Introduction to supersymmetry

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Talks by my colleagues

- 28th Jan: Wolfgang Frisch: Higgs sector in MSSM
- 4th Feb: Helmut Eberl: What is mSUGRA
- 25th Mar: Elena Ginina: CP violation in the MSSM
- 6th May: Sebastian Frank: Flavour Physics beyond the SM
- ?th ???: H.H.: Sfermions and their decays in the MSSM

References

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Outline

- Motivation: defects of the SM, hierarchy problem
- Symmetries, supersymmetries
- Weyl spinors and Grassman variables
- Superspace and superfields: chiral superfields, vector superfields
- Supersymmetric lagrangians: Wess-Zumino lagrangian, Susy QED

Motivation

defects of the SM

- If one accepts the rather bizarre set of group representations and hypercharges, the SM contains 19 parameters: $g_{1,2,3}, \theta_3, 6\, m_q, 3\, m_l, \mu, \lambda, \delta$ and 3 mixing angles. More parameters are needed to accomodate non-accelerator observations (neutrino masses, cosmological inflation, cosmological baryon assymetry, cosmological constant)
- Problem of mass: Do masses originate from a Higgs boson? If so, why they are not close to the Planck mass?
- Question of unification: Can all the particles be unified in the single gauge group? Will it predict new phenomena (baryon decay, neutrino masses, relations between SM parameters)?
- Problem of flavour: What is the origin of 6 flavours? What explains their mixing and CP violation?
- TOE: seems most promising in the context of string theory. TOE should reconcile QM with GTR, explain the origin of space-time and the number of dimensions.
- What is dark matter? No SM particle can candidate for being dark matter particle.

Supersymmetry may adress some of the above issues. The most significant indication that the supersymmetry is real is the **naturalness problem**.

Motivation

Naturalness problem

- Why $m_W, m_H \ll m_P$?

  radiative corrections are quadratically divergent:

$$\delta m^2_{H,W} = O\left(\frac{g^2}{16\pi^2}\right)\int^{\Lambda} d^4k \frac{1}{k^2} = O\left(\frac{\alpha}{\pi}\right)\Lambda^2,$$

Λ - validity scale of SM

If $\Lambda \sim m_P$, the quantum corrections $\gg$ physical value of $m_{H,W}$. One could fine-tune the value of $m^0_{H,W}$ but

  - it seems unnatural
  - would have to be repeated order by order in perturbation theory

- corrections to fermion masses are only logarithmically divergent

$$\delta m_f = O\left(\frac{g^2}{16\pi^2}\right) m_f \int^{\Lambda} d^4k \frac{1}{k^4} O\left(\frac{\alpha}{\pi}\right) \ln\left(\frac{\Lambda}{m_f}\right)$$

(chiral symmetry behind)

- Is there a symmetry which would protect boson masses from large corrections?

$\Longrightarrow$ **Supersymmetry**

Motivation

Naturalness problem

- Boson and fermion loop diagrams have opposite sign. Equal number of fermions and bosons with equal couplings would lead to cancelation of quadratic divergences

$$\delta m_{H,W}^2 = -\left(\frac{g_F^2}{16\pi^2}\right)\left(\Lambda^2 + m_F^2\right) + \left(\frac{g_B^2}{16\pi^2}\right)\left(\Lambda^2 + m_B^2\right) = \left(\frac{\alpha}{4\pi}\right)\left|m_B^2 - m_F^2\right|$$

corrections are small provided that $\left|m_B^2 - m_F^2\right| \leqslant 1\,TeV$

- „This naturalness argument is the only available theoretical motivation for thinking that supersymmetry may manifest itself at an accesible energy scale“ (J. Ellis)

- „However, this argument is qualitative, and a matter of taste. It does not tell us whether sparticles should appear at 900 GeV, 1 TeV or 2 TeV, and some theorists reject it altogether. They say that, in renormalizable theory such as the SM, one needs not to worry about the fine-tuning of a bare parameter, since it is not physical. However, I take naturalness seriously as a physical argument … (J. Ellis)

Symmetries

- Def.: Symmetry is a group of transformations that leaves the lagrangian invariant
- Symmetries are very important
  - to each continuous symmetry there is a conserved quantity
  - Nature seems to respect them

Ex: rotations and translations in 3D

$$\vec{x} \rightarrow \vec{x}\,' = R(\vec{\vartheta}) \cdot \vec{x} + \vec{a}$$

In a quantum mechanical system a state $\psi(\vec{x})$ under rotations and translations transforms as

$$\psi(\vec{x}) \rightarrow \psi\,'(\vec{x}) = e^{-i\vec{a}\vec{P}} e^{-i\vec{\vartheta}\vec{J}} \psi(\vec{x})$$

where J_i and P_i are generators of rotations and translations ($i=1,2,3$) and they satisfy

$$[P_i, P_j] = 0$$
$$[J_i, J_j] = i\varepsilon_{ijk} J_k$$
$$[P_i, J_j] = i\varepsilon_{ijk} P_k$$

- Nature respects rotational and translational symmetry
- Lagrangian of any fundamental theory must be invariant under rotations and translations

Symmetries

- The symmetry group that lies at the heart of every QFT is the Poincaré group

$$x^\mu \rightarrow x'^\mu = x^\mu + \omega^{\mu\nu} x_\nu + a^\mu$$

The transformation of an arbitrary scalar classical field is

$$\Phi(x) \rightarrow \Phi'(x) = e^{i a^\rho P_\rho} \Phi(x) \qquad \text{translations}$$

$$\Phi(x) \rightarrow \Phi'(x) = e^{(i/2)\omega^{\rho\sigma} M_{\rho\sigma}} \Phi(x) \qquad \text{Lorentz transformations}$$

where M_μ are generators of Lorentz transformations and they satisfy together with P_μ

$$[P^\rho, P^\sigma] = 0$$
$$[P^\rho, M^{\nu\sigma}] = i(g^{\rho\nu} P^\sigma - g^{\rho\sigma} P^\nu)$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho} M^{\nu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho} - g^{\nu\rho} M^{\mu\sigma})$$

NOTE: All generators mix; the space-time translations and LT are linked together

3D ⟶ 4D
6 generators ⟶ 10 generators
3 coordinates ⟶ 4 coordinates
nontrivial mixing of 'old' generators J^i with 'new' generators M^{0i}

- Nature respects Poincaré symmetry.
- Can the symmetry be extended even further?

Symmetries

- trivial extension by gauge symmetry

SU(N) group:

- we add generators T^a with $a \in \{1 \dots N^2 - 1\}$

- 'new' generators commute with all 'old' generators

$$[T^a, T^b] = i f^{abc} T^c$$
$$[T^a, P^\rho] = 0$$
$$[T^a, M^{\rho\sigma}] = 0$$

Extended symmetry group is a direct product of the Poincaré group with a gauge (or internal symmetry) group.

- non - trivial extension

Can we extend the Poincaré group in a nontrivial way?

- No. - Coleman – Mandula no-go theorem (1967) : Any symmetry compatible with an interacting relativistic QFT is of the form of a direct product of the Poincaré algebra with an internal symmetry group.
- Yes. - Haag – Lopuszanski – Sohnius (1975): allow for fermionic generators

Supersymmetries

- fermionic generators

  - transform a bosonic state into fermionic
  - we allow for generators that change the spin of a state by ½
  - they must have a spinor label α

$$Q_\alpha |bos\rangle = |ferm\rangle_\alpha , \qquad Q_\alpha |ferm\rangle^\alpha = |bos\rangle$$

- N=1 super Poincaré algebra

  - 1 set of fermionic generators $Q, \bar{Q};\ (Q^\dagger = \bar{Q})$
  - additional (anti-) commutation relations:

$$[Q_\alpha, P^\rho] = 0$$
$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2(\sigma^\rho)_{\alpha\dot\beta} P_\rho$$
$$[M^{\rho,\sigma}, Q_\alpha] = -i(\sigma^{\rho\sigma})_\alpha^\beta Q_\beta$$
$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

• addition of fermionic generators → the set of coordinates will increase
(space → superspace)

consequences of Susy: solution to hierarchy problem, gauge coupling unification, DM

Weyl spinors and Grassmann variables

Weyl spinors

- In susy theories it is more convenient to work with Weyl spinors

Dirac spinor: $\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} \chi^\alpha & \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$

For Weyl spinors the bar indicates that if ψ_α transforms with a certain matrix M under LT, $\bar{\psi}_{\dot{\alpha}}$ transforms with M^*.

$$\bar{\psi}_{\dot{\alpha}} = [\psi_\alpha]^\dagger, \qquad \chi^\alpha = [\bar{\chi}^{\dot{\alpha}}]^\dagger, \qquad \alpha, \dot{\alpha} = 1, 2$$

The Weyl spinors ψ_α and $\bar{\chi}^{\dot{\alpha}}$ are called left - handed and right - handed.

$$P_L \Psi = (1/2)(1-\gamma_5)\Psi = \psi_\alpha, \qquad P_R \Psi = (1/2)(1+\gamma_5)\Psi = \bar{\chi}^{\dot{\alpha}}$$

The indices on Weyl spinors can be raised / lowered using ϵ - tensor.

$$\chi\psi \equiv \chi^\alpha \psi_\alpha = \chi^\alpha \epsilon_{\alpha\beta} \psi^\beta$$ - Lorentz invariants

$$\bar{\chi}\bar{\psi} \equiv \bar{\chi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}} \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}}$$ - Note the position of indices!

We define:

$$(\sigma^\mu)_{\alpha\dot{\alpha}} \equiv \{1, \sigma^1, \sigma^2, \sigma^3\}_{\alpha\dot{\alpha}}, \qquad (\bar{\sigma}^\mu)^{\dot{\alpha}\beta} \equiv \{1, -\sigma^1, -\sigma^2, -\sigma^3\}^{\dot{\alpha}\beta}$$

• terms in lagrangians:

$$\bar{\Psi}\Psi = \chi\psi + \bar{\psi}\bar{\chi} \equiv \chi^\alpha \psi_\alpha + \bar{\psi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}$$

$$\bar{\Psi}\gamma^\mu \Psi = \chi \sigma^\mu \bar{\chi} - \psi \sigma^\mu \bar{\psi} \equiv \chi^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} - \psi^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}$$

Weyl spinors and Grassmann variables

- Lagrangian for a free Dirac spinor

$$\mathscr{L} = i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - m\bar{\Psi}\Psi = i\chi\sigma^{\mu}\partial_{\mu}\bar{\chi} + i\psi\sigma^{\mu}\partial_{\mu}\bar{\psi} - m\chi\psi - m\bar{\psi}\bar{\chi}$$

- Lagrangian for a free Majorana spinor

$$\mathscr{L} = (i/2)\bar{\Psi}_M\gamma^{\mu}\partial_{\mu}\Psi_M - (m/2)\bar{\Psi}_M\Psi_M = (i/2)\left(\psi\sigma^{\mu}\partial_{\mu}\bar{\psi} - (\partial_{\mu}\psi)\sigma^{\mu}\bar{\psi}\right) - (m/2)\left(\psi\psi + \bar{\psi}\bar{\psi}\right)$$

Grassman variables and Grassman spinors

- A Grassman variable anticommutes with other Grassman variables.
- A Grassman spinor θ^{α} or $\bar{\theta}^{\dot{\alpha}}$ is made of two Grassman variables.

$$\theta^{\alpha} = \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix}, \qquad \bar{\theta}^{\dot{\alpha}} = \begin{pmatrix} \bar{\theta}^1 \\ \bar{\theta}^2 \end{pmatrix}$$

  - the product of a Grassman spinor with itself: $\theta\theta = \theta^1\theta_1 + \theta^2\theta_2 = -2\theta^1\theta^2 \neq 0$
  - but adding one more factor of θ^{α} gives zero: $(\theta\theta)\theta^{\alpha} = 0$

- function of a Grassman spinor:

$$\phi(\theta) = c + \theta\zeta + f(\theta\theta)$$

derivative: $\partial_{\alpha}\phi \equiv \partial/\partial\theta^{\alpha}\phi = \zeta_{\alpha} + 2f\theta_{\alpha}$

integration: $\int \phi(\theta)\,d^2\theta = f$

  - with help of Grassman spinors: $[\theta Q, \bar{\theta}\bar{Q}] \equiv [\theta^{\alpha}Q_{\alpha}, \bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}] = 2\theta\sigma^{\mu}\bar{\theta}P_{\mu}$

Superspace and superfields

- We enlarged symmetry group with fermionic generators $\rightarrow$ we need fermionic coordinates that change in a certain way under the enlarged group of transformations.

$Q_\alpha, \bar{Q}_{\dot\alpha} \leftrightarrow \theta^\alpha, \bar\theta^{\dot\alpha}$ enlarged space - *superspace*

- Field that depends on the enlarged set of coordinates is called a *superfield.*

$$\Omega = \Omega(X) = \Omega(x, \theta, \bar\theta)$$

Consider a susy transformation with $\omega^{\mu\nu} = 0$

$$S(a, \zeta, \bar\zeta) \equiv e^{-i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha} \bar{Q}^{\dot\alpha} + a^\mu P_\mu\right)}$$

Two susy transformations give

$$S(a, \zeta, \bar\zeta) S(x, \theta, \bar\theta) = S(x^\mu + a^\mu + i\zeta\sigma^\mu\bar\theta - i\theta\sigma^\mu\bar\zeta, \theta + \zeta, \bar\theta + \bar\zeta)$$

- even if $a^\mu = x^\mu = 0$ we induce a translation.
- thus under a susy transformation: $X = (x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}) \rightarrow X' = (x^\mu + a^\mu + i\zeta\sigma^\mu\bar\theta - i\theta\sigma^\mu\bar\zeta, \theta + \zeta, \bar\theta + \bar\zeta)$

- We look for differential operators $Q, \bar{Q}, P$ such that

$$\Omega(x^\mu + a^\mu + i\zeta\sigma^\mu\bar\theta - i\theta\sigma^\mu\bar\zeta, \theta + \zeta, \bar\theta + \bar\zeta) = e^{-i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar{Q}^{\dot\alpha} + a^\mu P_\mu\right)} \Omega(x, \theta, \bar\theta)$$

- change in the sign and/or i factors $\rightarrow$ different conventions for $Q, \bar{Q}$

$$P_\mu = i\partial_\mu$$
$$Q_\alpha = i\partial_\alpha - \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$
$$\bar{Q}_{\dot\alpha} = -i\bar\partial_{\dot\alpha} + \theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

Superspace and superfields

The change of a superfield under a susy transformation:

$$\Omega \to \Omega' = \Omega + \delta\Omega = \Omega - i(\zeta^\alpha Q_\alpha + \bar{\zeta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}} + a_\mu P^\mu)\Omega$$

- Covariant derivatives:

$$D_\alpha \equiv \partial_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu, \qquad \bar{D}_{\dot{\alpha}} \equiv \bar{\partial}_{\dot{\alpha}} - i\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu$$

- they satisfy: $\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = 0$

- origin of name: $D_\alpha \Omega$ transforms in the same way under susy trasformation as Ω

- The most general superfield

$$\Omega(x,\theta,\bar{\theta}) = c(x) + \theta\psi(x) + \bar{\theta}\bar{\psi}'(x) + (\theta\theta)F(x) + (\bar{\theta}\bar{\theta})F'(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x)$$
$$+ (\theta\theta)\bar{\theta}\bar{\lambda}'(x) + (\bar{\theta}\bar{\theta})\theta\lambda(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)$$

- component fields: 4 Weyl spinors: $\psi(x), \bar{\psi}'(x), \lambda(x), \bar{\lambda}'(x)$
  4 scalar fields: c, F, F', D
  1 vector field: v^μ

- superfield contains a collection of component fields $\to$ supermultiplet

- there are 8 complex fermionic and 8 complex bosonic degrees of freedom – this equality is not a coincidence

- general superfield does not form an irreducible representation of the susy algebra – there are smaller building blocks $\to$ *chiral superfields, vector superfields*

Superspace and superfields

Chiral superfields

- Left-handed chiral superfield: $\bar{D}_{\dot{\alpha}} \phi(x, \theta, \bar{\theta}) = 0$ $\phi \ldots LH_{\chi} SF$

  Right-handed chiral superfield: $D_{\alpha} \left(\phi^{\dagger}(x, \theta, \bar{\theta}) \right) = 0$ $\phi^{\dagger} \ldots RH_{\chi} SF$

Since $\bar{D}_{\dot{\alpha}} \theta_{\alpha} = 0, \bar{D}_{\dot{\alpha}} y^{\mu} = 0$ where $y^{\mu} \equiv x^{\mu} - i \theta \sigma^{\mu} \bar{\theta}$

$$\phi(y, \theta) \equiv \varphi(y) + \sqrt{2} \theta \psi(y) - (\theta \theta) F(y)$$

- 2 complex fermionic and 2 complex bosonic degrees of freedom
- left-handed Weyl spinor as a fermionic component field
- $\psi \rightarrow$ l-h quarks, $\varphi \rightarrow$ squarks, F - unphysical field

Under a susy transformation: $\varphi \rightarrow \varphi + \delta \varphi, \psi \rightarrow \psi + \delta \psi, F \rightarrow F + \delta F$ where

$$\delta \varphi = \sqrt{2} \zeta \psi$$
$$\delta \psi_{\alpha} = -\sqrt{2} F \zeta_{\alpha} - i \sqrt{2} \sigma^{\mu}_{\alpha \dot{\alpha}} \partial_{\mu} \varphi$$
$$\delta F = -i \sqrt{2} \partial_{\mu} \psi \sigma^{\mu} \bar{\zeta} = \partial_{\mu} \left(-i \sqrt{2} \psi \sigma^{\mu} \zeta \right)$$

- change in the bosonic field is proportional to the fermionic and vice versa
- change in F is a total derivative – crucial for lagrangian construction

Hermitian conjugate of $LH_{\chi} SF$ is $RH_{\chi} SF$.

Superspace and superfields

Vector superfields

- Vector superfield: $V(x,\theta,\bar{\theta})=V^{\dagger}(x,\theta,\bar{\theta})$ (preserved under susy transformation)

$$\begin{aligned} V(x,\theta,\bar{\theta}) = {} & c(x)+i\theta\chi(x)-i\bar{\theta}\bar{\chi}(x)+\theta\sigma^{\mu}\bar{\theta}v_{\mu}(x)+i(\theta\theta)N(x)-i(\bar{\theta}\bar{\theta})N^{\dagger}(x) \\ & +i(\theta\theta)\bar{\theta}\left(\bar{\lambda}(x)+(i/2)\partial_{\mu}\chi(x)\sigma^{\mu}\right)-i(\bar{\theta}\bar{\theta})\theta\left(\lambda(x)-(i/2)\sigma^{\mu}\partial_{\mu}\bar{\chi}(x)\right) \\ & +(1/2)(\theta\theta)(\bar{\theta}\bar{\theta})\left(D(x)-(1/2)\partial^{\mu}\partial_{\mu}c(x)\right) \end{aligned}$$

- 8 real bosonic and fermionic degrees of freedom
- physical fields: v and λ (gauge boson, gaugino)

Under a susy transformation: $D\rightarrow D+\delta D$ where

$$\delta D=\zeta\sigma^{\mu}\partial_{\mu}\bar{\lambda}(x)+\partial_{\mu}\lambda(x)\sigma^{\mu}\bar{\zeta}=\underbrace{\partial_{\mu}\left(\zeta\sigma^{\mu}\bar{\lambda}(x)+\lambda\sigma^{\mu}\bar{\zeta}\right)}_{\text{total derivative}}$$

- Particle content of the supersymmetric SM is now straightforward (having the information that no 2 known particles are superpartners)
- What are the allowed interactions?

Supersymmetric lagrangians

- The $\theta\theta$ - component of a $LH_\chi SF$ or the $\bar{\theta}\bar{\theta}$ - component of a $RH_\chi SF$ and the $\theta\theta\bar{\theta}\bar{\theta}$ - component of a VSF transform into themselves plus a total derivative under susy transformations.

$\longrightarrow$ action $\int d^4x\,\mathscr{L}$ does not change

- supersymmetric lagrangian: $\mathscr{L}=\mathscr{L}_F+\mathscr{L}_D$ (theory – invariant under susy trafos)

made of F-terms (of chiral SF) made of D-terms (of vector SF)

The Wess-Zumino lagrangian

- the simplest susy lagrangian
- contains only chiral superfields

Product of two LH chiral SFs is again a LH chiral SF because

$$\bar{D}_{\dot{\alpha}}(\phi_i\phi_j)=(\bar{D}_{\dot{\alpha}}\phi_i)\phi_j+\phi_i(\bar{D}_{\dot{\alpha}}\phi_j)$$

• We define the superpotential:

$$W(\phi_i)\equiv a_i\phi_i+(1/2)m_{ij}\phi_i\phi_j+(1/3!)y_{ijk}\phi_i\phi_j\phi_k$$ sum $\sum_{ijk}$ is understood

• The F-part of the W-Z lagrangian:

$$\mathscr{L}_{F,WZ}=\int d^2\theta\, W(\phi_i)+\int d^2\bar{\theta}\, W^\dagger(\phi_i^\dagger)\equiv[W(\phi_i)]_{\theta\theta}+[W^\dagger(\phi_i^\dagger)]_{\bar{\theta}\bar{\theta}}$$

Supersymmetric lagrangians

- product of 4 LH chiral superfields would lead to a non-renormalizable theory
- F-part of the W-Z lagrangian contains mass terms and Yukawa couplings but no kinetic terms

Product of one LH chiral SF with its conjugated field gives a vector superfield: $(\phi \phi^\dagger)^\dagger = \phi \phi^\dagger$

• The D-part of the W-Z lagrangian:

$$\mathscr{L}_{D,WZ} = \int d^2\theta \, d^2\bar{\theta} \, \phi_i \phi_i^\dagger \equiv [\phi_i \phi_i^\dagger]_{\theta\theta\bar{\theta}\bar{\theta}}$$

- The full W-Z lagrangian is:

$$\mathscr{L}_{WZ} = \mathscr{L}_{F,WZ} + \mathscr{L}_{D,WZ} = [W(\phi_i)]_{\theta\theta} + [W^\dagger(\phi_i^\dagger)]_{\bar{\theta}\bar{\theta}} + [\phi_i \phi_i^\dagger]_{\theta\theta\bar{\theta}\bar{\theta}}$$

- this W-Z theory is obviously supersymmetric (usefulness of the superfield formalism)
- to see the particle content and interactions we have to rewrite it in terms of component fields

Consider the case with only one chiral SF ($a_1 = a, m_{11} = m, y_{111} = y$):

$$\mathscr{L}_{D,WZ} = F^\dagger F + (\partial_\mu \varphi)(\partial^\mu \varphi)^\dagger + (i/2)(\partial_\mu \psi)\sigma^\mu \bar{\psi}$$
$$\mathscr{L}_{F,WZ} = -aF - m\varphi F - (m/2)(\psi\psi) - (y/2)\varphi\varphi F - (y/2)\varphi(\psi\psi) + h.c.$$

- the D-term contains the kinetic term of the φ and the ψ component fields
- there is no kinetic term for the F -field

Supersymmetric lagrangians

Equation of motion for the field F:

$$0=\partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu)F}-\frac{\partial \mathscr{L}}{\partial F}=\frac{-\partial \mathscr{L}}{\partial F}=-F^\dagger+a+m\varphi+\frac{y}{2}\varphi\varphi$$

Solving that we get

$$F^\dagger F-\left(aF+m\varphi F+(y/2)\varphi\varphi F+h.c.\right)=-\left|a+m\varphi+(y/2)\varphi\varphi\right|^2=-\left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2$$

We can eliminate the $a\varphi$ term: $\varphi\rightarrow\varphi+(M-m)/y\,,\ M\equiv\sqrt{m^2-2ay}$

$$\mathscr{L}_{WZ}=(\partial_\mu\varphi)(\partial^\mu\varphi)+(i/2)\psi\sigma^\mu(\partial_\mu\bar{\psi})-(i/2)(\partial_\mu\psi)\sigma^\mu\bar{\psi}$$
$$-|M|^2\varphi\varphi^\dagger-(|y|^2/4)\varphi\varphi\varphi^\dagger\varphi^\dagger-\left((M/2)\psi\psi+(M^\star y/2)\varphi\varphi\varphi^\dagger+(y/2)\varphi\psi\psi+h.c.\right)$$

- the W-Z theory contains a spin-0 particle and spin-½ particle of the same mass
- there is a 3-point and 4-point scalar interaction and s-s-f interaction
- couplings are related

We can rewrite the lagrangian in the following way

$$\mathscr{L}_{WZ}=(\partial_\mu\varphi)(\partial^\mu\varphi)+(i/2)\psi\sigma^\mu(\partial_\mu\bar{\psi})-(i/2)(\partial_\mu\psi)\sigma^\mu\bar{\psi}$$
$$-\sum_i\left|\frac{\partial W(\varphi_i)}{\partial\varphi_i}\right|^2-\frac{1}{2}\left(\frac{\partial^2 W(\varphi_i)}{\partial\varphi_i\partial\varphi_j}\right)\psi_i\psi_j-\frac{1}{2}\left(\frac{\partial^2 W^\dagger(\varphi_i)}{\partial\varphi_i^\dagger\partial\varphi_j^\dagger}\right)\bar{\Psi}_i\bar{\Psi}_j$$

- the supepotential determines all interactions and masses

Supersymmetric lagrangians

Susy QED

- the W-Z model does not contain spin-1 particles
- we are going to include VSF
- since D-term does not provide kinetic term we define new constructs

$$U_\alpha \equiv (-1/4)(\bar{D}\bar{D})D_\alpha V\,, \qquad \bar{U}_{\dot{\alpha}} \equiv (-1/4)(DD)\bar{D}_{\dot{\alpha}} V$$

  - U is a LH-chiral-SF ($\bar{D}_{\dot{\alpha}}\bar{D}\bar{D}=0$) and $\bar{U}$ is a RH-chiral-SF
  - products $U^\alpha U_\alpha$ and $U_{\dot{\alpha}} U^{\dot{\alpha}}$ are Lorentz-invariant
  - corresponding F-terms contain kinetic terms for v^μ and λ

• Combination of gauge symmetry and susy

Under U(1) global gauge symmetry component fields transform as

$$\varphi \rightarrow \varphi' = e^{-i\Lambda}\varphi\,, \qquad \Lambda \text{ - real constant} \qquad \varphi^\dagger\varphi \text{ - gauge independent}$$

Note: a real constant $\Lambda = \Lambda^\dagger$ is a special case of chiral SF (LH- and RH- chiral-SF)

Thus $\quad \phi \rightarrow \phi' = e^{-i\Lambda}\phi\,,$ where ϕ' is still a LH-chiral-SF if ϕ is

and $\quad [\phi\phi^\dagger]_{\theta\theta\bar{\theta}\bar{\theta}}$ is supersymmetric and invariant under gauge transformations

Supersymmetric lagrangians

- local gauge

Λ will have to be a funtion of variable x
But it is impossible to have x-dependent superfield that is LH and RH at the same time.

therefore $\phi^\dagger \phi \rightarrow \phi'^\dagger \phi' = \phi^\dagger e^{i\Lambda^\dagger(x)} e^{-i\Lambda(x)} \phi \neq \phi^\dagger \phi$ since $\Lambda \neq \Lambda^\dagger$

To restore a local gauge invariance we introduce a gauge VSF which transforms as

$e^V \rightarrow e^{-i\Lambda^\dagger(x)} e^V e^{i\Lambda(x)}$ and in abelian case: $V \rightarrow V' = V - i\Lambda(x)^\dagger + i\Lambda(x)$

then $[\phi^\dagger e^V \phi]_{\theta\theta\bar\theta\bar\theta} \rightarrow [\phi'^\dagger e^{V'} \phi']_{\theta\theta\bar\theta\bar\theta} = [\phi^\dagger e^V \phi]_{\theta\theta\bar\theta\bar\theta}$ - is supersymmetric and invariant under local gauge trafos

We can use the gauge freedom to obtain a convenient representation of gauge VSF:

$$V_{WZ}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta v_\mu(x) + i(\theta\theta)\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\theta\lambda(x) + (1/2)(\theta\theta)(\bar\theta\bar\theta)D(x)$$

- we can still eliminate 1 degree of freedom in v^μ through a choice of $\Re(\varphi)$
- we are left with: 4 real bosonic and 4 real fermionic degrees of freedom

Note: U_α and $\bar U_{\dot\alpha}$ are gauge independent

Abelian gauge invariant and susy lagrangian:

$$\mathscr{L} = (1/4)[U^\alpha U_\alpha]_{\theta\theta} + (1/4)[\bar U_{\dot\alpha} \bar U^{\dot\alpha}]_{\bar\theta\bar\theta} + [\phi_i^\dagger e^{2gV} \phi]_{\theta\theta\bar\theta\bar\theta} + [W(\phi_i)]_{\theta\theta} + [W^\dagger(\phi_i^\dagger)]_{\bar\theta\bar\theta}$$

providing that the superpotential is also gauge invariant

Supersymmetric lagrangians

- Abelian gauge invariant and susy lagrangian

$$\mathscr{L}=(1/4)[U^{\alpha}U_{\alpha}]_{\theta\theta}+(1/4)[\bar{U}_{\dot{\alpha}}\bar{U}^{\dot{\alpha}}]_{\bar{\theta}\bar{\theta}}+[\phi_i^{\dagger}e^{2gV}\phi]_{\theta\theta\bar{\theta}\bar{\theta}}+[W(\phi_i)]_{\theta\theta}+[W^{\dagger}(\phi_i^{\dagger})]_{\bar{\theta}\bar{\theta}}$$

- first two terms: photon, photino and the scalar field D, kinetic terms for photon and photino
- third term: part is the same as $\mathscr{L}_{D,WZ}$ (kinetic terms for leptons and sleptons)
  other part: interactions between leptons (and sleptons) with the photon (and photino)
- last two terms: as before (interaction involving only component fields of the chiral SF)

$$(1/4)[U^{\alpha}U_{\alpha}]_{\theta\theta}+(1/4)[\bar{U}_{\dot{\alpha}}\bar{U}^{\dot{\alpha}}]_{\bar{\theta}\bar{\theta}}=-(1/4)F^{\mu\nu}F_{\mu\nu}-i/2(\partial_{\mu}\lambda)\sigma^{\mu}\bar{\lambda}+(i/2)\lambda\sigma^{\mu}(\partial_{\mu}\bar{\lambda})+(1/2)D^2$$

- no kinetic term for the D-field, it is an auxiliary field, eliminated through equation of motion

$[\phi_i^{\dagger}e^{2gV}\phi]_{\theta\theta\bar{\theta}\bar{\theta}}=g\varphi_i^{\dagger}\varphi_i D+$ terms without D

Equation of motion for the D-field:

$$0=D+g\varphi_i^{\dagger}\varphi_i$$

By elimination of the D-field from the Lagrangian we get

$$D^2/2+Dg\varphi_i^{\dagger}\varphi_i=-(1/2)(g\varphi_i^{\dagger}\varphi_i)^2$$