Amplitude analysis of $B^0 \to (\pi^+\pi^-)(K^+\pi^-)$ decays

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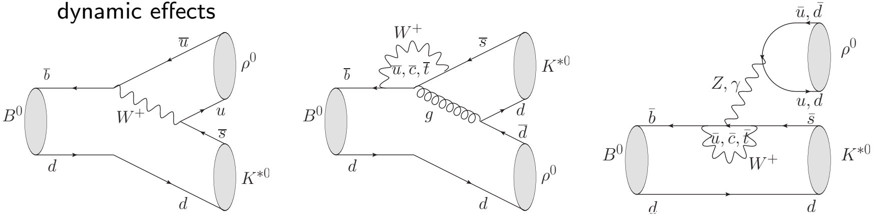
Phenomenology of the $B^0 \to \rho^0 K^*(892)^0$ decay

Charmless B^0 **meson** decay reconstructed as $\boldsymbol{B^0 \to \rho^0(\pi^+\pi^-)K^*(892)^0(K^+\pi^-)}$

- **Proceeds via**:
  - ◇ A **doubly Cabibbo suppressed tree**
  - ◇ A **gluonic $b \to s$ penguin** (GP)
  - ◇ A **electro-weak $b \to s$ penguin** (EWP)
  - → Tree and GP diagrams have similar amplitudes → maximises interferences

- **Self-tagged** decay: $\begin{cases} B^0 \to (\pi^+\pi^-)(K^+\pi^-) \\ \overline{B}^0 \to (\pi^-\pi^+)(K^-\pi^+) \end{cases}$

- **Vector** resonances → **additional** CP-violating **observables** and sensitivity to QCD dynamic effects

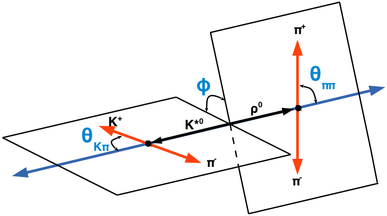


Observables in an amplitude analysis of $B \to VV$ decays

$B \to (p_a p_b)_1 (p_c p_d)_2$ decays

Can be **fully described** in terms of:

- ◇ **Three helicity angles**: θ_1, θ_2, ϕ
- ◇ **Two invariant masses**: m_1, m_2



A **$B \to VV$ proceeds via three amplitudes** $\to$ three **spin configurations**:
P-odd $S_{VV} = 1$ and P-even $S_{VV} = 0, 2$, rotated into the transversity basis $\lambda = L, ||, \perp$.

EWP diagram contributes differently to each amplitude: rich pattern of interferences.

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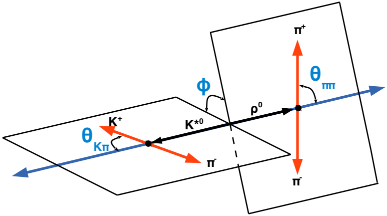

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EWP diagram contributes differently to each amplitude: rich pattern of interferences.

Observables: number of events per amplitude (**polarisation fractions**), f^{λ}, and their **phase differences**:

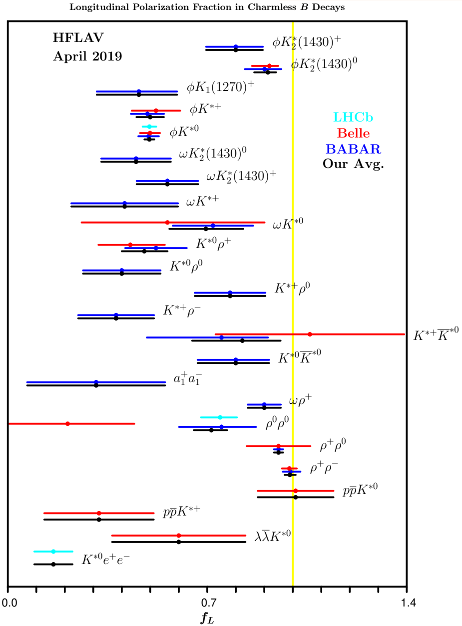
$$f^{\lambda} \equiv \frac{|A^{\lambda}|^2}{|A^L|^2 + |A^{||}|^2 + |A^{\perp}|^2} \qquad \delta^{\lambda_i - \lambda_j} \equiv (\delta^{\lambda_i} - \delta^{\lambda_j})$$

$\to$ **Sensitivity to CPV by comparing B and $\overline{B}$ parameters**

The landscape of longitudinal polarisations

Available results:

- All available measurements are CP-averaged
- Precise predictions unavailable, general dynamics not fully understood $\rightarrow$ **polarisation puzzle**
- Large f_L values confirmed in $b \rightarrow u$ tree dominated decays
- Penguin dominated modes span wider ranges



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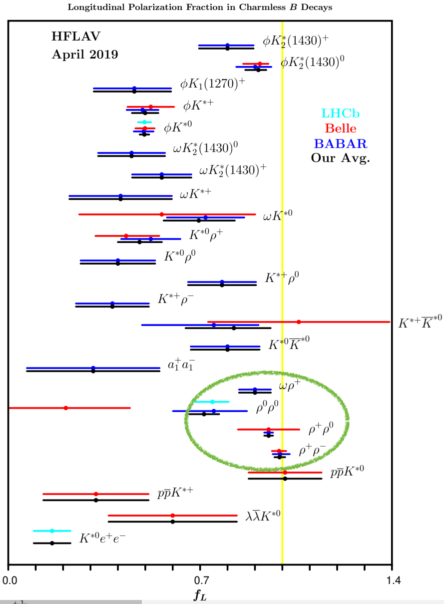

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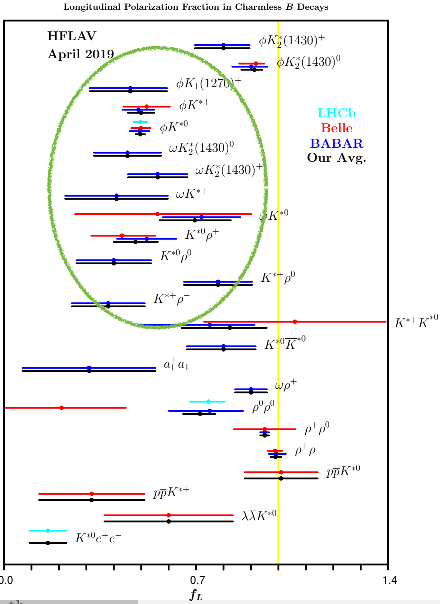

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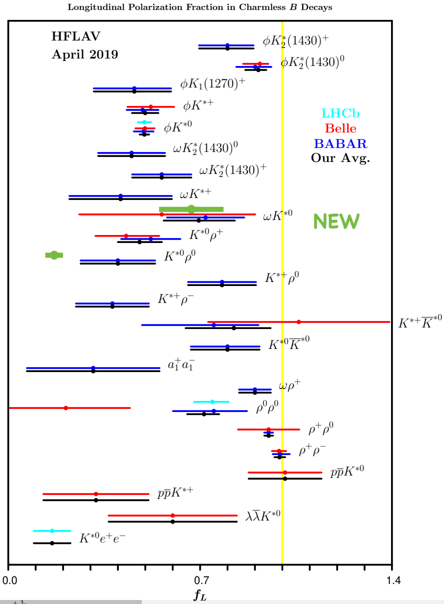
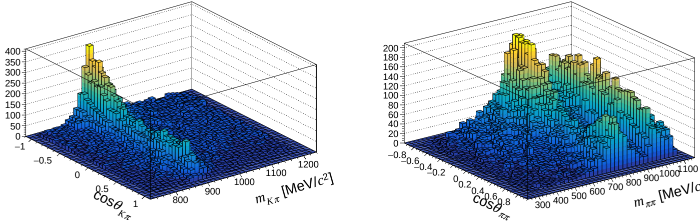
★ **Two new measurements, first observation of CPV in f_L for VV decays** JHEP 05 (2019) 026 (LHCb Run I dataset)



Observables in an amplitude analysis of 4-body decays

In general, a ***VV*** final state **cannot be uniquely selected** and other possible decay channels must be accounted for:



← Illustrative toys

Generalise to N amplitudes (isobar model):

$$\boldsymbol{d^5\Gamma} \propto \Phi_4 \left| \sum_{i=1}^{N} A_i \cdot g_i(\cos\theta_1, \cos\theta_2, \phi) \cdot M_i(m_1, m_2) \right|^2$$

More observables: $+1$ amplitude, $+1$ phase difference per new contribution

An amplitude analysis disentangles the final state!

$A_i \rightarrow$ **physical parameters**

$g_i(\theta_1, \theta_2, \phi) \rightarrow$ **spherical harm.**

$M_i(m_1, m_2) \rightarrow$ **mass prop.**

Partial waves in the $B^0 \to \rho^0 K^*(892)^0$ channel

Remarks:

◇ Analyse a large phase-space: testing many variations of strong phase differences.

Sensitive to localised CP-violating effects!

Partial waves:

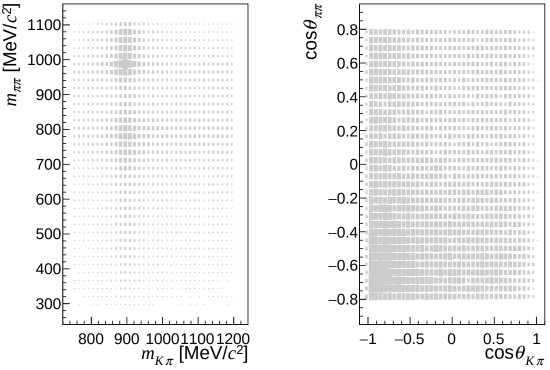
VV: $\rho K^*, \omega K^*$, **VS**: $\rho(K\pi), \omega(K\pi)$,

SV: $[f_0(500), f_0(980), f_0(1370)]K^*$,

SS:$[f_0(500), f_0(980), f_0(1370)](K\pi)$

◇ The **invariant mass dependence** disentangles different **resonances** with the same spin

◇ The **angular dependence** separates contributions in **partial waves**



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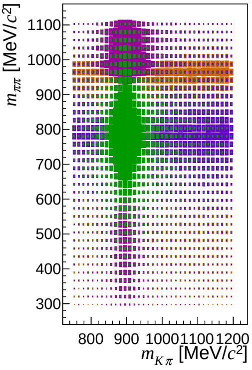
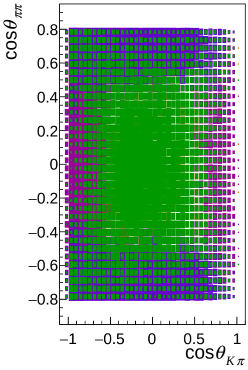
*toy generated without interferences, only contains $\sum_i |A_i|^2$
(and $(a+b)^2 \neq a^2 + b^2$)





Signal selection: four-body mass fit

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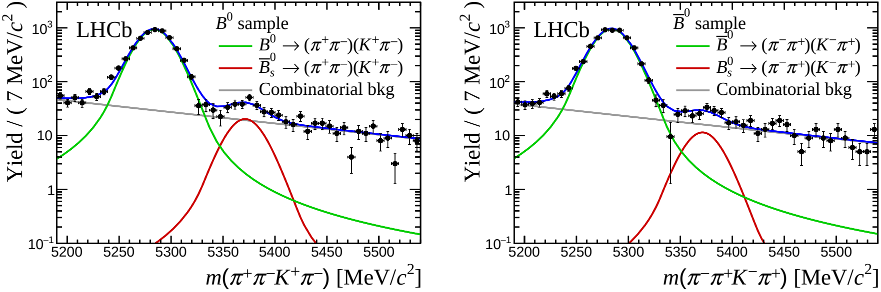
$\rightarrow$ **Used to obtain signal weights**, which allows the amplitude fit to account for the signal PDF only.

The fit is performed simultaneously in 8 categories, arising from B-meson flavour, kinematics and selection requirements (trigger).



Modelling

- $B^0_{(s)}$ peaks: Hypatia function
- Combinatorial: exponential function

$\sim$ **11k signal events in $B^0 + \overline{B}^0$**

Amplitude fit (I): projections on the helicity angles

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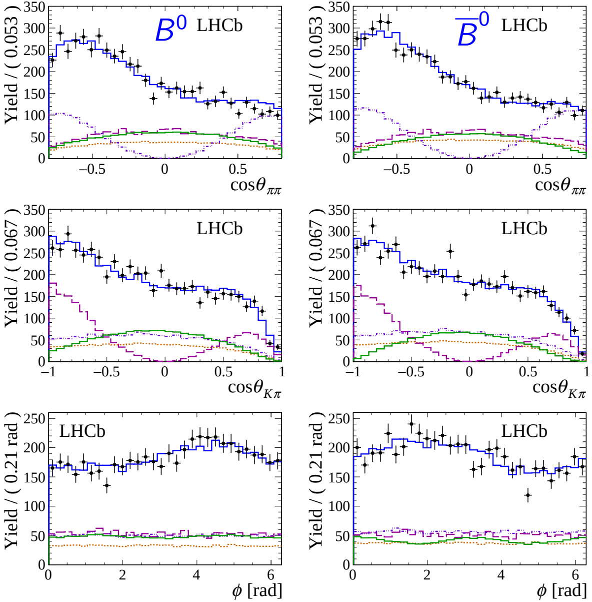

$\leftarrow \cos\theta_{\pi\pi}$

$\leftarrow \cos\theta_{K\pi}$

$\leftarrow \phi$

VV: $\rho K^*, \omega K^*$

VS: $\rho(K\pi), \omega(K\pi)$

SV: $[f_0(500), f_0(980), f_0(1370)]K^*$

SS:$[f_0(500), f_0(980), f_0(1370)](K\pi)$

Amplitude fit(II): projections on the invariant masses

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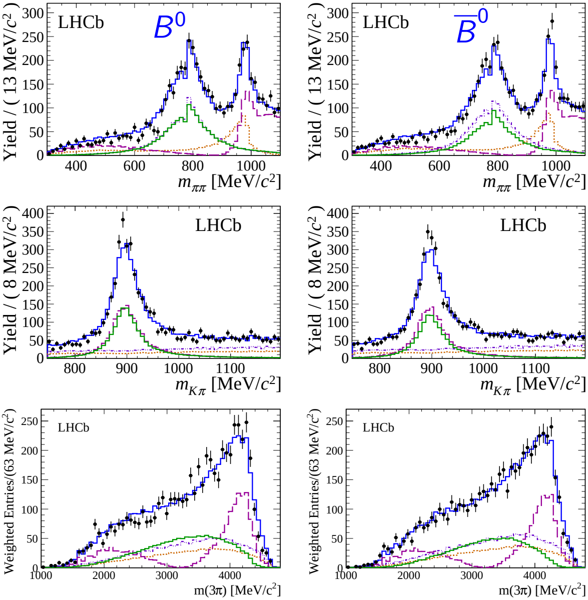

← $m(\pi\pi)$

← $m(K\pi)$

← $m(\pi\pi\pi)$ (Not fitted)

VV: $\rho K^*, \omega K^*$

VS: $\rho(K\pi), \omega(K\pi)$

SV: $[f_0(500), f_0(980), f_0(1370)]K^*$

SS: $[f_0(500), f_0(980), f_0(1370)](K\pi)$

Full set of numerical results

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Parameter	CP average, $\tilde{f}$	CP asymmetry, $\mathcal{A}$
$\lvert A^{0}_{\rho K^*}\rvert^2$	$0.32 \pm 0.04 \pm 0.07$	$-0.75 \pm 0.07 \pm 0.17$
$\lvert A^{\parallel}_{\rho K^*}\rvert^2$	$0.70 \pm 0.04 \pm 0.08$	$-0.049 \pm 0.053 \pm 0.019$
$\lvert A^{\perp}_{\rho K^*}\rvert^2$	$0.67 \pm 0.04 \pm 0.07$	$-0.187 \pm 0.051 \pm 0.026$
$\lvert A^{0}_{\omega K^*}\rvert^2$	$0.019 \pm 0.010 \pm 0.012$	$-0.6 \pm 0.4 \pm 0.4$
$\lvert A^{\parallel}_{\omega K^*}\rvert^2$	$0.0050 \pm 0.0029 \pm 0.0031$	$-0.30 \pm 0.54 \pm 0.28$
$\lvert A^{\perp}_{\omega K^*}\rvert^2$	$0.0020 \pm 0.0019 \pm 0.0015$	$-0.2 \pm 0.9 \pm 0.4$
$\lvert A_{\omega(K\pi)}\rvert^2$	$0.026 \pm 0.011 \pm 0.025$	$-0.47 \pm 0.33 \pm 0.45$
$\lvert A_{f_0(500)K^*}\rvert^2$	$0.53 \pm 0.05 \pm 0.10$	$-0.06 \pm 0.09 \pm 0.04$
$\lvert A_{f_0(980)K^*}\rvert^2$	$2.42 \pm 0.13 \pm 0.25$	$-0.022 \pm 0.052 \pm 0.023$
$\lvert A_{f_0(1370)K^*}\rvert^2$	$1.29 \pm 0.09 \pm 0.20$	$-0.09 \pm 0.07 \pm 0.04$
$\lvert A_{f_0(500)(K\pi)}\rvert^2$	$0.174 \pm 0.021 \pm 0.039$	$0.30 \pm 0.12 \pm 0.09$
$\lvert A_{f_0(980)(K\pi)}\rvert^2$	$1.18 \pm 0.08 \pm 0.07$	$-0.083 \pm 0.066 \pm 0.023$
$\lvert A_{f_0(1370)(K\pi)}\rvert^2$	$0.139 \pm 0.028 \pm 0.039$	$-0.48 \pm 0.17 \pm 0.15$
$f^{0}_{\rho K^*}$	$0.164 \pm 0.015 \pm 0.022$	$-0.62 \pm 0.09 \pm 0.09$
$f^{\parallel}_{\rho K^*}$	$0.435 \pm 0.016 \pm 0.042$	$0.188 \pm 0.037 \pm 0.022$
$f^{\perp}_{\rho K^*}$	$0.401 \pm 0.016 \pm 0.037$	$0.050 \pm 0.039 \pm 0.015$
$f^{0}_{\omega K^*}$	$0.68 \pm 0.17 \pm 0.16$	$-0.13 \pm 0.27 \pm 0.13$
$f^{\parallel}_{\omega K^*}$	$0.22 \pm 0.14 \pm 0.15$	$0.26 \pm 0.55 \pm 0.22$
$f^{\perp}_{\omega K^*}$	$0.10 \pm 0.09 \pm 0.09$	$0.3 \pm 0.8 \pm 0.4$

Parameter	CP average, $\frac{1}{2}(\delta_{\overline{B}} + \delta_B)$ [rad]	CP difference, $\frac{1}{2}(\delta_{\overline{B}} - \delta_B)$ [rad]
$\delta^{0}_{\rho K^*}$	$1.57 \pm 0.08 \pm 0.18$	$0.12 \pm 0.08 \pm 0.04$
$\delta^{\parallel}_{\rho K^*}$	$0.795 \pm 0.030 \pm 0.068$	$0.014 \pm 0.030 \pm 0.026$
$\delta^{\perp}_{\rho K^*}$	$-2.365 \pm 0.032 \pm 0.054$	$0.000 \pm 0.032 \pm 0.013$
$\delta^{0}_{\omega K^*}$	$-0.86 \pm 0.29 \pm 0.71$	$0.03 \pm 0.29 \pm 0.16$
$\delta^{\parallel}_{\omega K^*}$	$-1.83 \pm 0.29 \pm 0.32$	$0.59 \pm 0.29 \pm 0.07$
$\delta^{\perp}_{\omega K^*}$	$1.6 \pm 0.4 \pm 0.6$	$-0.25 \pm 0.43 \pm 0.16$
$\delta_{\omega(K\pi)}$	$-2.32 \pm 0.22 \pm 0.24$	$-0.20 \pm 0.22 \pm 0.14$
$\delta_{f_0(500)K^*}$	$-2.28 \pm 0.06 \pm 0.22$	$-0.00 \pm 0.06 \pm 0.05$
$\delta_{f_0(980)K^*}$	$0.39 \pm 0.04 \pm 0.07$	$0.018 \pm 0.038 \pm 0.022$
$\delta_{f_0(1370)K^*}$	$-2.76 \pm 0.05 \pm 0.09$	$0.076 \pm 0.051 \pm 0.025$
$\delta_{f_0(500)(K\pi)}$	$-2.80 \pm 0.09 \pm 0.21$	$-0.206 \pm 0.088 \pm 0.034$
$\delta_{f_0(980)(K\pi)}$	$-2.982 \pm 0.032 \pm 0.057$	$-0.027 \pm 0.032 \pm 0.013$
$\delta_{f_0(1370)(K\pi)}$	$1.76 \pm 0.10 \pm 0.11$	$-0.16 \pm 0.10 \pm 0.04$
$\delta^{\parallel-\perp}_{\rho K^*}$	$3.160 \pm 0.035 \pm 0.044$	$0.014 \pm 0.035 \pm 0.026$
$\delta^{\parallel-0}_{\rho K^*}$	$-0.77 \pm 0.09 \pm 0.06$	$-0.109 \pm 0.085 \pm 0.034$
$\delta^{\perp-0}_{\rho K^*}$	$-3.93 \pm 0.09 \pm 0.07$	$-0.123 \pm 0.085 \pm 0.035$
$\delta^{\parallel-\perp}_{\omega K^*}$	$-3.4 \pm 0.5 \pm 0.7$	$0.84 \pm 0.52 \pm 0.16$
$\delta^{\parallel-0}_{\omega K^*}$	$-1.0 \pm 0.4 \pm 0.6$	$0.57 \pm 0.41 \pm 0.17$
$\delta^{\perp-0}_{\omega K^*}$	$2.4 \pm 0.5 \pm 0.8$	$-0.28 \pm 0.51 \pm 0.24$

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$\lvert A^0_{\omega K}$…	[illegible]	[illegible]
$\lvert A^{\parallel}_{\omega K}$…	[illegible]	[illegible]
$\lvert A^{\perp}_{\omega K}$…	[illegible]	[illegible]
$\lvert A_{\omega(K}$…	[illegible]	[illegible]
$\lvert A_{f_0(500}$…	[illegible]	[illegible]
$\lvert A_{f_0(980}$…	[illegible]	[illegible]
$\lvert A_{f_0(137}$…	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible] ± 0.14
[illegible]	[illegible]	[illegible] ± 0.05
[illegible]	[illegible]	[illegible] ± 0.022
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Amplitudes and phase differences
measured for 13 waves ($\mathcal{CP}$-av. and asym.)

- ✓ **First measurements for several modes**
- ✓ **First measurements of weak phases per channel**
- ✓ **First observation of CPV in angular distributions of *VV* decays**

Detailed systematical uncertainties for the VV

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- The $B^0 \to a_1(1260)^- K^+$, being sensitive to polarisations too, **dominates the systematics for the *VV* parameters**. *S*-waves are mostly affected by the parameters used in the mass propagators and the experimental resolution.

	Systematic uncertainty	$f^0_{\rho K^*}$	$f^{\parallel}_{\rho K^*}$	$f^{\perp}_{\rho K^*}$	$\delta^{\parallel-\perp}_{\rho K^*}$	$\delta^{\parallel-0}_{\rho K^*}$	$\delta^{\perp-0}_{\rho K^*}$
CP averages	Centrifugal barrier factors	0.001	0.001	0.002	0.001	–	–
	Hypatia parameters	0.001	0.001	0.001	0.001	–	–
	$B^0_s \to K^{*0}\overline{K}^{*0}$ bkg.	0.005	0.003	0.005	0.018	0.02	0.02
	Simulation sample size	0.004	0.004	0.004	0.009	0.02	0.02
	Data-Simulation corrections	–	–	–	0.001	–	–
CP asym.	Centrifugal barrier factors	–	0.001	0.002	0.004	0.007	0.004
	Hypatia parameters	–	0.003	0.002	0.001	0.002	0.002
	$B^0_s \to K^{*0}\overline{K}^{*0}$ bkg.	0.03	0.007	0.011	0.024	0.020	0.026
	Simulation sample size	0.02	0.010	0.009	0.011	0.027	0.023
	Data-Simulation corrections	–	0.001	0.001	–	0.002	0.002
Common (B^0, $\overline{B}^0$)	Mass propagators parameters	0.011	0.005	0.006	0.004	0.028	0.024
	Masses and angles resolution	0.010	0.016	0.018	0.031	0.029	0.040
	Fit method	0.003	0.001	0.002	0.003	0.005	0.004
	$a_1(1260)$ pollution	0.015	0.040	0.031	0.024	0.035	0.032
	Symmetrised $(\pi\pi)$ PDF	0.004	–	0.004	0.005	0.001	0.001

Dominant and second dominant systematic uncertainties.

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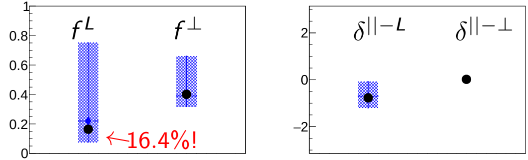
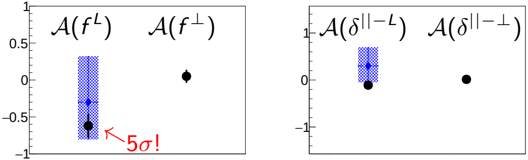
Remarks

- $B^0 \to \rho^0(K^+\pi^-)$ amplitude fixed (normalisation)
- Measurements of the relative amplitudes and phases for the remaining 13 waves

CP-averages



CP-asymmetries



Fit results (stats. and syst. uncertainties included)

Theoretical predictions (QCDF) with uncertainties

Nucl.Phys. B774 (2007) 64-101

VV dominated angular distributions

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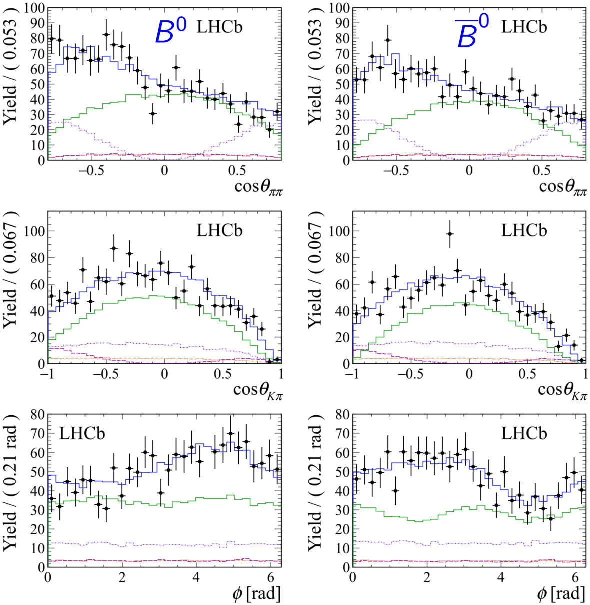


CP-violating effects can be seen in:

Different shapes of the inverted green parabola for the *VV*

Different oscillation in the *VV*

Summary

- **Amplitude analyses**
  - Give access to large sets of observables probing structures of potential new contributions
  - Exp.: high technicality, require careful treatment of correlations and very good understanding of the detector effects
  - Th.: calculations still affected by very large uncertainties
- **Analysis of $B^0 \to (\pi^+\pi^-)(K^+\pi^-)$ decays**
  - New results from the **$\mathcal{CP}$ averages and asymmetries** of the **polarisation fractions** together with their **phase differences**: first evidence of CPV in differential distributions of VV decays!
  - Important **input to the theory community**: tests reliability of QCDF vs pQCD hypotheses (polarisation puzzle) and relevance of the EW penguin diagrams ($B \to K\pi$ puzzle)
  - This work hints towards large EWP influence and is in agreement with the expectation: $f_L(\rho^0\overline{K}^{*0}) < f_L(\rho^-\overline{K}^{*0}) < f_L(\rho^0 K^{*-})$

Summary

- **Amplitude analyses**
  - Give access to large sets of observables probing structures of potential new contributions
  - Exp.: high technicality, require careful treatment of correlations and very good understanding of the detector effects
  - Th.: calculations still affected by very large uncertainties

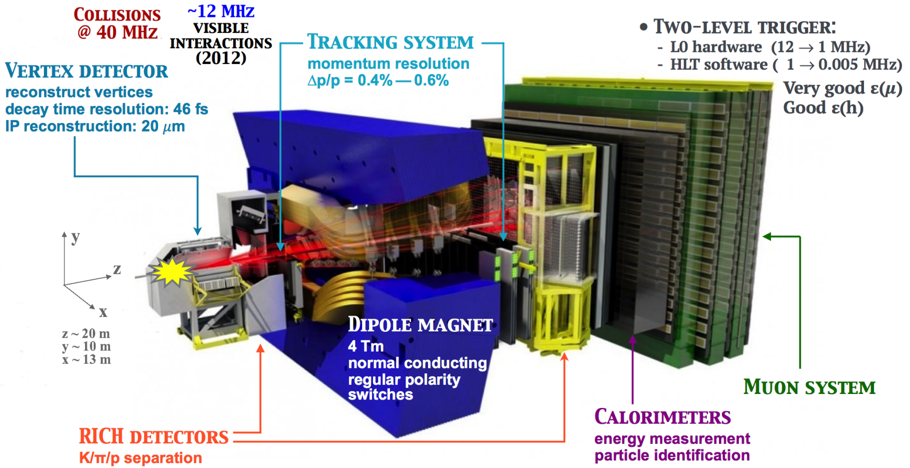
- **Analysis of $B^0 \to (\pi^+\pi^-)(K^+\pi^-)$ decays**
  - New results from the $\mathcal{CP}$ **averages and asymmetries** of the **polarisation fractions** together with their **phase differences**: first evidence of CPV in differential distributions of VV decays!
  - Important **input to the theory community**: tests reliability of QCDF vs pQCD hypotheses (polarisation puzzle) and relevance of the EW penguin diagrams ($B \to K\pi$ puzzle)
  - This work hints towards large EWP influence and is in agreement with the expectation: $f_L(\rho^0\overline{K}^{*0}) < f_L(\rho^-\overline{K}^{*0}) < f_L(\rho^0 K^{*-})$

Thank you for your attention!

...comments, questions

Backup slides

The LHCb detector



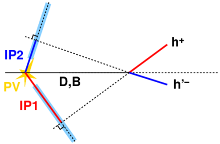
Selection summary

Event **selection** is performed in three steps:

1.- Stripping + loose preselection cuts

Geometry of B decays is preselected using soft cuts on the p_T, IP and a good track quality is required.

Soft PID cuts allow to **reconstruct ρ^0 and $K^*(892)^0$ candidates.**

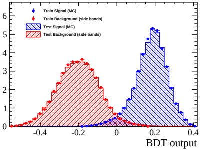


2.- Multivariate analysis + PID

A **BDT** is used to reduce the **combinatorial background**.

Charm decays are rejected by eliminating their phase space.

Tighter PID cuts on $\pi^{\pm}$ and $K^{\pm}$ are applied and $\mu^{\pm}$ are vetoed.



3.- s-Weights&Injection of simulated events

Obtain a **background substracted** sample via **s-Weights**$\rightarrow$ $M(K\pi\pi\pi)$ spectrum.

The **topologically similar** $B_s^0 \rightarrow K^*(892)^0 \bar{K}^*(892)^0$ decay is cancelled by **injecting simulated** $(K^+\pi^-)(K^-\pi^+)$ events.

Build a PDF describing all waves

The $B^0 \to (\pi\pi)(K\pi)$ **amplitude model accounts for 10 decay channels (14 contributions):**

Mass propagators

- ρ^0: Gounaris-Sakurai
- ω, $K^*(892)^0$: relativistic spin-1 Breit-Wigners
- $f_0(500)$: spin-0 Breit-Wigner
- $f_0(980)$: Flattè
- $f_0(1370)$: spin-0 Breit-Wigner
- $(K\pi)_0$: LASS with a Form Factor

i	Type	A_i	$g_i(\theta_1, \theta_2, \phi)$	$M_i(m_1, m_2)$
1		$A^0_{\rho K*}$	$cos\theta_1 cos\theta_2$	$M_\rho(m_1)M_{K*}(m_2)$
2	$V_1 V$	$A^{\|\|}_{\rho K*}$	$\frac{1}{\sqrt{2}} sin\theta_1 sin\theta_2 cos\phi$	$M_\rho(m_1)M_{K*}(m_2)$
3		$A^{\perp}_{\rho K*}$	$\frac{i}{\sqrt{2}} sin\theta_1 sin\theta_2 sin\phi$	$M_\rho(m_1)M_{K*}(m_2)$
4		$A^0_{\omega K*}$	$cos\theta_1 cos\theta_2$	$M_\omega(m_1)M_{K*}(m_2)$
5	$V_2 V$	$A^{\|\|}_{\omega K*}$	$\frac{1}{\sqrt{2}} sin\theta_1 sin\theta_2 cos\phi$	$M_\omega(m_1)M_{K*}(m_2)$
6		$A^{\perp}_{\omega K*}$	$\frac{i}{\sqrt{2}} sin\theta_1 sin\theta_2 sin\phi$	$M_\omega(m_1)M_{K*}(m_2)$
7	$V_1 S$	$A^0_{\rho(K\pi)}$	$\frac{1}{\sqrt{3}} cos\theta_1$	$M_\rho(m_1)M_{(K\pi)}(m_2)$
8	$V_2 S$	$A^0_{\omega(K\pi)}$	$\frac{1}{\sqrt{3}} cos\theta_1$	$M_\omega(m_1)M_{(K\pi)}(m_2)$
9	$S_1 V$	$A^0_{f_0(500)K*}$	$\frac{1}{\sqrt{3}} cos\theta_2$	$M_{f_0(500)}(m_1)M_{K*}(m_2)$
10	$S_2 V$	$A^0_{f_0(980)K*}$	$\frac{1}{\sqrt{3}} cos\theta_2$	$M_{f_0(980)}(m_1)M_{K*}(m_2)$
11	$S_3 V$	$A^0_{f_0(1370)K*}$	$\frac{1}{\sqrt{3}} cos\theta_2$	$M_{f_0(1370)}(m_1)M_{K*}(m_2)$
12	$S_1 S$	$A^0_{f_0(500)(K\pi)}$	$\frac{1}{3}$	$M_{f_0(500)}(m_1)M_{(K\pi)}(m_2)$
13	$S_2 S$	$A^0_{f_0(980)(K\pi)}$	$\frac{1}{3}$	$M_{f_0(980)}(m_1)M_{(K\pi)}(m_2)$
14	$S_3 S$	$A^0_{f_0(1370)(K\pi)}$	$\frac{1}{3}$	$M_{f_0(1370)}(m_1)M_{(K\pi)}(m_2)$

Account for the $\bar{B}$ decay: $A_i \to \eta_i \bar{A}_i$; with η_i the parity of each amplitude:

$$\eta_{\mathcal{A}_i} = 1 \text{ except for } \eta_{\mathcal{A}_\perp} = -1$$

Sources of systematic uncertainties

$$\texttt{PDF}\ \text{term} \sim \frac{\mathcal{A}_i \cdot g_i(\theta_1, \theta_2, \phi) \cdot \mathcal{M}_i(m_1, m_2) \times (...)^*_j}{\sum_{i,j} \mathcal{A}_i \mathcal{A}^*_j nw_{ij}}$$

Normalisation: $\sum_{i,j} \mathcal{A}_i \mathcal{A}^*_j nw_{ij}$

- $\mathcal{A}_i \mathcal{A}^*_j \rightarrow$ polarisation affects acceptance.
- nw_{ij} obtained from MC sample, limited statistics
- nw_{ij}: data-simulation corrections (PID, p_T^B and Ntracks)

Mass propagators: $\mathcal{M}(m_1, m_2)$

- Vary the parameters in the propagators: $BW(m, L, m_0, \Gamma_0, r_0) \rightarrow x_0 \rightarrow Gauss(x_0, \sigma_{x_0})$

Pull distributions: to estimate possible model-induced biases

Neglected contributions in the model:

- $\mathcal{A}_i \mathcal{A}^*_j$: identical π exchange, $B^0 \rightarrow (\pi^+\pi^-)(K^+\pi^-)$, and $B^0 \rightarrow a_1(1240)^- K^+$ pollution
- $\theta_1, \theta_2, \phi, m_1, m_2$: experimental resolution and orbital angular momentum barriers

Data sample:

- Negative weights cancelling the $B^0_s \rightarrow K^*(892)^0 \bar{K}^*(892)^0$ contribution (yield and shapes)
- Signal weights from the *s*Fit

Fitting frameworks

The $B^0 \to (\pi\pi)(K\pi)$ PDF model was implemented in three different frameworks:

- **Minuit + CPU: RooFit based**
  - Fully implemented in ROOT, was the first option for historical reasons
  - Slow: fits toy-MC in 15min
  - Has trouble converging with many (>20) free parameters in several dimensions with weighted data (spoil $\log \mathcal{L}$ smoothness)
  - ✓ **toy-MC generation**

- **Minuit + GPU: Ipanema based**

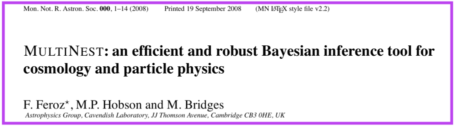
  →Ipanema@arXiv
  - Same methods as above, but implemented in Python + pyCUDA
  - Very fast: fits toy-MC in 18s
  - Still relies on Minuit → same issues with weighted data as above
  - ✓ **toy-MC based systematics (fits)**

- **MultiNest + GPU: Ipanema based**

  →Multinest project
  - Implemented in Python + pyMultinest + pyCUDA
  - Uses nested sampling → performs good with weighted samples
  - Scans the whole parameter space → very slow (fits toy MC in 3h)
  - ✓ **nominal fit + data based systematics**

A glimpse into MultiNest

Uses **clustered nested sampling**: a **Monte Carlo** method targetted at the efficient calculation of the probability for a set of parameter values given a data sample


Mon. Not. R. Astron. Soc. **000**, 1–14 (2008) Printed 19 September 2008 (MN LaTeX style file v2.2)

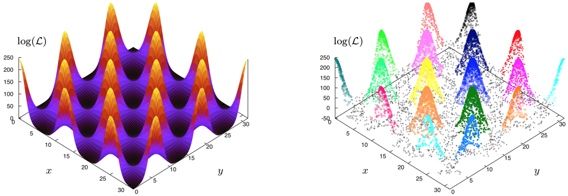
MultiNest: an efficient and robust Bayesian inference tool for cosmology and particle physics

F. Feroz*, M.P. Hobson and M. Bridges
Astrophysics Group, Cavendish Laboratory, JJ Thomson Avenue, Cambridge CB3 0HE, UK

Highlighted characteristics:

- Defines "high dimensionality" as $>$ 50D :-)
- **Nested sampling**: new algorithm type ($\sim$ 2004) performing better (less evaluations needed) than MC-Markov-Chain reference
- **Clustered** nested sampling: very good finding several modes in the posterior distributions (induced by non smoothness of the $\log \mathcal{L}$ in our case)
- Very slow **but**: parameter estimation, uncertainties, $\log \mathcal{L}$ profiles, iso-$\log \mathcal{L}$ contours, correlations, ... **all produced at once**

Example of MultiNest performance finding peaks in a multimodal $\log \mathcal{L}$ distribution. Toy (left) vs fit (right).



PDF inside LHCb: acceptance

Goal: perform a maximum likelihood fit of the PDF model → compute the sum

$$\frac{1}{N}\sum_{e}^{N}\log\left(\frac{|\mathrm{PDF}_e|^2}{\int_{\mathcal{D}}|\mathrm{PDF}|^2}\right)$$

- PDF_e is the PDF evaluated for event e and N, the total number of events
- $\mathcal{D}$: 5D integration domain → shaped by the LHCb detector acceptance and the selection requirements ⇒ not easy to parametrise as $f(\theta_1, \theta_2, \phi, m_1, m_2)$.

Relevance of $\mathcal{D}$:

- Defines the **normalisation of the PDF**
- Lack of analytical expression for $\mathcal{D}$: the 5D integral has to be done **numerically**.

In general, it will be needed to:

→ Rely on simulated samples (MC) to characterise $\mathcal{D}$

→ Analyse different domains separately

→ Use an approximation to obtain an analytical expression allowing to generate toys

→ Control the normalisation of the PDF in the fit