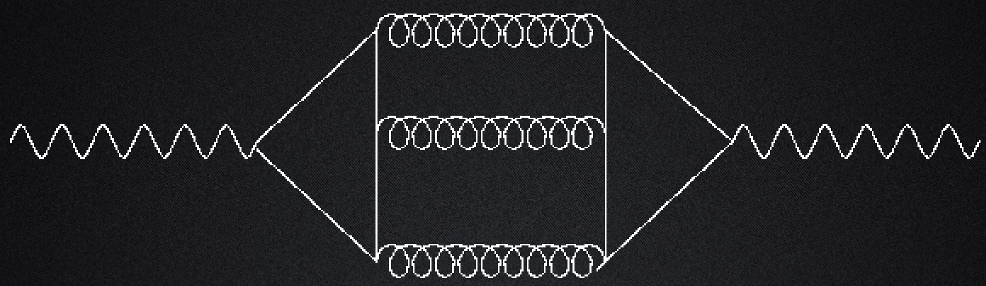
Using the master equation, we can easily propagate this uncertainty to the low energy weak mixing angle.

$$\delta \sin^2\hat{\theta}_W(0) = \left(\frac{1}{2} - \sin^2\hat{\theta}_W\right)\delta\Delta^{(3)}\alpha(2.0\,\text{GeV})$$

$$\delta \sin^2\hat{\theta}_W(0) = \pm 1.2 \times 10^{-5}$$

Disconnected contributions

These comes from diagrams like



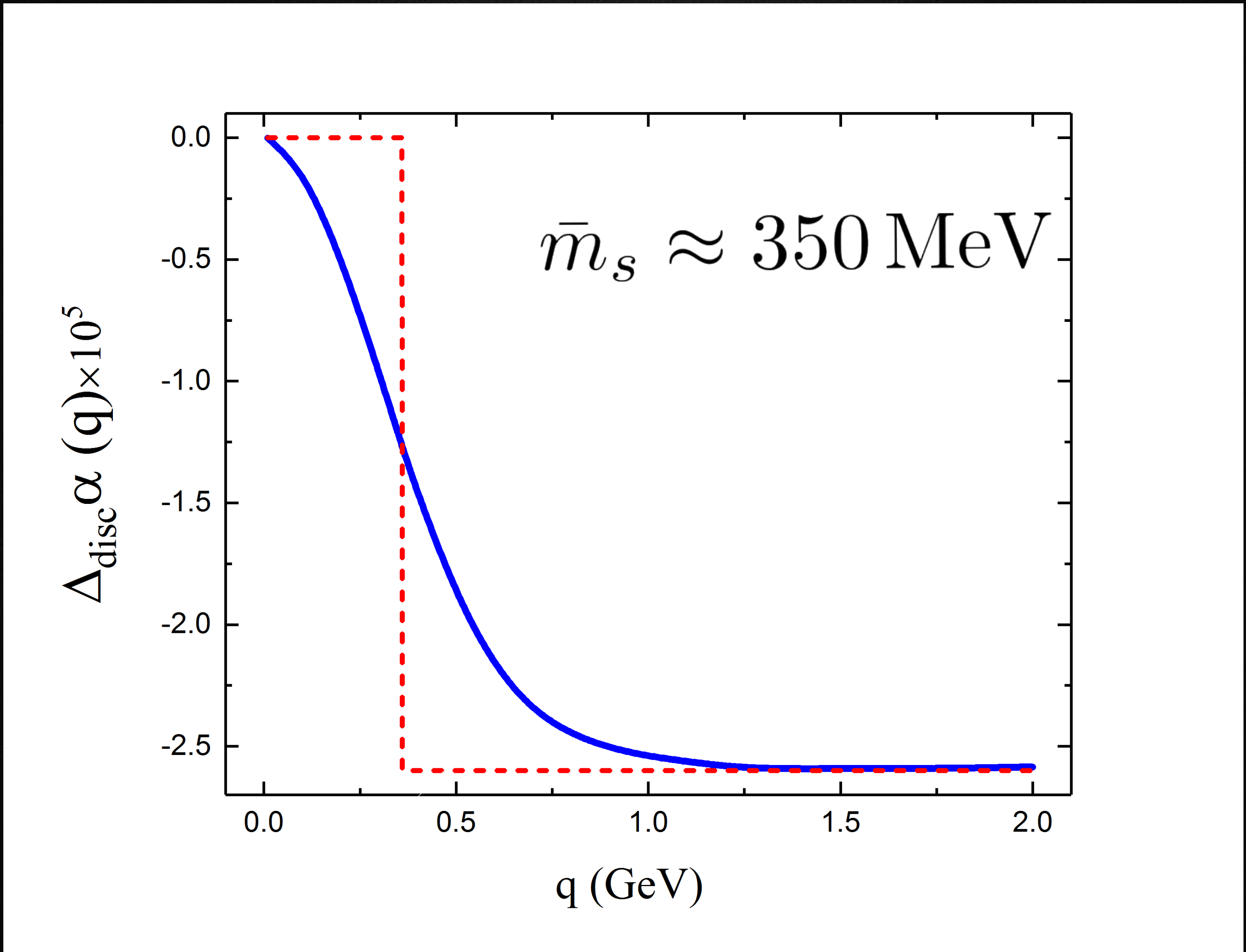
Known to be explicitly small in the perturbative limit. In low energies suppression of ϕ going to three pions are explained with it.

$$\tilde{\sigma}(\mu) - \tilde{\sigma}(\mu_0) = -\lambda_1 \frac{\pi}{\alpha}[\Delta_{\text{disc}}\alpha(\mu) - \Delta_{\text{disc}}\alpha(\mu_0)]$$

We can use lattice calculations to constraint these contribution! In T. Blum et al 2016 , they calculate these contributions for a_μ.

Disconnected contributions

Using the lattice results, the dependence on q^2 for $\Delta\alpha$ can be obtained



Flavor separation

How much dœs the strange contributes relative to the up and down quarks? essentially we want to split the total contribution as:

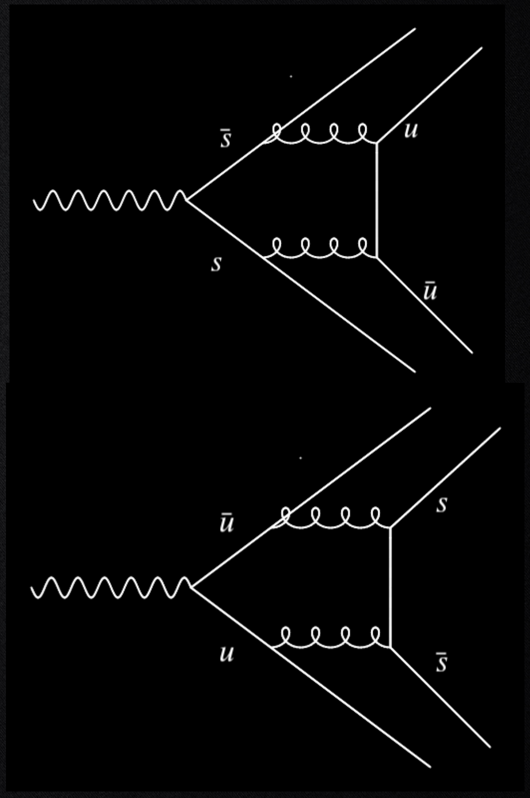
$$\Delta\hat{\alpha}^{(3)}\left(\bar{m}_c\right) = \Delta_s\hat{\alpha}\left(\bar{m}_c\right) + \Delta_{u,d}\hat{\alpha}\left(\bar{m}_c\right) = 6\Delta_s\hat{\alpha}\left(\bar{m}_c\right) + \Delta_{u,d}\hat{\alpha}\left(\bar{m}_s\right)$$

In the past this was constrained using the SU(3) limit, and the heavy quark limit. This is where the largest uncertainty came from.

Flavor separation

Now we use a data driven approach. First we look to specific channels that we can associate with a strange quark current. using the results from M. Davier et al (2017).

channel	$a_\mu \times 10^{10}$	$\Delta\alpha \times 10^4$
ϕ	38.43	5.13
$K\bar{K}\pi$	2.45	0.78
$\eta\phi$	0.36	0.13
PQCD [?] ($> 1.8\,\mathrm{GeV}$)	7.30	—
Total	48.54	6.04
$K\bar{K}$ (non $-\,\phi$)	3.62	0.76
$K\bar{K}2\pi$	0.85	0.30
$K\bar{K}3\pi$	-0.03	-0.01
$K\bar{K}\eta$	0.01	0.00
$K\bar{K}\omega$	0.01	0.00
Total	4.46	1.05



Hard to distinguish!!

Flavor separation

Nevertheless we expect this kaon contributions to come from strange currents.

From lattice calculation (T. Blum et al 2016), we are able to assign an error to this assumption.

$$\Delta_s \alpha(1.8 \text{ GeV}) = (7.09 \pm 0.11 \pm 0.19 \pm 0.23) \times 10^{-4} = (7.09 \pm 0.32) \times 10^{-4},$$

From another lattice calculation B. Chakraborty et al we estimate

$$\Delta_{\mathrm{s}}^{lattice} \alpha(2.0\,\mathrm{GeV}) \approx (6.9 \pm 0.5) \times 10^{-4}$$

Flavor separation

Now we can propagate this uncertainty to the weak mixing angle.

$$\delta \hat{s}^2(0) \simeq \frac{1}{20}\, \delta \Delta \hat{\alpha}^{(2)}(\bar{m}_c) = \pm 1.0 \times 10^{-5},$$

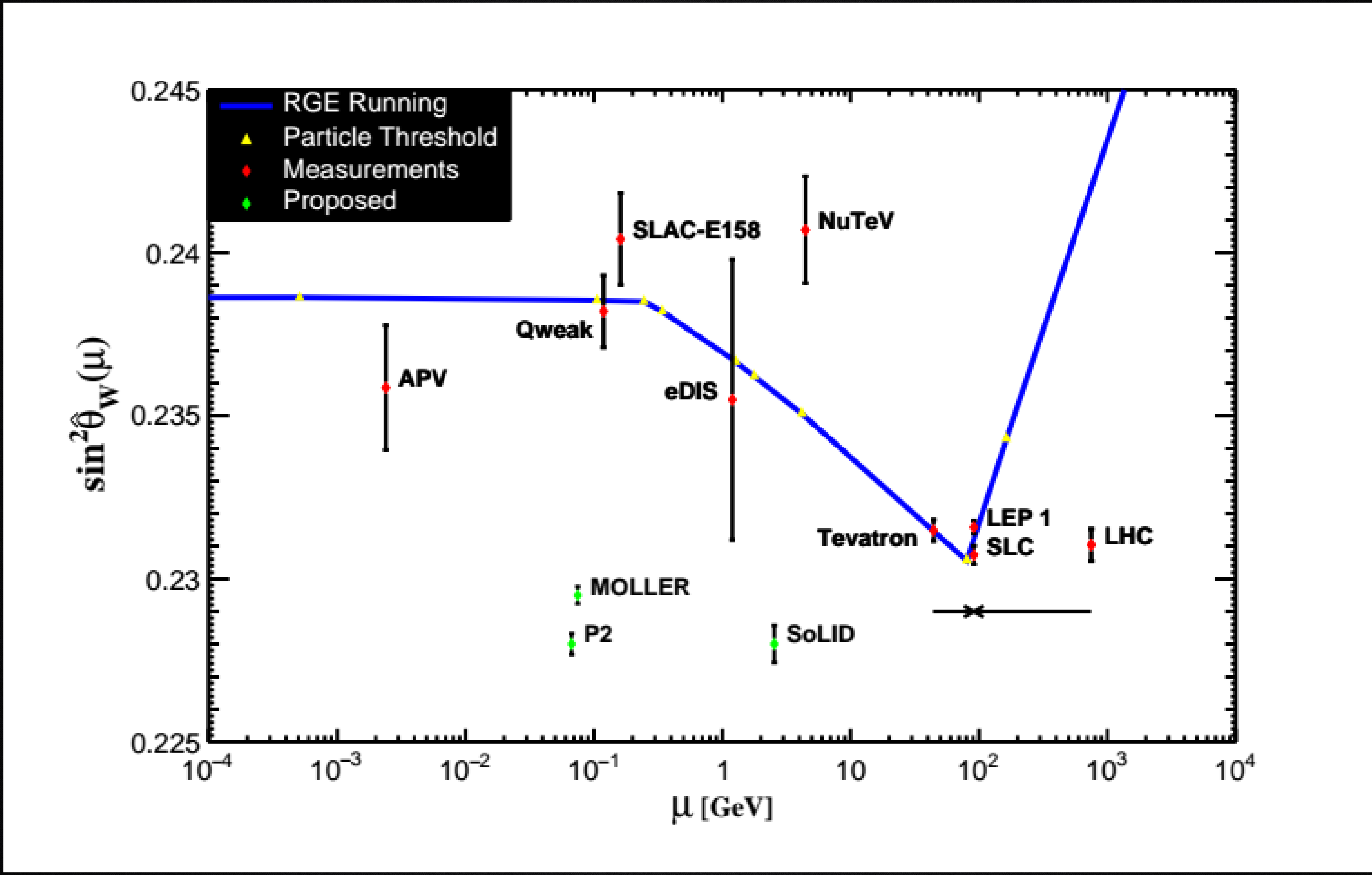
Final results

Using the result $\sin\hat{\theta}_w(M_Z) = 0.23129(5)$ from a global fit to electroweak data we get

$$\sin\hat{\theta}_w(0) = 0.23868(5)(2)$$

In terms of $\sin\hat{\theta}_W(0) \equiv \hat{\kappa}(0)\sin\hat{\theta}_W(M_Z)$ this result reads

$$\hat{\kappa}(0) = 1.03196 \pm 0.00006 + 1.14\,\tilde{\Delta}\alpha + 0.025\,\Delta\hat{\alpha}_s - 0.0016\,\Delta\hat{m}_c - 0.0012\,\Delta\hat{m}_b$$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Conclusions.

- We updated the inputs for different data.
- Included next order contributions for QCD in the RGE.
- Different method to handle OZI contribution.
- Different method to handle the flavour separation.
- The uncertainty is now four times smaller.

Matching conditions

At each threeshold we have to match the electromagnetic coupling constant. This is very similar to the QCD mathcing. Using results from Chetyrkin 2006 we get

$$
\begin{aligned}
\frac{\pi}{\hat{\alpha}(m_f)^+} &= \frac{\pi}{\hat{\alpha}(m_f)^-} - \frac{15}{16} N_f^c \frac{\hat{\alpha}(m_f)}{\pi} Q_f^4 \\
&- \frac{N_f^c - 1}{2} \left[\frac{13}{12} \frac{\hat{\alpha}_s^+}{\pi} + \left(\frac{655}{144}\zeta_3 - \frac{3847}{864} + \frac{361}{1296} n_q \right) \frac{\hat{\alpha}_s^{+2}}{\pi^2} \right. \\
&+ \left. \left(-0.55739 - 0.92807\, n_q + 0.01928\, n_q^2 \right) \frac{\hat{\alpha}_s^{+3}}{\pi^3} \right] Q_f^2 \\
&- \frac{N_f^c - 1}{2} \left[\frac{295}{1296} \frac{\hat{\alpha}_s^{+2}}{\pi^2} + (\mathcal{K}_1 + \mathcal{K}_2 n_q) \frac{\hat{\alpha}_s^{+3}}{\pi^3} \right] \sum_\ell Q_\ell^2 \; .
\end{aligned}
$$

Threshold masses

We define a threshold mass as the t' hooft scale where the matching conditions become trivial. We obtain it up to next order in QCD. We computed this in the perturbative regime

$$\begin{aligned}\bar{m} &= \hat{m}\left\{1-\frac{13}{24}\frac{\hat{\alpha}_s}{\pi}+\left(\frac{10073}{3456}-\frac{655}{288}\zeta_3-\frac{361}{2592}n_q\right)\frac{\hat{\alpha}_s^2}{\pi^2}\right.\\
&+ \left(1.61024+0.59599\,n_q-0.00964\,n_q^2\right)\frac{\hat{\alpha}_s^3}{\pi^3}\\
&+ \left.\left[-\frac{295}{2592}\frac{\hat{\alpha}_s^2}{\pi^2}+\left(\frac{5767}{62208}-\frac{\mathcal{K}_1+\mathcal{K}_2 n_q}{2}\right)\frac{\hat{\alpha}_s^3}{\pi^3}\right]\frac{\sum Q_\ell^2}{Q_h^2}\right\}.\end{aligned}$$

Threshold masses light quarks

This defintion implies that for the light quarks we must have

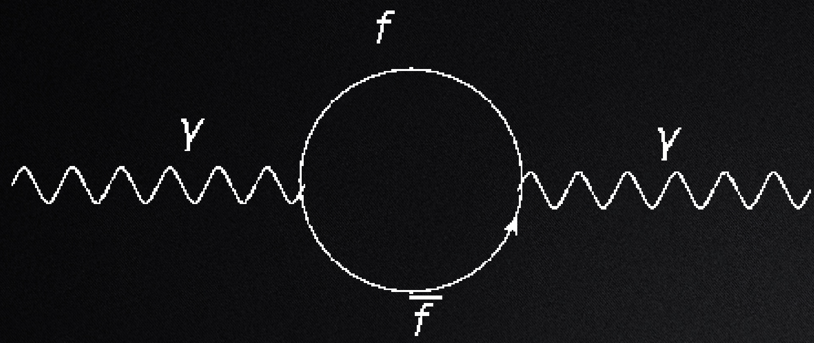
$$\Delta_s \hat{\alpha}(\bar{m}_c) = Q_s^2 \frac{\alpha}{\pi} K_{\mathrm{QCD}}^s(\bar{m}_c) \ln \frac{\bar{m}_c^2}{\bar{m}_s^2}$$

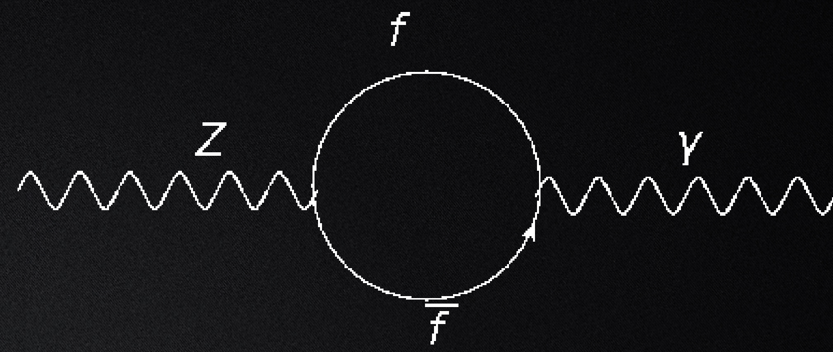
$$K_{\mathrm{QCD}}^c(\bar{m}_c) < K_{\mathrm{QCD}}^s(\bar{m}_c).$$

$$\bar{m}_s < 390 \text{ MeV},$$

$$K_{\mathrm{QCD}}^s(\bar{m}_c) = 1.34 \pm 0.16,$$

$$\bar{m}_s = 342^{+48}_{-53} \text{ MeV}.$$





$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$

$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha}}{24\pi} Q_f \left[\sum K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q \hat{v}_q \right) \left(\sum_q Q_q \right) \right]$$

Renormalization group evolution

Key idea: We mix both equations in order to absorb the explicit dependence on α_s in the RGE of the vector coupling.

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$

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$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} \hat{s}^2(\mu_0) + \lambda_1 \left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} \right] + \frac{\hat{\alpha}(\mu)}{\pi} \left[\frac{\lambda_2}{3} \ln \frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4} \ln \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu) \right]$$

This is the solution to the running of the weak angle in terms of alpha.

Δα from hadronic data

We computed the integral, and did a cross check the RGE and the perturbative expantion for R.

$$\begin{aligned}
4\pi I^{(3)} &= 2\alpha \int_0^{2\pi} d\theta\, \hat{\Pi}^{(3)}(\mu^2 e^{i\theta}) \\
&= \frac{2\alpha}{3\pi}\left[\frac{5}{3} + \left(\frac{55}{12} - 4\zeta(3) + 2\frac{\hat{m}_s^2}{\mu^2}\right)\left(\frac{\hat{\alpha}_s}{\pi} + \frac{\hat{\alpha}}{4\pi}\right)\right. \\
&+ \left(\frac{34525}{864} - \frac{9}{4}\zeta(2) - \frac{715}{18}\zeta(3) + \frac{25}{3}\zeta(5) + \frac{125}{12}\frac{\hat{m}_s^2}{\mu^2} + F(\hat{m}_c, \hat{m}_b)\right)\frac{\hat{\alpha}_s^2}{\pi^2} \\
&+ \left(\frac{7012579}{13824} - \frac{961}{16}\zeta(2) - \frac{76681}{144}\zeta(3) + \frac{12515}{288}\zeta(5)\right. \\
&\left.\left. - \frac{665}{36}\zeta(7) + \frac{81}{2}\zeta(2)\zeta(3) + \frac{155}{2}\zeta(3)^2\right)\frac{\hat{\alpha}_s^3}{\pi^3}\right] \\
&= (24.86 \pm 0.18 - 43\,\Delta\hat{\alpha}_s) \times 10^{-4},
\end{aligned}$$

Perturbative uncertainty

Where does the uncertainty come from?

$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\hat{s}^2(\mu_0) + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

- We need to calculate the fine structure constant from hadronic data.
- How much does the strange contributes? different λ in this case!!
- What is the size of the explicit α_s dependent OZI contribution?
- What is the perturbative error?

Disconnected contributions

We can roughly estimate the contribution of these diagrams assuming α_s to be of order one. This implies,

$$\Delta_{\rm disc}\sin\hat{\theta}(0) \approx 10^{-6}$$

By construction the contribution of the disconnected diagrams to the weak angle is related to the one in α. For μ less than the charm mass we get the useful relation

$$\tilde{\sigma}(\mu) - \tilde{\sigma}(\mu_0) = -\lambda_1 \frac{\pi}{\alpha}[\Delta_{\rm disc}\alpha(\mu) - \Delta_{\rm disc}\alpha(\mu_0)]$$

Flavor separation

As a very conservative approach we can take 50%±50% of the contribution of these channels to come from the strange.

$$\Delta_s \alpha(1.8\ \mathrm{GeV}) = (6.56 \pm 0.11 \pm 0.19 \pm 0.53) \times 10^{-4} = (6.56 \pm 0.57) \times 10^{-4}.$$

Experimental uncertainties

Kaon channels.

Parametrization uncertainties.

Extra stuff starts here….

To estimate the size of the SU(2) breaking we assume that the SU(2) breaking is as large as the SU(3) one. This gives us

$$\Delta\alpha^{(1)}(\bar{m}_d) < 14.8 \times 10^{-4}.$$

We propagate this to the weak mixing angle, giving us

$$\delta\hat{s}^2(0) = -\frac{3}{40}\Delta\alpha^{(1)}(\bar{m}_d) > -1.1 \times 10^{-4}.$$

Using as a measure if the SU(2) breaking relative to the SU(2) breaking the ratio

$$\left|\frac{M^2_{K*\pm} - M^2_{K*0}}{M^2_{K*\pm} - M^2_{\rho^0}}\right| \approx 0.06,$$

$$\delta\hat{s}^2(0) =^{+0}_{-7} \times 10^{-6}.$$