Exploring the landscape of eta-deformed AdS superstrings

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- Integrability, duality and beyond -

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based on [arXiv:1811.07841] with B. Hoare and work in progress

Motivation

2018 — Hoare Seibold, supergravity solutions for the $AdS_5 \times S^5$ superstrings

2016 — Borsato Wulff , target space supergeometry and condition on R-matrix to give a supergravity solution

— Arutyunov Frolov Hoare Roiban Tseytlin; Tseytlin Wulff, solution of generalised supergravity equations of motion

2015 — Arutyunov Borsato Frolov, RR fluxes for a particular choice of R-matrix: not a supergravity solution!

— Lunin Roiban Tseytlin, supergravity backgrounds for deformed $AdS_n \times S^n$ supercosets

2014 — Arutyunov Borsato Frolov, metric and B-field of the η-deformed $AdS_5 \times S^5$ superstring

2013 — Delduc Magro Vicedo, η-deformation of the symmetric space σ model
action for the η-deformed $AdS_5 \times S^5$ superstring

2008 — Klimčík, integrability of the η-deformed PCM

2002 — Klimčík, η-deformation of the PCM

Motivation

2018 — Hoare Seibold, supergravity solutions for the $AdS_5 \times S^5$ superstrings

also not the good mirror limit

Arutyunov Borsato Frolov, RR fluxes for a particular choice of R-matrix: not a supergravity solution!

perturbative calculation does not match the q-deformed S-matrix, in particular not good mirror limit

2015

Arutyunov de Leeuw van Tongeren, mirror duality

2014

Arutyunov Borsato Frolov, metric and B-field of the η-deformed $AdS_5 \times S^5$ superstring

for bosons perturbative calculation matches the q-deformed S-matrix

2008 — Beisert Koroteev, q-deformed S-matrix

q-deformed S-matrix
$q = e^{-\kappa/T}$, $\kappa = \frac{2\eta}{1-\eta^2}$

$\kappa \to 0$ → S-matrix of undeformed theory

$\kappa \to \infty, T \to T\kappa^2$ → S-matrix of mirror theory

- The η deformation
  - The semi-symmetric space sigma model
  - The η-deformed semi-symmetric space sigma model
- Example of the η-deformed $AdS_2 \times S^2 \times T^6$ superstring
  - The choice of R-matrix
  - The different backgrounds
- Example of the 2 parameter deformation of $AdS_3 \times S^3 \times T^4$
  - The choice of R-matrix
  - The supergravity backgrounds
- Conclusion and open problems

The η-deformed semi-symmetric space sigma model

Sigma model on the supercoset $\hat{G}/H$:

- $\hat{G}$ is a supergroup with basic Lie superalgebra $\hat{\mathfrak{g}}$
- $\hat{\mathfrak{g}}$ admits a $\mathbb{Z}_4$ grading

even grading odd grading

$$\hat{\mathfrak{g}} = \mathfrak{g}^{(0)} + \mathfrak{g}^{(1)} + \mathfrak{g}^{(2)} + \mathfrak{g}^{(3)} \qquad [\mathfrak{g}^{(i)}, \mathfrak{g}^{(j)}\} \in \mathfrak{g}^{(i+j \mod 4)}$$

$\mathfrak{g}^{(0)}$ → subalgebra, identified with the Lie algebra of H

- introduce projectors $P^{(i)}$ projecting onto the spaces $\mathfrak{g}^{(i)}$
- there exists an ad-invariant bilinear form on $\hat{\mathfrak{g}}$ that we denote by STr

Action of the semi-symmetric space sigma model on the supercoset $\hat{\mathrm{G}}/\mathrm{H}$:

$$S[g \in \hat{\mathrm{G}}] = -\frac{T}{4}(\gamma^{ij} - \epsilon^{ij}) \int \mathrm{d}^2\sigma \ \mathrm{STr}[g^{-1}\partial_i g\, P\, g^{-1}\partial_j g]$$

$$P = 2P^{(2)} + P^{(1)} - P^{(3)}$$

Properties:

- global left $\hat{\mathrm{G}}$ symmetry $g \to g_0 g$
- local right H gauge symmetry $g \to gh$
- diffeomorphism and Weyl invariance, fermionic κ symmetry
- classical integrability

Action of η-deformed SSSσM on the supercoset $\hat{\mathrm{G}}/H$:

Delduc Magro Vicedo
arXiv:1309.5850

$$S_\eta[g \in \hat{\mathrm{G}}] \sim (\gamma^{ij} - \epsilon^{ij}) \int \mathrm{d}^2\sigma \ \mathrm{STr}\Big[g^{-1}\partial_i g \, P \frac{1}{1-\eta R_g P} g^{-1}\partial_j g \Big]$$

$$P = \tfrac{2}{1-\eta^2} P^{(2)} + P^{(1)} - P^{(3)} \qquad\qquad R_g = \mathrm{Ad}_g^{-1} R \, \mathrm{Ad}_g$$

The deformation is governed by the linear operator $R : \hat{\mathfrak{g}} \to \hat{\mathfrak{g}}$, where

- R is skew-symmetric: $\mathrm{STr}[X R(Y)] = -\mathrm{STr}[Y R(X)]$
- R satisfies the mcYBE: $[R(X), R(Y)\} - R([R(X), Y\} + [X, R(Y)\}) = [X, Y\}$

Strength of deformation $\eta \in \mathbb{R}$, $\eta \to 0$ gives undeformed SSSσM

This deformation preserves classical integrability

- Defines an integrable deformation of the Green Schwarz string on various spaces, $AdS_5 \times S^5$, $AdS_3 \times S^3 \times T^4$, $AdS_2 \times S^2 \times T^6 \dots$

- Are the resulting theories critical string theories (do they satisfy the standard supergravity equations of motion)?

- The R-matrix should be unimodular

Borsato Wulff
arXiv:1608.03570

$$[X, Y\}_R = [R(X), Y\} + [X, R(Y)\}$$

$$\sum_b (-1)^{[b]} \tilde{f}_{ab}{}^b = 0$$

$[b] = 0$ for bosonic generators
$[b] = 1$ for fermionic generators

- A particular solution of the modified classical YBE is given by the Drinfel'd Jimbo R-matrix

- Relies on the choice of a Cartan-Weyl basis $\hat{\mathfrak{g}} = \{\text{Cartan}, E^+, E^-\}$

$$R(\text{Cartan}) = 0 \,, \qquad R(E^+) = -iE^+ \,, \qquad R(E^-) = +iE^-$$

- Superalgebras admit several inequivalent Dynkin diagrams

$\Rightarrow$ Inequivalent η-deformations! Supergravity solutions!

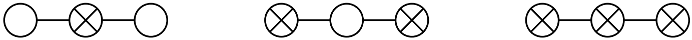
Example of the η-deformed $AdS_2 \times S^2 \times T^6$ superstring

η-deformation of $AdS_2 \times S^2$

- Curved part of the background described by the supercoset

$$\frac{\hat{\mathrm{G}}}{\mathrm{H}} = \frac{\mathrm{PSU}(1,1|2)}{\mathrm{SO}(1,1) \times \mathrm{SO}(2)} \quad \sim \quad \mathrm{AdS}_2 \times \mathrm{S}^2 + 8 \text{ fermions}$$

- Dynkin diagrams of the complexified isometry algebra $\mathfrak{sl}(2|2)$



- Most of the literature is based on the R-matrix associated to the distinguished Dynkin diagram

$$R_0(M)_{ij} = -i\epsilon_{ij}M_{ij} \ , \qquad \epsilon = \left(\begin{array}{cc|cc} 0 & \oplus & + & + \\ - & 0 & \oplus & + \\ \hline - & - & 0 & \oplus \\ - & - & - & 0 \end{array}\right)$$

How to construct the R-matrices associated to the other diagrams?

$$R = \mathrm{Ad}^{-1}_{\mathrm{Permut}} R_0 \, \mathrm{Ad}_{\mathrm{Permut}}$$

$$\left(\begin{array}{cc|cc} 0 & \oplus & + & + \\ - & 0 & \oplus & + \\ \hline - & - & 0 & \oplus \\ - & - & - & 0 \end{array}\right) \xrightarrow[\text{r and c (2,3)}]{\text{permutation}} \left(\begin{array}{cc|cc} 0 & + & \oplus & + \\ - & 0 & - & \oplus \\ \hline - & \oplus & 0 & + \\ - & - & - & 0 \end{array}\right) \xrightarrow[\text{r and c (1,3)}]{\text{permutation}} \left(\begin{array}{cc|cc} 0 & \oplus & - & + \\ - & 0 & - & \oplus \\ \hline \oplus & + & 0 & + \\ - & - & - & 0 \end{array}\right)$$

How many inequivalent solutions do we expect?

- permutation of 4 elements $\rightarrow 24$
- Action on $\mathfrak{su}(1,1)$ and $\mathfrak{su}(2)$ should be left unchanged $\rightarrow 6$
- Identify R-matrices related by AdS $\longleftrightarrow$ S $\rightarrow 3$

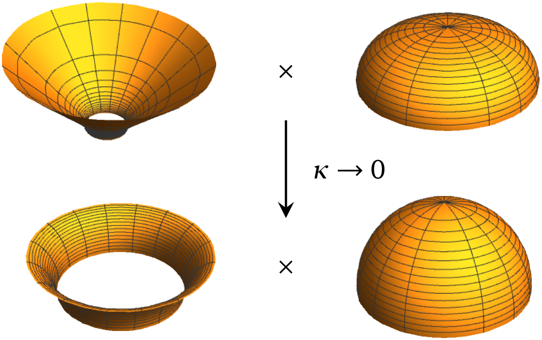
$\Rightarrow$ The 3 R-matrices correspond to the 3 inequivalent Dynkin diagrams

η-deformation of $AdS_2 \times S^2$

- The 3 chosen R-matrices give the same metric for the η-deformed $\mathrm{AdS}_2 \times \mathrm{S}^2$ space

$$\kappa = \frac{2\eta}{1-\eta^2}$$

$$\mathrm{d}s^2 = \frac{1}{1-\kappa^2\rho^2}\left(-(1+\rho^2)\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{1+\rho^2}\right) + \frac{1}{1+\kappa^2 r^2}\left((1-r^2)\mathrm{d}\phi^2 + \frac{\mathrm{d}r^2}{1-r^2}\right) + \mathrm{d}x^i \mathrm{d}x^i$$



η-deformation of $AdS_2 \times S^2$: unimodularity

- Unimodularity of the 3 different Drinfel'd Jimbo R-matrices

Dynkin diagram	$\bigcirc - \otimes - \bigcirc$	$\otimes - \bigcirc - \otimes$	$\otimes - \otimes - \otimes$
R-matrix $R(M)_{ij} = -i\epsilon_{ij}M_{ij}$	$\begin{pmatrix} 0 & \oplus & + & + \\ - & 0 & \oplus & + \\ - & - & 0 & \oplus \\ - & - & - & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \oplus & - & + \\ - & 0 & - & \oplus \\ \oplus & + & 0 & + \\ - & - & - & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & + & \oplus & + \\ - & 0 & - & \oplus \\ - & \oplus & 0 & + \\ - & - & - & 0 \end{pmatrix}$
Unimodular?	No	No	Yes!

Generalised supergravity backgrounds related by field redefinitions

Supergravity solution!

- One can construct a one-parameter family $a \in [0,1]$ of supergravity backgrounds supporting the metric and B-field

Lunin Roiban Tseytlin
arXiv:1411.1066

$$e^{-2\Phi} = e^{-2\Phi_0} \frac{(1-\kappa^2\rho^2)(1+\kappa^2 r^2)}{P_a(\rho,r)}$$
$$F_3 = \frac{1}{2} \mathrm{d}C_a \wedge J_2 + \frac{1}{12} \star (\mathrm{d}C_a \wedge J_2 \wedge J_2 \wedge J_2)$$
$$F_5 = \frac{1}{2}(1+\star)\mathrm{d}A_a \wedge \mathrm{Re}\,\Omega_3$$

$$P_a(\rho,r) = 1+\kappa^2(a^2(r^2-\rho^2)+r^2\rho^2) - 2\kappa\sqrt{1-a^2}\sqrt{1+a^2\kappa^2}\, r\rho$$
$$C_a = \frac{2}{a\sqrt{P(\rho,r)}}\left[\sqrt{1-a^2} - \kappa\sqrt{1+a^2\kappa^2}\,\rho r\right]$$
$$A_a = \frac{\sqrt{2}}{\sqrt{P(\rho,r)}}\left[\sqrt{1+a^2\kappa^2}(\rho \mathrm{d}t + r\mathrm{d}\varphi) + \kappa\sqrt{1-a^2}(r\mathrm{d}t - \rho\mathrm{d}\varphi)\right]$$

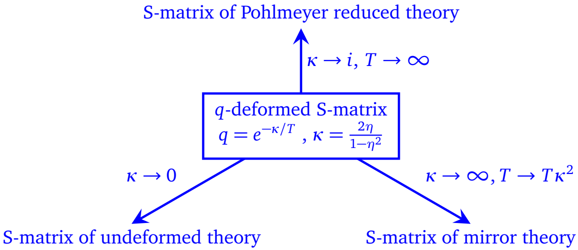
- The supergravity background corresponds to the $a=1$ point

η-deformation of $AdS_2 \times S^2$: limits

- Various limits of the 3 backgrounds

All three have the same plane-wave limit

pp wave background whose l.c.g.f. model is the Pohlmeyer reduced theory, namely the $\mathcal{N} = 2$ supersymmetric sine-Gordon model



Undeformed $\mathrm{AdS}_2 \times \mathrm{S}^2$

The 3 R-matrices lead to different maximal deformation limits, but none reproduces the mirror $\mathrm{AdS}_2 \times \mathrm{S}^2$ background!

What did this example teach us?

- R-matrices associated to inequivalent Dynkin diagrams may lead to different deformations

- The R-matrix associated to the fully fermionic Dynkin diagram is unimodular

- Accordingly, the resulting background solves the supergravity equations of motion

Questions that remain:

- Do the other supergravity backgrounds with $a \in [0, 1)$ correspond to integrable models?

- If yes, which R-matrix to use?

- Why is the maximal deformation limit not giving the mirror model?

Example of the 2-parameter deformation of the $AdS_3 \times S^3 \times T^4$ superstring

- Curved part of the background described by the supercoset

$$\frac{\hat{\mathrm{G}}}{\mathrm{H}} = \frac{\mathrm{PSU}(1,1|2)_L \times \mathrm{PSU}(1,1|2)_R}{\mathrm{SO}(1,2)\times \mathrm{SO}(3)} \quad \sim \quad \mathrm{AdS}_3 \times \mathrm{S}^3 + 16 \text{ fermions}$$

- Group-product structure $\hat{\mathrm{G}} = \hat{\mathrm{F}}_L \times \hat{\mathrm{F}}_R$ allows for a two-parameter deformation

Hoare arXiv:1411.1266

$$S_{\eta_L,\eta_R}[g \in \hat{\mathrm{G}}] \sim T(\gamma^{ij} - \epsilon^{ij}) \int \mathrm{d}^2\sigma \; \mathrm{STr}\left[g^{-1}\partial_i g \, P \frac{1}{1-\mathrm{diag}(\eta_L,\eta_R)\mathcal{R}_g P} g^{-1}\partial_j g \right]$$

$$P = \frac{2}{\sqrt{(1-\eta_L^2)(1-\eta_R^2)}} P^{(2)} + P^{(1)} - P^{(3)}$$

block-diagonal matrix realisation $\mathscr{M} = \mathrm{diag}(M_L, M_R) \in \hat{\mathfrak{g}}$

in $\hat{\mathfrak{f}}_L$ ← M_L, M_R → in $\hat{\mathfrak{f}}_R$

bi-Yang-Baxter deformation of $AdS_3 \times S^3$

Choice of Drinfel'd Jimbo R-matrix

- There are 2 unimodular R-matrices on $\mathfrak{psu}(1,1|2)$ with the desired action on the bosonic subalgebra

$$R: \left(\begin{array}{cc|cc} 0 & + & \oplus & + \\ - & 0 & - & \oplus \\ \hline - & \oplus & 0 & + \\ - & - & - & 0 \end{array}\right) \qquad \bar{R}: \left(\begin{array}{cc|cc} 0 & + & - & \oplus \\ - & 0 & - & - \\ \hline \oplus & + & 0 & + \\ - & \oplus & - & 0 \end{array}\right)$$

- They were not considered inequivalent in the $\mathrm{AdS}_2 \times \mathrm{S}^2$ case because they led to backgrounds related by analytic continuation $\mathrm{AdS} \longleftrightarrow \mathrm{S}$

- Can construct two unimodular R-matrices on $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$

$$\mathscr{R}_1 = \mathrm{diag}(R, R) \qquad \mathscr{R}_2 = \mathrm{diag}(R, \bar{R})$$

$\Rightarrow$ Do they give different supergravity backgrounds?

- Same metric and (closed) B-field

$$\kappa_{\pm} = \frac{\eta_L \pm \eta_R}{\sqrt{(1-\eta_L^2)(1-\eta_R^2)}}$$

- RR sector is different, in particular the dilaton

$$e^{-2\Phi} = e^{-2\Phi_0} \frac{F(\rho)\tilde{F}(r)}{P(\rho,r)^2} \qquad \begin{aligned} F(\rho) &= 1+\kappa_-^2(1+\rho^2)-\kappa_+^2\rho^2 \\ \tilde{F}(r) &= 1+\kappa_-^2(1-r^2)+\kappa_+^2 r^2 \end{aligned}$$

$\mathscr{R}_1$ $\quad P(\rho,r) = 1-\kappa_+^2(\rho^2-r^2-\rho^2 r^2)+\kappa_-^2(1+\rho^2)(1-r^2)$

+ a 3-form and a 5-form

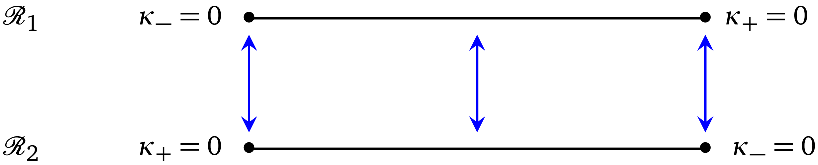
$\mathscr{R}_2$ $\quad P(\rho,r) = 1-\kappa_+^2\rho^2 r^2+\kappa_-^2(1+\rho^2 r^2)$

+ a 3-form and a 5-form

- The bosonic background is invariant under the transformations

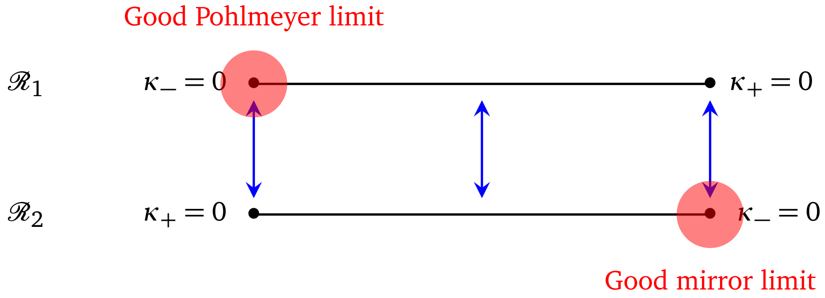
$$\rho \to i\sqrt{1+\rho^2}\ , \qquad r \to \sqrt{1-r^2}\ , \qquad t \leftrightarrow \psi\ , \qquad \varphi \leftrightarrow \phi\ , \qquad \kappa_+ \leftrightarrow \kappa_-\ .$$

- Relation between the two supergravity backgrounds



- Limits of the supergravity backgrounds at the four corners

Same plane-wave limit, but different Pohlmeyer ($\kappa \to i$) and maximal deformation ($\kappa \to \infty$) limit



What did this example teach us?

- A given bosonic background can be promoted to a supergravity background in different ways

- The two integrable supergravity backgrounds are related by an imaginary field redefinition and swapping of the deformation parameter

- One point gives the good Pohlmeyer limit, while another point gives the good mirror limit

Choice of Drinfel'd Jimbo R-matrix

- 2 R-matrices associated to the distinguished Dynkin diagram on $\mathfrak{psu}(1,1|2)$ with the desired action on the bosonic subalgebra

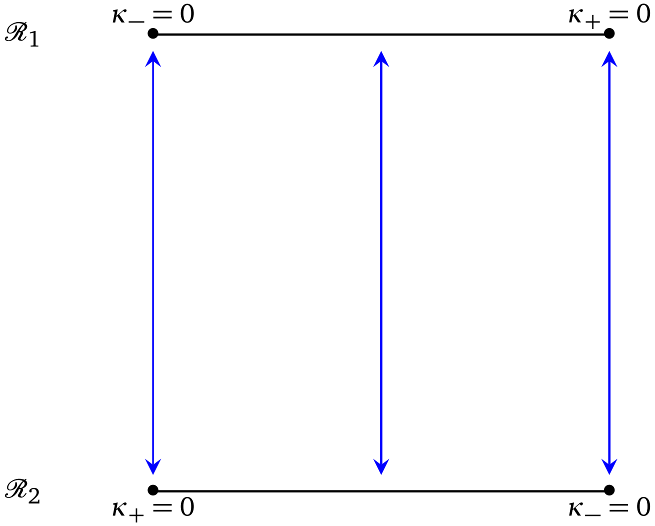
$$R_0 : \left(\begin{array}{cc|cc} 0 & \textcircled{+} & + & + \\ - & 0 & \textcircled{+} & + \\ \hline - & - & 0 & \textcircled{+} \\ - & - & - & 0 \end{array}\right) \qquad \bar{R}_0 : \left(\begin{array}{cc|cc} 0 & \textcircled{+} & - & - \\ - & 0 & - & - \\ \hline + & + & 0 & \textcircled{+} \\ \textcircled{+} & + & - & 0 \end{array}\right)$$

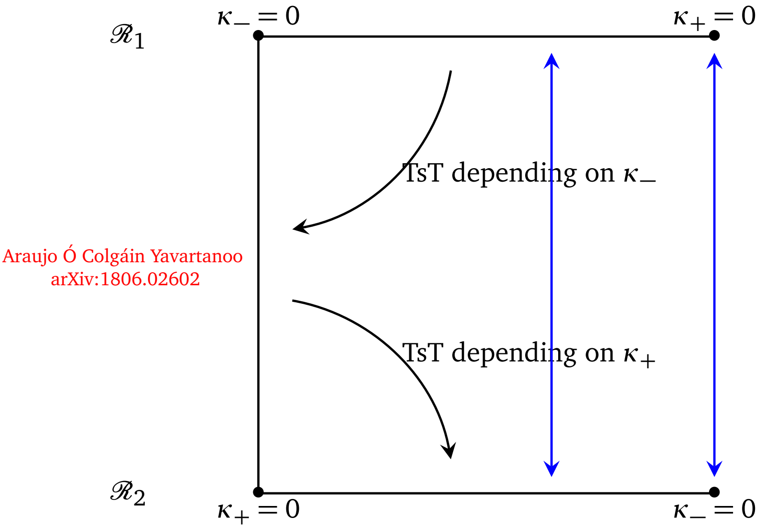
- They were not considered inequivalent in the $\mathrm{AdS}_2 \times \mathrm{S}^2$ case because they led to backgrounds related by analytic continuation $\mathrm{AdS} \longleftrightarrow \mathrm{S}$

- Can construct two R-matrices on $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$

$$\mathscr{R}_1 = \mathrm{diag}(R_0, R_0) \qquad \mathscr{R}_2 = \mathrm{diag}(R_0, \bar{R}_0)$$

$\Rightarrow$ Do they give different generalised supergravity backgrounds?


$\mathscr{R}_1$
$\kappa_- = 0$
$\kappa_+ = 0$
$\mathscr{R}_2$
$\kappa_+ = 0$
$\kappa_- = 0$


$\mathscr{R}_1$
$\kappa_- = 0$
$\kappa_+ = 0$
TsT depending on κ_-
Araujo Ó Colgáin Yavartanoo
arXiv:1806.02602
TsT depending on κ_+
$\mathscr{R}_2$
$\kappa_+ = 0$
$\kappa_- = 0$

Conclusions and open problems

- Drinfel'd Jimbo R-matrices associated to inequivalent Dynkin diagrams can lead to different backgrounds

- R-matrices associated to the fully fermionic Dynkin diagram are unimodular and give rise to a supergravity background, checked for $AdS_2 \times S^2 \times T^6$ and $AdS_5 \times S^5$

Open problems

- Is it possible to get a background that has good Pohlmeyer and mirror limit?

- Understand where the other solutions $a \in [0, 1)$ come from, within or outside the realm of Drinfel'd Jimbo solutions.

- Resolve issues with the S-matrix

Conclusions & Open problems

- Drinfel'd Jimbo R-matrices associated to inequivalent Dynkin diagrams can lead to different backgrounds

- R-matrices associated to the fully fermionic Dynkin diagram are unimodular and give rise to a supergravity background, checked for $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^6$ and $\mathrm{AdS}_5 \times \mathrm{S}^5$

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Thank You!