Continuum Naturalness

KAIST-KAIX Future Colliders Workshop, Daejeon, KR

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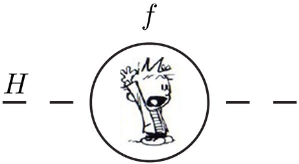
JHEP 1903 (2019) 142 [arXiv:1811.06019] with C. Csáki, S. Lombardo, S.J. Lee, O. Telem
ongoing work with same authors

July 16, 2019

Outline

One-slide naturalness



If the Higgs is a fundamental scalar, its mass is unprotected at the scale of new physics.
Quantum corrections to the mass-squared suffer from quadratic divergences:

$$\Delta m_H^2\Big|_f \sim \frac{\lambda_f^2}{8\pi^2}\Lambda^2\,.$$

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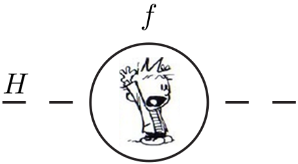

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One way out: supersymmetry introduces additional scalars to cancel the fermionic contribution

$$\Delta m_H^2\Big|_S \sim \frac{\lambda_S}{8\pi^2}\left(-\Lambda^2 + m_S^2 \log \frac{\Lambda^2}{m_S^2}\right) .$$

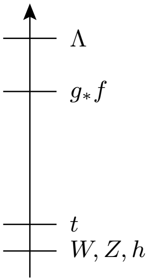
with $\lambda_S = \lambda_f^2$.

The composite Higgs framework

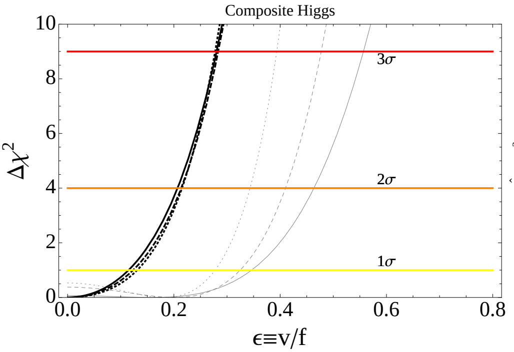
The Higgs is not a fundamental scalar.

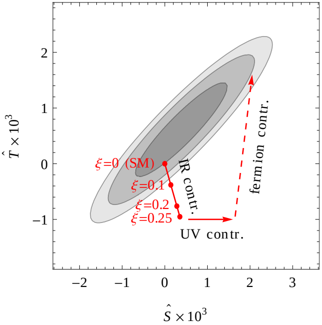
Georgi, Kaplan (1984)

- Strongly-interacting sector with global symmetry group $\mathcal{G}$ spontaneously broken to $\mathcal{H}$ at a confinement scale Λ.
- Composite states (e.g., mesons) have masses $g_* f$, with $g_* \sim \mathcal{O}(1)$ (where f is like pion decay constant).
- Higgs is pNGB in the coset $\mathcal{G}/\mathcal{H}$.
- Higgs potential generated radiatively, quadratic divergences cut off at scale $g_* f$ of composite partners.
- SM fermions and gauge bosons are a mixture of elementary and composite sector states (partial compositeness). Kaplan (1991)



Higgs couplings and EWPO





Falkowski, Riva, Urbano 1303.1812; Grojean, Matsedonskyi, Panico 1306.4655

Naturalness and the LHC

Naturalness suggests that we should see signs of new physics at the TeV scale.

Null results in LHC searches for new physics (some with 136 fb^{-1} of data):

- MET,
- new scalars,
- top partners via same-sign dileptons (e.g., ATLAS 1807.11883, CMS 1810.03188).

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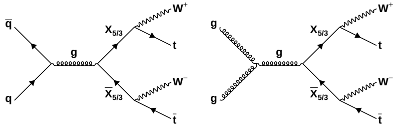

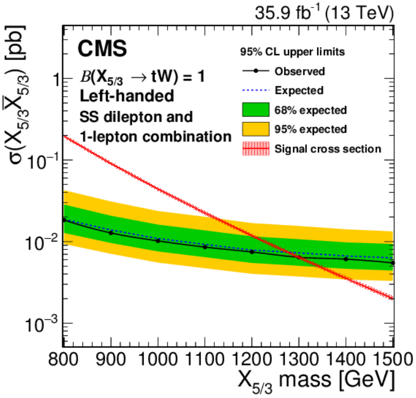

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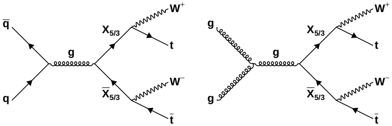

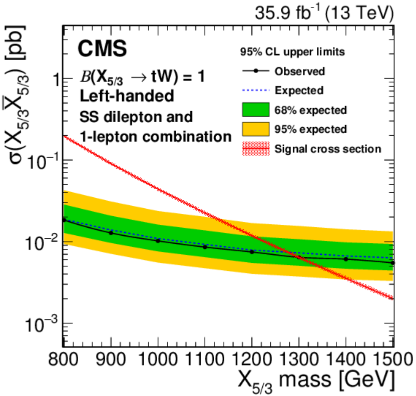

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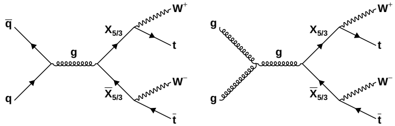

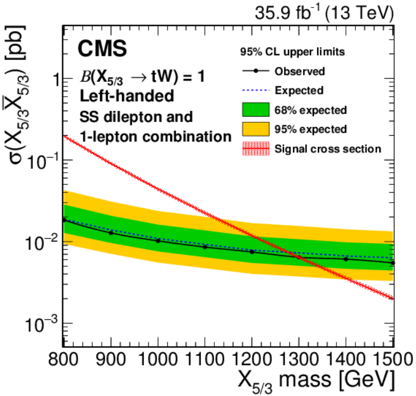

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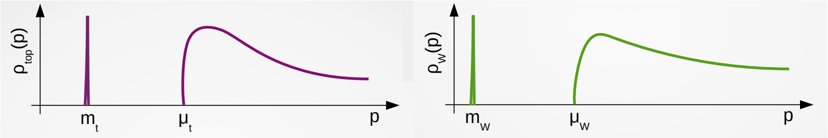
What if new states are not particles?

Goals



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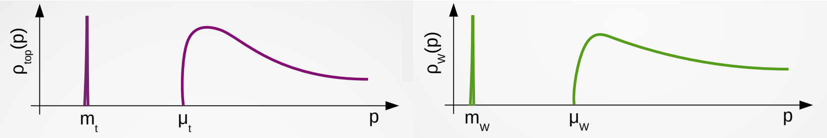
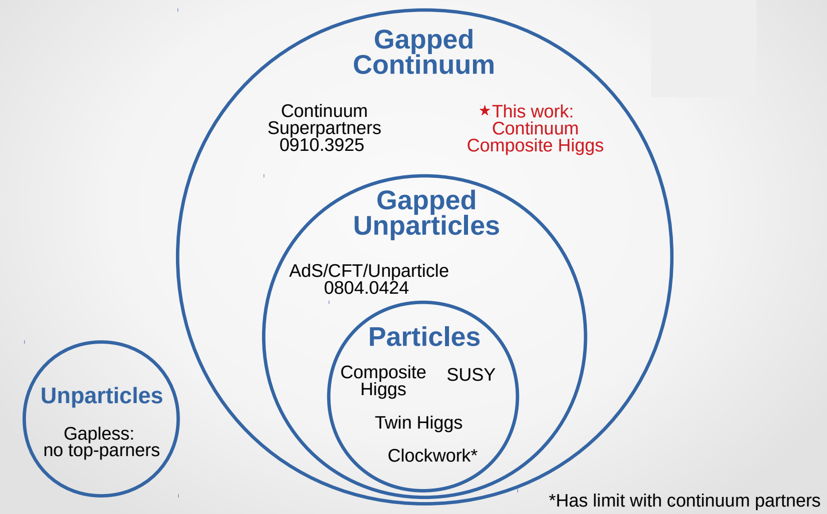


Our goal is to build a model such that the partners of tops and gauge bosons form continua.

- Can we achieve this in the context of confinement in a strongly-interacting model in a calculable way?
- Can it improve on existing models in terms of tuning for cancelling the quadratic divergence?
- How do we hide from existing searches at colliders?

Thinking outside the particle box (or circle)



slide by C. Csáki

4D Weyl (continuum) fermions

4D Lagrangian and two-point correlator for massless LH Weyl fermion

$$\mathcal{L}_\chi = \bar{\chi}\overline{\sigma}^\mu p_\mu \chi \quad \Rightarrow \quad \langle \bar{\chi}\chi \rangle = \frac{i\sigma^\mu p_\mu}{p^2} \, .$$

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- poles of $G \longleftrightarrow$ particles
- branch cuts of $G \longleftrightarrow$ continuum

We see this via a dispersion relation to express G in terms of its spectral density ρ:

$$G(p^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\operatorname{Im} G(s)}{s - p^2} \, ,$$

$$\rho(s) = \frac{1}{\pi}\operatorname{Im} G(s) = \sum_n \delta(s - m_n^2) + \rho_c(s) \, .$$

Obtaining G from 5D models: RS2 example

In 5D models, integrating out the bulk in 5D yields a 4D effective action on the UV brane.
In the 4D picture, we integrate out CFT operators to get a nonlocal effective action.

Csáki et al., hep-ph/0310355; Contino, Pomarol hep-th/0406257

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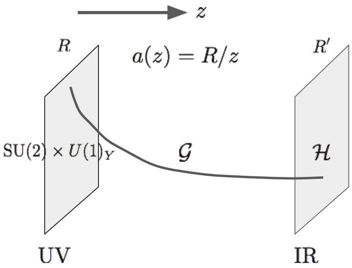
Take for example, RS2 with UV brane position taken to $R \to 0$ $(M_{\rm Pl} \to \infty)$ with a bulk fermion:

Caccipaglia, Marandella, Terning 0804.0424



$$ds^2 = a^2(z)(dx^\mu dx_\mu - dz^2)\,,$$
$$a(z) = R/z\,, \quad \Psi = \begin{pmatrix} \chi \\ \bar{\psi} \end{pmatrix}.$$

The Lagrangian in the Einstein frame is

$$\mathcal{L}_E = a^4(z)\left[\mathcal{L}_{\rm kin} + \frac{c}{z}(\psi\chi + \bar{\chi}\bar{\psi})\right]\,,$$
$$\mathcal{L}_{\rm kin} = -i\bar{\chi}\bar{\sigma}^\mu p_\mu \chi - i\psi\sigma^\mu p_\mu \bar{\psi} + \frac{1}{2}\left(\psi \overleftrightarrow{\partial}_5 \chi - \bar{\chi}\overleftrightarrow{\partial}_5 \bar{\psi}\right)\,,$$

where c is a bulk mass.

5D: bulk eom and UV effective action

The 5D bulk eom are

$$\chi'(z) - p\psi(z) + \frac{c-2}{z}\chi(z) = 0\,,$$
$$\psi'(z) + p\chi(z) + \frac{c+2}{z}\psi(z) = 0\,.$$

IR regular solutions are Hankel functions

$$\begin{pmatrix}\chi \\ \bar{\psi}\end{pmatrix}(p,z) = \begin{pmatrix}\chi_4 \\ \bar{\psi}_4\end{pmatrix} A\left(\frac{z}{R}\right)^{5/2} H^{(1)}_{c\pm\frac{1}{2}}(pz)\,.$$

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Integrating out the bulk eom to map onto the 4D effective action, the Green's function for a LH source (or a RH operator in the 4D CFT) is

$$G(p^2) = \frac{1}{p}\overline{\lim_{z\to 0}}\frac{\psi(z)}{\chi(z)} \propto \frac{\Gamma\left(\frac{1}{2}-c\right)}{4^c\Gamma\left(\frac{1}{2}+c\right)}\frac{1}{(-p^2)^{\frac{1}{2}-c}}$$

4D: ungapped unparticle

The 4D Green's function for a Weyl fermion operator with scaling dimension d is

$$G(p^2) \propto \frac{\Gamma\left(\frac{5}{2}-d\right)}{4^{d-2}\Gamma\left(d-\frac{1}{2}\right)} \frac{1}{\left(-p^2\right)^{\frac{5}{2}-d}}\,.$$

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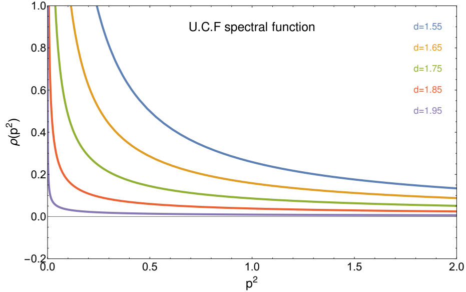
Compared with

$$G_{\text{eff}}(p^2) = \frac{1}{p}\overline{\lim_{z\to 0}}\frac{\psi(z)}{\chi(z)} \propto \frac{\Gamma\left(\frac{1}{2}-c\right)}{4^{c}\Gamma\left(\frac{1}{2}+c\right)}\frac{1}{(-p^2)^{\frac{1}{2}-c}} ,$$

the equivalence between the 4D and 5D pictures is manifest when $d = 2 + c$ for $c > -1/2$.



5D Schrödinger picture

Falkowski, Perez-Victoria 0806.1737, 0810.4940

Alternatively, taking another derivative of the χ bulk eom and using the ψ bulk eom, we obtain

$$-\chi''(z) + \frac{4}{z}\chi'(z) + \frac{c^2 + c - 6}{z^2}\chi(z) = p^2\chi(z)\,.$$

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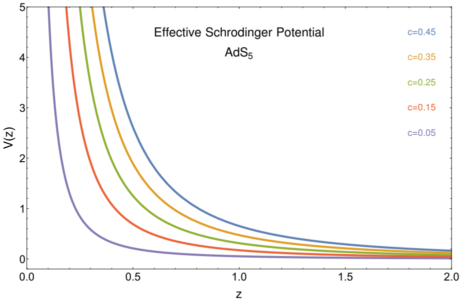
Defining $\hat{\chi}(z) = (R/z)^2\chi(z)$, the 2nd order ODE becomes

$$-\hat{\chi}''(z) \; + V(z)\hat{\chi}(z) = p^2\hat{\chi}(z)\,,$$

$$V(z) = \frac{c(c+1)}{z^2}\,.$$



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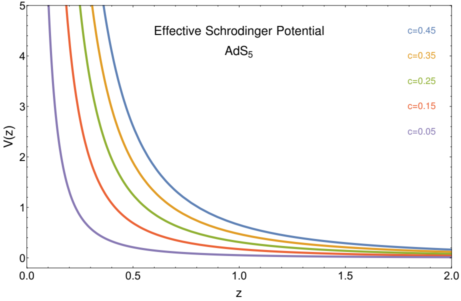
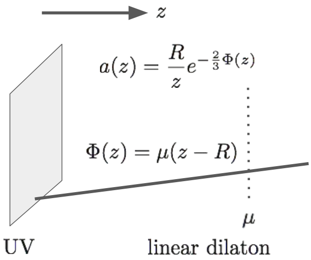


$V(z\to\infty)\to\text{const}\;\Rightarrow$ "QM" scattering states in Schrödinger picture $\longleftrightarrow$ 4D continuum

Enter the (linear) dilaton



String frame 5D Lagrangian:

$$\mathcal{L}_S = e^{-2\Phi(z)} \left(\frac{R}{z}\right)^5 \left[\frac{z}{R}\mathcal{L}_{\text{kin}} + \frac{1}{R}\left(c + y\,\Phi(z)\right)\left(\psi\chi + \bar{\chi}\bar{\psi}\right)\right],$$

c is bulk mass, y is bulk Yukawa coupling.

Einstein frame 5D Lagrangian:

$$\mathcal{L}_E = a^4(z)\mathcal{L}_{\text{kin}} + a^5(z)\frac{\hat{c}(z)}{R}\left(\psi\chi + \bar{\chi}\bar{\psi}\right),$$

$$\hat{c}(z) = (c + y\Phi(z))e^{\frac{2}{3}\Phi(z)}.$$

Can also gauge kinetic terms:

$$\mathcal{L}_E = a(z)e^{-2\Phi(z)}\frac{1}{4}F^{MN}F_{MN}.$$

Falkowski, Perez-Victoria 0806.1737; Batell, Gherghetta, Sword 0808.3977; Gutsche et al. 1108.0346

Back to Schrödinger

Returning to Schrödinger equation,

$$-\hat{\chi}''(z) + V_{\text{eff}}(z)\hat{\chi}(z) = p^2\hat{\chi}(z)\,,$$
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IR regular bulk solutions:

$$\chi(z) \;=\; A\,a^{-2}(z)\;W\left(-\frac{c\mu y}{\Delta}, c + \frac{1}{2}, 2\Delta z\right)\,,$$
$$\psi(z) = A\,a^{-2}(z)\;W\left(-\frac{c\mu y}{\Delta}, c - \frac{1}{2}, 2\Delta z\right)\frac{\mu y - \Delta}{p}\,,$$

with W a Whittaker function, $\Delta = \sqrt{y^2\mu^2 - p^2}$, imaginary for $p^2 > y^2\mu^2$.

Gapped continuum from linear dilaton

As before, the Green's function for the LH component χ of the bulk fermion is

$$G(p^2) = \frac{1}{p^2(y\mu - \Delta)} \frac{\Gamma(1+2c)}{\Gamma(1-2c)} \frac{\Gamma\left(1 + c\frac{y\mu-\Delta}{\Delta}\right)}{\Gamma\left(1 + c\frac{y\mu+\Delta}{\Delta}\right)} (2\Delta)^{-2c} .$$

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- Pole at $p^2 = 0$ (zero mode), branch cut for $p^2 > y^2\mu^2$ (continuum with gap $y\mu$).
- $0 \leq c < 1/2$ to avoid poles in Gamma functions.
- Can assign LH fermion a "dimension" of $d_\chi = 2 - c$, but p^2 scaling behaviour is more complicated.

For RH Weyl fermion: $\chi \leftrightarrow \psi$, $y \leftrightarrow -y$.

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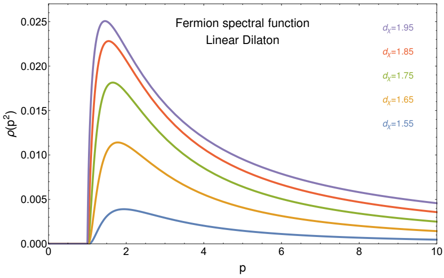
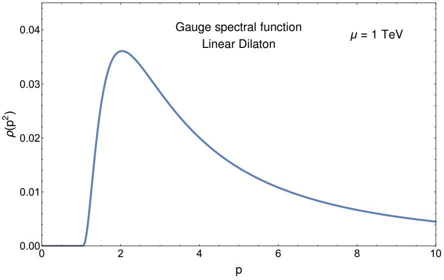
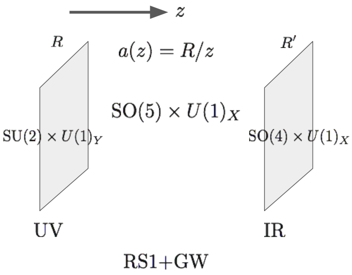




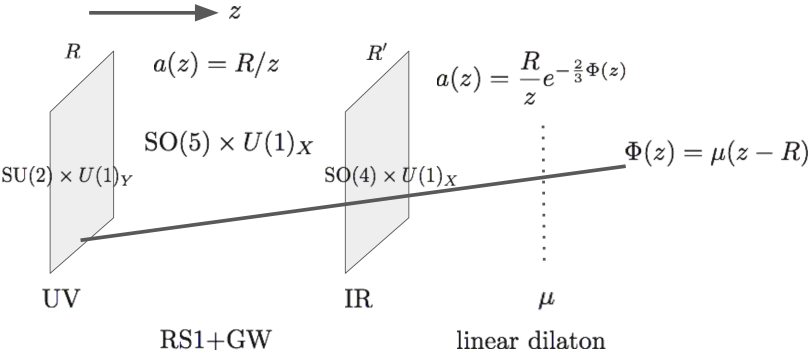
5D picture for (minimal) CH



Agashe, Contino, Pomarol hep-ph/0412089

- RS1 implementation, with $SO(5) \times U(1)_X$ bulk gauge symmetry, broken to $SU(2) \times U(1)_Y$ on the UV brane and $SO(4) \times U(1)_X$ on the IR brane, with $Y = T_{3R} + X$ on UV brane.
- Dirichlet BC's on IR brane for broken generators of $SO(5)/SO(4)$ yield zero modes in A_5 – Wilson line from R to R' corresponds to Higgs in 4D theory.
- Massless states become combinations of fermion bulk gauge multiplets with nonzero $SO(4)$-invariant IR brane mass terms – SM Yukawas in 4D eff theory.
- A_5 gets a vev. Perform a bulk gauge transformation using the $\langle A_5 \rangle$ Wilson line, rotating the IR boundary conditions and lifting the zero modes.
- Goldberger-Wise mechanism to stabilize brane positions in 5th dimension.

The full 5D setup for continuum CH



Roughly, combine

- 5D RS1 (with Goldberger-Wise) implementation of MCHM to solve hierarchy problem,
- linear dilaton in space beyond IR brane ("deep IR") responsible for continuum.

Solving for the spectral densities

Rundown:

- Take MCHM bulk and brane symmetries.
- Embed Q_L, T_R, B_R as $\mathbf{5_{2/3}} + \mathbf{5_{2/3}} + \mathbf{10_{2/3}}$ multiplets in the bulk.

Medina, Shah, Wagner; 0706.1281; Csáki, Falkowski, Weiler 0804.1954

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- Apply appropriate boundary conditions on UV and IR branes to obtain chiral SM zero modes and the Higgs as the A_5 of the broken generators of $SO(5)/SO(4)$, and demand "deep IR"-regular solutions.
- Implement rotation of IR boundary jump conditions (for fermions, inhomogeneous due to IR brane mass terms) by Higgs vev (Wilson line of $\langle A_5 \rangle$ from R to R') of broken generators $SO(5)/SO(4)$.

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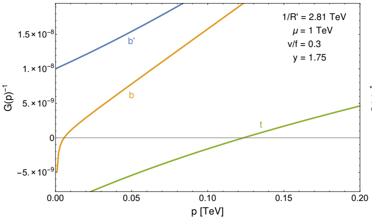
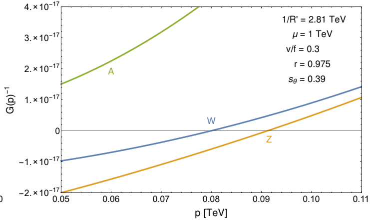
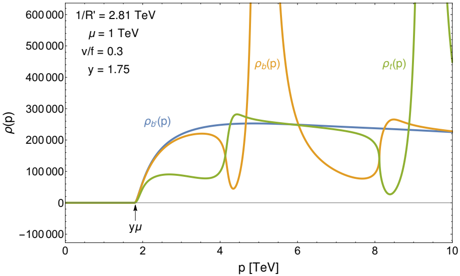
  Medina, Shah, Wagner; 0706.1281; Csáki, Falkowski, Weiler 0804.1954
- Apply appropriate boundary conditions on UV and IR branes to obtain chiral SM zero modes and the Higgs as the A_5 of the broken generators of $SO(5)/SO(4)$, and demand "deep IR"-regular solutions.
- Implement rotation of IR boundary jump conditions (for fermions, inhomogeneous due to IR brane mass terms) by Higgs vev (Wilson line of $\langle A_5 \rangle$ from R to R') of broken generators $SO(5)/SO(4)$.
- Solve for inhomogeneous eom in 5th dimension to obtain 5D Green's functions $G(z, z'; p^2)$.
- Take appropriate limit $z, z' \to R$ to obtain 4D Green's functions in effective theory.
- Diagonalize (20×20 for fermion, 4×4 for gauge) and take imaginary parts to obtain spectral densities.

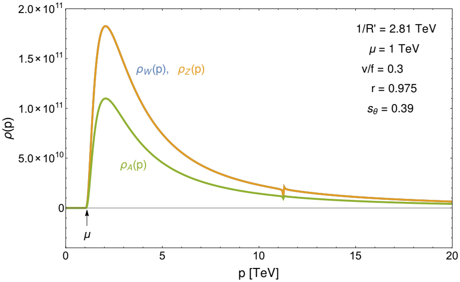
Results: spectral functions for benchmark point

$$R/R' = 10^{-16},\ 1/R' = 2.81\,\mathrm{TeV},\ \mu = 1\,\mathrm{TeV},\ y = 1.75,\ r = 0.975,\ \sin\theta = 0.39,$$
$$c_Q = 0.2,\ c_T = -0.22,\ c_B = -0.03,\ M_1 = 1.2,\ M_4 = 0,\ M_b = 0.017$$









Results: Higgs potential

Radiatively-generated Coleman-Weinberg potential from fermion and gauge Green's functions.

$$V(h) = \frac{3}{16\pi^2} \int dp\, p^3 \left[-4 \sum_{j=1}^{20} \log G_{f_j}(ip) + \sum_{k=1}^{4} \log G_{g_k}(ip) \right]$$

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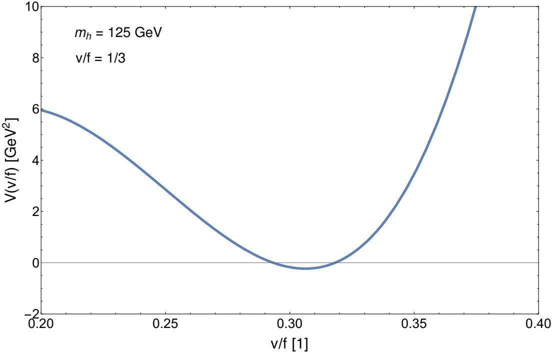
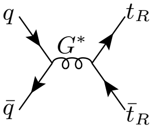


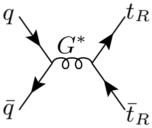
- Reproduce $v/f \sim 0.3$, typical for CH models that satisfy EWPT bounds. Agashe, Contino hep-ph/0510164
- In terms of model parameters, tuning is 1% calculated using Barbieri-Giudice, compared to 0.1% for standard CH models. Panico, Redi, Tesi, Wulzer 1210.7114

Pheno: s-channel resonances



At the LHC, KK gluons are resonances in Drell-Yan processes, e.g., $q\bar{q} \to t_R\bar{t}_R$.

Pheno: s-channel resonances



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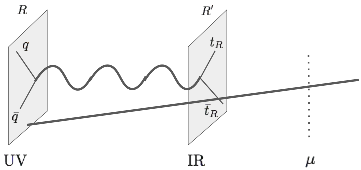
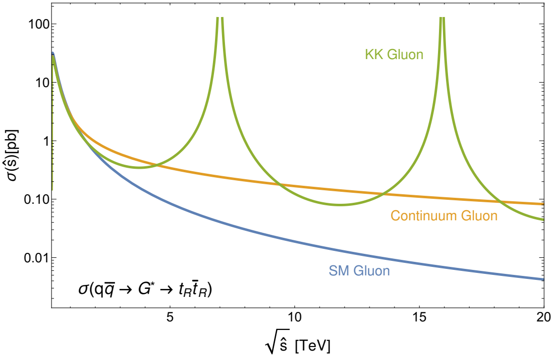


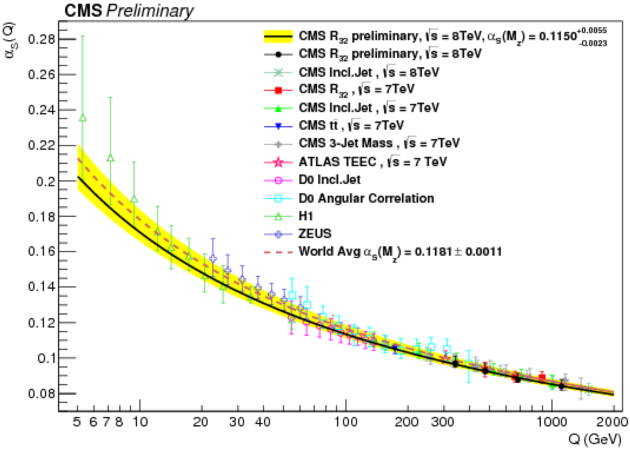


Instead, continuum gluons yield a smooth excess above the SM background, given by

$$\sigma(\hat{s}) \sim \sigma_{\rm SM}(\hat{s}) \times \hat{s}^2 |G(R, R'; \hat{s})|^2$$

Pheno: bounds from α_s running at the LHC

Jets at the LHC test perturbative QCD at the TeV scale, including the running of α_s.



CMS-PAS-SMP-16-008, CMS 1609.05331

see also talks by I.J. Watson, M. Mangano

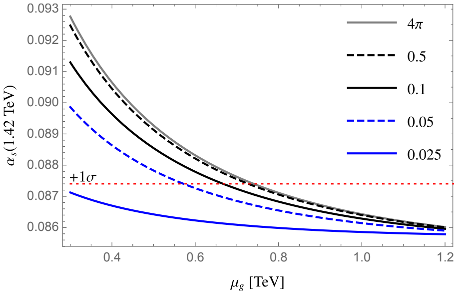
Pheno: bounds from α_s running at the LHC

We can use holographic techniques to calculate the running of the strong gauge coupling.

$$\frac{1}{g^2(Q)} = \frac{1}{g_5^2}\int_R^{1/Q} dz\, a(z) + \frac{1}{g_{\rm UV}^2} - \frac{b_{\rm UV}}{8\pi^2}\log\left(\frac{1}{RQ}\right),$$

Arkani-Hamed, Porrati, Randall hep-ph/0012148

Include UV brane-localized kinetic terms that maintain bulk perturbativity.



Bounds from CMS data from $\sqrt{s} = 7$ and 8 TeV runs up to $Q = 1.42\,\text{TeV} \Rightarrow \mu_g \gtrsim 600\,\text{GeV}$ (excluding error from assuming SM running).

Pheno: continuum partner pair production

$$q\bar{q} \to g \to \chi^\dagger \chi$$

We can calculate this using the optical theorem by first computing the gluon vacuum polarization.

$$\sigma(q\bar{q} \to \chi^\dagger \chi) \sim \operatorname{Im} \Pi_g(s)|_\chi$$

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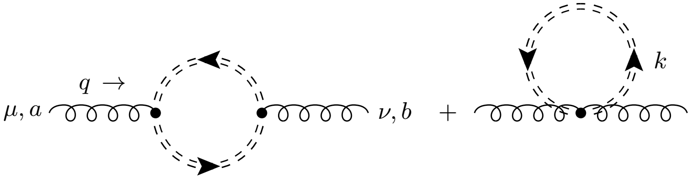

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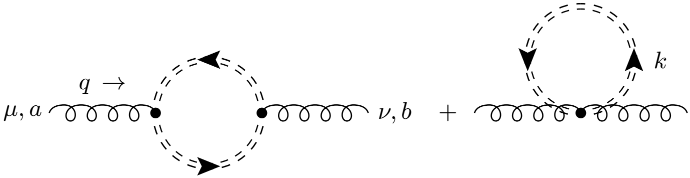
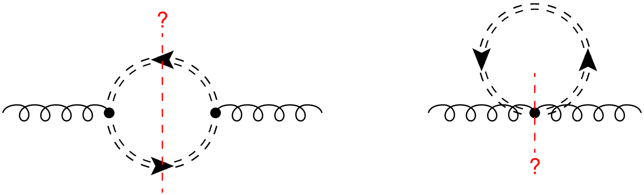


Looks simple - just use Cutkosky?

$$\operatorname{Im} \frac{1}{p^2 - m_\chi^2 + i\varepsilon} = -\pi\delta(p^2 - m_\chi^2)$$

Pheno: continuum partner pair production...?

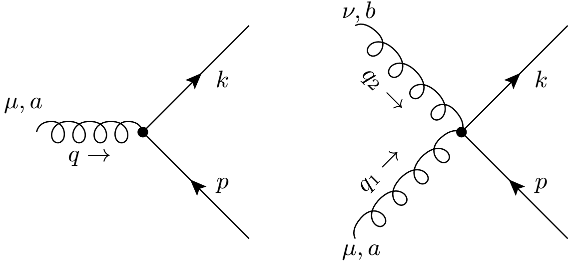
Unfortunately, no.



- "Generalized unitarity", since gluon-continuum fermion vertices are functions of momentum.
- Also, unknown final-state phase space for general continuum (tractable example: "fractional particles" for unparticles).

Return of the nonlocal action

$$S_{\text{eff},\psi} = \int \frac{d^4k}{(2\pi)^4}\, \bar{\psi}(k)(\not{k} - m)\Sigma_\psi(k^2)\psi(k)\,,$$



Mandelstam (1962), Terning (1991)

Gluon-fermion couplings constrained by gauge invariance.

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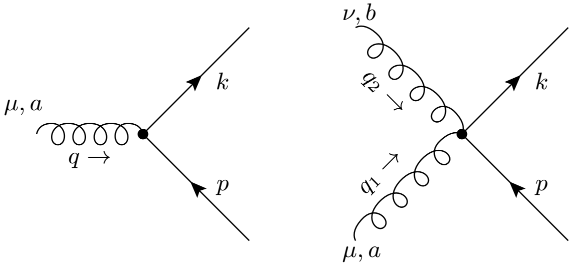

Mandelstam (1962), Terning (1991)

Gluon-fermion couplings constrained by gauge invariance.

- Go to position space.
- Insert Wilson line to preserve gauge invariance.
- Taylor expand and Fourier transform back to momentum space.
- Resulting vertex form factors satisfy generalized Ward-Takahashi identities.

Vacuum polarization with nonlocal actions

Typical terms in the vacuum polarization:

$$\mathrm{Im}\,\Pi \sim \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k+q)^2(q^2+2k\cdot q)^2} \left[\frac{\Sigma(k+q)}{\Sigma(k)} P_+ + \frac{\Sigma(k)}{\Sigma(k+q)} P_- \right]$$

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- Polynomial of momenta.
- Spectral densities (model-dependent, constrained by Lorentz invariance and unitarity).
- Denominator terms linear in loop momentum (like those appearing in EFT's such as SCET or HQET).

Integral reduction

Separate out model dependence of spectral densities via dispersion relations (possibly with subtractions for convergence):

$$\Sigma(k^2) = \frac{1}{\pi}\int ds\,\frac{\mathrm{Im}\,\Sigma(s)}{s-k^2}\,,$$

$$\Sigma^{-1}(k^2) = \Sigma^{-1}(k_0^2) - \frac{k^2-k_0^2}{\pi}\int_{\mu^2}^{\infty} ds\,\frac{\mathrm{Im}\,\Sigma^{-1}(s)}{(s-k_0^2)(s-k^2)}\,.$$

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Then vacuum polarization terms become

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$\mathcal{I}$ is (nontrivial) integral kernel for loop momenta, Feynman parameters, etc.

Ambition: master formula for s-channel pair production of general continuum partners.

Scalar continuum case factorizes similarly into a subset of terms of the fermionic case.

Conclusion

- We constructed a 5D model combining aspects of RS and soft-wall models that evades standard LHC searches for top partners and KK gluons by modelling them as continua.
- Embedding these into the composite Higgs framework yields a realistic Higgs potential with less tuning.
- Not completely hidden from collider searches: e.g., with standard final states, more likely to see smooth contributions in cross sections over SM background.
- Focus of ongoing work is on providing a generic formula for pair production of continuum partners.
- Further work in model building could yield interesting collider signals at a 100 TeV pp machine.

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Thank you.

Schrödinger for gauge bosons

$$\hat{A}(z) = \sqrt{\frac{R}{z}}\, e^{-\mu(z-R)}\, A(z)\,,$$
$$-\hat{A}''(z) + V_{\text{eff}}(z)\hat{A}(z) = p^2\hat{A}(z)\,,$$
$$V_{\text{eff}}(z) = \mu^2 + \frac{\mu}{z} + \frac{3}{4z^2} \quad \underset{z\to 0}{\longrightarrow} \quad \mu^2\,.$$

IR regular bulk solutions:

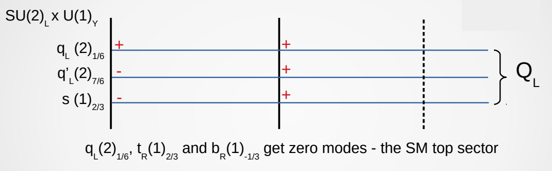
$$A(z) = A\sqrt{\frac{z}{R}}\, e^{\mu(z-R)}\, W\left(-\frac{\mu}{2\Delta}, 1; 2\Delta z\right)\,, \quad \Delta = \sqrt{\mu^2 - p^2}\,.$$

The Neumann Green's function has a pole at $p^2 = 0$ and a branch cut for $p^2 > \mu^2$ with the spectral density

$$\rho(s) = \frac{1}{\pi}\overline{\lim_{z\to 0}}\, \text{Im}\,\frac{A(z)}{A'(z)} = \frac{1}{2\pi s}\left[1 + i\psi\left(\frac{1}{2} + \frac{\mu}{2\Delta}\right) - i\psi\left(\frac{1}{2} - \frac{\mu}{2\Delta}\right)\right]\,,$$

$\psi(x)$ is the digamma function.

More calculational details



Bulk fermions:

$$\begin{aligned} Q_L(\mathbf{5})_{\frac{2}{3}} &\to q_L(\mathbf{2})_{\frac{1}{6}} + \tilde{q}_L(\mathbf{2})_{\frac{7}{6}} + y_L(\mathbf{1})_{\frac{2}{3}} , \\ T_R(\mathbf{5})_{\frac{2}{3}} &\to q_R(\mathbf{2})_{\frac{1}{6}} + \tilde{q}_R(\mathbf{2})_{\frac{7}{6}} + t_R(\mathbf{1})_{\frac{2}{3}} , \\ B_R(\mathbf{10})_{\frac{2}{3}} &\to q'_R(\mathbf{2})_{\frac{1}{6}} + \tilde{q}'_R(\mathbf{2})_{\frac{7}{6}} + x_R(\mathbf{3})_{\frac{2}{3}} + y_R(\mathbf{1})_{\frac{7}{6}} + \tilde{y}_R(\mathbf{1})_{\frac{1}{6}} + b_R(\mathbf{1})_{-\frac{1}{3}} . \end{aligned}$$

IR brane mass terms:

$$S_{\rm IR} = \int d^4x \sqrt{g_{\rm ind}} \left[M_1 \bar{y}_L t_R + M_4 \left(\bar{q}_L q_R + \bar{\tilde{q}}_L \tilde{q}_R \right) + M_b \left(\bar{q}_L q'_R + \bar{\tilde{q}}_L \tilde{q}'_R \right) \right] .$$

Tuning:

$$\text{tuning} = \left[\max_i \frac{d\log\, v}{d\log\, p_i} \right]^{-1} , \; p_i \in \left\{ R, R', \mu, r, \theta, y, c_Q, c_T, c_B, M_1, m_4, M_b \right\} .$$

Green's functions in full model

Gauge:

$$G(z) = \left(\frac{z}{R}\right) e^{\mu(z-R)} \left[A\, M\left(-\frac{\mu}{2\Delta}, 1; 2\Delta z\right) + B\, W\left(-\frac{\mu}{2\Delta}, 1; 2\Delta z\right)\right] .$$

Fermion:

$$\chi_{Q_L}(z) = a(z)^{-2} \left[A\, \hat{M}(c_Q, z) + B\, \hat{W}(c_Q, z)\right] ,$$

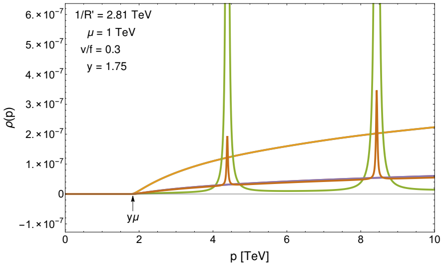
$$\psi_{Q_L}(z) = a(z)^{-2} \left[A\, \alpha(c_Q, p)\, \hat{M}(c_Q, z) + B\, \beta(p)\, \hat{W}(c_Q, z)\right] ,$$

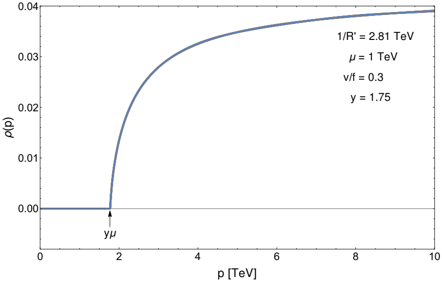
with $\alpha(c_Q, p) \equiv \frac{4\left(\frac{1}{2}+c_Q-R\mu y\right)\Delta}{p}$, $\beta(p) \equiv \frac{\mu y-\Delta}{p}$, and

$$\hat{M}(c_Q, z) = M\left(\frac{-\mu y\,(c_Q - \mu y R)}{\Delta}, \frac{1}{2} + c_Q - R\mu y, 2\Delta z\right) ,$$

$$\hat{W}(c_Q, z) = W\left(\frac{-\mu y\,(c_Q - \mu y R)}{\Delta}, \frac{1}{2} + c_Q - R\mu y, 2\Delta z\right) ,$$

Exotic partner spectral functions

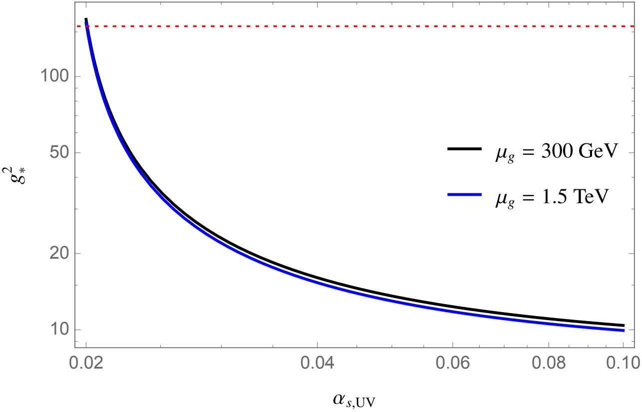




Bulk perturbativity

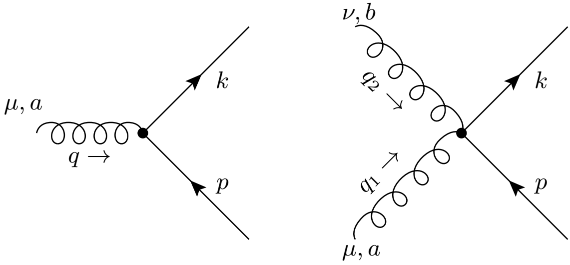
$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha_5}\int_R^{1/Q} dz\, a(z) + \frac{1}{\alpha_{\rm UV}} - \frac{b_{\rm UV}}{2\pi}\log\left(\frac{1}{RQ}\right) + \theta(1/R' - Q)\left[\frac{1}{\alpha_{\rm IR}} - \frac{b_{\rm IR}}{2\pi}\log\left(\frac{1}{R'Q}\right)\right].$$

- Use soft-wall metric $a(z) = R/z \exp\left(\frac{2}{3}(R-z)\mu\right)$.
- Assume SM fields localized at UV brane except for t_R.



Feynman rules from nonlocal actions

$$S_{\mathrm{eff},\psi} = \int \frac{d^4k}{(2\pi)^4}\, \bar{\psi}(k)(\not{k} - m)\Sigma_\psi(k^2)\psi(k)\,,$$



Mandelstam (1962), Terning (1991)

$$ig\Gamma^{a\mu}_\psi(p,q) = igT^a \left\{ \left(\not{p} + \frac{\not{q}}{2} - m \right)(2p+q)^\mu \mathcal{F}_\psi(p,q) + \gamma^\mu \left[\frac{\Sigma_\psi(p+q) + \Sigma_\psi(p)}{2} \right] \right\}$$

$$ig^2\Gamma^{ab,\mu\nu}_\psi(p,q_1,q_2) = ig^2 \left(\not{p} + \frac{\not{q}_1 + \not{q}_2}{2} - m \right) \Gamma^{ab,\mu\nu}_\phi(p,q_1,q_2)\Big|_{\Sigma_\phi \to \Sigma_\psi}$$

$$+ \frac{ig^2}{2}\gamma^\mu \Gamma^{b\nu,a}_\psi(p,q_2,q_1) + \frac{ig^2}{2}\gamma^\nu \Gamma^{a\mu,b}_\psi(p,q_1,q_2)$$

$$\Gamma^{a\mu,b}_\psi(p,q_1,q_2) = T^aT^b(2p+q_1)^\mu \mathcal{F}_\psi(p,q_1) + T^bT^a(2(p+q_2)+q_1)^\mu \mathcal{F}_\psi(p+q_2,q_1)$$

$$\mathcal{F}(p,q) \equiv \frac{\Sigma_\psi(p+q) - \Sigma_\psi(p)}{q^2 + 2p\cdot q}$$

Proof of principle: gapped fermionic unparticles

$$\left.\frac{g_{\mu\nu}\mathrm{Im}\,\widehat{\Pi}^{\mu\nu}_{\psi}(q;D)}{g_{\mu\nu}\mathrm{Im}\,\widehat{\Pi}^{\mu\nu}_{\psi}(q;0)}\right|_{\beta\to 0} = 1 + \frac{\sin D\pi}{D\pi}\bigg(\frac{1}{2} + B_{\frac{1}{2}}(2+D,1-D) + B_{\frac{1}{2}}(2-D,1+D) - B_{\frac{1}{2}}(1+D,2-D) - B_{\frac{1}{2}}(1-D,2+D)\bigg)$$