

Neutrino mass and the Dark side of the Universe

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the Dark Side of the Universe
Guadeloupe, March 10, 2020*



Particular properties of neutrinos:
smallness of their mass and specific large mixing
may indicate on origin of the mass connected to the
Dark sector

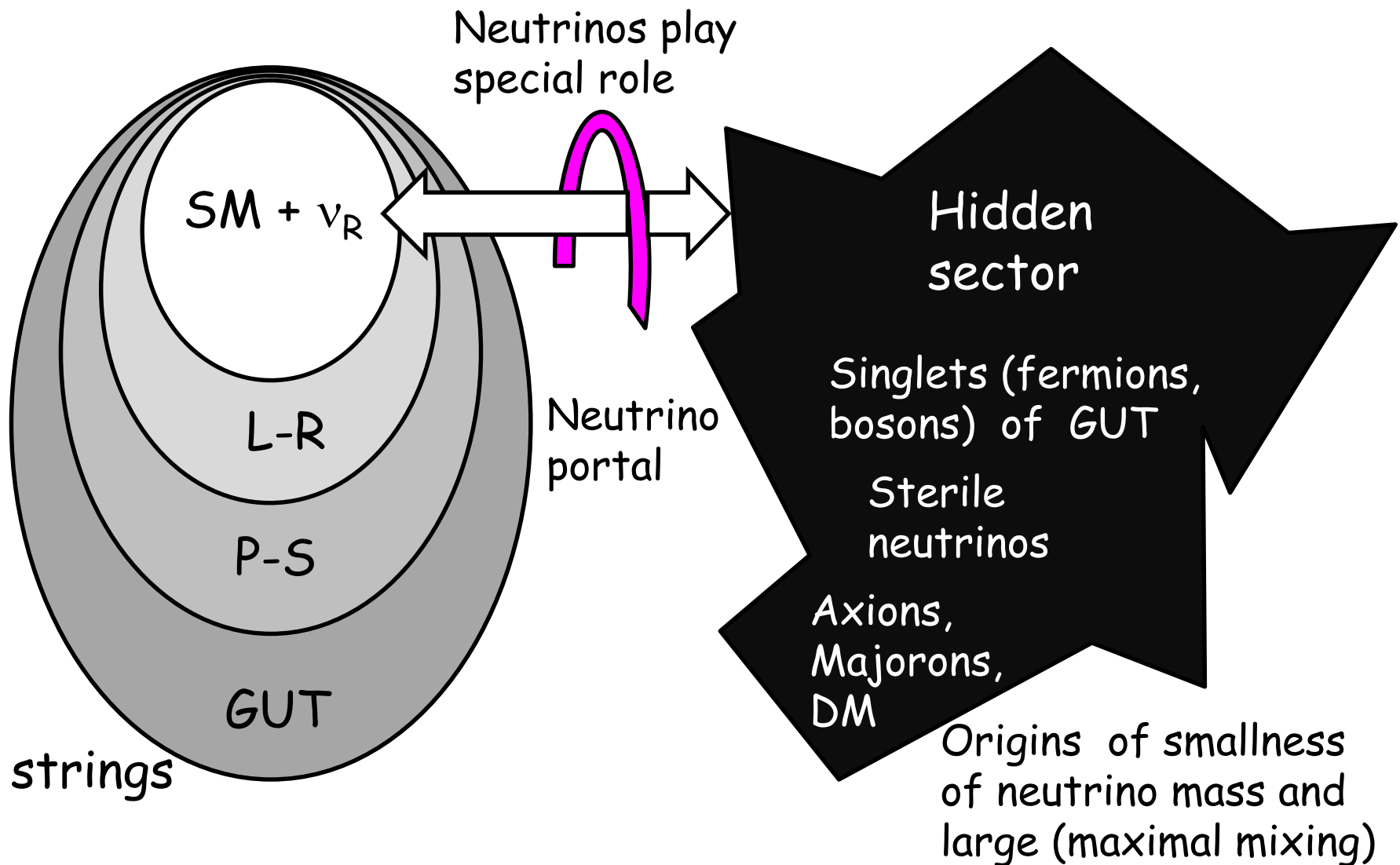
In this connection:

Neutrino mass from
the dark sector

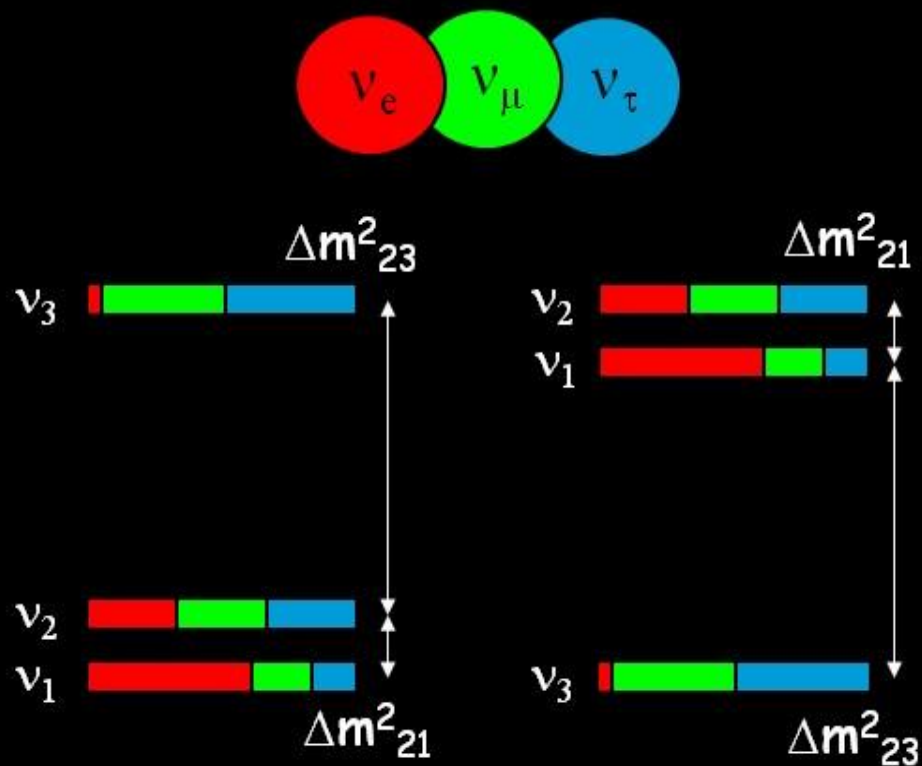
Massless neutrino
oscillations from
interactions with dark
universe

Show the range of possibilities under discussion

Mass and mixing from the hidden world



I. Neutrino mass from dark sector



Observation

*H. Minakata, A Y S, Z - Z. Xing
J Harada, S Antusch, S. F. King
Y Farzan, A Y S, M. Picariello ...*

The data are in a good agreement with the relation between the lepton and quark mixing matrices:

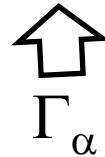
S. Petcov, AYS, 1993

C. Giunti, M. Tanimoto

$$U_{\text{PMNS}} \sim U_I^\dagger U_X$$

$$U_I \sim V_{\text{CKM}}$$

quark mixing
matrix



diagonal
phase matrix

$$U_X = U_{\text{BM}}, U_{\text{TBM}}$$

matrices of special form
dictated by symmetry

$$U_X = U_{23}(\pi/4)U_{12}, \theta_{13}^X \sim 0$$

reproduces approximately the quark-lepton complementarity, QLC
gives prediction for 1-3 mixing

$$\sin \theta_{13} \sim \sqrt{\frac{1}{2}} \sin \theta_c$$

$$\sin^2 \theta_{13} \sim \frac{1}{2} \sin^2 \theta_c$$

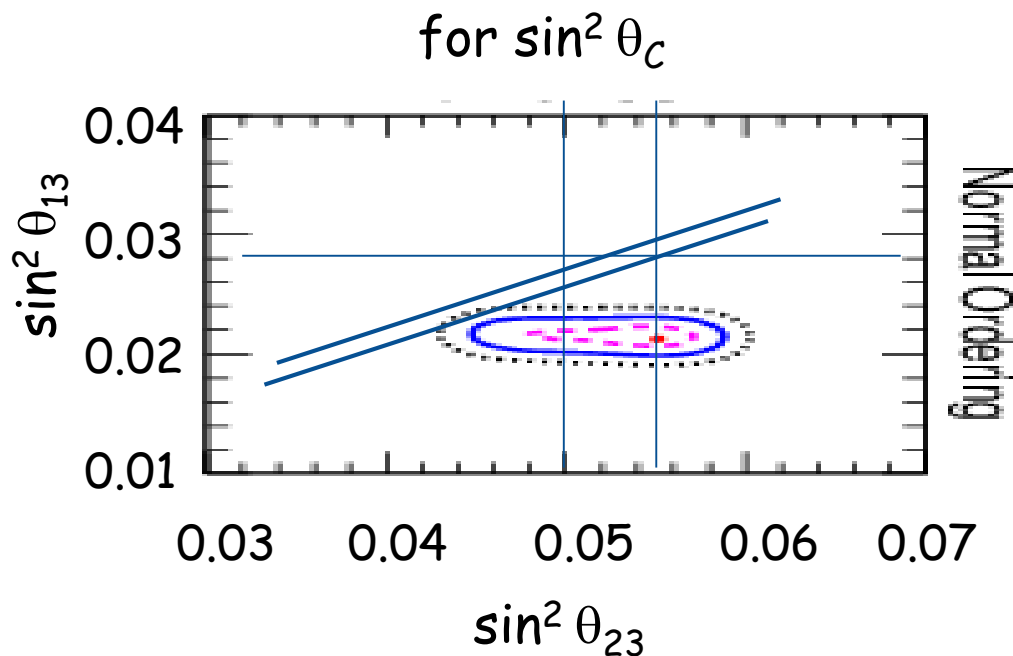
In general,

$$\sin^2 \theta_{13} = \sin^2 \theta_{23} \sin^2 \theta_c (1 + O(\lambda^2))$$

Experimental status

From global fit

*F. Capozzi, et al. Prog.Part.Nucl.Phys.
102 (2018) 48, arXiv:1804.09678 [hep-ph]*



$\sim 20\%$ deviation in $\sin^2 \theta_{13}$

can be due to deviation
of θ_{12}^l from θ_c
which in turn is related
to difference of q and l -
masses

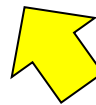
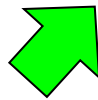
Renormalization (RGE)
effects from GUT
scales to low energies

$$\sin^2 \theta_{13} = \sin^2 \theta_{23} \sin^2 \theta_c (1 + O(\lambda^2))$$

lines: predictions from QLC

U_X from the dark sector

$$U_{\text{PMNS}} \sim V_{\text{CKM}}^\dagger U_X$$



Common sector for quarks and leptons. Implies

$$m_l \sim m_d \quad m_{\nu}^D \sim m_u$$

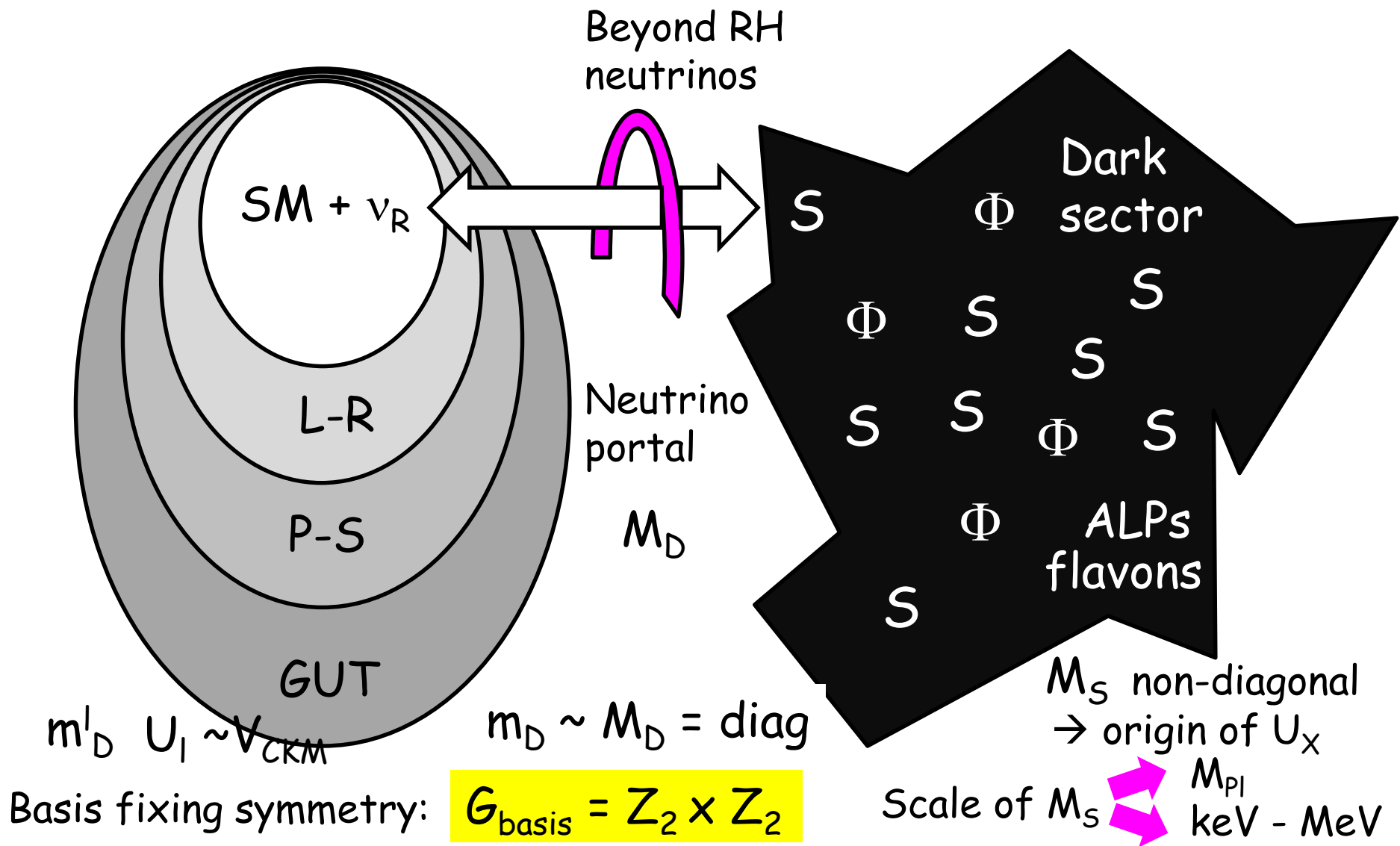
From the dark sector coupled to neutrinos. Responsible for large neutrino mixing
smallness of neutrino mass

Q - L unification, GUT

CKM physics, hierarchy, of masses and mixings
Froggatt-Nielsen (?), relations between masses and mixing

may have special symmetries which lead to BM or TBM mixing

Mass and mixing from the hidden world



Realizations: double or inverse seesaw

R.N. Mohapatra
J. Valle

Simplest case: three singlets S (combinations of S) which couple to RH neutrinos \rightarrow inverse or double seesaw

$$M_S \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_S \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ S \end{pmatrix}$$

$M_S \ll M_D$ - inverse seesaw

$M_S \gg M_D$ - double seesaw

RH neutrinos get mass via see-saw

$$M_R = M_D^T M_S^{-1} M_D$$

if $M_S \sim M_{\text{Pl}}$, $M_D \sim M_{\text{GUT}}$

For light neutrinos

$$m_\nu = m_D^T M_D^{-1T} M_S M_D^{-1} m_D$$

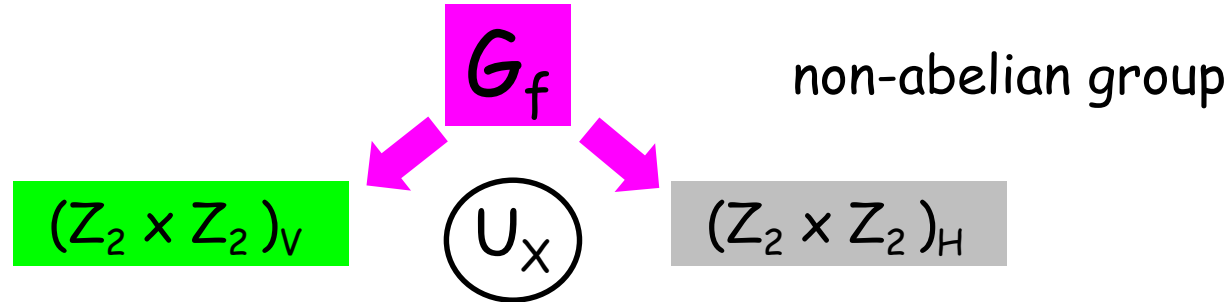
$$\text{If } m_D = A M_D \quad m_\nu = A^2 M_S$$

Structure of m_ν is determined by M_S , it does not depend on the Dirac mass matrix structure (Dirac screening)

Embedding

D. Hernandez, A.S. 1204.0445
B. Bajc, A.S.

To fix U_X one can use the residual symmetry approach:



Embedding leads to general expression (via symmetry group condition)

$$|(U_X)_{ij}|^2 = \cos^2 \frac{\pi n_{ij}}{p_{ij}} \quad p, n \text{ -integer}$$

Unitarity condition

$$\sum_i \cos^2 \frac{\pi n_{ij}}{p_{ij}} = 1 \quad \text{similar for the sum over } j$$

This allows to reconstruct the matrix U_X up to discrete number of possibilities. BM mixing is among them

A GUT scheme with $G_{\text{hidden}} = S_4$

Xun-Jie Xu, A.S

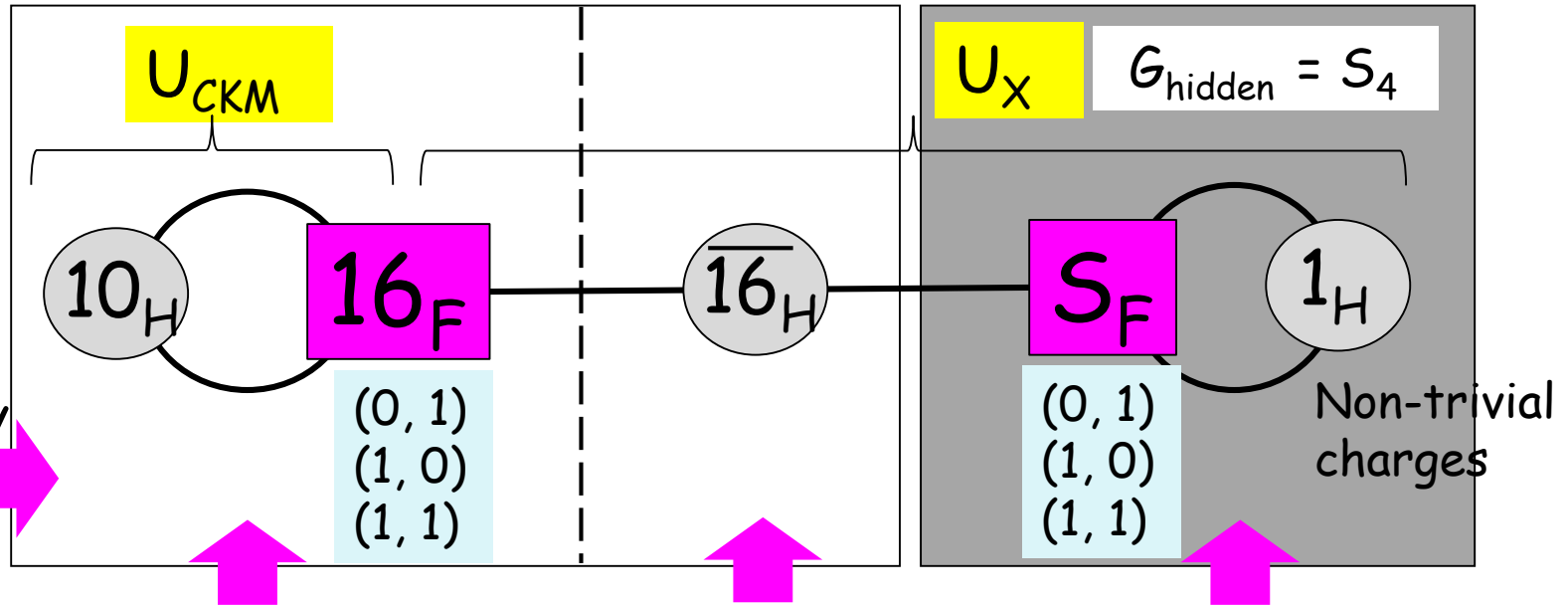
and BM mixing

$SO(10)$

Visible sector

Portal

Hidden sector



mass hierarchy

no mixing

$Z_2 \times Z_2 \subset S_4$

CKM mixing -
additional
structure

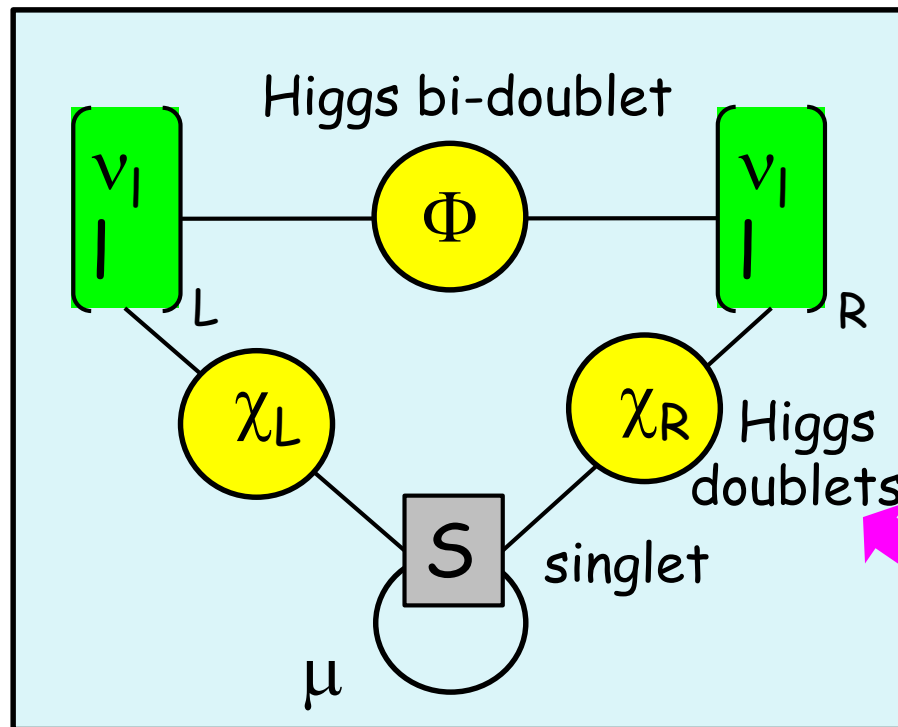
$M_S \sim M_{\text{Pl}}$
 $M_D \sim M_{\text{GUT}}$
 $m_D \sim M_D = \text{diag}$
 Double seesaw

Spontaneously
broken by flavons
 $M_S \sim M_{\text{BM}}$

Low scale Left-right symmetric model

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$$

with q-l similarity $m_q \sim m_l \sim m_{\nu}^D$ - inverse seesaw



with Majorana mass terms

Fields	L_L	L_R	χ_L	χ_R	S
$B - L$	-1	-1	1	1	0

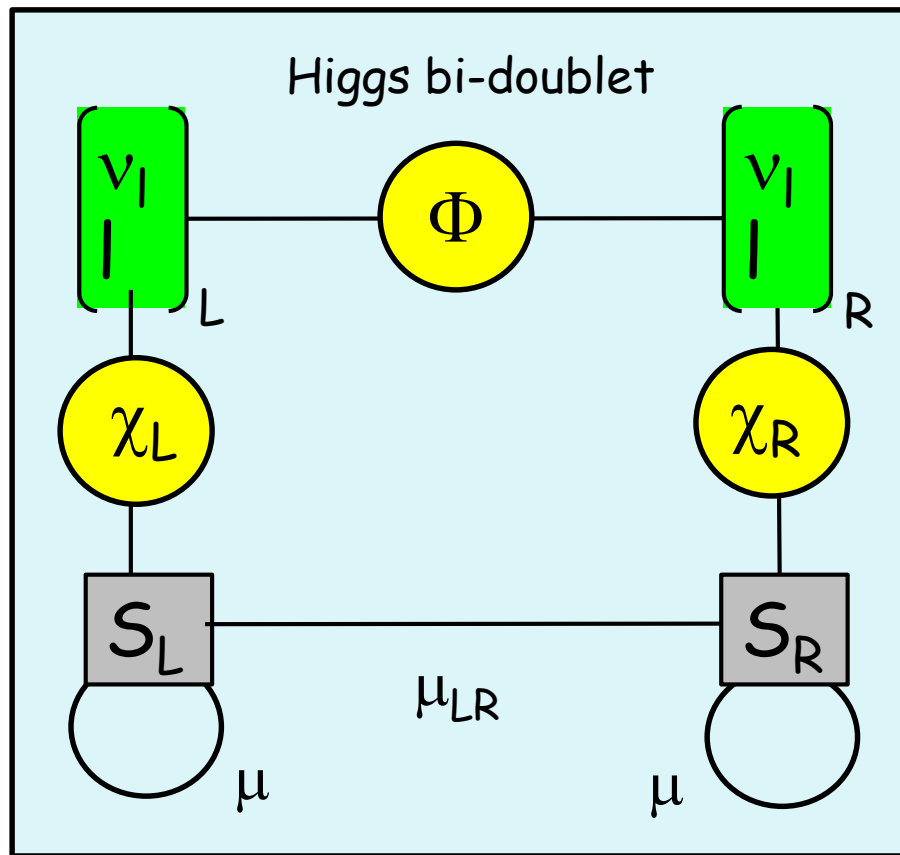
$$M_D \sim M_R \sim \text{PeV}$$

$$\mu \sim 10 \text{ keV}$$

inverse seesaw
flavor symmetry in μ

Model with "left and right" singlets

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$$



invariant under global $U(1)$

Fields	L_L	L_R	S_L	S_R
L	1	1	-1	-1

broken by μ -terms

keV scale sterile neutrino
- DM

Problems and solution?

No connection to masses

Mass hierarchy from additional symmetries

U(1) Froggatt- Nielsen mechanism

$$\left(\frac{\phi}{\Lambda}\right)^n$$

power n is determined by U(1) charges
of the corresponding operators

Discrete symmetries - restricted possibilities to explain
mass spectrum (degenerate, partially degenerate spectra)

modular symmetries

Non-linear realization of flavor symmetries

Yukawa couplings: functions of moduli fields

γ transform as superfields

modular forms

$$\varphi^I \rightarrow (c\tau + d)^{k_I} \rho(\gamma) \varphi^I$$

k_I is the weight of multiplet

$\rho(\gamma)$ is the representation of γ element of the group Γ_N

c, d parameters of transformation γ

For terms of potential $\gamma \varphi_1 \varphi_2 \varphi_3$ invariance requires

$$\rho_1 \times \rho_2 \times \rho_3 \times \rho_\gamma = \mathbb{I}$$

$$\sum_i k_i + k_\gamma = 0$$

for weights

Additional condition which acts as Froggatt- Nielsen factors

Yukawa couplings form multiplets they are fixed by symmetry

Model building

Another organization of Dark sector

It can be considered as special case neutrino mass generation with multiple RH neutrinos

Resembles generation due to extra dimension in deconstruction mode

Clockwork mechanism

fast
rotation



slow
rotation

Strong hierarchy (small quantities)
without small parameters

G. Giudice, et al

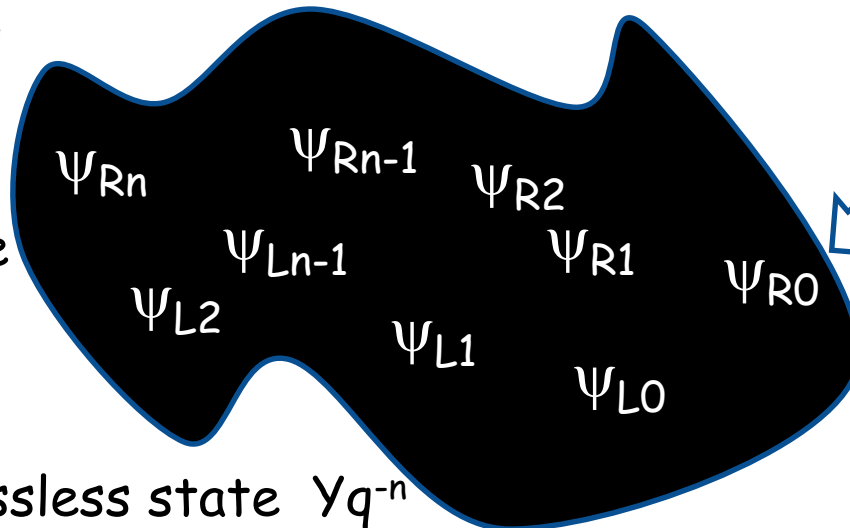
A. Ibarra et al
1711.02070 [hep-ph]

ν_L



Mixing of massless state
in ψ_{Rn} is suppressed
by factor q^n , $q > 1$

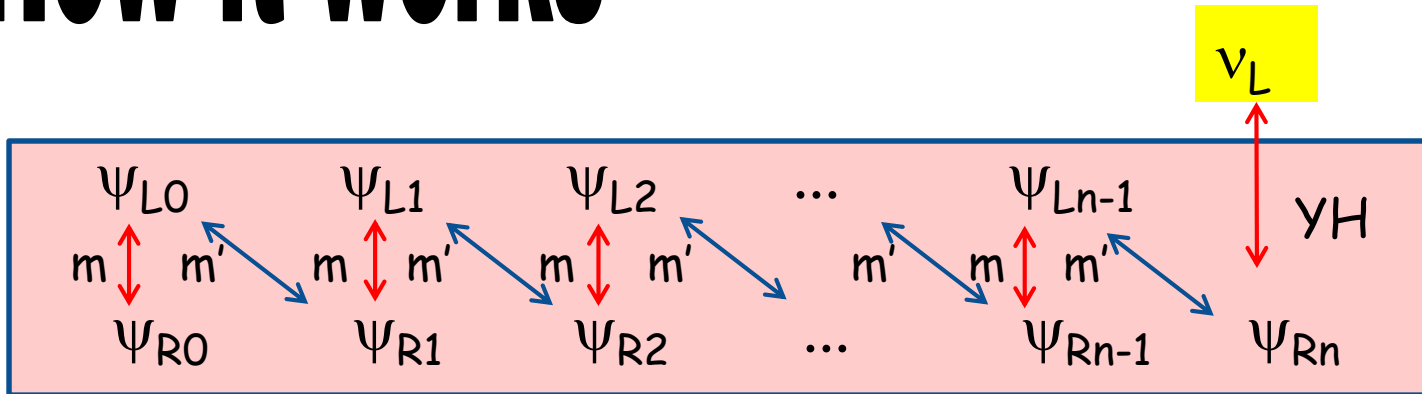
Yukawa coupling with massless state Yq^{-n}



one massless
state mostly
here

$$m_\nu = \frac{1}{q^n} Y \langle H \rangle$$

How it works



$$q = m'/m > 1 \quad \text{gear}$$

$$\begin{array}{c}
 \Psi_{R0} \\
 \Psi_{R1} \\
 \Psi_{R2} \\
 \dots \\
 \Psi_{Rn}
 \end{array}
 \begin{pmatrix}
 \Psi_{L0} & \Psi_{L1} & \Psi_{L2} & \dots & \Psi_{Ln-1} & \Sigma \\
 1 & 0 & 0 & 0 & \dots & q^n \\
 -q & 1 & 0 & 0 & \dots & q^{n-1} \\
 0 & -q & 1 & 0 & \dots & q^{n-2} \\
 \dots & & & & 1 & \dots \\
 0 & \dots & & 0 & \dots & -q & 1
 \end{pmatrix}
 \times$$

Mixing of massless
state in Ψ_{Rn}

$$1/N$$

$$m_v = Y \langle H \rangle / N$$

Suppression factor

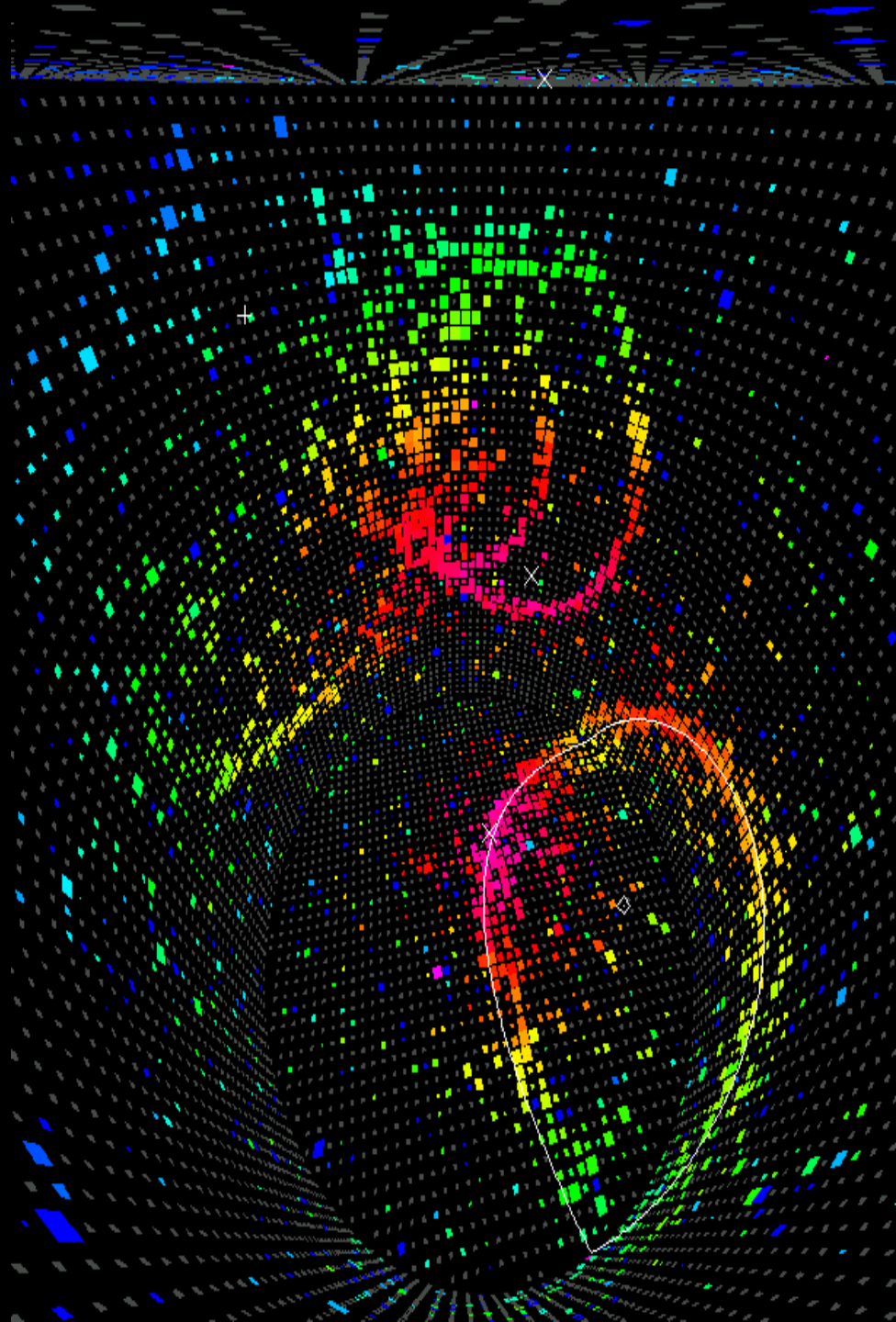
Massless state

$$(q^n \Psi_{R0} + q^{n-1} \Psi_{R1} + q^{n-2} \Psi_{R2} + \dots + \Psi_{Rn})/N$$

Normalization: $N^2 = \sum_{0 \dots n} q^{2j}$

$$\frac{1}{q^n} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2n}}}$$

III. Neutrino mass from interactions with dark matter



Question:

Do the observed oscillation effects imply neutrino mass?

Can oscillation data be described without neutrino mass?

Wolfenstein: oscillations of massless neutrinos.

Require non-standard interactions (NSI)

If mediators of NSI are heavy
(effective 4 fermion interactions)



Matter effects (potentials) do
not depend on neutrino energy

In contrast, experiments do show dependence of the oscillation
effects on neutrino energy - presence of terms in the Hamiltonian

$$H = \dots + \frac{m^2}{E}$$

Atmospheric neutrinos

Solar neutrinos*

KamLAND

Daya-Bay, RENO } 

Explicit oscillatory dependence
for fixed baseline

NSI with $1/E$ dependence

Changing
paradigm

$1/E$ dependence of the matter potential appears if

$$s \gg (m_{\text{mediator}})^2$$

"above resonance"

Square of total energy
in the center of mass

mass of
mediator

For SM weak interactions with W boson as mediator and scattering on relic neutrinos the condition is satisfied for $E > 10^{21}$ eV

C. Lunardini, A.S.

Existing oscillation data are for $E > E_b = 0.1$ MeV (solar pp-neutrinos)

$$s = 2Em_{\text{target}}$$

The $1/E$ condition can be satisfied for light enough mediators

S.F. Ge, H. Murayama, E. J. Chun, Ki-Yong Choi, Jongkuk Kim,,,

Can scattering with $1/E$ dependence of the potential substitute neutrino mass completely?

Interaction with background

Refraction on BG particles

Refraction on known particles due to new long range forces

Relic neutrinos

Dark matter particles, other particles of the Dark sector

Neutrino interaction with DM



Via light mediators

Immediately

Generic
problem

$$m_\nu(z) \sim n_{\text{DM}}(z)$$

$$m_\nu(z) \sim \sqrt{n_{\text{DM}}(z)}$$

Mass increases becoming $> 5 \text{ MeV}$ and $z = 1000$
Problem with structure formation

Mass in cosmology

$$m'(z) \sim \sqrt{n(z)} \quad n(z) = n_0 (1+z)^3$$

$$m'(z) \sim [\xi (1+z)^3]^{1/2} m'_{\text{local}}$$

$1/\xi \sim 10^5$ - enhancement factor for DN density in the Galaxy near the Earth

In the epoch of matter-radiation equality, $z = 1000$, DM should already be formed and structures start to form

For $m'_{\text{local}} = 0.05 \text{ eV}$ $m'(1000) \sim 5 \text{ eV}$

- violates cosmological bound on sum of neutrino masses

Inversely, if $m'(1000) \sim 0.3 \text{ eV}$, $m'_{\text{local}} = 0.003 \text{ eV}$ which is too small to explain neutrino masses. Still can give an observable corrections.

Effective mass due to interactions with dark matter

$$L = - g_X \phi \bar{X} X - g_\nu \phi \bar{\nu} \nu - m_X X \bar{X} - \frac{1}{2} m_\phi^2 \phi^2$$

*H. Davoudiasl, et al
1803.0001 [hep-ph]*

ϕ - very light scalar field producing long (astronomical) range forces
 X - Dark matter particle (fermion of GeV mass scale) source of the scalar field.


$$m_\nu = g_\nu \phi$$

From equation of motion for ϕ , with neutrino contribution to generation of ϕ neglected

$$\phi = - \frac{g_X n_X}{m_\phi^2}$$

$m_\phi = 10^{-20} - 10^{-26}$ eV is mass of scalar
 $n_X = \langle \bar{X} X \rangle$ is the number density of X

ρ_X - energy density of DM


$$m_\nu = \frac{g_X g_\nu \rho_X}{m_\phi^2 m_X}$$

$$g_X = g_\nu = 10^{-19} \rightarrow m_\nu = 0.1 \text{ eV}$$

Mass depends on local density of DM and different in different parts of the Galaxy and outside

Resolving problem of mass increase

If neutrinos, as source of ϕ , are not neglected, eq. of motion gives

$$\phi = \frac{-g_X n_X}{m_\phi^2 + g_v^2 n_v / \langle E_v \rangle}$$

*H. Davoudiasl, et al
1803.0001 [hep-ph]*

In the Early Universe before structure formation neutrino term (in denominator) can dominate

$$m_v = \frac{-g_X n_X \langle E_v \rangle}{g_v n_v}$$

$$m_v = g_v \phi$$

Forces between neutrinos and DM are repulsive. In later phases DM clumps can drive out the neutrino background

Density of relic neutrinos in Galaxy and in solar system can be strongly suppressed. PTOLEMY ?

Immediate interactions with Dark Matter

DM: complex scalar field ϕ with mass m_ϕ

$$L = g \bar{\nu}_L f_R \phi + m_f \bar{f}_L f_R + h.c.$$

f - Dirac fermion

If $m_f = 0$, f_L decouples, while ν_L and f_R can form Dirac neutrino

The interaction can be generated via mixing of ϕ with Higgs boson

We assume zero VEV $\langle \phi \rangle = 0$

In general (depending on production) the field has two components

$$\phi = \phi_c + \phi_q$$

Classical - coherent state
of quanta, condensate

$$\phi \sim \sqrt{n_\phi/m_\phi}$$

Quantum (incoherent)
state

Effect of classical component

A. Berlin,
1608.01307 [hep-ph]

Ultra-light scalar DM, large number density - as a classical field, solution

$$\phi(t, x) \sim \frac{\sqrt{2 \rho(x)}}{m_\phi} \cos(\omega t - k x)$$

$\omega \sim m_\phi$ $k = m_\phi v$ $v \sim 10^{-3}$ - virialized velocity in the Galaxy

generates the mass term

$$m' = g \phi_c$$

$$m' v_L f_R + m_f f_L f_R + h.c.$$

Oscillating mass with period $T_{osc} = \frac{2\pi}{m_\phi} = 4 \cdot 10^{-15} \text{ sec} (1 \text{ eV}/m_\phi)$

Lost of coherence due to velocity dispersion $\Delta v \sim v \Rightarrow \Delta\omega = m_\phi v \Delta v \sim m_\phi v^2$

Coherence time: $\tau_{coh} = \frac{2\pi}{\Delta\omega} = 4 \cdot 10^{-9} \text{ sec} (1 \text{ eV}/m_\phi)$

Coherence length: $L_{coh} = \frac{2\pi}{\Delta v m_\phi} = 1.2 \cdot 10^{-3} \text{ m} (1 \text{ eV}/m_\phi)$

System transforms in the cold gas of individual scatterers. Still in some aspects can be considered as classical field without t variations

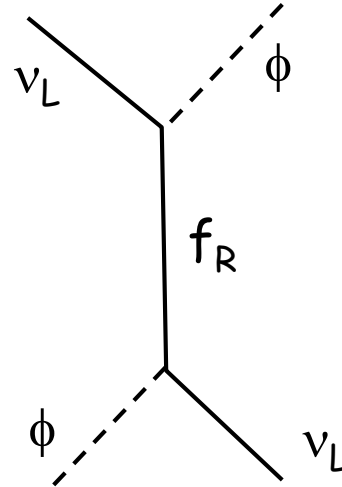
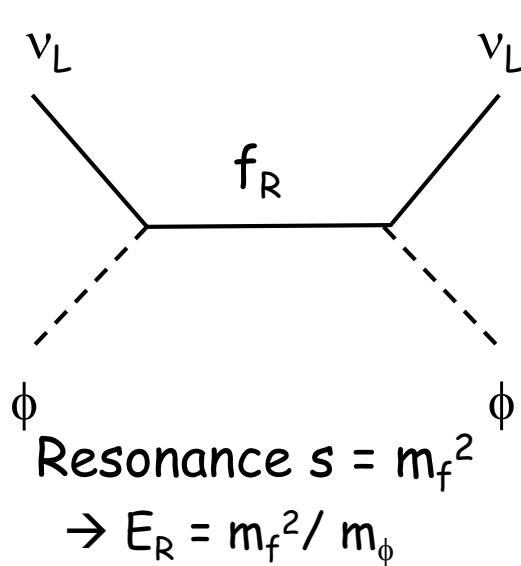
Neutrino Refraction on DM

*S. F Ge and H Murayama,
1904.02518 [hep-ph]*

*Ki-Yong Choi, Eung Jin Chun,
Jongkuk Kim, 1909.10478
[hep-ph]*

Neutrino scattering
on DM particles

+ scattering on ϕ^*



Elastic forward scattering - potential

$$V \sim \frac{s - m_f^2}{(s - m_f^2)^2 + s \Gamma^2} + \frac{1}{u - m_f^2}$$

$$\Gamma = \frac{g^2}{32 \pi} m_f$$

Matter potential

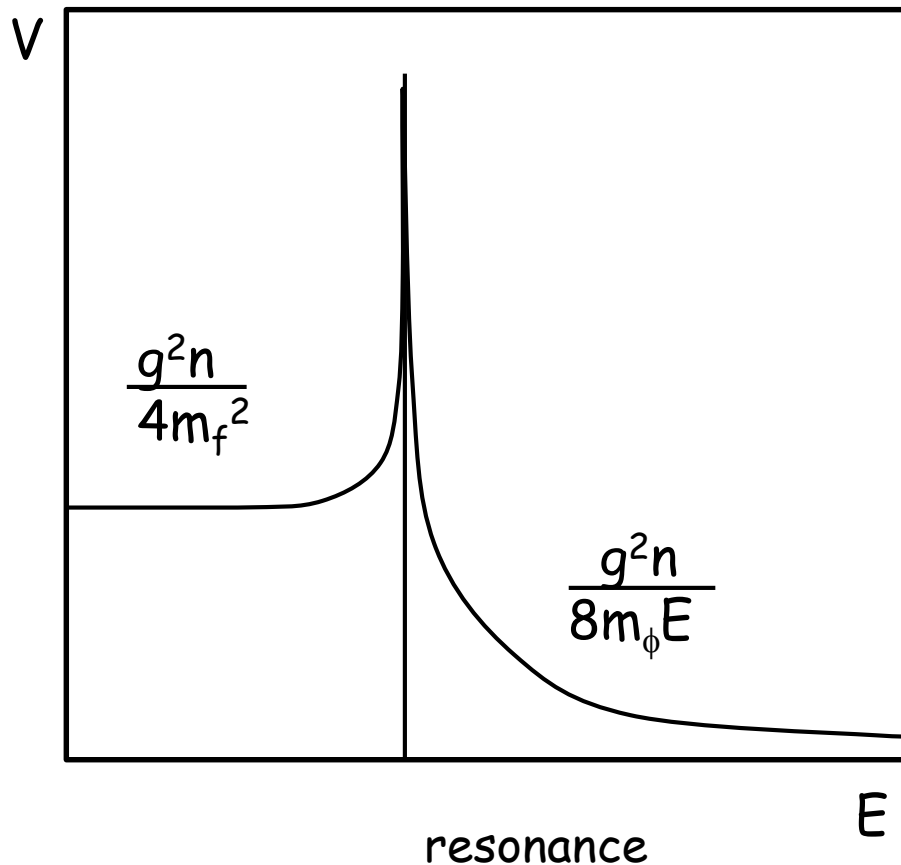
$$V \sim \frac{g^2 \rho}{4 m_\phi} \frac{+/- \varepsilon m_f^2 - 2 m_\phi E}{m_f^4 - 4 m_\phi^2 E^2}$$

$$\varepsilon = (n - \bar{n}) / (n + \bar{n})$$

- charge asymmetry of DM

V is non-zero even in C symmetric DM

$$m'^2 = 2 E V$$



Effective mass

$$\text{For } E > E_R \quad V = \frac{m'^2}{2E} \quad m'^2 = \frac{g^2 n}{4 m_\phi} = \frac{g^2 \rho}{4 m_\phi^2}$$

Condition for correct value of neutrino mass: $m'^2 = \Delta m_{31}^2$

$$\frac{g}{m_\phi} = \sqrt{\frac{\Delta m_{31}^2}{\rho}}$$

Condition for $1/E$ dependence: $E_R < E_B < 0.1 \text{ MeV}$

$$m_f < \sqrt{2 m_\phi E_B}$$

Phenomenology. Bounds on parameters

ν - DM inelastic scattering

$$\sigma = \frac{g^2}{16\pi} \begin{cases} \frac{s}{m_f^4} & s \ll m_f^2 \\ \frac{1}{s} & s \gg m_f^2 \end{cases}$$

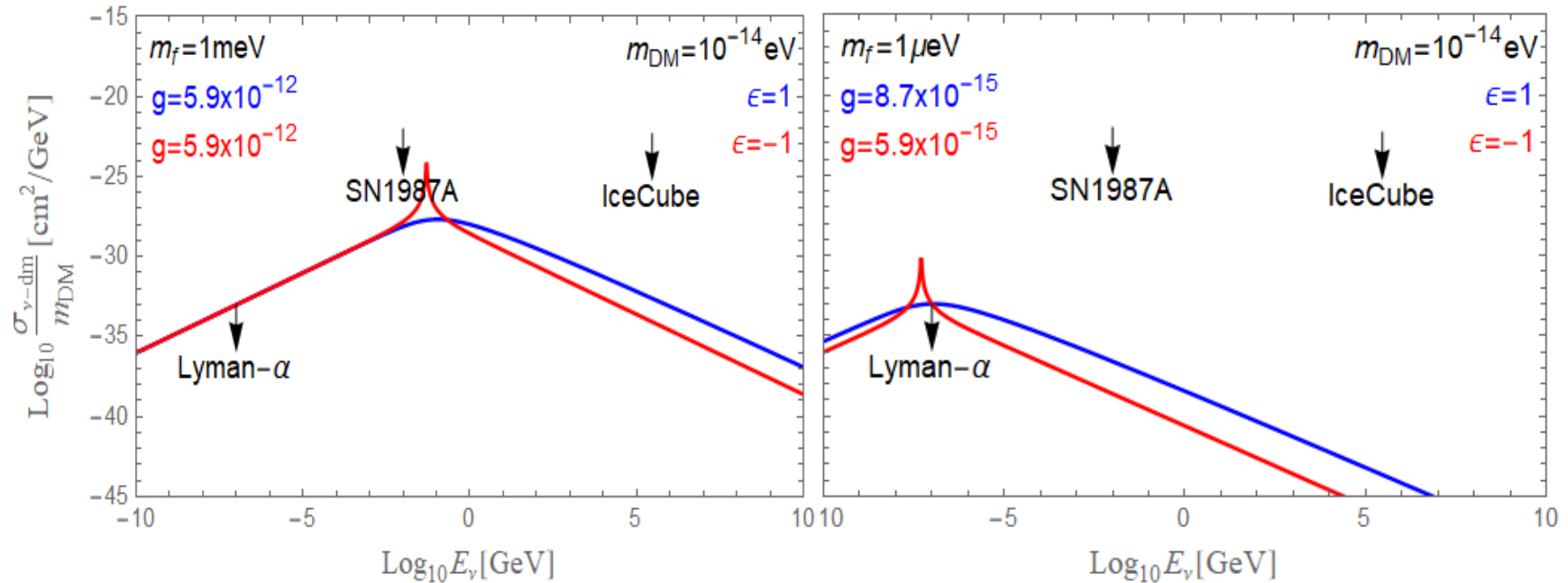
Upper phenomenological bounds on σ / m_ϕ

$\sigma / m_\phi < \xi$ for certain neutrino energies E_ξ

$$m_f > g \left(\frac{E_\xi}{8\pi \xi} \right)^{1/4} \quad m_\phi < m_f^2 / 2E_\xi$$

$$m_\phi > g^2 \left(\frac{1}{32\pi E_\xi \xi} \right)^{1/2} \quad m_\phi > m_f^2 / 2E_\xi$$

Bounds from neutrino DM interactions



The most stringent bound
from Ly α
relic neutrinos

$$\xi < 10^{-33} \text{ cm}^2 / \text{GeV}$$

R.J. Wilkerson, C. Boehm, L. Lesgourges
JCAP 1405 (2014) 011

SN87A, $E = 10 \text{ MeV}$

Ice Cube

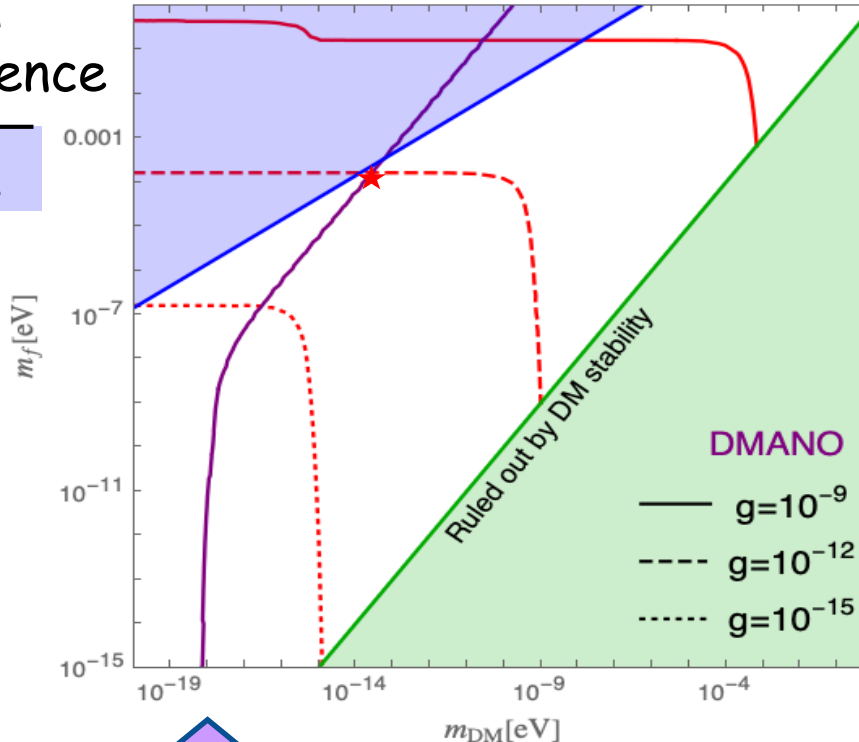
Relic SN neutrinos

Stability of DM

Bounds on parameters

Bound from
1/E dependence

$$m_f < \sqrt{2m_\phi E_B}$$



$$m_\phi > m_f$$

Red lines - lower bounds
on masses of DM and
mediator from Ly α - data
for different values of g

Lower bound for parameters which can reproduce
observed oscillation effects

Bounds

$$m_f < 10^{-4} \text{ eV} \quad m_\phi < 10^{-13} \text{ eV} \quad g < 10^{-12}$$

Effective neutrino masses

$$\begin{pmatrix} \nu \\ f_L \\ f_L^c \end{pmatrix} \begin{pmatrix} 0 & 0 & m' \\ 0 & 0 & m_f \\ m' & m_f & 0 \end{pmatrix} \quad m' = g \phi_c - \text{the induced mass}$$

Features:

massless state f_0

$\nu - f_L$ mixing $\nu = c\nu_m + sf_0 \quad \tan\theta = m_f / m'$

ν_m and f_L^c form Dirac neutrino with mass

$$m(\nu_m)^2 = m'^2 + m_f^2 = \Delta m^2$$

$\nu - f_L$ oscillations (active - sterile) with

$$\Delta m^2 = m(\nu_m)^2 = \Delta m_{31}^2$$

in the case of mass hierarchy

Effective mass vs. scattering

$$M M^+ = \begin{pmatrix} m'^2 & m_f m' & 0 \\ m_f m' & m_f^2 & 0 \\ 0 & 0 & m'^2 + m_f^2 \end{pmatrix}$$

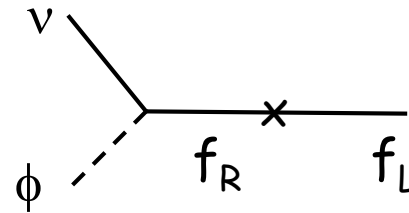
$$m (v_m)^2 = m'^2 + m_f^2$$

m'^2

$v \phi \rightarrow v \phi$

$m_f m'$

$v \phi \rightarrow f_R \rightarrow f_L$



Coherence:

States of medium with ϕ being absorbed from different space-time points are coherent once $\Delta x < \lambda_{DB} = 2\pi/v m_\phi$

Energy - momentum conservation ok within $\Delta p < 1/L$ baseline

Refraction in coherent field removes resonance

Cosmological mass increase . Way out:

Late phase transition ?

Lost of coherence in the field

transition $\nu \phi \rightarrow f_R \rightarrow f_L$ and off-diagonal term in MM^+ disappears

Resonance and dependence below resonance are restored

Below resonance (for $\varepsilon = 1$) :

$$V = \frac{g^2 n}{4 m_f^2} \quad m'^2 = 2VE \quad m' = \sqrt{\frac{g^2 n E}{2 m_f^2}}$$

Effective mass decreases with energy:

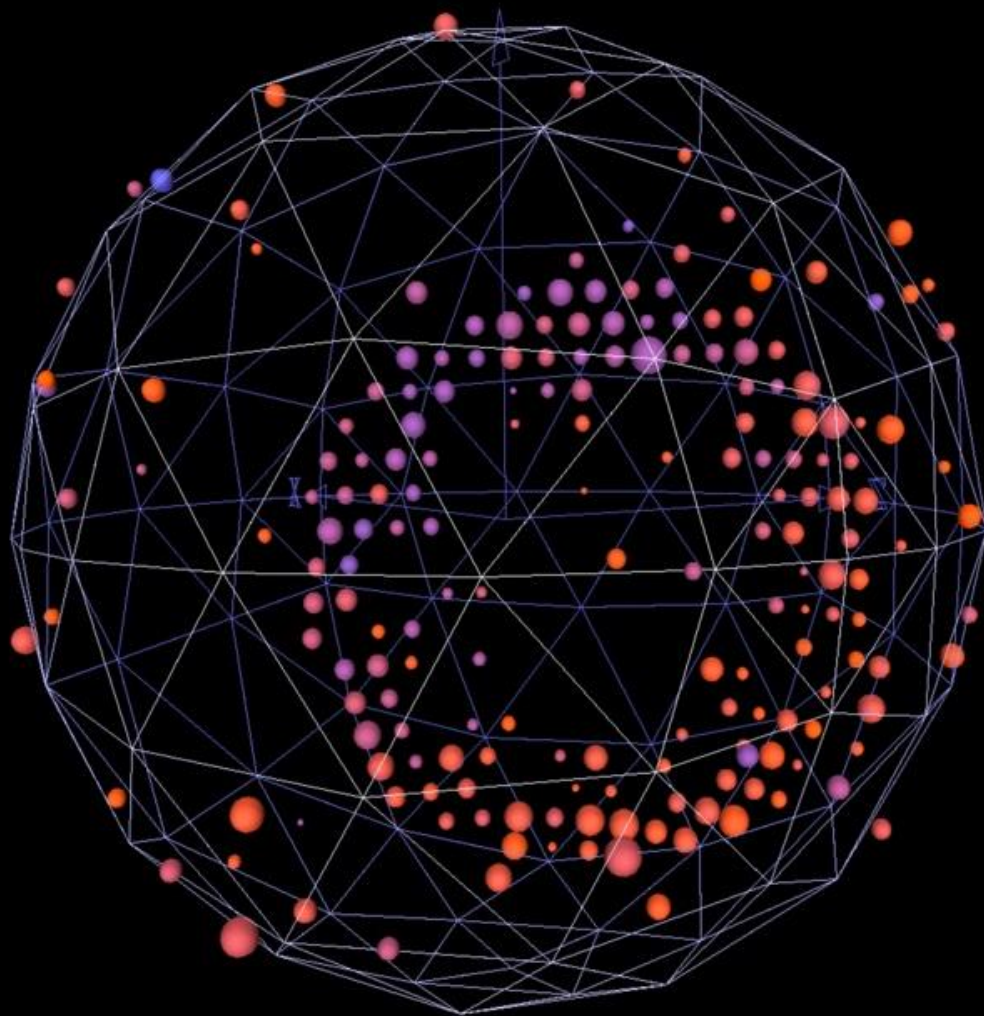
$$m'(E) = m'(> E_R) \sqrt{\frac{E}{E_R}}$$

$$\text{For } E_R = 0.1 \text{ MeV and } E = 1 \text{ eV} \quad m' < 2 \cdot 10^{-4} \text{ eV} \quad m'^2 = 2 V E$$

In KATRIN neutrino masses are not measurable

Relic neutrinos $m' < 2 \cdot 10^{-6} \text{ eV}$ no problem with structure formation

Conclusions



Nature and origins of ν - mass are still not known

There are some indications that neutrino properties, are related to the Dark sector of the Universe

Small mass and large mixing can be generated via RH neutrino portal by the Dark (Hidden) sector at the Planck scale, or at much lower energies of 10^5 GeV in the framework of L-R symmetric models

Nature of what we observe as a mass in oscillations may differ from masses of other particles

It can be generated due to refraction on particles of DM

Nontrivial neutrino mass dynamics which can depend on E and z : relevant for some cosmological problems?

Backup

Basis fixing symmetry and mixing

Higgs multiplets of visible sector are singlets of $G_{\text{basis}} = Z_2 \times Z_2$
the charges of generations can be selected such that

$$m_D \sim M_D = \text{diagonal}$$

Flavons Φ are charged with respect to G_{basis} and
spontaneously break $G_{\text{basis}} \rightarrow$ non-diagonal $M_S \rightarrow$ mixing U_X

$G_{\text{basis}} = Z_2 \times Z_2$ is a part of intrinsic symmetry of Majorana mass
mass matrix $(Z_2)^3$ which is always present!

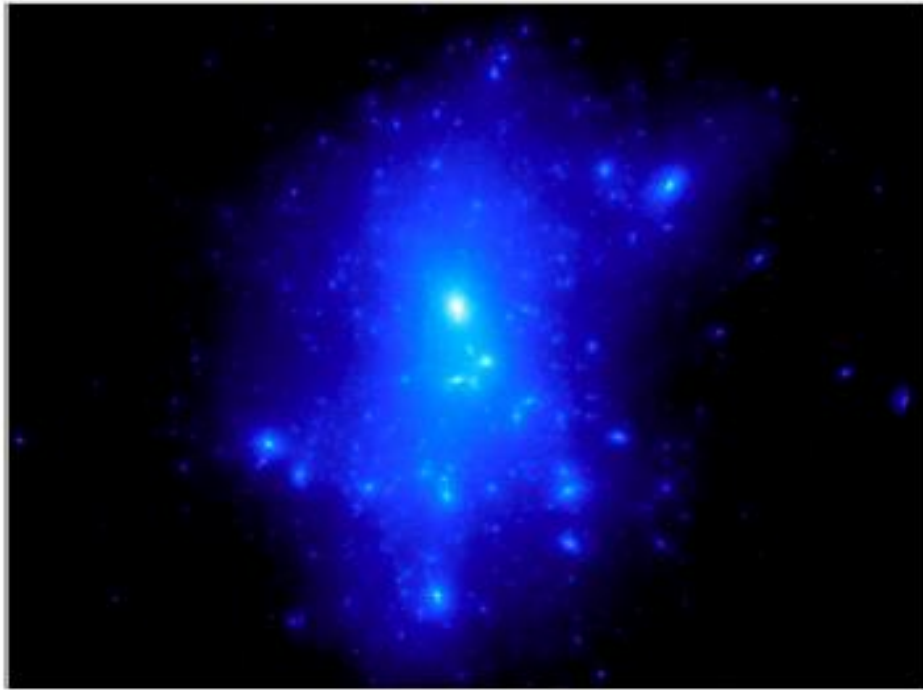
For free!

In the basis fixed by G_{basis} : m_D, M_D - diagonal, M_S is non-diagonal.
 M_S is diagonalized by U_X and has another unbroken symmetry $(Z_2 \times Z_2)_H$

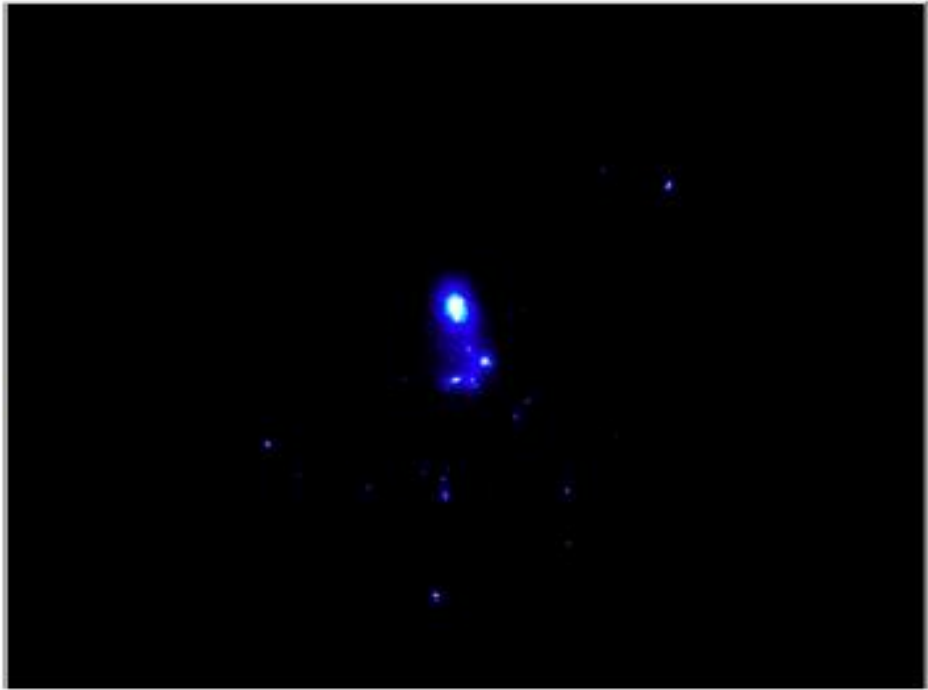
U_X connects bases determined by $(Z_2 \times Z_2)_V$ and $(Z_2 \times Z_2)_H$

Residual symmetry approach

Equivalence of classical field - state of quantum system



DM **decay** signal from a galaxy



DM **annihilation** signal from a galaxy

Medium generated mass

Due to interactions with new light scalar fields

m_ν



Interactions with usual matter (electrons, quarks) due to exchange by very light scalar



Interactions with scalar field sourced by DM particles



Interaction with "Fuzzy" dark matter

Strongly restricted

Interactions with fuzzy dark matter

A. Berlin,
1608.01307 [hep-ph]

Ultra-light scalar DM, huge density ρ - as a classical field, solution

$$\phi(t, x) \sim \frac{\sqrt{2\rho(x)}}{m_\phi} \cos(m_\phi t)$$

Mass
states
oscillate

Coupling to neutrinos $g_\phi \phi \nu_i \nu_j + \dots$

gives contribution to neutrino mass and modifies mixing

$$\delta m(t) = g_\phi \phi(t)$$

$$\Delta\theta_m(t) = g_\phi \phi(t) / \Delta m_{ij}$$

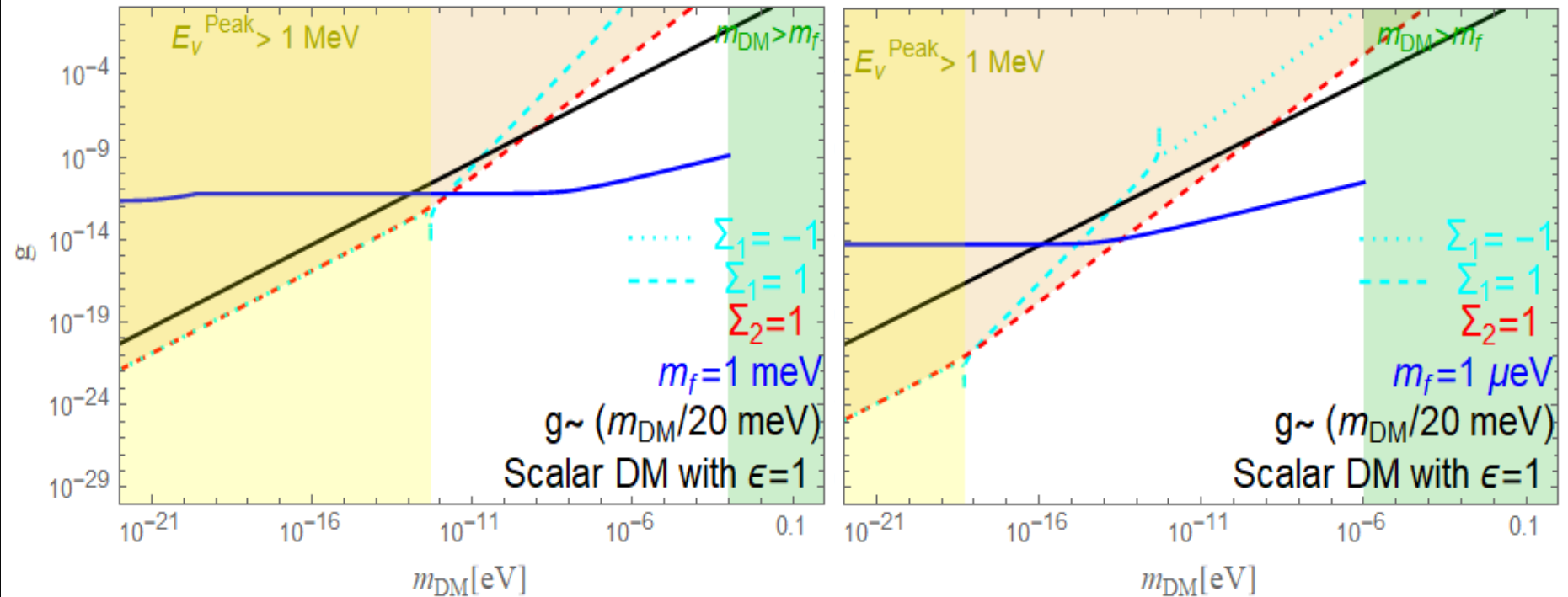
Neutrinos propagating in this field will experience variations of mixing in time with frequency given by m_ϕ

Period \sim month, bounds from solar neutrinos, lab. experiments

New observable effects (and not just renormalization of SM Yukawa and VEV) if the field has

- spatial dependence
- different sign for neutrinos and antineutrinos

Bounds on parameters



Black correct neutrino masses

CMB, LSS . affected above blue line

Perturbativity: cyan dashed - $|\Sigma_1| > 1$
 red dashed - $|\Sigma_2| > 1$

Resummation of diagrams with many scalar field insertions is needed

Relating to mass degeneracy

Symmetry which left mass matrices invariant for specific mass spectra:

Partially degenerate spectrum $m_1 = m_2, m_3$

D. Hernandez, A.S.

Transformation matrix $S_\nu = O_2$ $G_\nu = SO(2) \times Z_2$

Relation:

$$\sin^2 2\theta_{23} = \pm \sin \delta = \cos \kappa = \frac{m_1}{m_2} = 1$$

maximal

$\pm \pi/2$

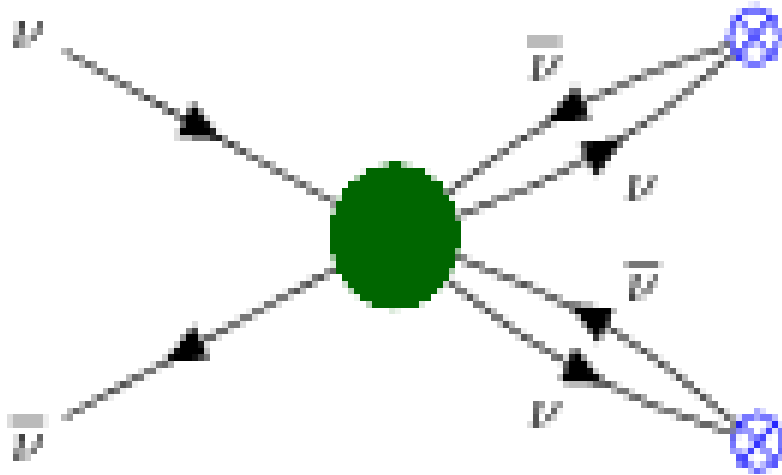


Majorana phase

1-2 mixing is undefined

Small corrections to mass matrix lead to 1-2 mass splitting and 1-2 mixing

Soft couplings and small VEV's



Neutrino mass generation through the condensate (crossed blue circles) via non-perturbative interaction (green circle).

Small neutrino masses from gravitational θ -term

*G. Dvali and L. Funcke,
Phys.Rev. D93 (2016) no.11, 113002
arXiv:1602.03191 [hep-ph]*

No $\beta\beta_{0\nu}$ decay due to large q^2
the vertex does not exist ?

$\beta\beta_{0\nu}$ decay - unique process where neutrinos are highly virtual

Certain generic features independent on specific scenario can be considered on phenomenological level

TBM: deviations and implications

Tri-bimaximal mixing

$$U_{\text{tbm}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

0.15
0.78
0.62

*P. F. Harrison, D. H. Perkins,
W. G. Scott*

$$U_{\text{tbm}} = U_{23}(\pi/4) U_{12}$$

$$\sin^2 \theta_{12} = 1/3$$

0.30 - 0.31

Accidental, numerology,
useful for bookkeeping

Accidental symmetry
(still useful)

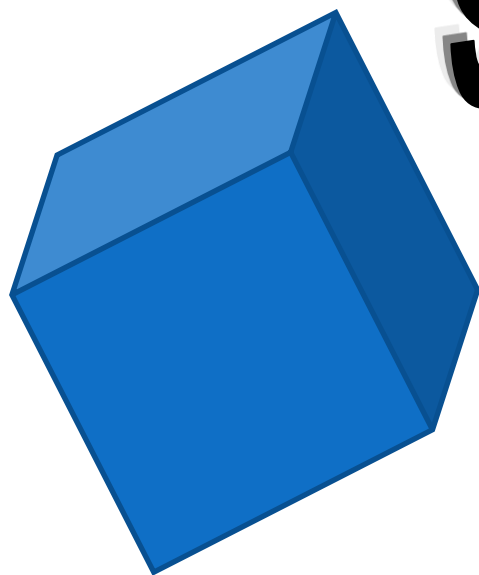
There is no relation of mixing
with masses (mass ratios)

Not accidental

Lowest order approximation
which corresponds to weakly
broken (flavor) symmetry
of the Lagrangian

with some other physics
and structures associated
flavons other new particles

S_4 - symmetry



Order 24, permutation of 4 elements

Symmetry of cube

Generators: S, T

Presentation:

$$S^4 = T^3 = (ST^2)^2 = 1$$

Irreducible representations:

$$1, 1', 2, 3, 3'$$

Products and invariants

$$3 \times 3 = 3' \times 3' = 1 + 2 + 3 + 3'$$

$$3 \times 3' = 1' + 2 + 3 + 3'$$

$$1' \times 1' = 1$$

$$2 \times 3 = 2 \times 3' = 3 + 3'$$

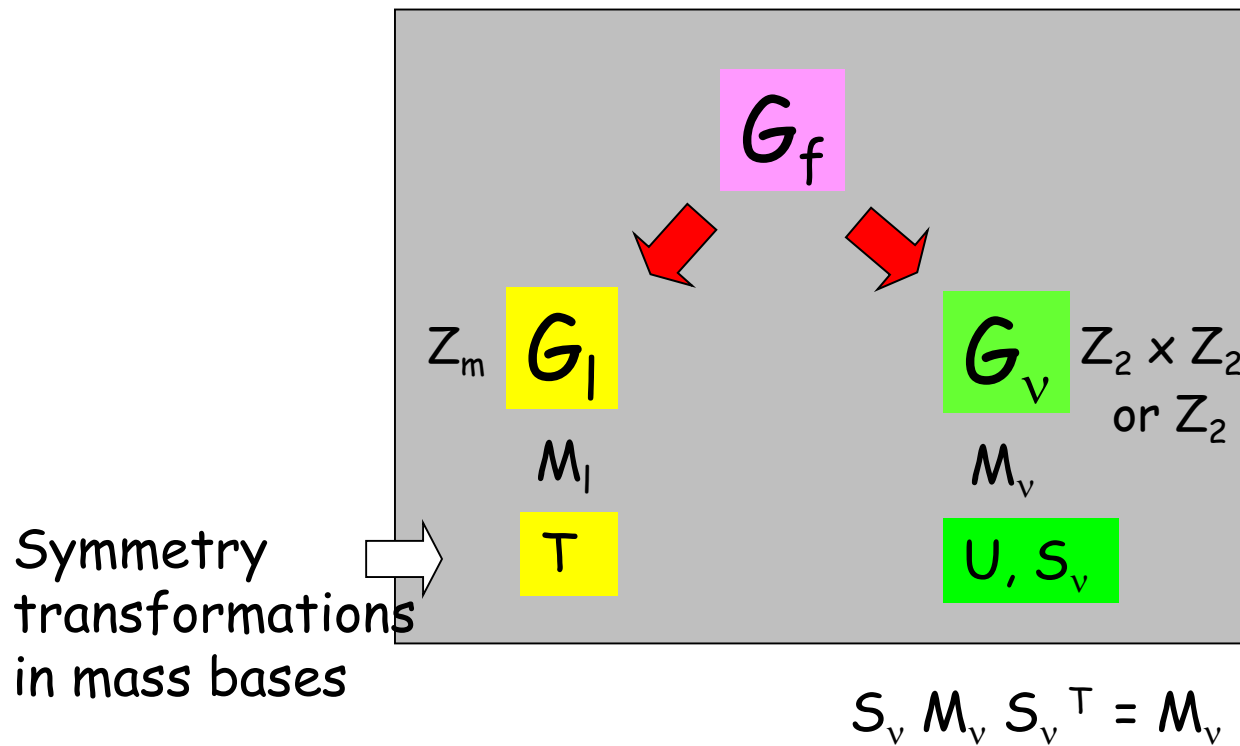
$$2 \times 2 = 1 + 1' + 2 \quad 1' \times 2 = 2$$

New flavor structure

Residual symmetries approach

E. Ma,
C. S. Lam
....

Mixing appears as a result of different ways of the flavor symmetry breaking in the neutrino and charged lepton (Yukawa) sectors.



Symmetry transformations
in mass bases

Discrete finite groups
Flavons to break
symmetries

Flavons

$$A_4 \quad S_4 \quad T_7 \quad T'$$

Residual symmetries
of the mass matrices

Generic symmetries
which do not depend
on values of masses

CP-transformations
can be added

Symmetry group condition

*D. Hernandez, A.S.
1204.0445*

If intrinsic symmetries are residual symmetries of the unique symmetry group (follow from breaking of unique group)
→ bounds on elements of mixing matrix

Inversely, S_i and T are elements of covering group.

By definition product of these elements (taken in the same basis) also belongs to the finite discrete group:

$$(U_{PMNS} S_i U_{PMNS}^\dagger T)^p = I$$

p -integer

For each i the equation gives two relations between mixing parameters
Two such equations for $i = 1, 2$ fix the mixing matrix completely → TBM

$Z_2 \times Z_2$



TBM

In general, allows to fix mixing matrix up to several possibilities

Generic problem

$$m = F(Y, v)$$

Mechanism of
mass generation

Yukawa
couplings

VEV's

VEV alignment

- different contributions
- high order corrections

follow from independent sectors:

Yukawa sector Scalar potential

tune by additional symmetries

TBM

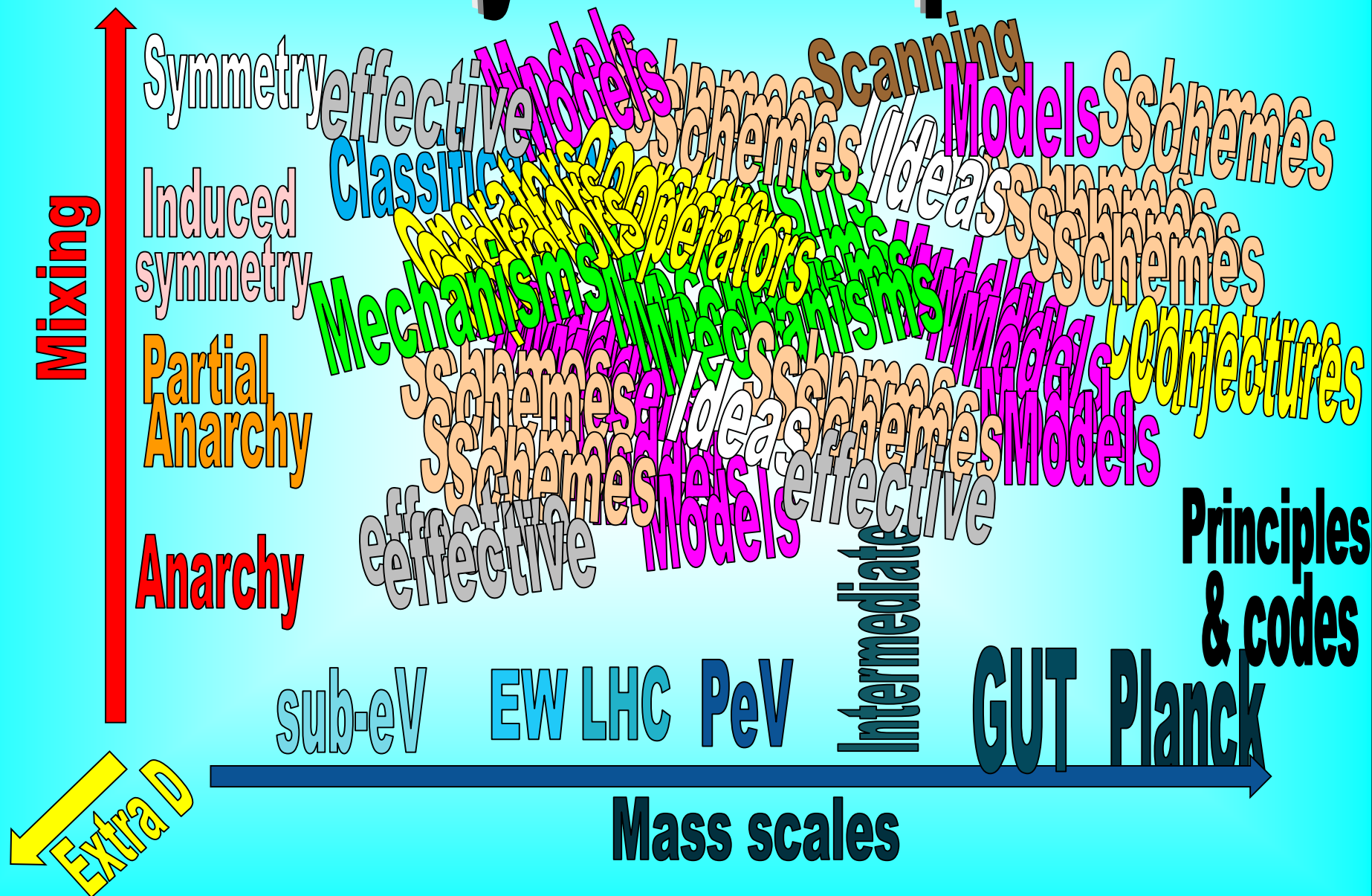
All these
components
should be
correlated

``Natural" - consequence
of symmetry?

one step constructions do not work

Multi
dimensional

"Theory" Landscape



Intrinsic symmetries

Realized for arbitrary values of neutrino and charged lepton mass
can not be broken, always exist

In the mass basis

for Majorana neutrinos

$$m = \text{diag}(m_1, m_2, m_3)$$

$$G_v$$

$$S_1 = \text{diag}(1, -1, -1)$$

$$S_2 = \text{diag}(-1, 1, -1)$$

$$S_i^2 = I$$

$$Z_2 \times Z_2$$

The Klein group

for charged leptons

$$m_l = \text{diag}(m_e, m_\mu, m_\tau)$$

$$G_l$$

$$T = \text{diag}(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})$$

$$\phi_\alpha = 2\pi k_\alpha / m$$

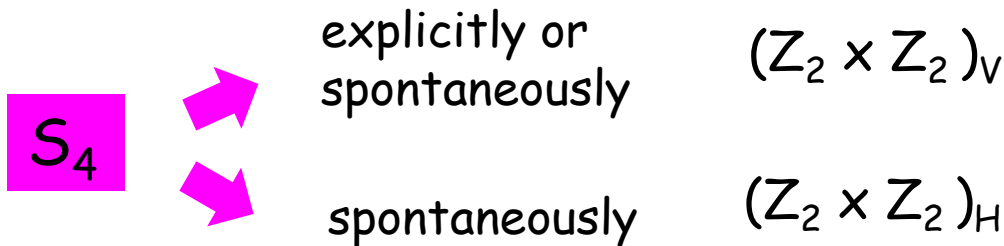
$$T^m = I$$

$$Z_m$$

$\sum \phi_\alpha = 0$ can use subgroup

Intrinsic symmetries as residual symmetries

Realization for BM mixing



Charge assignment

Fields	16_{Fi}	S_i	η	ξ	ϕ
S_4	3	3	1	2	3'

other fields are S_4 singlets

Flavons, singlets of $SO(10)$

VEV alignment

$$\langle \phi \rangle \sim (0, 0, 1), \quad \langle \xi \rangle \sim (0, 1)$$

$\Rightarrow M_S = M_{BM} \quad U_X = U_{BM}$

$$\delta_{CP} = (0.80 - 1.16) \pi$$

The theory?

$$\frac{1}{\Lambda} LLHH$$

S. Weinberg

Large scale
of new physics

Violation of
universality,
Unitarity?

or maybe:


$$h L \bar{\nu}_R H$$

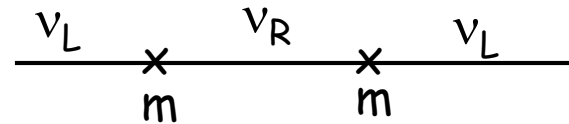
With very small
coupling $h \lll 1$

That's all?
Will we learn more?

Oscillations and masses

Oscillations and adiabatic conversion test the dispersion relations and not neutrino masses

$$p_i = \sqrt{E_i^2 - m_i^2}$$




In oscillations: no change of chirality, so e.g. V , A interactions with medium can reproduce effect of mass. Also interactions with scalar fields

It is consistency of results of many experiments in wide energy ranges and different environment: vacuum, matter with different density profiles that makes explanation of data without mass almost impossible.

proof of non-zero neutrino mass

Kinematical methods: distortion of the beta decay spectrum near end point - KATRIN

Neutrinoless double beta decay

Cosmology, Large scale structure of the Universe

Probing Nature of neutrino mass

Determination of masses, mass squared differences from processes at different conditions

Searches for dependence of mass on external variables:

Vacuum - media with different densities, fields

Solar - KamLAND: Δm_{21}^2
2-3 mixing: T2K - NOvA

Energies (in medium, or if Lorentz is violated)

Epochs (red shifts)

MAVAN

Momentum transfer

Virtuality: On shell - off shell

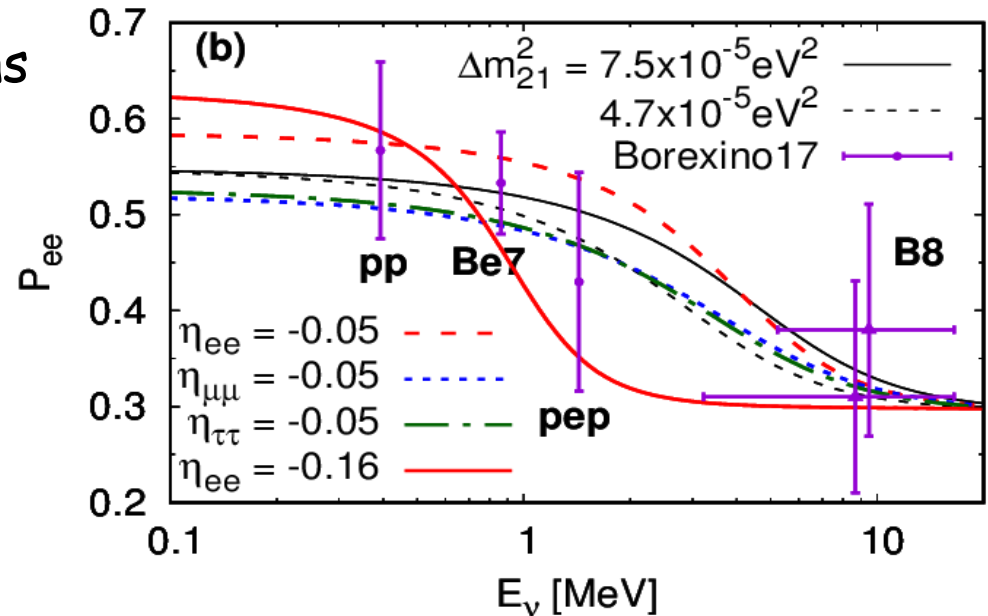
Neutrinoless Double beta decay - unique?

Refraction due to very light scalar mediator

Shao-Feng Ge,
S. Parke, 1812.08376
[hep-ph]

Neutrino scattering on electrons
via very light scalar exchange

The solar neutrino conversion
probabilities with scalar
NSIs vs. Borexino results.



To satisfy bounds on $h_\nu h_e$ (especially from searches of 5th force:

$$1/m_\phi \gg R_{\text{Earth}}$$

→ strong suppression of the potential $V = V_0 m_\phi R_{\text{Earth}}$

To avoid bounds - cancellations in 5th force experiments - not shown if this is possible

Modular symmetries

Another realization of symmetry approach inspired by string theory

Symmetry related to (orbifold) compactification of extra dimensions and primary realized on the moduli fields which describe geometry of the compactified space.

For single modulus field τ the modular transformation reads

$$\tau \rightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

The 2x2 matrices of integer numbers

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{with determinant } ad - bc = 1$$

Form the group $\Gamma = SL(2, \mathbb{Z})$ special linear ...

Finite subgroup of Γ : $\Gamma_N = \Gamma / \Gamma(N)$

Modular group

Finite subgroup of Γ is quotient group of level N : $\Gamma_N = \Gamma / \Gamma(N)$
where $\Gamma(N)$ - is congruence subgroup of Γ of level N .
 Γ_N is called the modular group

Γ_3 , Γ_4 , Γ_5 are isomorphic to A_4 , S_4 , A_5 , correspondingly

In SUSY, the chiral superfields transform as

$$\varphi^I \rightarrow (c\tau + d)^{k_I} \rho(\gamma) \varphi^I$$

k_I is the weight of multiplet

$\rho(\gamma)$ is the representation of γ element of the group Γ_N

Appearance of the weight factor in transformations is new element which leads to new consequences

Modular forms

Yukawa couplings are modular forms

Another key element of formalism

$f_i(\tau)$ - holomorphic functions of modulus field τ

Transformation properties are similar to those of superfields

$$f_i(\tau) \rightarrow f_i(\gamma\tau) = (c\tau + d)^{k_f} \rho(\gamma)_{ij} f_j(\tau)$$

Form multiplet of Γ_N whose dimension is determined by level N and weight k_f

For instance for $N = 3$ and $k_f = 2$, f_i form triplet with components

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + \dots \\ Y_2(\tau) &= -6q^{1/3} (1 + 7q + \dots) \\ Y_3(\tau) &= -18q^{2/3} (1 + \dots) \end{aligned}$$

$$q = e^{i2\pi\tau}$$

Data fit: $\tau = 0.0117 + i 0.995$

$Y_i = (1, -0.74, -0.27)$ weak hierarchy

Invariance

For terms of potential $\mathcal{Y} \phi_1 \phi_2 \phi_3$ invariance requires

$\rho_1 \times \rho_2 \times \rho_3 \times \rho_Y = \mathbf{I}$ for product of A_4 representations

$\sum_i k_i + k_Y = 0$ for weights

Additional condition which acts as Froggatt- Nielsen factors

Yukawa couplings form multiplets
they are fixed by symmetry

Yukawa couplings are modular forms

k - free parameters

Models

J Griado and F. Feruglio
1807.01125 [hep-ph]

flavon

	L	E^c	N^c	Y	φ
A_4	3	$1, 1', 1''$	3	3	3
k	1	-4	-1	2	3

Y lowest order
modular form
weights

τ and φ are fixed by fitting on m_3/m_2 , 12 mixing and 13 mixing

Predictions:

$$\sin^2 \theta_{23} = 0.46$$

$$\delta / \pi = 1.434$$

$$\alpha_{21} / \pi = 1.7$$

$$\alpha_{31} / \pi = 1.2$$

Models

Gui-Jun Ding, S.F. King,
Xiang-Gan Liu
1907.11714 [hep-ph]

	L	E^c	N^c	Y
A_4	3	$1, 1'', 1'$	3	3
k	2	2	0	-2

No flavons

Flavor from single modulus field τ

Higher order modular forms
 $Y^{(2)}, Y^{(4)}, Y^{(6)}$ constructed as
products of $Y^{(2)}$

All Yukawas are modular forms

τ is fixed by fitting 12 mixing and 13 mixing

Predictions:

$$\sin^2 \theta_{23} = 0.58$$

$$\delta / \pi = 1.6$$

$$\alpha_{21} / \pi = 0.15$$

$$\alpha_{31} / \pi = 1.00$$

$$m_1 = 0.0946 \text{ eV}$$

$$m_2 = 0.0950 \text{ eV}$$

$$m_3 = 0.1071 \text{ eV}$$

Normal ordering

$$m_{ee} = 0.095 \text{ eV}$$

Cosmological
bound?

Reconstructed forms of U_χ

A. Yu. S.,
B. Xun-Jie Xu

Taking into account that elements of U_χ are in general complex
5 matrices have been found. Among them

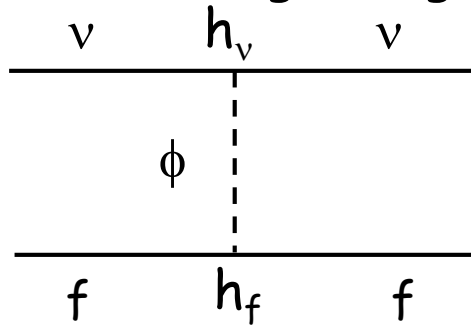
U_χ	Group
$U_{q/p} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \pi q/p & \sin \pi q/p \\ 0 & -\sin \pi q/p & \cos \pi q/p \end{pmatrix}$	D_4 (dihedral)
$U_{BM} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & -1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$	S_4
$U_{GR} \begin{pmatrix} \phi/2 & 1/2 & \phi^{-1}/2 \\ 1/2 & -\phi^{-1}/2 & -\phi/2 \\ \phi^{-1}/2 & -\phi/2 & 1/2 \end{pmatrix}$	A_5

$\phi = \frac{1}{2} (1 + \sqrt{5})$ is the Golden ratio

Refraction due to long range forces

Light dark sector scalars, vectors ...

Scattering via light mediators exchange:



$$A \sim \frac{h_v h_f}{q^2 - m_\phi^2}$$

With decrease of m_ϕ and the same decrease of h

refraction ($q^2 = 0$) $\sim h_v h_f / m_\phi^2$ does not change
inelastic scattering is suppressed as $h_v h_f / q^2$

Refraction effects dominate at small m_ϕ

Potential

$$V = \frac{h_v h_f}{m_\phi^2} n_f$$

number density of scatters