A non-universal $U(1)_X$ gauge extension to the MSSM

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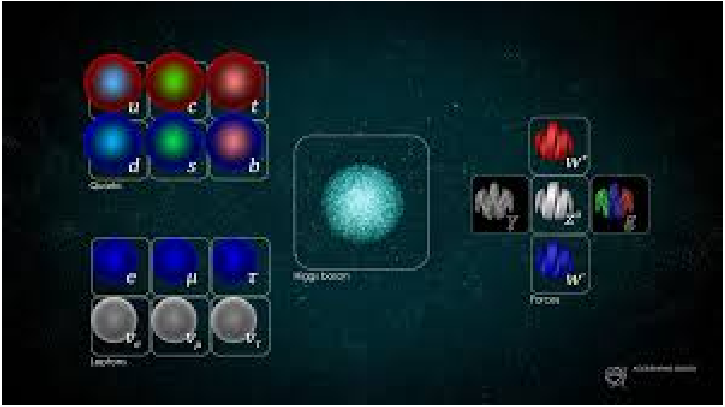
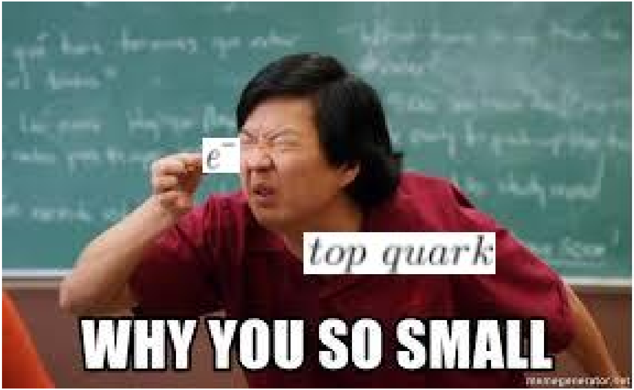
Standard Model



...and all its frustrating success

Fermion Mass Hierarchy (FMH)



It is a $\sim 10^6$ difference!

Neutrino masses

-Standard model : Why aren't you massless ?
-Neutrinos :

The non-SUSY model

Scalar bosons	X	$\mathbf{Z}_2$
Higgs Doublets		
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1 + v_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}$	$2/3$	$+$
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2 + v_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}$	$1/3$	$-$
Higgs Singlets		
$\chi = \frac{\xi_\chi + v_\chi + i\zeta_\chi}{\sqrt{2}}$	$-1/3$	$+$
σ	$-1/3$	$-$

TABLE – Non-universal X quantum number for Higgs fields.

The non-SUSY model

It is an anomaly free model, but its supersymmetric generalization would induce anomalies due to Higgsinos.

$$
\begin{aligned}
\left[\mathrm{SU}(3)_C\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_C &= \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
\left[\mathrm{SU}(2)_L\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_L &= \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} \\
\left[\mathrm{U}(1)_Y\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_{Y^2} &= \sum_{\ell,Q}\left[Y_{\ell_L}^2 X_{\ell_L} + 3Y_{Q_L}^2 X_{Q_L}\right] - \sum_{\ell,Q}\left[Y_{\ell_R}^2 X_{L_R} + 3Y_{Q_R}^2 X_{Q_R}\right] \\
\mathrm{U}(1)_Y \left[\mathrm{U}(1)_X\right]^2 \rightarrow \quad A_Y &= \sum_{\ell,Q}\left[Y_{\ell_L} X_{\ell_L}^2 + 3Y_{Q_L} X_{Q_L}^2\right] - \sum_{\ell,Q}\left[Y_{\ell_R} X_{\ell_R}^2 + 3Y_{Q_R} X_{Q_R}^2\right] \\
\left[\mathrm{U}(1)_X\right]^3 \rightarrow \quad A_X &= \sum_{\ell,Q}\left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q}\left[X_{\ell_R}^3 + 3X_{Q_R}^3\right] \\
\left[\mathrm{Grav}\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_G &= \sum_{\ell,Q}\left[X_{\ell_L} + 3X_{Q_L}\right] - \sum_{\ell,Q}\left[X_{\ell_R} + 3X_{Q_R}\right],
\end{aligned} \tag{1}
$$

Particle Content

TABLE – Scalar content of the model, non-universal X quantum number, $\mathbb{Z}_2$ parity and hypercharge

Higgs Scalar Doublets	$X^{\pm}$	Y	Higgs Scalar Singlets	$X^{\pm}$	Y
$\hat{\Phi}_1 = \begin{pmatrix} \hat{\phi}_1^+ \\ \frac{\hat{h}_1+v_1+i\hat{\eta}_1}{\sqrt{2}} \end{pmatrix}$	$+\,{}^2/_3{}^+$	$+1$	$\hat{\chi} = \frac{\hat{\xi}_\chi+v_\chi+i\hat{\zeta}_\chi}{\sqrt{2}}$	$-\,{}^1/_3{}^+$	0
$\hat{\Phi}_2 = \begin{pmatrix} \hat{\phi}_2^+ \\ \frac{\hat{h}_2+v_2+i\hat{\eta}_2}{\sqrt{2}} \end{pmatrix}$	$+\,{}^1/_3{}^-$	$+1$	$\sigma = \frac{\hat{\sigma}_\chi+i\hat{\zeta}_\sigma}{\sqrt{2}}$	$-\,{}^1/_3{}^-$	0
$\hat{\Phi}_1' = \begin{pmatrix} \frac{\hat{h}_1'+v_1'+i\hat{\eta}_1'}{\sqrt{2}} \\ \hat{\phi}_1^{-\prime} \end{pmatrix}$	$-\,{}^2/_3{}^+$	-1	$\hat{\chi}' = \frac{\hat{\xi}_\chi'+v_\chi'+i\hat{\zeta}_\chi'}{\sqrt{2}}$	$+\,{}^1/_3{}^+$	0
$\hat{\Phi}_2' = \begin{pmatrix} \frac{\hat{h}_2'+v_2'+i\hat{\eta}_2'}{\sqrt{2}} \\ \hat{\phi}_2^{-\prime} \end{pmatrix}$	$-\,{}^1/_3{}^-$	-1	$\sigma' = \frac{\hat{\xi}_\sigma'+i\hat{\zeta}_\sigma'}{\sqrt{2}}$	$+\,{}^1/_3{}^-$	0

Quark content

TABLE – Quark content of the abelian extension, non-universal X quantum number and parity $\mathbb{Z}_2$.

Left-Handed Fermions	$X^{\pm}$	Right-Handed Fermions	$X^{\pm}$
SM Quarks			
$\hat{q}_L^1 = \begin{pmatrix} \hat{u}^1 \\ \hat{d}^1 \end{pmatrix}_L$	$+1/3^{+}$	$\hat{u}_L^{1\ c}$	$-2/3^{+}$
		$\hat{u}_L^{2\ c}$	$-2/3^{-}$
$\hat{q}_L^2 = \begin{pmatrix} \hat{u}^2 \\ \hat{d}^2 \end{pmatrix}_L$	0^{-}	$\hat{u}_L^{3\ c}$	$-2/3^{+}$
		$\hat{d}_L^{1\ c}$	$+1/3^{-}$
$\hat{q}_L^3 = \begin{pmatrix} \hat{u}^3 \\ \hat{d}^3 \end{pmatrix}_L$	0^{+}	$\hat{d}_L^{2\ c}$	$+1/3^{-}$
		$\hat{d}_L^{3\ c}$	$+1/3^{-}$
Non-SM Quarks			
$\hat{\mathcal{T}}_L$	$+1/3^{-}$	$\hat{\mathcal{T}}_L^c$	$-2/3^{-}$
$\mathcal{J}_L^1$	0^{+}	$\hat{\mathcal{J}}_L^c$	$-1/3^{+}$
$\mathcal{J}_L^2$	0^{+}	$\hat{\mathcal{J}}_L^{c\ 2}$	$-1/3^{+}$

Lepton Content

TABLE – Lepton content of the abelian extension, non-universal X quantum number and parity $\mathbb{Z}_2$.

Left-Handed Fermions	$X^{\pm}$	Right-Handed Fermions	$X^{\pm}$
SM Leptons			
$\hat{\ell}_L^e = \begin{pmatrix} \hat{\nu}^e \\ \hat{e}^e \end{pmatrix}_L$	0^+	$\hat{\nu}_L^{e\,c}$	$-1/3^-$
		$\hat{\nu}_L^{\mu\,c}$	$-1/3^-$
$\hat{\ell}_L^\mu = \begin{pmatrix} \hat{\nu}^\mu \\ \hat{\mu}^\mu \end{pmatrix}_L$	0^+	$\hat{\nu}_L^{\tau\,c}$	$-1/3^-$
		$\hat{e}_L^{e\,c}$	$+4/3^-$
$\hat{\ell}_L^\tau = \begin{pmatrix} \hat{\nu}^\tau \\ \hat{\tau}^\tau \end{pmatrix}_L$	-1^+	$\hat{e}_L^{\mu\,c}$	$+1/3^-$
		$\hat{e}_L^{\tau\,c}$	$+4/3^-$
Non-SM Leptons			
$\hat{E}_L$	-1^+	$\hat{E}_L^c$	$+2/3^+$
$\hat{\mathcal{E}}_L$	$-2/3^+$	$\hat{\mathcal{E}}_L^c$	$+1^+$
Majorana Fermions		$\mathcal{N}_R^{1,2,3}$	0^-

The potential

$$W_\phi = -\mu_1 \hat{\Phi}_1' \hat{\Phi}_1 - \mu_2 \hat{\Phi}_2' \hat{\Phi}_2 - \mu_\chi \hat{\chi}' \hat{\chi} - \mu_\sigma \hat{\sigma}' \hat{\sigma} + \lambda_1 \hat{\Phi}_1' \hat{\Phi}_2 \hat{\sigma}' + \lambda_2 \hat{\Phi}_2' \hat{\Phi}_1 \sigma.$$

$$\begin{aligned} V_{soft} &= -m_1^2 \Phi_1^\dagger \Phi_1 - m_1'^2 \Phi_1'^\dagger \Phi_1' - m_2^2 \Phi_2^\dagger \Phi_2 - m_2'^2 \Phi_2'^\dagger \Phi_2' - m_\chi^2 \chi^\dagger \chi \\ &- m_\chi'^2 \chi'^\dagger \chi' - m_\sigma^2 \sigma^\dagger \sigma - m_\sigma'^2 \sigma'^\dagger \sigma' + \Big[\mu_{11}^2 \epsilon_{ij} (\Phi_1'^i \Phi_1^j) + \mu_{22}^2 \epsilon_{ij} (\Phi_2'^i \Phi_2^j) \\ &+ \mu_{\chi\chi}^2 (\chi \chi') + \mu_{\sigma\sigma}^2 (\sigma \sigma') - \tilde{\lambda}_1 \Phi_1'^\dagger \Phi_2 \sigma' - \tilde{\lambda}_2 \Phi_2'^\dagger \Phi_1 \sigma \\ &+ \frac{2\sqrt{2}}{9} (k_1 \Phi_1^\dagger \Phi_2 \chi' - k_2 \Phi_1^\dagger \Phi_2 \chi^* + k_3 {\Phi_1'}^\dagger \Phi_2' \chi - k_4 {\Phi_1'}^\dagger \Phi_2' \chi'^*) + h.c. \Big] \end{aligned}$$

$$\frac{1}{2} M_h^2 = \begin{pmatrix} M_{hh} & M_{h\xi} \\ M_{h\xi}^T & M_{\xi\xi} \end{pmatrix}. \tag{2}$$

$$M_{hh} \sim v_1, v_2, v_1', v_2', \mu_{11}, \mu_{22} \quad M_{h\xi} \sim \lambda_i \quad M_{\xi\xi} \sim v_\chi, v_\chi', \mu_{\chi\chi}, \mu_{\sigma\sigma} \tag{3}$$

$$So\ M_{\xi\xi} >> M_{hh}, M_{h\xi} \tag{4}$$

A seesaw implementation

A seesaw rotation through $\mathbb{V}_{\text{SS}}^{h} = \begin{pmatrix} 1 & (\Theta^{h\dagger})^{\dagger} \\ -M_{h\xi}(M_{\xi\xi})^{-1} & 1, \end{pmatrix}$

$$\tilde{M}_h \approx \begin{pmatrix} \tilde{M}_{hh} & 0 \\ 0 & M_{\xi\xi} \end{pmatrix},$$

$$\tilde{M}_{hh} \approx M_{hh} - \underbrace{M_{h\xi} M_{\xi\xi}^{-1} M_{h\xi}^{T}}_{very\ small} \approx M_{hh}.$$

$$\tilde{M}_{hh} \to m_{h1}^2 \approx m_Z^2 \left(cos^2 2\tilde{\beta} + \frac{4}{9} \frac{g_X^2}{g^2 + g'^2} (cos 2\beta_1 + \cos 2\beta_2)^2 \right)$$

$$\tan^2 \tilde{\beta} = \frac{v_1^2 + v_2^2}{v_1'^2 + v_2'^2} \qquad \tan \beta_1 = \frac{v_1}{v_1'} \qquad \tan \beta_2 = \frac{v_2}{v_2'}$$

$M_{\xi\xi} \to$ *heavy states via parity violating terms*

Up-like quarks

In the $(u, c, t, \mathcal{T})$ basis, the mass matrix reads :

$$M_U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & h_{2u}^{12} v_2 & 0 & h_{2T}^{1} v_2 \\ 0 & h_{1u}^{22} v_1 & 0 & h_{1T}^{2} v_1 \\ h_{1u}^{31} v_1 & 0 & h_{1u}^{33} v_1 & 0 \\ 0 & h_{\chi' u}^{2} & 0 & \frac{v_\chi'}{\sqrt{2}} g_{\chi' T} \end{pmatrix} \tag{5}$$

$$m_u^2 = 0, \qquad m_c^2 = \frac{1}{2} v_1^2 \frac{\left[h_{1u}^{22} g_{\chi' T} - h_{1T}^{2} h_{\chi' u}^{2}\right]^2}{(g_{\chi' T})^2 + (h_{\chi' u}^{2})^2},$$

$$m_t^2 = \frac{1}{2} v_1^2 \left[(h_{1u}^{31})^2 + (h_{1u}^{33})^2\right], \qquad m_T^2 = \frac{1}{2} v_\chi'^{\,2} \left[(g_{\chi' T})^2 + (h_{\chi' u}^{2})^2\right]. \tag{6}$$

$$7 \times 10^{-3} \approx \frac{h_{1u}^{22} g_{\chi' T} - h_{1T}^{2} h_{\chi' u}^{2}}{\sqrt{(g_{\chi' T})^2 + (h_{\chi' u}^{2})^2} \sqrt{(h_{1u}^{31})^2 + (h_{1u}^{33})^2}} \tag{7}$$

Down-like quarks

In the $(d, s, b, \mathcal{J}_1, \mathcal{J}_2)$ basis, the mass matrix reads :

$$M_D = \frac{v_2'}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\sqrt{2}} h_{1J}^{11} v_1' & h_{1J}^{12} v_1' \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} h_{2J}^{21} v_2' & h_{2J}^{22} v_2' \\ \frac{v_2'}{\sqrt{2}} h_{2d}^{31} & \frac{v_2'}{\sqrt{2}} h_{2d}^{32} & \frac{v_2'}{\sqrt{2}} h_{2d}^{33} & 0 & 0 \\ 0 & 0 & 0 & \frac{v_\chi}{\sqrt{2}} g_{\chi J}^{11} & \frac{v_\chi}{\sqrt{2}} g_{\chi J}^{12} \\ 0 & 0 & 0 & \frac{v_\chi}{\sqrt{2}} g_{\chi J}^{21} & \frac{v_\chi}{\sqrt{2}} g_{\chi J}^{22} \end{pmatrix} \tag{8}$$

$$m_d^2 = 0 \qquad m_s^2 = 0 \tag{9}$$

$$m_{J^1}^2 = \frac{1}{2} v_\chi^2 (g_{\chi J}^{11})^2 \qquad m_{J^2}^2 = \frac{1}{2} v_\chi^2 (g_{\chi J}^{22})^2 \tag{10}$$

$$m_b^2 = \frac{1}{2} {v_2'}^2 \left[(h_{2d}^{31})^2 + (h_{2d}^{32})^2 + (h_{2d}^{33})^2 \right] \tag{11}$$

Leptons

$$\mathcal{M}_E = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & h_{2e}^{e\mu} v_2' & 0 & h_{1e}^{E} v_1' & 0 \\ 0 & h_{2e}^{\mu\mu} v_2' & 0 & h_{1\mu}^{E} v_1' & 0 \\ h_{2e}^{\tau e} v_2' & 0 & h_{2e}^{\tau\tau} v_2' & 0 & 0 \\ \hline 0 & 0 & 0 & g_{\chi' E} v_\chi' & -\mu_E \\ 0 & 0 & 0 & -\mu_{\mathcal{E}} & g_{\chi\mathcal{E}} v_\chi \end{array} \right) \tag{12}$$

$$m_e^2 = 0 \qquad m_\mu^2 = \frac{1}{2} v_2'^{\,2} \left[(h_{2e}^{e\mu})^2 + (h_{2e}^{\mu\mu})^2 \right] \tag{13}$$

$$m_E^2 = \frac{1}{2} g_{\chi' E}^2 \, v_\chi'^{\,2} \qquad m_{\mathcal{E}}^2 = \frac{1}{2} g_{\chi\mathcal{E}}^2 \, v_\chi^2 \tag{14}$$

$$m_\tau^2 = \frac{1}{2} v_2'^{\,2} \left[(h_{2e}^{\tau e})^2 + (h_{2e}^{\tau\tau})^2 \right] \tag{15}$$

$$0.14 \approx \frac{\sqrt{(h_{2e}^{e\mu})^2 + (h_{2e}^{\mu\mu})^2}}{\sqrt{(h_{2e}^{\tau e})^2 + (h_{2e}^{\tau\tau})^2}} \tag{16}$$

Neutrinos

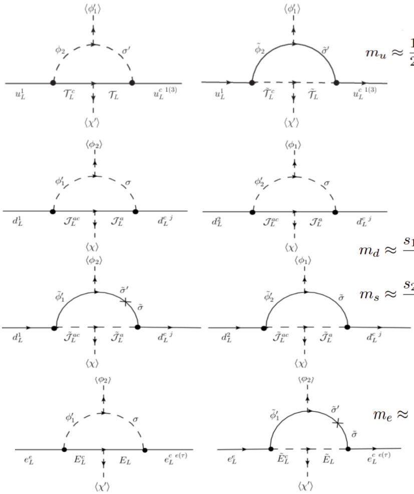
Left handed neutrinos couple to right handed and Majorana neutrinos. The resulting mass matrix has a structure compatible with inverse seesaw.

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_M \end{pmatrix}, \tag{17}$$

$$m_D = \frac{v_2}{\sqrt{2}} \begin{pmatrix} h_{2\nu}^{ee} & h_{2\nu}^{e\mu} & h_{2\nu}^{e\tau} \\ h_{2\nu}^{e\mu} & h_{2\nu}^{\mu\mu} & h_{2\nu}^{\mu\tau} \\ 0 & 0 & 0 \end{pmatrix}, \quad (M_D)^{ij} = \frac{v'_\chi}{\sqrt{2}} (h'^{\nu}_{\chi})^{ij}, \qquad (M_M)_{ij} = \frac{1}{2} M_{ij}. \tag{18}$$

Neutrino masses are proportional to v_2 with contribution proportional to $1/v'_\chi$

Radiative Corrections



$$m_u \approx \frac{1}{2} v_1'^2 ((\Sigma_{11}^{NS})^2 + (\Sigma_{11}^{NS})^2) \tag{19}$$

$$m_d \approx \frac{s_{11}(\Sigma) v_2'^2}{2 m_b^2} \tag{20}$$

$$m_s \approx \frac{s_{22}(\Sigma) v_1'^2}{2 m_b^2} \tag{21}$$

$$m_e \approx \frac{v_2^2}{2} ((\Sigma_{11}^{NS})^2 + (\Sigma_{11}^{S})^2) \tag{22}$$

Fermion sector

Overall, the fermion sector implies :

- $v_1 \sim m_t$
- $v_2' \sim m_b, m_\tau, m_\mu$
- $v_\chi \sim m_{J^a}, m_{\mathcal{E}}$
- $v_\chi' \sim m_T, m_E$
- 3 massless states at tree level. Thus, radiative corrections are needed.
- $v_2 \sim m_\nu$

Conclusions

- The model is compatible with a 125 GeV Higgs boson.
- Heavy scalar singlets couple entirely to exotic fermions.
- $\mathbf{Z}_2$ symmetry prevents scalar singlets to have an elevated mass value.
- A fermion mass hierarchy is justified due to the presence of unobserved scalar particles with different VEV whose couplings are restricted by a new gauge group.
- the 3 lightest particles are massless at tree level but receive contributions at one-loop level
- Neutrinos justify their small masses because of a coupling to heavy right handed and Majorana neutrinos leading to an inverse seesaw mass generation.

References

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- Alvarado, J. S., Diaz, C. E., & Martinez, R. (2019). A non-universal $U(1)_X$ gauge extension to the MSSM.

Thank you