Proton Bunch Analysis in the AWAKE Experiment

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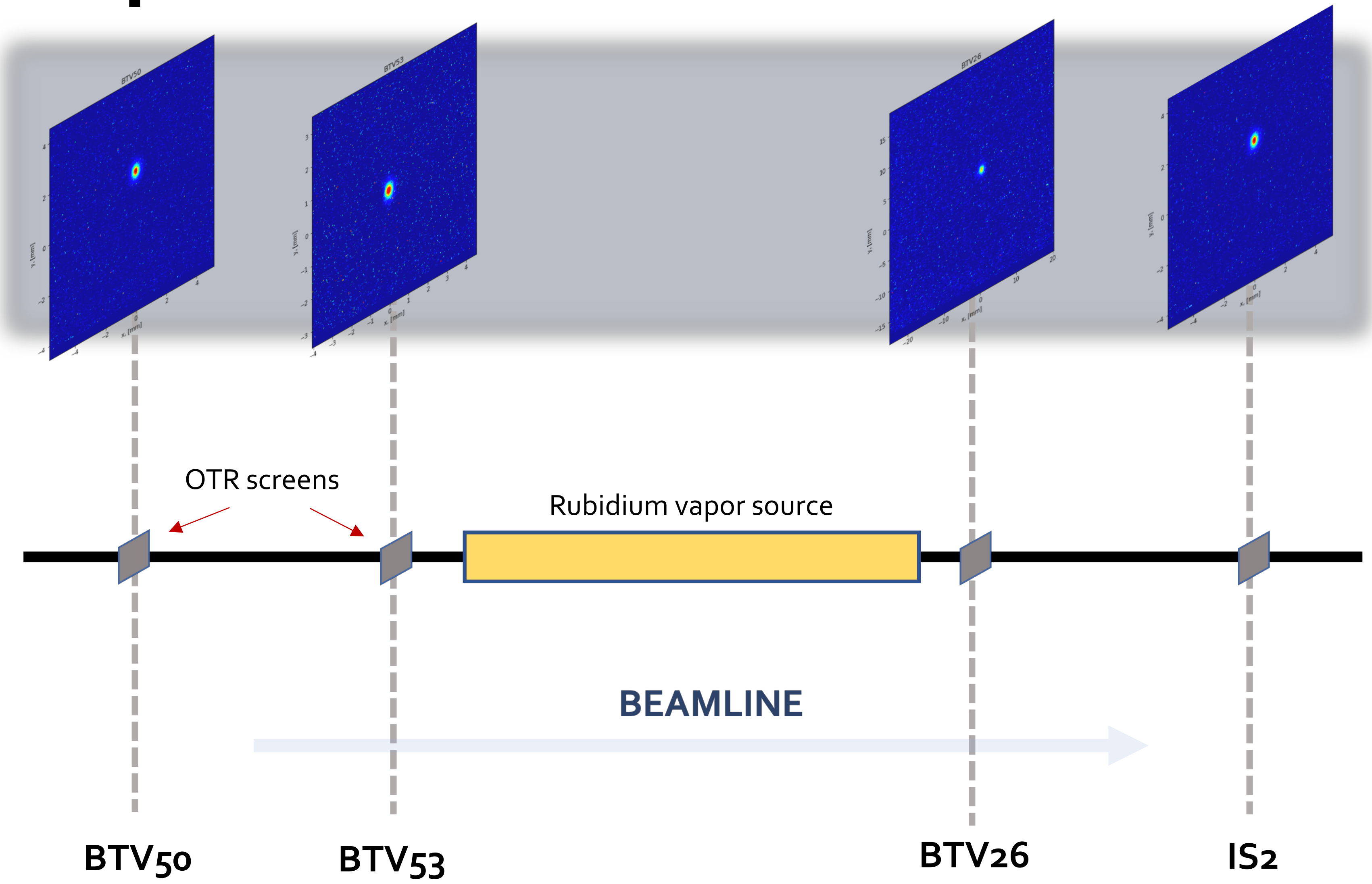
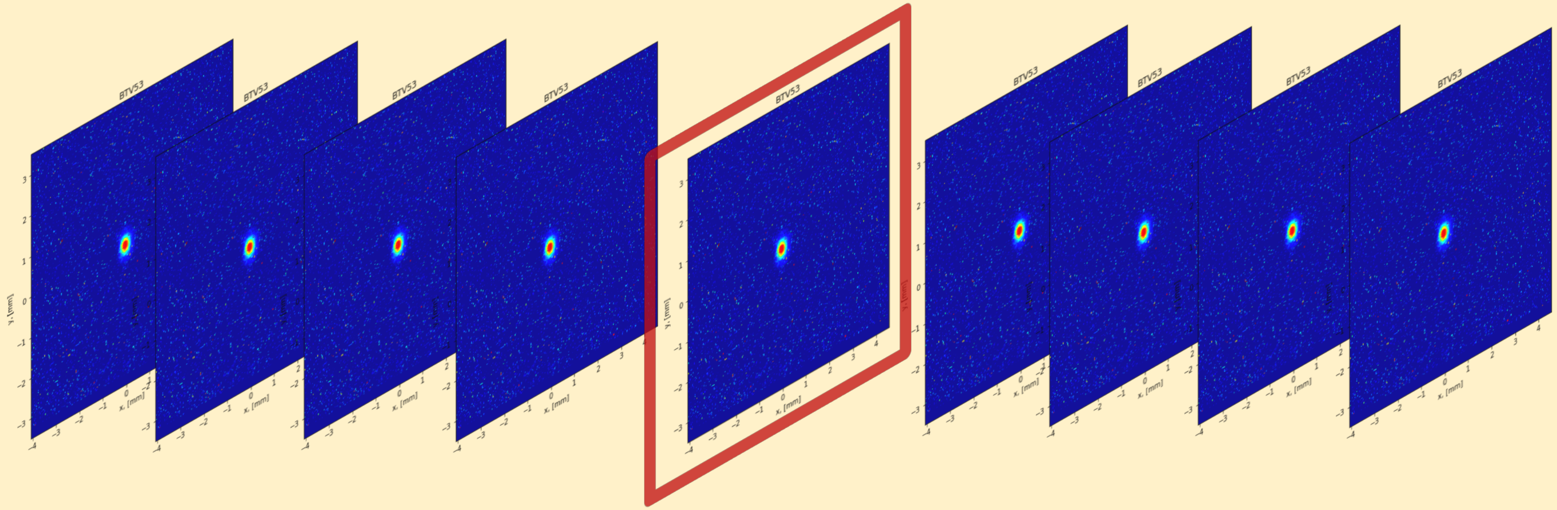
Experimental Setups

Optical transition radiation (OTR) light produced by protons crossing the OTR screen has been measured by using 4 SSD cameras:

1. BTV50 (3 [m] upstream).
2. BTV53 (1.5 [m] upstream).
3. BTV26 (2 [m] downstream).
4. IS2 (10 [m] downstream).



Noise Distribution



1400 events

Set 1
- Population 3e11 [p];
- Bunch rotation ON;
- 120 events;

Set 2
- Population 3e11 [p];
- Bunch rotation OFF;
- 120 events;

Set 3
- Population 1e11 [p];
- Bunch rotation ON;
- 120 events;

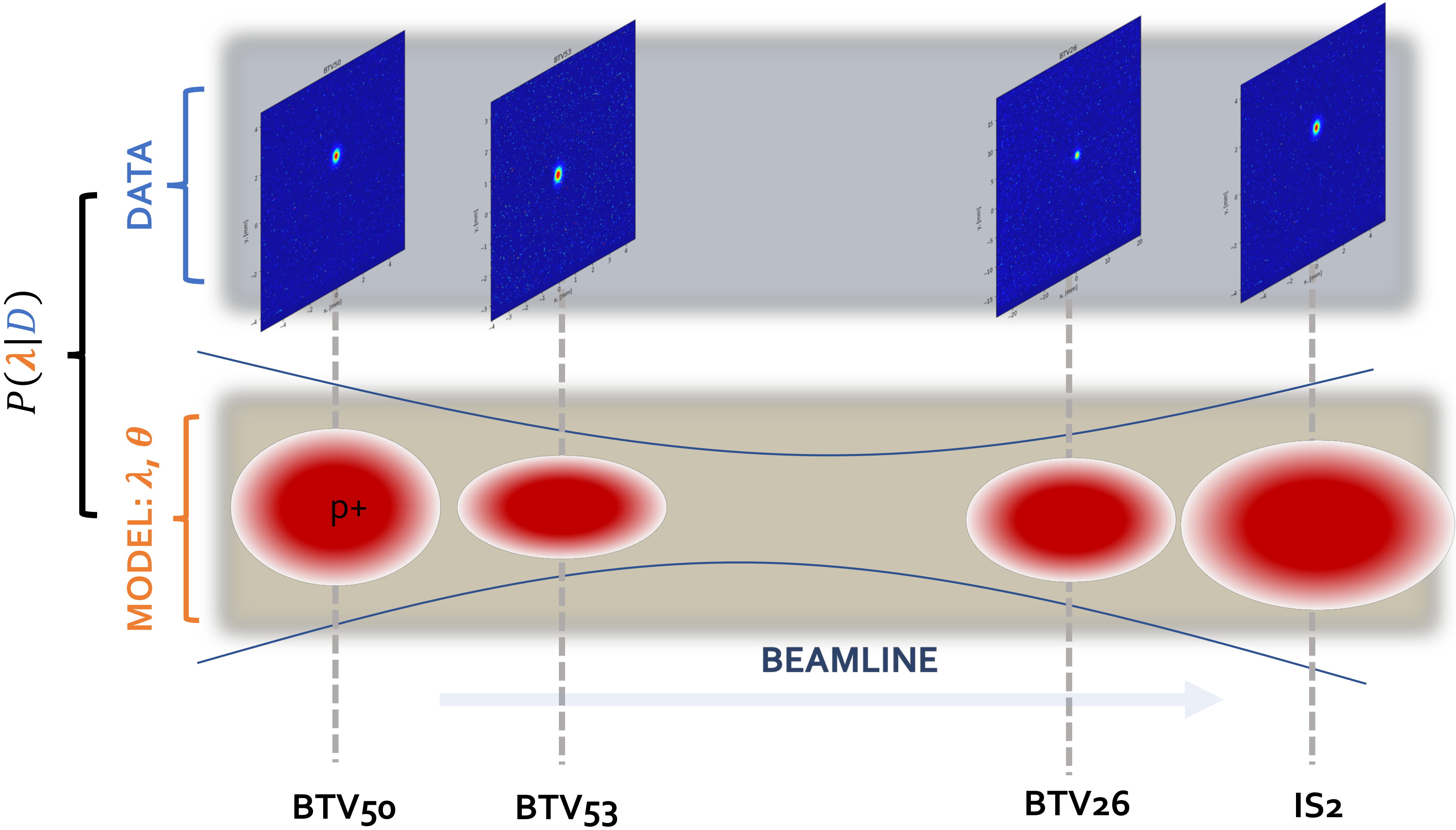
Set 4
- Population 1e11 [p];
- Bunch rotation OFF;
- 120 events;

Statistical Model

Assumption:
Proton bunch is Gaussian with parameters $\boldsymbol{\lambda}$, $\rho(\boldsymbol{x}, t|\boldsymbol{\lambda})$,
$\boldsymbol{\theta}$ is nuisance parameters.

$$\boldsymbol{\lambda} = \{\sigma_x, \sigma_y, \sigma_{x'}, \sigma_{y'}, s_w\}$$
$$\boldsymbol{\theta} = \{\sigma_{sx}, \sigma_{sy}, x_{ax}, x_{ay}, s_n\}$$

$\boldsymbol{\lambda}$ & $\boldsymbol{\theta}$ — **11 parameters**.



Statistical Model

We would like to learn $\boldsymbol{\lambda}$ from data. Bayes' theorem:

Likelihood Function, needs modelling!

$$P(\boldsymbol{\lambda}|D) = \frac{\int P(D|\boldsymbol{\lambda},\boldsymbol{\theta})P_0(\boldsymbol{\lambda})P_0(\boldsymbol{\theta})d\boldsymbol{\theta}}{\int P(D|\boldsymbol{\lambda},\boldsymbol{\theta})P_0(\boldsymbol{\lambda})P_0(\boldsymbol{\theta})d\boldsymbol{\theta}\, d\boldsymbol{\lambda}}$$

Where:

$P(D|\boldsymbol{\lambda},\boldsymbol{\theta}) = \prod_{i=1}^{N_{events}} P(D_i|\boldsymbol{\lambda},\boldsymbol{\theta})$ — product over different events.

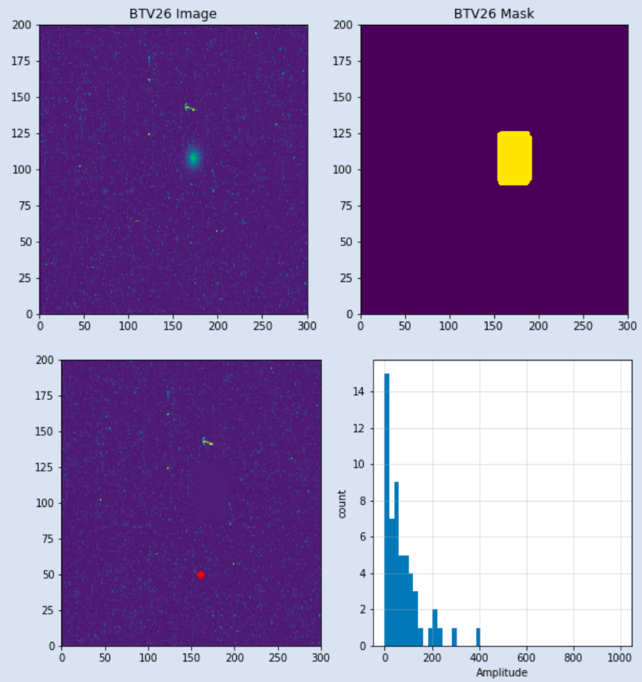
$P(D_i|\boldsymbol{\lambda},\boldsymbol{\theta}) = \prod_{j=1}^{N_{screens}} P\left(D_{ij}\middle|\boldsymbol{\lambda},\boldsymbol{\theta}\right)$ — product over different screens.

$D_{ij} = \{D_{ij}^{rc}\}$ — row and column index of a camera, $\boldsymbol{\theta}$ nuisance parameters.

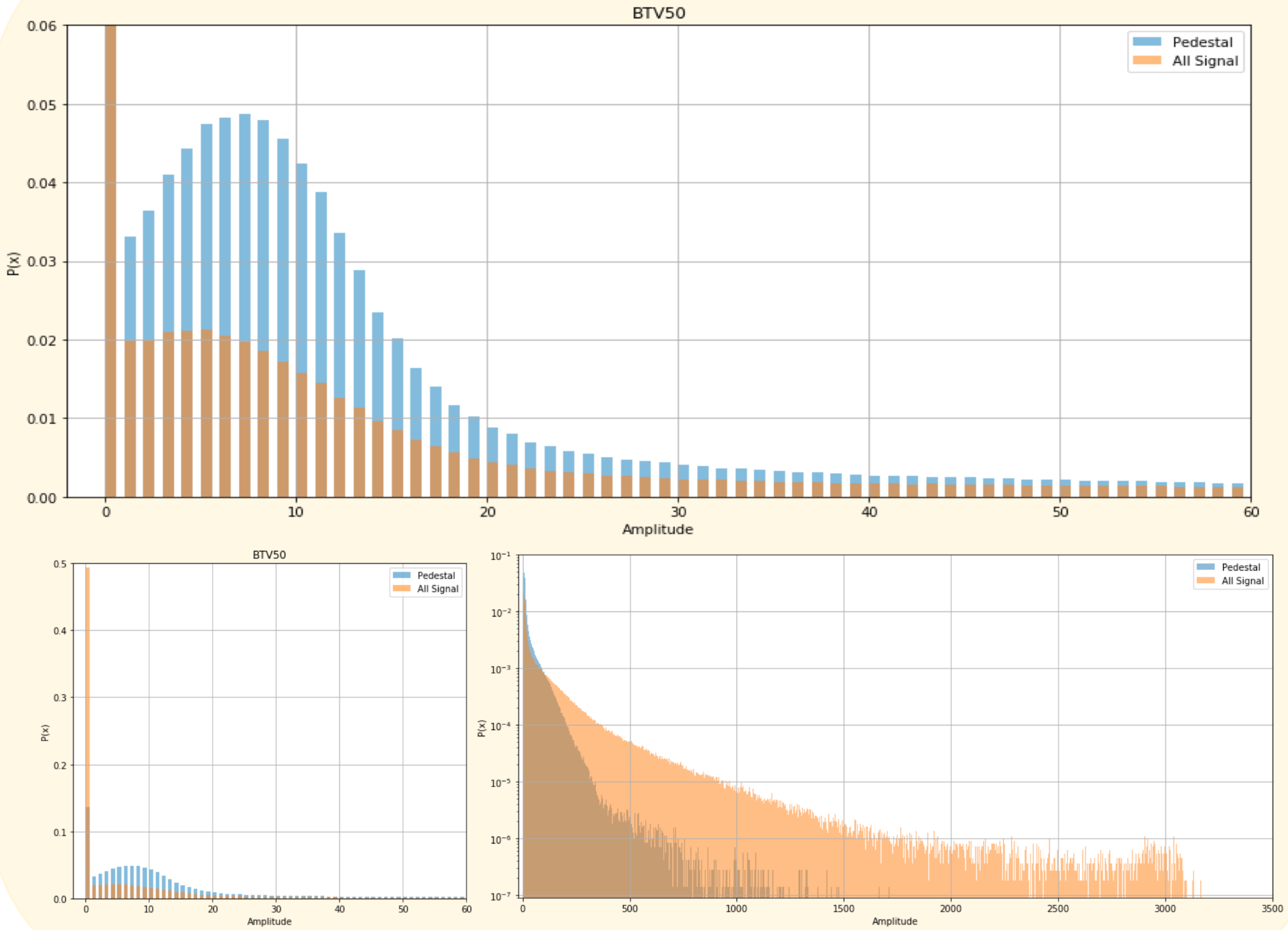
$P\left(D_{ij}^{rc}\right) = P\left(D_{ij}^{rc}\middle|\boldsymbol{S}(\boldsymbol{r},\boldsymbol{c}),\boldsymbol{N}(\boldsymbol{r},\boldsymbol{c})\right)$ — where $\boldsymbol{S}$ and $\boldsymbol{N}$ are resolution and noise effects respectively.

The task is generic, and one can consider similar analysis for different applications.

Noise Distribution



Noise distribution for a single pixel of the BTV50

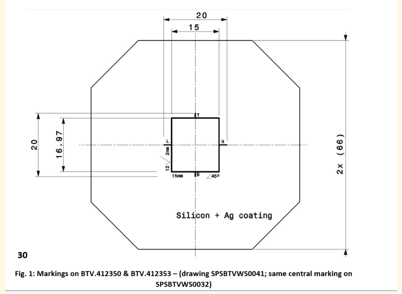


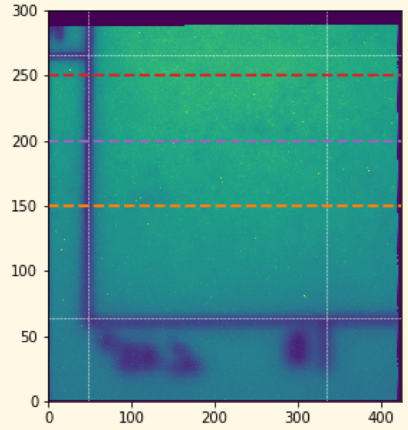
Resolution Function

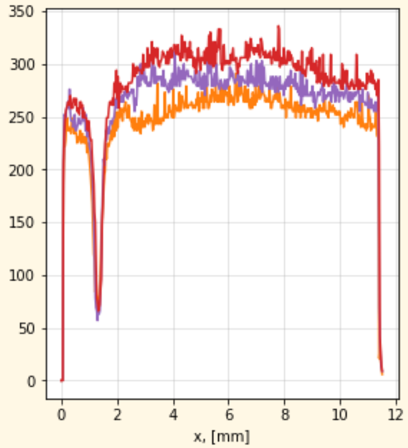
A task to be solved

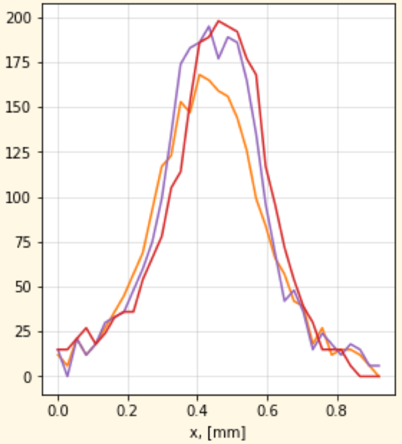
$$f_{resp}(x) = \int_{-\infty}^{\infty} g_{res}(x-\theta)\, f_{sign}(\theta)\, d\theta$$

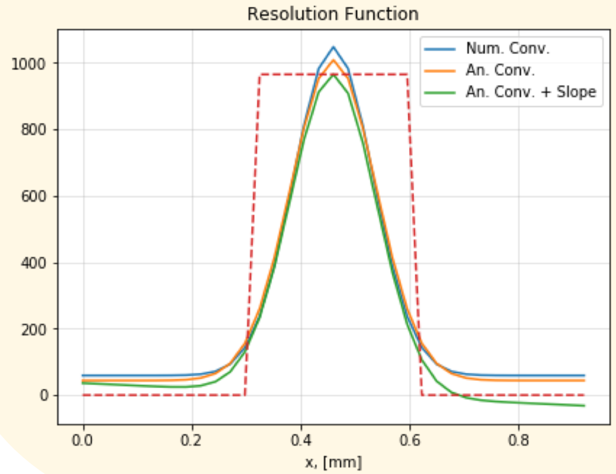
$g_{res}(x)$ is a resolution function.
$f_{sign}(x)$ is the original signal.
$f_{resp}(x)$ is a response of the camera.

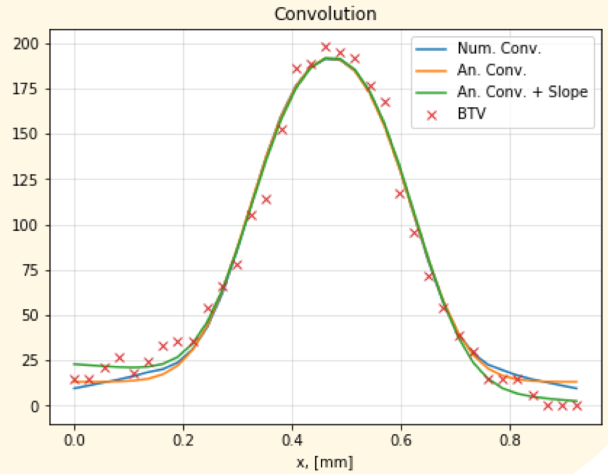












Resolution Function

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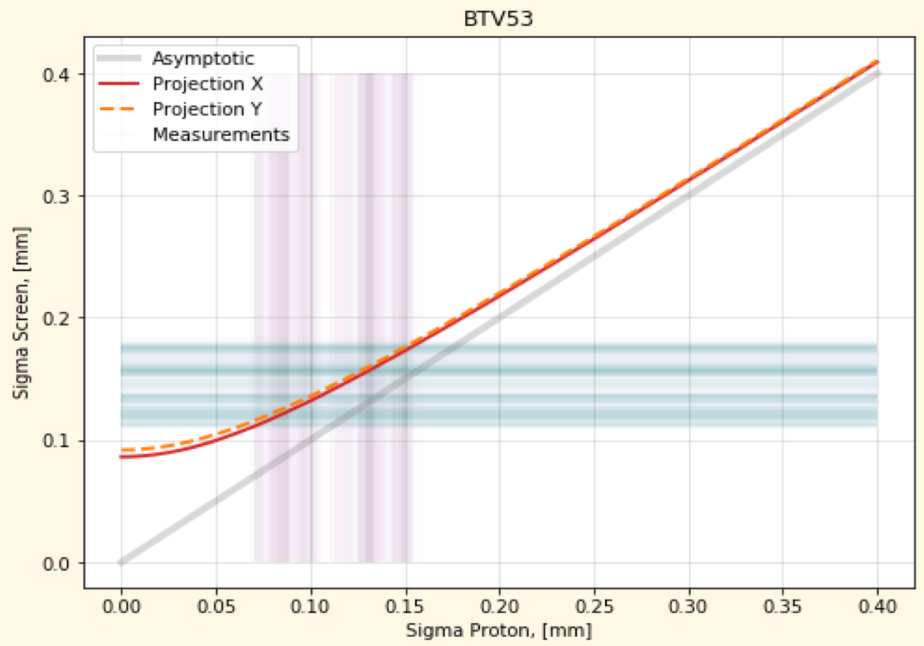
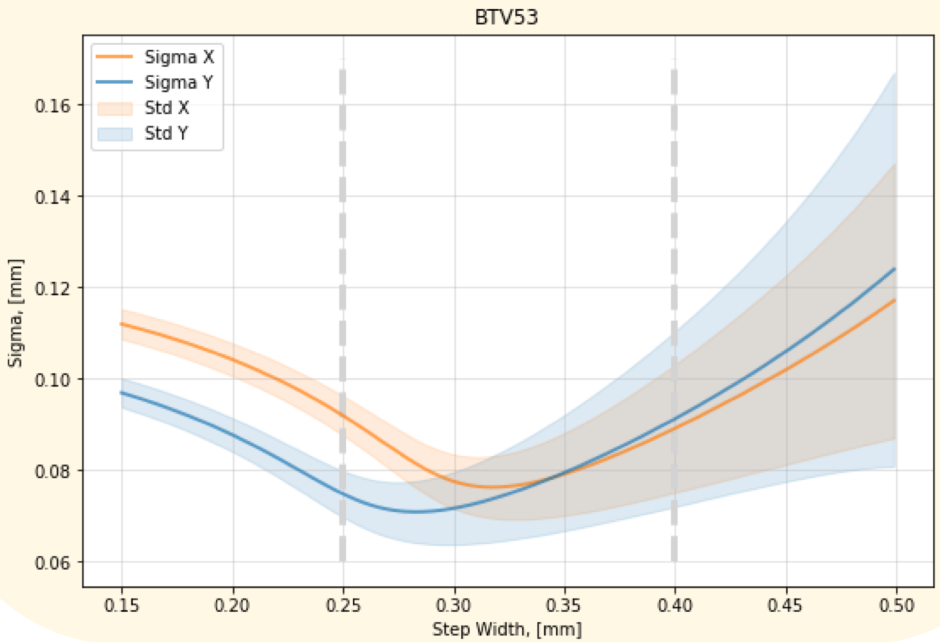
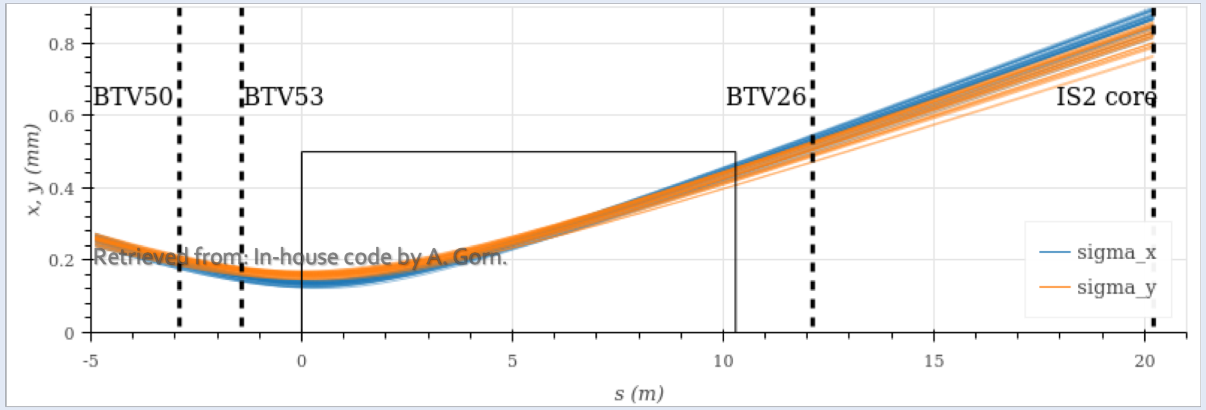
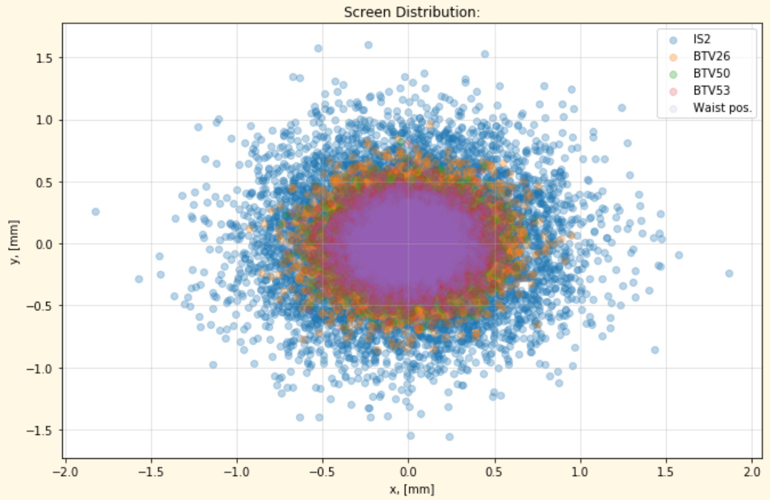
Smearing effects are important!

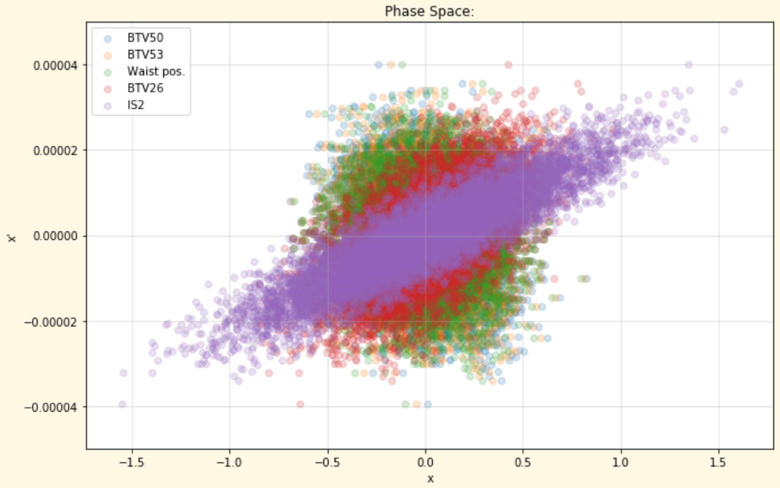




Modeling of a Camera Response







PDF of the proton bunch

Particle Simulation:

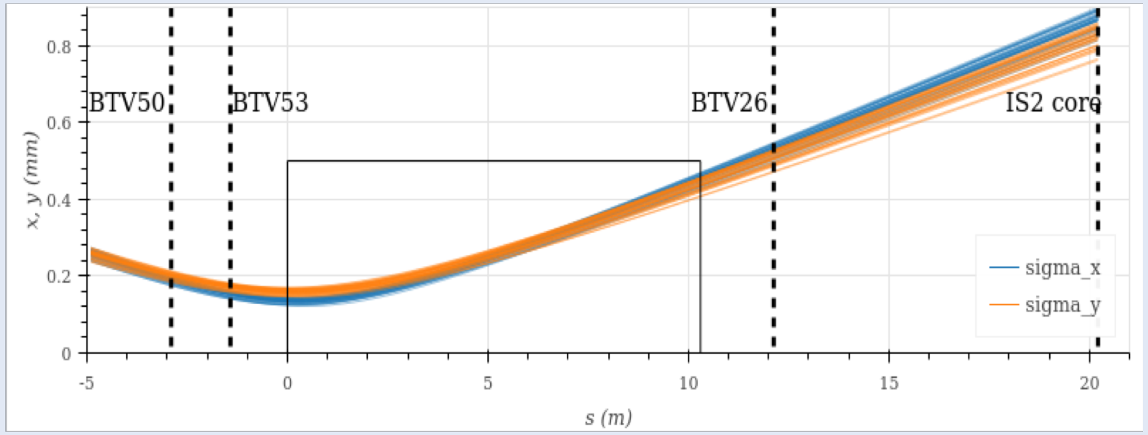
1) Select coordinate of the waist position s_{waist}.

2) Generate coordinates of N=8e6 particles according to the Gaussian distribution with parameters (μ_0, σ_0).

3) Generate N=8e6 random angular momentum according to the Gaussian distribution with parameters $(\mu = 0, \sigma_a)$.

4) Propagate each x_i according to the equation:
$$x_i(s) = x_i(0) + x_i' * s$$

5) Construct histogram of particle distribution at each screen.



Probability distribution of the proton bunch

Analytical equation:

1) Define analytical equation which describes $\sigma(s)$:

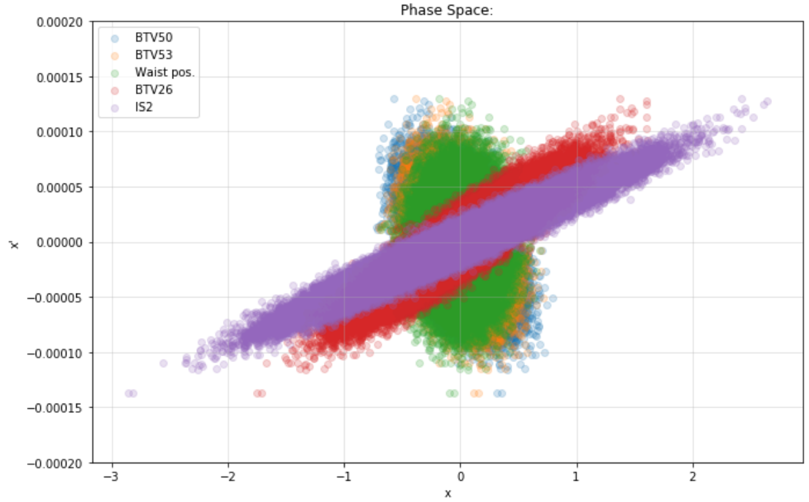
$$\sigma(s)^2 = \sigma_0^2 - 2\, s_{waist}\sigma_a^2 s + \sigma_a^2 s$$

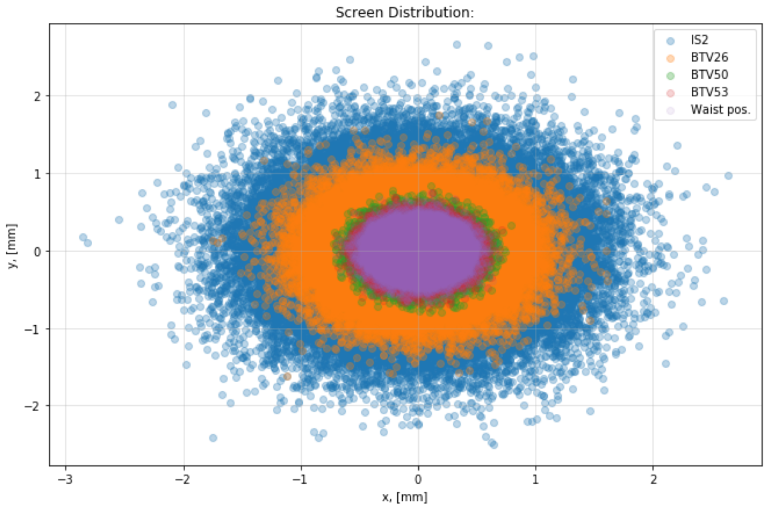
2) Generate $P(x, y|\sigma(s), \mu) = P(x, y|s, s_{waist}, \sigma_a, \sigma_0)$

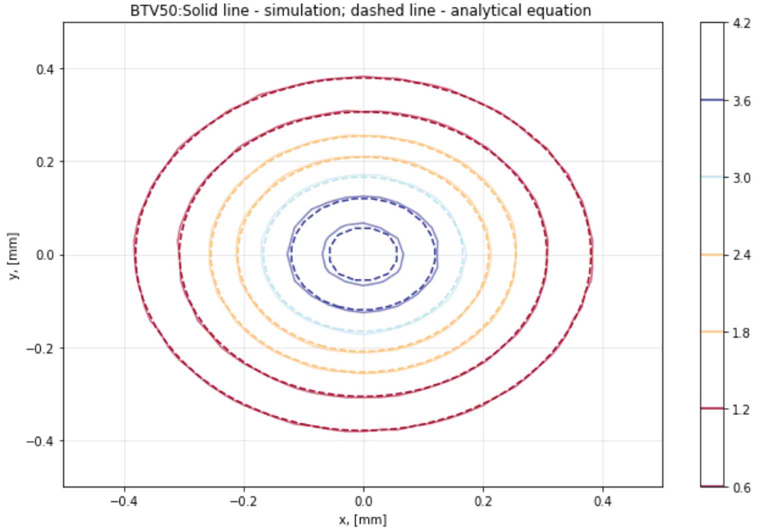
$$P(x, y|\sigma(s), \mu) = \frac{1}{2\, \pi\sigma_x(s)\sigma_y(s)} e^{-\frac{(x-\mu_x)^2}{2\sigma_x(s)^2}} \cdot e^{-\frac{\left(y-\mu_y\right)^2}{2\sigma_y(s)^2}}$$

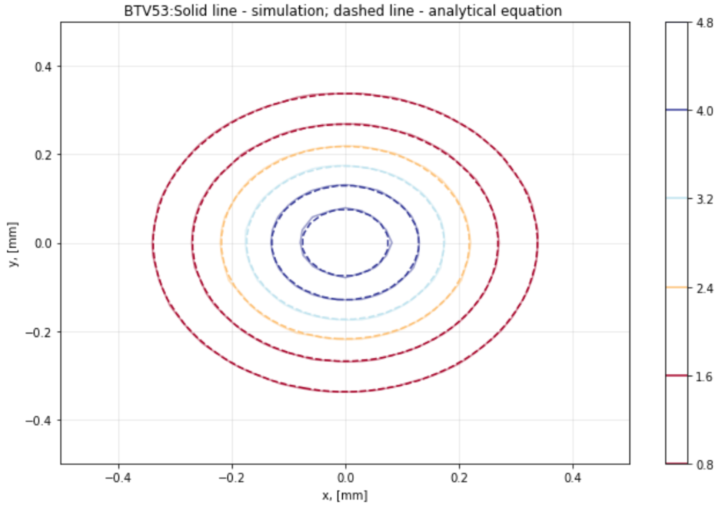
Probability distribution of the proton bunch

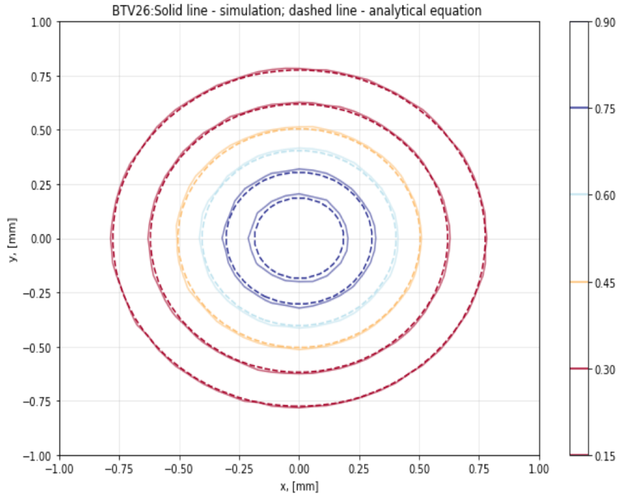
$$\{\sigma_x, \sigma_y\} = \{0.19, 0.19\}[mm]$$
$$\{\sigma_{x'}, \sigma_{x'}\} = \{3e^{-5}, 3e^{-5}\}$$









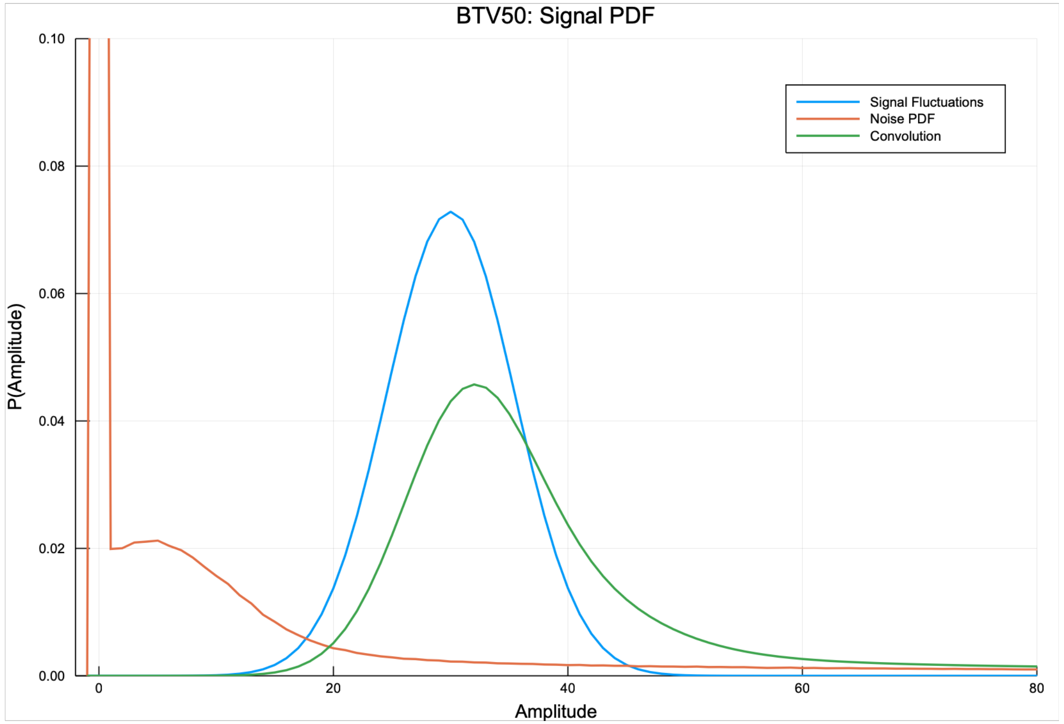


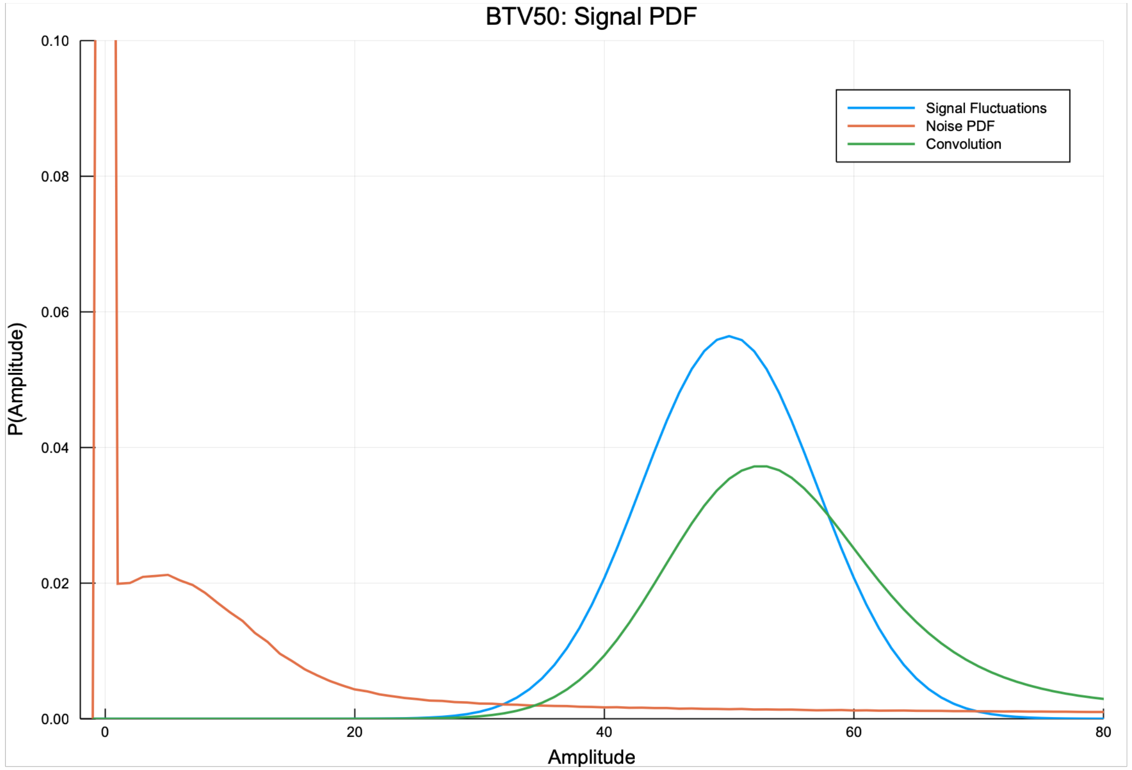
“Heavy” Likelihood: Tests results

Difficult task: Consider full numerical convolution of the expected light fluctuations with the noise PDF.

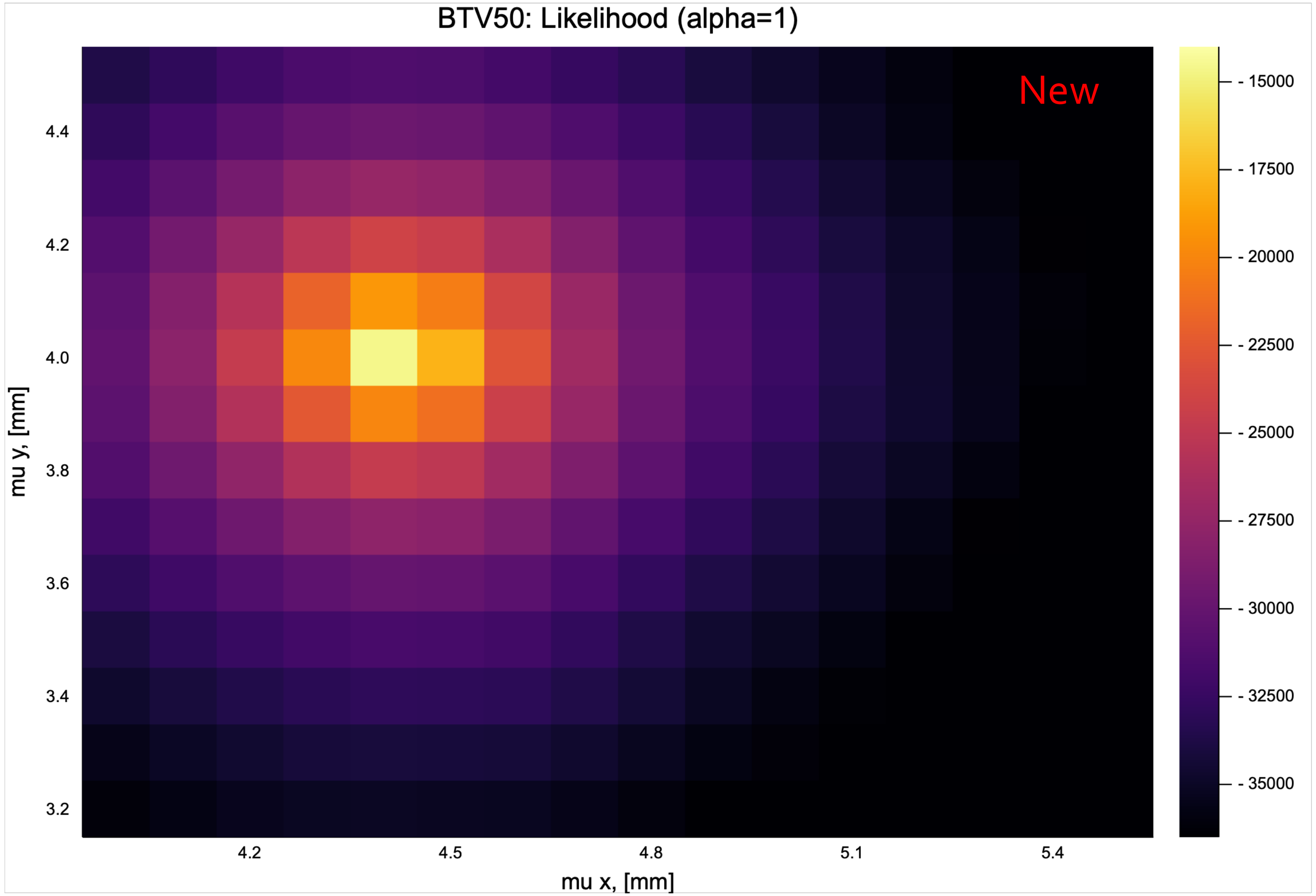
$$P_{light}(x|\lambda) = \frac{1}{\sqrt{2\pi}\alpha\sqrt{x}} e^{-\frac{(x-M(\lambda))^2}{2(\alpha\sqrt{x})^2}}$$

$$P_{rc}(x) = \int_{-\infty}^{\infty} P_{noise}(\theta - x) P_{light}(\theta|\lambda) d\theta$$

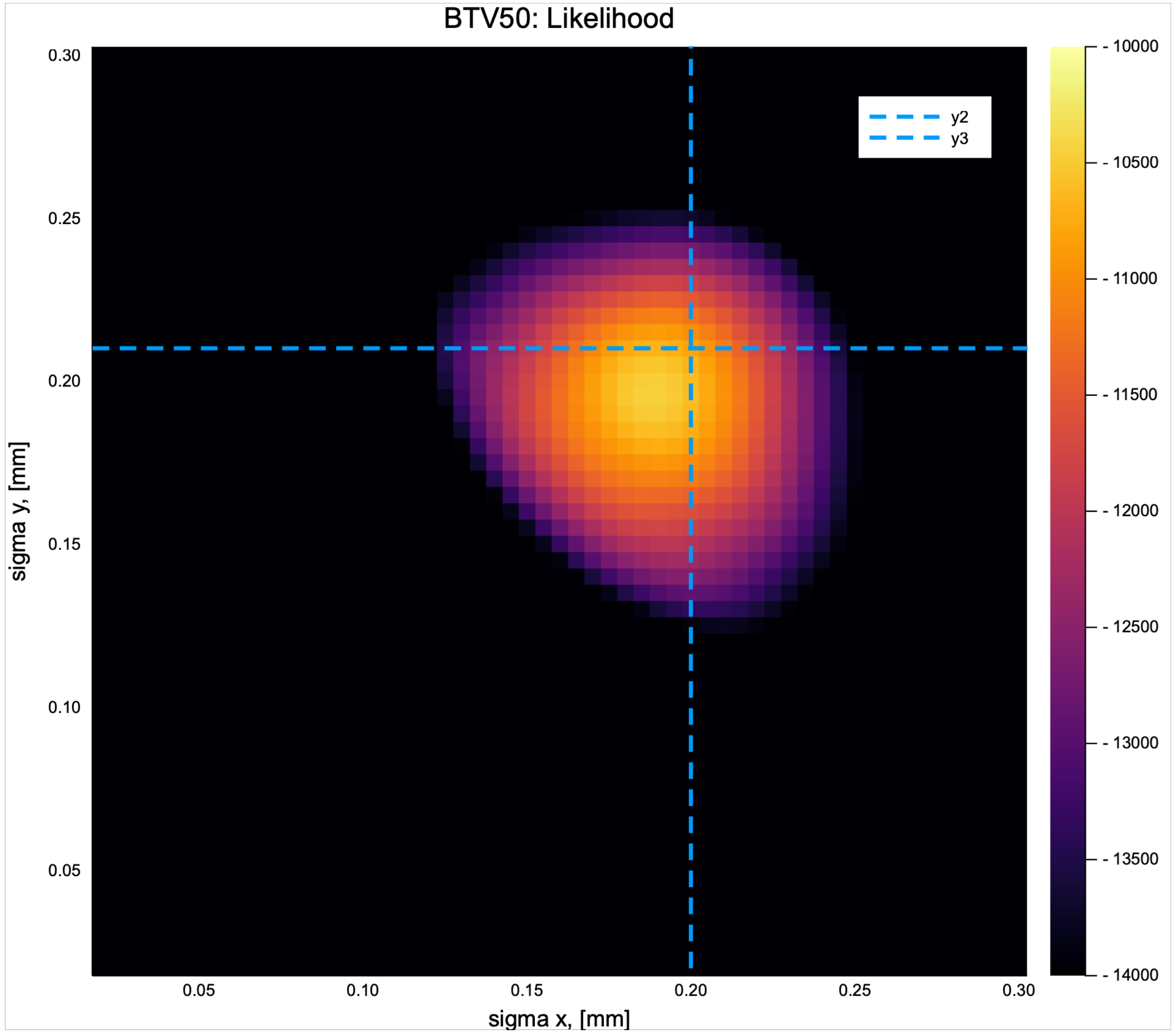




"Heavy" Likelihood: Tests results



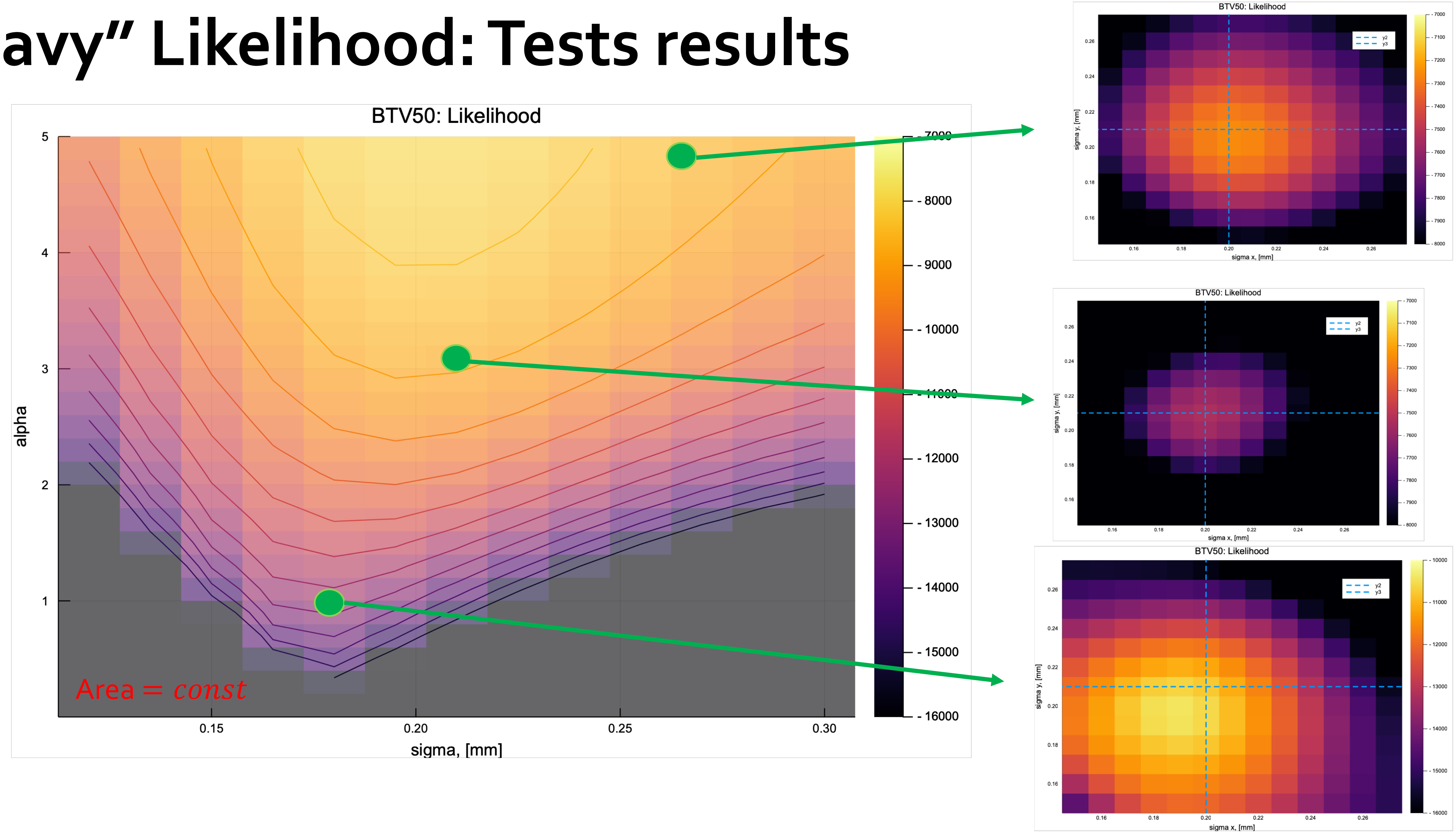
"Heavy" Likelihood: Tests results



Conclusions:

1) Likelihood function has to be optimized.
2) MCMC has to be performed.

“Heavy” Likelihood: Tests results



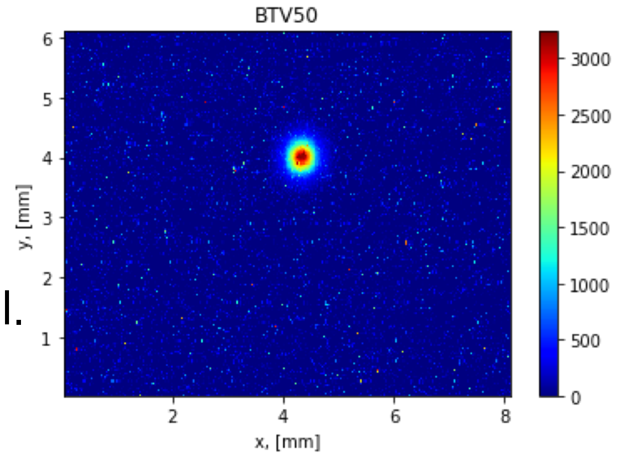
Likelihood Function

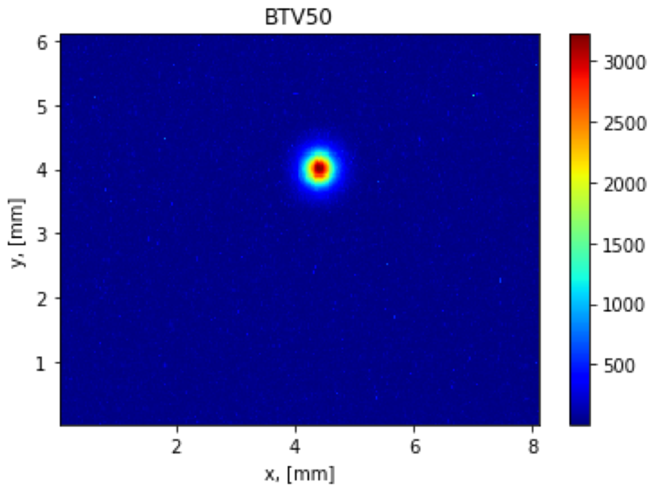
Alpha Coefficient: Residual Analysis

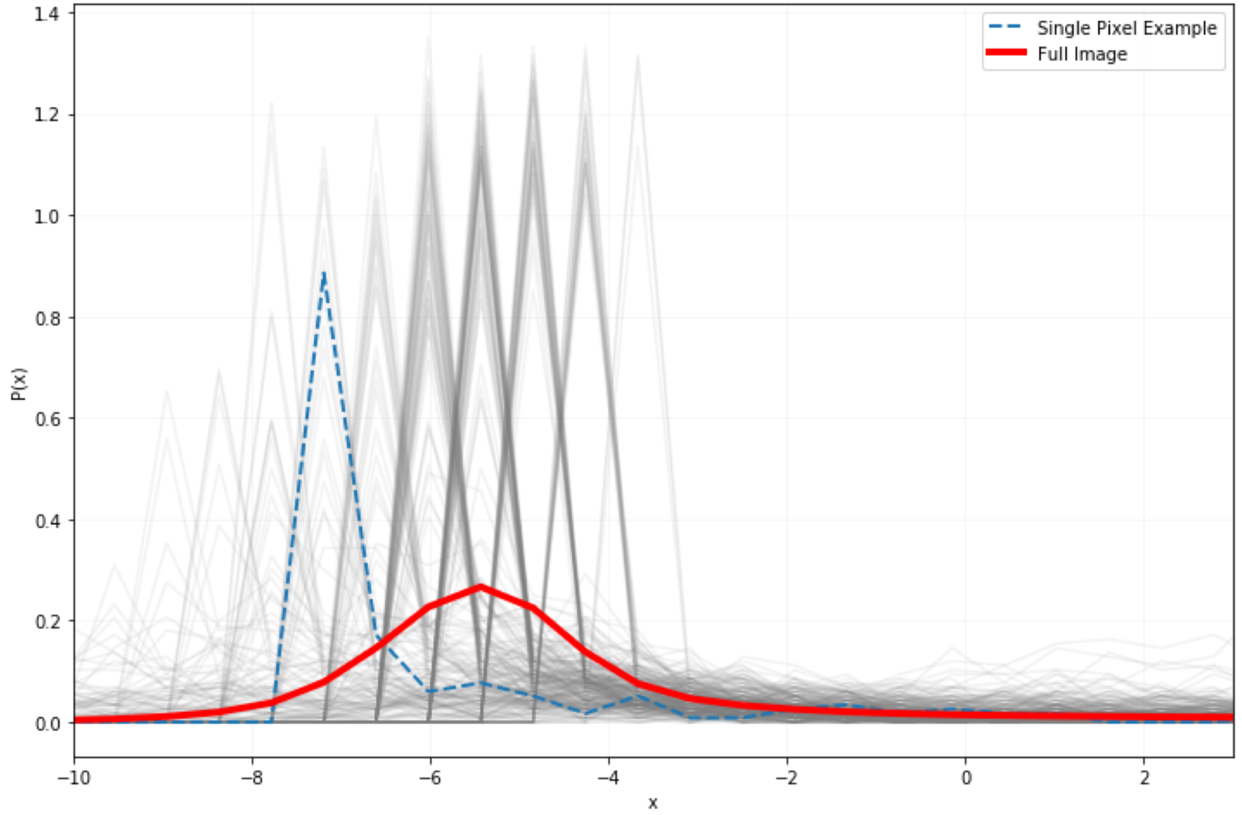
1) Find an average image over ~200 events => model signal.
2) Construct a histogram of a quantity x_i^{rc}

$$x_i^{rc} = \frac{D_i^{rc} - \mu^{rc}}{\alpha \sqrt{\mu^{rc}}}$$

- Where D_i^{rc} is an individual amplitude of the every row and column.
- μ^{rc} is an amplitude of the average image for the given row and column.



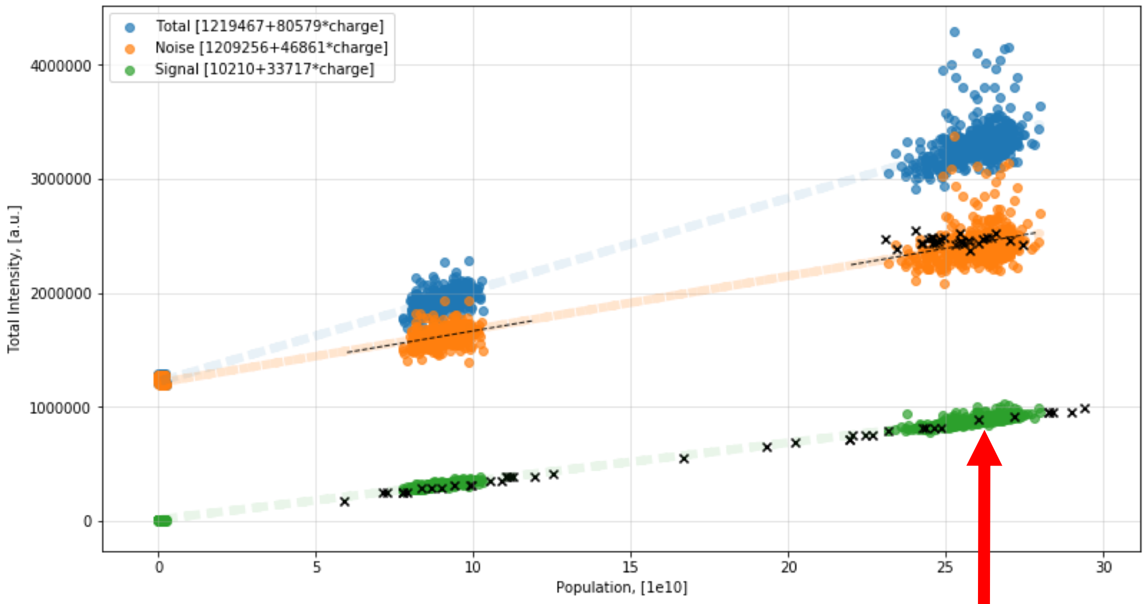


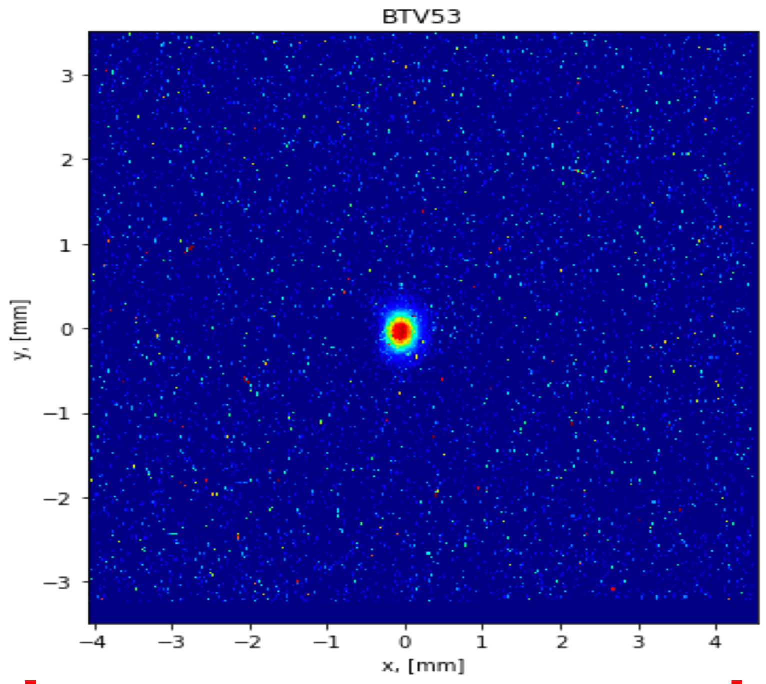


Likelihood function: Pseudocode

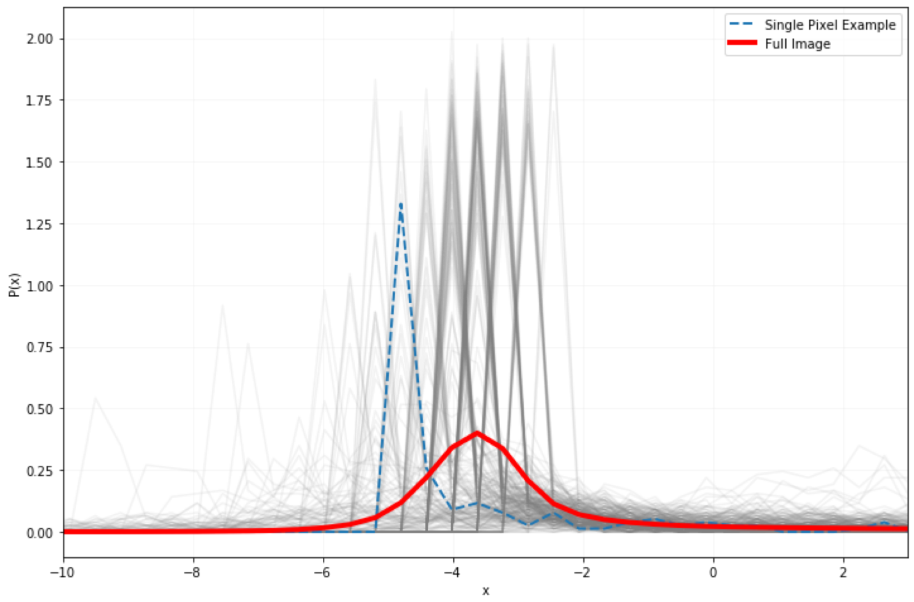
1. Generate Gaussian PDF that satisfies intensity rules for a given N:

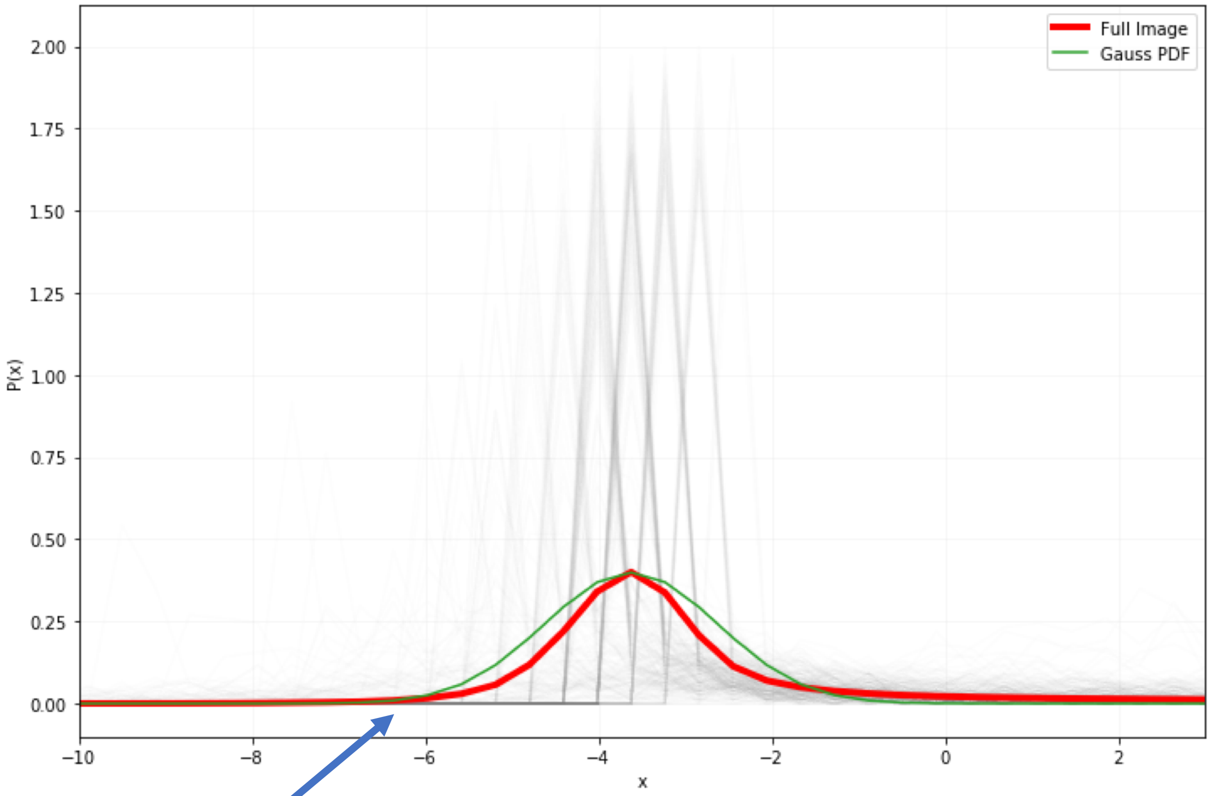
Preliminary Analysis: Total Intensity on the Camera





Likelihood Function





Fluctuations of light are (almost) Gaussian

BTV50 Likelihood function

1. Light Fluctuations → $$P_{light}(x|\lambda) = \frac{1}{\sqrt{2\pi}\alpha\sqrt{x}} e^{-\frac{(x-M(\lambda))^2}{2(\alpha\sqrt{x})^2}}$$

2. "Light" Likelihood: Tests results → $$P_{rc}(x) = \max\{\, P_{noise}(x), P_{light}(x|\lambda)\,\}$$

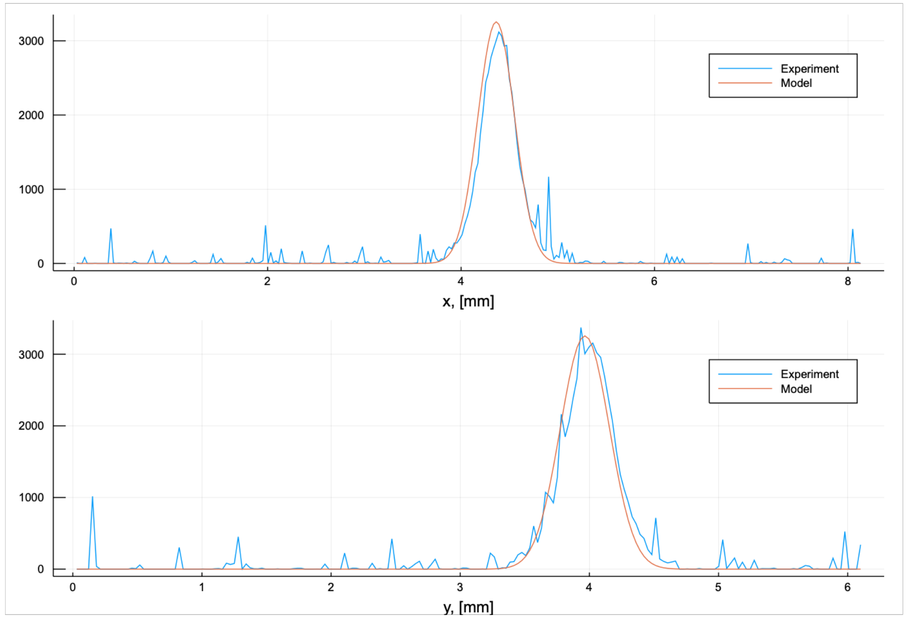
3. "Heavy" Likelihood: Tests Results → $$P_{rc}(x) = \int_{-\infty}^{\infty} P_{noise}(\theta - x) P_{light}(\theta|\lambda) d\theta c$$

“Light” Likelihood: Tests results

$$P_{rc}(x) = \max\{\, P_{noise}(x), P_{light}(x|\lambda) \,\}$$

Simple Test:

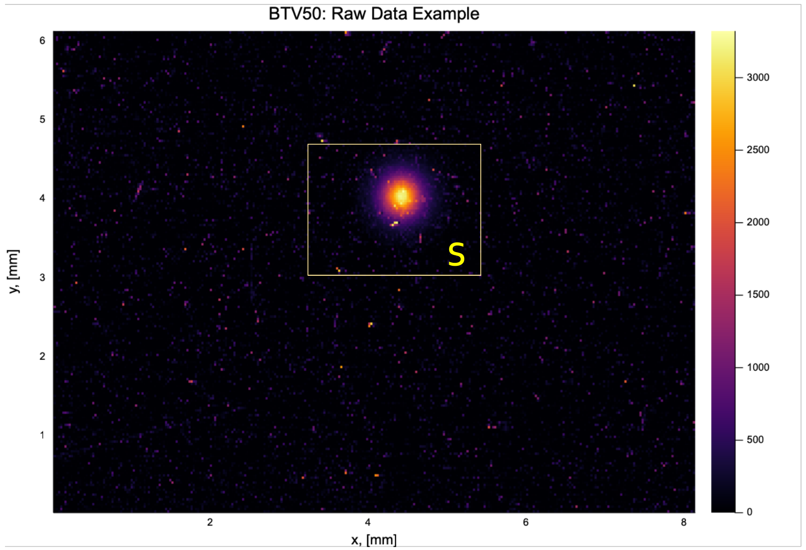
1) Find approximate parameters of the model:
2) Vary μ parameter to see how the likelihood changes.
3) Vary σ parameters to see how the likelihood changes.



Normalizing rules:

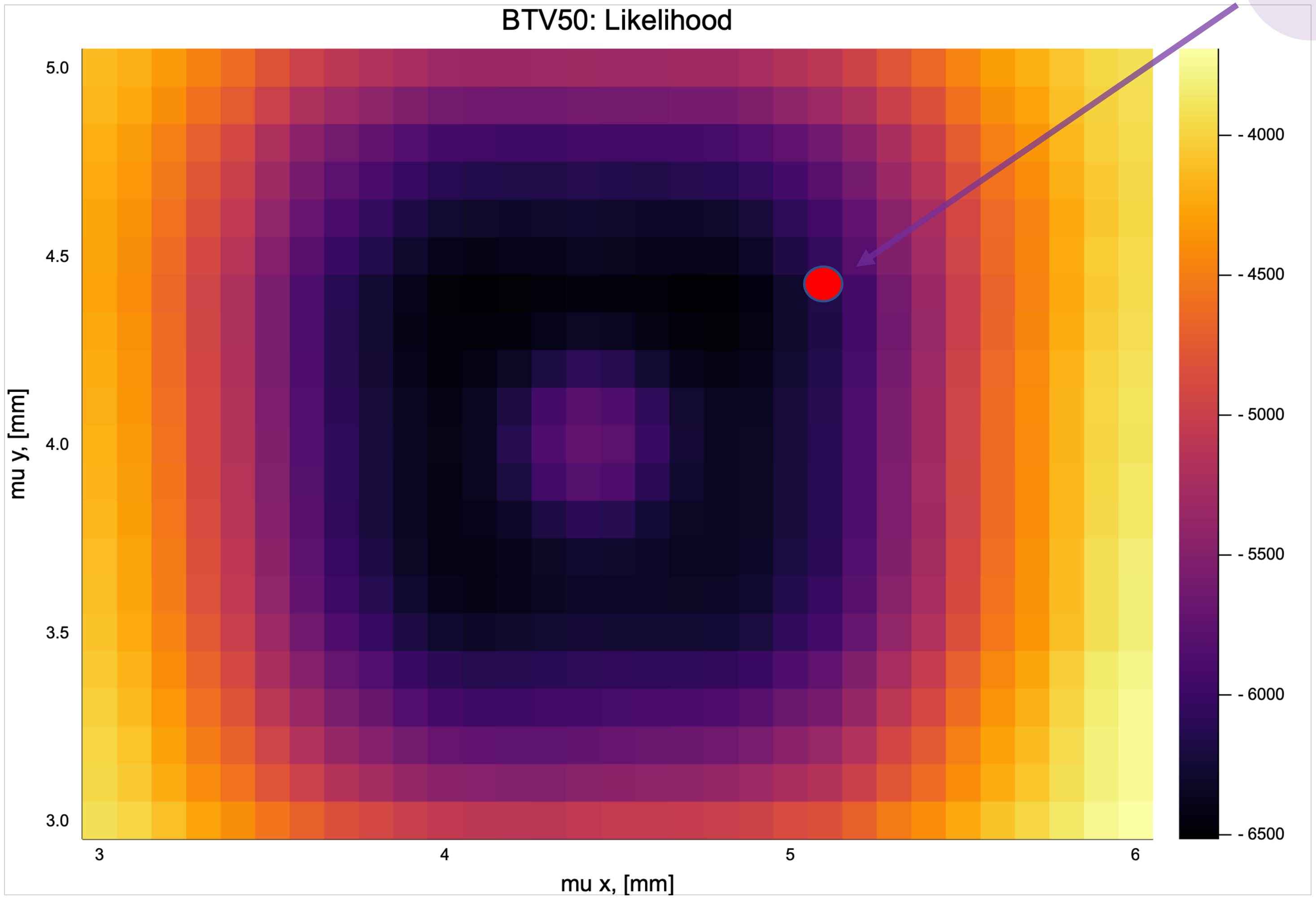
1. Likelihood is averages over pixels.
2. Likelihood is averaged over Data images.

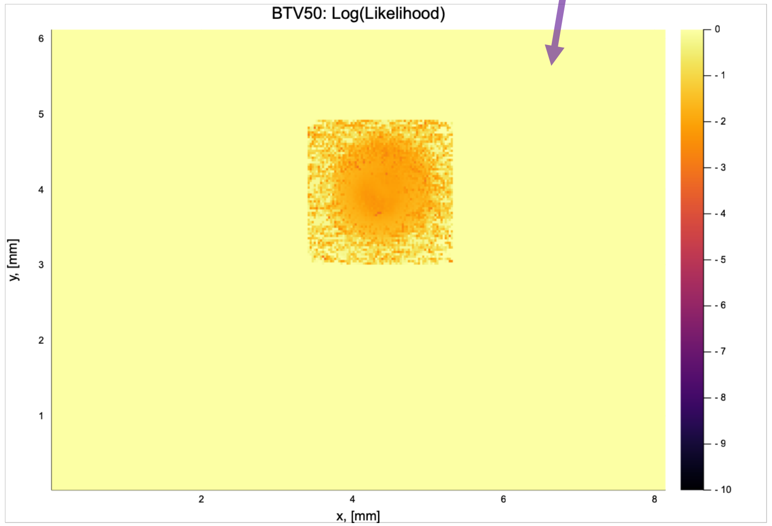
$$S(\lambda) = \frac{1}{N_{pixels\,\in\,S} \cdot N_{img}} \sum_{i}^{\in\,img} \sum_{r,c}^{\in S} \log[P^{i}_{rc}(Data|\lambda)]$$



“Light” Likelihood: Tests results

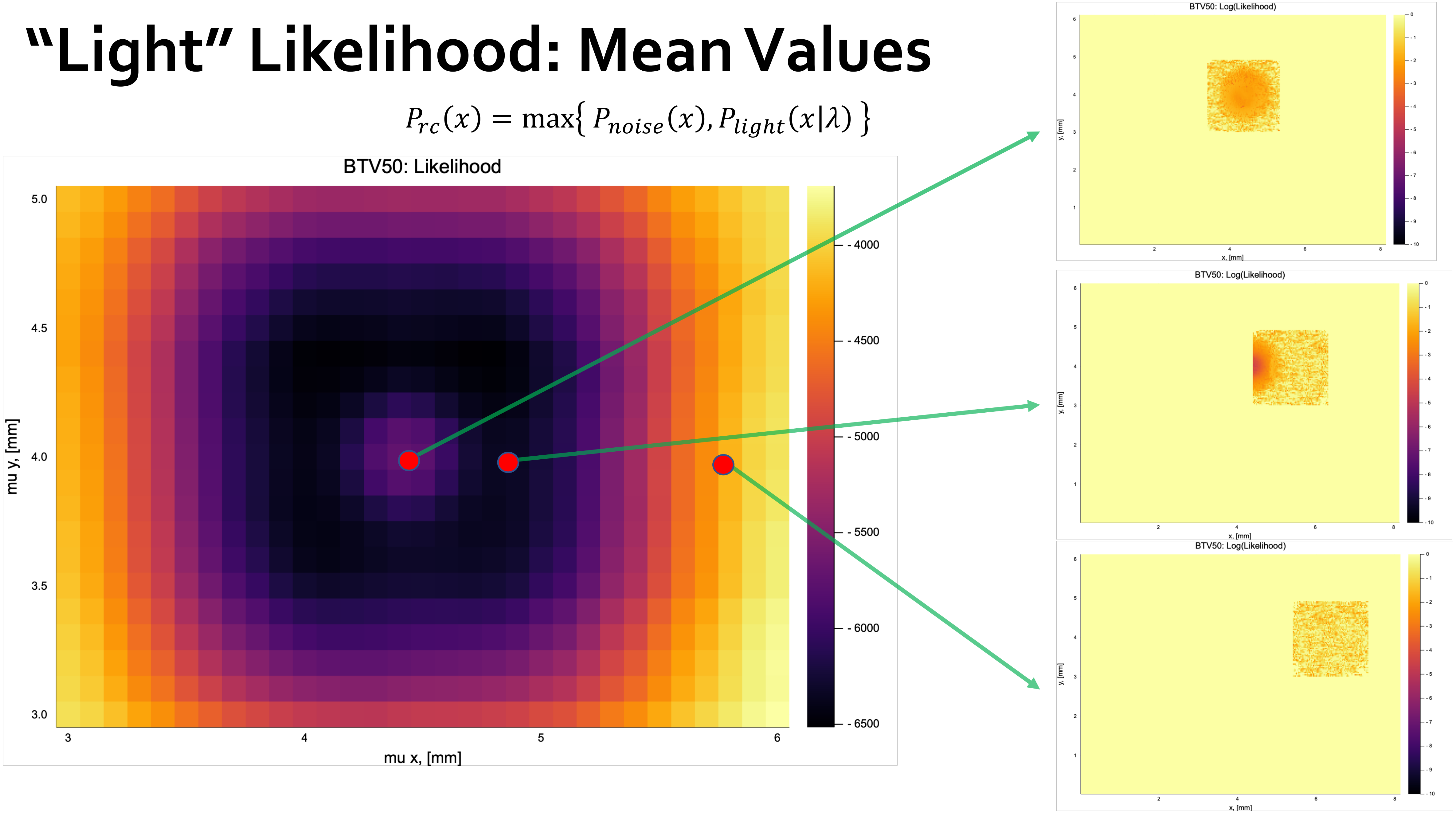
$$S(\lambda) = \frac{1}{N_{pixels \in S} \cdot N_{img}} \sum_{i}^{\in img} \sum_{r,c}^{\in S} \log[P^i_{rc}(Data|\lambda)]$$





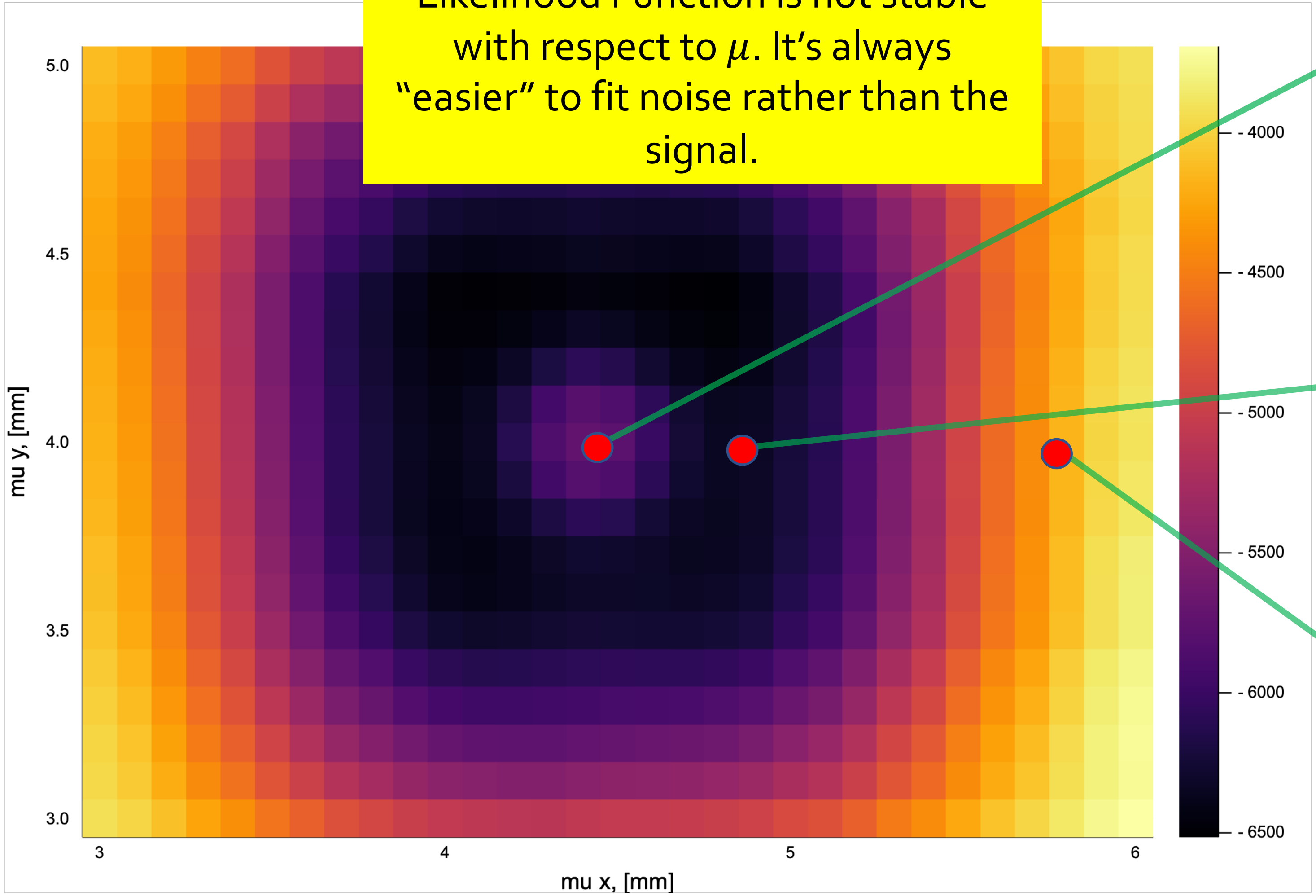
“Light” Likelihood: Mean Values

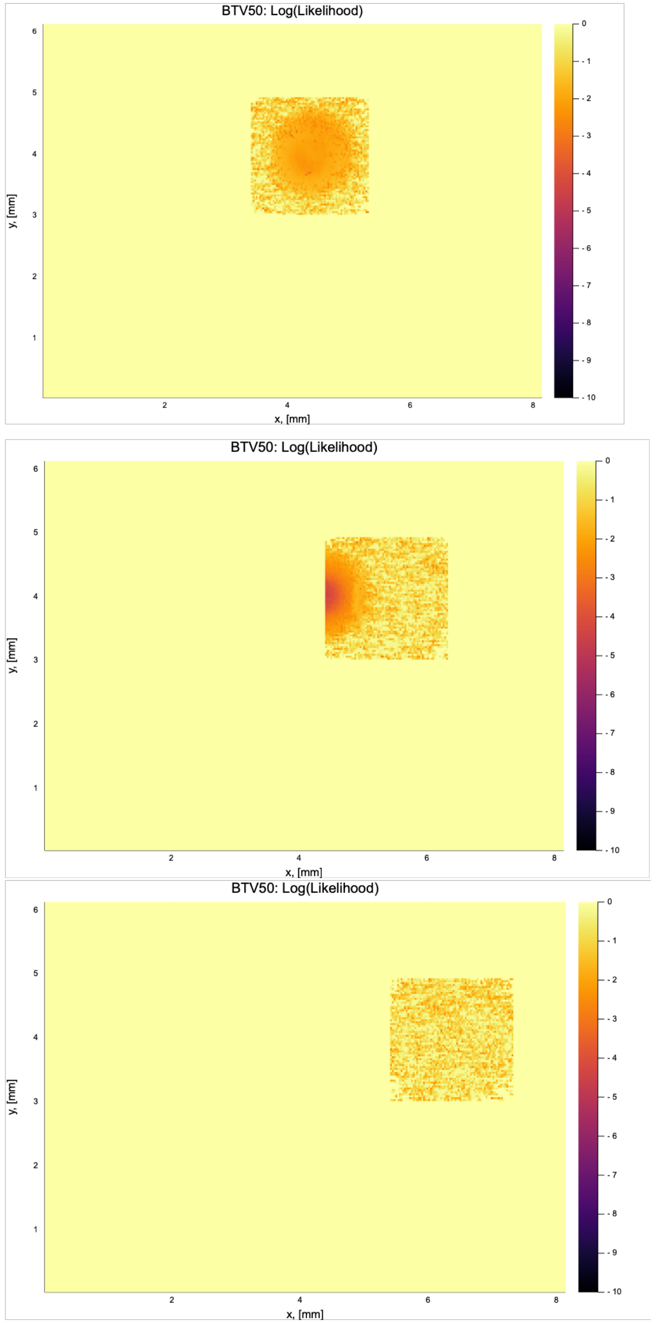
$$P_{rc}(x) = \max\{ P_{noise}(x), P_{light}(x|\lambda) \}$$



“Light” Likelihood: Mean Values

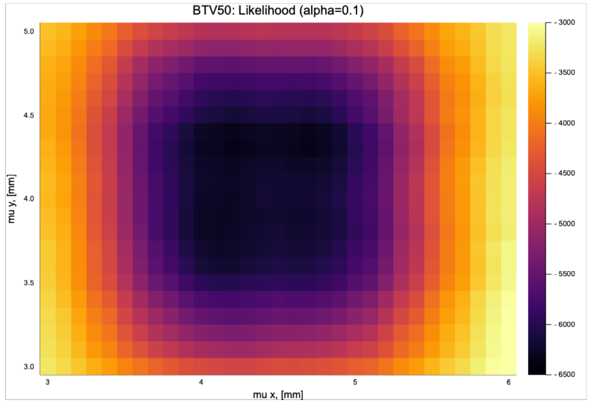
Likelihood Function is not stable with respect to μ. It’s always “easier” to fit noise rather than the signal.

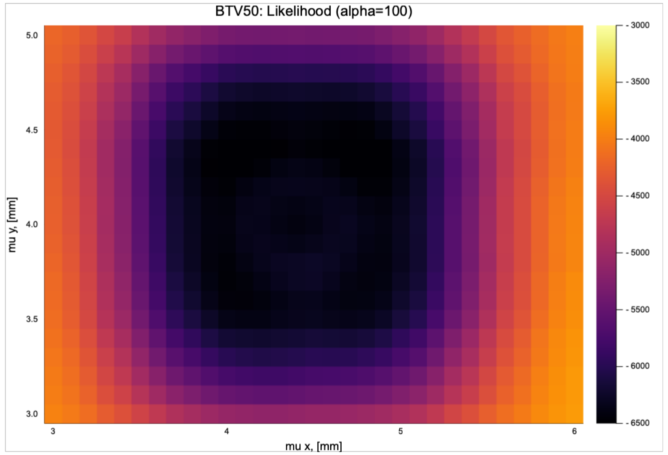


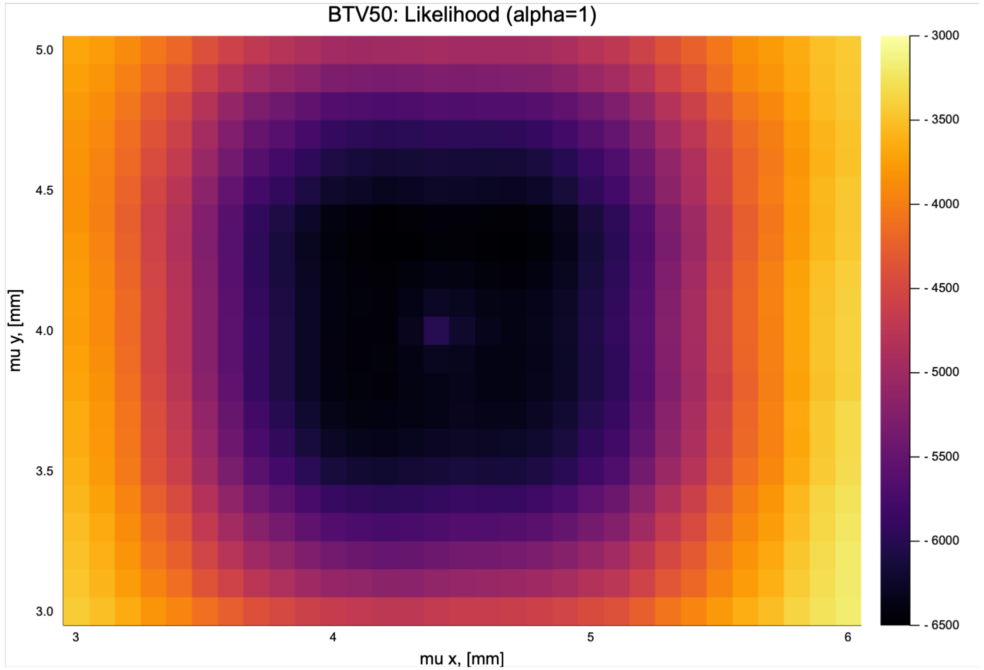


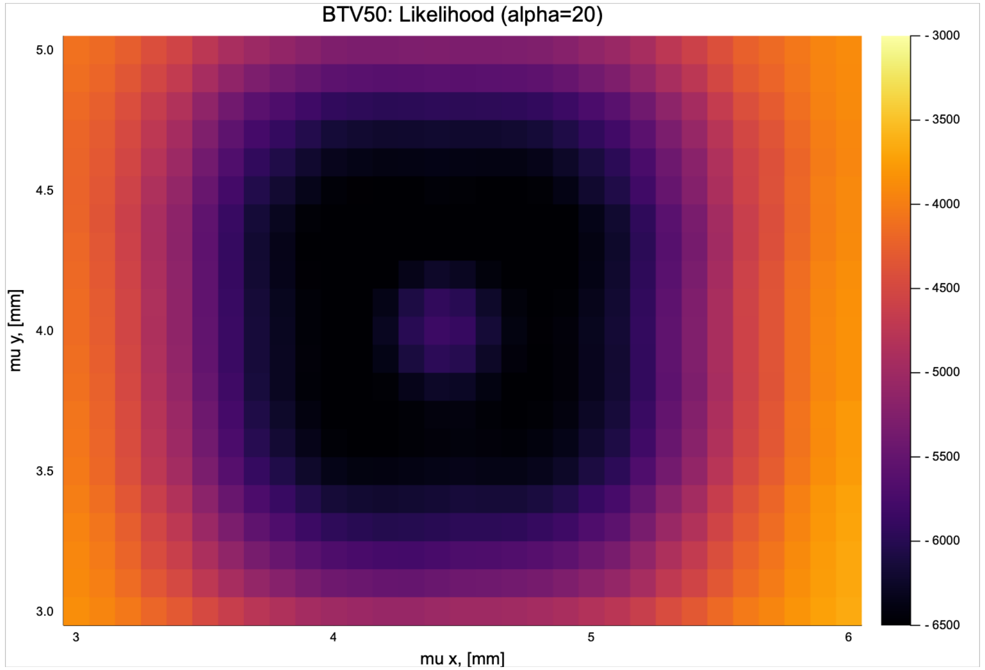
"Light" Likelihood: Mean Values

$$\alpha = \{0.1, 1, 20, 100\}$$

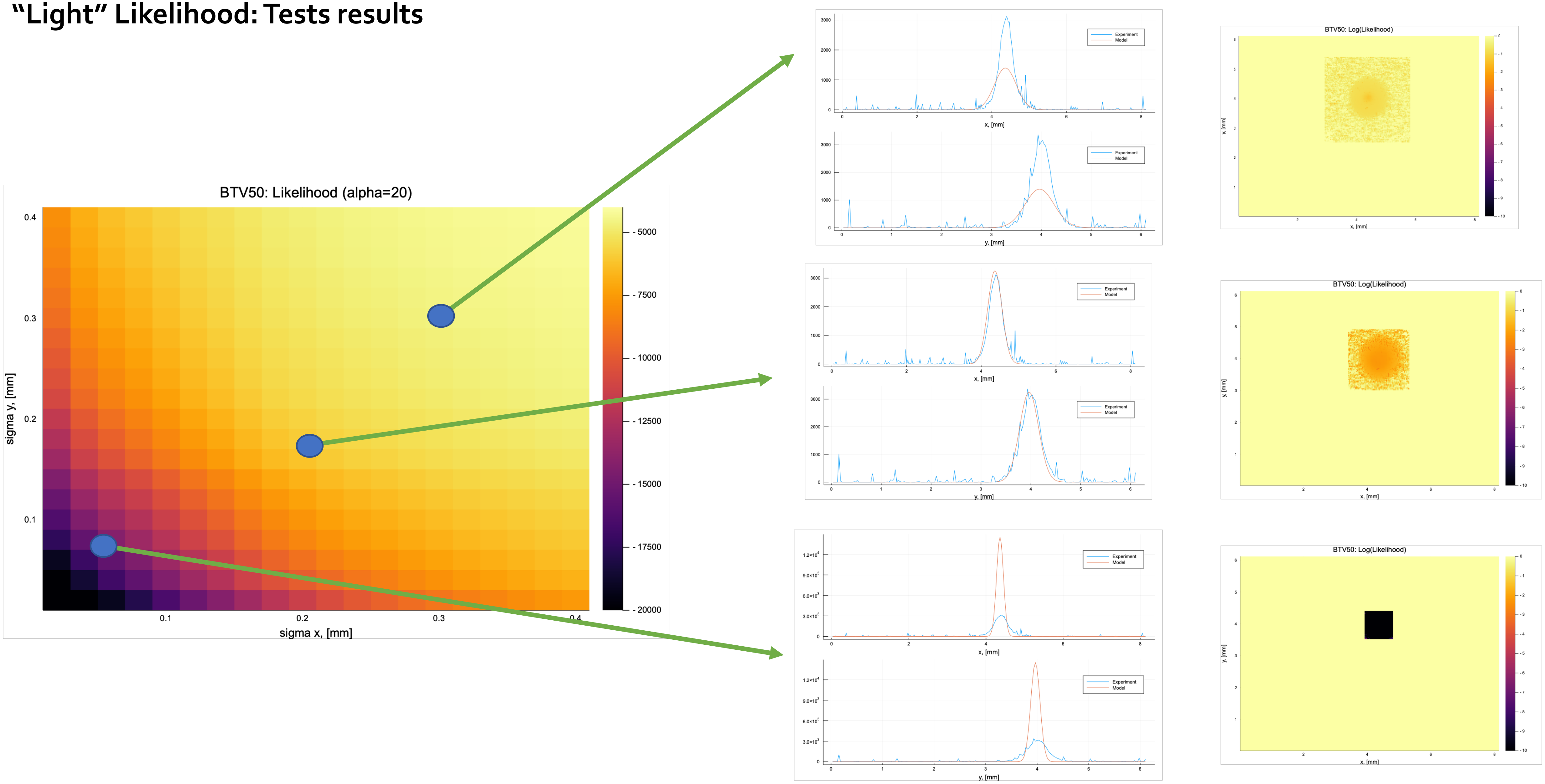






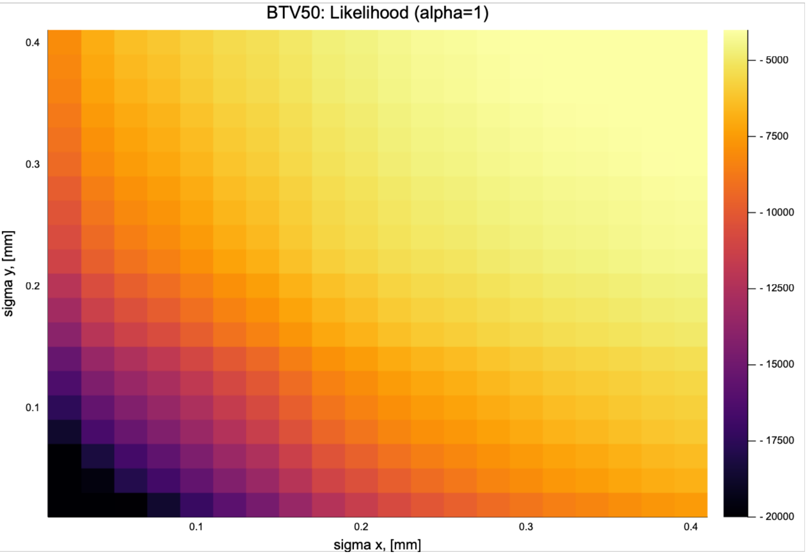


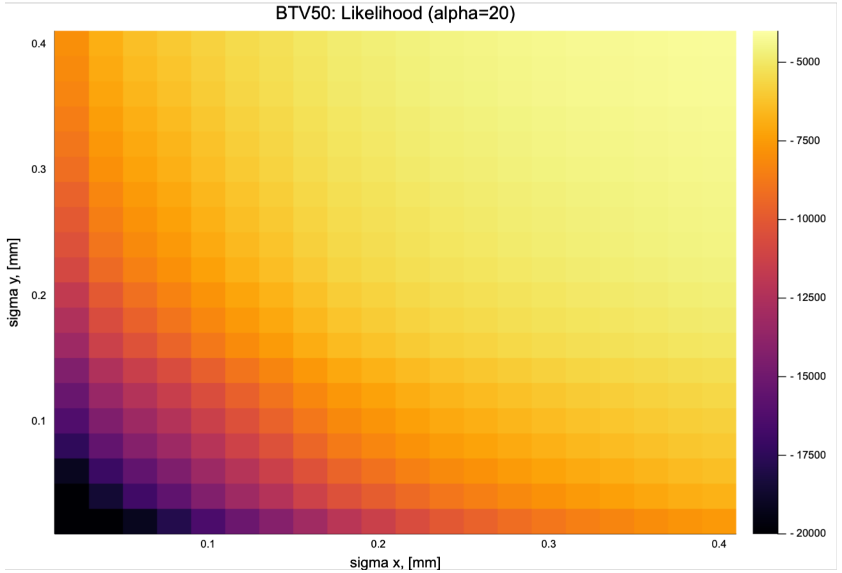
“Light” Likelihood: Tests results

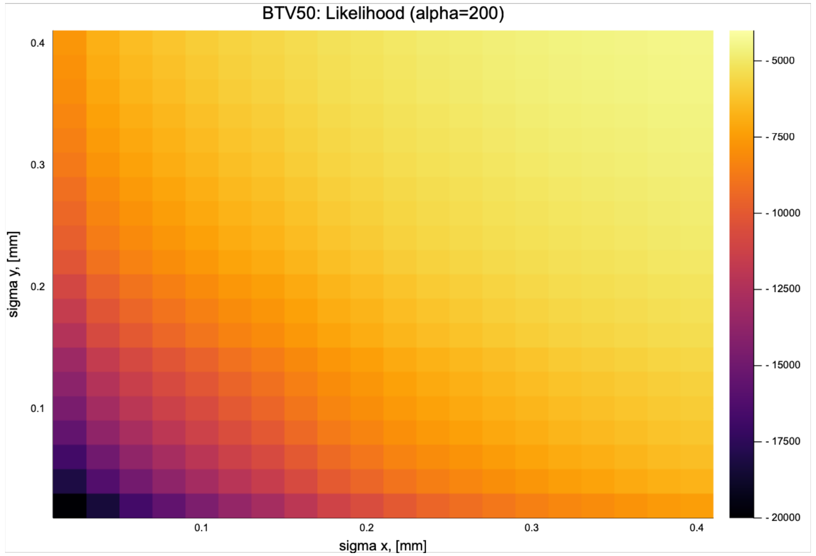


"Light" Likelihood: Tests results

Likelihood function is not stable.



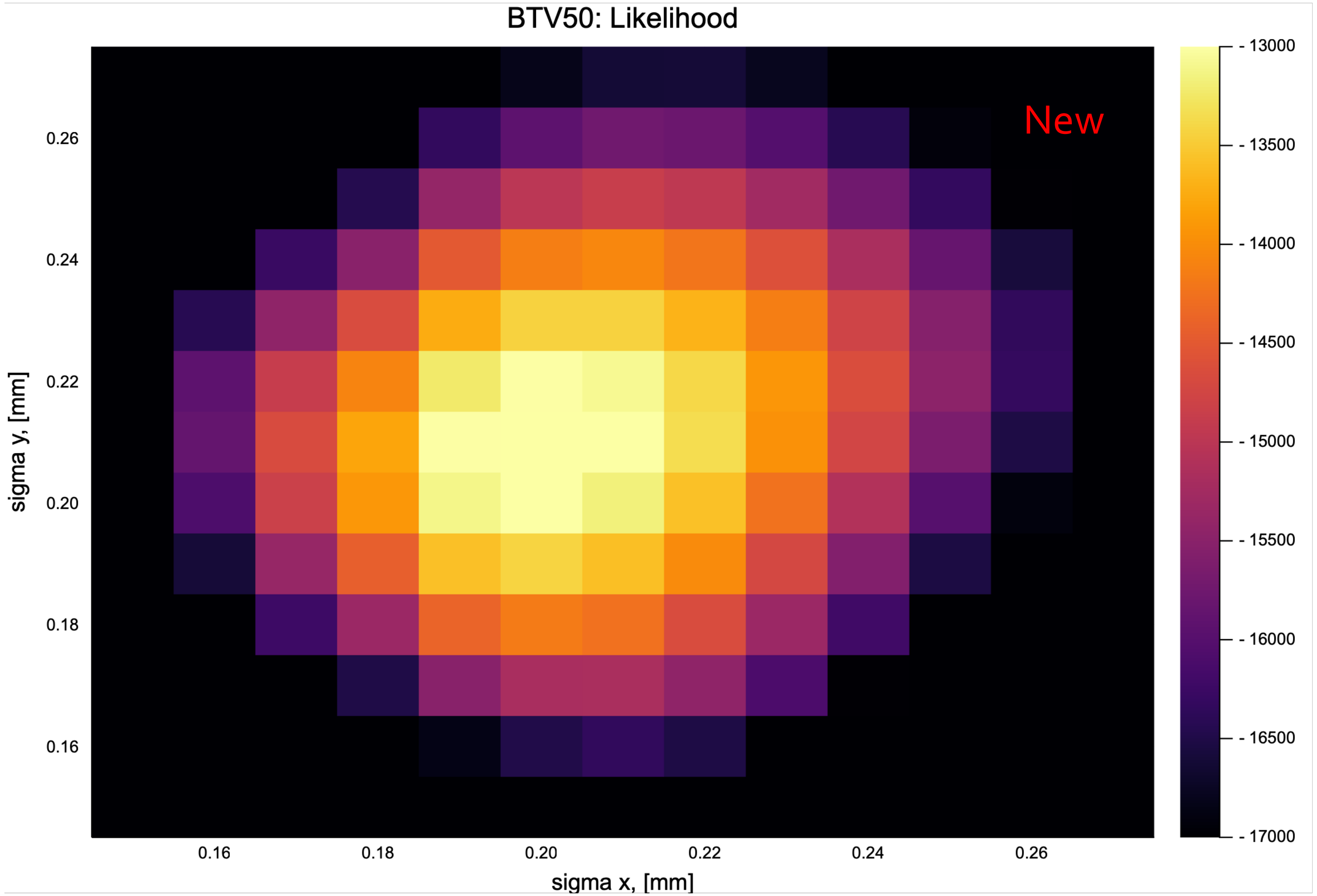


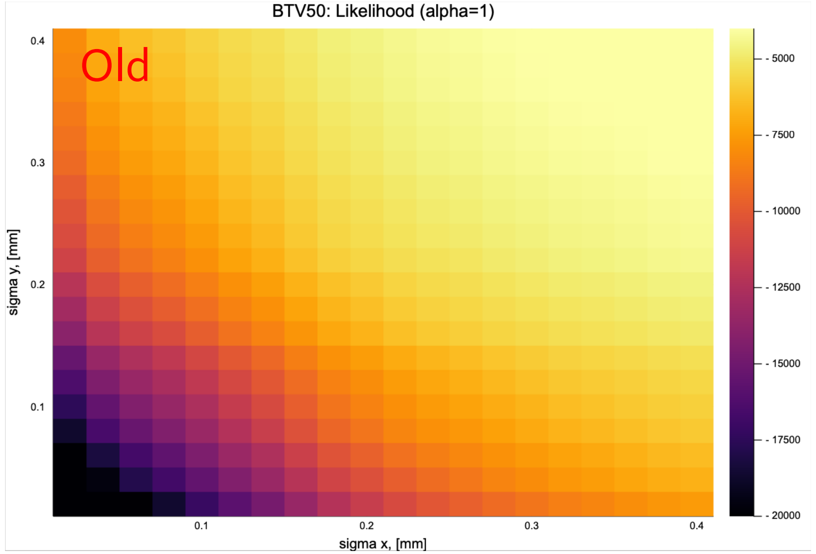


BTV50 Likelihood function

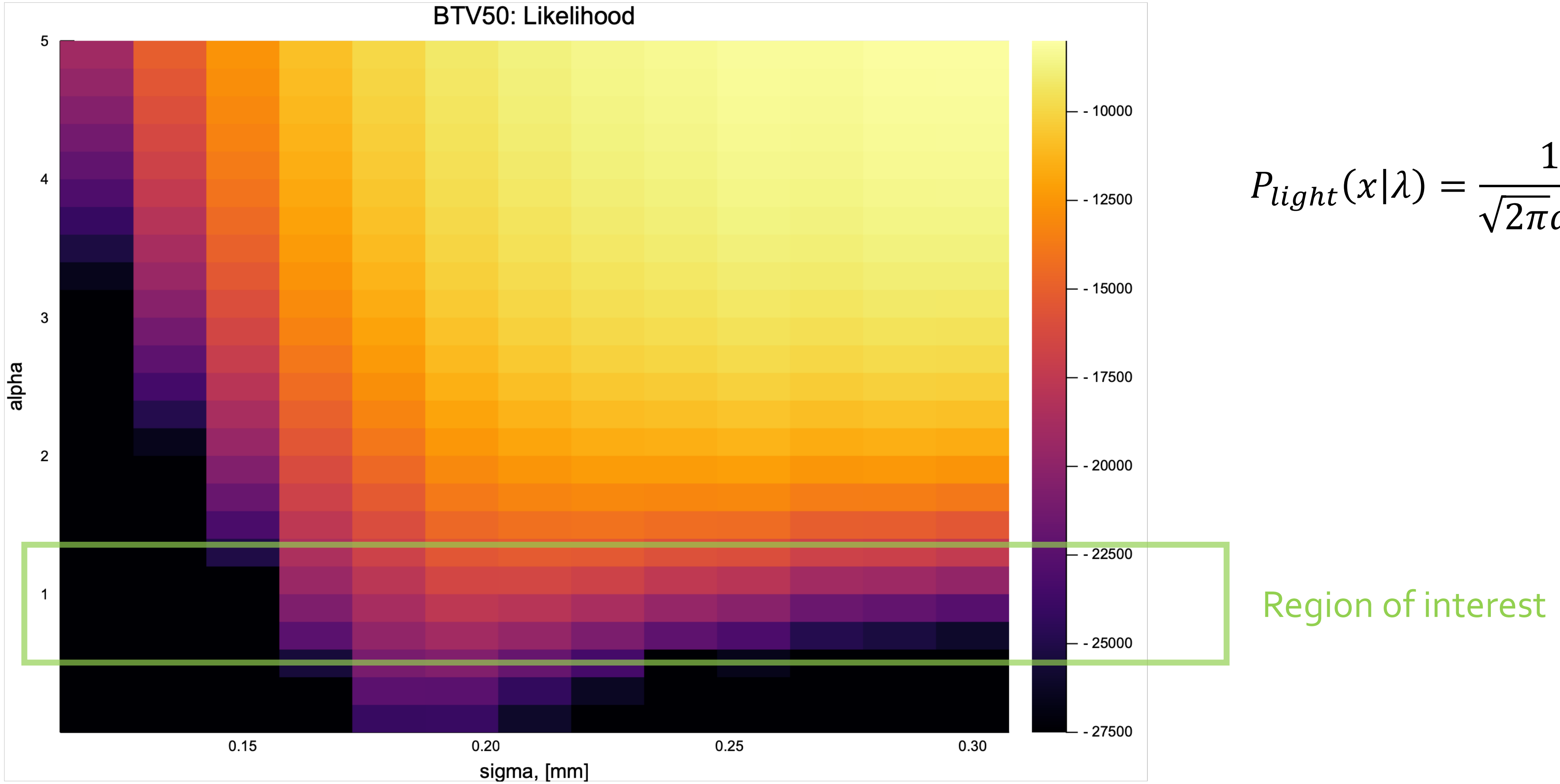
1. Light Fluctuations → $P_{light}(x|\lambda) = \frac{1}{\sqrt{2\pi}\alpha\sqrt{x}} e^{-\frac{(x-M(\lambda))^2}{2(\alpha\sqrt{x})^2}}$
2. "Light" Likelihood: Tests results → $P_{rc}(x) = \max\{ P_{noise}(x), P_{light}(x|\lambda) \}$
3. "Heavy" Likelihood: Tests Results → $P_{rc}(x) = \int_{-\infty}^{\infty} P_{noise}(\theta - x) P_{light}(\theta|\lambda) d\theta$

“Heavy” Likelihood: Tests results





“Heavy” Likelihood: Tests results



$$P_{light}(x|\lambda) = \frac{1}{\sqrt{2\pi}\alpha\sqrt{x}} e^{-\frac{(x-M(\lambda))^2}{2(\alpha\sqrt{x})^2}}$$

“Heavy” Likelihood: Tests results

Performance:

- ”Light” Likelihood: 198 images — 0.02 sec.
- “Heavy” Likelihood: 198 images (30,000 convolutions) – 1.8 sec.

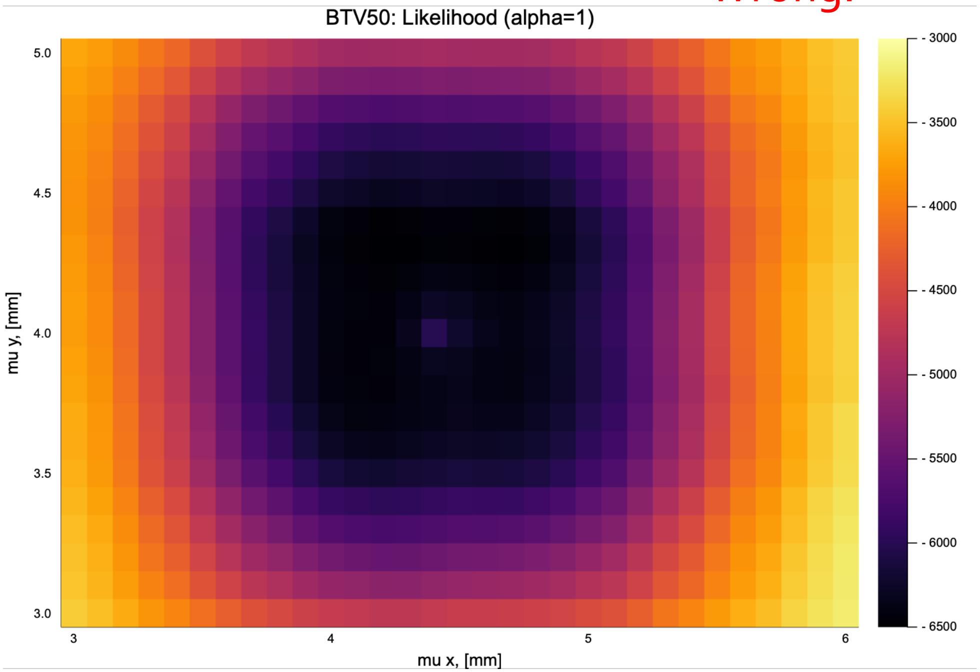
Possible improvement:

- Precompute 4000 convolutions for every histogram bin. Save it as a multidimensional array. Access precomputed array every iteration.
- The downside is that this requires discretization of signal amplitude.

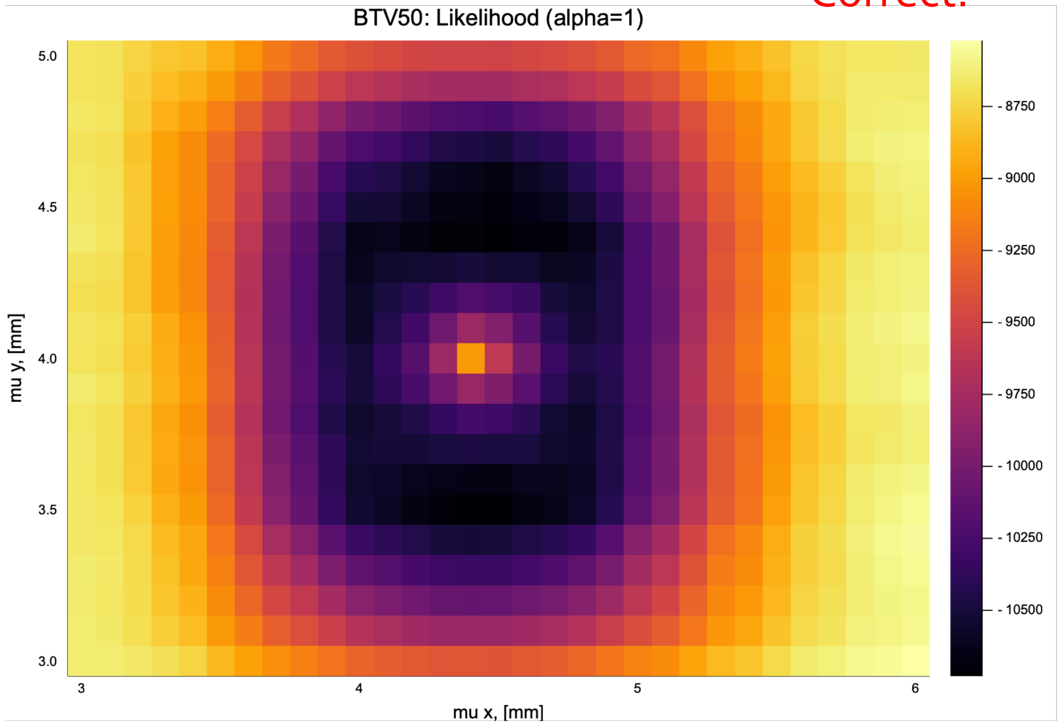
BTV50 Likelihood function

"Light" Likelihood: Tests results $P_{rc}(A) = \max\{ P_{noise}(A + A_{model}), P_{light}(A|\lambda) \}$

Wrong!

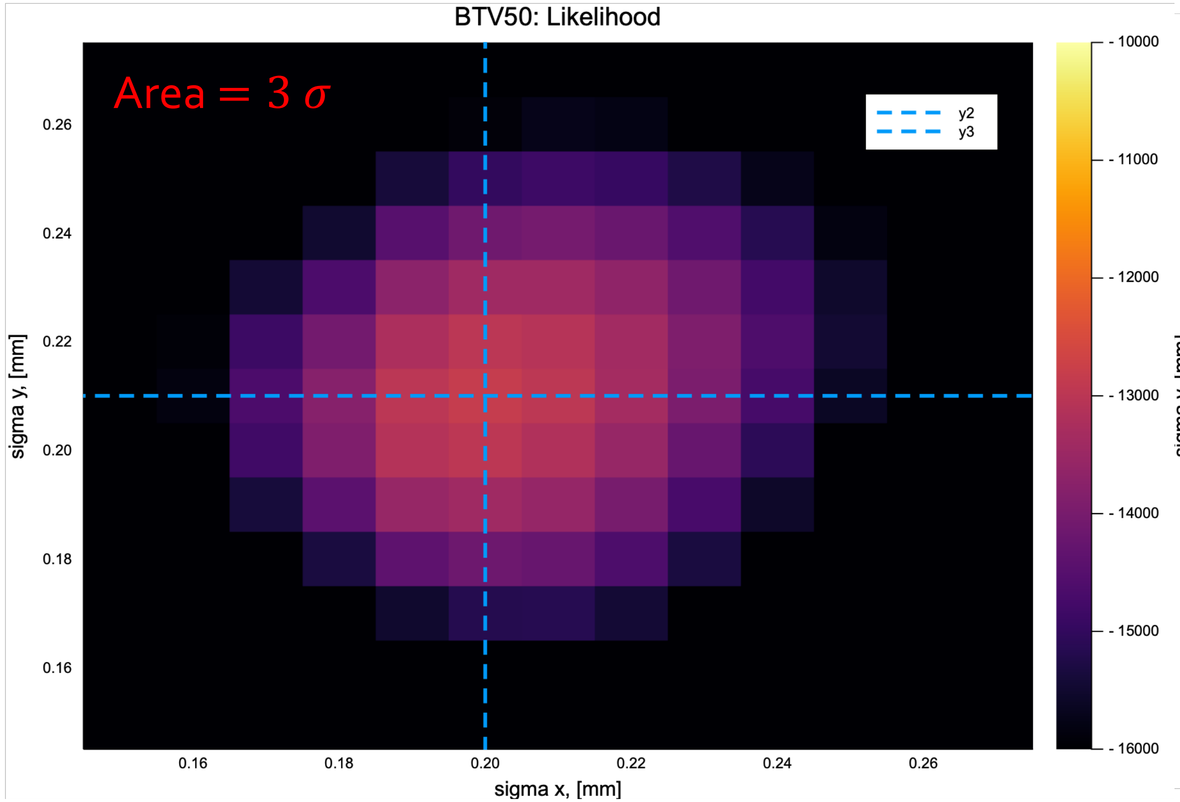


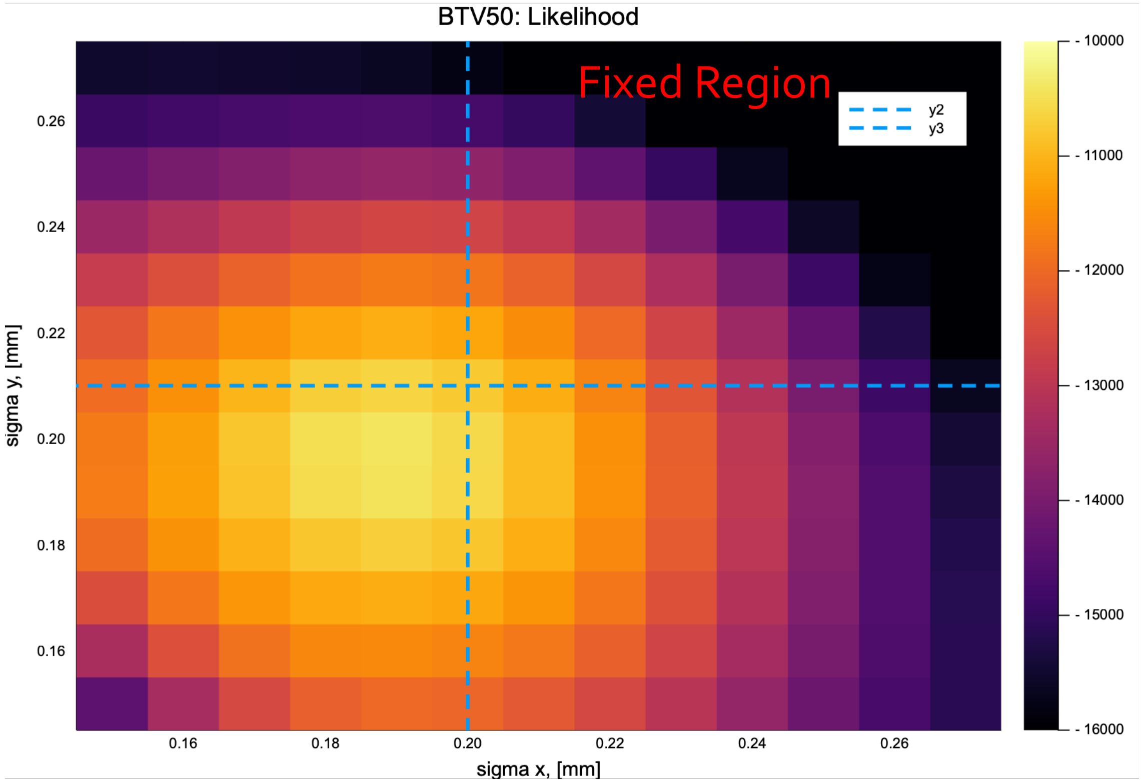
Correct!



BTV50 Likelihood function

$$S(\lambda) = \frac{1}{N_{pixels \in S} \cdot N_{img}} \sum_{i}^{\in img} \sum_{r,c}^{\in S} \log[P_{rc}^{i}(Data|\lambda)]$$

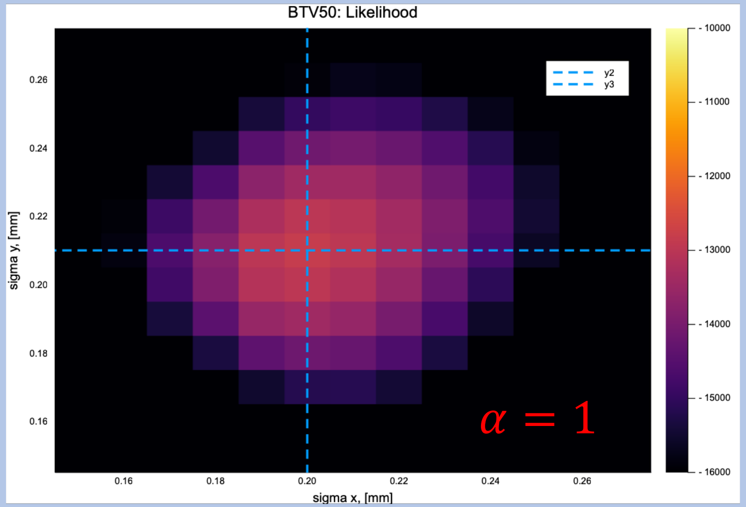


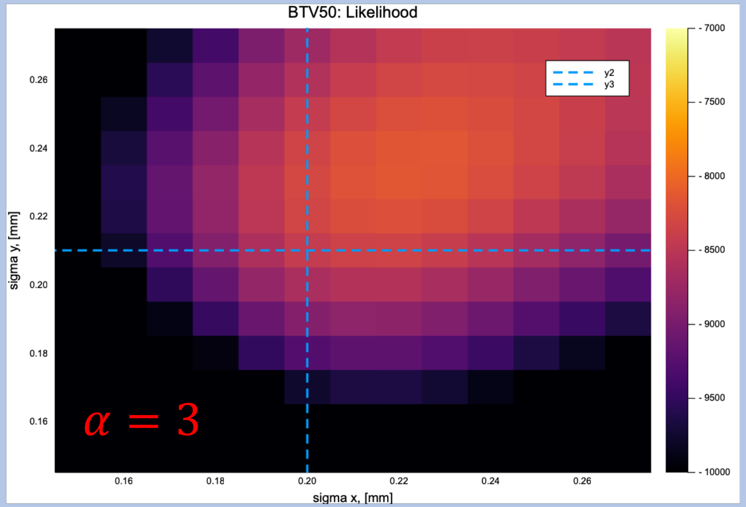


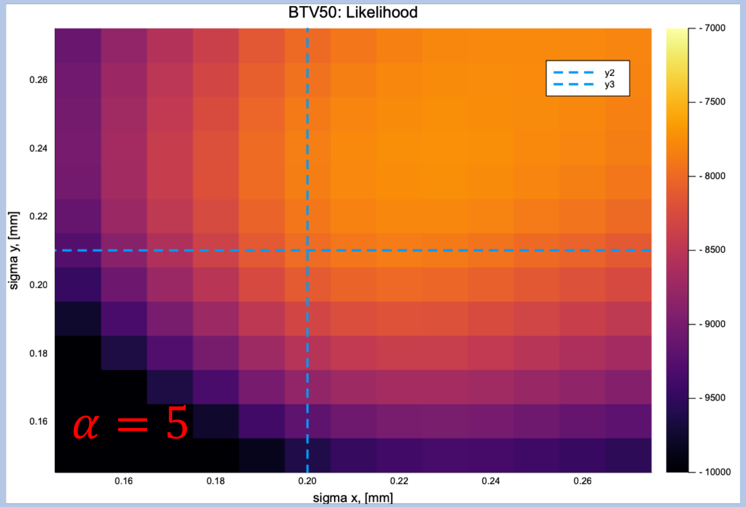
Why positions of maximum are different?

Area $= 3\,\sigma$

Position of the maximum is changing!

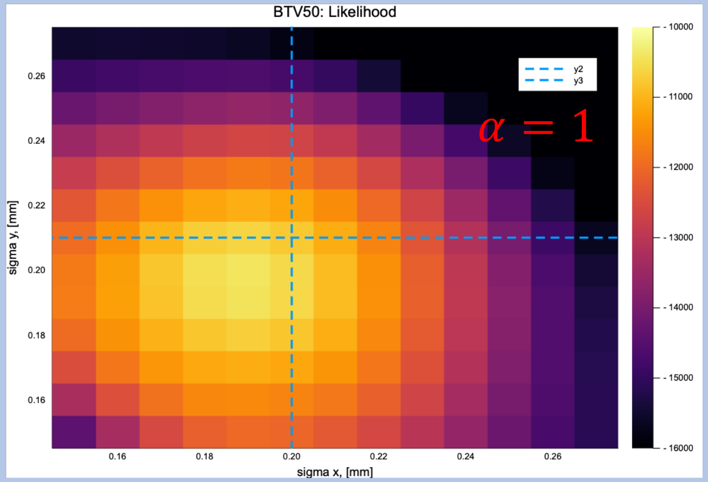


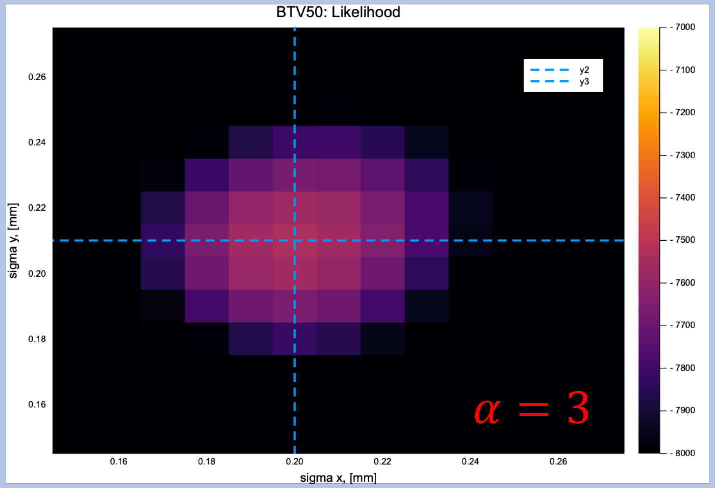


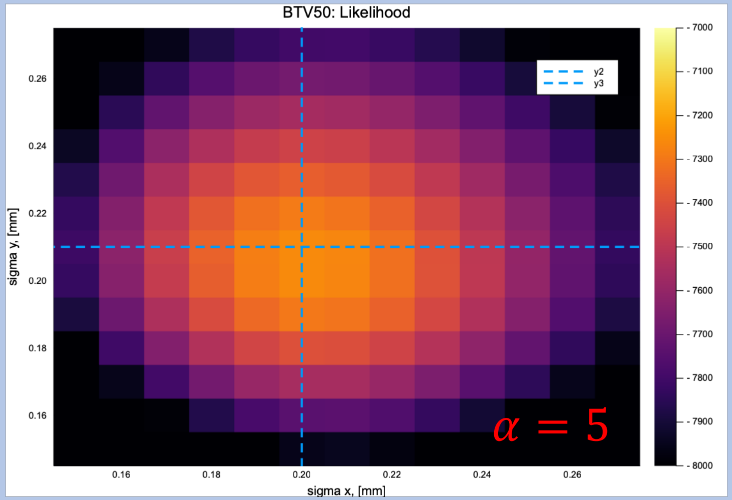


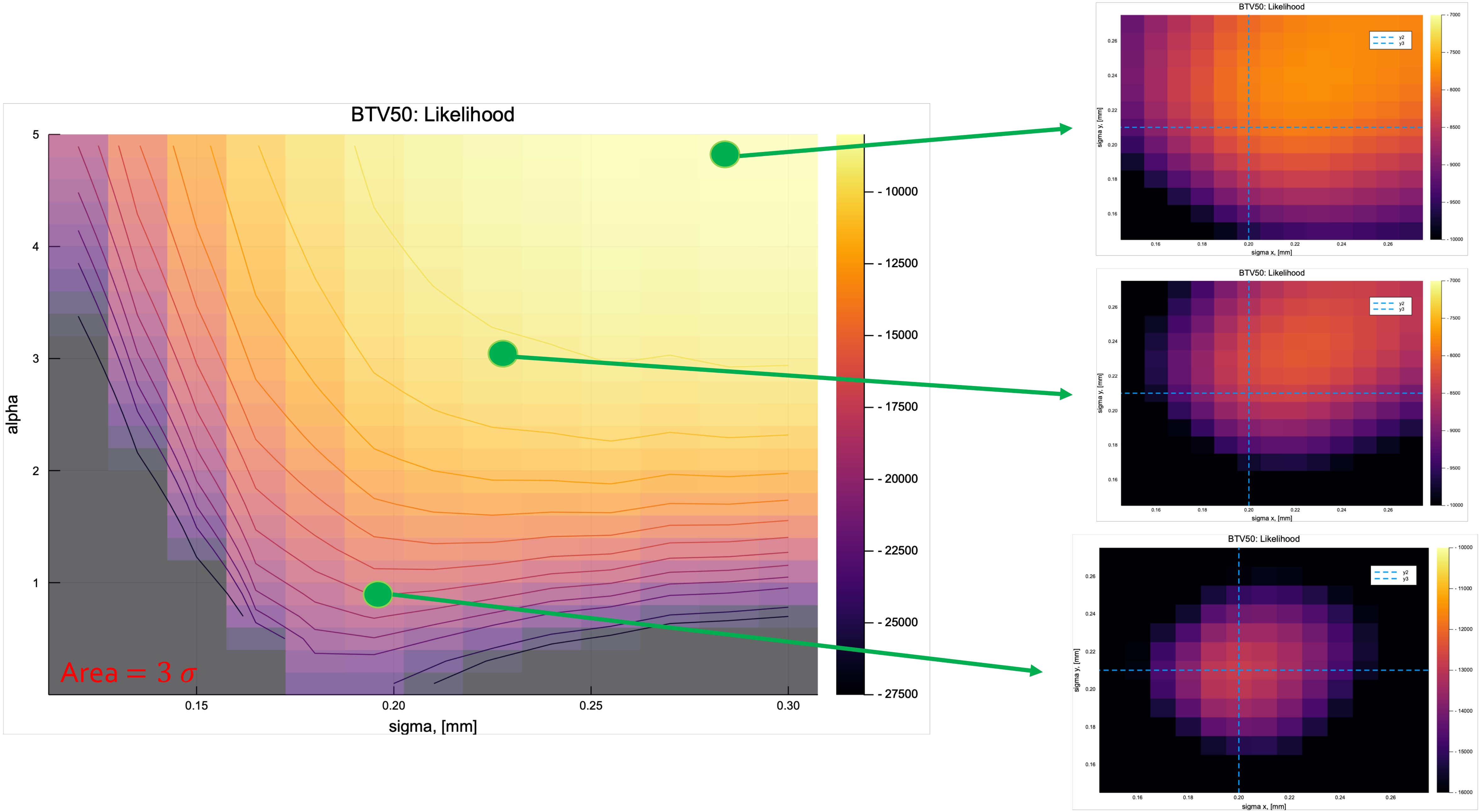
Area $= const$

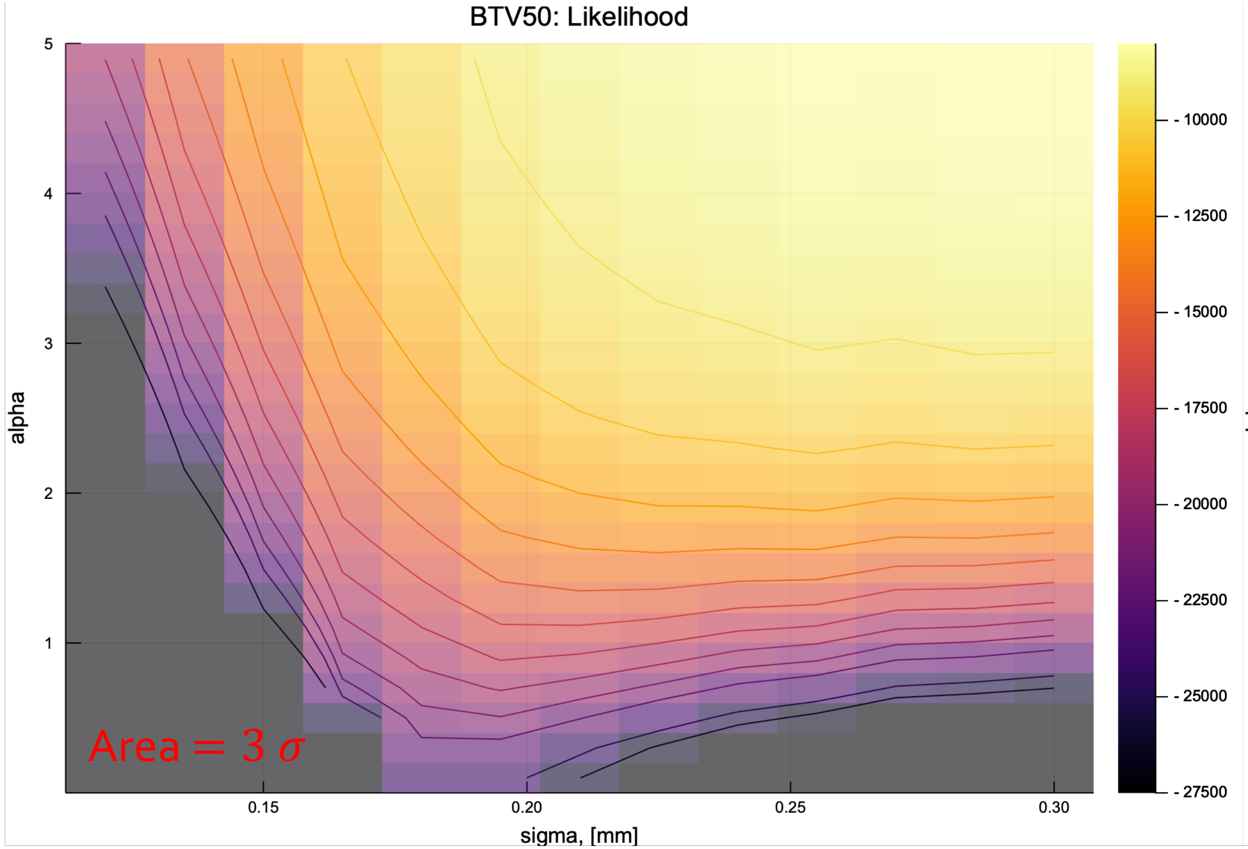
Position of the maximum is not changing!

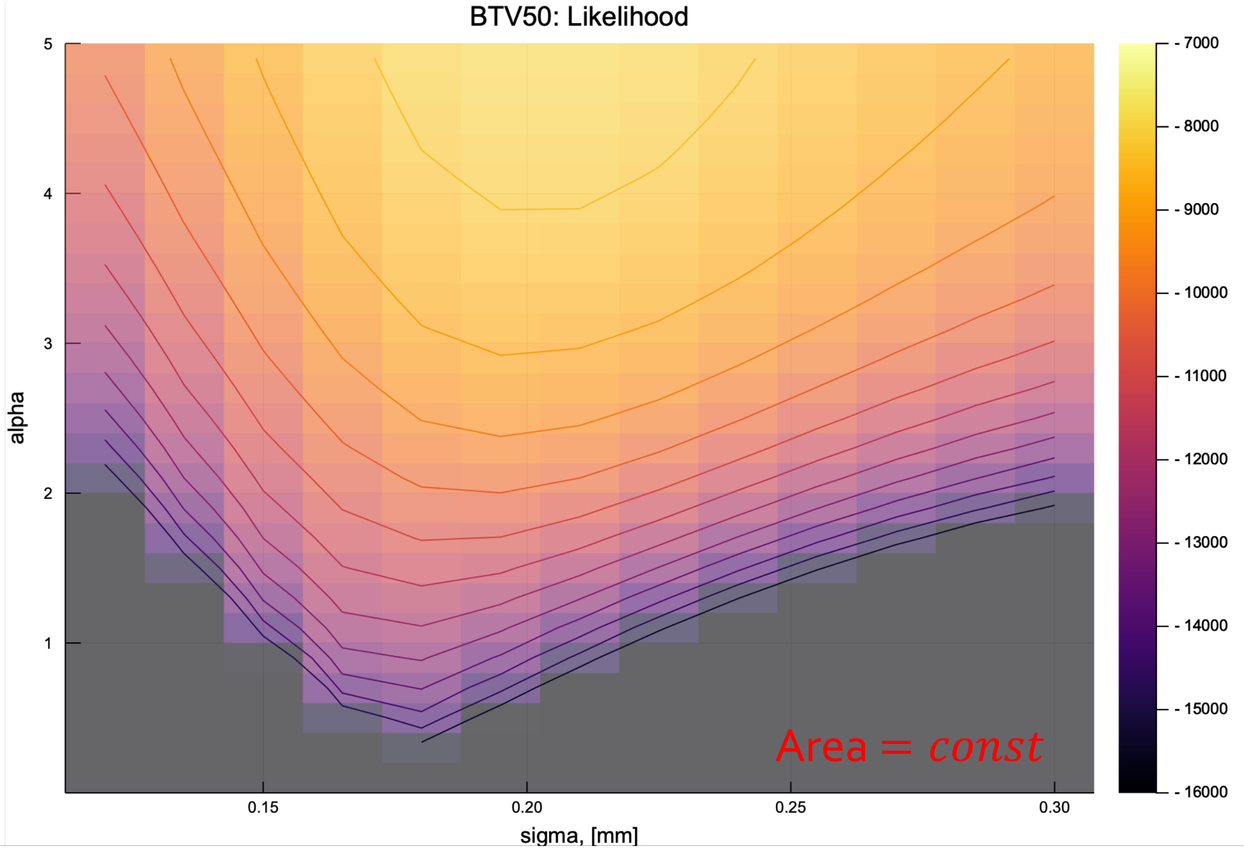


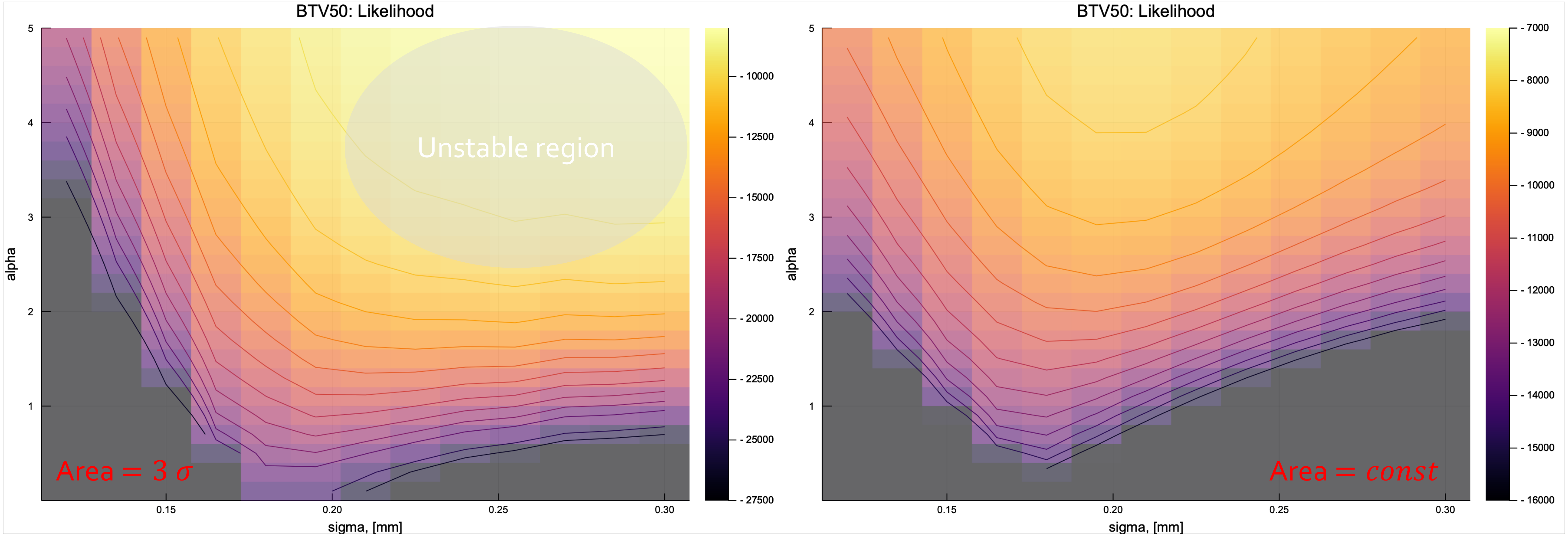





BTV50: Likelihood
alpha
sigma, [mm]
0.15
0.20
0.25
0.30
1
2
3
4
5
- 10000
- 12500
- 15000
- 17500
- 20000
- 22500
- 25000
- 27500
Area = 3 σ
BTV50: Likelihood
sigma x, [mm]
sigma y, [mm]
BTV50: Likelihood
sigma x, [mm]
sigma y, [mm]
BTV50: Likelihood
sigma x, [mm]
sigma y, [mm]

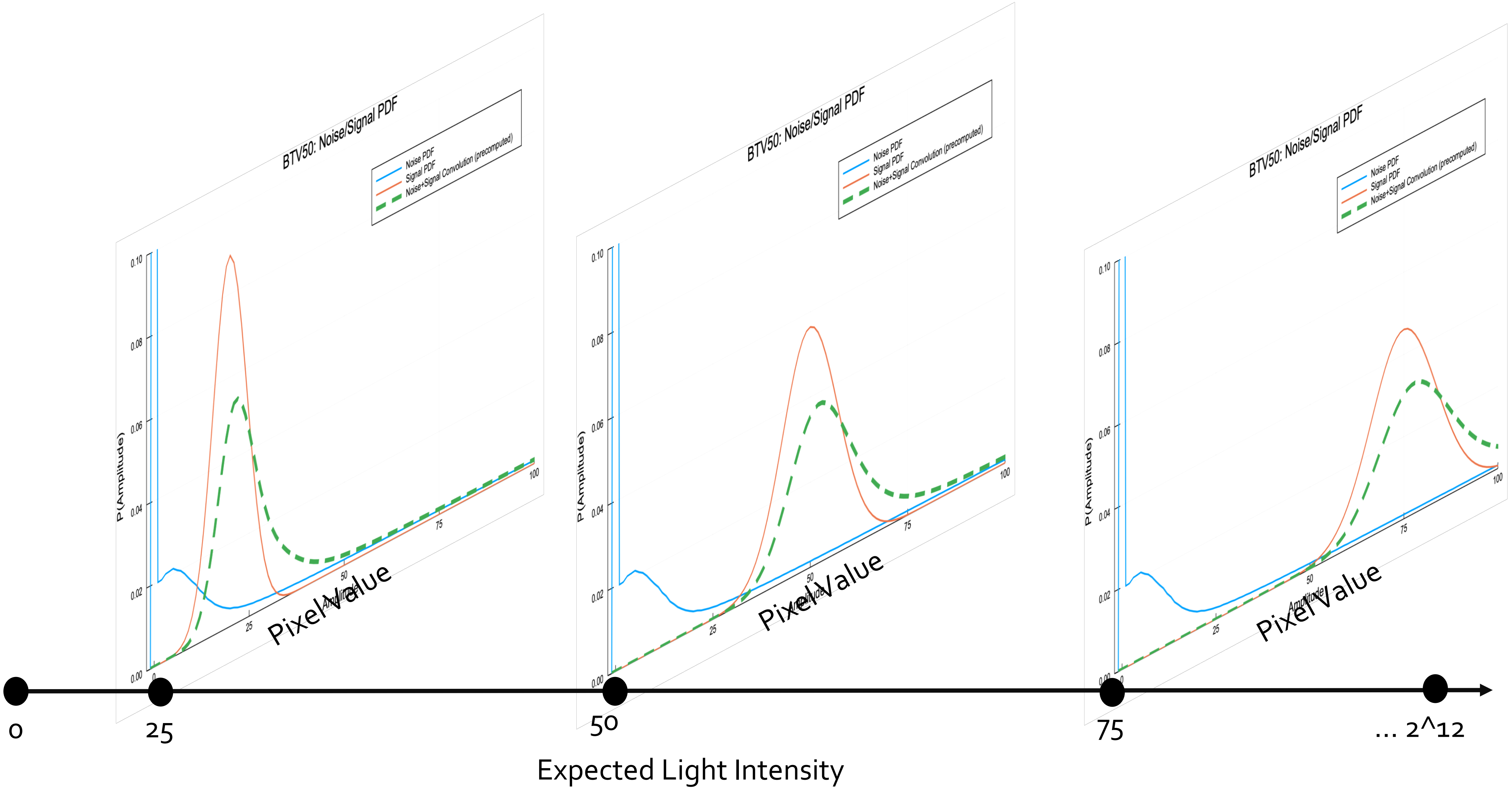

BTV50: Likelihood
alpha
sigma, [mm]
Area = 3 σ

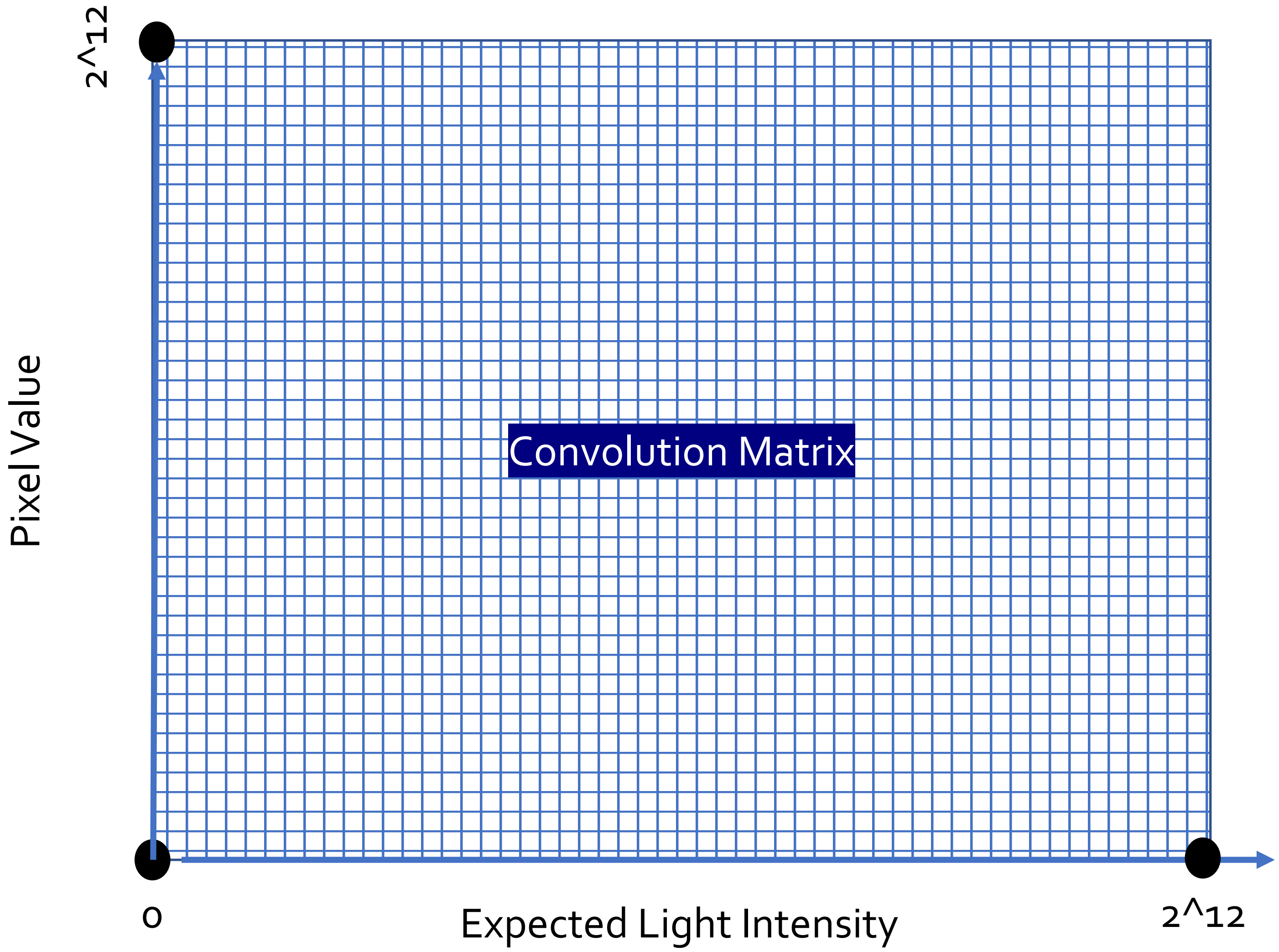

BTV50: Likelihood
alpha
sigma, [mm]
Area = const

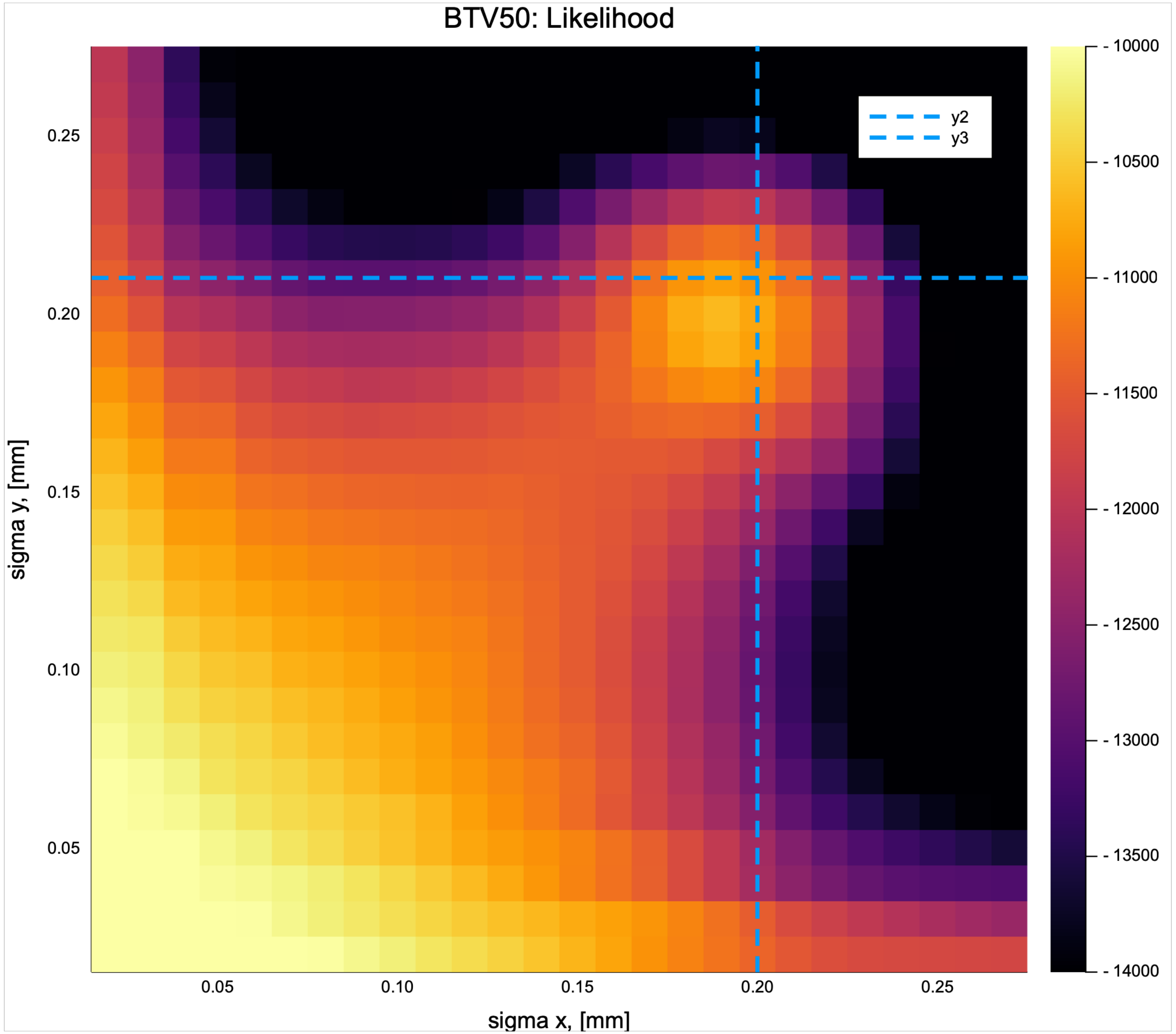


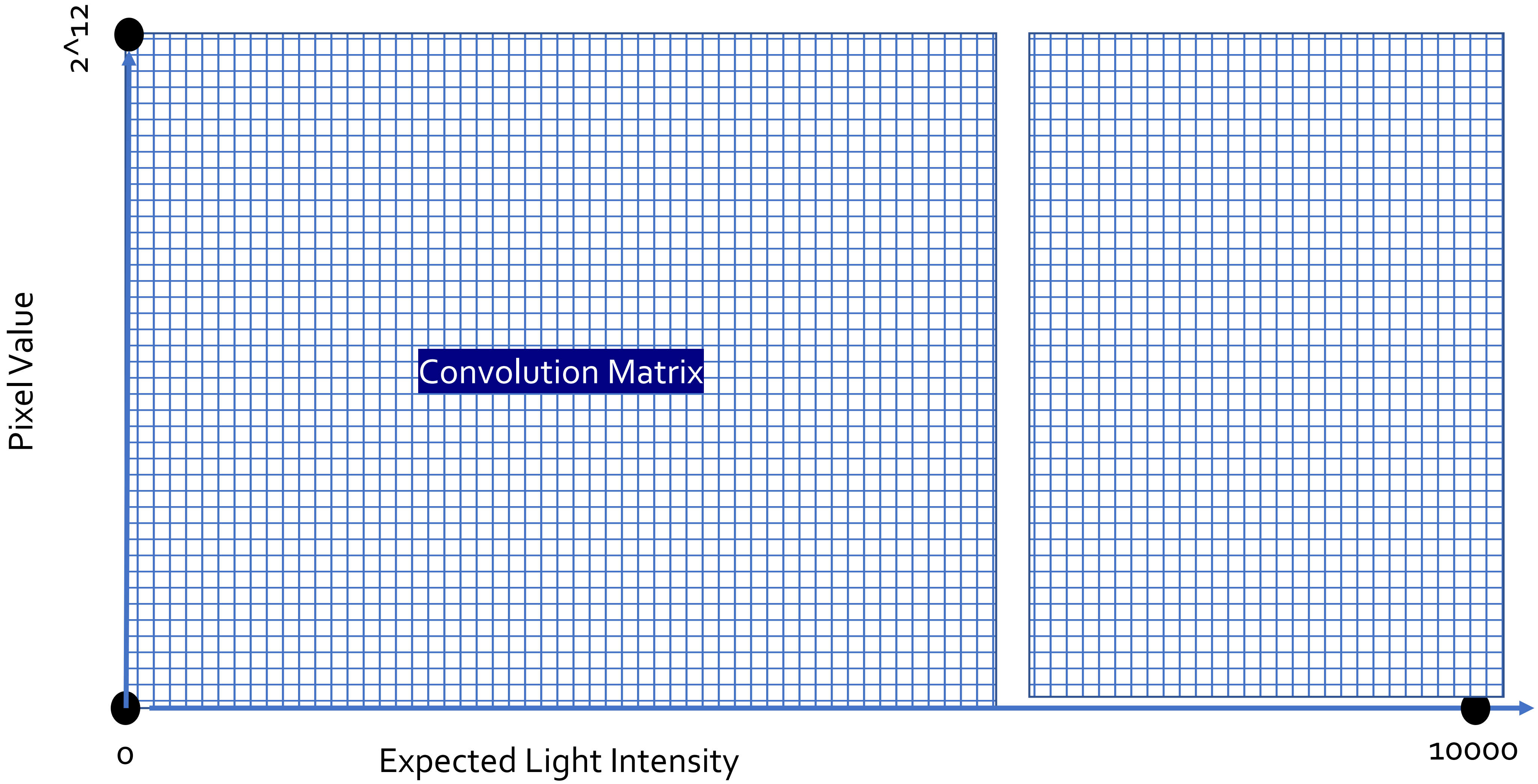
Nominal Settings:

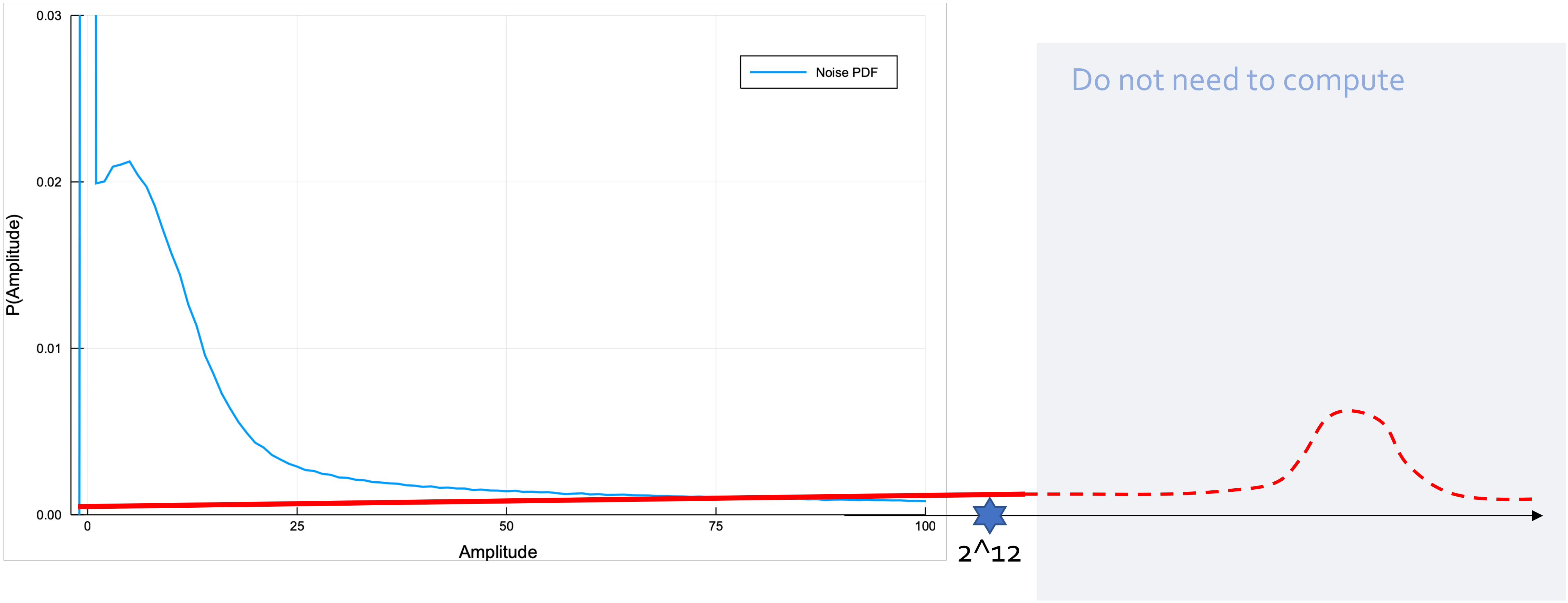
Alpha=1.0
Area = 50*50 pixels (1.4*1.4 mm)


BTV50: Noise/Signal PDF
Noise PDF
Signal PDF
Noise+Signal Convolution (precomputed)
P(Amplitude)
Amplitude
Pixel Value
0
25
50
75
... 2^12
Expected Light Intensity


2^12
Pixel Value
Convolution Matrix
0
Expected Light Intensity
2^12


BTV50: Likelihood
y2
y3
sigma y, [mm]
sigma x, [mm]
0.25
0.20
0.15
0.10
0.05
0.05
0.10
0.15
0.20
0.25
- 10000
- 10500
- 11000
- 11500
- 12000
- 12500
- 13000
- 13500
- 14000


2^12
Pixel Value
Convolution Matrix
0
Expected Light Intensity
10000



Expected Light Intensity