

Theoretical Prospective on Leptonic CP Violation

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Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics. It is part of the more general fundamental problem in particle physics of understanding the origins of flavour in the quark and lepton sectors, i.e., of the patterns of quark masses and mixing, and of the charged lepton and neutrino masses and of neutrino mixing.

“Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn't have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses.”

From Model Physicist, CERN Courier, 13 October 2017.

Of fundamental importance are also

- . the determination of the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos (which is one of the most challenging and pressing problems in present day elementary particle physics) (GERDA, CUORE, KamLAND-Zen, EXO, LEGEND, nEXO,...);
- . determining the status of CP symmetry in the lepton sector (T2K, NO ν A; T2HK, DUNE);
- . determination of the type of spectrum neutrino masses possess, or the “neutrino mass ordering” (T2K + NO ν A; JUNO; PINGU, ORCA; T2HKK, DUNE);
- . determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology).

The program of research extends beyond 2030.

- **BS 3ν RM: eV scale sterile ν 's; NSI's; ChLFV processes ($\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu^- - e^-$ conversion on (A,Z)); ν -related BSM physics at the TeV scale (N_{jR} , H^{--} , H^- , etc.).**

Reference Model: 3- ν mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary.

$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

Data: 3 ν s are light: $\nu_{1,2,3}, m_{1,2,3} \lesssim 0.5$ eV.

3- ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E ; at distance L : $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0, P(\nu_\mu \rightarrow \nu_\mu) < 1$

$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

n	2	3	4	
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6

CP-violating phases:

• ν_j – Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j – Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and

2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21} , α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2\dots$
S.M. Bilenky et al., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.37 \times 10^{-5}$ eV $^2 > 0$, $\sin^2 \theta_{12} \cong 0.297$, $\cos 2\theta_{12} \gtrsim 0.29$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.53$ (2.43) [2.56 (2.54)] $\times 10^{-3}$ eV 2 , $\sin^2 \theta_{23} \cong 0.437$ (0.569) [0.425 (0.589)], NO (IO) ,
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0214$ (0.0218) [0.0215 (0.0216)], NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1601.07777 [arXiv:1703.04471].

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31(32)}^2)$ not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$, normal mass ordering (NO)

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$, inverted mass ordering (IO)

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m_{31}^2(\text{NO}) = -\Delta m_{32}^2(\text{IO})$$

$$m_1 \ll m_2 < m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg |\Delta m_{31(32)}^2|, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}$, $m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}$, $m_2 = \sqrt{m_3^2 + \Delta m_{23}^2}$ - IO;

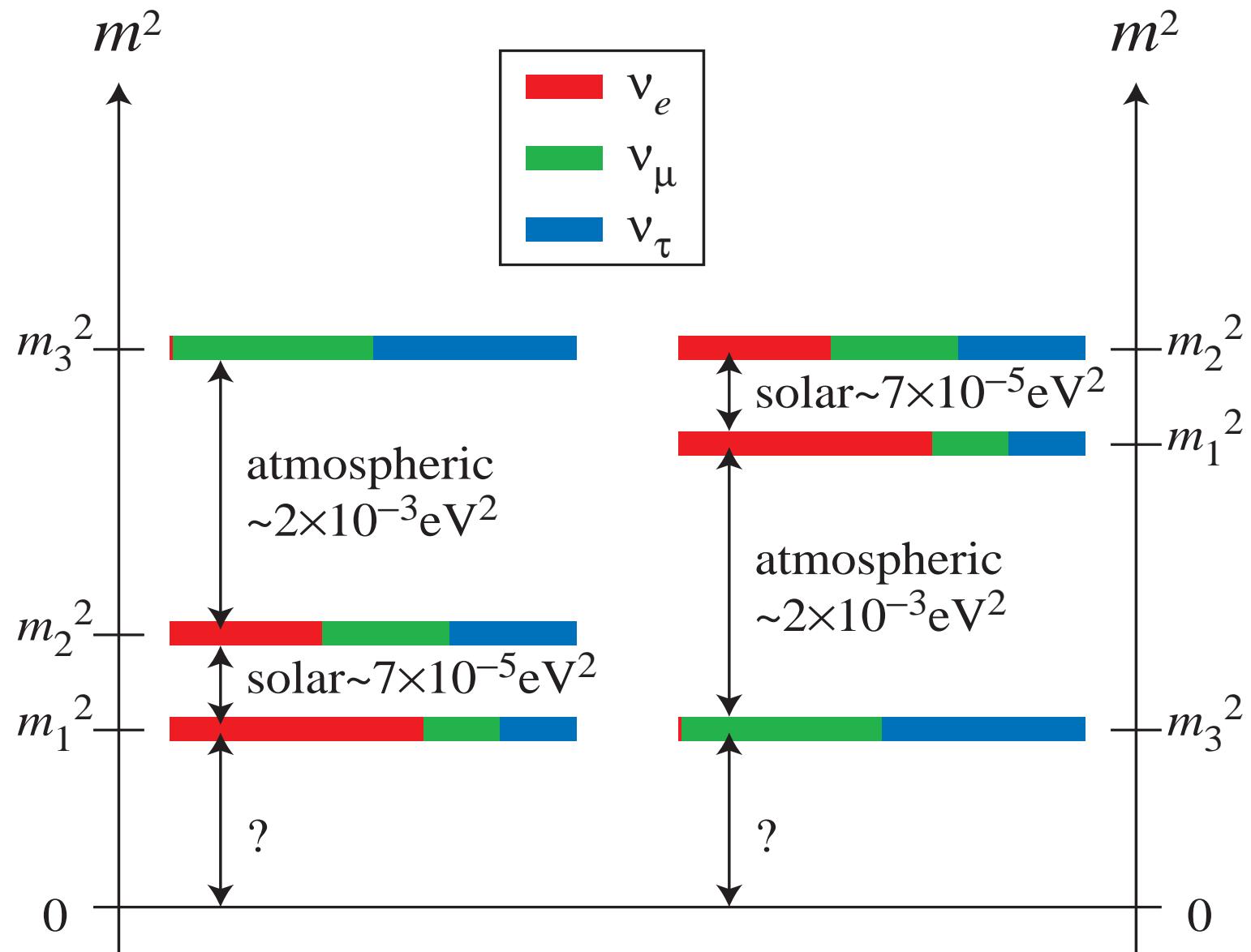


Table 3: Best fit values and allowed ranges at $N\sigma = 1, 2, 3$ for the 3ν oscillation parameters, in either NO or IO. The latter column shows the formal “ 1σ accuracy” for each parameter, defined as $1/6$ of the 3σ range divided by the best-fit value (in percent).

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	“ 1σ ” (%)
$\Delta m_{\odot}^2 / 10^{-5}$ eV 2	NO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
	IO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
$ \Delta m_A^2 / 10^{-3}$ eV 2	NO	2.49	2.46 – 2.53	2.43 – 2.56	2.39 – 2.59	1.4
	IO	2.48	2.44 – 2.51	2.41 – 2.54	2.38 – 2.58	1.4
$\sin^2 \theta_{12}$	NO	3.04	2.91 – 3.18	2.78 – 3.32	2.65 – 3.46	4.4
	IO	3.03	2.90 – 3.17	2.77 – 3.31	2.64 – 3.45	4.4
$\sin^2 \theta_{13} / 10^{-2}$	NO	2.14	2.07 – 2.23	1.98 – 2.31	1.90 – 2.39	3.8
	IO	2.18	2.11 – 2.26	2.02 – 2.35	1.95 – 2.43	3.7
$\sin^2 \theta_{23} / 10^{-1}$	NO	5.51	4.81 – 5.70	4.48 – 5.88	4.30 – 6.02	5.2
	IO	5.57	5.33 – 5.74	4.86 – 5.89	4.44 – 6.03	4.8
δ/π	NO	1.32	1.14 – 1.55	0.98 – 1.79	0.83 – 1.99	14.6
	IO	1.52	1.37 – 1.66	1.22 – 1.79	1.07 – 1.92	9.3

$$\Delta m_{\odot}^2 \equiv \Delta m_{21}^2; \quad \Delta m_A^2 \equiv \Delta m_{31(32)}^2, \text{ NO (IO).}$$

F. Capozzi et al. (Bari Group), arXiv:1804.09678.

- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

- $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;
- $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
- BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

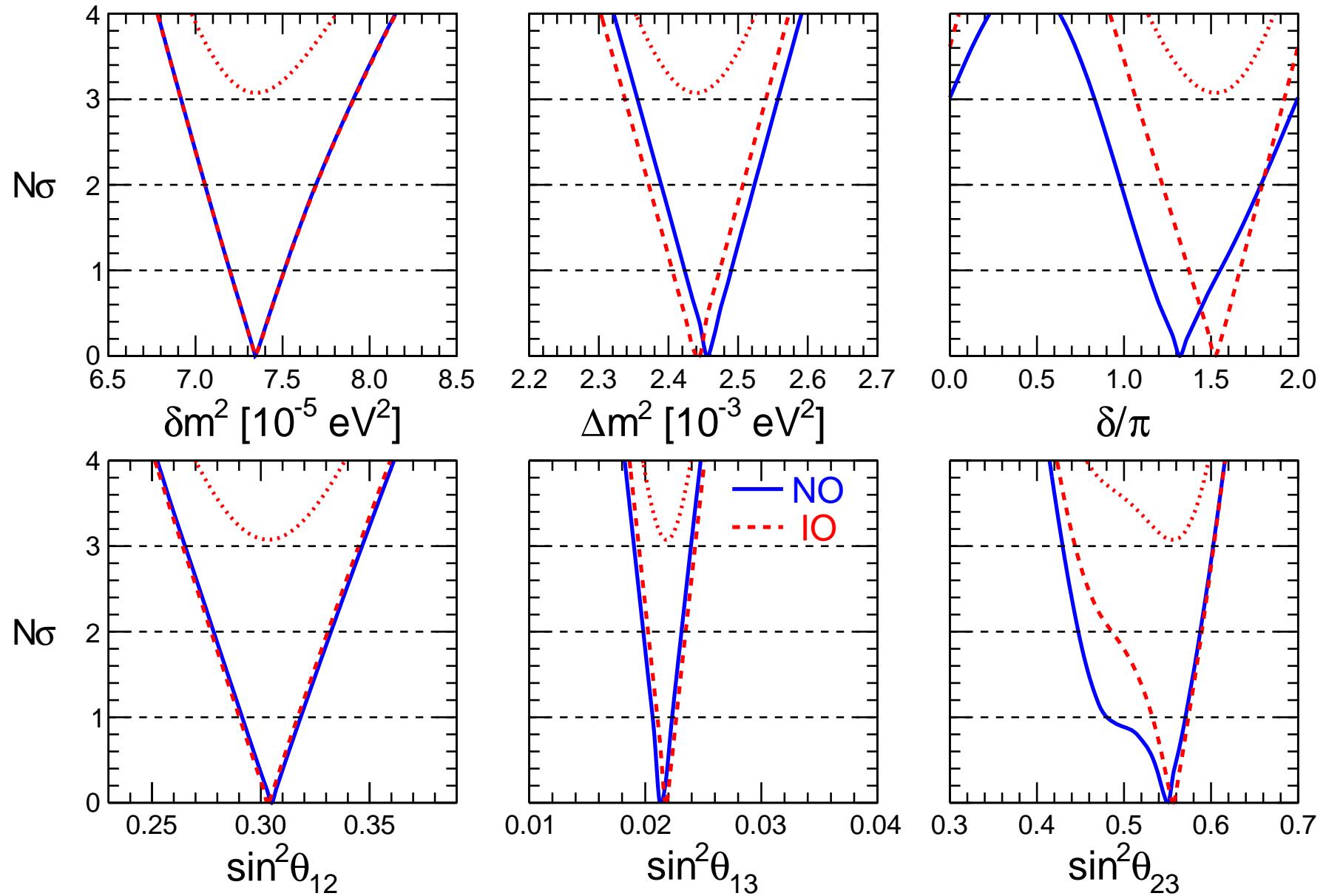
$$\delta \cong 3\pi/2?$$

$$\begin{aligned} J_{CP} &= \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \end{aligned}$$

- **Best fit value:** $\delta = 1.32(1.52)\pi$ [$1.30(1.54)\pi$];
- $\delta = 0$ or 2π are disfavored at $3.0(3.6)\sigma$ [$2.6(3.0)\sigma$];
- $\delta = \pi$ is disfavored at $1.8(3.6)\sigma$ [$1.7(3.3)\sigma$];
- $\delta = \pi/2$ is strongly disfavored at $4.4(5.2)\sigma$ [$4.3(5.0)\sigma$].
- **At 3σ :** δ/π is found to lie in **0.83-1.99 (1.07-1.92)** [**1.07-1.97 (0.80-2.08)**].

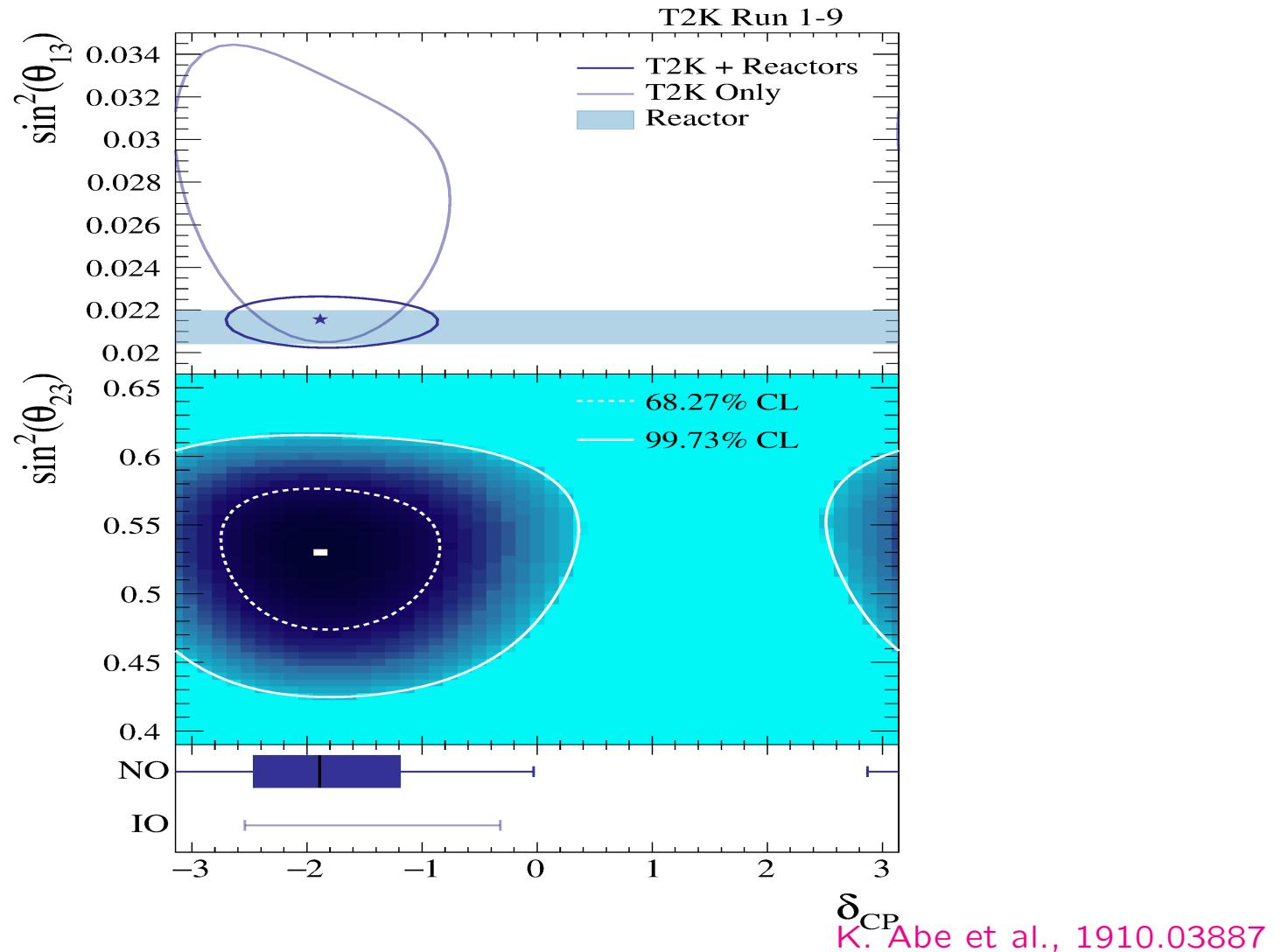
F. Capozzi, E. Lisi *et al.*, arXiv:1804.09678 [E. Esteban *et al.*, NuFit 3.2 (Jan., 2018)]

LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



F. Capozzi et al. (Bari Group), arXiv:1804.09678.

Latest results from T2K



Best fit value: $\delta = -1.89$ (-1.38), NO (IO).

$\delta = 0, \pi$ ruled out at 95% CL.

At 3σ : δ is found to lie in $[-3.41, -0.03]$ ($[-2.54, -0.32]$), NO (IO).

Latest global analysis: data favors NO

IO disfavored at 3.1σ .

F. Capozzi et al., 1804.09678.

Massive Dirac Neutrinos: the assumption $L=const.$

“In modern understanding of particle physics global symmetries are approximate.” Global $U(1)$ symmetry leading to $L=const.$ is expected to be broken by quantum gravity effects.

See E. Witten, 1710.01791; S. Weinberg, CERN Courier, 13 October 2017

Qualitative understanding of $m_{\nu_j} \lll m_{e,\mu,\tau}, m_q$

- Seesaw mechanisms of neutrino mass generation

P. Minkowski, 1977; T. Yanagida, 1979; M. Gell-Mann, P. Ramond, R. Slansky, 1979;
R. Mohapatra, G. Senjanovic, 1980 (type I).
W. Konetschny, W. Kummer 1977; M. Magg, C. Watterich, 1980; T.P. Cheng, L.-F. Li, 1980 (type II).
R. Foot *et al.*, 1989 (type III).

Explains the smallness of ν -masses (naturalness); connection to grand unification.

Through **leptogenesis theory** link the ν -mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Massive neutrinos ν_j - Majorana particles.

- Radiative generation of ν masses and mixing

A. Zee, 1980; K. Babu, 1985;...; recent review: Y.Cai *et al.*, 1706.08524

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH $\nu s'$ (heavy).

Type II seesaw mechanism: $H(x)$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

W. Konetschny, W. Kummer 1977; M. Magg, C. Wetterich, 1980; T.P. Cheng, L.-F. Li, 1980; J. Schechter, J.F.W. Valle, 1980 G. Lazarides, Q. Shafi, C. Wetterich, 1981.

Type III seesaw mechanism: $T(x)$ - a triplet of fermion fields.

R. Foot *et al.*, 1989

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$ -decay, LFV processes, etc.) and New Physics at LHC.

Radiative generation of ν masses and mixing

A. Zee, 1980; K.S. Babu, 1985;...; Y. Farzan *et al.*, 1208.2732; recent review: Y.Cai *et al.*, 1706.08524

Generic features

- Loop suppression helps explaining the smallness of ν -masses.
- New particles need not be super heavy - can be at the TeV scale.
- Models at the TeV scale - testable.
- No need to introduce ν_R .
- Typically includes extended scalar sector.

Understanding the Pattern of Neutrino Mixing

With the observed pattern of neutrino mixing Nature is sending us a Message. The Message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. We do not know at present what is the content of Nature's Message. However, on the basis of the current ideas about the origin of the observed pattern of neutrino mixing the Message can have two completely different contents:

ANARCHY or SYMMETRY.

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

U_{PMNS} from random draw of unbiased distribution of 3×3 unitary matrices.

θ_{ij} random quantities, no correlations whatsoever between the values of θ_{12} and/or θ_{13} and/or θ_{23} . Predicts distributions (not values) of θ_{ij} ; values of $\theta_{ij} \sim \pi/4$ most probable.

Three large mixing angles - most natural for the approach.

However, $\theta_{13} \cong 0.15\dots$

Values of m_j , Δm_{ij}^2 - not predicted.

The Quest for Nature's Message

Towards Quantitative Understanding of U_{PMNS} , m_j

The observed pattern of 3- ν mixing, two large and one small mixing angles,

$$\theta_{12} \cong 33^\circ, \theta_{23} \cong 45^\circ \pm 6^\circ \text{ and } \theta_{13} \cong 8.4^\circ,$$

can most naturally be explained by extending the Standard Model (SM) with a flavour symmetry corresponding to a non-Abelian discrete (finite) group G_f .

$$G_f = A_4, T', S_4, A_5, D_{10}, D_{12}, \dots$$

Vast literature; reviews: G. Altarelli, F. Feruglio, 1002.0211; H. Ishimori et al., 1003.3552; M. Tanimoto, AIP Conf. Proc. 1666 (2015) 120002; S. King and Ch. Luhn, 1301.1340; D. Meloni, 1709.02662; STP, 1711.10806

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}$, $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4} (?)$, $\theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1} \frac{1}{\sqrt{3}} - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$,
 $\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 \mp 0.10$.
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell) \ Q(\psi, \omega) U_{\text{TBM}, \text{BM}, \text{LC}, \dots} \ \bar{P}(\xi_1, \xi_2),$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM}, \text{BM}, \text{LC}, \dots} \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi, \omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

U_{LC} , U_{GRAM} , U_{GRBM} , U_{HGM} :

$$U_{LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c_{23}^\nu}{\sqrt{2}} & \frac{c_{23}^\nu}{\sqrt{2}} & s_{23}^\nu \\ \frac{s_{23}^\nu}{\sqrt{2}} & -\frac{s_{23}^\nu}{\sqrt{2}} & c_{23}^\nu \end{pmatrix}; \quad \mu - \tau \text{ symmetry : } \theta_{23}^\nu = \mp\pi/4;$$

$$U_{GR} = \begin{pmatrix} c_{12}^\nu & s_{12}^\nu & 0 \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta_{12}^\nu = \pi/6.$$

U_{GRAM} : $\sin^2 \theta_{12}^\nu = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$)

U_{GRBM} : $\sin^2 \theta_{12}^\nu = (3-r)/4 \cong 0.345$.

GRB and HG mixing: W. Rodejohann et al., 2009.

$U_{\text{TBM(BM)}}$: Groups A_4 , T' , S_4 (S_4),... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

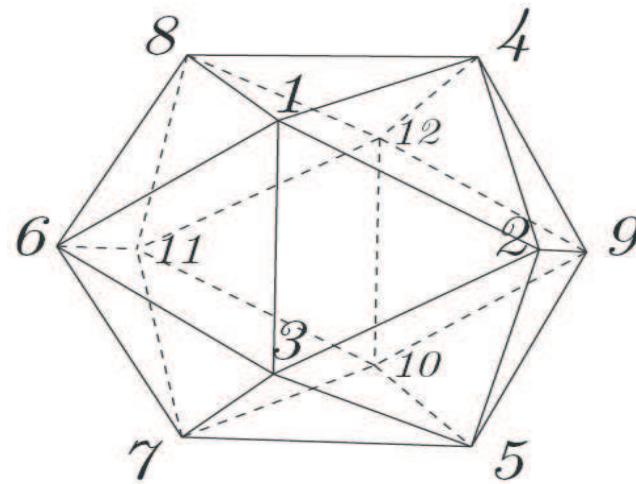
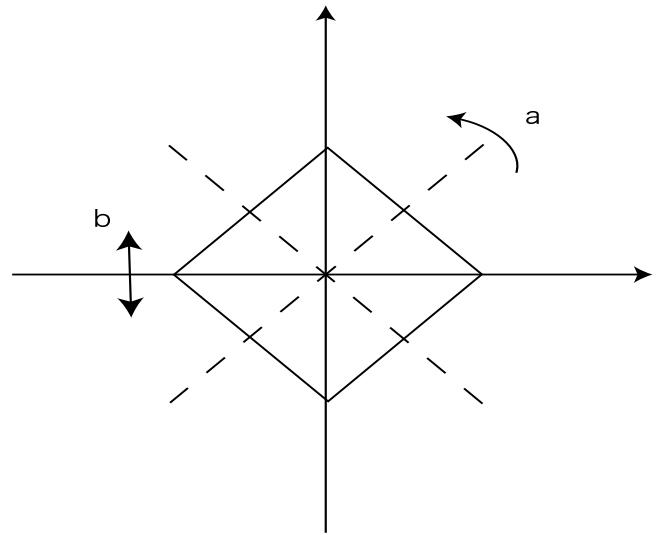
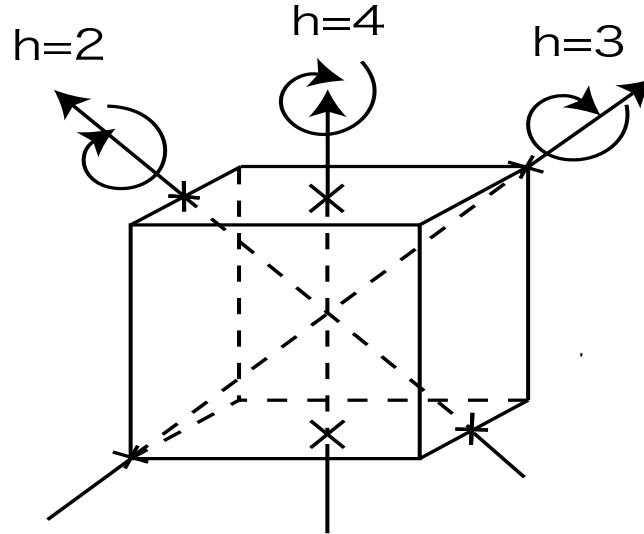
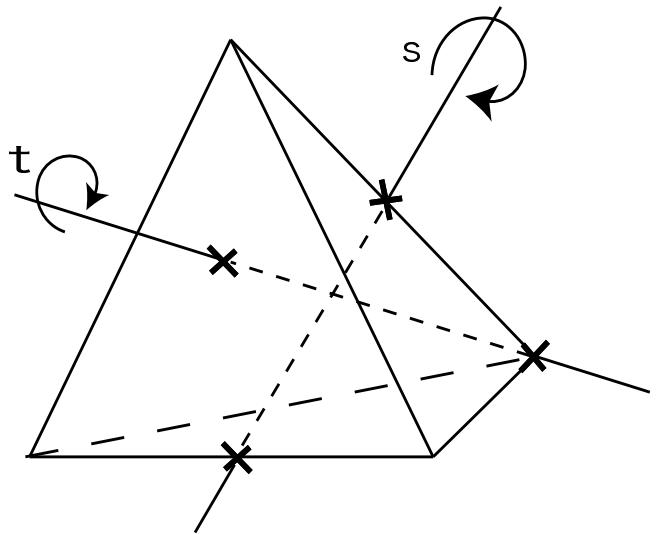
- U_{BM} : $s_{12}^2 = 1/2$, $s_{13}^2 = 0$, $s_{23}^2 = 1/2$;
 $s_{13}^2 = 0$ and $s_{12}^2 = 1/2$ must be corrected.
- U_{GRA} : Group A_5, \dots ; $s_{13}^2 = 0$ and possibly $s_{12}^2 = 0.276$ and $s_{23}^2 = 1/2$ must be corrected.
L. Everett, A. Stuart, arXiv:0812.1057;...
- U_{LC} : alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$
S.T.P., 1982
- U_{LC} : $s_{12}^2 = 1/2$, $s_{13}^2 = 0$, s_{23}^ν - free parameter;
 $s_{13}^2 = 0$ and $s_{12}^2 = 1/2$ must be corrected.

- U_{GRB} : **Group** D_{10}, \dots ; $s_{13}^2 = 0$ and possibly $s_{12}^2 = 0.345$ and $s_{23}^2 = 1/2$ must be corrected.
- U_{HG} : **Group** D_{12}, \dots ; $s_{13}^2 = 0$, $s_{12}^2 = 0.25$ and possibly $s_{23}^2 = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta_{13}^\nu = 0$, $\theta_{23}^\nu = \mp\pi/4$.

They differ by the value of θ_{12}^ν :

TBM, BM, GRA, GRB and HG forms correspond to $\sin^2 \theta_{12}^\nu = 1/3; 0.5; 0.276; 0.345; 0.25$.

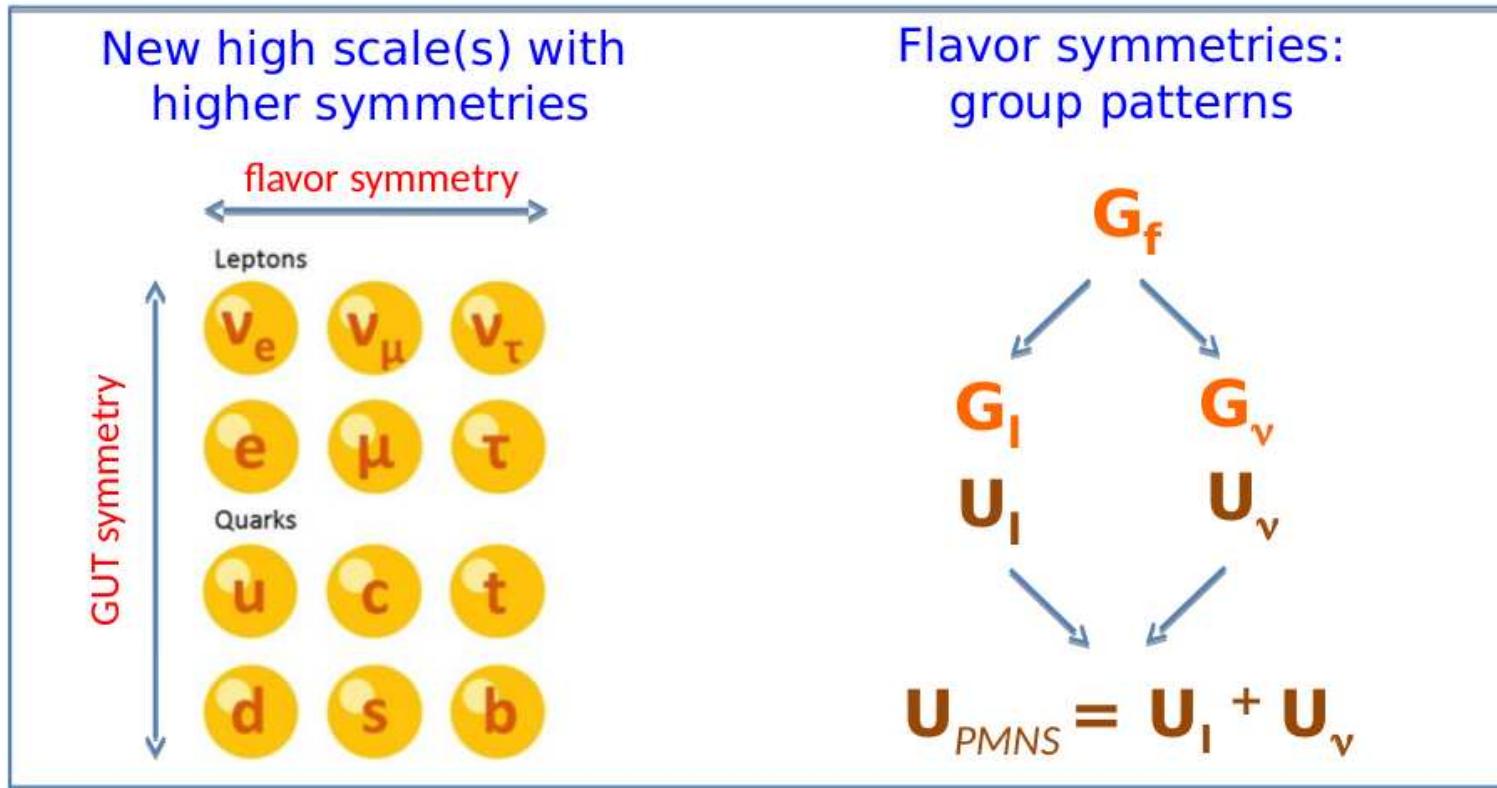


Examples of symmetries: A_4 , S_4 , D_4 , A_5

From M. Tanimoto et al., arXiv:1003.3552

How Does it Work

Model building with symmetries



E. Lisi, TAUP 2019

Predictions and Correlations

$U_\nu = U_{\text{TBM}, \text{BM}, \text{GRA}, \text{GRB}, \text{HG}} \bar{P}(\xi_1, \xi_2); \theta_{12}^\nu;$

$U_\ell^\dagger = R_{12}(\theta_{12}^\ell) Q, Q = \text{diag}(e^{i\varphi}, 1, 1)$

(the “minimal” = simplest case ($SU(5) \times T'$, ...))

$U_\ell^\dagger = R_{12}(\theta_{12}^\ell) R_{23}(\theta_{23}^\ell) Q, Q = \text{diag}(1, e^{-i\psi}, e^{-i\omega})$

(next-to-minimal case)

$\cos \delta = \cos \delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu, \dots),$

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu, \dots),$

θ_{12}^ν, \dots - known (fixed) parameters, depend on the underlying symmetry.

For arbitrary fixed θ_{12}^ν and any θ_{23}
("minimal" and "next-to-minimal" cases):

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} [\cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13})].$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2 \theta_{23} = \frac{1}{2} \frac{1 - 2 \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}}$.

In all cases TBM, BM (LC), GRA, GRB, HG:

- New sum rules relating $\theta_{12}, \theta_{13}, \theta_{23}$ and δ ;
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu)$.

S.T.P., arXiv:1405.6006

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu)$.
- **TBM case:** $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij} :
 $\delta \cong 263.5^\circ$ or 96.5° , $\cos \delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- **GRAM case, b.f.v. of θ_{ij} :** $\delta \cong 286.8^\circ$ or 73.2° ;
 $\cos \delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- **GRBM case, b.f.v. of θ_{ij} :** $\delta \cong 258.5^\circ$ or 101.5° ;
 $\cos \delta = -0.200$, $J_{CP} \mp 0.0333$.
- **HGM case, b.f.v. of θ_{ij} :** $\delta \cong 298.4^\circ$ or 61.6° ;
 $\cos \delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- **BM, LC cases:** $\delta \cong \pi$, $\cos \delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij} : F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos \delta$ or δ and using high precision data on θ_{12} , θ_{23} and θ_{13} , one can distinguish between different symmetry forms of \tilde{U}_ν !

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK) see, e.g., R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984; K. Abe *et al.* [T2HK Proto-Collab.], arXiv:1502.05199 (PTEP 2015 (2015) 053C02).

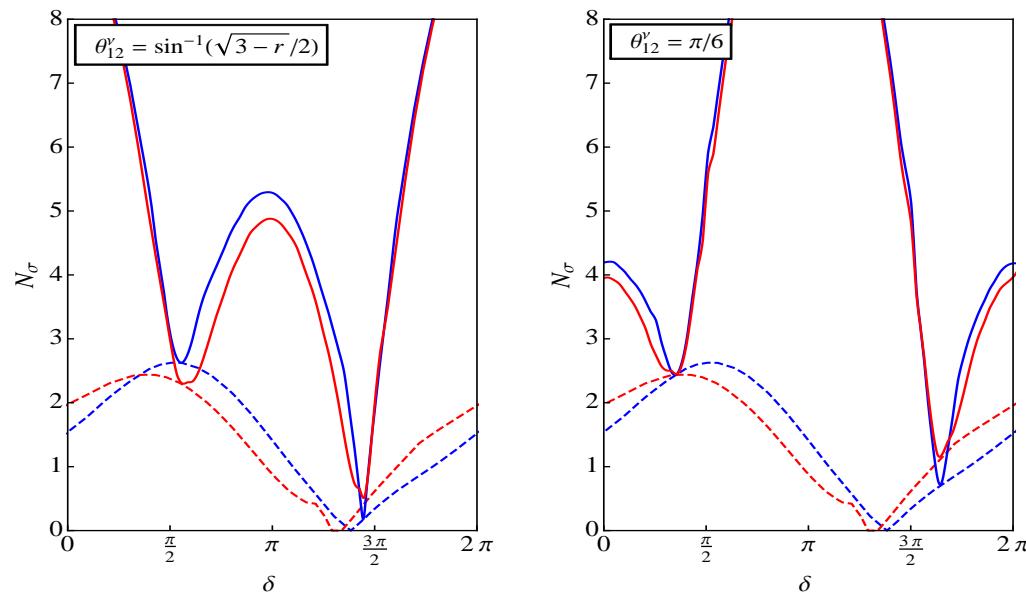
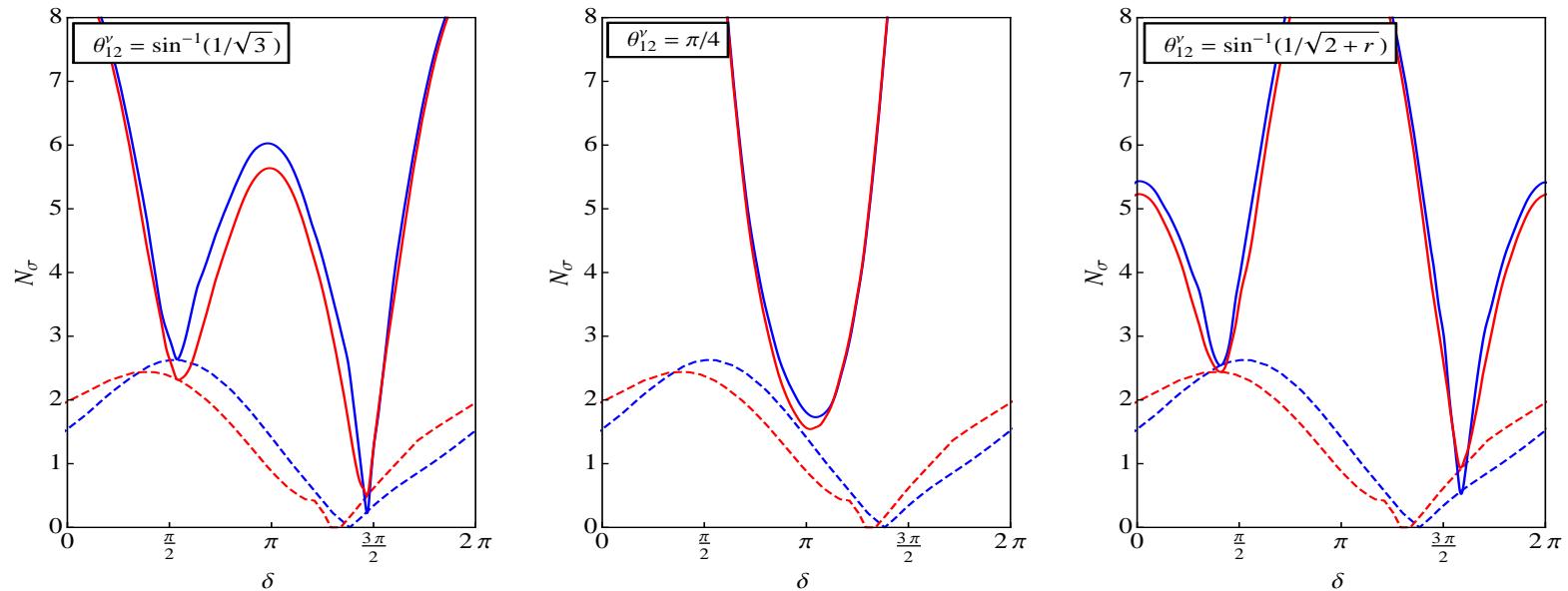
**Statistical analysis, likelihood method;
input “data”: $\sin^2 \theta_{13}$, $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, δ
from F. Capozzi et al., arXiv:1312.2878v2 (May 5,
2014).**

$$L(\cos \delta) \propto \exp\left(-\frac{\chi^2(\cos \delta)}{2}\right)$$

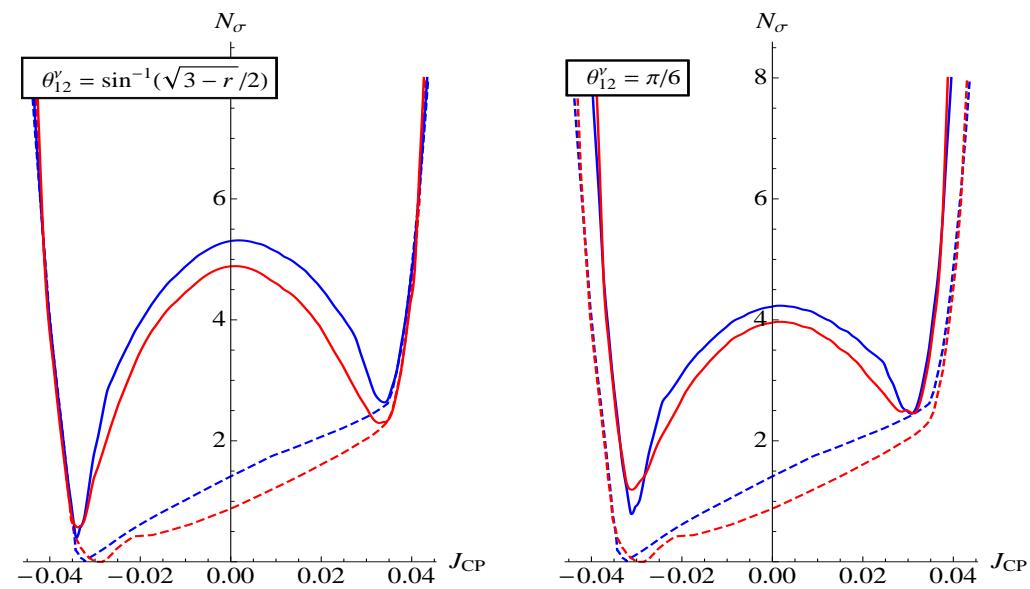
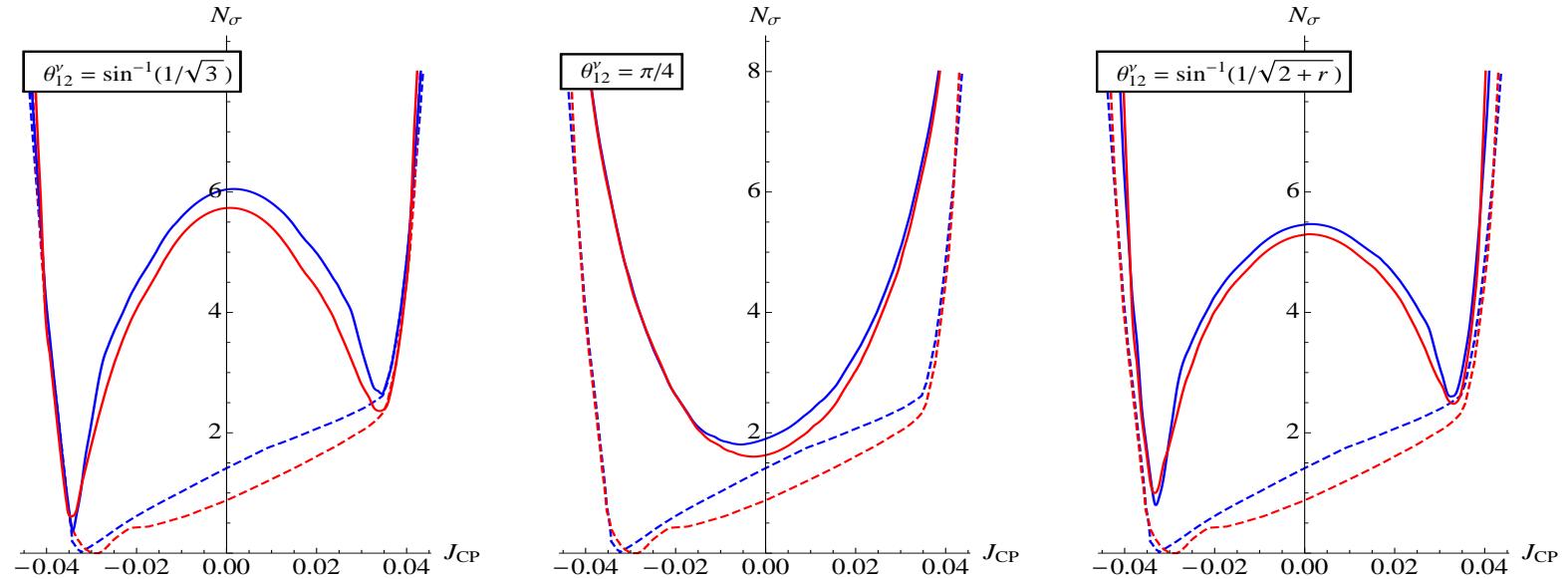
$n\sigma$ confidence level interval of values of $\cos \delta$:

$$L(\cos \delta) \geq L(\chi^2_{\min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J_{CP} = 0$ excluded at 5σ , 4σ , 4σ , 3σ confidence level.

At 3σ : $0.020 \leq |J_{CP}| \leq 0.039$.

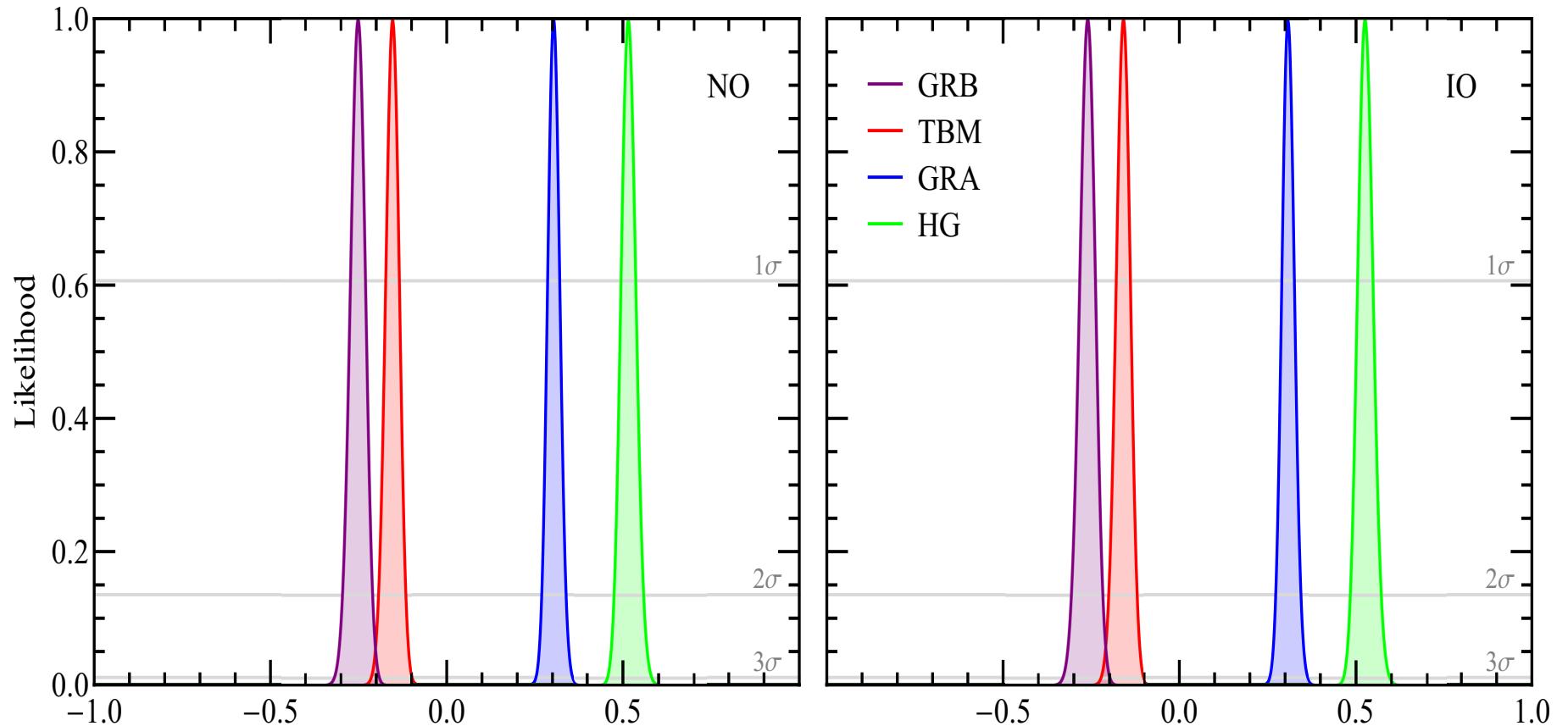
BM (LC), b.f.v.: $J_{CP} = 0$;
at 3σ : -0.026 (-0.025) $\leq J_{CP} \leq 0.021$ (0.023) for NO
(IO) neutrino mass spectrum.

Prospective precision:

$\delta(\sin^2 \theta_{12}) = 0.7\% \text{ (JUNO),}$

$\delta(\sin^2 \theta_{13}) = 3\% \text{ (Daya Bay),}$

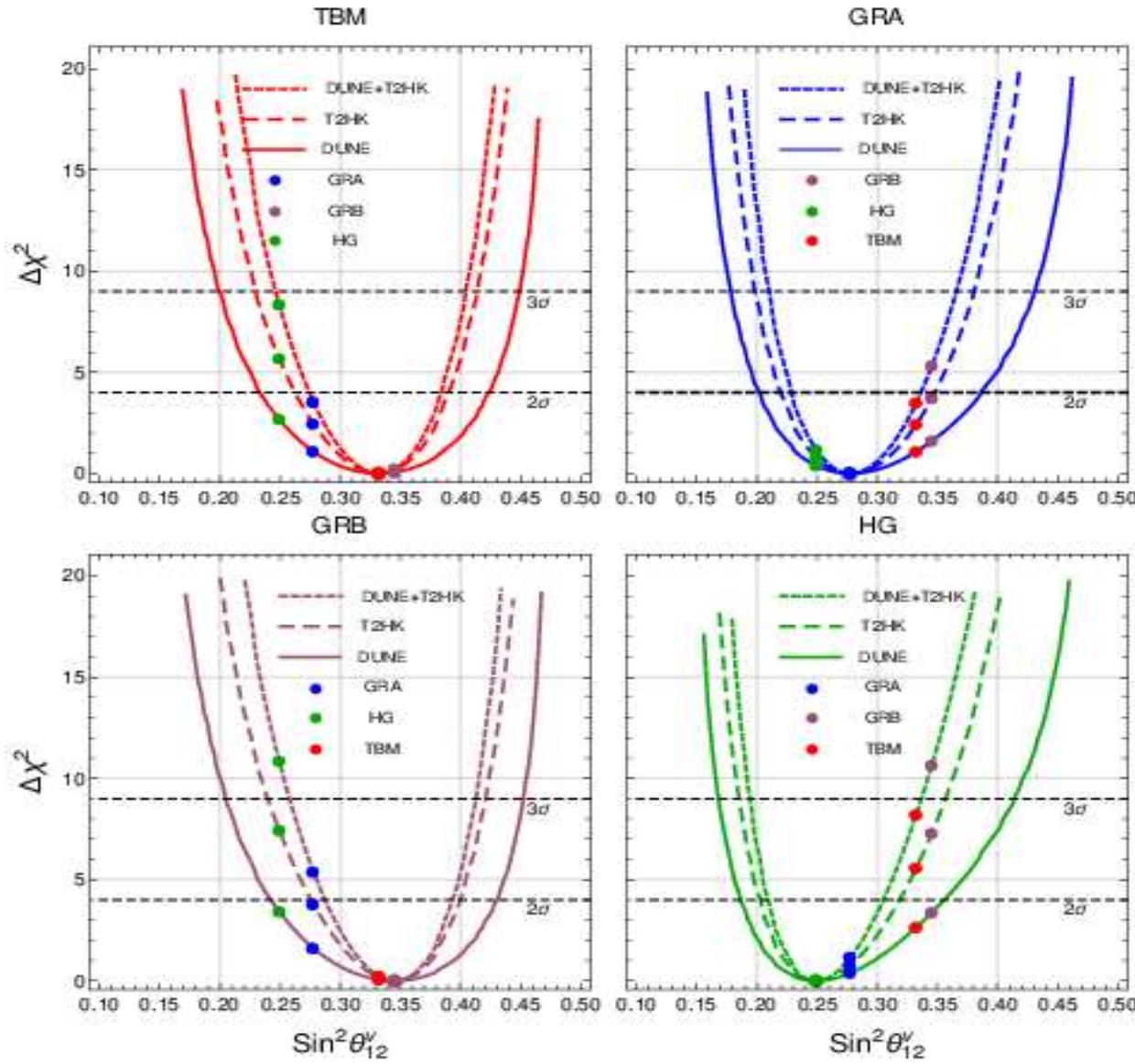
$\delta(\sin^2 \theta_{23}) = 5\% \text{ (T2K, NO}\nu\text{A combined).}$



b.f.v. of $\sin^2 \theta_{ij}$ (\cos^δ Esteban et al., Jan., 2018) + the prospective precision used.

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} [\cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13})] .$$

$\delta(\sin^2 \theta_{23}) = 3\%$ (T2HK, DUNE).



Agarwalla, Chatterjee, STP, Titov, arXiv:1711.02107

GRB - HG $> 3\sigma$; GRA - GRB $\geq 2\sigma$; TMB - HG $\cong 3\sigma$; TMB - GRA $\cong 2\sigma$.

With T2HKK data - better sensitivity.

Examples of Predictions and Correlations II.

- $\sin^2 \theta_{23} = \frac{1}{2}$.
- $\sin^2 \theta_{23} \cong \frac{1}{2}(1 \mp \sin^2 \theta_{13}) + O(\sin^4 \theta_{13}) \cong \frac{1}{2}(1 \mp 0.022)$.
- $\sin^2 \theta_{23} = 0.455; 0.463; 0.537; 0.545; 0.604$ (**small uncert.**).
- $\sin^2 \theta_{12} \cong \frac{1}{3}(1 + \sin^2 \theta_{13}) + O(\sin^4 \theta_{13}) \cong 0.340$.
- $\sin^2 \theta_{12} \cong \frac{1}{3}(1 - 2\sin^2 \theta_{13}) + O(\sin^4 \theta_{13}) \cong 0.319$.
- **and/or** $\cos \delta = \cos \delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu, \dots)$,

$$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu, \dots),$$

θ_{12}^ν, \dots - known (fixed) parameters, depend on the underlying symmetry.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12} , θ_{13} and θ_{23} , can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Prospective (useful/requested) precision:

$\delta(\sin^2 \theta_{12}) = 0.7\%$ (**JUNO**),

$\delta(\sin^2 \theta_{13}) = 3\%$ (**Daya Bay**),

$\delta(\sin^2 \theta_{23}) = 3\%$ (**T2HK, DUNE; T2K+NO ν A(?)**).

$\delta(\delta) = 10^\circ$ at $\delta = 3\pi/2$ (**THKK?**)

The Power of Data

Systematic analysis (I. Girardi et al., 2016):
all possible combinations of residual symmetries G_e and G_ν of the lepton flavour symmetry groups $G_f = S_4, A_4, T'$ and A_5 , leading to correlations between some of the three neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ , were considered.

- (A) $G_e = Z_2$ and $G_\nu = Z_k, k > 2$ or $Z_m \times Z_n, m, n \geq 2$;
- (B) $G_e = Z_k, k > 2$ or $Z_m \times Z_n, m, n \geq 2$ and $G_\nu = Z_2$;
- (C) $G_e = Z_2$ and $G_\nu = Z_2$.

In these cases U_e^\dagger and/or U_ν of $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$, are partially (or fully) determined by residual discrete symmetries of $G_f = S_4, A_4, T'$ and A_5 .

More specifically:

- A. $G_e = Z_2$, $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$;
 U_ν fixed; A1, A2 (A3): θ_{23} , $\cos \delta$ (θ_{12} , θ_{13}) predicted.
- B. $G_e = Z_n$, $n > 2$ or $G_e = Z_n \times Z_m$, $n, m \geq 2$, $G_\nu = Z_2$;
 U_e fixed; B1, B2 (B3): θ_{12} , $\cos \delta$ (θ_{23} , θ_{13}) predicted.
- C. $G_e = Z_2$ and $G_\nu = Z_2$: θ_{12} or θ_{23} or $\cos \delta$ predicted.

$G_f = A_4, S_4, T', A_5.$

A_4 : 3 Z_2 , 4 Z_3 , 1 $Z_2 \times Z_2$ subgroups (total 8).

T' : similar to A_4 .

S_4 : 9 Z_2 , 4 Z_3 , 3 Z_4 , 4 $Z_2 \times Z_2$ subgroups (total 20).

A_5 : has 15 Z_2 , 10 Z_3 , 6 Z_5 , 5 $Z_2 \times Z_2$ subgroups (36).

In the case of A_4 (T') symmetry only there are 64 models (up to permutation of rows and columns).

A_4 :

$$(G_e, G_\nu) = (Z_2, Z_3), \mathbf{A1} - \mathbf{A3};$$

$$(G_e, G_\nu) = (Z_2, Z_2), \mathbf{A1} - \mathbf{A3};$$

$$(G_e, G_\nu) = (Z_3, Z_2), \mathbf{B1} - \mathbf{B3};$$

$$(G_e, G_\nu) = (Z_2 \times Z_2, Z_2), \mathbf{B1} - \mathbf{B3};$$

$$(G_e, G_\nu) = (Z_2, Z_2), \mathbf{C1} - \mathbf{C9}.$$

For A_4 , S_4 and A_5 the total number of models to be analysed is extremely large. However, a total of only 14 models survive the 3σ constraints on $\sin^2 \theta_{ij}$ from the current data and the requirement $|\cos \delta| \leq 1$.

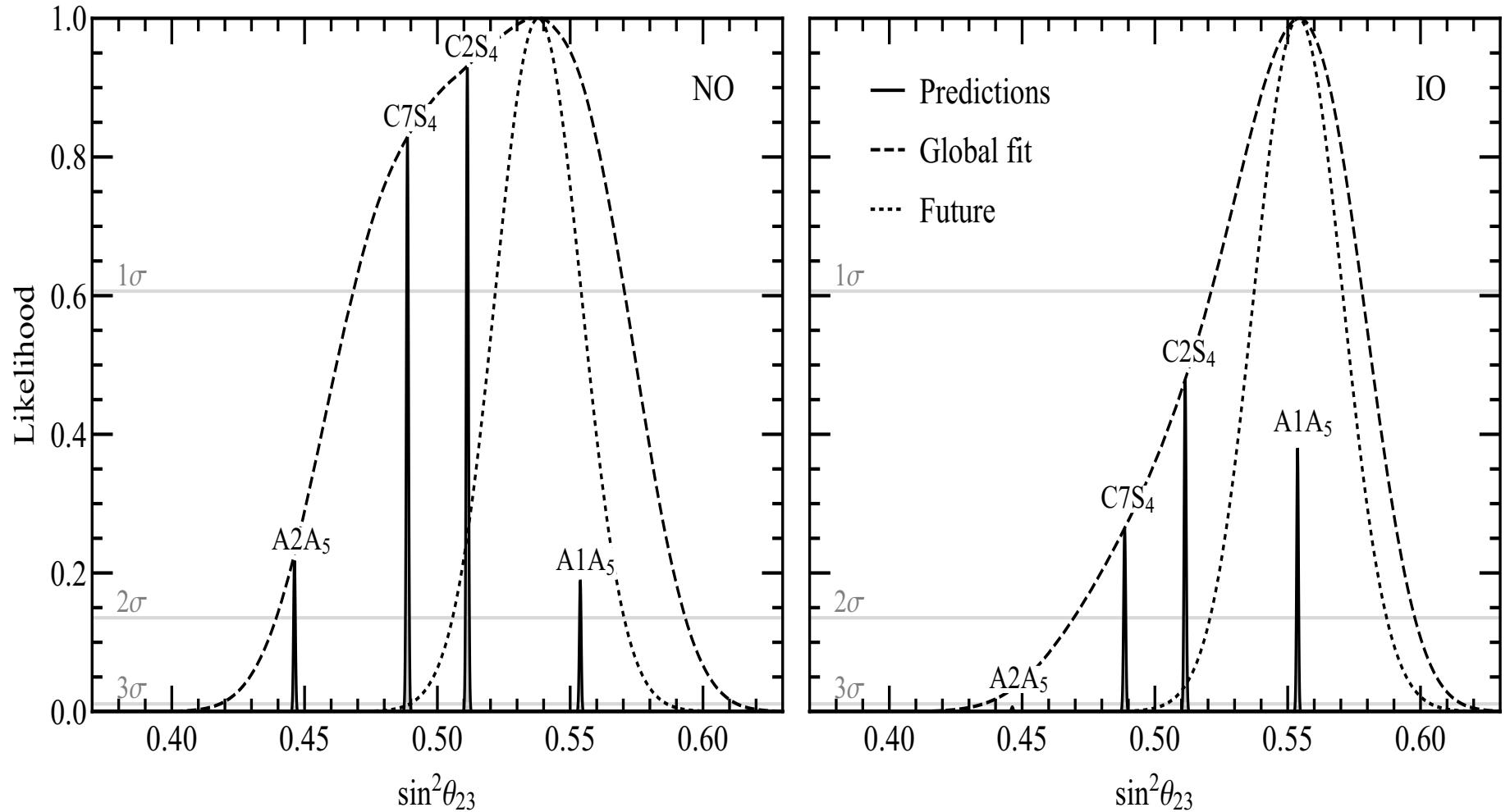
Phenomenologically Viable Predictions

A1 (A2), A_5 ($G_e = Z_2$, $G_\nu = Z_3$ (Dirac ν_j)): $\sin^2 \theta_{23} \cong 0.553$ (0.447); $\cos \delta \cong 0.716$ (-0.716).

A1, S_4 : $\sin^2 \theta_{23} \cong 0.5(1 - \sin^2 \theta_{13}) \cong 0.489$;
 $\cos \delta \cong -1$ requires $\sin^2 \theta_{12} \cong 0.348$ (!)

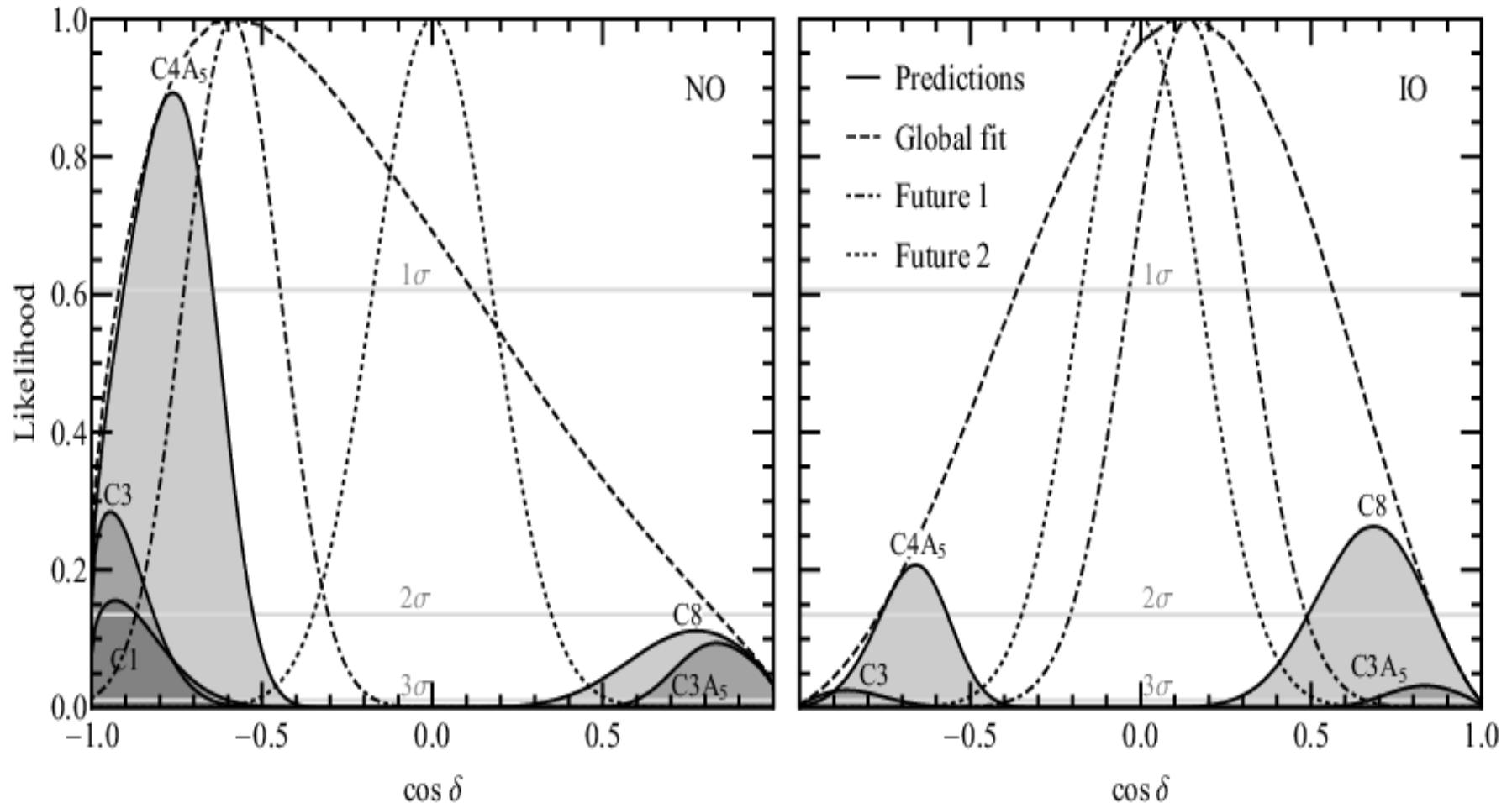
B1, A_4 (T' , S_4 , A_5) ($G_e = Z_3^T$, $G_\nu = Z_2^S$):
 $U_{\text{PMNS}} = U_{\text{TBM}} U_{13}(\theta_{13}^\nu, \delta_{13}) Q_0$;
 $\sin^2 \theta_{12} = 1/(3 \cos^2 \theta_{13}) \cong 0.340$; $\cos \delta \cong 0.570$.

B2, S_4 ($G_e = Z_3^T$, $G_\nu = Z_2^{SU}$):
 $\sin^2 \theta_{12} \cong (1 - 2 \sin^2 \theta_{13})/3 = 0.319$; $\cos \delta \cong -0.269$.



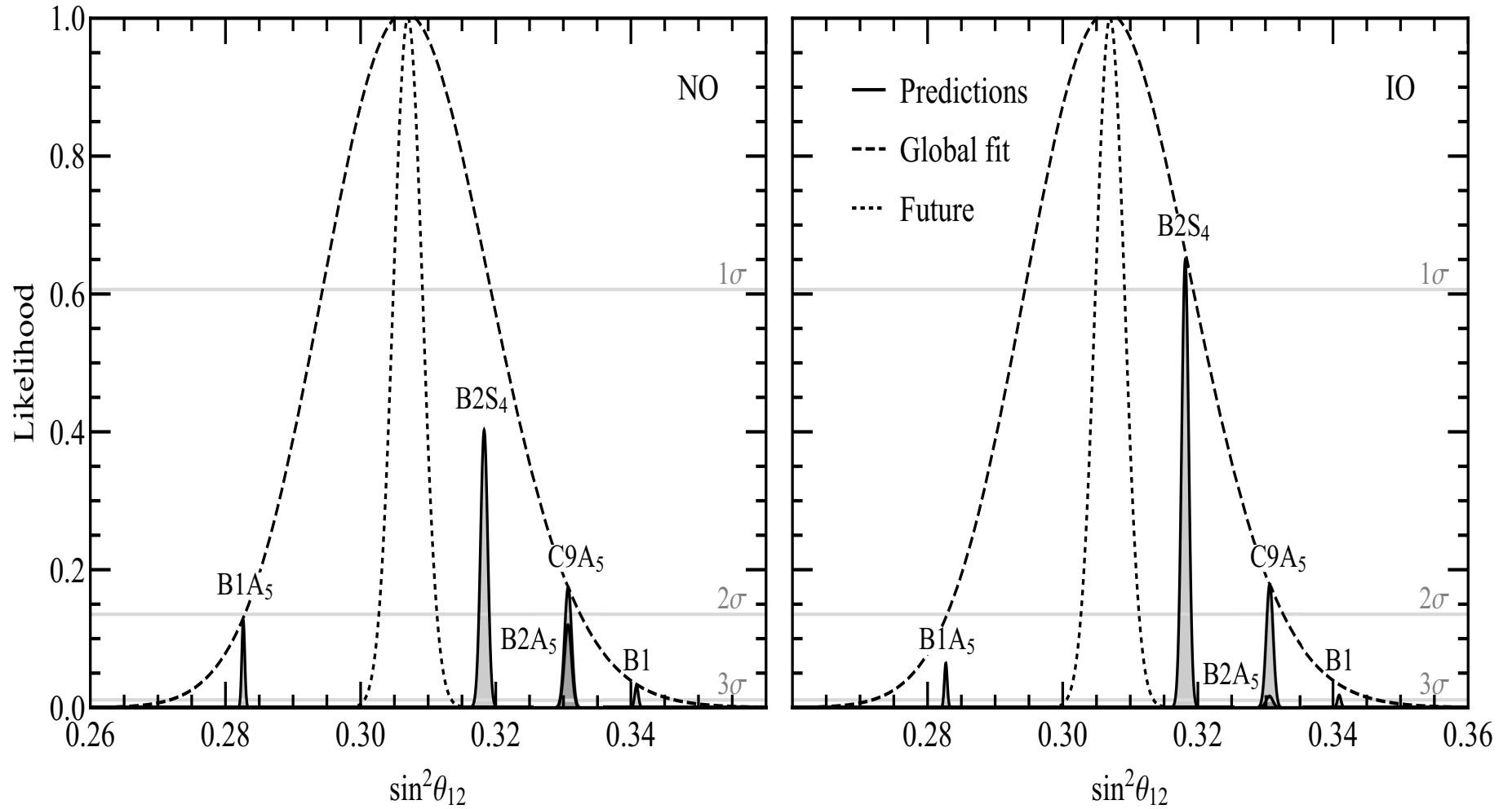
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2 \theta_{23}) = 3\% \text{ (T2HK, DUNE).}$



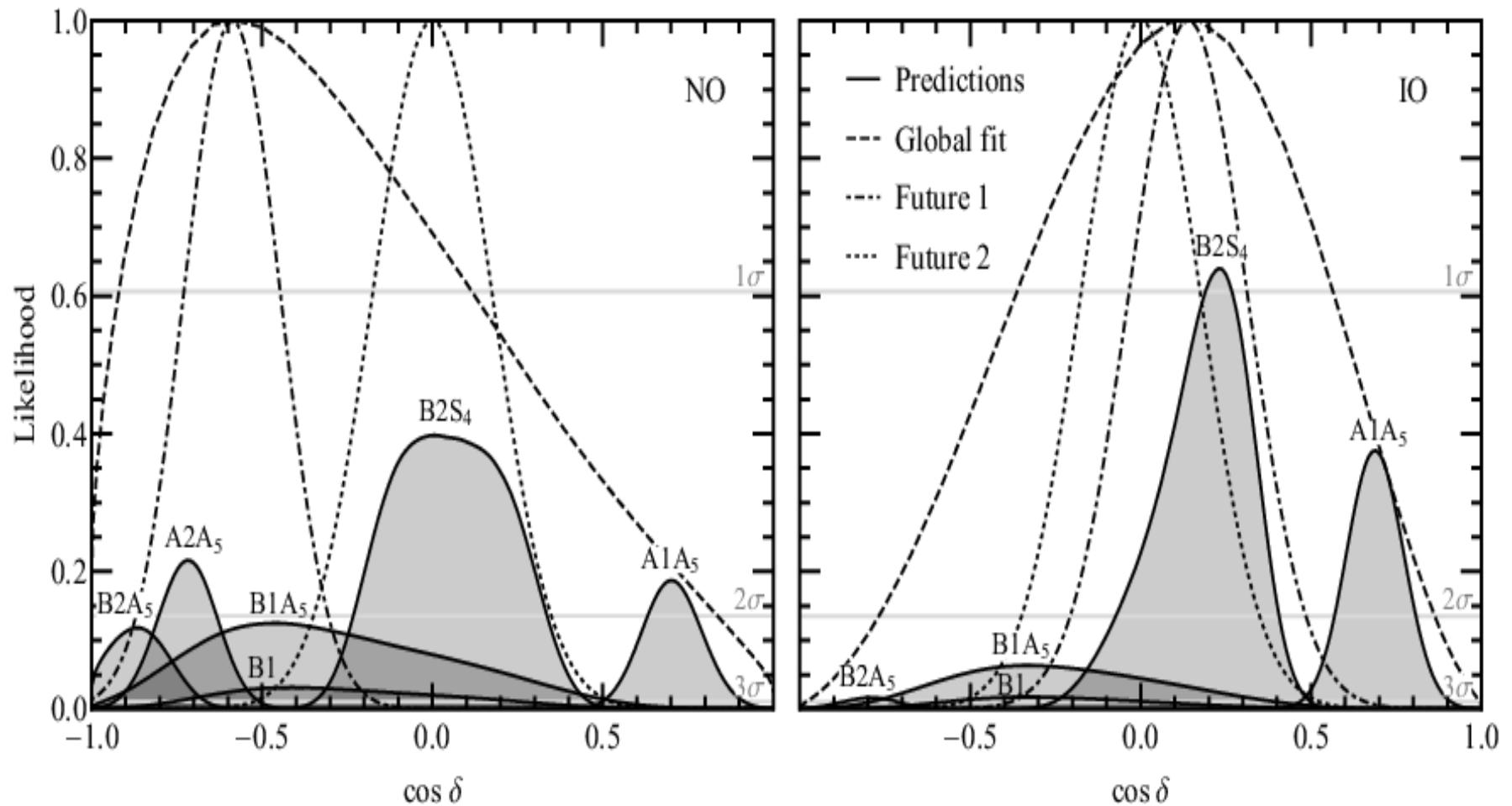
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\delta) = 10^\circ$.



S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2 \theta_{12}) = 0.7\%$ (JUNO).



S.T.P., A. Titov, arXiv:1804.00182

A total of 6 models would survive out of the currently viable 14 (of the extremely large number) considered if $\delta(\sin^2 \theta_{23}) = 3\%$, $\delta(\sin^2 \theta_{12}) = 0.7\%$ and the current b.f.v. would not change:

A1A₅, C2S₄, C3, C3A₅, C4A₅, C8.

Will be constrained further by the data on δ .

LEPTOGENESIS

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.

M. Gell-Mann, P. Ramond, R. Slansky, 1979;

T. Yanagida, 1979;

R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of ν -masses.
- Through **leptogenesis theory** links the ν -mass generation to the generation of baryon asymmetry of the Universe Y_B .
S. Fukugita, T. Yanagida, 1986; GUT's: M. Yoshimura, 1978.
- In SUSY GUT's with see-saw mechanism of ν -mass generation, the LFV decays

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma, \text{ etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}$ -decay is allowed.

See-Saw: Dirac ν -mass m_D + Majorana mass M_R for N_R

In GUTs, $M_{1,2,3} < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_{1,2,3} = (10^{11}, 10^{12}, 10^{13})$ GeV, $m_D \sim 1$ GeV.

TeV Scale Resonant Leptogenesis:

$M_{1,2,3} \sim (10^2 - 10^3)$ GeV (requires fine-tuning (severe)); observation of N_j at LHC - problematic (low production rates); observable LFV processes: $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu^- - e^-$ conversion (?).

GeV Scale ARS Leptogenesis:

$M_{1,2,3} \sim (1 - 50)$ GeV (requires fine-tuning (severe)); observation of N_j at LHC - problematic, signature: displaced vertices, requires dedicated experiment(s).

**Can the CP violation necessary for the generation
of the observed value of the Baryon Asymmetry of
the Universe (BAU) be provided exclusively by the
Dirac and/or Majorana CPV phases in the neutrino
PMNS matrix?**

Demonstrated in (incomplete list):

- S. Pascoli *et al.*, hep-ph/0609125 and hep-ph/0611338.
- E. Molinaro *et al.*, arXiv:0808.3534.
- A. Meroni *et al.*, arXiv:1203.4435.
- C. Hagedorn *et al.*, arXiv:0908.0240.
- J. Gehrlein *et al.*, arXiv:1502.00110 and arXiv:1508.07930.
- J. Zhang, Sh. Zhou, arXiv:1505.04858 (FGY 2002 model).
- P. Chen *et al.*, arXiv:1602.03873.
- C. Hegdorn, E. Molinaro, arXiv:1602.04206.
- P. Hernandez *et al.*, arXiv:1606.06719 and 1611.05000.
- M. Drewes *et al.*, arXiv:1609.09069.
- G. Bambahaniya *et al.*, arXiv:1611.03827.
- M. J. Dolan *et al.*, arXiv:1802.08373.
- K. Moffat *et al.*, arXiv:1804.05066.
- K. Moffat *et al.*, arXiv:1809.08251.
- I. Brivio et al, arXiv:1905.12642.

The Seesaw Lagrangian

$$\mathcal{L}^{\text{lept}}(x) = \mathcal{L}_{CC}(x) + \mathcal{L}_Y(x) + \mathcal{L}_M^N(x),$$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \bar{l}_L(x) \gamma_\alpha \nu_{lL}(x) W^{\alpha\dagger}(x) + h.c.,$$

$$\mathcal{L}_Y(x) = \lambda_{il} \bar{N}_{iR}(x) H^\dagger(x) \psi_{lL}(x) + Y_l H^c(x) \bar{l}_R(x) \psi_{lL}(x) + \text{h.c.},$$

$$\mathcal{L}_M^N(x) = -\frac{1}{2} M_i \bar{N}_i(x) N_i(x).$$

ψ_{lL} - **LH doublet**, $\psi_{lL}^\top = (\nu_{lL} \ l_L)$, l_R - **RH singlet**, H - **Higgs doublet**.

Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
 m_D generated by the Yukawa interaction:

$$-\mathcal{L}_Y^\nu = \lambda_{il} \bar{N}_{iR} H^\dagger(x) \psi_{lL}(x), \quad v = 174 \text{ GeV}, \quad v \lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2 \ \lambda^T D_N^{-1} \lambda = U_{\text{PMNS}}^* D_\nu U_{\text{PMNS}}^\dagger.$$

$$m_\nu \simeq v^2 \ \lambda^T D_N^{-1} \lambda = U_{\text{PMNS}}^* D_\nu U_{\text{PMNS}}^\dagger,$$

$$\lambda \equiv Y_\nu$$

$$Y_\nu \equiv \lambda = \sqrt{D_N} \ R \ \sqrt{D_\nu} \ (U_{\text{PMNS}})^\dagger / v_u, \text{ all at } M_R;$$

R -complex, $R^T R = 1$.

J.A. Casas and A. Ibarra, 2001

$$D_N \equiv \text{diag}(M_1, M_2, M_3), \ D_\nu \equiv \text{diag}(m_1, m_2, m_3).$$

Theories, Models:

- R - CP conserving ($SU(5) \times T'$, A. Meroni et al., arxiv:1203.4435; S_4 , P. Cheng et al., arXiv:1602.03873; C. Hagedorn, E. Molinaro, arXiv:1602.04206).
- CPV parameters in R determined by the CPV phases in U (e.g., class of A_4 theories).
- **Texture zeros in Y_ν :** CPV parameters in R determined by the CPV phases in U (Frampton, Glashow Yanagida (FGY), 2002: $N_{1,2}$, two texture zeros in Y_ν ; LG in FGY model: J. Zhang, Sh. Zhou, arXiv:1505.04858).

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.10 \pm 0.04) \times 10^{-10}, \quad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- B number non-conservation.
- Violation of C and CP symmetries.
- Deviation from thermal equilibrium.

Leptogenesis (via N_j Decays)

- The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.
- When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.
- If $\Gamma(N_1 \rightarrow \Phi^- \ell^+) \neq \Gamma(N_1 \rightarrow \Phi^+ \ell^-)$, a lepton asymmetry will be generated.
- Wash-out processes, like $\Phi^+ + \ell^- \rightarrow N_1$, $\ell^- + \Phi^+ \rightarrow \Phi^- + \ell^+$, etc. tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final result is a net (non-zero) lepton asymmetry.
- This lepton asymmetry is then converted into a baryon asymmetry by $(B + L)$ violating but $(B - L)$ conserving sphaleron processes which exist within the SM (at $T \gtrsim M_{\text{EWSB}}$) and are efficient at $T_{\text{EW}} \sim 140 \text{ GeV} < T < 10^{12} \text{ GeV}$.

S. Fukugita, T. Yanagida, 1986.

In order to compute Y_B :

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}$$

2. solve the Boltzmann (or similar) equation to account for the wash-out of the asymmetry:

$$Y_L = \kappa \varepsilon$$

where $\kappa = \kappa(\tilde{m})$ is the “efficiency factor”, \tilde{m} is the “the wash-out mass parameter” - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_B = -\frac{c_s}{g_*} \kappa \varepsilon, \quad c_s \cong 1/3, \quad g_* = 215/2$$

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \quad (n_\gamma: \sim 6.1 \times 10^{-10})$$

$$Y_B \cong -3 \times 10^{-3} \quad \varepsilon \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

κ — efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε : CP-, L- violating asymmetry generated in out of equilibrium N_{Rj} -decays in the early Universe,

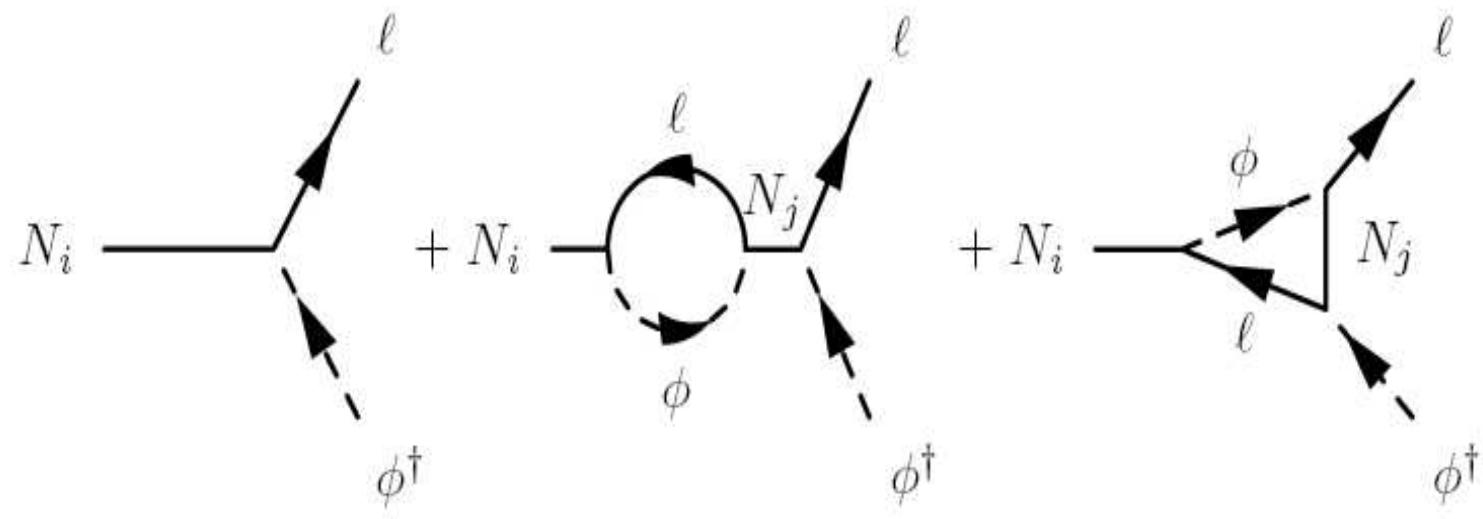
$$\varepsilon_1 = \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\tilde{m})$, \tilde{m} - determines the rate of wash-out processes:

$\Phi^+ + \ell^- N_1$, $\ell^- + \Phi^+ \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004



Low Energy Leptonic CPV and Leptogenesis

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (**NH**)

Dirac CP-violation

$\alpha_{32} = 0$ (2π), $\beta_{23} = \pi$ (0); $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}| \cong 0.86$, $|R_{13}|^2 = 1 - |R_{12}|^2$, $|R_{13}| \cong 0.51$ - **maximise** $|Y_B|$:

$$|Y_B| \cong 2.1 \times 10^{-13} |\sin \delta| \left(\frac{s_{13}}{0.15} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV **imply**

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.11, \quad \sin \theta_{13} \cong 0.15.$$

The lower limit corresponds to

$$|J_{CP}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0$ (2π), $\beta_{23} = 0$ (π):

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \sin \theta_{13} \cong 0.15; \quad |J_{CP}| \gtrsim 2.0 \times 10^{-2}$$

Realised in a theory based on the S_4 symmetry: P. Cheng et al.,
[arXiv:1602.03873](https://arxiv.org/abs/1602.03873).

The requirement $\sin \theta_{13} \gtrsim 0.09$ (0.11) - compatible with the Daya Bay, RENO, Double Chooz results: $\sin \theta_{13} \cong 0.15$.

$|\sin \theta_{13} \sin \delta| \gtrsim 0.11$ implies $|\sin \delta| \gtrsim 0.7$ - compatible with $\delta \cong 3\pi/2$.

$\sin \theta_{13} \cong 0.15$ and $\delta \cong 3\pi/2$ imply relatively large (**observable**) CPV effects in neutrino oscillations: $J_{CP} \cong -3.5 \times 10^{-2}$.

Conclusions.

- Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics.
- The observed pattern of neutrino mixing can be due to a new basic (approximate non-Abelian discrete) symmetry of particle interactions leading to an approximate symmetry form of the PMNS matrix.
- The most important testable consequence of the symmetry approach to understanding the pattern of neutrino mixing is the correlation between the values of some of the neutrino mixing angles and/or the value of $\cos\delta$ and the values of the neutrino mixing angles: $\delta = \delta(\theta_{12}, \theta_{13}, \theta_{23}; \theta_{12}^\nu)$. The second correlation depends on the underlying approximate symmetry form of the U_{PMNS} .

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12} , θ_{13} and θ_{23} , can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Conclusions (contd.)

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

Dirac and Majorana CPV may have the same source.

The see-saw mechanism provides a link between the ν -mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

These results underline further the importance of the experimental studies of Dirac (and searches for Majorana) leptonic CP-violation at low energies.