Stark effect

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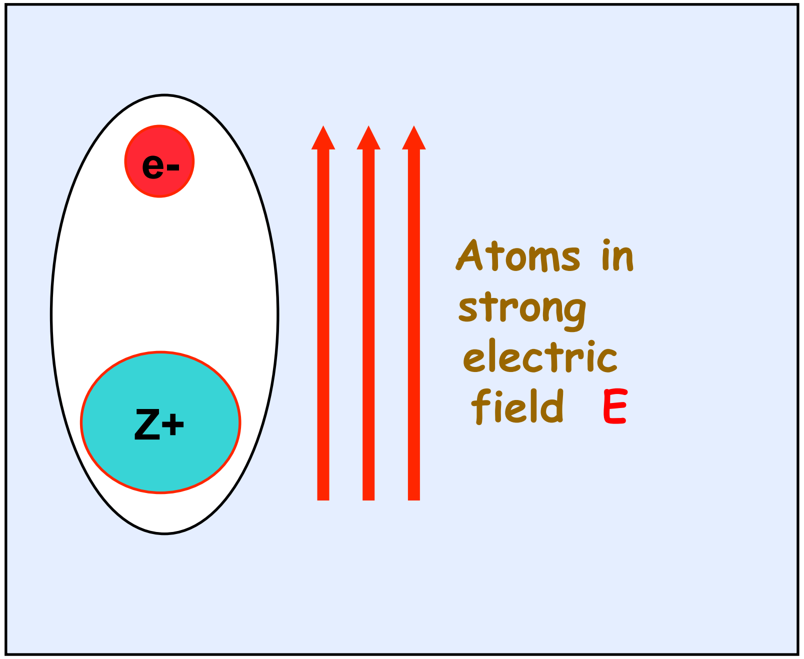
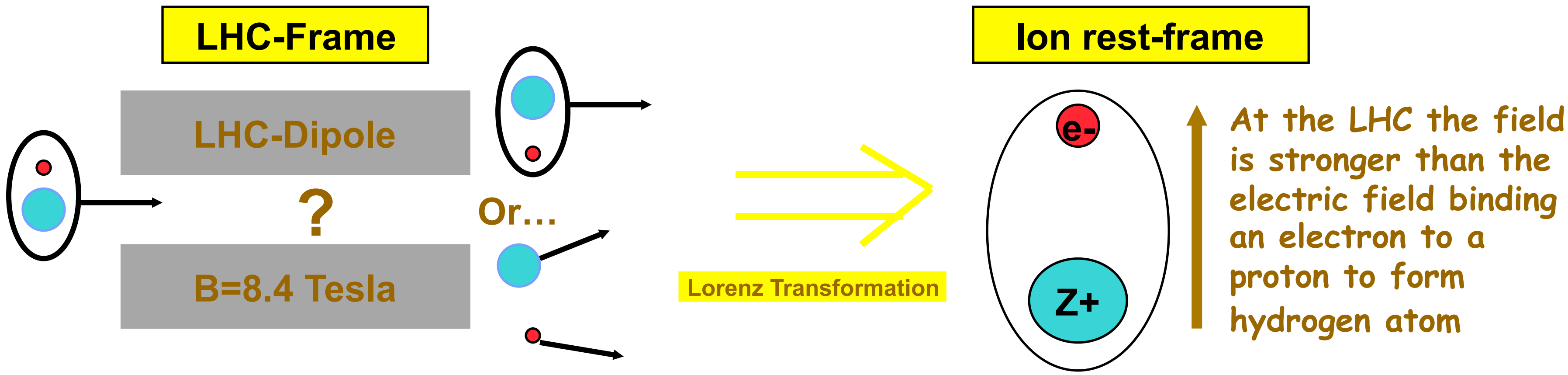


Survival of PSIs in the LHC ring



$$\vec{E} = \gamma \times \vec{\beta} \times c \times \vec{B}$$

The dipole field of the LHC storage ring of 8.4 Tesla generates -- for beam particles, moving with velocity corresponding to the Lorenz-factor γ_L of 3000 -- the electric field of a strength: $E = 7.3 \ 10^{10}$ V/cm.

NB. The B field of the SPS bending magnets is by a factor of 4, and the Lorenz- by a factor of 15 smaller. The electric field strength at the SPS is thus lower by a factor of 60!

Schrödinger equation

$$V = -\frac{Ze^2}{(4\pi\varepsilon_0)r} + e\mathcal{E}z$$

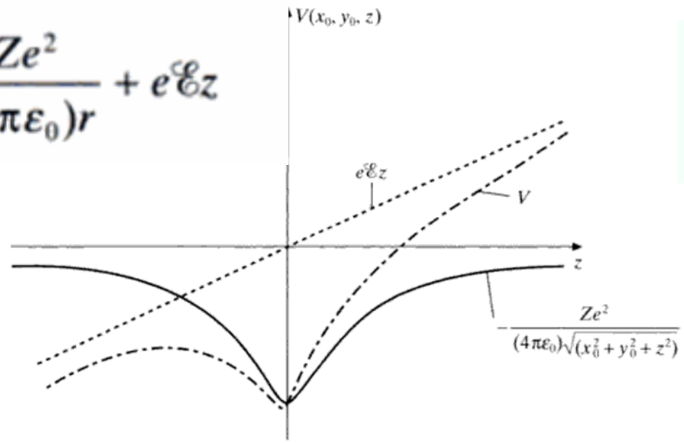
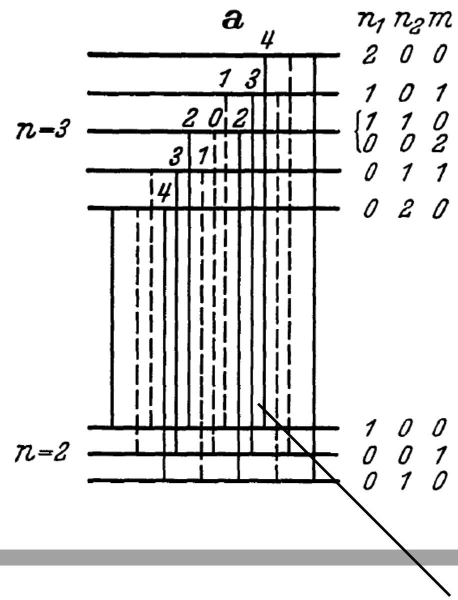


Figure 6.3 The potential V experienced by an electron interacting with a nucleus of charge Ze, in a uniform electric field of strength $\mathcal{E}$, as a function of z, for $x = x_0$ and $y = y_0$ fixed.

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$



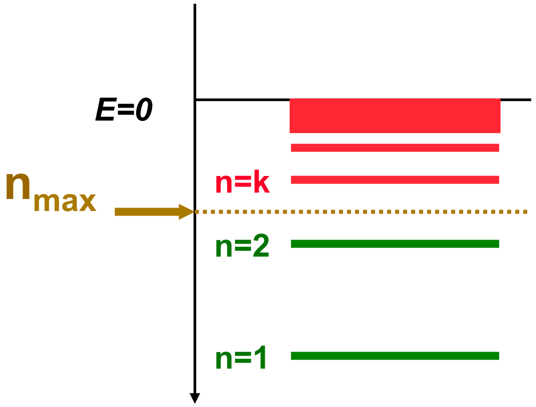
Solution (linear approximation), P.S. Epstein, Anlagen der Physik Vol 50 489 (1916)

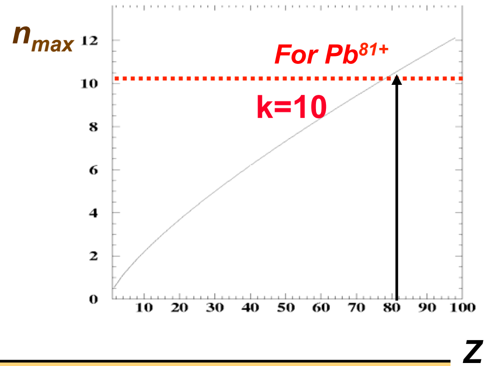
$$\text{Energy} = \left[-\frac{1.097\times10^5}{n^2}Z^2 + \frac{E}{15620}\,\frac{n}{Z}\,(n_1 - n_2)\right]\mathrm{cm}^{-1}.$$

Ionization condition: Energy > 0

Binding energy of hydrogen-like atoms

Ionization of hydrogen-like atoms



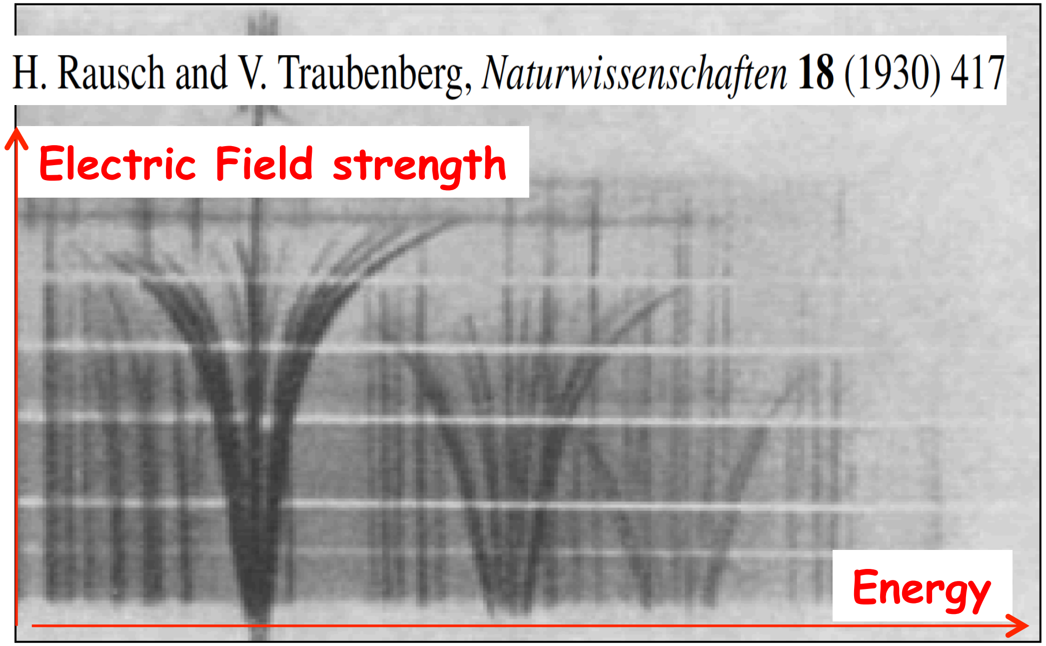


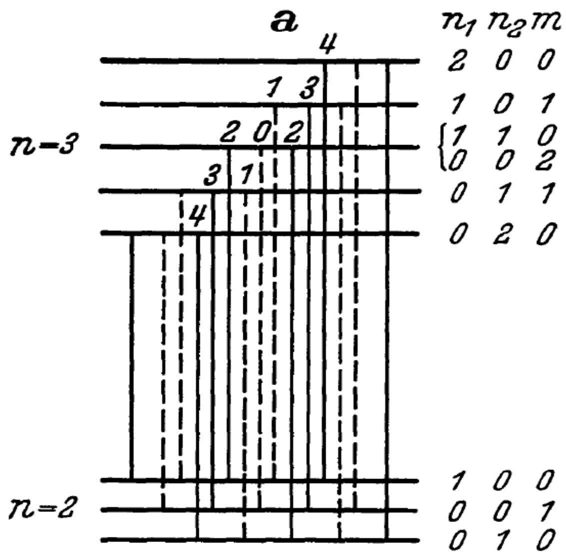
Result: For heavy ions only highly excited atomic states can be ionized (safe beams).

Light ions up to the beryllium can be stored in the LHC ring only in the fully stripped state. For e.g. oxygen only hydrogen-like and helium-like ions can be stored at the LHC.

…tunneling effects ???

The first observation of field ionisation (importance of tunnelling effects)

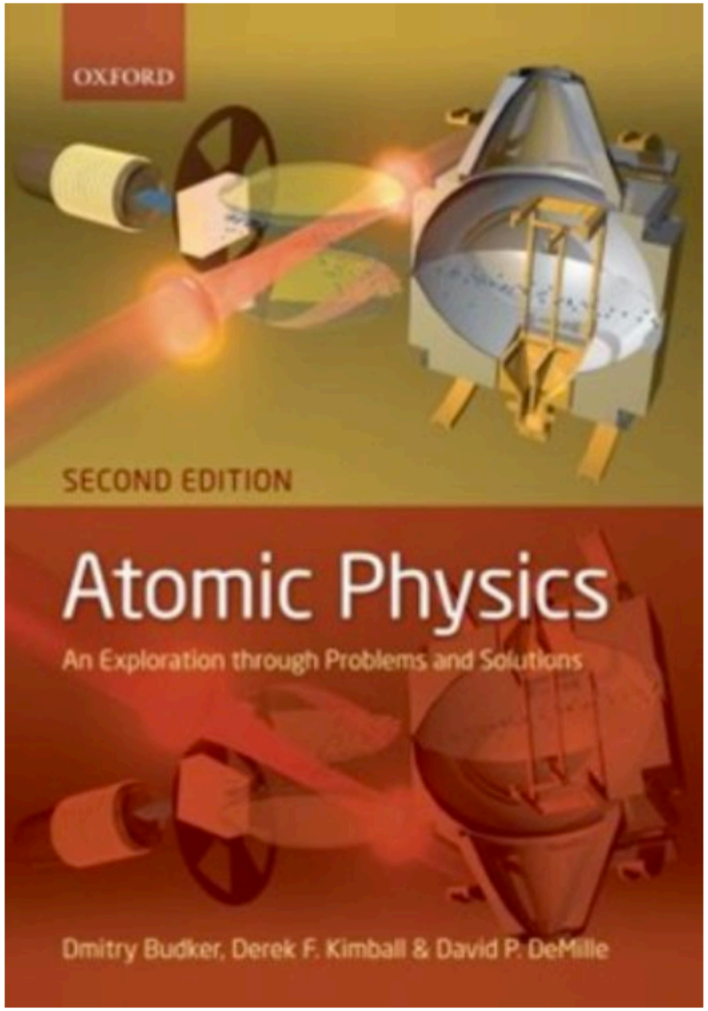




It turn out (C. Lanczos, Z. Phys. 62 (1930) 518, C. Lanczos, Z. Phys. 68 (1931) 204) that the lightest ion with negligible field ionization losses in the LHC is the hydrogen (helium)-like oxygen ion. All the partially stripped ion beam particles heavier than oxygen can also be safely stored in the LHC in the hydrogen(helium)-like charge state.

This conclusion has been cross-checked using the formulae presented --for hydrogen-like ions

Field ionisation for Hydrogen-like ions



Probability per unit time of field ionisation
For hydrogen-like ions in the ground state:

$$W = \frac{4 m_e^3 e^9 Z^5}{\mathcal{E}\hbar^7} \exp\left(-\frac{2 m_e^2 e^5 Z^3}{3\mathcal{E}\hbar^4}\right).$$

Slightly less stringent constraint: the lightest ion with negligible field ionization losses in the LHC is the hydrogen-like carbon ion

Summary

- *All the partially stripped ions heavier than oxygen can be safely stored in the LHC in the hydrogen and helium like states.*
- *The above conclusions holds for un-bunched beams and for a perfect vacuum in the LHC storage rings.*
- *The excitation of atomic levels of the beam particles in intra-beam collisions of the beam particles and in their collisions with the residual gas in the LHC beam pipe may promote the electrons to higher energy levels and the field ionization may potentially become important.*
- *Such a potential mechanism --even if expected to be negligible -- remains to be verified experimentally for the realistic achievable values of the bunch density of partially stripped and the measured concentration of the gas molecules in the LHC storage ring – the losses will have a very singular dependence upon the energy of the stored beam.*

Next order (quadratic) corrections to linear approximation

…typically below 10 % for small Z and n

22 cm^{-1}.

The perturbation expansion was carried through by ISHIDA and HIYAMA[1] up to terms of third order in the field strength F. Their result is

$$E_3 = E_2 + \frac{3}{32} F^3 \left(\frac{n}{Z}\right)^7 (n_1 - n_2)\left(23 n^2 - (n_1 - n_2)^2 + 11 m^2 + 39\right). \quad (52.4)$$

[1] Y. ISHIDA and S. HIYAMA: Sci. Pap. Inst. phys. and chem. Res., Tokyo **1928**, Nr. 152.