Discussion on the RF parameters for the SPL cavities: CEA vs. BNL II

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1. Definitions

The quality factor Q_0 is given by:

$$Q_0 = \frac{\omega W}{P_{cav}} = \frac{G}{Rs} \quad \textbf{(1)}$$

where P_{cav} is the power dissipated in the cavity walls, W the energy stored in the cavity , G the geometrical parameter and Rs the surface resistance.

The external quality factor is given by the following formula, for a loaded cavity, when the power dissipated in the cavity is small compared to the power transferred to the beam.

$$Q_{ext} = \frac{E_{acc}.L_{acc}}{\frac{r}{Q} * I_0 \cos\varphi_s} \quad \textbf{(2)}$$

The values of the accelerating field E_{acc}, the beam current I_0 and the synchronous phase φ_s are fixed by the accelerator. As a consequence, the external factor is increased when the r/Q is decreased:

$$\frac{Q_{ext}(BNL)}{Q_{ext}(CEA)} = \frac{r/Q(CEA)}{r/Q(BNL)} = \alpha \quad (3)$$

α represents the r/Q ratio between the two cavities.

In addition, the expression for the r/Q, corresponding to the “accelerator definition” of the shunt impedance, is:

$$\frac{r}{Q} = \frac{(E_{acc}.L_{acc})^2}{\omega W} \quad (4)$$

Finally, from equations (1) and **(4)** an expression for P_{cav} is:

$$P_{cav} = \frac{(E_{acc}.L_{acc})^2}{\frac{r}{Q} \times Q_0} \quad (5)$$

2. Comparison between BNL II and CEA cavities

The RF parameters have been calculated at CEA with the code Superfish for both cavities. For the BNL cavity, the calculations were made with the Superfish file given on Twiki/SPLWeb (1).

	BNL II	CEA	difference
Frequency (MHz)	704.2	704.4	≈
Lacc (N*βλ/2)	1.065	1.065	≈
Geometrical parameter G (Ohms)	273	270	≈
r/Q (Ohms)*	520	566	+8 %
Ep/Eacc	2.13	2	-6 %
Bp/Eacc (mT/MV/m)	4.4	4.2	-4.5%
K (%)		1.92	

Table 1 RF parameters calculated at CEA with Superfish *calculated with the "accelerator definition" **(2)**

a. Difference of the energy provided to the cavities

Equation **(3**) shows that $\frac{Q_{ext}(BNL)}{Q_{ext}(CEA)} = \alpha = 1.088$

The value for the external quality factor has been estimated for the CEA cavity to $Q_{ext} = 1.2\text{x}10^6$. Thus the BNL cavity has a Q_{ext} equal to $1.31\text{x}10^6$

The response time for the RF fields in a cavity is given by $\tau_{cav}=2Q_L/\omega_0$, where Q_L is the external factor of the loaded cavity:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} + \frac{1}{Q_{HOM}} \quad (6)$$

For these superconducting cavities, the quality factor due to the RF losses (Q_0) is very large compared to the external quality factor (Q_{ext}). Moreover, the coupling to the HOM couplers is very low compared to the coupling to the fundamental coupler, and $Q_{HOM} >> Q_{ext}$; consequently, $Q_L \approx Q_{ext}$, and **τ_{cav} (CEA)=0.54 ms** while **τ_{cav}(BNL)=0.59 ms**.

The SPL cavities are aimed to work in the pulsed mode, with a beam pulse length equal to Δt_{beam} = 0.9 ms for LP-SPL (and 0.8 ms for HP-SPL).

During each beam pulse, the accelerating field in the cavities varies as follow, (see Figure 1):

- $E_{acc}(t) = 2 \times E_{acc}(target)(1 - exp\left(-\frac{t}{\tau_{cav}}\right))$ before the beginning beam pulse. The value of E_{acc}(target) has to be reached when the beam pulse starts, at t=0 in Figure 1.
- $E_{acc}(t) = E_{acc}(target)exp(-t/\tau_{cav})$ after the beam pulse (*t=0.9 ms* in Figure 1).

E_{acc}(target) is the nominal accelerating field fixed by the SPL parameters.

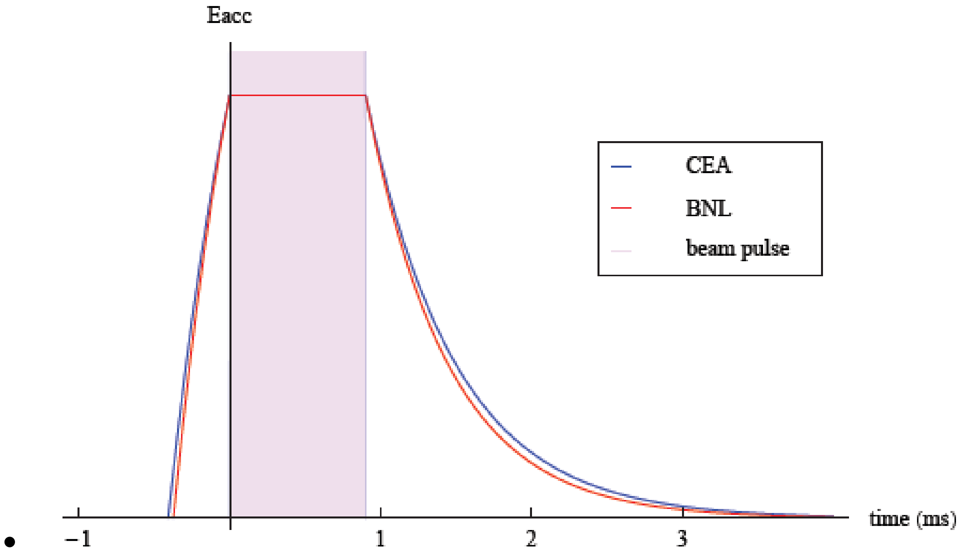

Figure 1 : Accelerating field Eacc in the cavity with the beam pulse.

The length of the RF pulse is equal to $\Delta t_{beam} + \tau_{cav}\, x Ln(2)$, which means that it depends on τ_{cav} and thus on *r/Q* as shown on Figure 2.

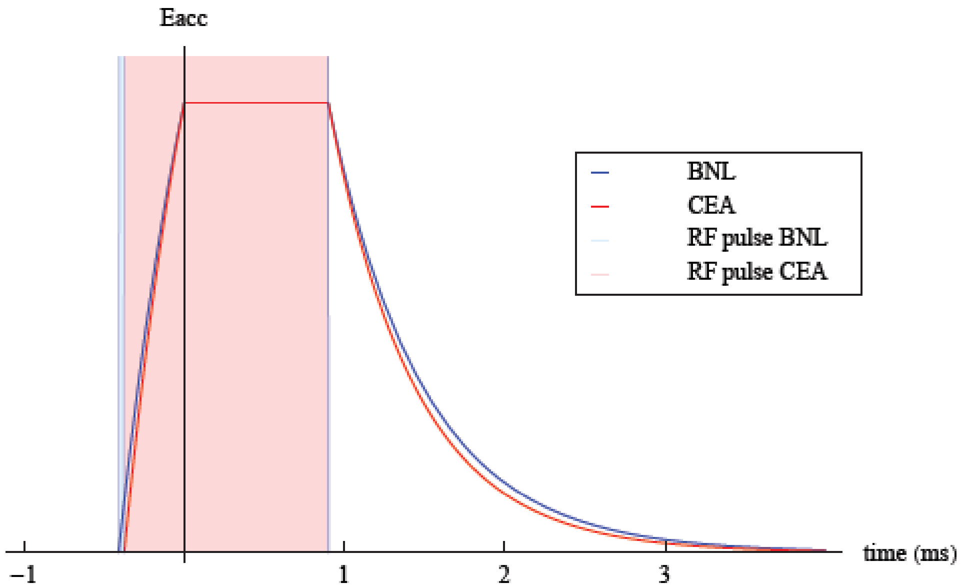

Figure 2: Accelerating field Eacc in the cavity with the beam pulse.

The RF pulse length is 1.27 ms for the CEA cavity and 1.31 ms for the BNL cavity, which corresponds to an increase of 2.7 % with the BNL cavity for the total RF energy to be provided.

b. Difference of the dissipated energy

The RF losses in the cavities, P_{cav}, should be minimized, since they have to be absorbed by the cryogenics.

In Table 1, it appears that both cavities have nearly the same geometrical parameter, and one can assume that they have the same surface resistance *Rs*. Thus, according to equation (1) we can consider that they have the same quality factor Q_0. The values for L_{acc} are also the same for the two cavities. Thus, according to equation **(5)** P_{cav} is only dependent on E_{acc}^2 and r/Q. P_{cav} is plotted in Figure 3.

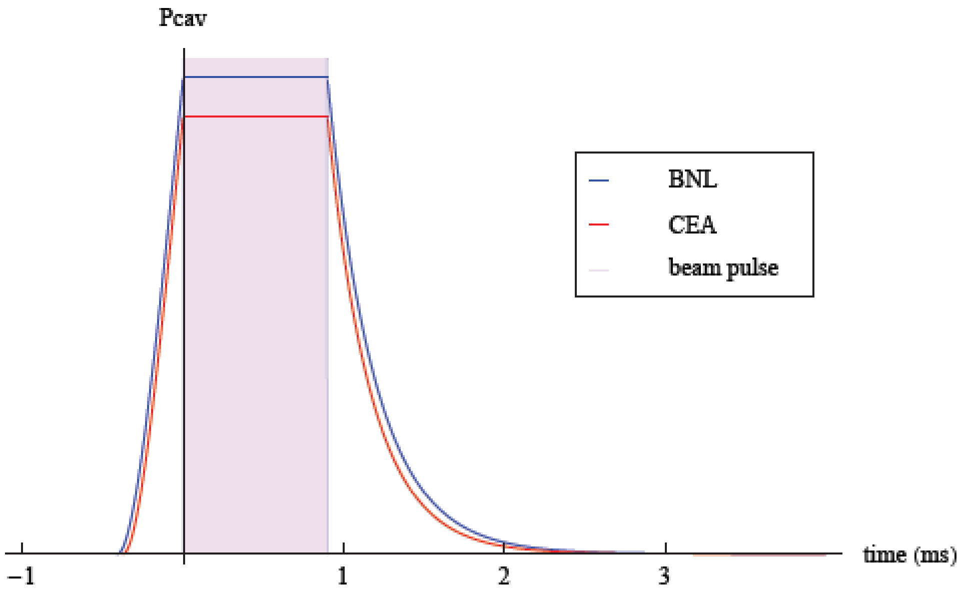

Figure 3: RF losses in the cavity (Pcav) during a beam pulse

The whole energy dissipated into the cavities (and absorbed by the cryogenics) corresponds to the integration of P_{cav} with time around the pulse. Calculations show that this energy is 10.7 % higher in the BNL case.

In conclusion, these calculations show how a lower value for the r/Q leads to an increase of the RF energy supplied to the cavities, and to an increase of the energy losses in the cavity walls, which have to be absorbed by the cryogenics.

The numerical results for the CEA and BNL cavities are summarized in the following table.

	r/Q	RF energy supplied	Cavity losses
CEA	566 Ω		
BNL	520 Ω	+ 2.7 %	+ 10.7 %

These considerations concern only the fundamental mode at 704 MHz, and a complete study should also take into account the contribution of the High Order Modes.

Bibliography

1. http://rcalaga.web.cern.ch/rcalaga/704MHz/. [Online]

2. **Padamsee et al.** *RF Superconductivity for accelerators.* 2007.