Exceptional geometry of supersymmetric AdS vacua and their consistent truncations

Emanuel Malek

Max-Planck-Institut für Gravitationsphysik
(Albert-Einstein-Institut).

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Based on EM: arXiv:1612.01692, arXiv:1612.01990, arXiv:1707.00714, EM, Samtleben, Vall Camell: arXiv:1808.05597, arXiv:1901.11039

Motivation from AdS/CFT

- AdS/CFT: window into strongly-coupled gauge theories
- $AdS_D \times M_{int}$ of 10-/11-dim SUGRA $\longleftrightarrow$ CFT_{D-1}

CFT_{D-1}	String theory on $AdS_D \times M_{int}$
(Exactly) marginal deformations	(Finite) moduli
RG flow	Domain-wall solutions
Thermal field theory	Asymptotically AdS black hole
Operators in short representations	Kaluza-Klein spectrum

- Geometry of M_{int} controls many properties of CFT.
- Practical applications: useful to study $AdS_D \times M_{int}$ using D-dimensional (gauged) SUGRA via "truncation".

Consistent truncations

- Which modes to keep on M_{int}?
- No scale separation ⇒ No effective action! [Kim, Romans, Nieuwenhuizen '85]

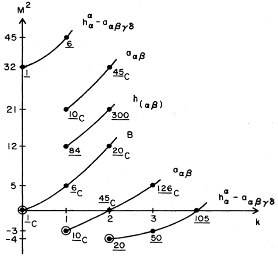
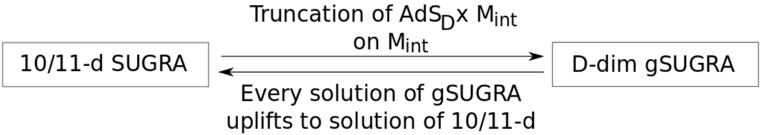


FIG. 2. Mass spectrum of scalars.

- Use "consistent truncation"



Consistent truncations

- Consistent truncations:
  - rare & difficult to construct
  - not necessarily unique
  - contain mixture of massless/massive modes
- Very few systematic constructions: group manifolds, singlets under group action
- Conjecture [Gauntlett, Varela '07] : For any SUSY $AdS_D \times_w M_{int}$ sol of 10-/11-dim SUGRA $\exists$ consistent truncation to only D-dim grav. supermultiplet ("minimal").

Outline

- Why are SUSY AdS geometries difficult?
- Exceptional field theory
- Exceptional geometry of supersymmetric $AdS_{6,7}$
- Classification of consistent truncations

Why is AdS hard?

- 10-/11-d SUGRA = GR + fluxes + spinors
- SUSY vacua $Mink_D$ / $AdS_D \times M_{int} \Rightarrow$ On M_{int}

$$\delta_\epsilon \psi \sim \nabla \epsilon + \not{F} \epsilon = 0\,.$$

- Generic properties for $Mink_D$ & $F = 0$ are well-understood:
  - Special holonomy (Calabi-Yau, G_2, etc.)
  - Moduli $\rightarrow$ cohomology of M_{int}
  - Effective action from integrating out massive modes
- SUSY AdS vacua have $F \neq 0$
  - Special holonomy??
  - Moduli – very difficult problem
  - Truncation mechanism?

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$$\delta_\epsilon \psi \sim \nabla \epsilon + \not{F} \epsilon = \nabla_{ExFT} \epsilon = 0\,.$$

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- SUSY AdS vacua have $F \neq 0$
  - Special holonomy??
  - Moduli – very difficult problem
  - Truncation mechanism?
- Unify fluxes + geometry into "larger" geometry

Unifying 1-form flux with geometry – Kaluza-Klein

- Einstein-Maxwell-Dilaton for *specific* α unified via Kaluza-Klein:
  $S = \int d^D x \sqrt{|g|} \left(R(g) - (\nabla\phi)^2 - e^{\alpha\,\phi} F^2 \right) \to \int d^{D+1} x \sqrt{|G|} \left(R(G) \right)$.
- Diffeomorphisms + gauge transformations

$$\delta g = L_v g \,, \quad \delta A = L_v A + d\lambda_{(0)} \,, \quad \delta\phi = L_v \phi \,,$$

combine into $\mathrm{GL}(D+1)$ diffeomorphisms:

$$V = v + \lambda_{(0)} \in \Gamma\left(TM \oplus C^\infty(M) \right) \,,$$

$$G = \begin{pmatrix} g + \phi^2 A^2 & \phi^2 A \\ \phi^2 A & \phi^2 \end{pmatrix} \in \frac{\mathrm{GL}(D+1)}{\mathrm{SO}(D+1)} \,,$$

$$\mathcal{L}_V G = V^M \partial_M G + (\partial \times_{adj} V) \cdot G = \left\{ L_v g,\, L_v A + d\lambda_{(0)},\, L_v \phi \right\} \,,$$

$$\partial_M = (\partial_i,\, 0) \,.$$

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$$V = v + \lambda_{(0)} \in \Gamma\left(TM \oplus C^\infty(M) \right) = \Gamma\left(T(M \times S^1) \right) ,$$

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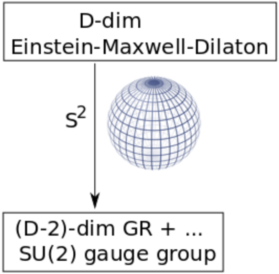
$$\partial_M = (\partial_i, 0) = (\partial_i, \partial_\psi) \,.$$

Benefits of unification

- Unified $\mathrm{GL}(D+1)$ perspective: new structures, systematics of results
- E.g. Einstein-Maxwell-Dilaton admits "remarkable" consistent truncation on S^2 for *specific* value of α [Cvetic, Lü, Pope '00].



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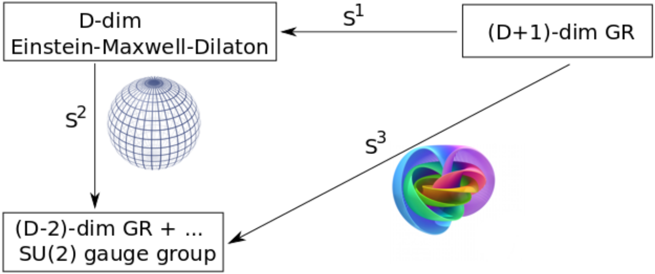

- Consistent truncation explained as Scherk-Schwarz truncation on group manifold $\mathrm{SU}(2) \sim S^3$ [Cvetic, Lü, Pope, Gibbons '03] .

Exceptional field theory

[Berman, Perry '10], [Berman, Godazgar^2, Perry '11], [Coimbra, Strickland-Constable, Waldram '11], [Berman, Cederwall, Kleinschmidt, Thompson '12], ...

- Consider KK-split of 11-d SUGRA:

$$M_{11} = M \times M_D \,.$$

- Unify diffeos + gauge symmetries of 11-d SUGRA on M

$$\delta g = L_v g \,, \quad \delta C_{(3)} = L_v C_{(3)} + d\lambda_{(2)} \,, \quad \delta C_{(6)} = L_v C_{(6)} + d\lambda_{(5)}$$

- Generalised tangent bundle

$$\mathcal{R}_1 \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus \ldots \,,$$
$$V = v + \lambda_{(2)} + \lambda_{(5)} + \ldots \in \Gamma\left(\mathcal{R}_1\right) \,.$$

- Form reps of $E_{d(d)}$:

$D = 11 - d$	7	6	5	4
$E_{d(d)}$	$\mathrm{SL}(5)$	$\mathrm{SO}(5,5)$	$E_{6(6)}$	$E_{7(7)}$

Generalised metric and other fields

- Internal bosonic fields on $M \longrightarrow$ generalised metric $\mathcal{M}_{MN}$.
- $\mathcal{M}_{MN}$ parameterises coset $E_{d(d)}/K(E_{d(d)})$:

$$\left\{g,\, C_{(3)},\, C_{(6)}\right\} = \mathcal{M}_{MN} \in \frac{E_{d(d)}}{K(E_{d(d)})}\,.$$

- Fields with mixed legs $\rightarrow$ generalised vector bundles with $E_{d(d)}$ action, e.g.
  - $\left\{g^{ij}g_{\mu j},\, C_{\mu\, ij},\, \ldots\right\} = \mathcal{A}_\mu{}^M \longrightarrow \mathcal{R}_1$.
  - $\left\{C_{\mu\nu\, i},\, C_{\mu\nu\, ijkl}\ldots\right\} = \mathcal{B}_{\mu\nu}{}^{MN} \longrightarrow \mathcal{R}_2$.

D	$E_{d(d)}$	R_1	R_2	R_3	R_4
7	$\mathrm{SL}(5)$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{10}}$
6	$\mathrm{Spin}(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	n/a
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	n/a	n/a
4	$E_{7(7)}$	$\mathbf{56}$	n/a	n/a	n/a

- Spinors form reps of $K(E_{d(d)})$ [Coimbra, Strickland-Constable, Waldram]

Generalised Lie derivative

- Generalised Lie derivative: local $E_{d(d)}$ action

$$\mathcal{L}_V = V^M \partial_M + (\partial \times_{adj} V) = \text{diffeo} + \text{gauge transf} \,,$$

with $\partial_M = (\partial_i \,,\, \partial^{ij} \,,\, \ldots) = (\partial_i \,,\, 0 \,,\, \ldots, 0)$.

- E.g.

$$\mathcal{L}_V \mathcal{M}_{MN} \longrightarrow \{ L_v g \,,\; L_v C_{(3)} + d\lambda_{(2)} \,,\; L_v C_{(6)} + d\lambda_{(5)} + \ldots \} \,.$$

- Construct generalised connection ∇_{ExFT}, etc. and unique action! SUSY [Godazgar^2, Hohm, Nicolai, Samtleben]

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- Construct generalised connection ∇_{ExFT}, etc. and unique action! SUSY [Godazgar[2], Hohm, Nicolai, Samtleben]
- Higher-dimensional origin? **No?!** Closure of algebra requires "section condition"

$$Y^{MN}_{PQ} \partial_M \otimes \partial_N = 0\,.$$

$E_{d(d)}$-covariant restriction to d (11-d SUGRA) or $d-1$ (IIB SUGRA) coordinates.

SUSY $\mathrm{AdS}_{6,7}$ vacua of type II SUGRA

- $\mathrm{AdS}_{6,7}$ vacua preserve 1/2-maximal SUSY; R-symmetry: $\mathrm{SU}(2)_R$
- IIA: $\mathrm{AdS}_7 \times S^2 \times I$ defined by $\dddot{t}(z) = m/2$, m Romans' mass.
  [Apruzzi, Fazzi, Rosa, Tomasiello '13], [Gaiotto, Tomasiello '14], [Apruzzi, Fazi, Passias, Rota, Tomasiello '15], [Cremonesi, Tomasiello '15]
  - Minimal consistent truncation [Passias, Rota, Tomasiello '15]
  - All AdS_7 vacua reduce to same 7-d vacuum $\rightarrow$ vector multiplets?

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  [Lozano, O Colgain, Rodriguez-Gomez, Sfetsos '12], [Apruzzi, Fazzi, Passias, Rosa, Tomasiello '14], [Kim, Kim, Suh '15], [D'Hoker, Gutperle, Karch, Uhlemann '16] .
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  - Minimal consistent truncation?
  - Vector multiplets?
- Systematics? Simplified construction? Consistent truncations?

Half-maximal SUSY in ExFT

- Half-maximal spinors [as reps of $K(E_{d(d)})$] define natural *universal* structures in ExFT [EM '16], [EM' 17]

$$\psi = \big(\underbrace{\psi_1, \ldots, \psi_N}_{\mathrm{SO}(d-1)_R}, \underbrace{0, \ldots, 0}_{\mathrm{SO}(d-1)_S} \big) \,,$$

e.g. $K(\mathrm{SL}(5)) = \mathrm{USp}(4) \longrightarrow \mathrm{SU}(2)_R \times \mathrm{SU}(2)_S$.

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- Spinor bilinears define "differential forms":

$$J_u{}^M \in \Gamma\left(\mathcal{R}_1\right) , \qquad \hat{K}_{MN} \in \Gamma\left(\mathcal{R}_{D-2}\right) ,$$

$u = 1, \ldots, d-1$ of $\mathrm{SO}(d-1)_R$, satisfying

$$\left(\delta_u^w \delta_v^x - \frac{1}{d-1} \delta_{uv} \delta^{wx} \right) J_w{}^M J_x{}^N Y^{PQ}_{MN} = 0 ,$$

$$\delta^{uv} J_u{}^M J_v{}^N \hat{K}_{MN} > 0 , \qquad \hat{K} \times_{R_c} \hat{K} = 0 . \qquad \left(R_c = \begin{cases} \emptyset , & D = 7 \\ \mathbf{1} , & D = 6 \end{cases} \right)$$

Half-maximal background in ExFT

- SUGRA fields captured by generalised metric
  [EM, Samtleben, Vall Camell '18]

$$\{\text{SUGRA fields}\} = \mathcal{M}_{MN} \sim J_u{}^P J^{u,Q} \hat{K}_{MP} \hat{K}_{NQ} + \hat{K}_{MN} + \epsilon^{u_1 \ldots u_{d-1}} \left(J_{u_1} \ldots J_{u_{d-1}} \right)_{MN} .$$

- $\mathrm{SO}(d-1)_R$ invariant combination of J_u and $\hat{K}$.

BPS equations

- BPS conditions $\Leftrightarrow$ universal differential conditions on J_u, $\hat{K}$: [EM' 17]

$$\mathcal{L}_{J_u} J_v = R_{uvw} J^w \,, \qquad \mathcal{L}_{J_u} \hat{K} = 0 \,,$$

$R_{uvw} R^{uvw} = -\Lambda$, and

  - $D = 7$: $d\hat{K}^{MN} = \epsilon^{uvw} R_{uvw} \, J^{x\,P} J_x{}^Q Y_{PQ}^{MN}$,
  - $D = 6$: $d\hat{K}^M = \epsilon^{uvwx} R_{uvw} J_x{}^M$,

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  - $D = 6$: $d\hat{K}^M = \epsilon^{uvwx} R_{uvw} J_x{}^M$,
- R_{uvw} breaks $\mathrm{SO}(d-1)_R \longrightarrow \mathrm{SU}(2)$ AdS R-symmetry
- **R-symmetry must be realised by isometries of M_{int}.**
  c.f. [Ashmore, Petrini, Waldram '16], [Coimbra, Strickland-Constable '17]
- $J_u \supset$ Killing vector fields, generate R-symmetry
- $\hat{K}$ invariant under R-symmetry

Summary of 1/2-max AdS structures

- Every 1/2-max SUSY AdS vacuum described by

$$d-1 \text{ gen vector fields } J_u{}^M \text{ and extra tensor } \hat{K}_{MN}\,,$$

- subject to algebraic conditions and differential conditions

$$\mathcal{L}_{J_u} J_v = R_{uvw} J^w\,, \qquad \mathcal{L}_{J_u} \hat{K} = 0\,, \qquad d\hat{K} \sim R\,.$$

- Can prove *universal* features:
  - R-symmetry must be realised by isometries of M_{int}
  - Every AdS vacuum admits consistent truncation (without matter)
  - Consistent truncations have max $N \leq d-1$ vector multiplets
  - Classification of consistent truncations with matter

Universal consistent truncation

- [EM '16], [EM '17] : **Proof of $1/2$-max version of** [Gauntlett, Varela '07]
- Y coordinates on M_{int}, x coordinates on M_D.
- Given $J_u(Y)$, $\hat{K}(Y)$ of $1/2$-max AdS, the linear Ansatz

$$\mathcal{J}_u(x, Y) = X^{-1}(x)\, J_u(Y)\,, \qquad \hat{\mathcal{K}}(x, Y) = X^2(x)\, \hat{K}(Y)\,,$$
$$\mathcal{A}_\mu(x, Y) = A_\mu{}^u(x)\, J_u(Y)\,, \qquad \ldots\,,$$

gives a *consistent* truncation.

- Consistency follows from BPS conditions of J_u, $\hat{K}$.
- $X(x)$ scalar, $A_\mu{}^u(x)$ $d-1$ vector fields of grav. supermultiplet.
- "Minimal" truncation $\Rightarrow$ no matter multiplets.

Consistent truncations with vector multiplets

[EM '16], [EM '17]

- Half-maximal SUSY $\Rightarrow$ N vector multiplets $\mathcal{M}_{scalar} = \frac{\mathrm{SO}(d-1,N)}{\mathrm{SO}(d-1)\times\mathrm{SO}(N)} \times \mathbb{R}^+$
- Need extra tensors to expand in:

$$\mathcal{J}_u(x, Y) = \phi_I(x) T^I(Y) , \quad \ldots$$

- $G_{stab}(T^I)$ dictates scalar manifold:

$$\mathcal{M}_{scalar} = \frac{Comm(G_{stab}, E_{d(d)})}{Comm(G_{stab}, K(E_{d(d)}))} \Rightarrow G_{stab} = \mathrm{Spin}(d-1-N) .$$

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- $d-1+N$ generalised vector fields:

$$J_A{}^M = \left(J_u{}^M, J_{\bar{u}}{}^M\right) \in \Gamma\left(\mathcal{R}_1\right) , \qquad \hat{K}_{MN} \in \Gamma\left(\mathcal{R}_{D-3}\right) ,$$

$A = (u, \bar{u}) = 1, \ldots, d-1+N$ of $\mathrm{SO}(d-1, N)$, s.t.

$$\left(\delta_A^C \delta_B^D - \frac{1}{d-1+N}\eta_{AB}\eta^{CD}\right) J_C{}^M J_D{}^N Y^{PQ}_{MN} = 0 , \quad \hat{K} \times_{R_c} \hat{K} = 0 , \quad \eta^{AB} J_A{}^M J_B{}^N \hat{K}_{MN} > 0 .$$

- **Can only have $N \leq d-1$ vector multiplets.**

Truncation Ansatz with vector multiplets

[EM '16], [EM '17]

- Consistent truncation

$$\mathcal{J}_u(x,Y) = X^{-1}(x)\, b_u{}^A(x) J_A(Y)\,, \qquad \hat{\mathcal{K}}(x,Y) = X^2(x)\,\hat{K}(Y)\,,$$
$$\mathcal{A}_\mu(x,Y) = A_\mu{}^A(x)\, J_A(Y)\,, \qquad \ldots\,,$$

- $b_u{}^A$ constrained algebraically by

$$b_u{}^A b_v{}^B \eta_{AB} = \delta_{uv}\,.$$

- Consistency requires differential conditions

$$\mathcal{L}_{J_A} J_B = f_{AB}{}^C J_C\,, \quad f_{AB}{}^C \text{ constant}\,.$$

- AdS vacua: $J_{\bar{u}}$ organise into reps of $\mathrm{SU}(2)_R$ symmetry $\rightarrow$ can fully classify!

Applications:
AdS_7 of mIIA
AdS_6 of IIB.

[EM, Samtleben, Vall Camell '18] : Solutions & derive "minimal" consistent truncation

[EM, Samtleben, Vall Camell '19] : Classify and construct consistent truncations with vector multiplets

Constructing AdS solutions from ExFT

- Geometric Ansatz $\rightarrow J_u,\ \hat{K}$
  - Algebraic conditions (existence of spinors)
  - Differential conditions (SUSY AdS vacuum)
- Construct generalised metric

$$\mathcal{M}_{MN} = \hat{J}_{u,M}\hat{J}^{u}{}_{N} - \hat{K}_{MN} + \left(J^{d-1}\right)_{MN}.$$

- ExFT dictionary: $\mathcal{M}_{MN} = \{\text{SUGRA fields}\}$.
- Minimal consistent truncation for free.
- Classify vector multiplets.

Example 1: AdS_7 from ExFT

[EM, Samtleben, Vall Camell '18]

- AdS_7 of mIIA in $\mathrm{SL}(5)$ ExFT.
- $\mathrm{SU}(2)_R$ must be realised by isometries of M_{int}
  - $M_{int} = S^2 \times I \quad (S^2 \longrightarrow \mathrm{SU}(2)_R)$
  - $M_{int} = S^3/\mathbb{Z}_k \quad (S^3/\mathbb{Z}_k \longrightarrow \mathrm{SU}(2)_R \times \mathrm{U}(1))$
- 3 J_u, 1 $\hat{K}$ satisfying algebraic conditions and

$$\mathcal{L}_{J_u} J_v = \sqrt{-\Lambda}\, \epsilon_{uvw} J^w \,, \qquad \mathcal{L}_{J_u} \hat{K} = 0 \,, \qquad d\hat{K} \sim \sqrt{-\Lambda} \,.$$

- Write most general $J_u \to \mathrm{SU}(2)_R$ triplets, $\hat{K} \to \mathrm{SU}(2)_R$ singlet.

$$J_u \in \Gamma\left(TM \oplus T^*M \oplus \Lambda^0 T^*M \oplus \Lambda^2 T^*M\right) \,,$$
$$\hat{K} \in \Gamma\left(\Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus \Lambda^3 T^*M\right) \,.$$
$$J_u = v_u + \ldots$$

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  - ~~$M_{int} = S^3/\mathbb{Z}_k \quad (S^3/\mathbb{Z}_k \longrightarrow \mathrm{SU}(2)_R \times \mathrm{U}(1))$~~
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$$\mathcal{L}_{J_u} J_v = \sqrt{-\Lambda}\, \epsilon_{uvw} J^w \,, \qquad \mathcal{L}_{J_u} \hat{K} = 0 \,, \qquad d\hat{K} \sim \sqrt{-\Lambda} \,.$$

- Write most general $J_u \to \mathrm{SU}(2)_R$ triplets, $\hat{K} \to \mathrm{SU}(2)_R$ singlet.

$$J_u \in \Gamma\left(TM \oplus T^*M \oplus \Lambda^0 T^*M \oplus \Lambda^2 T^*M\right) \,,$$
$$\hat{K} \in \Gamma\left(\Lambda^0 T^*M \oplus \Lambda^2 T^*M \oplus \Lambda^3 T^*M\right) \,.$$

$$J_u = v_u + \left(g(z) dy_u + k(z) y_u dz + l(z) \epsilon_{uvw} y^v dy^w\right) + \ldots \,, \qquad y_u y^u = 1 \,.$$

G_{half} structure on $S^2 \times I$

[EM, Samtleben, Vall Camell '18]

- Algebraic conditions $\to J_u$, $\hat{K}$ depend on 5 functions on I: $g(z)$, $h(z)$, $q(z)$, $s(z)$, $t(z)$; $\quad t(z)h(z) \geq 0$.
- Differential conditions:

$$\dot{g}(z) = -\frac{h(z)}{q(z)}\,, \quad 2\,q(z)\,\dot{q}(z) = m\,h(z)\,,$$
$$\dot{s}(z) = h(z)\,, \quad \dot{t}(z) = -\frac{h(z)\,s(z)}{q(z)}\,.$$

- Gauge choice $h(z) = q(z)$, wlog

$$g(z) = -z\,, \qquad s(z) = -\dot{t}(z)\,, \qquad q(z) = -\ddot{t}(z)\,,$$

and $\dddot{t}(z) = -m/2$.

AdS_7 solutions

[EM, Samtleben, Vall Camell '18]

Generalised metric $\mathcal{M}(J_u, \hat{K})$ and EFT $\leftrightarrow$ IIA SUGRA dictionary:

$$ds^2_{10} = 4\sqrt{-\frac{t}{\ddot{t}}}\, ds^2_{AdS_7} + \frac{1}{2}\sqrt{-\frac{\ddot{t}}{t}} \left(\frac{t^2}{\dot{t}^2 - 2\,\ddot{t}\,t} ds^2_{S^2} + dz^2 \right) ,$$

$$e^{\psi} = \left(-\frac{t}{\ddot{t}}\right)^{3/4} \frac{1}{\sqrt{\dot{t}^2 - 2\,\ddot{t}\,t}} ,$$

$$B_2 = \frac{1}{2\sqrt{2}} \left(z - \frac{\dot{t}\,t}{\dot{t}^2 - 2\,\ddot{t}\,t} \right) vol_2 ,$$

$$F_2 = \frac{1}{2\sqrt{2}} \left(2\ddot{t} + \frac{m\,\dot{t}\,t}{\dot{t}^2 - 2\,\ddot{t}\,t} \right) vol_2 ,$$

with $\dddot{t} = -m/2$, $t \geq 0$ with equality at ∂I.

These are the general SUSY AdS_7 solutions of mIIA [Apruzzi, Fazzi, Rosa, Tomasiello] in coords of [Cremonesi, Tomasiello] .

AdS_7 minimal consistent truncation

- Consistent truncation for AdS_7 vacua [EM, Samtleben, Vall Camell '18].
  $\mathcal{J}_u(x,Y) = X^{-1}(x) J_u(Y)$, $\hat{\mathcal{K}}(x,Y) = X^2(x)\hat{K}(Y)$:

$$ds^2_{10} = 4\sqrt{-\frac{t}{\ddot{t}}} X^{1/2} ds^2_7 + \frac{1}{2}\sqrt{-\frac{\ddot{t}}{t}} \left[X^{-5/2} dz^2 + X^{5/2} \frac{t^2}{\dot{t}^2 X^5 - 2\,\ddot{t}\,t} ds^2_{S^2} \right] ,$$

$$e^{\psi} = X^{5/4} \left(-\frac{t}{\ddot{t}}\right)^{3/4} \frac{1}{\sqrt{X^5 \dot{t}^2 - 2\,\ddot{t}\,t}} ,$$

$$B_2 = \frac{1}{2\sqrt{2}} \left(z - \frac{\dot{t}\,t X^5}{\dot{t}^2 X^5 - 2\,\ddot{t}\,t} \right) vol_{S^2} ,$$

$$F_2 = \frac{1}{2\sqrt{2}} \left(2\ddot{t} + X^5 \frac{m \dot{t}\,t}{\dot{t}^2 X^5 - 2\,\ddot{t}\,t} \right) vol_{S^2} ,$$

$$\dddot{t} = -m/2.$$

- Reproduces [Passias, Rota, Tomasiello].
- Explains universality of consistent truncation!

Example 2: AdS_6 of IIB from ExFT

[EM, Samtleben, Vall Camell '18]

- AdS_6 of IIB in $\mathrm{SO}(5,5)$ ExFT. $\mathrm{SU}(2)_R$ must be realised by isometries of M_{int}
  - $M_{int} = S^2 \times \Sigma_2 \quad (S^2 \longrightarrow \mathrm{SU}(2)_R)$
  - ~~$M_{int} = S^3/\mathbb{Z}_k \times I \quad (S^3/\mathbb{Z}_k \longrightarrow \mathrm{SU}(2)_R \times \mathrm{U}(1))$~~
- Write most general Ansatz for J_u, $\hat{K}$
- Algebraic + differential conditions + gauge choice: AdS_6 solutions defined by pair of harmonic functions $f^\alpha = -p^\alpha + ik^\alpha$ on Σ_2 with

$$df^\alpha \wedge d\bar{f}_\alpha \geq 0 \,, \quad (\text{equality on } \partial\Sigma_2).$$

AdS_6 solutions of IIB

[EM, Samtleben, Vall Camell '18]

- Compute SUGRA fields from $\mathcal{M}_{MN}(J_u, \hat{K})$:

$$ds^2 = \frac{\sqrt{2}\, r^{5/4}\, \Delta^{1/4}}{3^{3/4}\, |dk|^{1/4}} \left(\frac{12}{r} ds^2_{AdS_6} + \frac{|dk|}{\Delta} ds^2_{S^2} + \frac{4}{r^2} dk^\alpha \otimes dp_\alpha \right), \qquad C_{(4)} = 0\,,$$

$$C_{(2)}{}^\alpha = -\frac{4}{3} vol_{S^2} \left(k^\alpha + \frac{r\, p_\gamma\, \partial_\beta k^\gamma\, \partial^\beta p^\alpha}{2\Delta} |dk|^{1/2} \right), \qquad H_{\alpha\beta} = \frac{|dk|^{1/2}\, p_\alpha p_\beta + 6\, r\, \partial_\gamma k_\alpha \partial^\gamma p_\beta}{2\sqrt{3\,\Delta\, r}}\,,$$

$$\Delta = \frac{3}{4}\, r\, |dk| + \frac{1}{2} |dk|^{1/2}\, p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta\,, \qquad H_{\alpha\beta} = \frac{1}{\mathrm{Im}\,\tau} \begin{pmatrix} |\tau|^2 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & 1 \end{pmatrix},$$

$$r = -p_\alpha\, dk^\alpha\,.$$

- Matches [D'Hoker, Gutperle, Karch, Ulhemann 2016] upon field redefinition
- Consistent truncation??

AdS$_6$ minimal consistent truncations

- **Find new minimal consistent truncation for free** [EM, Samtleben, Vall Camell '18]:

$$\mathcal{J}_u(x,Y) = X^{-1}(x)\, J_u(Y)\,, \qquad \hat{\mathcal{K}}(x,Y) = X^2(x)\,\hat{K}(Y)\,.$$

- From generalised metric and ExFT/IIB dictionary:

$$ds^2 = \frac{\sqrt{2}\, r^{5/4}\,\Delta^{1/4}}{3^{3/4}\,|dk|^{1/4}}\left(\frac{12}{r}ds_6^2 + \frac{X^2\,|dk|^{1/2}}{\Delta}\,ds^2_{S^2} + \frac{4}{X^2\,r^2}dk^\alpha\otimes dp_\alpha\right)\,, \qquad C_{(4)} = 0\,,$$

$$C_{(2)}{}^\alpha = -\frac{4}{3}vol_{S^2}\left(k^\alpha + \frac{X^4\, r\, p_\gamma\,\partial_\beta k^\gamma\,\partial^\beta p^\alpha}{2\Delta}|dk|^{1/2}\right)\,, \quad H_{\alpha\beta} = \frac{X^4\,|dk|^{1/2}\,p_\alpha p_\beta + 6\,r\,\partial_\gamma k_\alpha \partial^\gamma p_\beta}{2\sqrt{3}\,\Delta\, r}\,,$$

$$\Delta = \frac{3}{4}\,r\,|dk| + \frac{1}{2}X^4|dk|^{1/2}\,p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta\,.$$

- Universal consistent truncation.
- Allows us to study AdS$_6$ solution using minimal 6d gSUGRA.
- c.f. [Hong, Liu, Mayerson '18] .

Minimal consistent truncation vs vector multiplets

- For AdS_7, AdS_6, "minimal" consistent truncations has universal form.
- All AdS_7, AdS_6 vacua are same vacuum in minimal 7-D/6-D SUGRA!
- Cannot construct flows between different $AdS_{6,7}$ vacua.
- Consistent truncation with vector multiplets to differentiate? [De Luca, Gnecchi, Lo Monaco, Tomasiello '18]

Consistent truncations with vector multiplets for AdS_7

[EM, Samtleben, Vall Camell '19]

- Need N additional generalised vector fields s.t.

$$
\begin{aligned}
J_A{}^M J_B{}^N Y^{PQ}_{MN} &= \frac{1}{d-1+N}\eta_{AB}\eta^{CD} J_C{}^M J_D{}^N Y^{PQ}_{MN}\,,\\
\mathcal{L}_{J_A} J_B &= f_{AB}{}^C J_C\,,\\
\mathcal{L}_{J_A}\hat{K} &= 0\,.
\end{aligned}
$$

- $N \leq d-1$ vector multiplets; organise themselves into reps of $\mathrm{SU}(2)_R$ symmetry.
- Fully classify possible consistent truncations with vector multiplets around AdS_7:
  - **NO** consistent truncations with vector multiplets when $m \neq 0$
  - 1 vector multiplet when $m = 0$.

Consistent truncations with vector multiplets for AdS_6

[EM, Samtleben, Vall Camell '19]

- Fully classify possible consistent truncations with vector multiplets around AdS_6:

N	$SU(2)_R$ rep	Consistent truncation
1	$\mathbf{1}$	**Only if** $\exists\ \chi$: $\partial\left(e^{i\chi}\partial f^\alpha\right) \in$ Real functions on Σ_2
2	$\mathbf{1}\oplus\mathbf{1}$	**NO** (no globally regular solutions)
3	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	**NO**
3	$\mathbf{3}$	**Only if** $d\pi^\alpha = 0$
4	$\mathbf{3}\oplus\mathbf{1}$	**Only if** $\exists\,\mathbf{3}$ and $\exists\,\mathbf{1}$ with $e^{i\chi} = \left(\frac{p_\alpha\bar\partial\bar f^\alpha}{p_\beta\partial f^\beta}\right)$

$$\pi^\alpha = \frac{1}{2}r^2\left(g\,\partial f^\alpha\, d\bar z + \bar g\,\bar\partial\bar f^\alpha dz\right), \quad g = i\left(\frac{p_\alpha\bar\partial\bar f^\alpha}{p_\beta\partial f^\beta}\right), \quad dr = -p_\alpha dk^\alpha$$

- Easily construct full non-linear truncation Ansätze.
- Can use results to uplift 6-D $F(4)$ gSUGRA results to IIB, e.g. [Gutperle, Kaidi, Raj '18], [Gutperle, Kaidi, Raj '17]

AdS$_6$ uplift formulae for 1 vector multiplet

Scalar fields $\in \frac{\mathrm{SO}(4,1)}{\mathrm{SO}(4)} \times \mathbb{R}^+$:

$$X \text{ and } m_A = (m_I,\, m_4,\, m_5)\,, \quad \text{s.t.} \quad \eta^{AB} m_A m_B = -1\,, \quad \eta_{AB} = \mathrm{diag}(1,1,1,1,-1)\,.$$

$$ds^2 = \frac{r^{5/4}\,\lambda^{3/2}}{2^{5/4}\bar{\Delta}^{3/4}} \left[\frac{2\sqrt{2}}{r\,\lambda^2} ds_6^2 + X^2 \left(ds_{S^2}^2 + w \otimes w - \frac{1}{r^2} p_\alpha p_\beta n^\alpha \otimes n^\beta \right) + \frac{2}{X^2\, r} n_\alpha \otimes X^\alpha + \frac{4\tilde{\Delta}}{X^2 r^2\, \lambda^2} dk^\alpha \otimes dp_\alpha \right],$$

$$C^\alpha_{(2)} = -vol_{S^2} \left(k^\alpha + \frac{X^4\, r\, \lambda}{2\bar{\Delta}} p_\beta \left[m_5 \partial_\gamma k^\beta \partial^\gamma p^\alpha + n^{\beta\gamma} \omega^\alpha{}_\gamma \right] \right) + \frac{\lambda^2}{4\bar{\Delta}} \left(2\, r \left[m_5 n^\alpha - \omega^\alpha \right] - X^4\, p^\alpha\, p_\beta \star n^\beta \right) \wedge dy^I y^J m^K \epsilon_{IJK}\,,$$

$$H^{\alpha\beta} = \frac{X^4\, p^\alpha\, p^\beta\, \lambda + 4\, r \left(m_5\, \partial_\gamma k^\alpha \partial^\gamma p^\beta + n^{\alpha\gamma} \omega^\beta{}_\gamma \right)}{2\sqrt{2}\, r\, \bar{\Delta}}\,,$$

$$\omega^\alpha = m_I y^I dk^\alpha + m_4 dp^\alpha\,, \qquad X^\alpha = m_I y^I dp^\alpha - m_4 dk^\alpha\,, \qquad w = m_I dy^I + \frac{p_\alpha}{r} \left(m_5 n^\alpha - \omega^\alpha \right),$$

$$\bar{\Delta} = \frac{1}{2} r\, \lambda^2 \left(m_5^2 - m_4^2 - \left(m_I y^I \right)^2 \right) + \frac{1}{2} X^4 \lambda\, p_\alpha\, p_\beta \left(m_5 \partial_\gamma k^\alpha \partial^\gamma p_\alpha + n^{\alpha\gamma} \omega^\beta{}_\gamma \right),$$

$$\tilde{\Delta} = \frac{1}{2} r\, m_5 \lambda^2 + \frac{1}{2} X^4\, \lambda\, p_\alpha p_\beta \partial_\gamma k^\alpha \partial^\gamma p^\beta\,, \quad n^\alpha = \frac{1}{2} \left(e^{i\chi} \partial f^\alpha d\bar{z} + c.c \right).$$

Conclusions

- ExFT useful new tool to analyse AdS vacua.
- Reproduce all mIIA $\text{AdS}_7 \times S^3$, IIB $\text{AdS}_6 \times S^2 \times \Sigma_2$ solutions.
- Construct universal "minimal" consistent truncation (grav. multiplet).
- Understand universality of results.
- Classification and construction of consistent truncations with vector multiplets.

Outlook

- Lower dimensions.
- Study/uplift of moduli, e.g. $\mathcal{N} = 4$ AdS_5 vacua. Zamolodchikov metric, ...
- Construct new AdS vacua
- Other amounts of SUSY, e.g. $\mathcal{N} = 2$ [Ashmore, Gabella, Graña, Petrini, Waldram '16].

  In the absence of extra isometries beyond R-symmetry, infinitesimal deformations (marginal) $\rightarrow$ finite deformations (exactly marginal).
- Natural language to study deformations?
- Other holographic properties: "a" / "c"-minimisation, etc.