Cooling of a 7-TeV LHC proton-beam via electron/proton collision

Stéphane Fartoukh, Lily Blondel

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Physics Constants

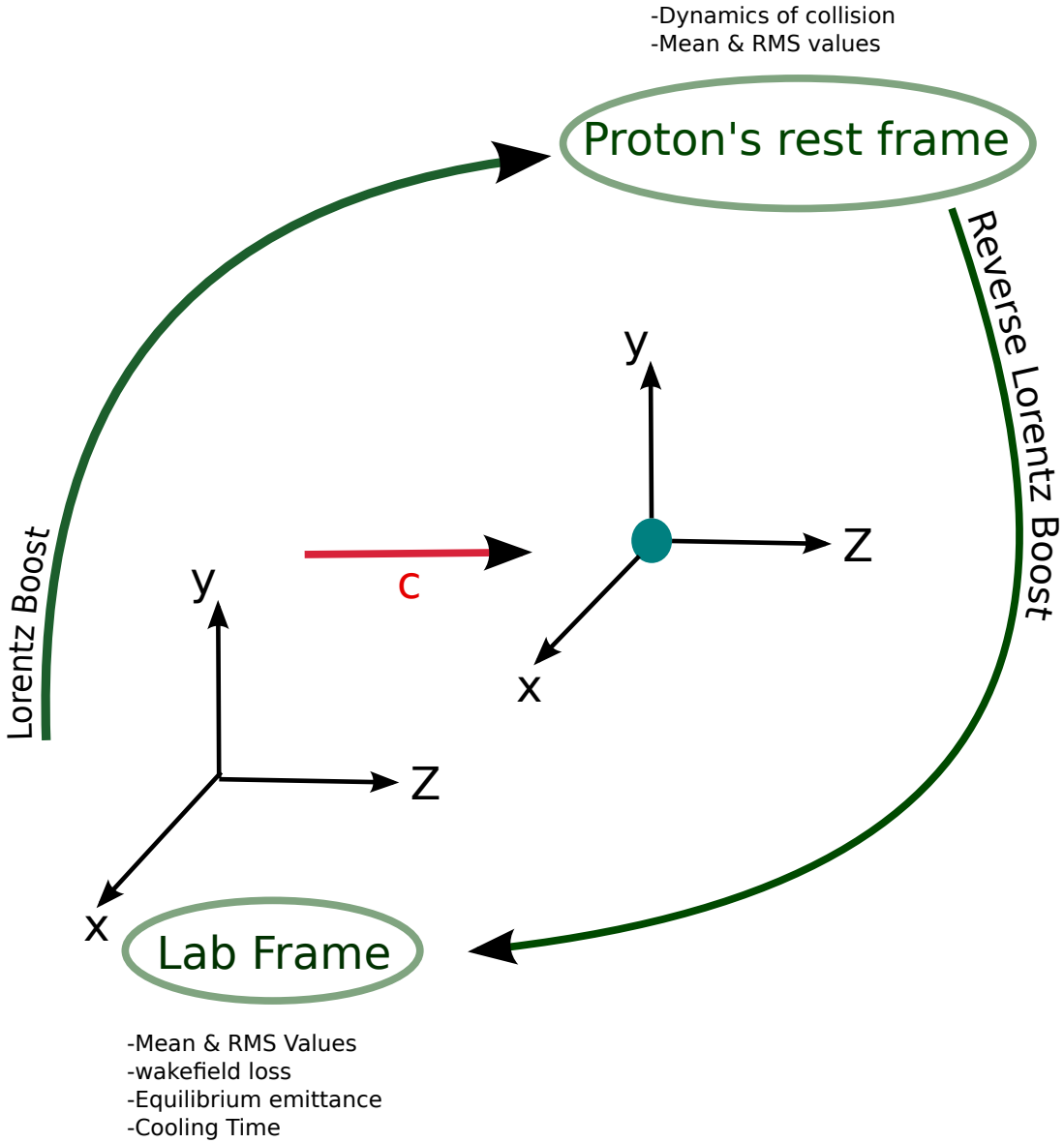
M	Proton mass	
m_e	Electron mass	
ε_0	Vacuum permittivity	
$r_e = \dfrac{e^2}{4\pi\epsilon_0 m_e c^2}$	Classical electron radius	

Parameters in the Laboratory Frame

E_p^L	Proton total energy	(typically 7 TeV)
$E_e^L = m_e c^2 + E_{k,e}$	Electron total energy	($E_{k,e}$ typically 10 keV)
$p_{e,p} = \dfrac{1}{c}\sqrt{E_{e,p}^2 - m_{e,p}^2 c^4}$	Electron, proton momentum	
$\gamma_{e,p} = \dfrac{E_{e,p}}{m_{e,p} c^2}$	Electron, proton Lorentz factor	
$v_{e,p} = \dfrac{p_{e,p}}{m_{e,p}\gamma_{e,p}} = \beta_{e,p} c$	Electron, proton velocity	

Introduction

- HL-LHC : Electron lenses are being proposed for multiple projects (halo cleaning for example)
- So far : Electron cooling already exists with the 2 beams going **in the same direction**. Particularly efficient for low-energy ion-beam. However : heating rate overcomes the cooling rate at high energy.
- Aim : **Collide electrons and ion** beams and assess if the coulomb scattering effect induced when going through the electron-lens could cool the beam after a few hours.
- Methodology:
  - colliding a 7-TeV proton beam against a DC current of low-energy electrons
  - computing the mean energy loss of the proton per LHC-turn together with its transverse divergence
  - if this energy loss is much larger than the transverse momentum kick taken by the proton, this will cool the proton beam, since the proton is re-accelerated only in the longitudinal plane in the RF cavity.
  - Challenge: not to break too many protons as the inelastic cross-section is not negligible.


-Dynamics of collision
-Mean & RMS values
Proton's rest frame
Reverse Lorentz Boost
Lorentz Boost
y
Z
x
c
y
Z
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Lab Frame
-Mean & RMS Values
-wakefield loss
-Equilibrium emittance
-Cooling Time

Presentation Outline

1. To the proton rest frame
   - Lorentz boost
   - Energy lost by the electron in the elastic scattering
2. Mean values
   - Mott-type cross-section
   - Minimum angle
   - Mean energy loss
   - RMS transverse moments
3. Back to the laboratory frame
   - Mean energy loss
   - RMS transverse moments
4. In practice
   - Equilibrium emittance
   - Cooling time
5. Wakefield loss component
6. Transverse instead of Head-on collision

Going to the proton rest frame

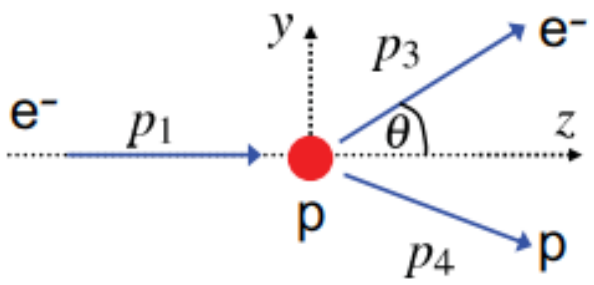
- Dynamics of the collision is easier to derive in the proton rest frame as the proton is initially at rest.
- Relative velocity between the two reference frames: $v = v_p$

$$E_e^R = \gamma_p(E_e^L + \beta_p p_e^L c)$$

$$E_p^R = \gamma_p(E_p^L - \beta_p p_p^L c)$$

with $\beta_p = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma_p^2}}$

Energy lost by the electron in the elastic scattering



$$\begin{cases} p_1 = 1/c\,\begin{pmatrix} E_e^R & 0 & 0 & -E_e^R \end{pmatrix} \\ p_2 = 1/c\,\begin{pmatrix} E_p^R & 0 & 0 & 0 \end{pmatrix} \\ p_3 = E_3/c\,\begin{pmatrix} 1 & 0 & \sin\theta & -\cos\theta \end{pmatrix} \\ p_4 = 1/c\,\begin{pmatrix} E_4 & \vec{p}_4 \end{pmatrix} \end{cases}$$

- Assumption of a **mass-less electron**
- The energy lost by the electron is fully transferred to the proton:

$$\begin{cases} E_3 = E_e^R - \Delta E_p \\ E_4 = E_p^R + \Delta E_p \end{cases}$$

Derivation of the energy loss via the Mandelstam variable t

- Approximation of the electron as a mass-less particle

$$\Rightarrow c|p_1| = \sqrt{(E_e^R)^2 - m_e^2 c^4} = E_e^R \quad \& \quad c|p_3| = E_e^R - \Delta E_p$$

- Mandelstam variable t:

$$t = (p_1 - p_3)^2 \equiv (p_2 - p_4)^2$$

$$\Rightarrow \Delta E_p = \frac{(E_e^R)^2(1 - \cos\theta)}{E_p^R + E_e^R(1 - \cos\theta)} \tag{1}$$

and within the small angle approximation:

$$\Rightarrow \boxed{\Delta E_p \underset{\theta \to 0}{\simeq} \frac{(E_e^R)^2}{2Mc^2}\,\theta^2} \tag{2}$$

Mott-type cross-section

- Following the methodology of Prof M. Thomson's lectures [1], we derived a Mott-type cross-section:

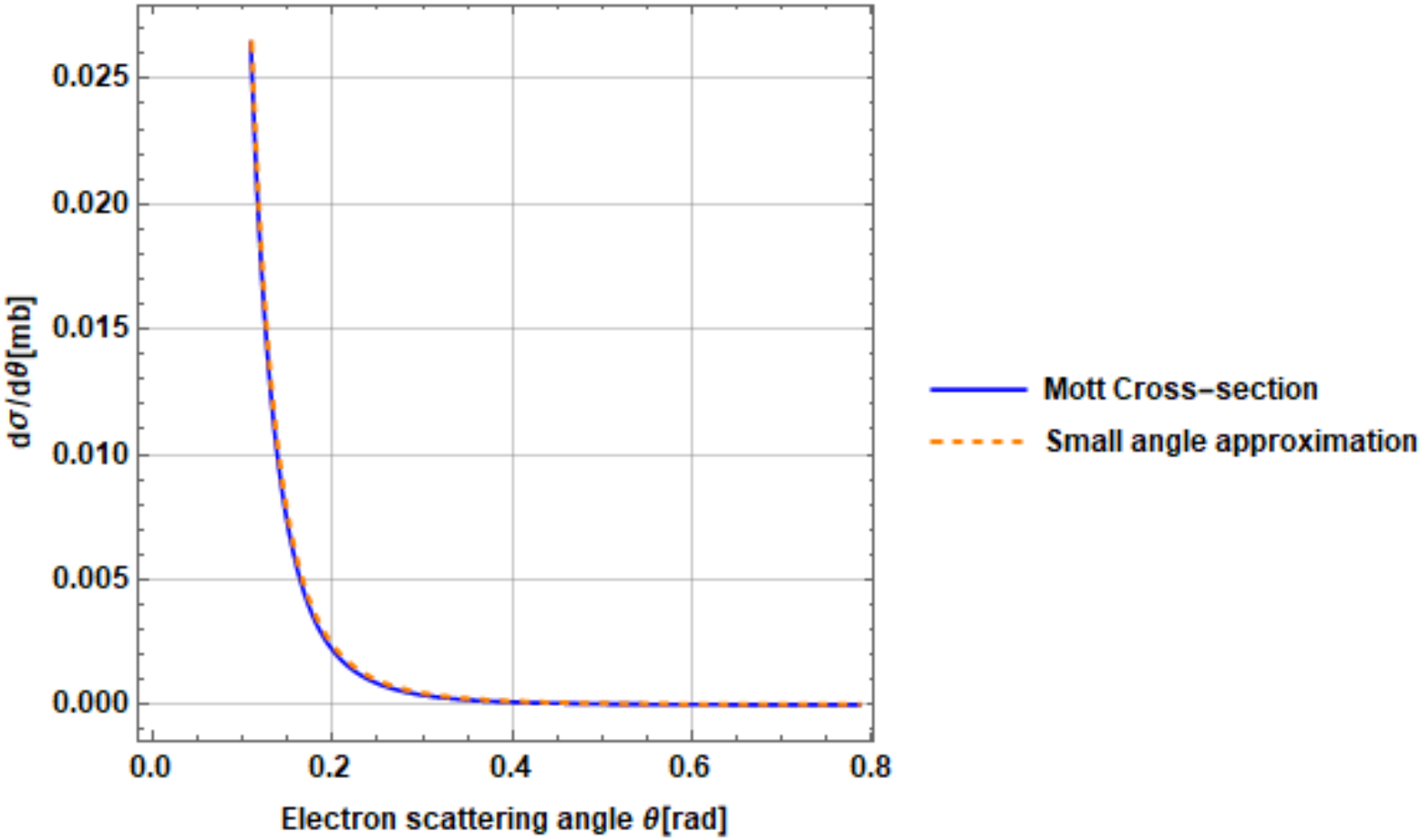
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{e^4}{\epsilon_0^2\, 64\pi^2 E_1 E_3 \sin^4\theta/2}\left(\frac{E_3}{E_1}\right)^2\left[\cos^2\theta/2 - \frac{q^2}{2M^2c^2}\sin^2\theta/2\right]$$

where $q^2 = \frac{-4E_1E_3}{c^2}\sin^2\theta/2$ and $\mathrm{d}\Omega = 2\pi\sin\theta\mathrm{d}\theta$

- within the small angle approximation, the cross-section simplifies as:

diverge in 0

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \underset{\theta\to 0}{=} \frac{8\pi\, r_e^2}{(\gamma_e^R)^2}\boxed{\frac{1}{\theta^3}} + O\left(\frac{1}{\theta}\right) \tag{3}$$



- Divergence at 0
- Is non-zero only for small scattering angle

Introduction of a minimum scattering angle

- Introduction of the classical impact parameter b to determine a minimum cut-off angle.

$$\theta_{min} = 2 \arctan\left(\frac{b_{90}}{b_{max}}\right)$$

where b_{max} is the radius of the incident electron beam.

- b_{90} is the impact parameter corresponding to a deflection by $90°$:

$$b_{90} = \frac{|q_1 q_2|}{4\pi\epsilon_0 2E_k} = \frac{|q_1 q_2|}{4\pi\epsilon_0 E_{tot}} \text{ as } E_k = \frac{E_{tot}}{2}$$

q_1, q_2 being the target's and projectile's charge respectively.

- After simplification:

$$\theta_{min} \simeq \frac{2\, r_e}{\gamma_e^R\, b_{max}} \simeq 3.1 \times 10^{-16}\mathrm{rad} \tag{4}$$

Mean energy loss

- Energy transferred from the electron to the proton

$$\langle \Delta E_p \rangle = \frac{1}{\pi b_{max}^2} \int_{\theta_{min}}^{\pi} \Delta E_p \frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \mathrm{d}\theta \simeq \frac{4\mathrm{r_e^2}}{\mathrm{b_{max}^2}} \frac{\mathrm{m_e}}{\mathrm{M}} \mathrm{m_e c^2} \ln\left(\frac{\pi}{\theta_{\mathrm{min}}}\right) \tag{5}$$

- Longitudinal proton's momentum after collision

$$\langle p_{4z} \rangle = \frac{1}{\pi b_{max}^2} \int_{\theta_{min}}^{\pi} p_{4z}(\theta) \frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \mathrm{d}\theta \simeq - \frac{\langle \Delta \mathrm{E_p} \rangle}{\mathrm{c}} \left(1 + \frac{\mathrm{E_p^R}}{\mathrm{E_e^R}}\right) \tag{6}$$

- Transverse 2D proton's moment after collision

$$\langle p_{4t} \rangle = \frac{1}{\pi b_{max}^2} \int_{\theta_{min}}^{\pi} p_{4y}(\theta) \frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \mathrm{d}\theta \simeq - \frac{4\mathrm{r_e m_e c}}{\mathrm{b_{max}}} \tag{7}$$

RMS transverse moments

Considering now separately the horizontal and vertical transverse momentum of the proton:

$$\begin{cases} p_x(\theta,\phi) = p_t(\theta)\cos\phi \\ \\ p_y(\theta,\phi) = p_t(\theta)\sin\phi \end{cases} \tag{8}$$

While these two quantities are zero when averaged over ϕ, their *RMS* values are not:

$$p_x RMS = \sqrt{\frac{1}{\pi b_{max}^2}\int_{\theta_{min}}^{\pi} p_t^2(\theta)\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\sin\theta\,\mathrm{d}\theta\int_0^{2\pi}\cos^2(\phi)\,\mathrm{d}\phi} = p_y RMS \tag{9}$$

It follows that:

$$p_{tx} RMS \simeq \frac{2\,r_e m_e c}{b_{max}}\sqrt{\ln\left(\frac{\pi}{\theta_{min}}\right)} \tag{10}$$

Back to the Lab frame - Mean Energy & RMS Value

Performing a reverse Lorentz-Boost (p_{4z} have negative value):

- Mean Energy

$$\begin{aligned}\langle \Delta E_p \rangle^{Lab} &= \gamma_p \left(\langle \Delta E_p \rangle^R + \beta_p c \langle p_{4z} \rangle^R \right) \\ &\simeq -4 \frac{m_e c^2 \, r_e^2}{(1+\beta_e^L)\, b_{max}^2} \ln \left(\frac{\pi}{\theta_{min}} \right) \qquad (11)\end{aligned}$$

$$\simeq 10^{-16}\mathrm{eV} \text{ per collision}$$

- RMS Value

$$p_{x,RMS}^{lab} = p_{x,RMS}^{R} \simeq \frac{2 r_e}{b_{max}} m_e c \sqrt{\ln \left(\frac{\pi}{\theta_{min}} \right)} \simeq 10^{-14}\mathrm{eV/c} \qquad (12)$$

Equilibrium Emittance

- Emittance is defined as the determinant of the beam matrix:

$$\varepsilon_{x,y} = \sqrt{\det \Sigma_{\mathrm{x,y}}} \tag{13}$$

which writes as:

$$\Sigma_{x,y} = \varepsilon_{x,y} \begin{pmatrix} \beta^* & 0 \\ 0 & \dfrac{1}{\beta^*} \end{pmatrix}_{x,y} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}_{identity} \tag{14}$$

- The whole emittance along x after N turns then writes as:

$$\boxed{\varepsilon_x(N) = \varepsilon_0(N) + \beta^* \frac{N}{2} x'^2_{RMS}} \tag{15}$$

with $x'_{RMS} = \dfrac{p_{x\,RMS}}{p_0}$ given by equation (12).

- Its derivative with respect to time can be derived as:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{dt}} = \frac{\mathrm{d}\varepsilon}{\mathrm{dN}}\frac{\mathrm{dN}}{\mathrm{dt}} = \underbrace{\boxed{\frac{\beta^*}{2} f_{rev}\, x'^2_{RMS}}}_{\text{heating}} - \underbrace{\boxed{\frac{\varepsilon}{\tau}}}_{\text{cooling}} \tag{16}$$

where τ is the cooling time and f_{rev} the revolution frequency.

- The equilibrium emittance can then be computed as:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{dt}} = 0 \Rightarrow \varepsilon_{eq} = \frac{\frac{\beta^*}{2}\left(\frac{p^2_{x,RMS}}{p_0^2}\right)}{\frac{\langle \Delta E \rangle}{E_0}} \tag{17}$$

- Plugging in the respective expressions for $p_{x,RMS}$ and $\langle \Delta E \rangle$ and knowing that $p_0^2 = \gamma_p^2 M^2 (\beta_p^L)^2 c^2 = (E_p^L \beta_p^L)^2 / c^2$ and $E_0 = E_p^L$, the equilibrium emittance writes finally as:

$$\Rightarrow \boxed{\varepsilon_{eq} = \frac{\beta^*}{2} \frac{m_e c^2}{E_p^L} \frac{(1+\beta_e^L)}{(\beta_p^L)^2}} \tag{18}$$

- To get the numerical value, one assume a waist $\beta^* = 1m$ at the electron lens.

$$\Rightarrow \begin{cases} \varepsilon_{eq} \simeq 43.62 \text{ nm} \text{ for protons} \\ \\ \varepsilon_{eq} \simeq 0.585 \text{ nm} \text{ for } Pb^{82+}\text{-ions} \end{cases} \tag{19}$$

$\Longrightarrow$ The equilibrium emittance for Pb^{82+}-ions is basically reduced by the ion-charge $Z_{Pb} = 82$.
$\Longrightarrow$ Both values are too large compared to $\varepsilon_{eq}^{LHC} = 0.5$ nm.

Cooling Time

From the time-derivative of the emittance (equation (16)):

$$\tau = \frac{E_0}{\langle \Delta E \rangle} \frac{1}{f_{rev} \mathcal{N}_{e^-}} \text{ where } \mathcal{N}_{e^-} \simeq 10^{11} \text{ electrons} \tag{20}$$

$$\Rightarrow \begin{cases} \tau_p & = \dfrac{(1+\beta_e^L)\, b_{max}^2}{4\, m_e\, c^2\, r_e^2} \dfrac{1}{\ln\left(\dfrac{\pi}{\theta_{min}}\right)} \dfrac{E_p^L}{f_{rev} \mathcal{N}_{e^-}} = 4.7624 \times 10^{13} \text{ s} \\ \tau_{Pb} & = \dfrac{(1+\beta_e^L)\, b_{max}^2}{4\, m_e\, c^2\, r_e^2\, Z^2} \dfrac{1}{\ln\left(\dfrac{\pi}{Z\, \theta_{min}}\right)} \dfrac{E_{Pb}^L}{f_{rev} \mathcal{N}_{e^-}} = 6.0029 \times 10^{11} \text{ s} \end{cases} \tag{21}$$

$\Longrightarrow$ Cooling time is by far too large.

$\Longrightarrow$ Additional energy-loss due to the external structure ?

Additional Loss - Short range resistive wakefield

- The longitudinal wakefield potential derived by O. Napoly in article [2] obeys the following expansion:

$$W_z(r,\theta,s,\sigma_z) = \frac{2Z_0 c}{\pi^2 a^2} \sum_{m=0}^{\infty} \left(\frac{r}{a}\right)^m \left(\frac{r_0}{a}\right)^m \cos(m\theta)\, w_z^{(m)}(s/s_0, \sigma_z/s_0) \tag{22}$$

Point-like charge $\Rightarrow \sigma_z = 0$. The first order longitudinal wakefield potential becomes:

$$W_z(r,\theta,s,0) = \frac{2Z_0 c}{\pi^2 a^2} w_z^{(0)}(u = s/s_0, 0) \tag{23}$$

where the dimensionless function $w_z^{(m=0)}$ is given by the integral:

$$w_z^{(0)}(u) = \frac{1}{3}\int_{-1}^{\infty} dx\, \frac{x \sin\left[u\, 2^{\frac{1}{3}}(x+1)^{\frac{2}{3}}\right] + \cos\left[u\, 2^{\frac{1}{3}}(x+1)^{\frac{2}{3}}\right]}{x^2+1} \tag{24}$$

The energy lost by the proton between z_1 and z_2 writes:

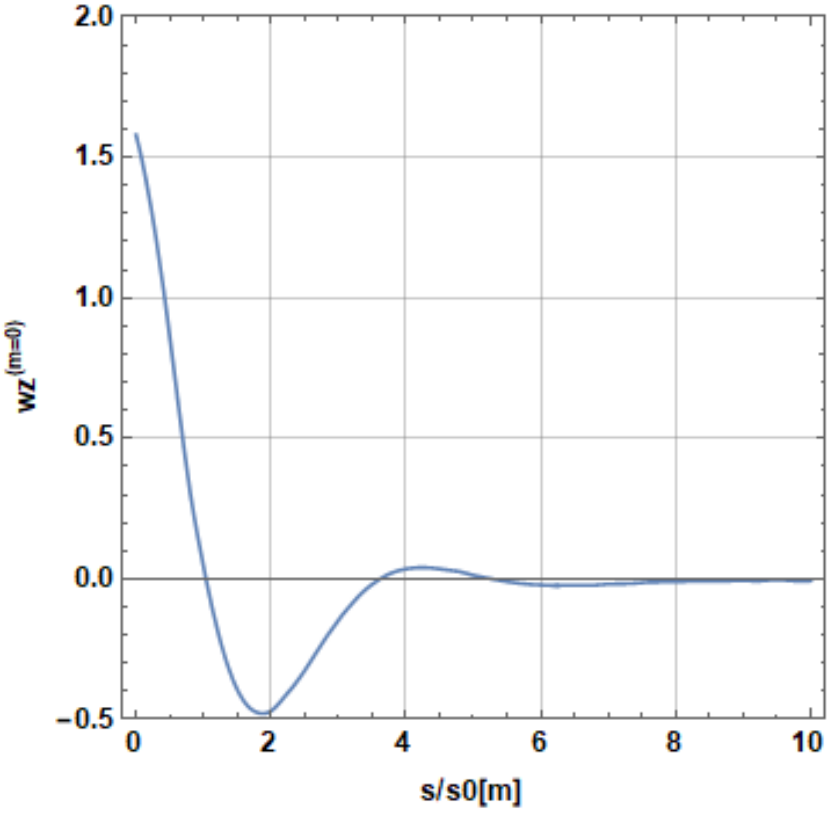
$$\Delta U(r, \theta, s, \sigma_z) = qq' \int_{s_1}^{s_2} ds\, W_z(r, \theta, s, 0) \tag{25}$$

where q is the proton's charge, q' the electron's charge and s is the distance between the proton and the electron:

$$s(t) = z_{e^-}(t) - z_p(t) = 1 - c\, t(1 + \beta_e^L) \tag{26}$$

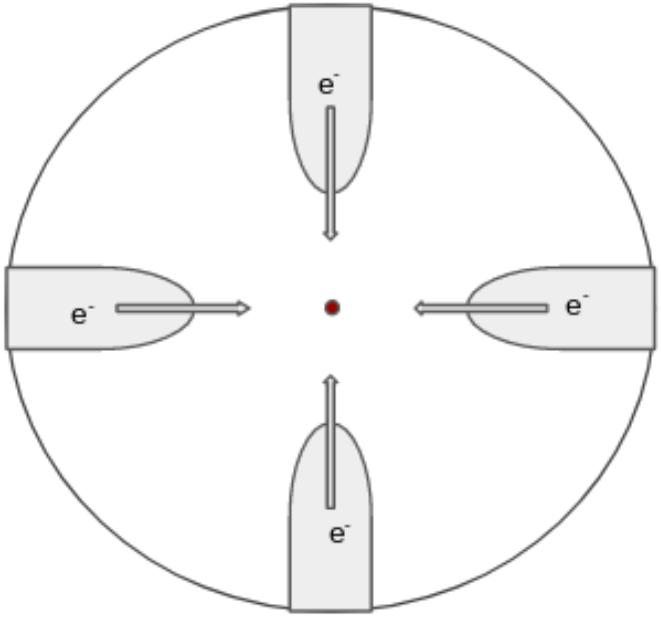
Collision happens when $s = 0$:

$$\begin{cases} z_{collision} = \dfrac{z_0}{1 + \beta_e^L} \\ \\ t_{collision} = \dfrac{z_0}{c(1 + \beta_e^L)} \end{cases} \tag{27}$$



- The longitudinal wake oscillates twice between decelerating and accelerating values before it reaches it's accelerating long-range behaviour.
- Integrating between $s_1 = 0$ and $s_2 = z_0 - z_{collision}$, a first estimation of the longitudinal loss would be approximately 10^{-7} eV per collision.
- This requires further calculation together with the computation of the transverse wakefield loss.

Transverse electron-proton collision



- Idea : use a certain number (at least 4) radial electron beams spread over a few meters instead of a longitudinal one.
- This way, the electric part of the Lorentz Force is still transverse but the magnetic part will be located in the longitudinal plane.
- By Momemtum conservation, this should reduce the transverse emittance of the beam.
- Calculations on-going...

Conclusion & Ways forward

- The transverse kick received by the proton being too big compared to the mean energy loss, cooling cannot be efficient without taking into consideration the external structure of the experiment.
  - Equilibrium emittance too high
  - Unrealistic cooling time
- The longitudinal wakefield following the proton will accelerate the electrons and give a longitudinal back-kick to the proton. This longitudinal loss might be quite consequent and would require further inverstigations.
  The transverse wakefield loss needs to be computed as well and the calculation will be quite tedious.
- To be continued...

The End

Thank you for your attention !

References

- *Handout 5 : Electron-Proton Elastic Scattering*, Prof M. Thomson, University of Cambridge (2011).
- *The Short-Range Resistive wall Wakefields*, K. Bane and M. Sands, SLAC-PUB-95-7074, Stanford University, California (December 1995).
- *The resistive-pipe wake potentials for short bunches*, O.Henry, O. Napoly, CLIC Note 142, DPhN-STAS, CE Sacley, Orme des Merisiers (May 1991).