Amplitude analysis of

\[ B^0 \rightarrow (\pi^+ \pi^-)(K^+ \pi^-) \] decays

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École Polytechnique Fédérale de Lausanne

Joint Annual Meeting of the
Swiss Physical Society
and the Austrian Physical Society

26–30 August, Universität Zürich
Phenomenology of the $B^0 \rightarrow \rho^0 K^*(892)^0$ decay

Charmless $B^0$ meson decay reconstructed as $B^0 \rightarrow \rho^0(\pi^+\pi^-)K^*(892)^0(K^+\pi^-)$

- **Proceeds via:**
  - A doubly Cabibbo suppressed tree
  - A gluonic $b \rightarrow s$ penguin (GP)
  - A electro-weak $b \rightarrow s$ penguin (EWP)
  - Tree and GP diagrams have similar amplitudes $\rightarrow$ maximises interferences

- **Self-tagged** decay: \[
\begin{align*}
B^0 & \rightarrow (\pi^+\pi^-)(K^+\pi^-) \\
\bar{B}^0 & \rightarrow (\pi^-\pi^+)(K^-\pi^+)
\end{align*}
\]

- **Vector** resonances $\rightarrow$ **additional** CP-violating **observables** and sensitivity to QCD dynamic effects

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M. Vieites Díaz  (EPFL)  
Annual SPS meeting, 28th August 2019
Observables in an amplitude analysis of $B \rightarrow VV$ decays

$B \rightarrow (p_a p_b)_1 (p_c p_d)_2$ decays

Can be **fully described** in terms of:
- Three helicity angles: $\theta_1, \theta_2, \phi$
- Two invariant masses: $m_1, m_2$

A $B \rightarrow VV$ proceeds via three amplitudes $\rightarrow$ three spin configurations:
P-odd $S_{VV} = 1$ and P-even $S_{VV} = 0, 2$, rotated into the transversity basis $\lambda = L, ||, \perp$.

EWP diagram contributes differently to each amplitude: rich pattern of interferences.
Observables in an amplitude analysis of $B \rightarrow VV$ decays

$B \rightarrow (p_a p_b) (p_c p_d)$ decays

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EWP diagram contributes differently to each amplitude: rich pattern of interferences.

**Observables**: number of events per amplitude (polarisation fractions), $f^\lambda$, and their phase differences:

$$f^\lambda \equiv \frac{|A^\lambda|^2}{|A^L|^2 + |A^||^2 + |A^\perp|^2}$$

$$\delta \lambda_i - \lambda_j \equiv (\delta \lambda_i - \delta \lambda_j)$$

→ Sensitivity to CPV by comparing $B$ and $\bar{B}$ parameters
Available results:

- All available measurements are CP-averaged
- Precise predictions unavailable, general dynamics not fully understood → polarisation puzzle
- Large $f_L$ values confirmed in $b \to u$ tree dominated decays
- Penguin dominated modes span wider ranges
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The landscape of longitudinal polarisations

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Two new measurements, first observation of CPV in $f_L$ for VV decays (JHEP 05 (2019) 026) (LHCb Run I dataset)
In general, a $VV$ final state cannot be uniquely selected and other possible decay channels must be accounted for:

Generalise to $N$ amplitudes (isobar model):

$$d^5\Gamma \propto \Phi_4 \left| \sum_{i=1}^{N} A_i \cdot g_i(\cos \theta_1, \cos \theta_2, \phi) \cdot M_i(m_1, m_2) \right|^2$$

More observables: +1 amplitude, +1 phase difference per new contribution

An amplitude analysis disentangles the final state!

$A_i \rightarrow$ physical parameters

$g_i(\theta_1, \theta_2, \phi) \rightarrow$ spherical harm.

$M_i(m_1, m_2) \rightarrow$ mass prop.
Partial waves in the $B^0 \rightarrow \rho^0 K^*(892)^0$ channel

Remarks:
- Analyse a large phase-space: testing many variations of strong phase differences.
- Sensitive to localised CP-violating effects!

- The invariant mass dependence disentangles different resonances with the same spin
- The angular dependence separates contributions in partial waves

Partial waves:
- $VV$: $\rho K^*, \omega K^*$, $VS$: $\rho(K\pi), \omega(K\pi)$
- $SV$: $[f_0(500), f_0(980), f_0(1370)]K^*$
- $SS$: $[f_0(500), f_0(980), f_0(1370)](K\pi)$
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SS: $[f_0(500), f_0(980), f_0(1370)](K\pi)$

*toy generated without interferences, only contains $\sum_i |A_i|^2$
(and $(a + b)^2 \neq a^2 + b^2$)
Signal selection: four-body mass fit

→ **Used to obtain signal weights**, which allows the amplitude fit to account for the signal PDF only.

**The fit is performed simultaneously in 8 categories**, arising from $B$-meson flavour, kinematics and selection requirements (trigger).

![Graphs showing mass distributions with signal and background components](image)

**Modelling**
- $B^0_{(s)}$ peaks: Hypatia function
- Combinatorial: exponential function

$\sim 11k$ signal events in $B^0 + \bar{B}^0$
Amplitude fit (1): projections on the helicity angles

\[ \cos \theta_{\pi\pi} \]

\[ \cos \theta_{K\pi} \]

\[ \phi \]

**VV:** \( \rho K^*, \omega K^* \)

**VS:** \( \rho(K\pi), \omega(K\pi) \)

**SV:** \( [f_0(500), f_0(980), f_0(1370)] K^* \)

**SS:** \( [f_0(500), f_0(980), f_0(1370)](K\pi) \)
Amplitude fit(II): projections on the invariant masses

:\( B^0 \) \hspace{1cm} \bar{B}^0

\( m(\pi\pi) \)

\( m(K\pi) \)

\( m(\pi\pi\pi) \) (Not fitted)

\( VV: \rho K^*, \omega K^* \)

\( VS: \rho(K\pi), \omega(K\pi) \)

\( SV: [f_0(500), f_0(980), f_0(1370)]K^* \)

\( SS: [f_0(500), f_0(980), f_0(1370)](K\pi) \)
### Full set of numerical results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$CP$ average, $\tilde{f}$</th>
<th>$CP$ asymmetry, $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>A^0_{pK^*}</td>
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<td>$</td>
<td>A^0_{(500)K^*}</td>
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<td>A^0_{(500)K^*}</td>
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</tr>
<tr>
<td>$</td>
<td>A^0_{(500)K^*}</td>
<td>^2$</td>
</tr>
<tr>
<td>$f_{pK^*}$</td>
<td>0.164 ± 0.015 ± 0.022</td>
<td>-0.62 ± 0.09 ± 0.09</td>
</tr>
<tr>
<td>$f_{pK^*}$</td>
<td>0.435 ± 0.016 ± 0.042</td>
<td>0.188 ± 0.037 ± 0.022</td>
</tr>
<tr>
<td>$f_{pK^*}$</td>
<td>0.401 ± 0.016 ± 0.037</td>
<td>0.050 ± 0.039 ± 0.015</td>
</tr>
<tr>
<td>$f_{pK^*}$</td>
<td>0.68 ± 0.17 ± 0.16</td>
<td>-0.13 ± 0.27 ± 0.13</td>
</tr>
<tr>
<td>$f_{pK^*}$</td>
<td>0.22 ± 0.14 ± 0.15</td>
<td>0.26 ± 0.55 ± 0.22</td>
</tr>
<tr>
<td>$f_{pK^*}$</td>
<td>0.10 ± 0.09 ± 0.09</td>
<td>0.3 ± 0.8 ± 0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$CP$ average, $\frac{1}{2}(\delta_B + \delta_B)$ [rad]</th>
<th>$CP$ difference, $\frac{1}{2}(\delta_B - \delta_B)$ [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^0_{pK^*}$</td>
<td>1.57 ± 0.08 ± 0.18</td>
<td>0.12 ± 0.08 ± 0.04</td>
</tr>
<tr>
<td>$\delta^0_{pK^*}$</td>
<td>0.795 ± 0.030 ± 0.068</td>
<td>0.014 ± 0.030 ± 0.026</td>
</tr>
<tr>
<td>$\delta^0_{pK^*}$</td>
<td>-2.365 ± 0.032 ± 0.054</td>
<td>0.000 ± 0.032 ± 0.013</td>
</tr>
<tr>
<td>$\delta^0_{pK^*}$</td>
<td>-0.86 ± 0.29 ± 0.71</td>
<td>0.03 ± 0.29 ± 0.16</td>
</tr>
<tr>
<td>$\delta^0_{pK^*}$</td>
<td>-1.83 ± 0.29 ± 0.32</td>
<td>0.59 ± 0.29 ± 0.07</td>
</tr>
<tr>
<td>$\delta^0_{pK^*}$</td>
<td>1.6 ± 0.4 ± 0.6</td>
<td>-0.25 ± 0.43 ± 0.16</td>
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<tr>
<td>$\delta^0_{pK^*}$</td>
<td>-2.32 ± 0.22 ± 0.24</td>
<td>-0.20 ± 0.22 ± 0.14</td>
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<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-2.28 ± 0.06 ± 0.22</td>
<td>-0.00 ± 0.06 ± 0.05</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>0.39 ± 0.04 ± 0.07</td>
<td>0.018 ± 0.038 ± 0.022</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-2.76 ± 0.05 ± 0.09</td>
<td>0.076 ± 0.051 ± 0.025</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-2.80 ± 0.09 ± 0.21</td>
<td>-0.206 ± 0.088 ± 0.034</td>
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<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-2.982 ± 0.032 ± 0.057</td>
<td>-0.027 ± 0.032 ± 0.013</td>
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<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>1.76 ± 0.10 ± 0.11</td>
<td>-0.16 ± 0.10 ± 0.04</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>3.160 ± 0.035 ± 0.044</td>
<td>0.014 ± 0.035 ± 0.026</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-0.77 ± 0.09 ± 0.06</td>
<td>-0.109 ± 0.085 ± 0.034</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-3.93 ± 0.09 ± 0.07</td>
<td>-0.123 ± 0.085 ± 0.035</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-3.4 ± 0.5 ± 0.7</td>
<td>0.84 ± 0.52 ± 0.16</td>
</tr>
<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>-1.0 ± 0.4 ± 0.6</td>
<td>0.57 ± 0.41 ± 0.17</td>
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<tr>
<td>$\delta^0_{(500)K^*}$</td>
<td>2.4 ± 0.5 ± 0.8</td>
<td>-0.28 ± 0.51 ± 0.24</td>
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Amplitudes and phase differences measured for 13 waves (CP-av. and asym.)

✓ First measurements for several modes
✓ First measurements of weak phases per channel
✓ First observation of CPV in angular distributions of VV decays
Detailed systematic uncertainties for the VV

- The $B^0 \rightarrow a_1(1260)^- K^+$, being sensitive to polarisations too, dominates the systematics for the VV parameters. $S$-waves are mostly affected by the parameters used in the mass propagators and the experimental resolution.

| Systematic uncertainty                  | $f_{pK^*}^0$ | $f_{pK^*}^0$ | $f_{pK^*}^0$ | $\delta_{pK^*}^{||-\perp}$ | $\delta_{pK^*}^{||-0}$ | $\delta_{pK^*}^{\perp-0}$ |
|-----------------------------------------|---------------|---------------|---------------|-----------------------------|--------------------------|--------------------------|
| Centrifugal barrier factors             | 0.001         | 0.001         | 0.002         | 0.001                       | −                        | −                        |
| Hypatia parameters                      | 0.001         | 0.001         | 0.001         | 0.001                       | −                        | −                        |
| $B_s^0 \rightarrow K^{*0} \bar{K}^{*0}$ bkg. | 0.005         | 0.003         | 0.005         | 0.018                       | 0.02                     | 0.02                     |
| Simulation sample size                  | 0.004         | 0.004         | 0.004         | 0.009                       | 0.02                     | 0.02                     |
| Data-Simulation corrections             | −             | −             | −             | 0.001                       | −                        | −                        |

| CP averages                             |               |               |               |                             |                          |                          |
| Centrifugal barrier factors             | −             | 0.001         | 0.002         | 0.004                       | 0.007                    | 0.004                    |
| Hypatia parameters                      | −             | 0.003         | 0.002         | 0.001                       | 0.002                    | 0.002                    |
| $B_s^0 \rightarrow K^{*0} \bar{K}^{*0}$ bkg. | 0.03          | 0.007         | 0.011         | 0.024                       | 0.020                    | 0.026                    |
| Simulation sample size                  | 0.02          | 0.010         | 0.009         | 0.011                       | 0.027                    | 0.023                    |
| Data-Simulation corrections             | −             | 0.001         | 0.001         | −                          | 0.002                    | 0.002                    |

| CP asym.                                |               |               |               |                             |                          |                          |
| Centrifugal barrier factors             | −             | 0.001         | 0.002         | 0.004                       | 0.028                    | 0.024                    |
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| Data-Simulation corrections             | −             | 0.001         | 0.001         | −                          | 0.002                    | 0.002                    |

| Mass propagators parameters             | 0.011         | 0.005         | 0.006         | 0.004                       | 0.028                    | 0.024                    |
| Masses and angles resolution            | 0.010         | 0.016         | 0.018         | 0.031                       | 0.029                    | 0.040                    |
| Fit method                              | 0.003         | 0.001         | 0.002         | 0.003                       | 0.005                    | 0.004                    |
| $a_1(1260)$ pollution                   | 0.015         | 0.040         | 0.031         | 0.024                       | 0.035                    | 0.032                    |
| Symmetrised $(\pi \pi)$ PDF             | 0.004         | −             | 0.004         | 0.005                       | 0.001                    | 0.001                    |

Dominant and second dominant systematic uncertainties.
**Remarks**

- $B^0 \to \rho^0(K^+\pi^-)$ amplitude fixed (normalisation)
- Measurements of the relative amplitudes and phases for the remaining 13 waves

**CP-averages**

![CP-averages](chart.png)

![CP-averages](chart.png)

- Fit results (stats. and syst. uncertainties included)

**CP-asymmetries**

![CP-asymmetries](chart.png)

![CP-asymmetries](chart.png)

- Theoretical predictions (QCDF) with uncertainties

**VV dominated angular distributions**

CP-violating effects can be seen in:

- Different shapes of the inverted green parabola for the $V V$
- Different oscillation in the $V V$
Summary

- **Amplitude analyses**
  - Give access to large sets of observables probing structures of potential new contributions
  - Exp.: high technicality, require careful treatment of correlations and very good understanding of the detector effects
  - Th.: calculations still affected by very large uncertainties

- **Analysis of $B^0 \rightarrow (\pi^+\pi^-)(K^+\pi^-)$ decays**
  - New results from the $CP$ averages and asymmetries of the polarisation fractions together with their phase differences: first evidence of CPV in differential distributions of $VV$ decays!
  - Important input to the theory community: tests reliability of QCDF vs pQCD hypotheses (polarisation puzzle) and relevance of the EW penguin diagrams ($B \rightarrow K\pi$ puzzle)
  - This work hints towards large EWP influence and is in agreement with the expectation: $f_L(\rho^0K^{*0}) < f_L(\rho^-K^{*0}) < f_L(\rho^0K^{*-})$
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Thank you for your attention!

...comments, questions
Backup slides
The LHCb detector

**Collision**
- @ 40 MHz

**Visible Interactions**
- 12 MHz

**Vertex Detector**
- Reconstruct vertices
- Decay time resolution: 46 fs
- IP reconstruction: 20 μm

**Tracking System**
- Momentum resolution
  - Δp/p = 0.4% — 0.6%

**Dipole Magnet**
- 4 Tm
- Normal conducting regular polarity switches

**Calorimeters**
- Energy measurement
- Particle identification

**Muon System**

**Rich Detectors**
- K/π/p separation

**Two-level Trigger**
- L0 hardware (12 → 1 MHz)
- HLT software (1 → 0.005 MHz)

Very good ϵ(μ)
Good ϵ(h)
Selection summary

Event selection is performed in three steps:

1.- Stripping + loose preselection cuts

Geometry of B decays is preselected using soft cuts on the $p_T, IP$ and a good track quality is required. Soft PID cuts allow to reconstruct $\rho^0$ and $K^*(892)^0$ candidates.

2.- Multivariate analysis + PID

A BDT is used to reduce the combinatorial background. Charm decays are rejected by eliminating their phase space. Tighter PID cuts on $\pi^\pm$ and $K^\pm$ are applied and $\mu^\pm$ are vetoed.

3.- s-Weights&Injection of simulated events

Obtain a background substracted sample via $s$-Weights $\rightarrow M(K\pi\pi\pi)$ spectrum. The topologically similar $B_s^0 \rightarrow K^*(892)^0\bar{K}^*(892)^0$ decay is cancelled by injecting simulated $(K^+\pi^-)(K^-\pi^+)$ events.
The $B^0 \rightarrow (\pi \pi)(K \pi)$ amplitude model accounts for 10 decay channels (14 contributions):

<table>
<thead>
<tr>
<th>i</th>
<th>Type</th>
<th>$A_i$</th>
<th>$g_i(\theta_1, \theta_2, \phi)$</th>
<th>$M_i(m_1, m_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\rho^0$</td>
<td>$A^0_{\rho K^*}$</td>
<td>$\cos \theta_1 \cos \theta_2$</td>
<td>$M_{\rho}(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>2</td>
<td>$V_1 V$</td>
<td>$A^0_{\omega K^*}$</td>
<td>$\frac{1}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \cos \phi$</td>
<td>$M_{\rho}(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$A^0_{\omega K^*}$</td>
<td>$\frac{i}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \sin \phi$</td>
<td>$M_{\rho}(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>$A^0_{\omega K^*}$</td>
<td>$\cos \theta_1 \cos \theta_2$</td>
<td>$M_{\omega}(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>5</td>
<td>$V_2 V$</td>
<td>$A^0_{\omega K^*}$</td>
<td>$\frac{1}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \cos \phi$</td>
<td>$M_{\omega}(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>$A^0_{\omega K^*}$</td>
<td>$\frac{i}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \sin \phi$</td>
<td>$M_{\omega}(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>7</td>
<td>$V_1 S$</td>
<td>$A^0_{\rho(K\pi)}$</td>
<td>$\frac{1}{\sqrt{3}} \cos \theta_1$</td>
<td>$M_{\rho}(m_1)M_{(K\pi)}(m_2)$</td>
</tr>
<tr>
<td>8</td>
<td>$V_2 S$</td>
<td>$A^0_{\omega(K\pi)}$</td>
<td>$\frac{1}{\sqrt{3}} \cos \theta_1$</td>
<td>$M_{\omega}(m_1)M_{(K\pi)}(m_2)$</td>
</tr>
<tr>
<td>9</td>
<td>$S_1 V$</td>
<td>$A^0_{f_0(500)K^*}$</td>
<td>$\frac{1}{\sqrt{3}} \cos \theta_2$</td>
<td>$M_{f_0}(500)(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>10</td>
<td>$S_2 V$</td>
<td>$A^0_{f_0(980)K^*}$</td>
<td>$\frac{1}{\sqrt{3}} \cos \theta_2$</td>
<td>$M_{f_0}(980)(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>11</td>
<td>$S_3 V$</td>
<td>$A^0_{f_0(1370)K^*}$</td>
<td>$\frac{1}{\sqrt{3}} \cos \theta_2$</td>
<td>$M_{f_0}(1370)(m_1)M_{K^*}(m_2)$</td>
</tr>
<tr>
<td>12</td>
<td>$S_1 S$</td>
<td>$A^0_{f_0(500)(K\pi)}$</td>
<td>$\frac{1}{3}$</td>
<td>$M_{f_0}(500)(m_1)M_{(K\pi)}(m_2)$</td>
</tr>
<tr>
<td>13</td>
<td>$S_2 S$</td>
<td>$A^0_{f_0(980)(K\pi)}$</td>
<td>$\frac{1}{3}$</td>
<td>$M_{f_0}(980)(m_1)M_{(K\pi)}(m_2)$</td>
</tr>
<tr>
<td>14</td>
<td>$S_3 S$</td>
<td>$A^0_{f_0(1370)(K\pi)}$</td>
<td>$\frac{1}{3}$</td>
<td>$M_{f_0}(1370)(m_1)M_{(K\pi)}(m_2)$</td>
</tr>
</tbody>
</table>

Account for the $\bar{B}$ decay: $A_i \rightarrow \eta_i \bar{A}_i$; with $\eta_i$ the parity of each amplitude:

$$\eta_{A_i} = 1$$ except for $\eta_{A_\perp} = -1$
Sources of systematic uncertainties

\[ \text{PDF term} \sim \frac{A_i \cdot g_i(\theta_1, \theta_2, \phi) \cdot M_i(m_1, m_2) \times (...)^*}{\sum_{i,j} A_i A_j^{*} n_{wij}} \]

Normalisation: \( \sum_{i,j} A_i A_j^{*} n_{wij} \)
- \( A_i A_j^{*} \) → polarisation affects acceptance.
- \( n_{wij} \) obtained from MC sample, limited statistics
- \( n_{wij} \): data-simulation corrections (PID, \( p_B \) and \( N_{tracks} \))

Mass propagators: \( M(m_1, m_2) \)
- Vary the parameters in the propagators: \( BW(m, L, m_0, \Gamma_0, r_0) \rightarrow x_0 \rightarrow Gauss(x_0, \sigma_{x_0}) \)

Pull distributions: to estimate possible model-induced biases

Neglected contributions in the model:
- \( A_i A_j^{*} \): identical \( \pi \) exchange, \( B^0 \rightarrow (\pi^+ \pi^-)(K^+ \pi^-) \), and \( B^0 \rightarrow a_1(1240)^- K^+ \) pollution
- \( \theta_1, \theta_2, \phi, m_1, m_2 \): experimental resolution and orbital angular momentum barriers

Data sample:
- Negative weights cancelling the \( B_s^0 \rightarrow K^*(892)^0 \bar{K}^*(892)^0 \) contribution (yield and shapes)
- Signal weights from the sFit
Fitting frameworks

The $B^0 \rightarrow (\pi\pi)(K\pi)$ PDF model was implemented in three different frameworks:

- **Minuit + CPU: RooFit based**
  - Fully implemented in ROOT, was the first option for historical reasons
  - **Slow**: fits toy-MC in 15min
  - Has trouble converging with many (>20) free parameters in several dimensions with weighted data (spoil log $\mathcal{L}$ smoothness)
  - ✓ toy-MC generation

- **Minuit + GPU: Ipanema based**
  - Same methods as above, but implemented in Python + pyCUDA
  - **Very fast**: fits toy-MC in 18s
  - Still relies on Minuit → same issues with weighted data as above
  - ✓ toy-MC based systematics (fits)

- **MultiNest + GPU: Ipanema based**
  - Implemented in Python + pyMultinest + pyCUDA
  - Uses nested sampling → performs good with weighted samples
  - Scans the whole parameter space → **very slow** (fits toy MC in 3h)
  - ✓ nominal fit + data based systematics
A glimpse into MultiNest

Uses **clustered nested sampling**: a **Monte Carlo** method targeted at the efficient calculation of the probability for a set of parameter values given a data sample.

Highlighted characteristics:

- Defines "high dimensionality" as > 50D :-)
- **Nested sampling**: new algorithm type (~ 2004) performing better (less evaluations needed) than MC-Markov-Chain reference
- **Clustered** nested sampling: very good finding several modes in the posterior distributions (induced by non smoothness of the log $\mathcal{L}$ in our case)
- Very slow but: parameter estimation, uncertainties, log $\mathcal{L}$ profiles, iso-log $\mathcal{L}$ contours, correlations, ... all produced at once

Example of MultiNest performance finding peaks in a multimodal log $\mathcal{L}$ distribution. Toy (left) vs fit (right).
**PDF inside LHCb: acceptance**

**Goal:** perform a maximum likelihood fit of the PDF model → compute the sum

\[
\frac{1}{N} \sum _{e}^{N} \log \left( \frac{|PDF_e|^2}{\int _D |PDF|^2} \right)
\]

- PDF\(_e\) is the PDF evaluated for event \(e\) and \(N\), the total number of events
- \(D\): 5D integration domain → shaped by the LHCb detector acceptance and the selection requirements ⇒ not easy to parametrise as \(f(\theta_1, \theta_2, \phi, m_1, m_2)\).

Relevance of \(D\):

- Defines the **normalisation of the PDF**
- Lack of analytical expression for \(D\): the 5D integral has to be done **numerically**.

**In general, it will be needed to:**

→ Rely on simulated samples (MC) to characterise \(D\)
→ Analyse different domains separately
→ Use an approximation to obtain an analytical expression allowing to generate toys
→ Control the normalisation of the PDF in the fit