Phase diagram of QCD-like matter from exotic PNJL model

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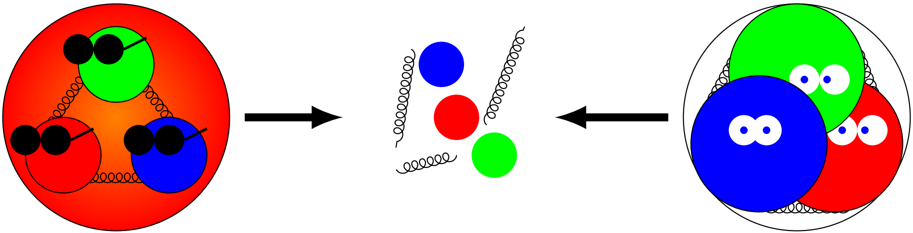
Two phases predicted for QCD matter :

- Hadronic phase :
  Quarks and gluons are bound into hadrons : confinement
  This is nuclear matter, we can observe it experimentally
- QGP phase :
  Quarks and gluons are free in the medium
  We don't directly observe this phase experimentally



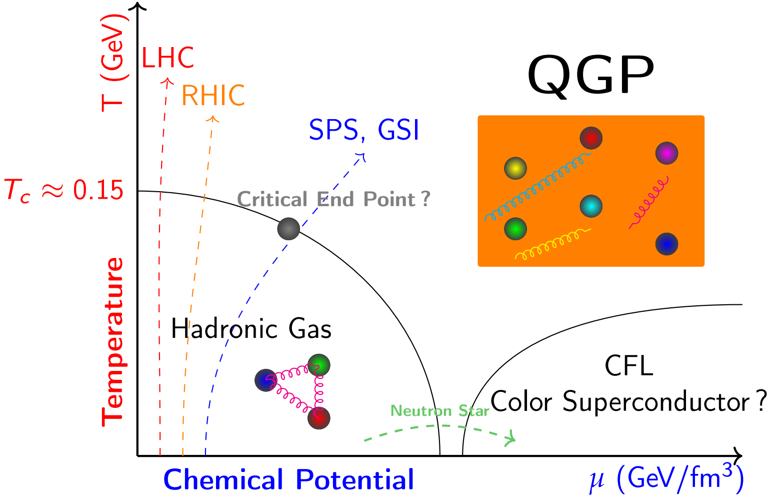


Figure – Phase Diagram of nuclear matter

QCD lagrangian : life is tough

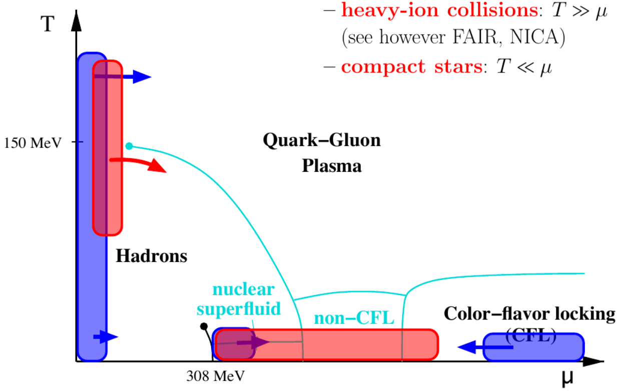
$$\mathscr{L}_{QCD} = i\delta_{ij}\bar{\psi}^i_k\gamma^\mu\partial_\mu\psi^j_k + g_s\bar{\psi}^i_k\gamma^\mu\lambda^a_{ij}A^a_\mu\psi^j_k - m_k\bar{\psi}^i_k\psi^j_k - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Perturbative approach pQCD

- Need of a small coupling constant for convergence of the perturbative series, works at high energy / high T, μ.
- not working at phase transition, the coupling constant is strong.

Lattice approach lQCD

- Space-time discretized on a lattice. Matter on the node, gluons are the lines connecting the nodes
- Static study, no dynamics on lattice, only thermodynamics
- Does not work at finite chemical potential, only at finite temperature.



- data
  - heavy-ion collisions: $T \gg \mu$ (see however FAIR, NICA)
  - compact stars: $T \ll \mu$

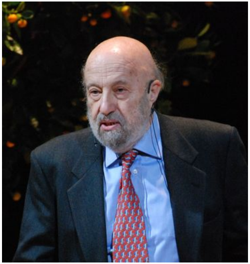
A. Schmitt from ect* summer school lectures

- GSI, FAIR
- NICA
- BES program (RHIC)
- SPS (CERN)

Lower temperature and higher density : search for critical end point, phase transitions and neutron star physics.

Needs prediction to know where to search. Those predictions can only be made using effective model

(P)NJL Model

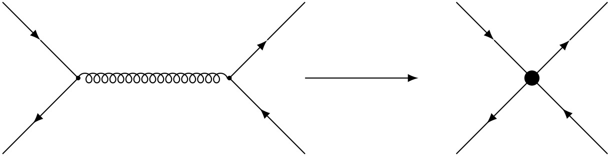


Effective model

Works only in a special domain of energy but allows finite chemical potential studies.

Contact interaction

Static approximation : no gluons propagating the interaction



Frozen gluons

$$\frac{1}{p^2 - \epsilon_g^2} = -\frac{1}{\epsilon_g^2}$$

$$\text{if } p << \epsilon_g^2$$

Nambu-Jona-Lasinio (NJL) Lagrangian

$$\mathscr{L}_{NJL} = \delta_{ij}\overline{\psi}^i_k(i\gamma^\mu\partial_\mu - m)\psi^j_k + G(\overline{\psi}^i_k\lambda_{ij}\psi^j_k)^2 + \text{'t Hooft term}$$

Symmetries

- Chiral symmetry $SU_L(3) \otimes SU_R(3)$
- Color symmetry $SU_c(3)$ (but global)
- Flavour symmetry $SU_f(3)$

Problem

Center symmetry is missing

Confinement is not described

Free parameters

$m^0_q = 0.0055\,GeV$

$m^0_s = 0.134\,GeV$

$\Lambda = 0.569\,GeV$

$G = \frac{2.3}{\Lambda^2}\,GeV^{-2}$

$K = \frac{11}{\Lambda^5}\,GeV^{-5}$

Polyakov loop

Confinement is taken into consideration using an effective potential $U(\phi, \bar{\phi}, T)$, function of the Polyakov loop ϕ.

Polyakov extended NJL Lagrangian

$$\mathscr{L}_{PNJL} = \overline{\psi}_k(i\not{D}_\mu - m)\psi_k + G(\overline{\psi}_k\lambda_i\psi_k)^2 + \text{'t Hooft} - U(\phi, \bar{\phi}, T)$$

Static gluon field

Covariant derivative : $D_\mu = \partial_\mu - iA_\mu$ and $A_\mu = \delta^0_\mu A_0$ (Polyakov gauge).

The Polyakov loop field is $\phi = \frac{1}{N_c} Tr << L >>$

Polyakov line $L = \mathscr{P} \exp(i \int_0^\beta dt A_4)$ with $A_0 = -iA_4$

Imaginary time Wilson line. No spatial componants, no dynamical gluons.
PNJL = Frozen gluons + Thermal gluons.

Still no gluons in the interaction

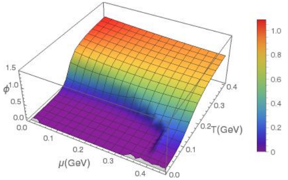
$\mathrm{U}(\phi, \bar{\phi}, T)$ is a mean field in which quarks propagate and give a pressure to the medium. It corresponds to the thermodynamic of the $\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu}$ term in the QCD lagrangian.
The parameters are determined by fitting with the P_{YM} of lQCD.

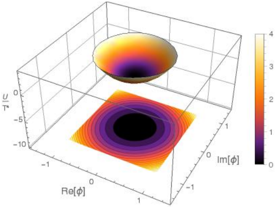
$$\frac{U(\phi,\bar{\phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{6}(\bar{\phi}^3 + \phi^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2$$

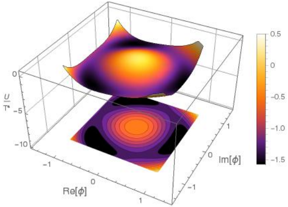
with the parameters : $b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3$

a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV

- T_0, critical temperature of center symmetry breaking.
- Below T_0 ,the Polyakov loop is 0. The center symmetry is not broken. The quarks are confined.
- Above T_0, the Polyakov loop is not zero. The center symmetry is broken. The quarks are deconfined.







- Modified quarks distributions :

$$f_\phi(E_i-\mu_i)=\frac{(\phi+2\bar{\phi}\exp(\frac{-E_i-\mu_i}{T}))\exp(-\frac{E_i-\mu_i}{T})+\exp(-3\frac{E_i-\mu_i}{T})}{1+3\phi\exp(-\frac{E_i-\mu_i}{T})+3\bar{\phi}\exp(-2\frac{E_i-\mu_i}{T})+\exp(-3\frac{E_i-\mu_i}{T})}$$

- For $\phi=\bar{\phi}=0$, $E_N=3E, \mu_N=3\mu$
- Leads to quarks suppression below T_c :
  "poor man's nucleon" = statistical confinement.
- For $\phi=\bar{\phi}=1$, deconfinement : NJL limit.

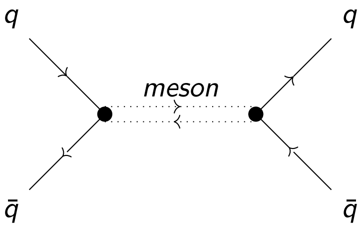
Hubert Hansen, https ://www.brown.edu/conference/12th-workshop-non-perturbative-quantum-chromodynamics/sites/files/uploads/Hansen.pdf

Hadrons and hadronic matter in chiral quarks model, David Blaschke, Dubna 2011

From quarks to hadrons : mesons

Quark-antiquark bound states

In NJL, degrees of freedom are quarks. Mesons need to be build from quark-antiquarks bound states



Amplitude

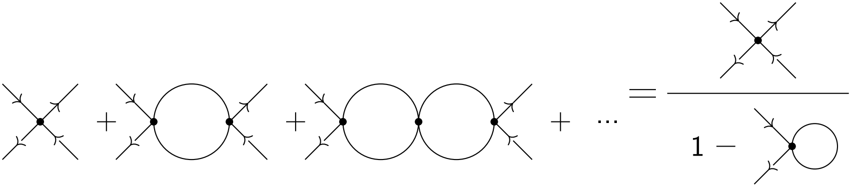
$$iU(k^2) = \Gamma \frac{-ig_m^2}{k^2 - m^2} \Gamma$$

Mesons masses

The mass is given by the poles : $m = k$

Bethe-Salpeter equation

$$iU(k^2) = \Gamma \frac{2ig_m}{1-2g_m\Pi(k^2)}\Gamma$$



Mesons masses

By analogy, the mass is given by the poles :

$$1 - 2G\Pi(k^2 = m^2) = 0$$

Limitations of the model

Good things

- ✓ Lagrangian which shares roughly the symmetries of the QCD lagrangian
- ✓ Works at finite density and in the phase transition region
- ✓ Degrees of freedom = quarks but nuclear matter made from bound states

Bad things

- Dynamical gluons do not participate in the interaction : low energy approximation.
- 4-point interactions are non renormalizable : need of a cut-off.

Equation of State

Partition function

As always in statistical physics, we need the partition function :

$$Z[\bar{q}, q] = \int \mathscr{D}_{\bar{q}} \mathscr{D}_q \left\{ \int_0^\beta d\tau \int_V d^3x \mathscr{L}_{NJL} \right\}$$

Grand potential

Using the bosonisation procedure, we obtain the mean field partition function :

$$Z[\bar{q}, q] = \exp\left\{ - \int_0^\beta d\tau \int_V \frac{\sigma_{MF}^2}{4G} + Tr \ln S_{MF}^{-1} \right\}$$

$$\Omega_{NJL}(T, \mu) = -\frac{T}{V} \ln Z[\bar{q}, q]$$

NJL grand potential

$$\Omega_{NJL} = -2\int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} E_p$$

$$+2T\int_0^{\infty}(\ln[1+\exp(-\beta(E_p-\mu))] + \ln[1+\exp(-\beta(E_p+\mu))]$$

$$+2G\sum_k <\bar{\psi}_k\psi_k>^2 -4K\Pi_i <\bar{\psi}_k\psi_j>)$$

PNJL grand potential

$$\Omega_{PNJL} = -2\int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} E_p$$

$$+2T\int_0^{\infty}(\ln[1+L^{\dagger}\exp(-\beta(E_p-\mu))] + \ln[1+L\exp(-\beta(E_p+\mu))]$$

$$+2G\sum_k <\bar{\psi}_k\psi_k>^2 -4K\Pi_i <\bar{\psi}_k\psi_j> +U_{PNJL})$$

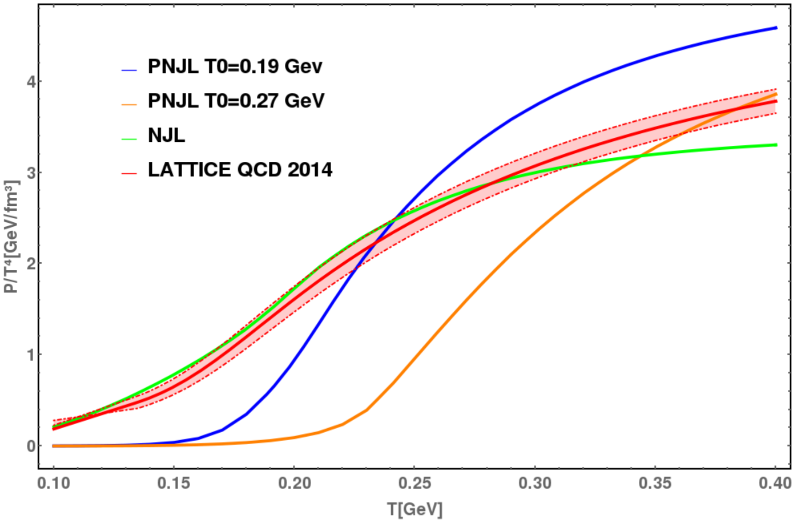
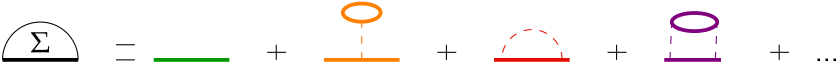


Figure from 3^{rd} student Laurence Pied '**t Hooft scaling** : $g\bar{\psi}A_\mu\psi \to gN_c\bar{\psi}\frac{A_\mu}{N_c}\psi$ with $gN_c = cst$

$$g^{2l}N_c^k \equiv (gN_c)^{2l}{N_c}^{k-2l}$$

k is the number of fermion lines and l is the number of interaction lines.

$$iS_\Sigma(p) = iS(p)\left(\boxed{O(1)O(N_c)} + \boxed{O((gN_c)^2)O(1)} + \boxed{O((gN_c)^2)O(\frac{1}{N_c})} + \boxed{O((gN_c)^4)O(\frac{1}{N_c})} + \ldots\right)$$



- The grand potential associated to this last diagram is :

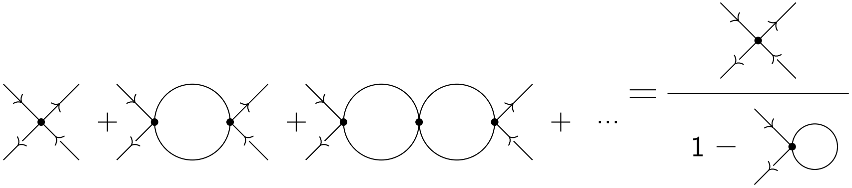
$$\Omega_M^{(0)} = \frac{g_M}{2} \int \frac{d^3p}{(2\pi)^3} \int_0^\infty d\omega \left(1 + \frac{1}{\exp(\beta(\omega-\mu_M))-1} + \frac{1}{\exp(\beta(\omega+\mu_M))-1}\right) \times \ln\left[\frac{1-2G\Pi(\omega-\mu_M+i\epsilon,p)}{1-2G\Pi(\omega-\mu_M-i\epsilon,p)}\right]$$

E. Quack and S. P. Klevansky, PRC, 49, 6 (1994)

Torres Rincon J., Aichelin J., PRC, 96, 0425205 (2017)

Bethe-Salpeter equation

$$iU(k^2) = \Gamma \frac{2ig_m}{1-2g_m\Pi(k^2)}\Gamma$$



Mesons masses

By analogy, the mass is given by the poles :

$$1 - 2G\Pi(k^2 = m^2) = 0$$

Beth-Uhlenbeck approach

- Express the 2nd virial coefficient of the Kamerlingh-Onnes equation of state for non ideal gas in terms of two body scattering phase shift.

- The same analogy can be done here. We use the S matrix, connecting in and out state, to determine the expression of the phase shift.

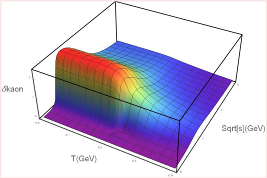
Mesonic grand potential

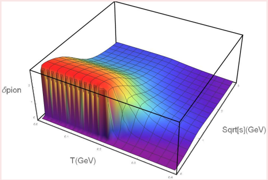
$$\Omega_M = -\frac{g_M}{8\pi^3}\int dp p^2 \int \frac{ds}{\sqrt{s+p^2}}\left[\frac{1}{\exp(\beta(\sqrt{s+p^2}-\mu)-1)}+\frac{1}{\exp(\beta(\sqrt{s+p^2}+\mu)-1)}\right]\delta_M$$

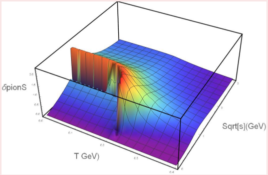
Phase shift : the physics

The phase shift depends on the mesons masses

$$\delta_M = -Arg[1-2K_M\Pi_M]$$







a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV

Traditional PNJL - Before

One of the parameter is $T_0 = 270 MeV$, the critical temperature for confinement.
This is the pure Yang-Mills critical temperature.

Quarks are here too ! - Better

Slight change in the critical temperature. We use the reduced temperature to quantify it. *https ://arxiv.org/abs/1302.1993, Haas and al.*

$T^{eff} = \frac{T-T_c}{T_c} \rightarrow T^{eff}_{YM} \simeq 0.57\, T^{eff}_{rs}$

This rescale the critical temperature to $T_0 = 190 MeV$

Different quark-gluons interaction

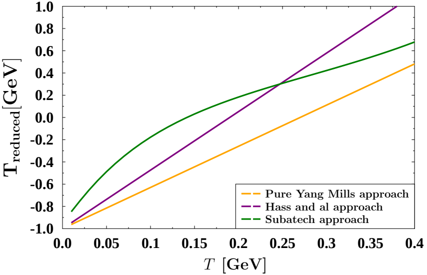
We include a temperature dependance in the rescaling :

$$\tau = 0.57\frac{T - T_0(T)}{T_0(T)}$$

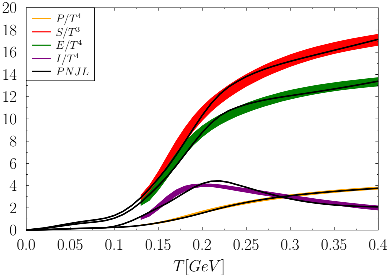
where : $T_0 = a + bT + cT^2 + dT^3 + e\frac{1}{T}$

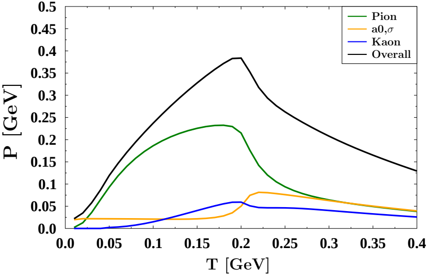
and : $b_2(T) = a_0 + \frac{a_1}{1+\tau} + \frac{a_2}{(1+\tau)^2} + \frac{a_3}{(1+\tau)^3}$

a	b	c	d	e
0.082	0.36	0.72	-1.6	-0.0002



Equation of state at zero μ



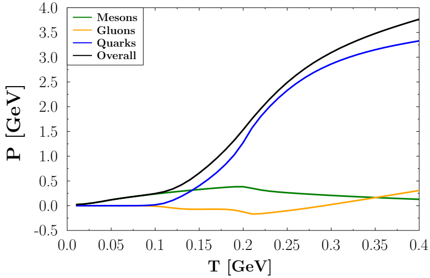


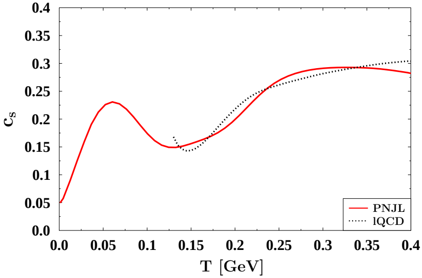
https ://arxiv.org/abs/1407.6387v2, HotQCD Collaboration

We reproduce lattice results at 0 μ

We have an effective model based on a lagrangian that shares QCD symmetry and match lattice results.
This is an effective theory, no sign problem, we can expand to finite chemical potential.

Equation of state at zero μ





Mesonic contributions to the pressure

As expected, Mesons have significant contribution at low temperature.

Critical temperature

Minimum of speed of sound : localisation of the cross over region.

Lattice at finite μ

Lattice can perform Taylor expansion at zero chemical potential.

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \ldots$$

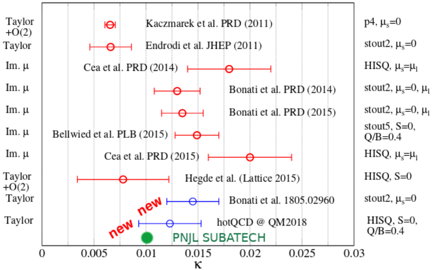
The κ coefficient is the second order derivative of our function :

$$\kappa = \left. \frac{\partial^2 \frac{T_c(\mu_B)}{T_c(0)}}{\partial \mu_B^2} \right|_{\mu_B = 0}$$

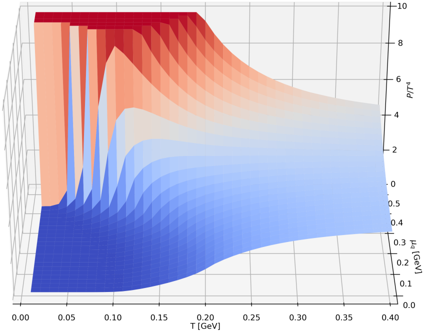
"On the critical line of 2+1 flavor QCD" Cea, Cosmai,Papa

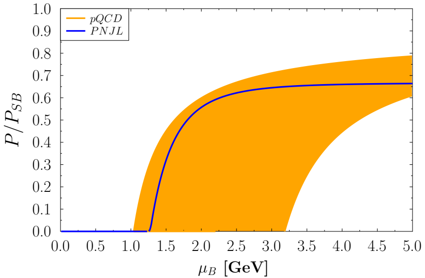
Our critical temperature

At $\mu_B = 0$, we get the critical temperature : $T_c = 138 MeV$



Aleksi Kurkela and Aleksi Vuorinen, Cool quark matter, Phys. Rev. Lett. 11

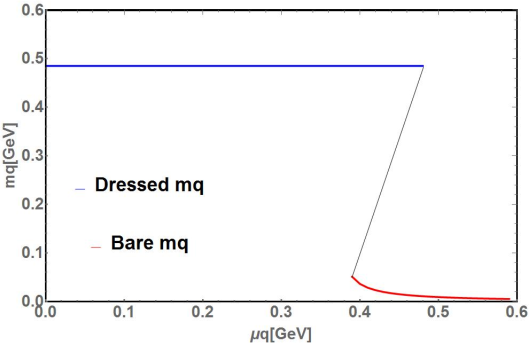




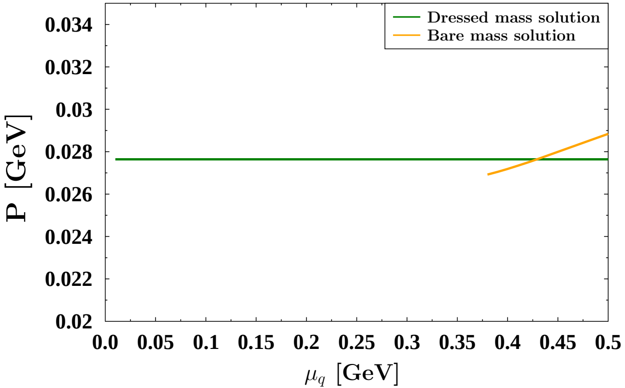
Large μ comparison

- Match pQCD predictions at large μ

- To determine the critical chemical potential, we first calculate the two solutions for bare and dressed quarks mass.
- Region with three solutions, meaning that we have a first order transition



To determine precisely the value of μ_{crit}, we use the same process but for the grand potential.



Critical chemical potential

The value obtained is $\mu_q = 0.425$ GeV for T=0.

Critical End Point

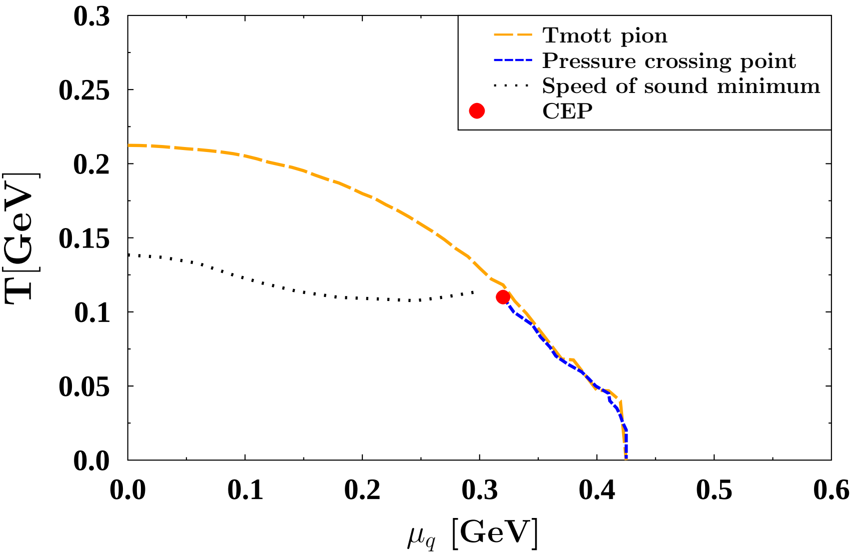
$$g_u(\mu, T, mq, ms, \phi, \bar{\phi}) = 0 \quad g_s(\mu, T, mq, ms, \phi, \bar{\phi}) = 0$$

$$\frac{\partial \Omega_{PNJL}(\mu, T, mq, ms, \phi, \bar{\phi})}{\partial \phi} = 0 \quad \frac{\partial \Omega_{PNJL}(\mu, T, mq, ms, \phi, \bar{\phi})}{\partial \bar{\phi}} = 0$$

$$\frac{\partial \mu}{\partial mq} = 0 \quad \frac{\partial^2 \mu}{\partial mq^2} = 0$$

The solution obtained has the coordinates :
$(T_{CEP} = 0.11 GeV, \mu_{CEP} = 0.32 GeV)$.

Alexandre Biguet, PhD thesis, https ://tel.archives-ouvertes.fr/tel-01453184/document


Tmott pion
Pressure crossing point
Speed of sound minimum
CEP
T[GeV]
μ_q [GeV]
0.0
0.05
0.1
0.15
0.2
0.25
0.3
0.0
0.1
0.2
0.3
0.4
0.5
0.6

Conclusion :

- PNJL : effective model to study the phase diagram at finite μ.

PNJL + T0(T) + Pressure beyond mean field (mesons)

=

- ✓ Lattice equation of state at $\mu = 0$.
- ✓ Lattice equation of state at $\mu \simeq 0$.
- ✓ PQCD results for pressure at large μ
- ✓ Cross over transition for T (speed of sound, Tmott)
- ✓ First order transition localized at $\mu = 0.425$ GeV at T = 0
- ✓ Critical End Point coordinates : $(T_{CEP} = 0.11 GeV, \mu_{CEP} = 0.32 GeV)$
- ✓ Phase diagram of QCD matter

What's next for thermodynamics ?

- Pressure beyond mean field, but chiral condensate calculated in mean field. *E. Quack and S. P. Klevansk, Phys rev C, V49, nb6 (1994)*

  O(10%) and 16% for the masses

- Apply our equation of state to event generators.

- Apply our equation of state to Neutron Star description.

Thank you for your attention ! !

Sign problem

- Partition function : $Z = \int \mathscr{D}_U \mathscr{D}_{\bar{\psi}} \mathscr{D}_\psi \exp(-S)$
- With the action :
  $S = \int d^4x \bar{\psi}(\gamma_\nu(\partial_\nu + iA_\nu) + \mu\gamma_4 + m)\psi = \int d^4x \bar{\psi} M \psi$
- μ appears as an A_4 imaginary quadrivector and :
  $M = \gamma_\nu \partial_\nu + i\gamma_\nu A_\nu + \mu\gamma_4 + m$
- We then have :
  $M^\dagger(\mu) = M(-\mu^*)$
- The action is now complex. It can be seen using the hermiticity of the γ_5 matrix. M hermiticity valide at $\mu = 0$ and but not for finite μ.

$U_A(1)$ anomaly

- Classical action invariant $\rightarrow$ symmetry.
- Quantum action not invariant $\rightarrow$ symmetry broken.
- Symmetry broken by quantum fluctuation : Anomalies !

S matrix

$S(p, E) = \exp(2i\delta(\vec{p}, E)) = \frac{F_J(\vec{k}, E^*)}{F_J(\vec{k}, E)}$

The zeroes of the Jost function are the poles of the S-matrix.

- S-matrix has a pole at k $= +\mathrm{i}\kappa$: Bound states have exponentially decaying solutions.
- Poles in the lower half plane can be written as k = -i$\kappa + \gamma$
  - γ= 0, resonances
  - γ = 0, antibound or virtual states.