What is dark matter made of?

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1. Motivation

Non-relativistic dark matter, when density perturbations are still relatively small:

$$v_{h\text{rms}}(a) = \frac{v_{h\text{rms}}(1)}{a}, \qquad \rho_h(a) = \frac{\rho_h(0)}{a^3}, \qquad \frac{v_{h\text{rms}}(a)}{\rho_h(a)^{1/3}} = \text{constant}$$

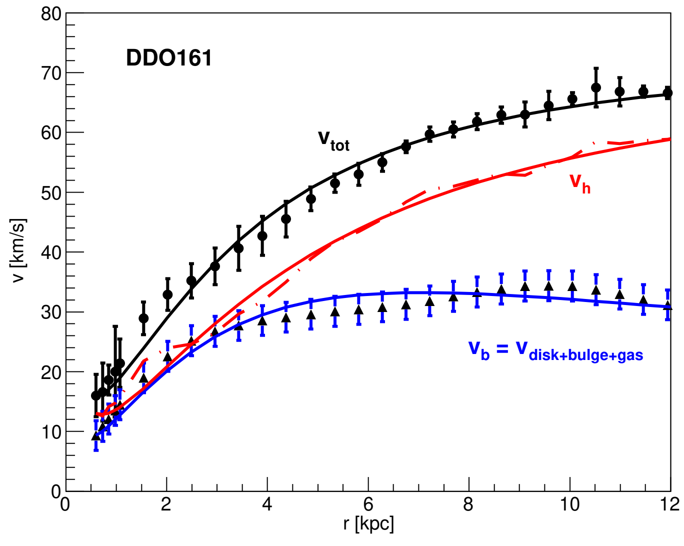
Consider free observer in density peak.

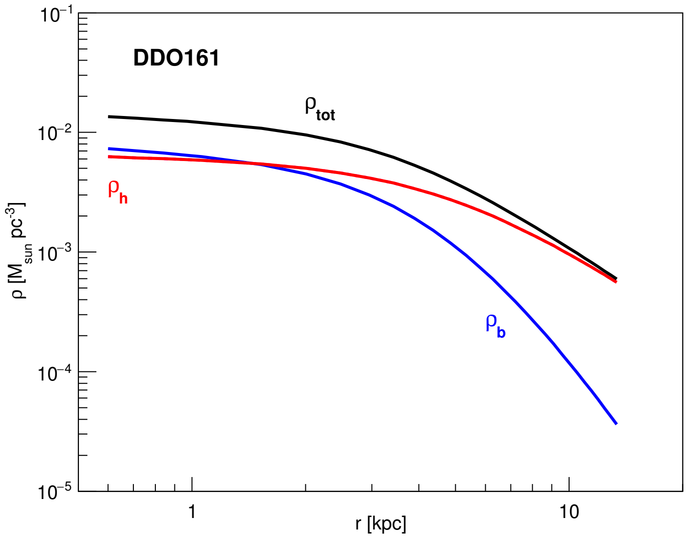
$v_{h\text{rms}}(1)$ can be measured with spiral galaxy rotation curves:

$$\frac{v_{h\text{rms}}(a)}{[\rho_h(a)]^{1/3}} = \frac{v_{h\text{rms}}(1)}{[\Omega_c \rho_{\text{crit}}]^{1/3}} = \frac{\sqrt{3}\left\langle v_{\text{r}h}^2 \right\rangle^{1/2}}{[\rho_h(r \to 0)]^{1/3}}$$

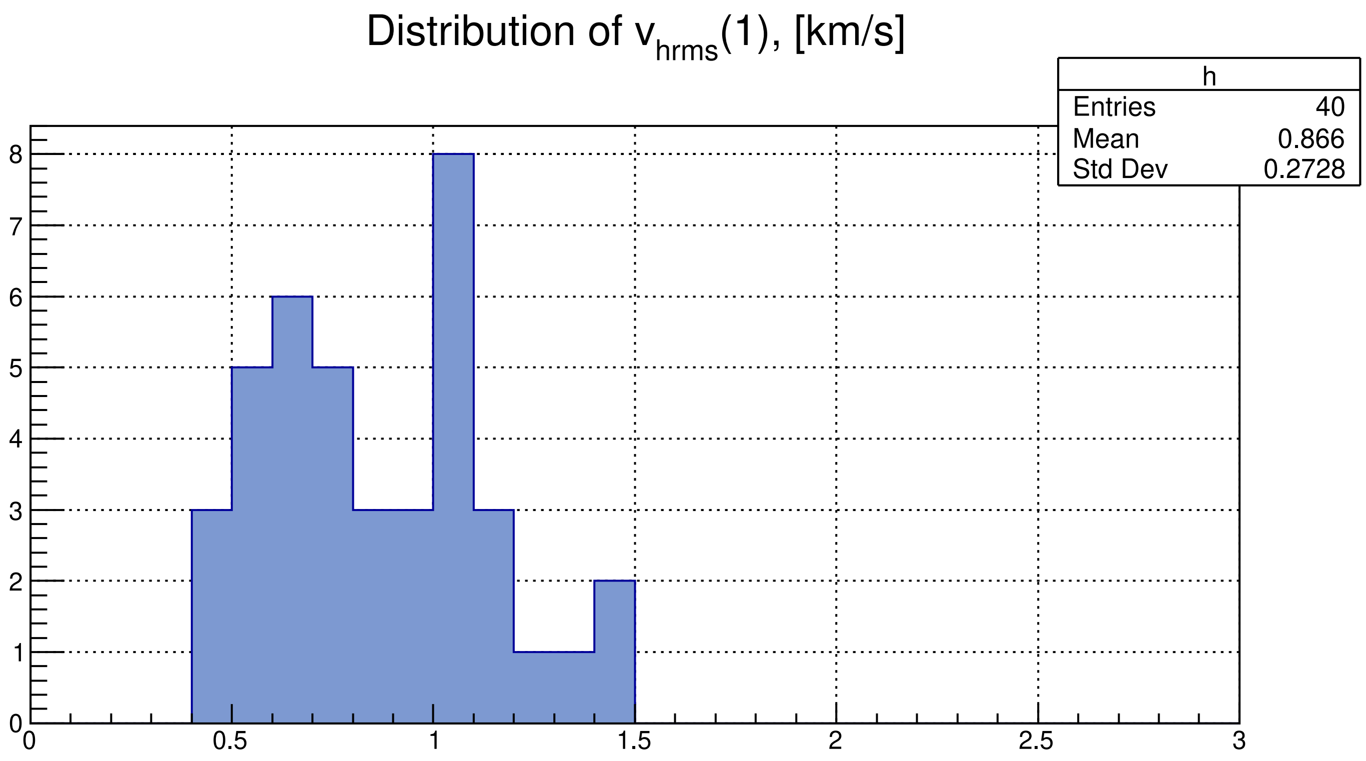
We predict that $v_{h\text{rms}}(1)$ is of cosmological origin, and is the same for all steady-state, relaxed, spiral galaxies.

2. Validation: Spiral galaxy adiabatic invariant





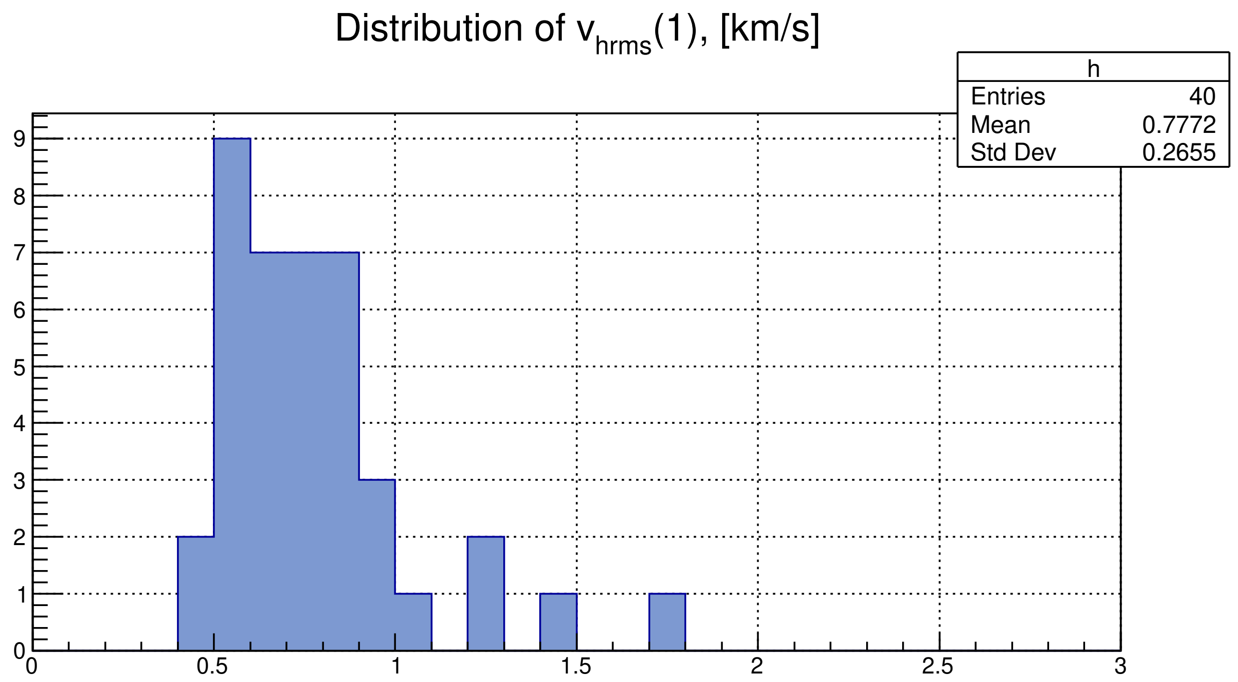
Left: Observed rotation curve $v_{\text{tot}}(r)$ (dots) and the baryon contribution $v_b(r)$ (triangles) of galaxy DDO161 (SPARC data). The solid lines are obtained by numerical integration. Right: Mass densities of baryons and of dark matter.



Distribution of the adiabatic invariant $v_{hrms}(1)$ corresponding to fits with large m_h, i.e. negative chemical potential. SPARC data.

Mean and standard deviation of $v_{h\mathrm{rms}}(1)$ for several galaxy selections. The galaxy classes are 5 = Sc, 6 = Scd, 7 = Sd, 9 = Sm, 10 = Im. SPARC data.

Galaxy selection	N	Mean $v_{h\mathrm{rms}}(1)$ [km/s]	Std. dev. [km/s]
All	40	0.866	0.273
$L_{3.6} < 1 \times 10^9 L_\odot$	11	0.838	0.297
$L_{3.6} > 4 \times 10^9 L_\odot$	11	1.036	0.192
$M_{HI} < 1 \times 10^9 M_\odot$	17	0.714	0.239
$\left\langle v_{\mathrm{r}h}^2 \right\rangle^{1/2} < 50$ km/s	17	0.786	0.259
$\left\langle v_{\mathrm{r}h}^2 \right\rangle^{1/2} > 60$ km/s	16	0.969	0.227
de Vaucouleurs class 5, 6 or 7	15	0.820	0.277
de Vaucouleurs class 9 or 10	18	0.869	0.258
SBdisk $< 100 \times 10^9 L_\odot/\mathrm{pc}^2$	10	0.843	0.174
$\rho_h(0) > \rho_b(0)$	37	0.842	0.255



Distribution of the adiabatic invariant $v_{hrms}(1)$ assuming dark matter has zero chemical potential. SPARC data.

Conclusion:

The small relative standard deviation of $v_{h\mathrm{rms}}(1)$ is noteworthy given that the galaxies in this sample have luminosities, central densities, and central surface brightnesses that span three orders of magnitude.

We conclude that $v_{h\mathrm{rms}}(1)$ is of cosmological origin.

The adiabatic invariant $v_{h\mathrm{rms}}(1)$ determines the ratio of dark matter temperature-to-mass.

3. Warm dark matter free-streaming, cut-off wavenumber k_{fs}, and transition mass M_{fs}

- Free-streaming of warm dark matter suppresses the power spectrum of linear density perturbations $P(k)$ for wavenumbers $k > k_{\mathrm{fs}}$.

- The expansion parameter at which dark matter becomes non-relativistic is $a'_{h\mathrm{NR}} \equiv v_{h\mathrm{rms}}(1)/c$.

- The free-streaming smoothing length is $2\sigma(d_{\mathrm{fs}}) = \lambda_{\mathrm{fs}}/2 = \pi/k_{\mathrm{fs}}$.

- The transition mass is $M_{\mathrm{fs}} = 4\pi R^3 \Omega_m \rho_{\mathrm{crit}}/3$, with $R = 1.555/k_{\mathrm{fs}}$.

- $\sigma(d_{\mathrm{fs}})$ is a function of $a'_{h\mathrm{NR}}$, so measuring any of these quantities obtains all the others.

- $v_{h\mathrm{rms}}(1) = 0.76 \pm 0.20$ [km/s], after DM rotation correction.

- $a'_{h\mathrm{NR}} = (2.54 \pm 0.97) \times 10^{-6}$.

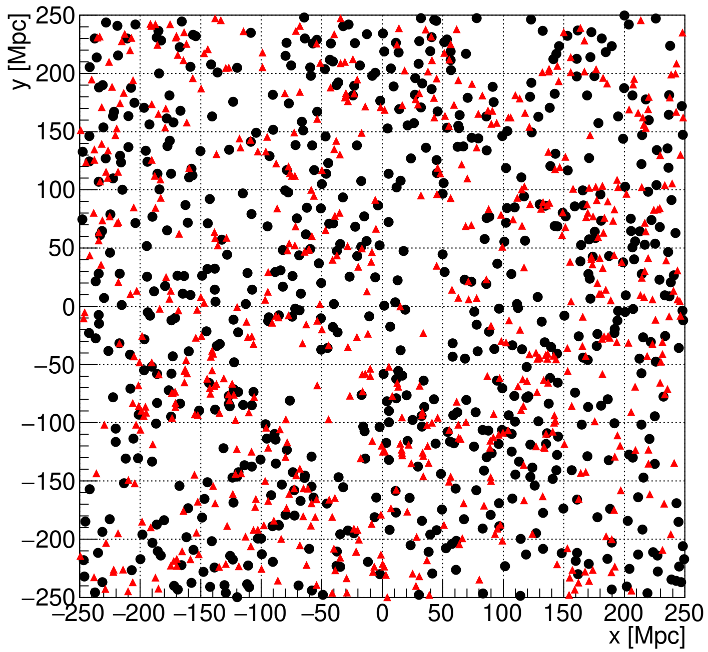
- Free streaming smoothing length: $\pi/k_{\mathrm{fs}} = 4.0 \pm 1.5$ Mpc (for the fermion case).

- Warm dark matter cut-off wavenumber $k_{\mathrm{fs}} = 0.80^{+0.42}_{-0.24}$ Mpc^{-1}.

- Warm dark matter transition mass $M_{\mathrm{fs}} = 10^{12.08 \pm 0.50} M_{\odot}$, corresponding to $R = 2.0 \pm 0.8$ Mpc. M_{fs} is approximately equal to the Milky Way mass, and hence addresses the "small scale crisis".

Limits from the Lyman-α forest

Several analysis of the Lyman-α forest, and of gravitational lensing, of light from distant quasars, have set lower limits on the *thermal relic mass*, typically in the range 2000 eV to 4000 eV. Such thermal relics are assumed to self-annihilate and freeze-out to obtain the present mean dark matter density of the universe. These limits are equivalent to setting lower limits on k_{fs} in the range 10 Mpc^{-1} to 21 Mpc^{-1}, with a sensitivity starting at $k \approx 0.2$ Mpc^{-1}, in disagreement with our measurement $k_{\text{fs}} = 0.80^{+0.42}_{-0.24}$ Mpc^{-1}.



(x, y) distribution of galaxies in a slice of thickness 40 Mpc for the simulation with redshift $z = 0.5$, size $L = 500$ Mpc, and $k_{\text{fs}} = 0.466$ Mpc^{-1}, corresponding to $m_h = 48$ eV, $N_f = 2$, and $M_{\text{fs}} = 6.1 \times 10^{12} M_\odot$. The filled circles are galaxies with $k < k_{\text{fs}}$, while triangles are smaller *stripped down galaxies* with $k > k_{\text{fs}}$.

4. Dark matter with zero chemical potential

The adiabatic invariant $v_{h\text{rms}}(1)$ obtains the ratio of dark matter temperature $T(a)$ to mass M_h. To obtain $T(a)$ and M_h separately, we need one more constraint.

It turns out that, if we assume that dark matter has zero chemical potential, and decouples (from the Standard Model sector, and from self-annihilation) while still ultra-relativistic, then dark matter is also in thermal equilibrium with the Standard Model sector in the early universe *with the measured values of* $v_{h\text{rms}}(1)$ and T_0!!!

The case of no freeze-in and no freeze-out: (T_h/T) is the ratio of dark matter-to-photon temperatures after e^+e^- annihilation while dark matter is still ultra-relativistic. m_{th} is the dark matter particle mass needed to obtain the present mean dark matter density $\Omega_c \rho_{\text{crit}}$, assuming no dark matter self-annihilation, for the case of chemical potential $\mu = 0$, $N_b = 0$, and $N_f = 2$.

Decoupling temperature range	(T_h/T) in range m_h to m_e	m_{th}
m_H to m_t	0.344	99.9 ± 3.1 eV
m_W to m_H	0.345	98.9 ± 3.1 eV
m_b to m_W	0.357	89.5 ± 2.8 eV
m_τ to m_b	0.372	78.6 ± 2.5 eV
m_c to m_τ	0.378	75.0 ± 2.4 eV
m_s to m_c	0.399	64.1 ± 2.0 eV
T_c to m_s	0.424	53.2 ± 1.7 eV
m_π to T_c	0.610	17.9 ± 0.6 eV
m_μ to m_π	0.650	14.8 ± 0.5 eV
m_e to m_μ	0.714	11.2 ± 0.4 eV

For fermions with chemical potential $\mu = 0$,

$$m_h = 78.5 \left(\frac{0.76 \text{ km/s}}{v_{h\mathrm{rms}}(1)} \right)^{3/4} \left(\frac{2}{N_f} \right)^{1/4} \text{ eV}. \qquad (1)$$

For bosons with chemical potential $\mu = 0$,

$$m_h = 51.1 \left(\frac{0.76 \text{ km/s}}{v_{h\mathrm{rms}}(1)} \right)^{3/4} \left(\frac{1}{N_b} \right)^{1/4} \text{ eV}. \qquad (2)$$

If dark matter decouples from the Standard Model sector, and from self-annihilation, while ultra-relativistic, the ratio of dark matter-to-photon temperatures after e^+e^- annihilation, while dark matter is still ultra-relativistic, is

$$\frac{T_h}{T} = 0.373 \left(\frac{v_{h\text{rms}}(1)}{0.76\ \text{km/s}}\right)^{1/4} \left(\frac{2}{N_f}\right)^{1/4} \quad (3)$$

for fermions with $\mu = 0$, and

$$\frac{T_h}{T} = 0.492 \left(\frac{v_{h\text{rms}}(1)}{0.76\ \text{km/s}}\right)^{1/4} \left(\frac{1}{N_b}\right)^{1/4} \quad (4)$$

for bosons with $\mu = 0$.

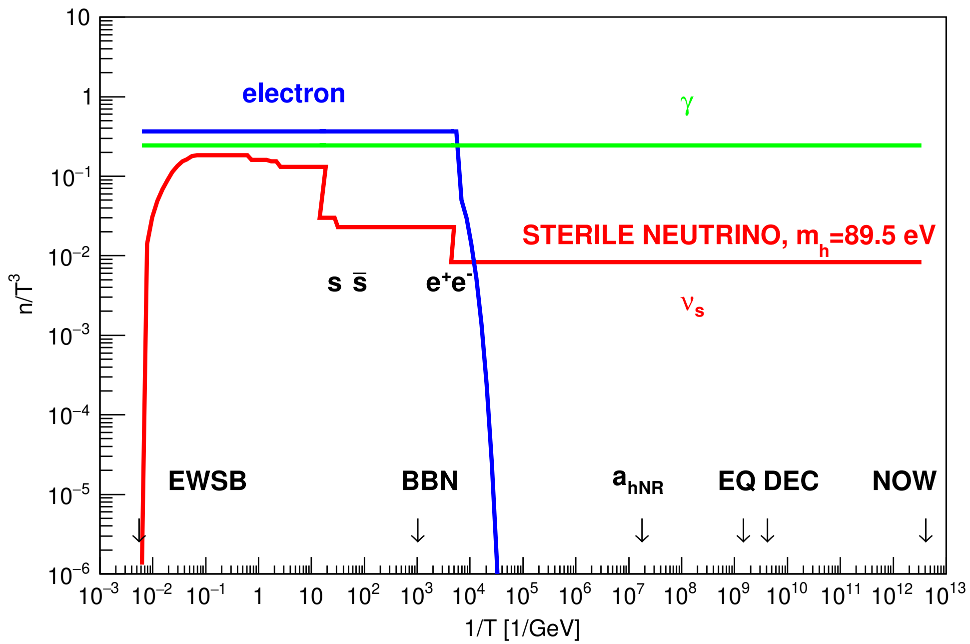
$$\frac{T_h}{T} = 0.373 \left(\frac{v_{h\mathrm{rms}}(1)}{0.76\ \mathrm{km/s}}\right)^{1/4} \left(\frac{2}{N_f}\right)^{1/4},$$

is strong evidence that dark matter was in diffusive, i.e. $\mu = 0$, and thermal equilibrium with the Standard Model sector at some time in the early Universe, and decoupled while ultra-relativistic, i.e. no freeze-in and no freeze-out.

The case of negative μ/kT_h, with large $m_h c^2 \approx kT_h(a_{h\text{NR}})$, with the same measured $a_{h\text{NR}}$ and $v_{h\text{rms}}(1)$, can not be ruled out, but is not compelling: it requires a *coincidence* of $m_{\text{th}} = m_h$ with the measured $v_{h\text{rms}}(1)$ and T_0, implies that dark matter was never in thermal or diffusive equilibrium with the Standard Model sector, yet requires self-annihilation and freeze-out to obtain the observed dark matter density.

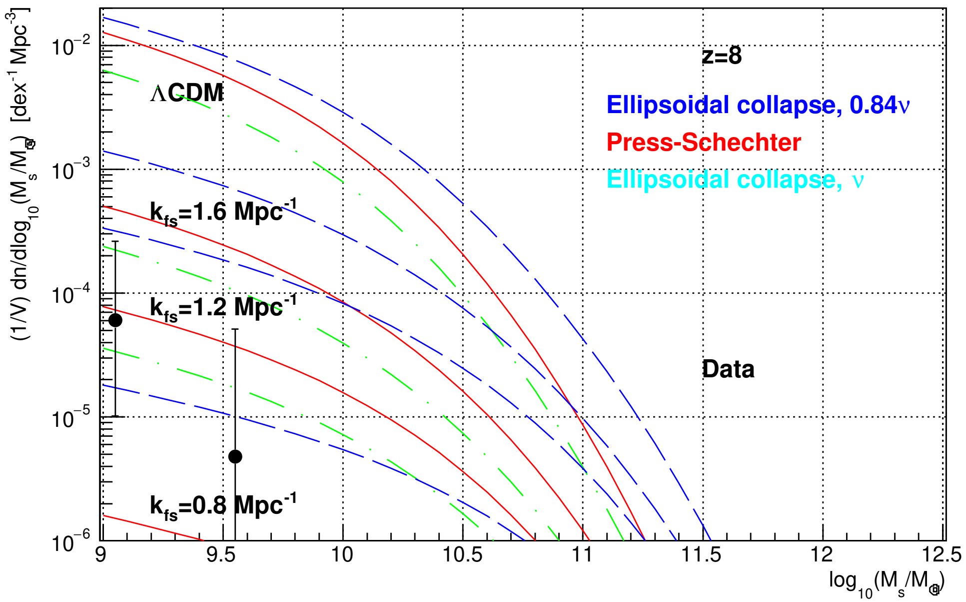
Large $m_h c^2 \approx kT_h(a_{h\text{NR}})$ does not solve Lyman-α, etc, limits.

5. Majorana sterile neutrino dark matter

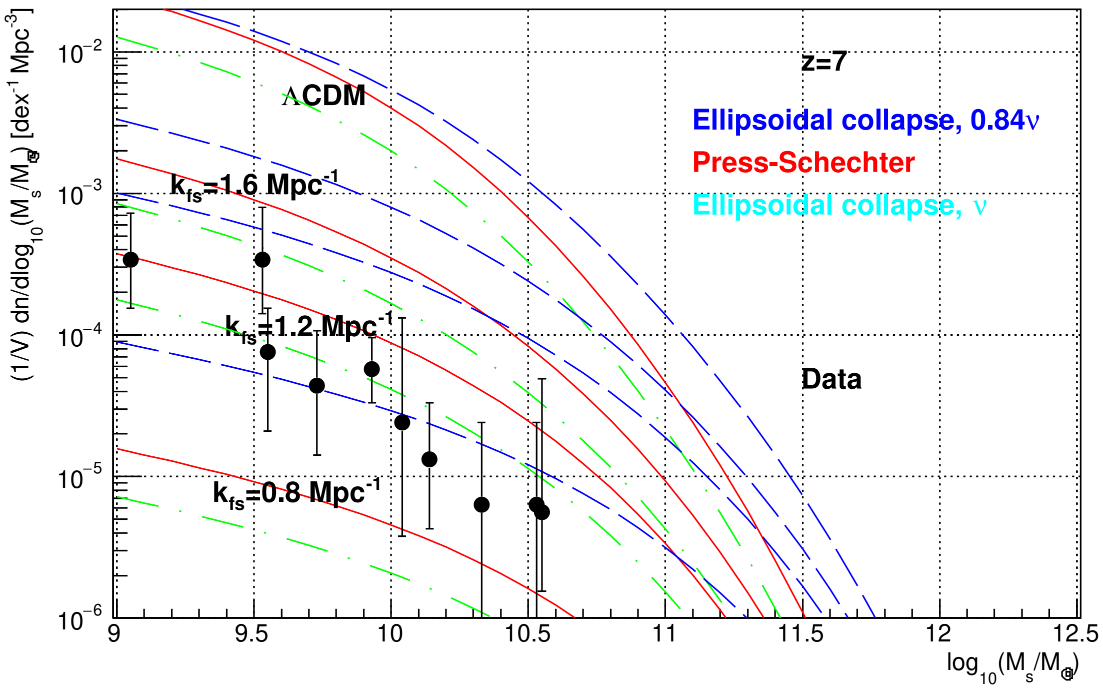


Example. Number density n of photons, electrons, and Majorana sterile neutrino dark matter particles with $m_h = 89.5$ eV, divided by T^3, as a function of $1/T$. T is the photon temperature. Lifetime of sterile Majorana neutrino $= 7 \times 10^{27}$ yr. BBN is satisfied.

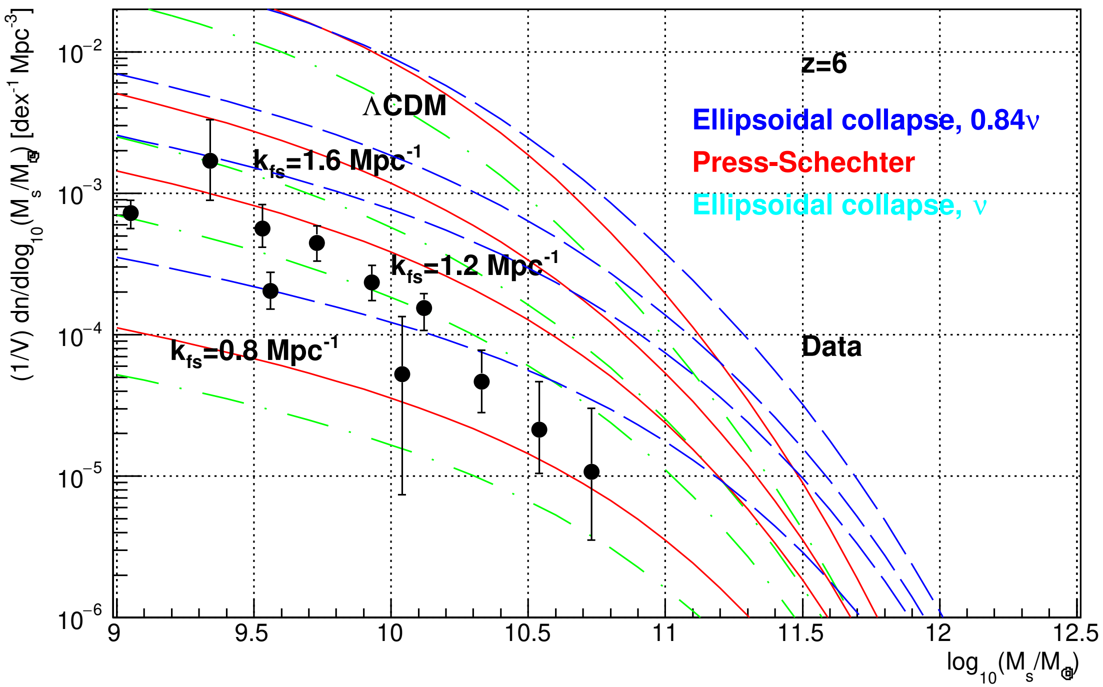
6. Cut-off wavenumber k_{fs} from galaxy stellar mass distributions



Calculated stellar mass functions with the Press-Schechter, Ellipsoidal Collapse with $\tilde{\nu} = \nu$, and Ellipsoidal Collapse with $\tilde{\nu} = 0.84\nu$, approximations, for ΛCDM, and ΛWDM with $k_{fs} = 1.6, 1.2$, and 0.8 Mpc^{-1}, at redshift $z = 8$, compared with observations.



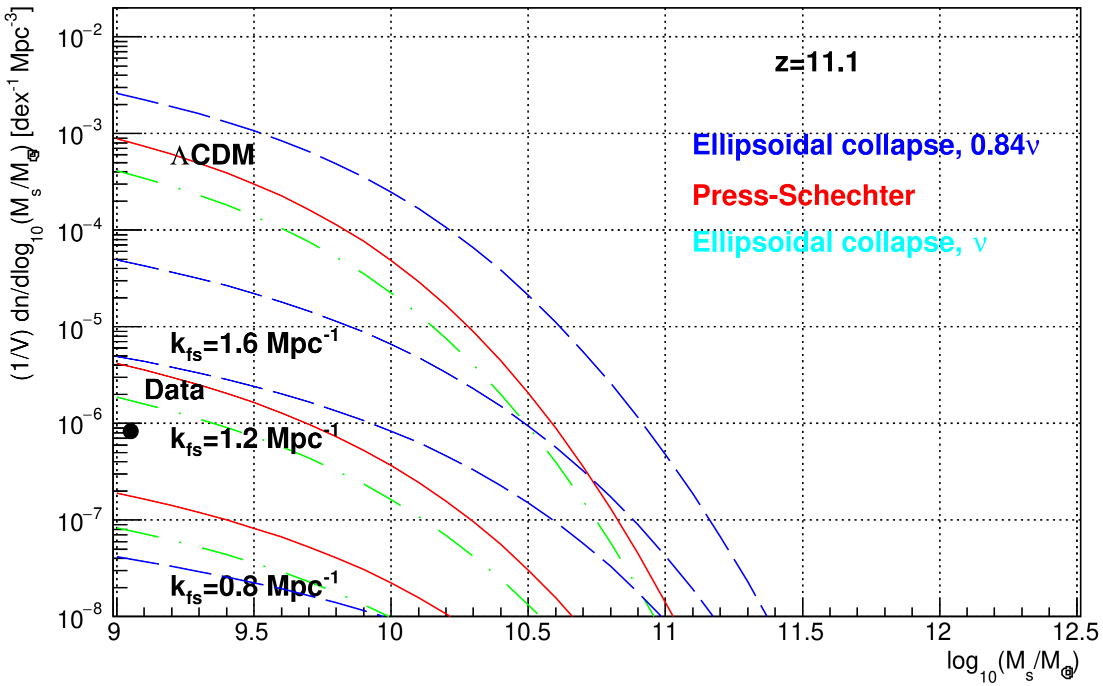
Calculated stellar mass functions with the Press-Schechter, Ellipsoidal Collapse with $\tilde{\nu} = \nu$, and Ellipsoidal Collapse with $\tilde{\nu} = 0.84\nu$, approximations, for ΛCDM, and ΛWDM with $k_{\text{fs}} = 1.6, 1.2$, and 0.8 Mpc^{-1}, at redshift $z = 7$, compared with observations.



Calculated stellar mass functions with the Press-Schechter, Ellipsoidal Collapse with $\tilde{\nu} = \nu$, and Ellipsoidal Collapse with $\tilde{\nu} = 0.84\nu$, approximations, for ΛCDM, and ΛWDM with $k_{\text{fs}} = 1.6, 1.2$, and 0.8 Mpc^{-1}, at redshift $z = 6$, compared with observations.

z	k_{fs} [Mpc^{-1}] Press-Schechter	k_{fs} [Mpc^{-1}] Ellipsoidal collapse, ν	k_{fs} [Mpc^{-1}] Ellipsoidal collapse, 0.84ν
≈ 8	1.10 ± 0.30	1.10 ± 0.40	0.80 ± 0.30
≈ 7	1.10 ± 0.30	1.25 ± 0.35	0.85 ± 0.25
≈ 6	1.10 ± 0.30	1.25 ± 0.35	0.80 ± 0.30

Measurements of k_{fs} from galaxy stellar mass functions (SMF).



Calculated stellar mass functions with the Press-Schechter, Ellipsoidal Collapse with $\tilde{\nu} = \nu$, and Ellipsoidal Collapse with $\tilde{\nu} = 0.84\nu$, approximations, for ΛCDM, and ΛWDM with $k_{\text{fs}} = 1.6, 1.2$, and 0.8 Mpc^{-1}, at redshift $z = 11.1$, compared with one observed galaxy GN-z11 (assuming one similar galaxy per dex). This graph obtains k_{fs} of the order of 1.1 Mpc^{-1}.

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7. Summary of four independent measurements

Fermions Observable	$v_{h\text{rms}}(1)$ [km/s]	$a'_{h\text{NR}} \times 10^6$	m_h [eV]	k_{fs} [Mpc^{-1}]	$\log_{10}(M_{\text{fs}}/M_\odot)$
Spiral galaxies	0.76 ± 0.29	2.54 ± 0.97	79^{+35}_{-17}	$0.80^{+0.42}_{-0.24}$	12.08 ± 0.50
No f.i., no f.o.	$0.81^{+0.47}_{-0.25}$	$2.69^{+1.57}_{-0.84}$	75 ± 23	0.76 ± 0.31	12.14 ± 0.52
First galaxies	$0.52^{+0.38}_{-0.19}$	$1.75^{+1.29}_{-0.63}$	104 ± 38	1.10 ± 0.40	11.67 ± 0.50
SMF				$0.90^{+0.44}_{-0.34}$	11.93 ± 0.56

Bosons Observable	$v_{h\text{rms}}(1)$ [km/s]	$a'_{h\text{NR}} \times 10^6$	m_h [eV]	k_{fs} [Mpc^{-1}]	$\log_{10}(M_{\text{fs}}/M_\odot)$
Spiral galaxies	0.76 ± 0.29	2.54 ± 0.97	51^{+22}_{-11}	$0.51^{+0.28}_{-0.15}$	12.66 ± 0.50
No f.i., no f.o.	$0.26^{+0.16}_{-0.08}$	$0.88^{+0.52}_{-0.28}$	113 ± 35	1.26 ± 0.50	11.49 ± 0.52
First galaxies	$0.31^{+0.23}_{-0.11}$	$1.04^{+0.74}_{-0.38}$	100 ± 39	1.10 ± 0.40	11.67 ± 0.50
SMF				$0.90^{+0.44}_{-0.34}$	11.93 ± 0.56