Study of quantum correlations in relativistic Møller scattering with electron Mott polarimetry

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PART 1

2POL Experiment

Correlation function

$$C(A,B) = \sum_{\alpha\beta} \alpha\beta P_{\alpha\beta}$$

$P_{\alpha\beta}$ – probability of obtaining α and β as a result of measurement of observables A and B, respectively

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$P_{\alpha\beta}$ – probability of obtaining α and β as a result of measurement of observables A and B, respectively

Observables A and B – spin projections on given directions, measured by two distant observers

$$C(\vec{a}, \vec{b}) = P_{++} + P_{--} - P_{+-} - P_{-+}$$

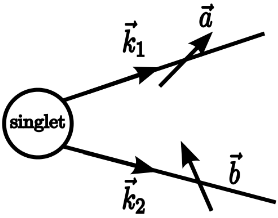
Experiments with massive particles

- experiments with protons:

Lamehi, Rachti, Mittig	1976	Saclay (France)
Hamieh *et al.*	2004	KVI (Holland)
Sakai *et al.*	2006	RIKEN (Japan)

- measurement of correlation function and violation of Bell inequalities for massive non–relativistic particles
- non–relativistic quantum mechanics only (too low energies)

Example: fermions in singlet state

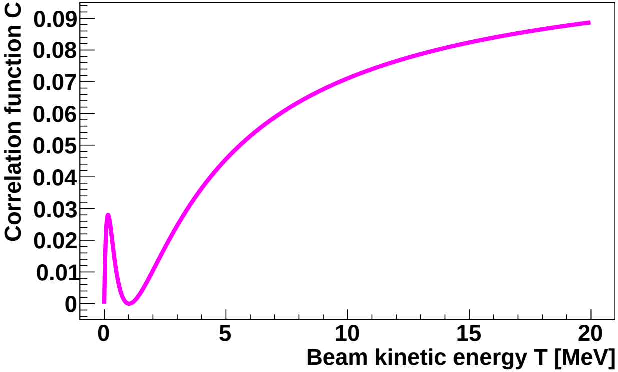


- spin $\frac{1}{2}$
- k_1, k_2 – particles four–momenta
- relativistic correction to the correlation function dependent on particles momenta

$$\text{singlet} \ \rightarrow k_1 \ k_2$$

$$\mathcal{C}(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} \ + \\ + \ \underbrace{\frac{\vec{k}_1 \times \vec{k}_2}{m^2 + k_1 k_2}\left[\vec{a}\times\vec{b} + \frac{(\vec{a}\cdot\vec{k}_1)(\vec{b}\times\vec{k}_2) - (\vec{b}\cdot\vec{k}_2)(\vec{a}\times\vec{k}_1)}{(k_1^0 + m)(k_2^0 + m)}\right]}_{\text{relativistic correction}}$$

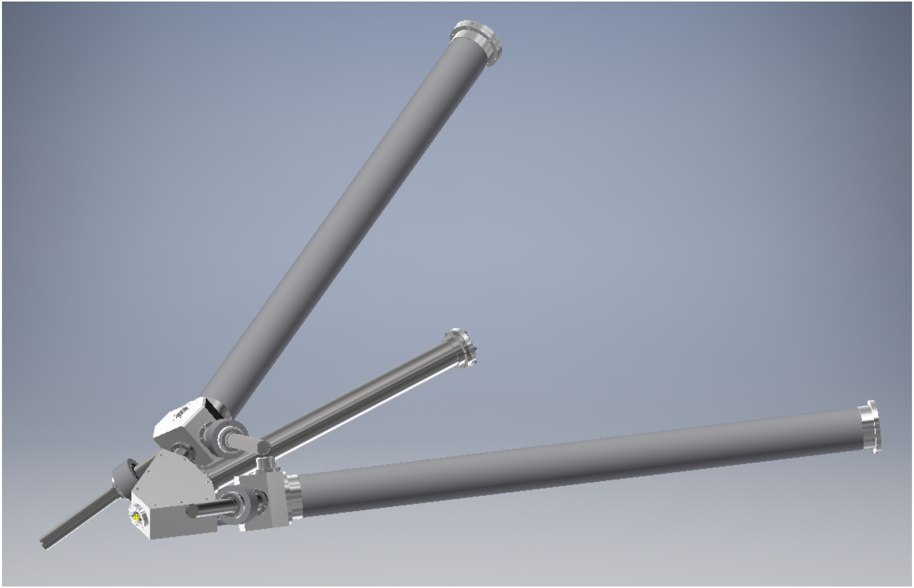
Møller scattering



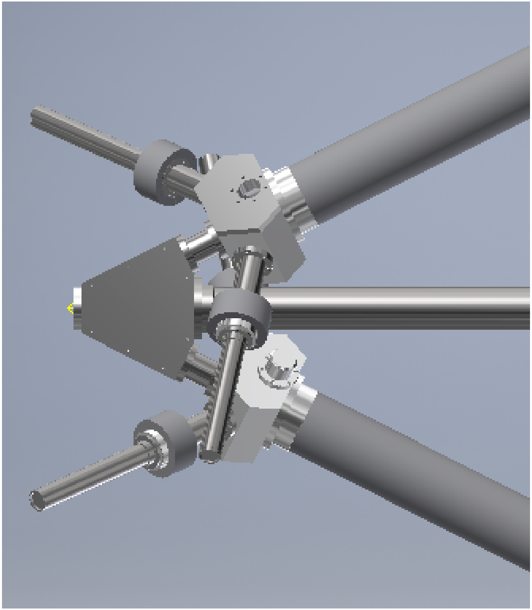
- polarized beam, unpolarized target
- $\vec{a}$ i $\vec{b}$ in Møller scattering plane
- symmetric scattering
- C does not depend on beam polarization, but $P_{\pm\pm}$ do

P. Caban, J. Rembieliński and M. Włodarczyk, *Møller scattering and Einstein–Podolsky–Rosen spin correlations*, *Phys. Rev. A* 88, 032116 (2013)

2POL Experiment

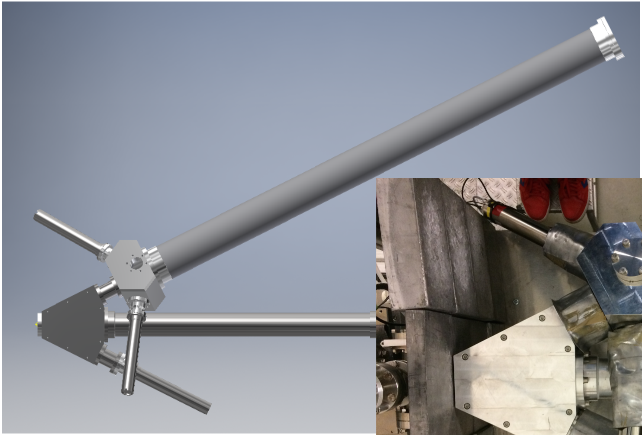


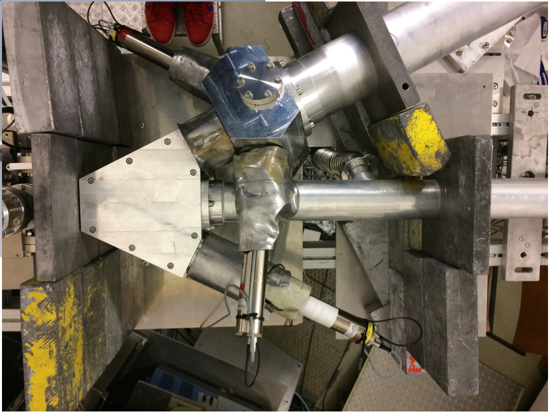
2POL Experiment



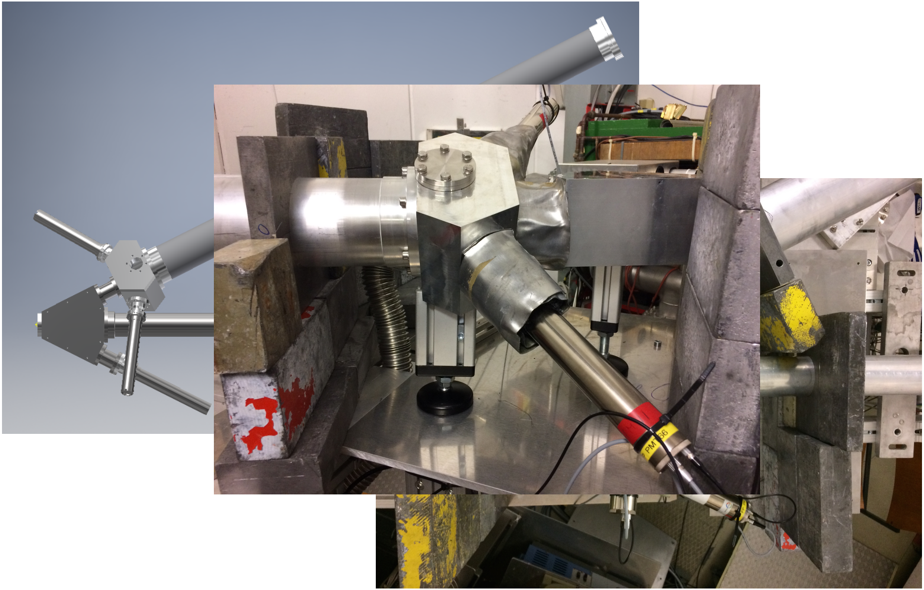
- directions $\vec{a}$ and $\vec{b}$ defined by selecting regions of counting
- double polarimeter (2 Mott polarimeters)
- detectors: scintillator + PMT

Test setup



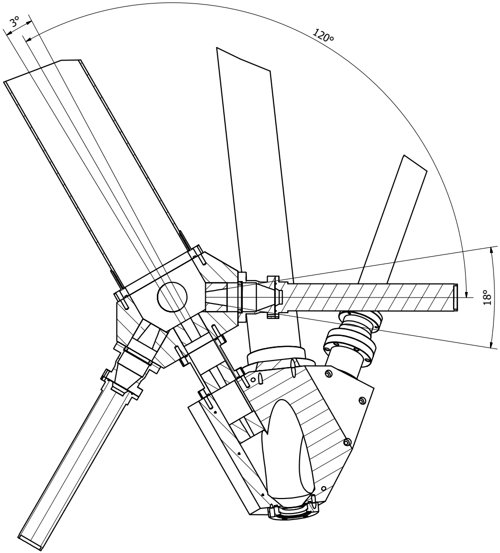


Test setup

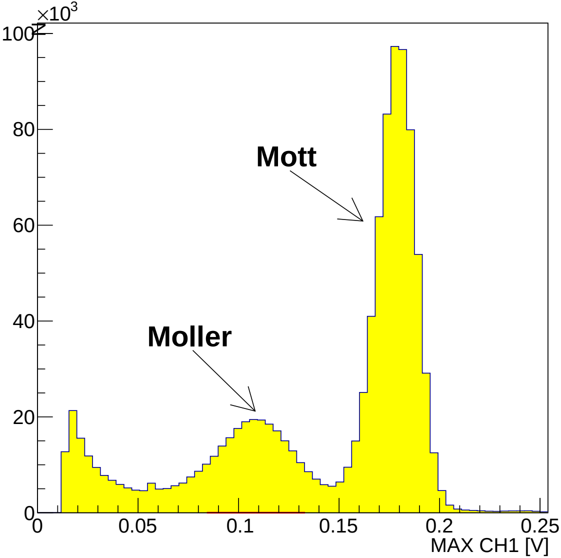


Test setup

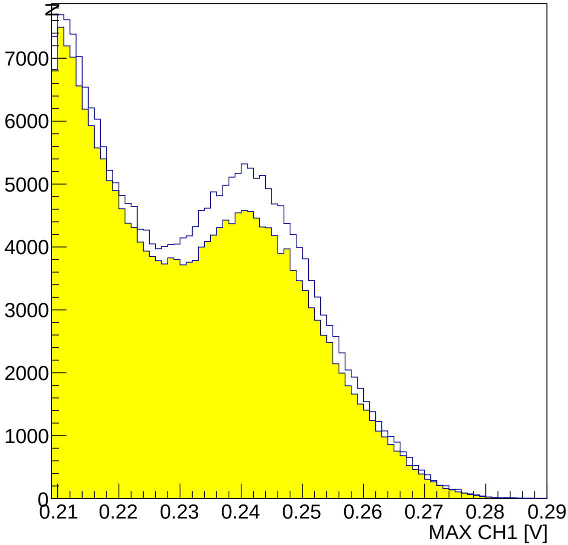
CROSS SECTION 2 - MOTT SCATTERING PLANE



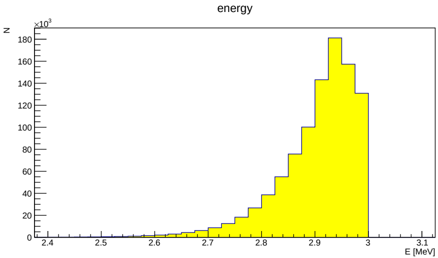
Energy spectrum

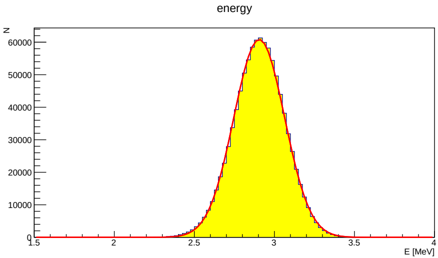


Beam polarization



Beam polarization





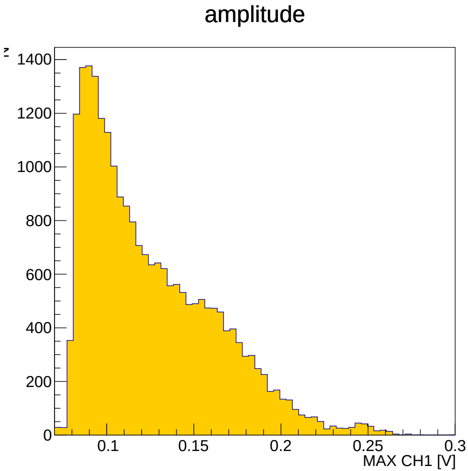
$$A_{exp}^{Mott} = -0.068 \pm 0.004$$

$$A_{theory}^{Mott} = SP \cos 30^\circ = -0.062 \pm 0.002 \ \text{(stat.)}$$

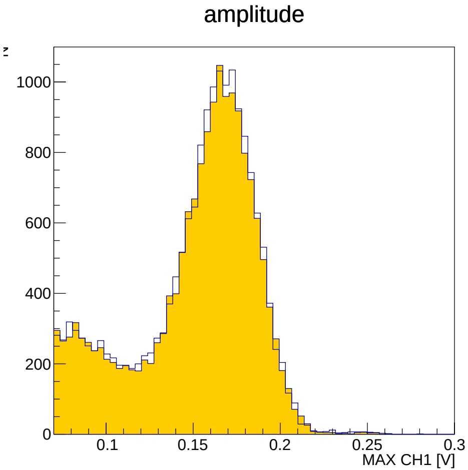
$$S(3 \ \text{MeV}) = -0.0890 \pm 0.0020 \ \text{(stat.)}$$

$$P = 0.81 \pm 0.02$$

Møller electrons



raw data



coincidence + timing selection

Summary of Part 1

1. we observe Møller electrons backscattered off the Au target
2. we measure the asymmetry arising due to beam polarization
3. the principle of the double polarimeter operation has been confirmed
4. issues: background reduction, PMT lifetime

PART 2

Theory of Electron Interactions with Matter

Interaction of electrons with matter

1. Mott scattering
   elastic scattering off target nuclei
2. Møller scattering
   elastic scattering off quasi-free target electrons
3. ionization
   scattering off bound target electrons
4. bremsstrahlung
   photon emission – negligible polarization change

Mott scattering

azimuthal asymmetry due to spin – orbit interaction

$$\frac{\mathrm{d}\sigma_{\mathrm{Mott}}}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma_{\mathrm{Mott}}}{\mathrm{d}\Omega}\right)_0 (1 + S(\theta)\vec{P}\cdot\vec{n})$$

$$\left(\frac{\mathrm{d}\sigma_{\mathrm{Mott}}}{\mathrm{d}\Omega}\right)_0 = \text{unpolarized cross section}$$

$$S(\theta) = \text{Sherman function}$$

$$\vec{P}\vec{n} = \frac{A_{LR}}{S_{\mathrm{eff}}}$$

the theoretical value of S is replaced with its effective value S_{eff}

N. F. Mott, *Proc. R. Soc.* A 124, 425 (1929)

Mott scattering

$$\frac{d\sigma_{\text{Mott}}}{d\Omega} = \left(\frac{d\sigma_{\text{Mott}}}{d\Omega}\right)_0 (1 + S(\theta)\vec{P} \cdot \vec{n})$$

$$\left(\frac{d\sigma_{\text{Mott}}}{d\Omega}\right)_0 = |f(\theta)|^2 + |g(\theta)|^2$$

$$S(\theta) = i\frac{fg^* - f^*g}{|f(\theta)|^2 + |g(\theta)|^2}$$

$$f(\theta) = \text{spin–conserving amplitude}$$

$$g(\theta) = \text{spin–flip amplitude}$$

Mott scattering

$$\vec{P'} = \frac{(\vec{P}\cdot\vec{n} + S(\theta))\vec{n} + T(\theta)\vec{n}\times(\vec{n}\times\vec{P}) + U(\theta)(\vec{n}\times\vec{P})}{1 + S(\theta)\vec{P}\cdot\vec{n}}$$

$$S(\theta) = i\frac{fg^* - f^*g}{|f(\theta)|^2 + |g(\theta)|^2}$$

$$T(\theta) = \frac{|f(\theta)|^2 - |g(\theta)|^2}{|f(\theta)|^2 + |g(\theta)|^2}$$

$$U(\theta) = \frac{fg^* + f^*g}{|f(\theta)|^2 + |g(\theta)|^2}$$

J. Kessler, *Polarized Electrons*, Springer, Berlin (1985)

Møller scattering

$$s = (p_1 + p_2)^2 = (k_1 + k_2)^2$$
$$t = (p_1 - k_1)^2 = (p_2 - k_2)^2$$
$$u = (p_1 - k_2)^2 = (p_2 - k_1)^2$$

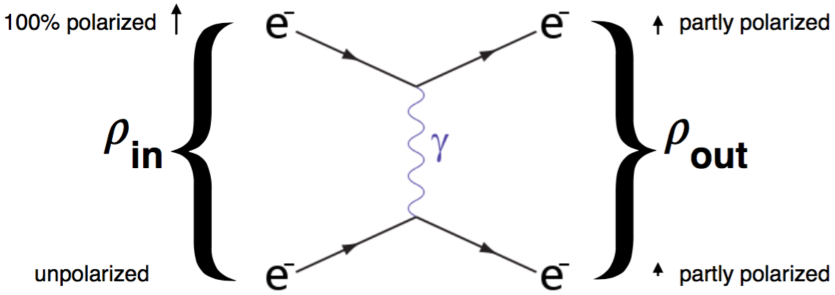
$$\mathrm{d}\sigma_{\mathrm{Møller}} = \frac{4\pi e^4 m^4}{s(s - 4m^2)} |F|^2 \mathrm{d}t \frac{\mathrm{d}\phi}{2\pi}$$

Stationary target only ($\vec{p_2} = 0$):

$$|F|^2 = \frac{1}{4m^4}\left[\frac{1}{t^2}\left(\frac{s^2+u^2}{2} + 4m^2(t-m^2)\right) + \right.$$
$$\left. + \frac{1}{u^2}\left(\frac{s^2+t^2}{2} + 4m^2(u-m^2)\right) + \frac{4}{tu}\left(\frac{s}{2} - m^2\right)\left(\frac{s}{2} - 3m^2\right)\right]$$

W. B. Berestetzki, E. M. Lifschitz, and L. P. Pitajewski, *Relativistic Quantum Theory*, Nauka, Moscow (1968)

Møller scattering



statistical mixture of singlet and triplet state

$$\rho_i = \mathrm{Tr}_j \rho_{out}, \quad i,j = 1,2$$

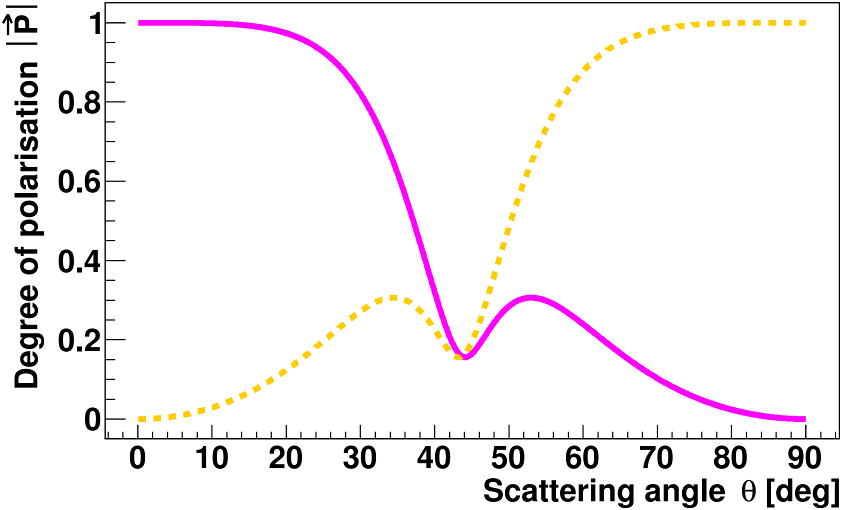
$$\vec{P}_i = \mathrm{Tr}\left(\rho_i \cdot \boldsymbol{\sigma}\right), \quad \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

Møller scattering

$$\vec{P} = x(\sin\chi\cos\psi, \sin\chi\sin\psi, \cos\chi)$$

$$P_1' = \Big[4mx(m+T)\cos^2(\theta)\left(m\sin(\chi)\cos(\psi)\left(T\left(-2\left(16m^2+20mT\right.\right.\right.\right.$$
$$\left.+3T^2\right)\cos(6\theta) - 128m^2 - 8T(6m+T)\cos(4\theta) + T(4m+3T)\cos(8\theta)$$
$$\left.-148mT - 27T^2\right) + \left(512m^3 + 672m^2T + 232mT^2 + 38T^3\right)\cos(2\theta)\Big)$$
$$+T\cos(\chi)\left(-T\left(4m^2+5mT+2T^2\right)\sin(8\theta) + \left(544m^3 + 552m^2T\right.\right.$$
$$\left.+210mT^2 + 28T^3\right)\sin(2\theta) - 2\left(96m^3 + 188m^2T + 91mT^2 + 14T^3\right)$$
$$\sin(4\theta) + 2\left(16m^3 + 36m^2T + 29mT^2 + 6T^3\right)\sin(6\theta)\Big)\Big)\Big]$$
$$\Big[1280m^6 + 3840m^5T + 5044m^4T^2 + 3674m^3T^3 + 1611m^2T^4 - 8T(4m+T)$$
$$\left(58m^4 + 145m^3T + 125m^2T^2 + 44mT^3 + 7T^4\right)\cos(2\theta) - 8T(4m+T)$$
$$\left(6m^4 + 15m^3T + 11m^2T^2 + 4mT^3 + T^4\right)\cos(6\theta) + T^2\left(28m^4 + 46m^3T\right.$$
$$\left.+25m^2T^2 + 6mT^3 + T^4\right)\cos(8\theta) + 4\left(192m^6 + 576m^5T + 652m^4T^2\right.$$
$$\left.+478m^3T^3 + 247m^2T^4 + 66mT^5 + 7T^6\right)\cos(4\theta) + 370mT^5 + 35T^6\Big]^{-1}$$

Møller scattering (100 keV)



PART 3

Monte Carlo

Motivation

1. scarce experimental data
2. complexity of experiment optimization
3. beyond the scope of general purpose Monte Carlo codes

ELSEPA

- ELSEPA = ELastic Scattering of Electrons and Positrons by neutral Atoms and positive ions
- scattering amplitudes f and g determining $\left(\frac{\mathrm{d}\sigma_{\mathrm{Mott}}}{\mathrm{d}\Omega}\right)_0$, S, T and U
- relativistic (Dirac) partial–wave analysis in a central potential:

$$f(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}\{(\ell+1)[\exp(2i\delta_{\kappa=-\ell-1})-1]+\ell[\exp(2i\delta_{\kappa=\ell})-1]\}P_\ell(\cos\theta)$$

$$g(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}[\exp(2i\delta_{\kappa=\ell})-\exp(2i\delta_{\kappa=-\ell-1})]P_\ell^1(\cos\theta).$$

- assuming Fermi distribution for nuclear charge density
  + numerical description of atomic electrons density
  + approximate exchange correction

F. Salvat, A. Jablonski, C. J. Powell, *Comput. Phys. Commun.* 165, 157 (2005)

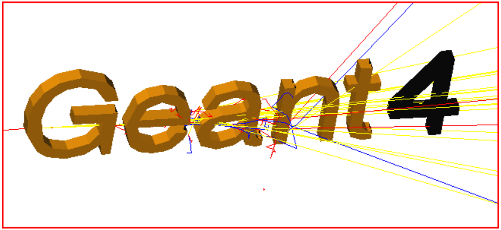
PEBSI Monte Carlo simulation

- PEBSI = Polarized Electron Bremsstrahlung SImulator
- simulation of bremsstrahlung emission from polarized electrons in thin solid state targets
- scattering amplitudes f and g determining $\left(\frac{\mathrm{d}\sigma_{\mathrm{Mott}}}{\mathrm{d}\Omega}\right)_0$, S, T and U from ELSEPA code
- analytical formulae for Møller scattering cross section and polarization transfer

G. Weber *et al.*, *Nucl. Instr. Meth. Phys. Res. B* 279, 155 (2012)
M. Drągowski *et al.*, *Nucl. Instr. Meth. Phys. Res. B* 389, 48 (2016)

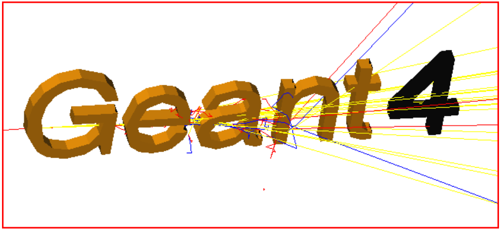
Geant4 Monte Carlo simulation

toolkit for the simulation of the passage of particles through matter



Geant4 Monte Carlo simulation

toolkit for the simulation of the passage of particles through matter

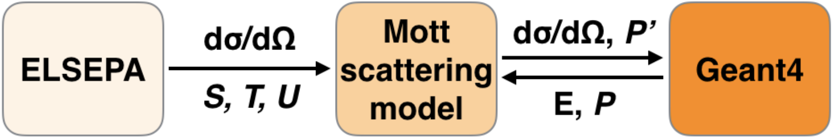


Coulomb Scattering:

- multiple and single scattering algorithms
- no dependence on polarization
- no polarization transfer

Sung Hun Kim *et al.*, *IEEE Trans. Nucl. Sci.* 62, 451 (2015)

Geant4 Extension

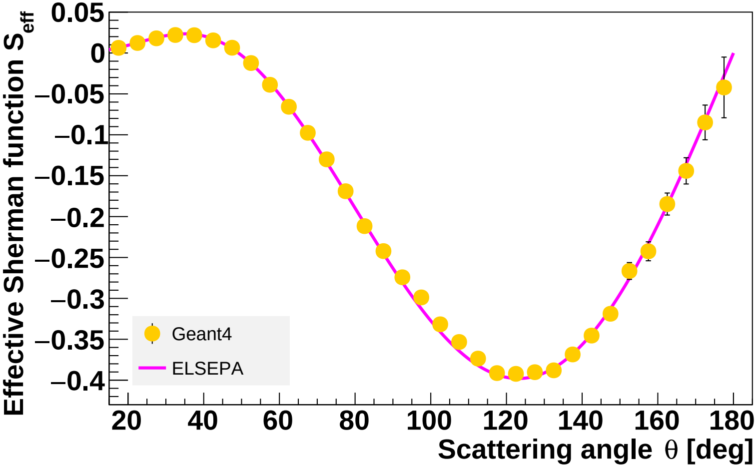


interaction model to replace the default Coulomb scattering model

cross section, momentum and polarization change calculated for given energy, momentum and polarization

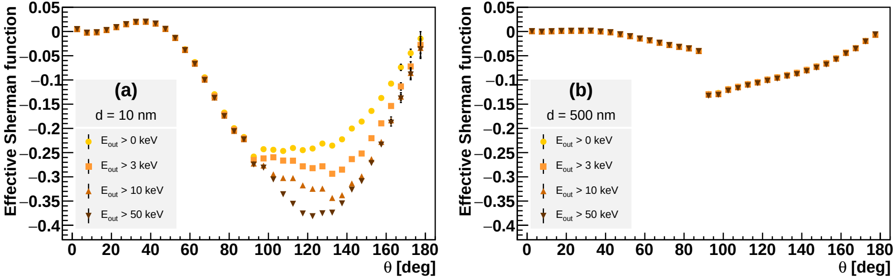
data from ELSEPA imported only at initialization

Simulated effective Sherman function
Comparison with theory



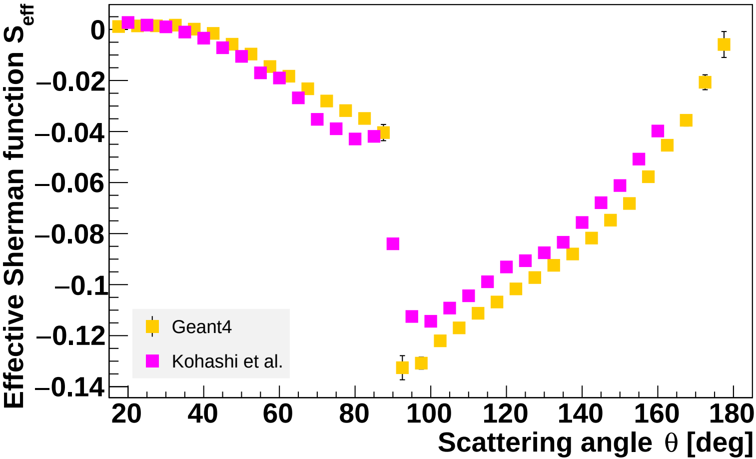
100 keV, 2 nm Au

Simulated effective Sherman function
Effect of energy cuts



100 keV, 10 and 500 nm Au

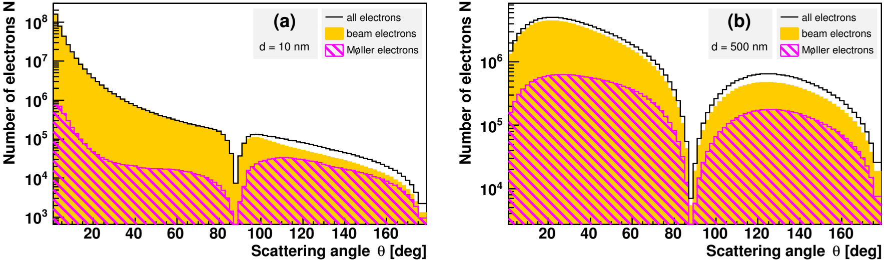
Simulated effective Sherman function
Comparison with measurement



100 keV, 500 nm Au

T. Kohashi, M. Konoto and K. Koike, Jpn. J. Appl. Phys. 45, 6468 (2006)

Scattering angle distribution



100 keV, 10 and 500 nm Au

Parametrizations

Two commonly used parametrizations:

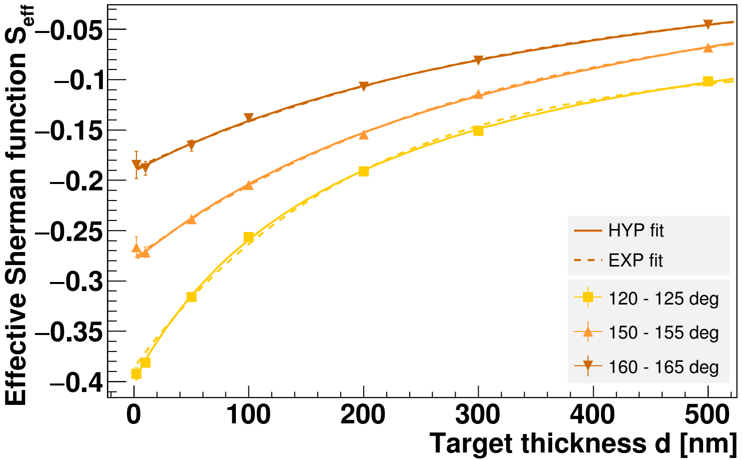
$$\text{hyperbolic: } S_{\text{eff}}^{\text{HYP}}(d) = S_c + \frac{S_0 - S_c}{1 + \alpha_H\, d}$$

$$\text{exponential: } S_{\text{eff}}^{\text{EXP}}(d) = S_c + (S_0 - S_c)\exp(-\alpha_E\, d)$$

Optimal target thickness for a given energy and scattering angle: maximum of the Figure-of-Merit

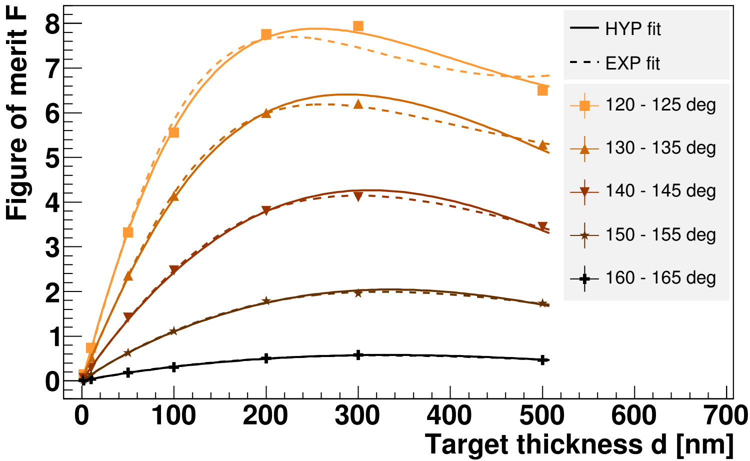
$$\text{FoM}(E, \theta, d) \propto S_{\text{eff}}^2(E, \theta, d)\, N(E, \theta, d)$$

Simulated effective Sherman function
Dependence on target thickness



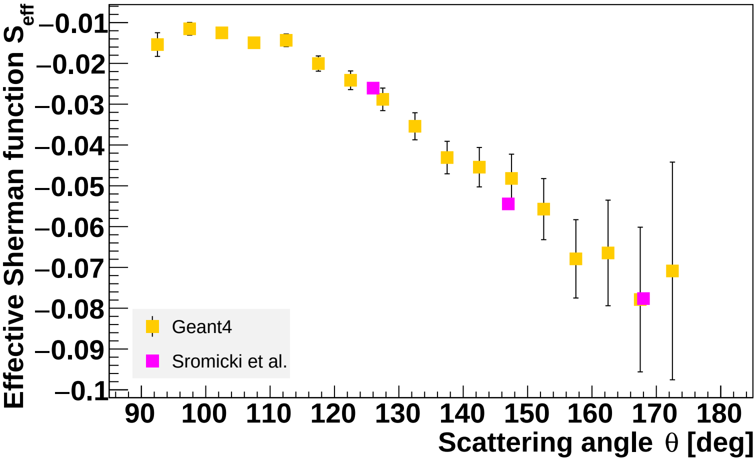
100 keV, 2 – 500 nm Au

Figure-of-Merit $= S_{eff}^2 N$



100 keV, 2 – 500 nm Au

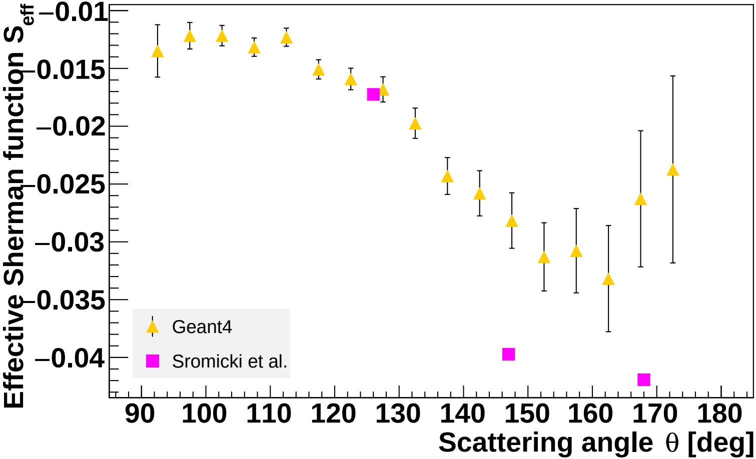
Simulated effective Sherman function
Comparison with measurement



14 MeV, 75 μm Pb

J. Sromicki et al., Phys. Rev. Lett. 82, 57 (1999)

Simulated effective Sherman function
Comparison with measurement

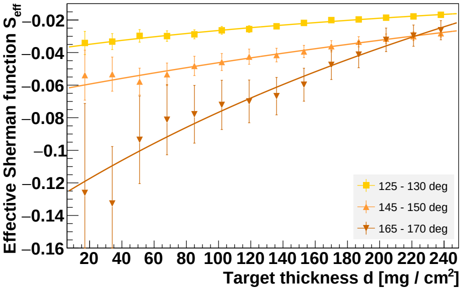


14 MeV, 210 μm Pb

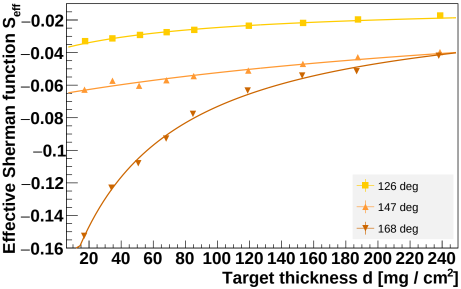
J. Sromicki et al., Phys. Rev. Lett. 82, 57 (1999)

Simulated effective Sherman function
Dependence on target thickness

Geant4



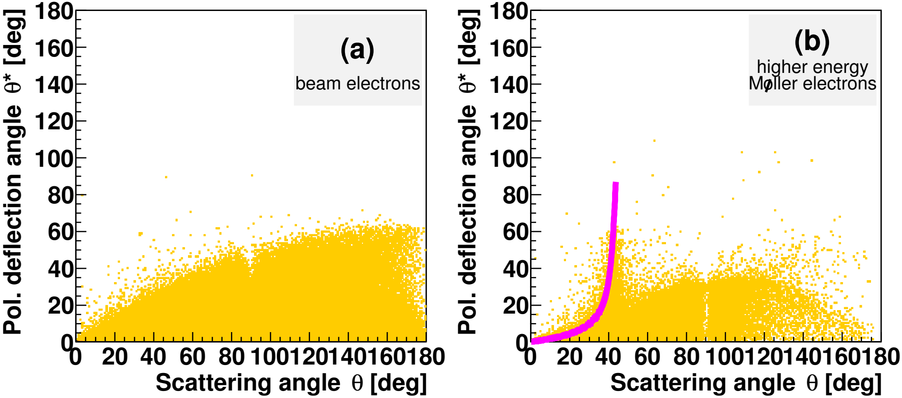
Sromicki et al.



14 MeV, 15 – 210 μm Au

J. Sromicki et al., Phys. Rev. Lett. 82, 57 (1999)

Depolarization



100 keV, 10 nm Au

Summary of Part 3

1. encouraging results of comparison with experimental data
2. applications:
   - computation of the effective Sherman function
   - experiment optimization
   - depolarization of electron beams passing through matter
3. further validation ongoing (higher energies)