Leptonic CP Measurement & New Physics Alternatives

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SFG [arXiv:1704.08518]

SFG, Hitoshi Murayama [arXiv:1904.02518]
CP Measurement @ Accelerator Exps

- T2K
- NOνA
- DUNE/T2KII/T2HK/T2HKK/T2KO; MOMENT/ADS-CI/DAEδALUS; Super-PINGU
The Dirac CP Phase $\delta_D$ @ Accelerator Exp

Accelerator experiment, such as T2(H)K, uses off-axis beam to compare $\nu_e$ & $\bar{\nu}_e$ appearance @ the oscillation maximum.

- **Disadvantages:**
  - **Efficiency:**
    - Proton accelerators produce $\nu$ more efficiently than $\bar{\nu}$ ($\sigma_\nu > \sigma_{\bar{\nu}}$).
    - The $\bar{\nu}$ mode needs more beam time [$T_{\bar{\nu}} : T_{\nu} = 2 : 1$].
    - Undercut statistics $\Rightarrow$ Difficult to reduce the uncertainty.
  - **Degeneracy:**
    - Only $\sin \delta_D$ appears in $P_{\nu_\mu \rightarrow \nu_e}$ & $P_{\nu_\mu \rightarrow \bar{\nu}_e}$.
    - Cannot distinguish $\delta_D$ from $\pi - \delta_D$.
  - **CP Uncertainty** $\frac{\partial P_{\mu e}}{\partial \delta_D} \propto \cos \delta_D \Rightarrow \Delta(\delta_D) \propto \frac{1}{\cos \delta_D}$.

- **Solution:**
  - Measure $\bar{\nu}$ mode with $\mu^+$ decay @ rest ($\mu$DAR)
A cyclotron produces 800 MeV proton beam @ fixed target.
Produce $\pi^\pm$ which stops &
  - $\pi^-$ is absorbed,
  - $\pi^+$ decays @ rest: $\pi^+ \rightarrow \mu^+ + \nu_\mu$.$\mu^+$ stops & decays @ rest: $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$.

$\bar{\nu}_\mu$ travel in all directions, oscillating as they go.
A detector measures the $\bar{\nu}_e$ from $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation.
Accelerator + $\mu$DAR Experiments

Combining $\nu_\mu \rightarrow \nu_e$ @ accelerator [narrow peak @ 550 MeV] & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ @ $\mu$DAR [wide peak $\sim$ 45 MeV] solves the 2 problems:

- **Efficiency:**
  - $\nu$ @ high intensity, $\mu$DAR is plentiful enough.
  - Accelerator Exps can devote all run time to the $\nu$ mode. With same run time, the statistical uncertainty drops by $\sqrt{3}$.

- **Degeneracy:** (decomposition in propagation basis [1309.3176])
It’s the **FIRST** proposal along this line:

- 3 $\mu$DAR with 3 high-intensity cyclotron complexes.
- 1 detector.
- Different baselines: **1.5, 8 & 20 km** to break degeneracies.

**Disadvantages:**

- The final-state lepton from IBD @ low energy is **isotropic**.
- **Cannot** distinguish $\bar{\nu}_e$ from different sources
- Baseline **cannot be measured**.
- Cyclotrons **cannot** run simultaneously (20~25% duty factor).
- **Large** statistical uncertainty.
- **Higher intensity** is necessary.
- **Expensive & Technically challenging.**
New Proposals

1 $\mu$DAR source + 2 detectors

Advantages

- Full (100%) duty factor!
- Lower intensity: $\sim 9\text{mA}$ [$\sim 4 \times$ lower than DAE$\delta$ALUS]
- Not far beyond the current state-of-art technology of cyclotron [2.2mA @ Paul Scherrer Institute]
- MUCH cheaper & technically easier.
  - Only one cyclotron.
  - Lower intensity.

Disadvantage?

- A second detector!
  - $\mu$DAR with Two Scintillators ($\mu$DARTS) [Ciuffoli, Evslin & Zhang, 1401.3977] also Smirnov, Hu, Li & Ling [1802.03677, 1808.03795]
  - Tokai 'N Toyama to(2) Kamioka (TNT2K) [Evslin, Ge & Hagiwara, 1506.05023]
TNT2K

- $T2(H)K + \mu SK + \mu HK$

$\mu$DAR is also useful for **material**, **medicine** industries in Toyama
$\delta_D$ Precision @ TNT2K

Evslin, Ge & Hagiwara [1506.05023]

![Graph showing precision vs baseline length for NH and IH cases. The graph demonstrates the variation of average $\Delta(\delta)$ with baseline length for different values of $\delta$. The baseline length is given in kilometers (km).](image)
$\delta_D$ Precision @ TNT2K

Evslin, Ge & Hagiwara [1506.05023]
Non-Unitarity Mixing (NUM)

Ge, Pasquini, Tortola & Valle [1605.01670]

\[ N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ |\alpha_{21}|e^{i\phi} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U. \]

\[ P_{\mu e}^{NP} = \alpha_{11}^2 \left\{ \alpha_{22}^2 \left[ c_a^2 |S_{12}'|^2 + s_a^2 |S_{13}'|^2 + 2c_a s_a (\cos \delta_D R - \sin \delta_D I)(S_{12}' S_{13}'^*) \right] + |\alpha_{21}|^2 P_{ee} \right\} + 2\alpha_{22} |\alpha_{21}| \left[ c_a (c_\phi R - s_\phi I) (S_{11}' S_{12}'^*) + s_a (c_\phi + \delta_D R - s_\phi + \delta_D I)(S_{11}' S_{13}'^*) \right]. \]

The effect of including non-unitarity at T2K \[ \delta_{CP}^{true} = -90^\circ, \text{NH} \]
\[ P^{NP}_{\mu e} (L \to 0) = \alpha_{11}^2 |\alpha_{21}|^2 P_{ee} \approx \alpha_{11}^2 |\alpha_{21}|^2 \approx |\alpha_{21}|^2 \]

Event Spectrum at \( \mu \text{Near} \) [20ton, \( L = 20m \)]

\[ |\alpha_{12}|^\text{true} = 0.02 \]

\( \mu^-\text{DAR}/\mu^+\text{DAR} = 5 \times 10^{-4} \)
The effect of including non-unitarity at T2K+$\mu$SK: $\delta_{CP}^{true} = -90^\circ$, NH

The effect of including non-unitarity at T2HK+$\mu$HK: $\delta_{CP}^{true} = -90^\circ$, NH

The effect of including non-unitarity at T2K+$\mu$SK+$\mu$Near: $\delta_{CP}^{true} = -90^\circ$, NH

The effect of including non-unitarity at T2HK+$\mu$HK+$\mu$Near: $\delta_{CP}^{true} = -90^\circ$, NH
Non-Standard Interaction

\[ \mathcal{H} \equiv \frac{1}{2E_{\nu}} U \begin{pmatrix} 0 & \Delta m_s^2 \\ \Delta m_a^2 & \end{pmatrix} U^\dagger + V_{cc} \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix} \]

- **Standard Interaction** – \( V_{cc} \) (also \( V_{nc} \))
- **Non-Standard Interaction** – \( \epsilon_{\alpha\beta} \)
  - **Diagonal** \( \epsilon_{\alpha\alpha} \) are real
  - **Off-diagonal** \( \epsilon_{\alpha\neq\beta} \) are complex
  - Both can fake CP
- \( Z' \) in LMA-Dark model with \( L_\mu - L_\tau \) gauged as \( U(1) \)
  - \( M_{Z'} \sim \mathcal{O}(10) \text{MeV} \)
  - \( g_{Z'} \sim 10^{-5} \)
The effect of NSI on the CP sensitivity at T2K $[\delta_D^{\text{true}} = -90^\circ]$ 

The effect of NSI on the CP sensitivity at $\mu$SK $[\delta_D^{\text{true}} = -90^\circ]$

The effect of NSI on the CP sensitivity at T2K+$\mu$SK $[\delta_D^{\text{true}} = -90^\circ]$

The effect of NSI on the CP sensitivity at $\nu$T2K+$\mu$SK $[\delta_D^{\text{true}} = -90^\circ]$
Scalar NSI

Vector NSI

\[ \mathcal{L}^{\text{eff}}_{cc} = \frac{g_\alpha g_\beta^*}{2} \left( \frac{1}{-m_V^2} \right) (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta) (\bar{\ell}_\sigma \gamma^\mu P_L \ell_\rho), \]

which is vector-vector type vertex.

Scalar Mediator

\[ -\mathcal{L} = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} M_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta + y_{\alpha\beta} \phi \bar{\nu}_\alpha \nu_\beta + Y_{\alpha\beta} \phi \bar{f}_\alpha f_\beta + \text{h.c.}, \]

Due to forward scattering, the effective Lagrangian is

\[ \mathcal{L}^{s}_{\text{eff}} \propto y_{\alpha\beta} Y_{ee} [\bar{\nu}_\alpha(p_3)\nu_\beta(p_2)] [\bar{e}(p_1)e(p_4)], \]

which is a scalar-scalar type vertex \Rightarrow significant phenomenological consequences.

\[ \mathcal{H} \approx E_\nu + \frac{(M + M_S)(M + M_S)\dagger}{2E_\nu} \pm V_{SI}, \]
**Solar Neutrino**

![Graph of solar neutrino fluxes and spectrum]

**Spectrum of solar neutrinos**

Flux at 1 AU (cm$^{-2}$ s$^{-1}$) vs. Neutrino energy (MeV) for various reactions:
- $^{13}$N
- $^{15}$O
- $^{17}$F
- $^{7}$Be
- $^{7}$Be pep
- hep

**Diagram showing**
- Spectrum of solar neutrinos
- Flux at 1 AU (cm$^{-2}$ s$^{-1}$)
- Neutrino energy (MeV)

**Equation**

$$P_{ee}^{\text{sun}} = \left| U_{ei}^{\text{prod}} (U_{ei}^{\text{vac}})^* \right|^2$$

**Graph showing**
- $P_{ee}$ vs. $E_{\nu}$ [MeV]
- $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV$^2$
- $4.7 \times 10^{-5}$ eV$^2$
- $\eta_{ee} = -0.05$
- $\eta_{\mu\mu} = -0.05$
- $\eta_{\tau\tau} = -0.05$
- $\eta_{ee} = -0.16$

**Legend**
- pp
- Be7
- pep
- B8

**Caption**

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Scalar NSI @ Accelerator Neutrino Oscillation

$E_
u$ [MeV]

$P_{\mu e}$

$\nu_{T2K}$

$\mu_{SK}$

$DUNU$ [nu]

$DUNU$ [NU]

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The DM mass can span almost 100 orders

\[ m_{\text{DM}} \]

For light bosonic DM

\[-\mathcal{L} = \frac{1}{2} m_{\phi}^2 \phi^2 + \frac{1}{2} M_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta + y_{\alpha\beta} \phi \bar{\nu}_\alpha \nu_\beta + h.c.,\]

leading to forward scattering

\[ \nu(p_\nu) \rightarrow \nu(p_\nu) \]
\[ \bar{\nu}(-p_\nu) \rightarrow \bar{\nu}(-p_\nu) \]

\[ \phi/V_\mu \]
\[ \phi/V_\nu \]
Effective Mass Correction from Dark Matter

- The forward scattering with the DM background

\[ \nu(p_{\nu}) \rightarrow \nu(p_{\nu}) \]
\[ \bar{\nu}(-p_{\nu}) \rightarrow \bar{\nu}(-p_{\nu}) \]
\[ \phi/V_{\mu} \rightarrow \phi/V_{\nu} \]

modifies the neutrino kinetic term

\[ i\delta\Gamma_{\alpha\beta} = \frac{i\rho_\phi(v_\phi)}{m_\phi^2} \sum_j y_{\alpha j} y_{j\beta}^* \left[ \frac{p_{\nu} + p'_{\phi} + m_{\nu}}{p_{\phi}^2 + 2p_{\nu} \cdot p_{\phi}} + \frac{p_{\nu} - p'_{\phi} + m_{\nu}}{p_{\phi}^2 - 2p_{\nu} \cdot p_{\phi}} \right] \]

with \( p_{\phi} \sim m_{\phi}(1, \tilde{v}_{\phi}) \), the correction

\[ \delta\Gamma_{\alpha\beta} \approx \sum_j y_{\alpha j} y_{j\beta}^* \frac{\rho_{\chi}}{m_{\phi}^2 E_{\nu}} \gamma_0 \]

appears as dark potential.

SFG, Hitoshi Murayama [arXiv:1904.02518]
The dark potential

$$\delta \Gamma_{\alpha \beta} \approx \sum_j y_{\alpha j} y_{j \beta}^{*} \frac{\rho_{\chi}}{m_{\phi}^{2} E_{\nu}} \gamma_{0}$$

is a correction to the Hamiltonian, same as the matter potential.

Due to $1/E_{\nu}$ dependence, the dark potential is promoted to mass correction

$$H = \frac{M^{2}}{2E_{\nu}} - \frac{1}{E_{\nu}} \sum_{j} y_{\alpha j} y_{j \beta}^{*} \frac{\rho_{\chi}}{m_{\phi}^{2}} \equiv \frac{M^{2} + \delta M^{2}}{2E_{\nu}}$$

which is totally different from the scalar NSI.

With mass term correction, any neutrino oscillation cannot see the original variables. Neutrino oscillation can happen even if the original mass term $M^{2}$ vanishes.
Dark NSI & Faked CP

- With just 3% of dark NSI

- The biprobability contour can totally change.

SFG, Hitoshi Murayama [arXiv:1904.02518]
Summary

- **Better CP measurement than T2K**
  - Much larger event numbers
  - Much better CP sensitivity around maximal CP
  - Solve degeneracy between $\delta_D$ & $\pi - \delta_D$
  - Guarantee CP sensitivity against NUM
  - Guarantee CP sensitivity against NSI (vector, scalar, dark)

- **Better configuration than DAE$d$ALUS**
  - Only one cyclotron
  - 100% duty factor
  - Much lower flux intensity
  - Much easier
  - Much cheaper
  - Single near detector
Thank You!
LHC & Daya Bay changed Physics in 2012

- **Higgs boson** ⇒ electroweak symmetry breaking & mass.
- **Chiral symmetry breaking** ⇒ majority of mass.
- **The world seems not affected by the tiny neutrino mass?**
  - Neutrino mass ⇒ Mixing
  - 3 Neutrino ⇒ possible **CP violation**
  - CP violation ⇒ **Leptogenesis**
  - **Leptogenesis** ⇒ **Matter-Antimatter Asymmetry**
  - There is something left in the Universe.
  - Baryogenesis from quark mixing is not enough.

- Majorana $\nu$ ⇔ **Lepton Number Violation**

- **Residual $\mathbb{Z}_2$ Symmetries:**
  \[
  \cos \delta_D = \frac{(s^2_s - c^2_s s^2_r)(c^2_a - s^2_a)}{4c_as_a c_s s_s s_r}
  \]

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<table>
<thead>
<tr>
<th>Oscillation Data</th>
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(for NH) \( -1\sigma \)  \( \text{Best Value} \)  \( +1\sigma \)

\[
\Delta m_s^2 \equiv \Delta m_{12}^2 (10^{-5} \text{eV}^2) \quad 7.37 \quad 7.56 \quad 7.75
\]

\[
|\Delta m_a^2 \equiv \Delta m_{13}^2| (10^{-3} \text{eV}^2) \quad 2.51 \quad 2.55 \quad 2.59
\]

\[
\sin^2 \theta_s (\theta_s \equiv \theta_{12}) \quad 0.305 (33.5^\circ) \quad 0.321 (34.5^\circ) \quad 0.339 (35.6^\circ)
\]

\[
\sin^2 \theta_a (\theta_a \equiv \theta_{23}) \quad 0.412 (39.9^\circ) \quad 0.430 (41.0^\circ) \quad 0.450 (42.1^\circ)
\]

\[
\sin^2 \theta_r (\theta_r \equiv \theta_{13}) \quad 0.02080 (8.29^\circ) \quad 0.02155 (8.44^\circ) \quad 0.02245 (8.62^\circ)
\]

\( \delta_D, \delta_{Mi} \quad ?, ??? \quad ?, ??? \quad ?, ?? \)
To leading order in $\alpha = \frac{\delta M^2_{21}}{|\delta M^2_{31}|} \sim 3\%$, the oscillation probability relevant to measuring $\delta_D@T2(H)K$,

$$P_{\nu_\mu \rightarrow \nu_e} \approx 4s_a^2c_r^2s_r^2\sin^2 \phi_{31} - 8c_a s_a c_r^2 s_r c_s s_s \sin \phi_{21} \sin \phi_{31} \cos \delta_D \cos \phi_{31} \pm \sin \delta_D \sin \phi_{31}$$

for $\nu$ & $\bar{\nu}$, respectively. [$\phi_{ij} \equiv \frac{\delta m^2_{ij}L}{4E_\nu}$]

$\nu_\mu \rightarrow \nu_\mu$ Exps measure $\sin^2(2\theta_a)$ precisely, but not $\sin^2 \theta_a$.

Run both $\nu$ & $\bar{\nu}$ modes @ first peak [$\phi_{31} = \frac{\pi}{2}, \phi_{21} = \alpha \frac{\pi}{2}$],

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} + P_{\nu_\mu \rightarrow \nu_e} = 2s_a^2c_r^2 s_r^2,$$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} - P_{\nu_\mu \rightarrow \nu_e} = \alpha \pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos \theta_r \sin \delta_D.$$
Lowest Atmospheric Neutrino Background

US/UK World Magnetic Model -- Epoch 2010.0
Main Field Horizontal Intensity (H)
Backgrounds to IBD ($\bar{\nu}_e + p \rightarrow e^+ + n$)

- Reactor $\bar{\nu}_e$: $E_\nu < 10$ MeV
- Accelerator $\nu_e$: $E_\nu > 100$ MeV
- Spallation: $E_\nu \lesssim 20$ MeV
- Supernova Relic Neutrino: $E_\nu \lesssim 20$ MeV

Cut with $30$ MeV < $E_\nu$ < 55 MeV

- Accelerator $\nu_\mu \rightarrow \text{Invisible muon}$
- Atmospheric Neutrino Background
  - **Invisible muon** (below Cherenkov limit)
    - $E_\mu \lesssim 1.5 \times m_\mu$, $\mu^\pm \rightarrow e^\pm$
    - $E_\pi \lesssim 1.5 \times m_\pi$, $\pi^+ \rightarrow \mu^+ \rightarrow e^+$
    - 1 neutron
    - No prompt photon
  - Irreducible $\bar{\nu}_e$: 30 MeV < $E_\nu$ < 55 MeV
  - Reducible $\nu_e$: 60 MeV < $E_\nu$ < 100 MeV
    - 1 neutron
    - No prompt photon

**Lowest** at $\mu$DARTS & TNT2K sites
Expected $\mu$DAR IBD signal from 6 yrs of running @ SK (15km) & HK (23km) with NH.

Simulated by NuPro, http://nupro.hepforge.org/
NUM vs Seesaw Mechanism

- **Heavy neutrinos**

  \[ \bar{\nu} M_D N + h.c. + \overline{N} M_N N = \begin{pmatrix} \bar{\nu} \\ \overline{N} \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix} \begin{pmatrix} \nu \\ \overline{N} \end{pmatrix} \]

- **Seeaw Mechanism**

  \[ M_\nu = -M_D M_N^{-1} M_D^T, \quad \nu' = \nu + M_D M_N^{-1} N \]
$1\sigma$ Upper Limit on $|\alpha_{21}|$ at $\mu$Near

- Detector Size [ton]
- Background-Signal Flux Ratio [$10^{-4}$]
Corrections to $\delta_D$ [Degree] at T2K

$|\varepsilon_{\mu\mu}|$

$\text{Arg}[\varepsilon_{\mu\mu}]$

Corrections to $\delta_D$ [Degree] at T2K

$|\varepsilon_{\mu\tau}|$

$\text{Arg}[\varepsilon_{\mu\tau}]$

Corrections to $\delta_D$ [Degree] at T2K

$|\varepsilon_{\mu\tau}|$

$\text{Arg}[\varepsilon_{\mu\tau}]$
EOM & Effective Hamiltonian with Scalar NSI

- **Two-Point Correlation Function**

\[
\delta \Gamma_S = \frac{y_{\alpha' \beta'} y_{\text{ee}}}{m_\phi^2} \langle \nu_\alpha | \bar{\nu}_{\alpha'} \nu_{\beta'} | \nu_\beta \rangle \langle e | \bar{e} e | e \rangle ,
\]

\[
\delta \bar{\Gamma}_S = \frac{y_{\beta' \alpha'} y_{\text{ee}}}{m_\phi^2} \langle \bar{\nu}_\alpha | \bar{\nu}_{\alpha'} \nu_{\beta'} | \bar{\nu}_\beta \rangle \langle e | \bar{e} e | e \rangle .
\]

- **Equation of Motion**

\[
\bar{\nu}_\beta \left[ i \partial_\mu \gamma^\mu + \left( M_{\beta \alpha} + \frac{n_{\text{ee}} y_{\text{ee}} Y_{\alpha \beta}}{m_\phi^2} \right) \right] \nu_\alpha = 0 ,
\]

- **Effective Hamiltonian**

\[
\mathcal{H} \approx E_\nu + \frac{(M + M_\text{S})(M + M_\text{S})^\dagger}{2E_\nu} \pm V_{\text{SI}} ,
\]

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The effective mass term is a combination
\[ \text{MM}^\dagger \rightarrow (\text{M} + \text{M}_S)(\text{M} + \text{M}_S)^\dagger = \text{MM}^\dagger + \text{MM}_S^\dagger + \text{M}_S\text{M}^\dagger + \text{M}_S\text{M}_S^\dagger \]

The absolute neutrino mass can enter neutrino oscillation!
\[ \text{MM}_S^\dagger + \text{M}_S\text{M}^\dagger \]

The unphysical CP phases can also enter neutrino oscillation!
\[ \text{M} \equiv R_\nu D_\nu R_\nu^\dagger \quad \text{&} \quad R_\nu \equiv P_\nu U_\nu Q_\nu \]

The Majorana rephasing matrices \( Q_\nu = \{e^{i\delta_{M1}/2}, 1, e^{i\delta_{M3}/2}\} \) can be absorbed, \( Q_\nu D_\nu Q_\nu^\dagger = D_\nu \) while the unphysical rephasing matrix \( P_\nu \equiv \{e^{i\alpha}, e^{i\beta}, e^{i\gamma}\} \) can not be simply rotated away now:
\[ M \rightarrow \tilde{M} = U_\nu D_\nu U_\nu^\dagger, \quad M_S \rightarrow \tilde{M}_S = P_\nu^\dagger M_S P_\nu \]
Use characteristic scale $\Delta m_a^2$ to parametrize scalar NSI

$$\tilde{M}_S \equiv \sqrt{\Delta m_a^2} \begin{pmatrix} \eta_{ee} & \eta_{\mu e}^* & \eta_{\tau e}^* \\ \eta_{\mu e} & \eta_{\mu\mu} & \eta_{\tau\mu}^* \\ \eta_{\tau e} & \eta_{\tau\mu} & \eta_{\tau\tau} \end{pmatrix},$$

where $\Delta m_a^2 \equiv \Delta m_{31}^2 = 2.7 \times 10^{-3}$ eV$^2$.

We first need input for $\tilde{M}$ which is not directly measured.

However, the directly measured from terrestial experiments is always a combination, $\tilde{M} + \tilde{M}_S(\rho_s \approx 3 \text{g/cm}^3)$. It is then necessary to first substract a constant term:

$$\tilde{M} \rightarrow \tilde{M} + \tilde{M}_S \frac{\rho - \rho_s}{\rho_s}$$

where $\tilde{M} = U_\nu D_\nu U_\nu^\dagger$ is reconstructed in terms of the measured mixing matrix while $\tilde{M}_S$ is the scalar NSI @ typical constant subtraction density $\rho_s$. 
Density Subtraction for Reactor Anti-Neutrinos

Since the reactor anti-neutrino experiments (Daya Bay & JUNO) are the most precise ones, we do substraction according to them:

\[
\tilde{\mathcal{M}} \rightarrow \tilde{\mathcal{M}} + \tilde{\mathcal{M}}_S \frac{\rho - \rho_s}{\rho_s}
\]

Then **no constraint** on scalar NSI from reactor experiments!
Scalar NSI @ Atmospheric Neutrino Oscillation

(a) SI $\nu$
(b) vector NSI $\varepsilon_{ee} = 1$
(c) scalar NSI $\eta_{ee} = 0.1$
(d) scalar NSI $\eta_{ee} = -0.16$

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