Flavor and CP violation Beyond the LHC

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Why flavour?

In the SM all flavour and CP violation is encoded the Higgs Yukawas and θ_{QCD}

Open questions

- "*The flavour Puzzle*"
  Is there a dynamical origin for the observed masses and mixings?
- "*The New Physics flavour Problem*"
  If New Physics is light it must have non-trivial flavour structure
- "*Baryogenesis*"
  Where are the CP violating sources beyond the CKM phase necessary for Baryogenesis?
- "*Strong CP Problem*"
  Why is the QCD vacuum CP conserving?

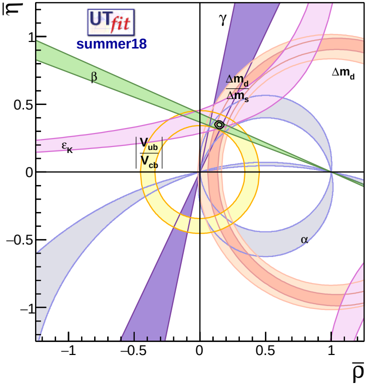
What is flavour?

Traditional flavour

- Quark-sector provides multiple observables, overconstrained system

➜ test the SM by testing the CKM unitarity triangle

SM-fit



[UTfit 18]

A success so far

Note that

- This is a SM-fit, more freedom if this is relaxed
- 20% deviations are still allowed in various observables sensitive to NP
- Still not sensitive to many observables (FCNCs/CP-violation with τ's, $B_q \to ee/\tau\tau, \ldots$)

➜Experimental efforts continue
LHCb upgrade, Belle-2, NA62, KOTO, …

What is flavour?

Flavour is more than the test of CKM unitarity

Higgs

- A new era for flavour: Higgs couplings to fermions
- Higgs Flavour [see talk by Felix Yu]
- CP violation in Higgs Yukawas and EDMs [this talk]

Light New Physics

- Most of the time the emphasis is laid on probing high-scale NP
- NP can be light if weakly interacting
- Important measurements/analyses for flavour experiments
- Example: the QCD axion [(this talk)]

Flavour observables constrain naively unrelated flavour-conserving parameters

(e.g., rare B/K decays constrain anomalous ttZ couplings more than $3000\,\mathrm{fb}^{-1}$ at HL-LHC)

Outline: searching under the 2019 lampposts

Part 1: EDMs and CP violation in Higgs yukawas

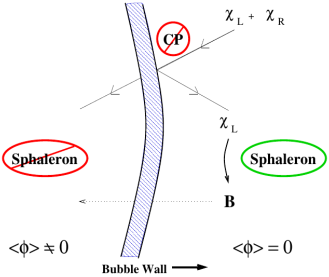
Part 2: Axions at precision flavour experiments

Part 3: Flavour at a tera-Z factory (discussion)

Part I

Probing the CP structure of the Higgs with EDMs

Electroweak Baryogenesis



Electroweak Baryogenesis possible if

- additional sources of CP violation
- allow for a first-order electroweak phase transition

Minimally two EFT operators suffice

[see Huber, Pospelov, Ritz 06 and Refs. within]

$$\mathcal{L}_{\text{dim-6}} = \frac{1}{\Lambda_{\text{CP}}^2}(H^\dagger H)^2 + \frac{1}{\Lambda_{\cancel{\text{CP}}}^2}(H^\dagger H)\bar{Q}_3\tilde{H}t_R + \text{h.c.}$$

- $\Lambda_{\text{CP}} \approx \Lambda_{\cancel{\text{CP}}} \approx 500 - 1000$ GeV gives the correct photon-baryon ratio, η_b
- Generically more operators and more phases

➜Phenomenologically attractive scenario with **signals both at colliders and low-energy observables**

[see, e.g., review Morrisey, Ramsey-Musolf 12]

Flavor-diagonal CP violation in the Higgs–quark sector

Parameterization

$$\mathcal{L} = -\frac{y_q^{\text{SM}}}{\sqrt{2}}\, \kappa_q\, h\, \bar{q}\, (\cos\phi_q + i \sin\phi_q)\, q$$

- $\kappa_q \geq 0$ is a CP conserving parameter — SM : $\kappa_q = 1$
- $\phi_q \in [-\pi, +\pi]$ is a CP violating NP phase, — SM : $\phi_q = 0$
- In total, 6 new CP conserving parameters, and 6 new phases in quark sector

In SMEFT they originate from $\frac{1}{\Lambda^2}(H^\dagger H)Q_L\tilde{H}u_R$ and $\frac{1}{\Lambda^2}(H^\dagger H)Q_L H d_R$ operators.

Example values for $\kappa = 2$:
$\Lambda = 240\,\text{GeV}(t), 1.5\,\text{TeV}(b), 3\,\text{TeV}(c), 10\,\text{TeV}(s), 45\,\text{TeV}(d), 70\,\text{TeV}(u)$

EDMs experimental status/prospects

Multiple experimental target measurements

- **“Elementary”** : electron, neutron, proton, deuteron
- **Atomic** : mercury, radium, xenon, …
- **Molecular** : ThO (essentially electron)

[e cm]	d_e	d_n	$d_{p,D}$
current	1.1×10^{-29}	2.9×10^{-26}	–
expected	5.0×10^{-30}	1.0×10^{-28}	1.0×10^{-29}

	d_{Hg}	d_{Xe}	d_{Ra}
current	7.4×10^{-30}	5.5×10^{-27}	4.2×10^{-22}
expected	–	5.0×10^{-29}	1.0×10^{-27}

[Hewett et al 12; Baker et al 06; ACME 18, Graner et al 16]

- Expect order of magnitude improvements

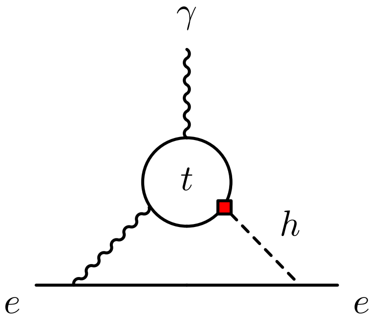
➜ ongoing work on lattice (chromoEDM operator)

Top-quark

Top quark: electronic EDM

Electron dipole induced via 2loop Barr–Zee type diagrams

[Weinberg 89; Barr, Zee 90]

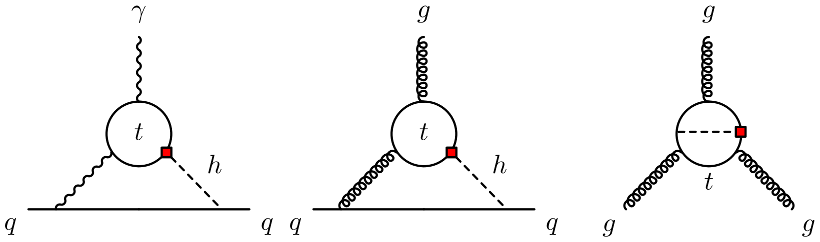


- 2018 result: $|d_e| < 1.1 \times 10^{-29}$ e cm 90% CL [ACME 18]
- No constraint if $\kappa_e = 0$

$$\rightarrow \kappa_t |\sin \phi_t| < 0.001$$

Top quark: hadronic EDM

Similar to electronic case, but for hadronic EDM more operators



- qEDM, qCEDM, and Weinberg all induced at the two-loop level

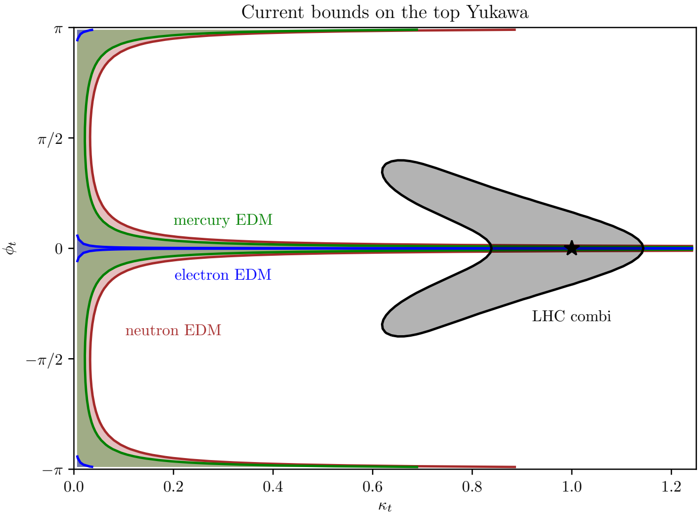
[Brod et al 13; Brod, Skodras 18]

- QCD RGE: mixing under QCD at 1loop [Hisano et al 12]

$$\frac{d_n}{e} = \boldsymbol{(\# + \# \log \frac{m_t}{m_h})}(\, \kappa_t^2 \sin\phi_t \cos\phi_t \text{ or } \kappa_t \kappa_q \sin\phi_t \cos\phi_q \,)$$

- Constraints remain even if $\kappa_q = 0$

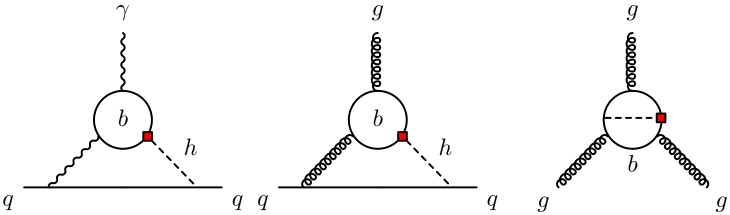
Top quark: combination



Bottom/Charm quarks

CP violation in Higgs-bottom/charm

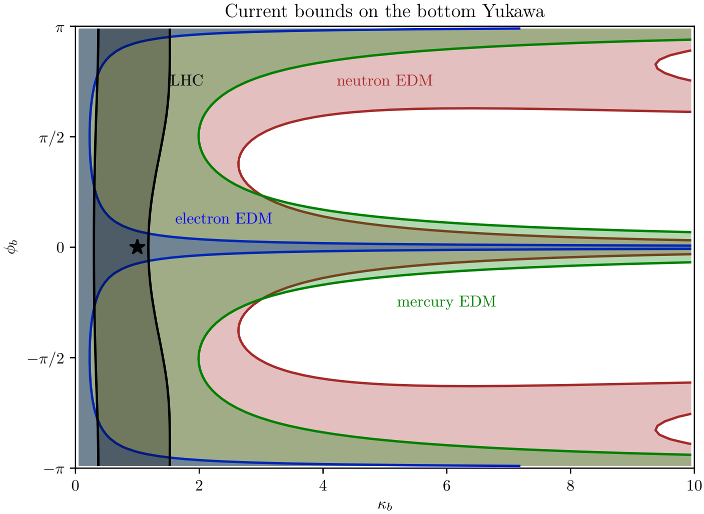
“Naive” Barr–Zee type computation



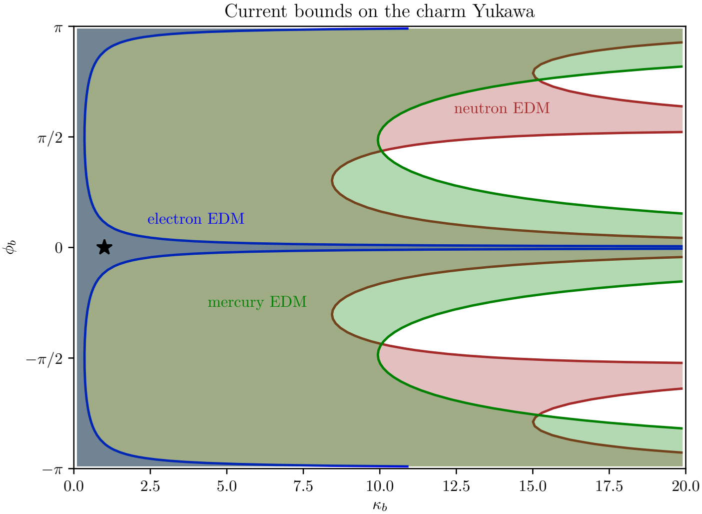
$$\tilde{d}_q = g_s^3(\# + \#\,\boldsymbol{\log \frac{m_b}{m_h}})(\ \kappa_b \sin\phi_b\ )$$

- Naive computation is a **bad** approximation **(factor 5 uncertainty in qCEDM)**
- Reason: multi-scale problem ➜$\alpha_s \log(m_b/m_h) \sim 1$!
- **Precision:** use EFT to resum the large logs
  - Sum all $\alpha_s^n \log^n(m_b/m_h)$ (LL) [Brod et al 13]
  - Sum all $\alpha_s^n \log^{n-1}(m_b/m_h)$ (NLL) [Brod, ES 18]

Bottom quark: combination



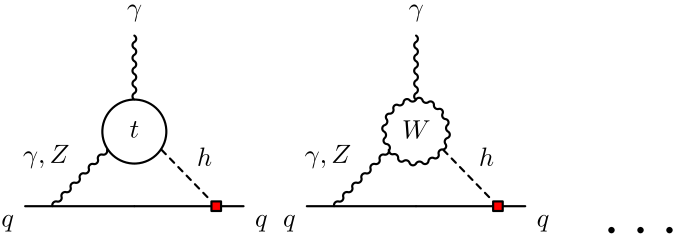
Charm quark: combination



Light quarks: u, d, s

Light quarks: hadronic EDMs

- Similar to electron case
- $\mathcal{O}(100)$ electroweak 2-loop diagrams [Brod, Skodras 18]

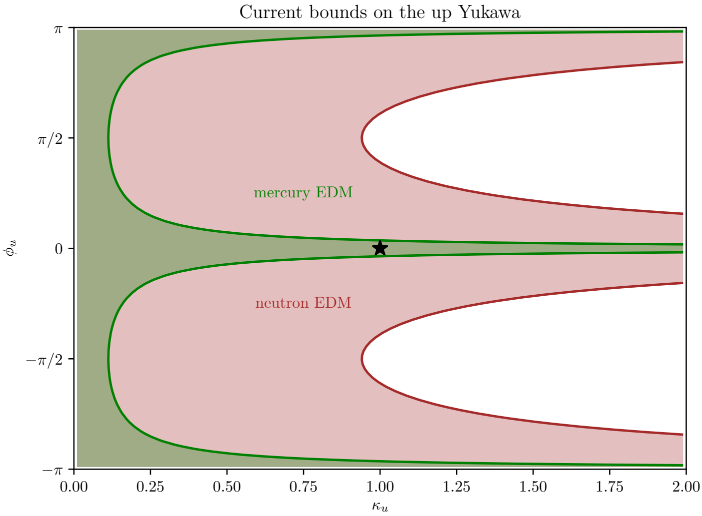


- Constraint only from hadronic EDMs, no constraints from electronic EDM

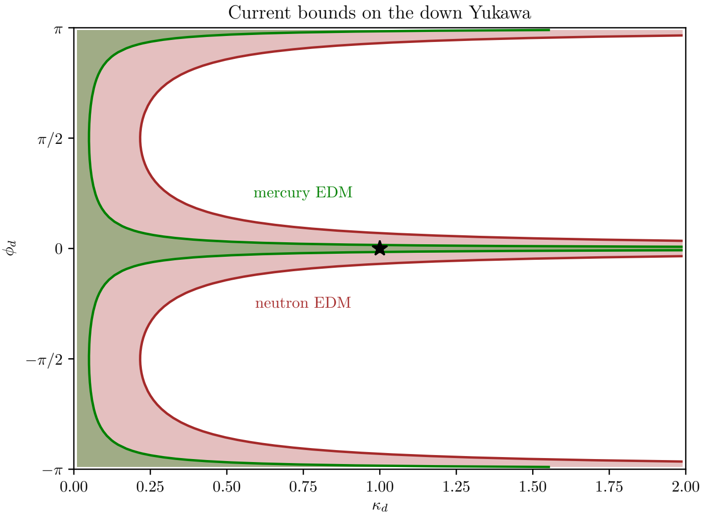
➜ $d_n/e = 1.1(\tilde{d}_d + 0.5\tilde{d}_u) + (g_T^u \frac{d_u}{e} + g_T^d \frac{d_d}{e} + g_T^s \frac{d_s}{e})$ [hadronic input Gupta et al 18]

➜ $d_{\mathrm{Hg}}/e = -1.8 \times 10^{-4}(4^{+8}_{-2})(\tilde{d}_u - \tilde{d}_d)$ [hadronic input Pospelov, Ritz 05]

Up-quark: combination

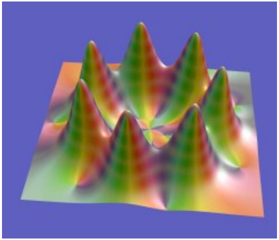


Down-quark: combination



Part 1: summary

- Expect order of magnitude improvements in EDM measurements
- EDMs strongly constrain new CP phases in the higgs sector / TeV scale
- **Important goal for the high-intensity community**
  ➜ Measure and combine info from many observables
  (neutron, proton, electron, atomic, . . . EDMs)
  ➜ Control theory uncertainties, **lattice QCD** and **pertubative errors**
- Global picture from global fit still missing [in progress]
- Multiple questions still open (CP violation in top-FCNCs, SMEFT interpretation. . .)



Part 2

QCD axions and precision flavour

Axion solution to Strong CP Problem

A dynamical way to address the Strong CP problem is via the QCD axion

[Peccei, Quinn 77; Wilczek 78; Weinberg 78]

- “elegant” $=$ solution via IR dynamics
- **Bonus:** viable DM candidate in large parameter space

[Preskill et al 83; Abbott, Sikivie 83; Dine, Fischler 83]

Minimal Requirements

A Goldstone boson of a spontaneously broken global U(1) that has a QCD anomaly $(=$ axion)

The axion effective Lagrangian

$$\mathcal{L} = \underbrace{N \frac{a}{\Lambda_{\text{PQ}}} \frac{\alpha_s}{4\pi} G\tilde{G}}_{(i)} + \underbrace{E \frac{a}{\Lambda_{\text{PQ}}} \frac{\alpha}{4\pi} F\tilde{F}}_{(ii)} + \underbrace{\frac{\partial_\mu a}{\Lambda_{\text{PQ}}} \bar{f}_i \gamma^\mu (C^V_{ij} + \gamma_5 C^A_{ij}) f_j}_{(iii)}$$

(i) Generates a potential for the axion ➜ dynamically sets $\theta = 0$ and small m_a

(ii) Provides coupling to photon, most common axion search channel

Haloscopes (ADMX), Helioscopes (CAST/IAXO)

(iii) $3 + 6$ couplings in each fermion sector

➜ flavor conserving constrained by **astrophysics**

➜ flavor violating constrained by **rare decays**

Origin of flavour violating couplings: missalignment of PQ and mass basis

e.g., non-universal DFSZ models [Celis et al 14; di Luzio et al 17]

e.g., PQ = FN (“axiflavon”/“flaxion”) [Wilczek 82; Calibbi et al 16; Ema et al 16]

Model-independent constraints

$$\mathcal{L}_{\text{eff}} = \frac{\partial_\mu a}{F^V_{ij}} \bar{f}_i \gamma^\mu f_j + \frac{\partial_\mu a}{F^A_{ij}} \bar{f}_i \gamma^\mu \gamma_5 f_j$$

	F^V_{ij} [GeV]	F^A_{ij} [GeV]
sd	6.9×10^{11}	2.3×10^6
cu	3.3×10^5	2.4×10^6
bd	1.0×10^8	1.4×10^6
bs	1.2×10^8	3.0×10^5

[Feng et al, 97; Björkeroth et al 18]

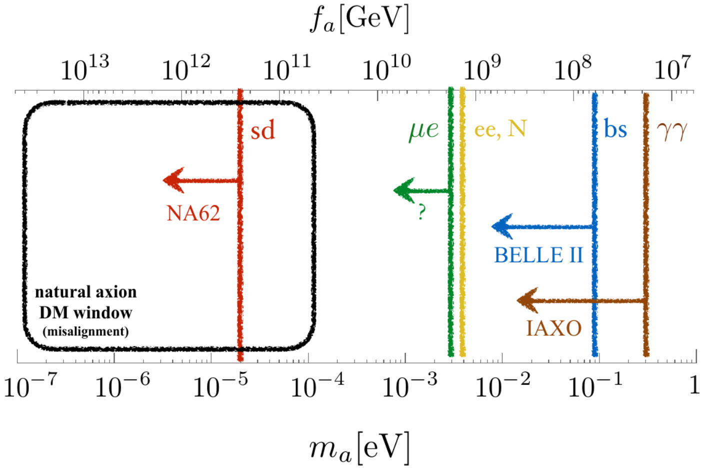
F^V_{ij}: strong constraints from $M \to Pa$, excellent prospects [NA62, Belle 2]
($K^+ \to \pi^+ a$ [E787+E949], $D^+ \to \pi^+$ [BR only], $B^+ \to K^+/\pi^+ a$ [CLEO]

F^A_{ij}: weaker (and less robust) constraints from $M - \overline{M}$ mixing, However, also similarly constrained by LHCb's sideband of $B_q \to \mu\mu$

[in progress with Albrecht, Ziegler, Zwicky]

Present and future constraints

From R. Ziegler @ La Thuille 2019



(for $C_i = 1$)

- Kaon sector, $K^+ \to \pi^+ \nu\nu$ already probes the Axion DM window
- no dedicated analyses yet from Belle for two-body $B \to M + a$

Part 2: summary

Flavour and the light-NP frontier

- The **QCD axion** can have flavor violating couplings

- They are **generic and sizable** whenever fermions have **non-universal PQ charges**

- **Precision-flavor** experiments offer a **complementary** way to search for the QCD axion.
  (Most stringent constraints from Kaons; NA62 will probe a PQ scale of 10^{12} GeV)

- Progress expected, but require dedicated analyses from the collaborations

Part 3

Flavour at a tera-Z factory

Qualitative differences of Z factories to LHCb and Belle

Tera-Z at FCC-ee or CEPC

- **production of 10^{12} Z's**

✓ no phase-space limitations like at Belle-2

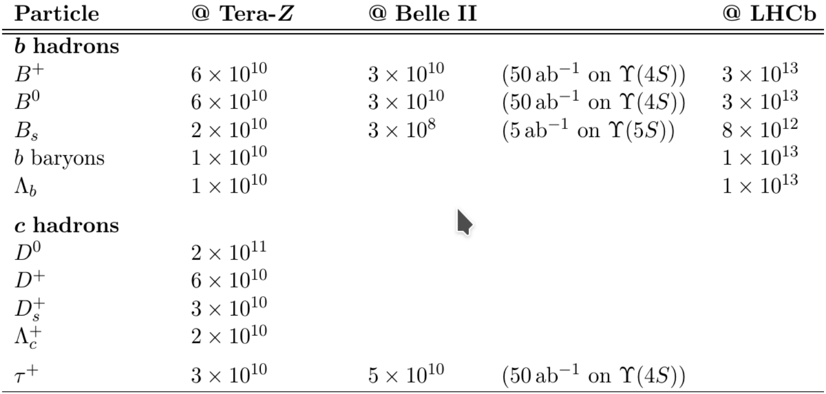
✓ LEP environment, less hadronic activity than at LHCb

✘ larger $\sqrt{s}$ than at Belle-2, more hadronic activity

✓ decay products of Z **more boosted** than at Belle 2
more separation in lab-frame, better experimental resolution?

It is **not clear (and process specific)** whether the combination of higher **hadronic activity** but larger **boost** is **beneficial** for the tera-Z. Input and dedicated studies needed.

Particle production



Particle	@ Tera-Z	@ Belle II		@ LHCb
***b* hadrons**				
B^+	6×10^{10}	3×10^{10}	($50\,\mathrm{ab}^{-1}$ on $\Upsilon(4S)$)	3×10^{13}
B^0	6×10^{10}	3×10^{10}	($50\,\mathrm{ab}^{-1}$ on $\Upsilon(4S)$)	3×10^{13}
B_s	2×10^{10}	3×10^{8}	($5\,\mathrm{ab}^{-1}$ on $\Upsilon(5S)$)	8×10^{12}
b baryons	1×10^{10}			1×10^{13}
Λ_b	1×10^{10}			1×10^{13}
***c* hadrons**				
D^0	2×10^{11}			
D^+	6×10^{10}			
D_s^+	3×10^{10}			
Λ_c^+	2×10^{10}			
τ^+	3×10^{10}	5×10^{10}	($50\,\mathrm{ab}^{-1}$ on $\Upsilon(4S)$)	

From CEPC's CDR using fragmentation ratios from Amhis et al, 17

- Similar statistical sample of $B^{0,\pm}$, τ's at Belle 2 and a tera-Z
- Two order of magnitude more B_s at tera-Z wrt to Belle 2
- b-baryon physics possible
- Limited possibilities for charm physics at Belle 2

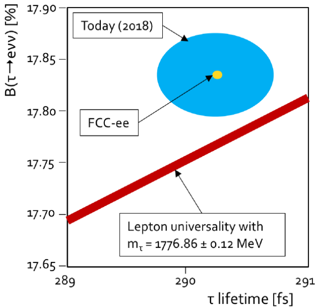
Flavour at a tera-Z: highlights

- Simulation-wise the study of flavour at FCC-ee or CEPC has only just begun
- Given the statistics, the tera-Z will compete well with both Belle-2 and LHCb

- **But in some cases tera-Z is expected to outperform both:**
  - τ **Physics**
    BRs and lifetime, lepton-flavour violating decays
  - **lepton flavour violating Z decays**
  - $\boldsymbol{B_0 \to K^* \tau\tau}$ and $\boldsymbol{B_s \to \tau\tau}$
  - $\boldsymbol{b \to s\nu\nu}$ transition
    access to $B_s \to \phi\nu\nu$ and $\Lambda_b \to \Lambda\nu\nu$ [ongoing CEPC study]
  - $\boldsymbol{B_c}$ **physics**
    so far uncharted territory, determination of V_{cb}, relation to R_{D^*} anomaly
    [ongoing CEPC study (Soeren Prell)]

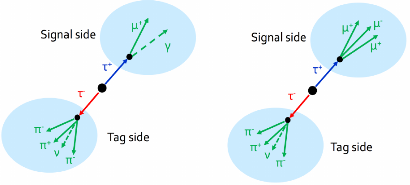
τ's and LFV at the tera-Z

- lifetime measurement, 3 orders of magnitude better than LEP
- Lepton flavour universality tests



- Z limits from rescaling LEP, 3 orders of magnitude improvement
- τ limits 1-2 orders of magnitude improvement [FCC-ee study by M. Dam]

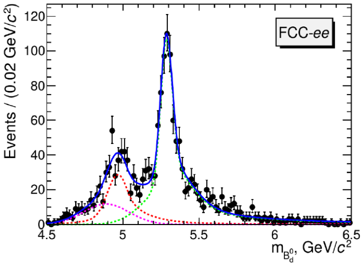
Decay	Present bound	FCC-ee sensitivity
$\mathrm{Z} \to \mu\mathrm{e}$	0.75×10^{-6}	10^{-10}–10^{-8}
$\mathrm{Z} \to \tau\mu$	12×10^{-6}	10^{-9}
$\mathrm{Z} \to \tau\mathrm{e}$	9.8×10^{-6}	10^{-9}
$\tau \to \mu\gamma$	4.4×10^{-8}	2×10^{-9}
$\tau \to 3\mu$	2.1×10^{-8}	10^{-10}



[FCC-ee study by M. Dam with some simulation, see also CEPC-CDR]

$B_0 \to K^* \tau\tau$ at the tera-Z

- Expected sensitivities from LHCb and Belle II far from SM expectations
- Important test of LFU violation given present R_K and R_{K^*} tensions
- **$B \to K^* \tau\tau$ a golden mode for the tera-Z**



- Fully reconstruct the decay
  Z vertex from primary tracks, B vertex from $K\pi$, τ vertices from 3 prong decays
- Expect $\mathcal{O}(10^3)$ reconstructed events
  [Kamenik et al 17]
- Angular analysis possible

- Thus $B_s \to \tau\tau$ also accesible for the first time at the tera-Z
- Together with $B_s \to \phi\nu\nu$ and $\Lambda_b \to \Lambda\nu\nu$ possible to disentangle chiral-structure of operators

Conclusions

Higgs and EDMs

- Low-energy observables (EDMs) complement the LHC in probing the Higgs–fermion interactions
- Good experimental prospects, expected progress from lattice

QCD axion and flavor

- Precision-flavor observables do probe parameter space of the QCD axion
- Kaon physics probe PQ scales in which the axion can be DM

Flavour at a tera-Z factory

- Competitive with LHCb and Belle II
- In some observables the tera-Z is beyond competition
- Dedicated studies needed

Appendix

Origin of axion–fermion couplings

- axion couples to PQ current
- PQ basis $\neq$ mass basis

$$C^{V,A}_{u_i,u_j} = (V^\dagger_{UL} \mathrm{PQ}_q V_{UL})_{ij} \pm (V^\dagger_{UR} \mathrm{PQ}_u V_{UR})_{ij}$$

- **If PQ charge non-universal ➜flavor violating couplings**
  e.g., non-universal DFSZ models [Celis et al 14; di Luzio et al 17]
  e.g., PQ = FN (“axiflavon”/“flaxion”) [Wilczek 82; Calibbi et al 16; Ema et al 16]

➜**Flavor violating couplings** offer another way to search for the **QCD axion**

➜Need (often neglected) dedicated analyses

EDMs SM

Sources of CP violation in the SM

QCD $\theta_{\text{QCD}} G\tilde{G}$

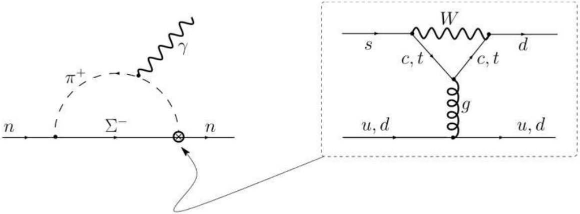
- neglect for now, see Part 2

EW δ_{CKM}

- Any CP violating observable involving δ_{SM} must involve all 3 generations

➜ EDM_{SM} small

Example: neutron EDM, $d_n^{\text{SM}} \sim 10^{-32}$ e cm [Khriplovich, Zhitnitsky 82]



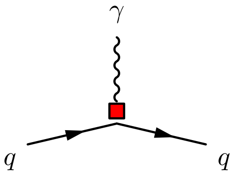
EDMs theory: operators

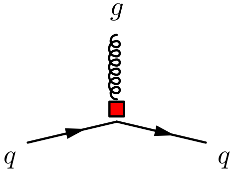
The (most) relevant operators

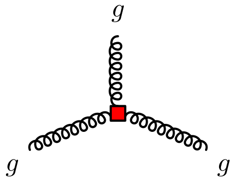
- qEDM: $\bar{q}\sigma^{\mu\nu}\gamma_5 q F_{\mu\nu}$
- qCEDM: $\bar{q}\sigma^{\mu\nu}\gamma_5 T^a q G^a_{\mu\nu}$
- Weinberg: $f^{abc}\epsilon_{\mu\nu\rho\sigma}G^a_{\rho\sigma}G^b_{\mu\tau}G^{c,\tau}_{\nu}$

Hadronic matrix elements

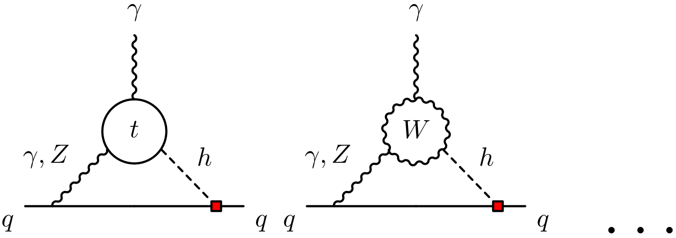
- qEDM → Lattice, “easy” because bilinear
  [Battacharya et al 15,15]
- qCEDM → ChPT and NDA
  [Pospelov, Ritz 05]
- ➜ next target for lattice
- Weinberg: no systematic computation, sign unknown
  [NDA: Weinberg 89; Sum rules: Demir et al 02]







Electron: electronic EDM

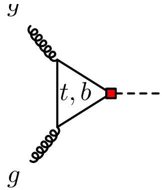


- $\mathcal{O}(100)$ electroweak 2-loop diagrams [Altmannshofer et al 15; Czarnecki, Gribouk 05]
- 2018 result: $|d_e| < 1.1 \times 10^{-29}$ e cm 90% CL [ACME 18]

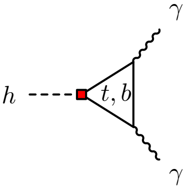
$$\rightarrow \kappa_e |\sin\phi_e| < 0.002$$

Top quark: CP violation at LHC

- CP violation in $h-t-t$ affects Higgs production and Higgs decay
- gluon fusion and $h \to \gamma\gamma$, loop effect, LHC has some sensitivity



$$\mu_{gg} = \frac{\sigma(gg \to h)}{\sigma(gg \to h)_{\text{SM}}} \approx (\kappa_t \cos\phi_t)^2 + 2.6(\kappa_t \sin\phi_t)^2 + 0.11\kappa_t \cos\phi_t(\kappa_t \cos\phi_t - 1)$$



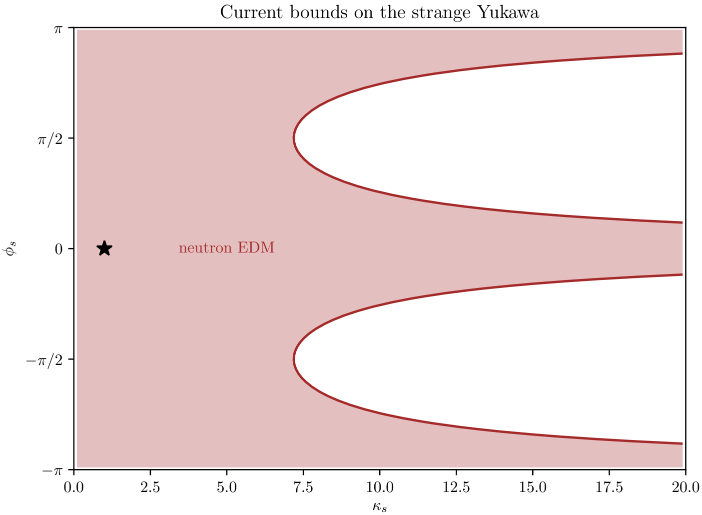
$$\mu_{\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\text{SM}}} \approx (1.28 - 0.28\kappa_t \cos\phi_t)^2 + (0.43\kappa_t \sin\phi_t)^2$$

[Brod et al 13]

- More sophisticated analyses, e.g., angular analysis of final state jets

[Del Luca et al 06; Klamke et al 07]

Strange-quark



The effective CP-odd flavor conserving Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{\sqrt{2}G_F}{M_h^2}\Big\{ m_q^2 \sum_{i=1,\dots,4} C_i^q O_i^q + C_5 O_5 + \sum_{q\neq q'} m_q\, m_{q'} \Big[\sum_{i=1,2} C_i^{qq'} O_i^{qq'} + \frac{1}{2}\sum_{i=3,4} C_i^{qq'} O_i^{qq'} \Big] \Big\}$$

$$O_1^q = (\bar q q)\,(\bar q\, i\gamma_5 q)$$

$$O_2^q = (\bar q \sigma_{\mu\nu} q)\,(\bar q\, i\sigma^{\mu\nu}\gamma_5 q)$$

$$O_3^q = \frac{ieQ_q}{2}\frac{m_q}{g_s^2}\,\bar q \sigma^{\mu\nu}\gamma_5 q\, F_{\mu\nu}$$

$$O_4^q = -\frac{i}{2}\,\frac{m_q}{g_s}\,\bar q \sigma^{\mu\nu} T^a \gamma_5 q\, G^a_{\mu\nu}$$

$$O_1^{qq'} = (\bar q q)\,(\bar q'\, i\gamma_5 q')$$

$$O_2^{qq'} = (\bar q\, T^a q)\,(\bar q'\, i\gamma_5 T^a q')$$

$$O_3^{qq'} = (\bar q \sigma_{\mu\nu} q)\,(\bar q'\, i\sigma^{\mu\nu}\gamma_5 q')$$

$$O_4^{qq'} = (\bar q \sigma_{\mu\nu} T^a q)\,(\bar q'\, i\sigma^{\mu\nu}\gamma_5 T^a q')$$

$$O_5 = -\frac{1}{3\,g_s} f^{abc}\, G^a_{\mu\sigma} G^{b,\sigma}_{\nu} \widetilde{G}^{c,\mu\nu}$$

In 5-flavor theory: $20 + 6 \times 10 + 1$ operators

Peculiarities of the 2-loop computation I

- We extract UV poles of diagrams using *Dimensional regularization*

$$4 \to 4 - 2\epsilon$$

The basis of operators is then infinitely large, **evanescent operators**
Their definition affects the 2-loop ADM

$$\begin{aligned}
E_1^q &= (\bar{q}T^a q)(\bar{q}i\gamma_5 T^a q) + \Big(\frac{1}{4} + \frac{1}{2n_c}\Big)O_1^q + \frac{1}{16}O_2^q \\
E_2^q &= (\bar{q}\sigma^{\mu\nu}T^a q)(\bar{q}\sigma_{\mu\nu}i\gamma_5 T^a q) + 3O_1^q - \Big(\frac{1}{4} - \frac{1}{2n_c}\Big)O_2^q \\
E_3^q &= (\bar{q}\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^{\sigma]}q)\,(\bar{q}\gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_{\sigma]}\,i\gamma_5 q) - 24O_1^q \\
E_4^q &= (\bar{q}\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^{\sigma]}\,T^a q)\,(\bar{q}\gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_{\sigma]}\,i\gamma_5\,T^a q) + 6\Big(1 + \frac{2}{n_c}\Big)O_1^q + \frac{3}{2}O_2^q \\
E_5^q &= (\bar{q}\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau\gamma^{\upsilon]}q)\,(\bar{q}\gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau\gamma_{\upsilon]}\,i\gamma_5 q) \\
E_6^q &= (\bar{q}\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau\gamma^{\upsilon]}\,T^a q)\,(\bar{q}\gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau\gamma_{\upsilon]}\,T^a\,i\gamma_5 q) \\
E_1^{qq'} &= (\bar{q}\gamma^\mu\gamma^\nu\sigma^{\rho\tau}q)\,(\bar{q}'\gamma_\mu\gamma_\nu\sigma_{\rho\tau}\,i\gamma_5 q') + 24(O_1^{qq'} + O_1^{q'q}) - 12O_3^{qq'} \\
E_2^{qq'} &= (\bar{q}\gamma^\mu\gamma^\nu\sigma^{\rho\tau}\,T^a q)\,(\bar{q}'\gamma_\mu\gamma_\nu\sigma_{\rho\tau}\,i\gamma_5\,T^a q') + 24(O_2^{qq'} + O_2^{q'q}) - 12O_4^{qq'} \\
E_3^{qq'} &= (\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\sigma^{\tau\upsilon}q)\,(\bar{q}'\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\sigma_{\tau\upsilon}\,i\gamma_5 q') + 384(O_1^{qq'} + O_1^{q'q}) - 192O_3^{qq'} \\
E_4^{qq'} &= (\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\sigma^{\tau\upsilon}\,T^a q)\,(\bar{q}'\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\sigma_{\tau\upsilon}\,i\gamma_5\,T^a q') + 384(O_2^{qq'} + O_2^{q'q}) - 192O_4^{qq'}
\end{aligned}$$

Peculiarities of the 2-loop computation II

- In $d = 4$

$$(\bar{q}\sigma^{\mu\nu}q)(\bar{q}'\sigma_{\mu\nu}i\gamma_5 q') = (\bar{q}\sigma^{\mu\nu}i\gamma_5 q)(\bar{q}'\sigma_{\mu\nu}) = (\bar{q}\sigma^{\mu\nu}q)(\bar{q}'\sigma^{\rho\tau}q')\epsilon_{e\mu\nu\rho\tau}$$

- In $d = 4 - 2\epsilon$

$$(\bar{q}\sigma^{\mu\nu}q)(\bar{q}'\sigma_{\mu\nu}i\gamma_5 q') \neq (\bar{q}\sigma^{\mu\nu}i\gamma_5 q)(\bar{q}'\sigma_{\mu\nu}) \neq (\bar{q}\sigma^{\mu\nu}q)(\bar{q}'\sigma^{\rho\tau}q')\epsilon_{\mu\nu\rho\tau}$$

  Operators differ by **evanescent** structures ➜ different ADM

- We have traces with γ_5 for which $[\gamma^\mu, \gamma_5] = 0$ (NDR) is **inconsistent**
- Need to use 't Hooft Veltaman (HV) scheme with mixed (anti-) commutation relations

$$[\tilde{\gamma}^\mu, \gamma_5] = 0 \qquad \{\hat{\gamma}^\mu, \gamma_5\} = 0$$

- **Make sure that physical results are independent of such choices**