Average model
fast simulation of gamma factory PoP@SPS

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Boundary conditions

Heavily inspired from Alexey spreadsheets especially for

- proper accounting of saturation effect
- purely longitudinal dynamics

Excitation rate of each ion in the 6D phase space is split into 2 contributions

- Luminosity-like contribution related to the spatio-temporal average of laser intensity as seen by the considered ion
- Cross-section like contribution related to the spectal average of laser intensity as seen by the considered ion – accounting for its linewidth

Product of these must account for saturation of the excitation probability

Further assume:

- No phase-space correlations…
- Simplifying density distributions
- Lithium-like Lead
- g1=g2

Spectral contribution

$$\bar{\sigma} = \int_{E=0}^{+\infty} \int_{\nu=0}^{+\infty} \int_{\gamma=0}^{+\infty} \sigma(E)\,\delta(E - h\nu\gamma(1+\beta\cos\theta))\,f(\nu)\,g(\gamma)\,dE\,dh\nu\,d\gamma.$$

Resonance cross-section — Laser-beam spectrum — Ion-beam spectrum

Some (semi-)analytical expressions are available with gaussian ion beam spectrum and flat laser-beam spectrum

Center ion-beam energy/Mic2

$$\bar{\sigma} \approx \sigma_{\mathrm{max}} \frac{\Gamma\pi}{4} \frac{\lambda_0^2}{hc\Delta\lambda\gamma_i(1+\beta_i\cos\theta)} \left(\mathrm{erf}\left(\frac{E_{\mathrm{i}}(\gamma_+ - \gamma_{\mathrm{i}})}{\gamma_i \Delta E\sqrt{2}} \right) - \mathrm{erf}\left(\frac{E_{\mathrm{i}}(\gamma_- - \gamma_{\mathrm{i}})}{\gamma_i \Delta E\sqrt{2}} \right) \right)$$

Ion-beam energy spread

$$\gamma_{\pm} = \frac{E_{\mathrm{t}} + \sqrt{E_{\mathrm{t}}^2 + 2(h\nu_{\mp})^2\cos\theta(1+\cos\theta)}}{2h\nu_{\mp}(1+\cos\theta)}$$

$$\nu_{\mp} = c/(\lambda_0 \pm \Delta\lambda/2).$$

This is the average « cross section » over the ion beam spectrum if the laser beam spectrum were continuous…

Single ion interaction model

But laser-beam spectrum is not continuous ! → Comb spectrum

$$f(\nu) = \sum_{n=N_0-N}^{N_0+N} \tilde{f}(\nu) \frac{1}{2\pi} \frac{\Delta\nu}{(\nu - n f_{\text{rep.}} - f_{\text{ceo}})^2 + \Delta\nu^2/4}$$

$$\tilde{f}(\nu) = \frac{1}{2N+1} \simeq \frac{\lambda_0^2}{\Delta\lambda} \frac{f_{\text{rep.}}}{c}.$$

Constant enveloppe distribution
→ Any other distribution easy to implement

Linewidth of the each comb's peak is about 1kHz (determined by laser and optical cavity)

$$2\gamma h \Delta\nu \simeq 8.3 \cdot 10^{-10}\ \text{eV}$$

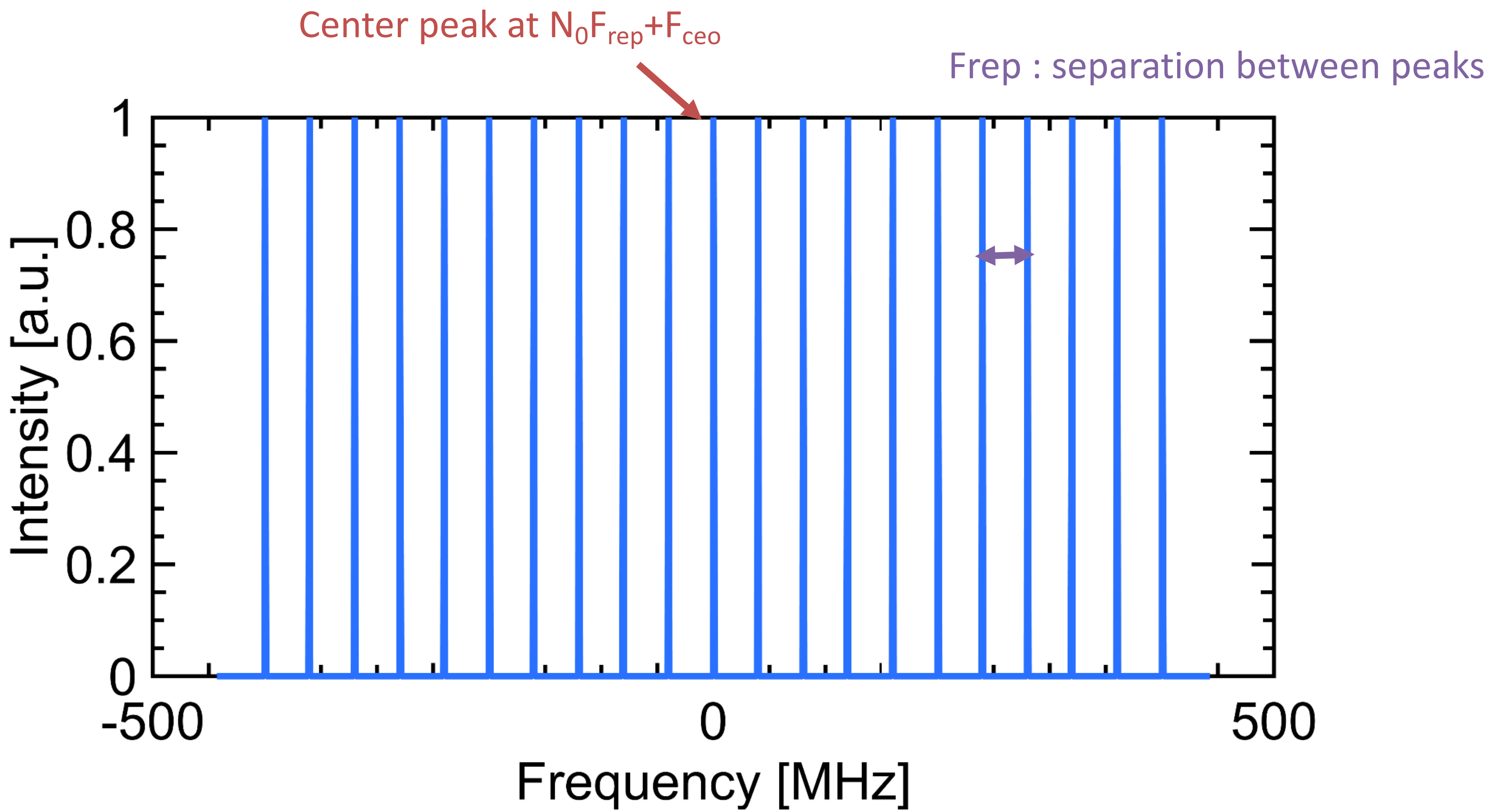
Single ray linewidth infinitely thin
Contradictory with previous model

$$\Gamma = \hbar/\tau \simeq 8.7 \cdot 10^{-6}\ \text{eV}$$

$$2\gamma h f_{\text{rep.}} \simeq 3.3 \cdot 10^{-5}\ \text{eV}$$

0.3 peak within atomic resonance FWHM

(truncated) Laser comb



Single-ion spectral overlap

The interaction probability for a single ion reads

$$\sigma_{\mathrm{i}} = \int_{E=0}^{+\infty} \int_{h\nu=0}^{+\infty} \sigma(E)\delta(E - h\nu\gamma(1+\beta\cos\theta)) f(\nu) dE dh\nu.$$

which simplifies, since $\gamma(1+\beta\cos\theta)h\Delta\nu \ll \Gamma$,

$$\sigma_{\mathrm{i}} = \sum_{n=N_0-N}^{N_0+N} \frac{\sigma_{\mathrm{max.}}}{2N+1} \frac{1}{1+4/\Gamma^2(\gamma(1+\beta\cos\theta)h(nf_{\mathrm{rep.}}+f_{\mathrm{ceo}})-E_T)^2}.$$

This can be written is a slightly different form

$$\sigma_{\mathrm{i}}(\gamma M_{\mathrm{i}}c^2) = \sum_{n=N_0-N}^{N_0+N} \frac{\sigma_{\mathrm{max.}}}{2N+1} \frac{1}{1+4(\gamma M_{\mathrm{i}}c^2 - \widetilde{E_T}[n])^2/\widetilde{\Gamma}[n]^2}$$

where

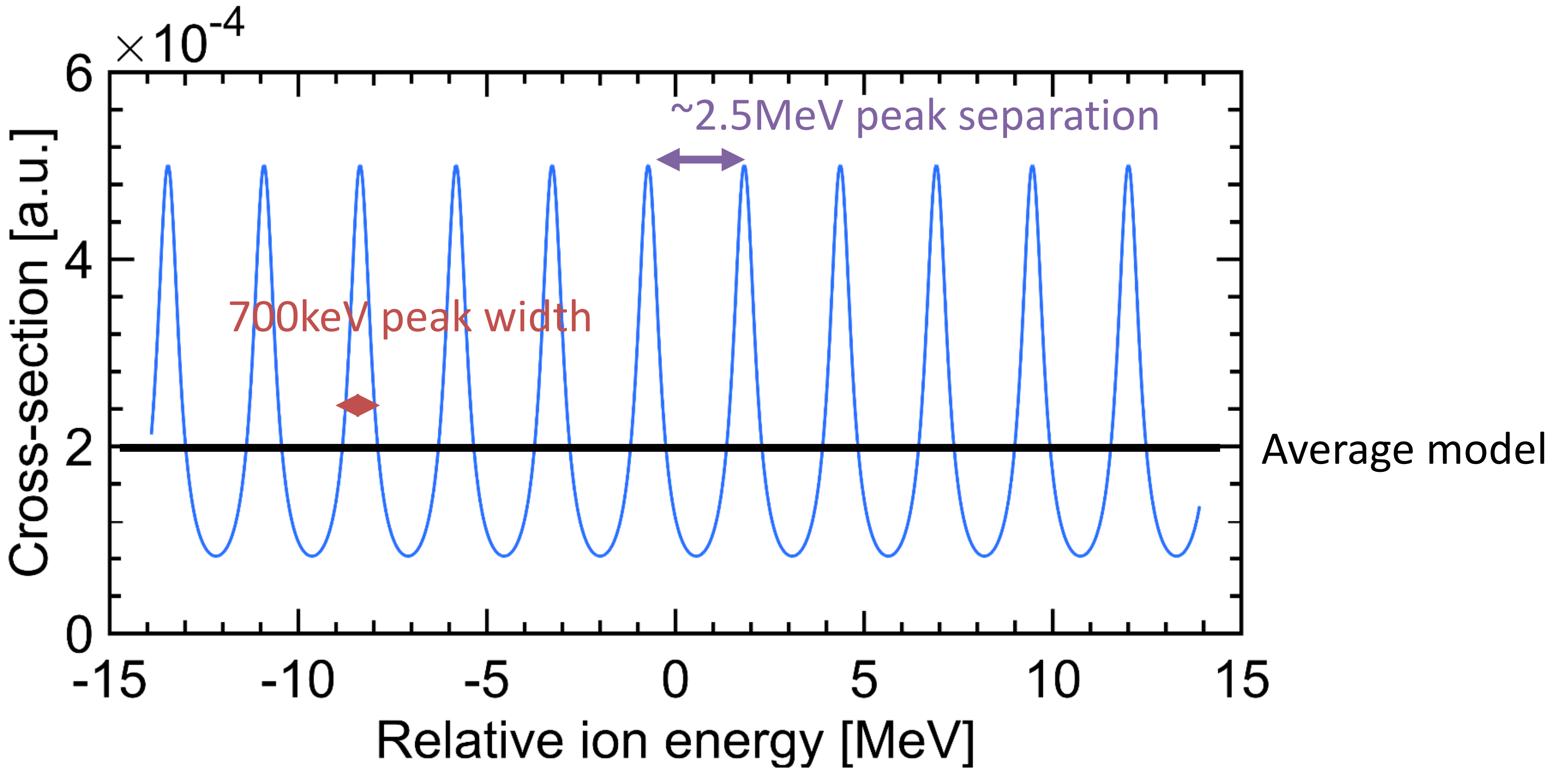
$$\widetilde{\Gamma}[n] = \Gamma\frac{M_{\mathrm{i}}c^2}{(1+\beta\cos\theta)h(nf_{\mathrm{rep.}}+f_{\mathrm{ceo}})} \simeq \frac{\Gamma M_{\mathrm{i}}c^2}{(1+\beta\cos\theta)h(N_0 f_{\mathrm{rep.}}+f_{\mathrm{ceo}})} \simeq \frac{\Gamma M_{\mathrm{i}}c^2\lambda_0}{2hc}$$

and

$$\widetilde{E_T}[n] = \frac{E_T M_{\mathrm{i}}c^2}{(1+\beta\cos\theta)h(nf_{\mathrm{rep.}}+f_{\mathrm{ceo}})} \approx \frac{E_T M_{\mathrm{i}}c^2}{(1+\beta\cos\theta)h(N_0 f_{\mathrm{rep.}}+f_{\mathrm{ceo}})}(1+(n-N_0)\frac{f_{\mathrm{rep.}}}{N_0 f_{\mathrm{rep.}}+f_{\mathrm{ceo}}})$$

Spectral overlap contribution versus energy

τ=76 ps



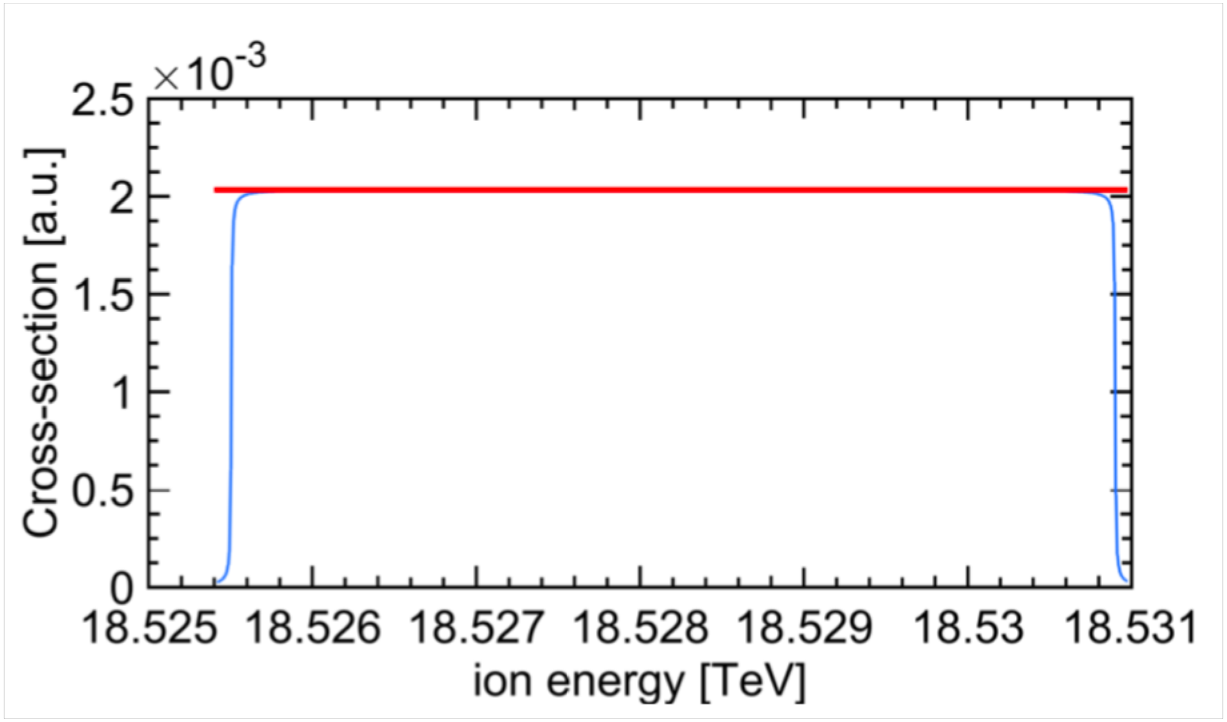
This anyway shll not be an ’issue’ thanks to synchrtotron oscillations

→ Average spectral overlap recovered when averaging over several turns

Spectral overlap contribution versus energy

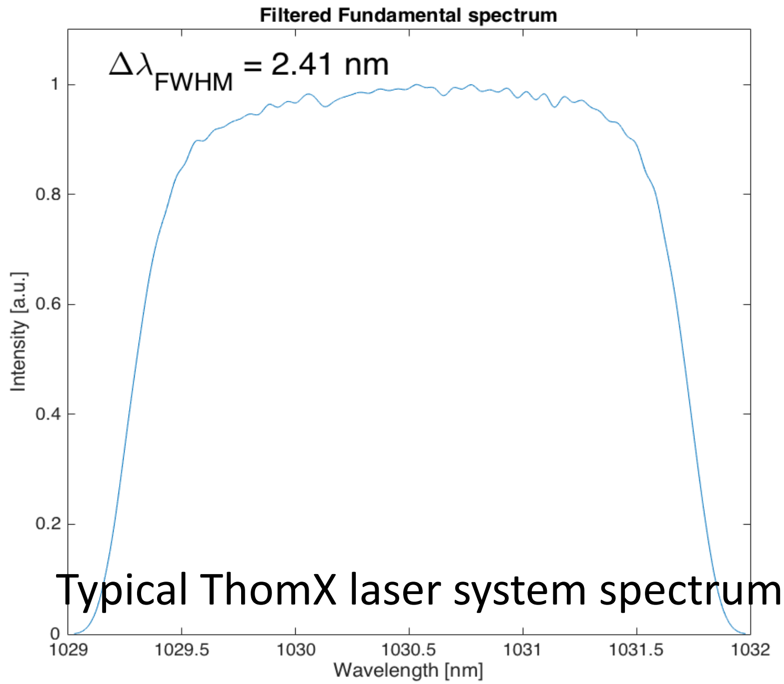
τ=760 ps

Effect is totally washed out if lifetime is an order of magnitude larger.



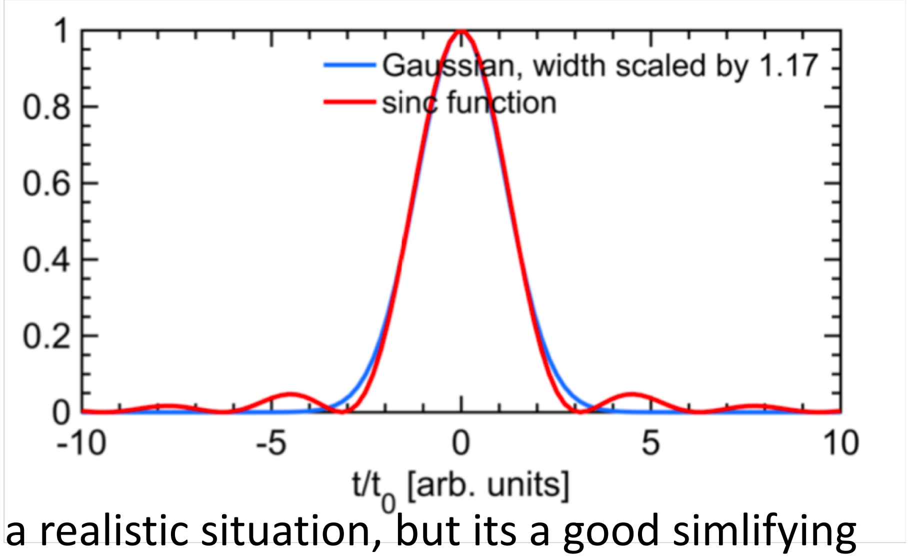
Discussion about flat spectrum

I assume flat spectrum, this is motivated by what we usually get out of the laser amplifier typically implemented in the system we (LAL) are used of



Typical ThomX laser system spectrum

This could 'theoretically' be compressed temporally to a sine cardinal-like shape that can be approximated by a gaussian



There will certainly be some resiudal chirp in a realistic situation, but its a good simlifying starting point

Spatial overlap contribution

$$\widetilde{\mathcal{L}} = 2n_L c\cos^2(\theta/2)\int\int\int\int n_{\rm laser}(x_{\rm L}, y_{\rm L}, z_{\rm L} - ct)\delta(x_{\rm i} - \delta x, y_{\rm i} - \delta y, z_{\rm i} - \beta ct - \delta z)dxdydzdt,$$

$$\begin{aligned}
x_{\rm L} &= -x\cos\theta\cos\alpha - z\sin\theta\cos\alpha + y\sin\alpha \\
y_{\rm L} &= x\cos\theta\sin\alpha + z\sin\theta\sin\alpha + y\cos\alpha \\
z_{\rm L} &= x\sin\theta - z\cos\theta \\
x_{\rm i} &= x \\
y_{\rm i} &= y \\
z_{\rm i} &= z
\end{aligned}$$

$\theta=2\Phi$: crossing angle

Account for arbitrary crossing plane and laser beam sizes.

$$\begin{aligned}
\widetilde{\mathcal{L}}_\alpha &= \frac{n_L}{2\pi\sigma_x\sigma_y\sqrt{1 + \frac{\tan^2\phi\sigma_z^2}{\sigma_x^2\sigma_y^2}(\sigma_x^2\sin^2\alpha + \sigma_y^2\cos^2\alpha)}} \\
&\times \exp\left(-\frac{(\delta y + \delta z\tan\phi\sin\alpha)^2\sigma_x^2 + (\delta x + \delta z\tan\phi\cos\alpha)^2\sigma_y^2 + (\delta y\cos\alpha - \delta x\sin\alpha)^2\sigma_z^2\tan^2\phi}{2\sigma_x^2\sigma_y^2(1 + \frac{\tan^2\phi\sigma_z^2}{\sigma_x^2\sigma_y^2}(\sigma_x^2\sin^2\alpha + \sigma_y^2\cos^2\alpha))}\right)
\end{aligned}$$

Saturation effect

in order to properly account for saturation of excitation probability ion per ion

- Remove the ion spectrum average from the « spectral overlap » contribution

For a given ion with boost γ, the maximum excitation probability reads

$$p_{max} = 0.5\left(\mathrm{erf}\left(\frac{E_\mathrm{i}(\gamma_+ - \gamma_\mathrm{i})}{\gamma_i \Delta E\sqrt{2}}\right) - \mathrm{erf}\left(\frac{E_\mathrm{i}(\gamma_- - \gamma_\mathrm{i})}{\gamma_i \Delta E\sqrt{2}}\right)\right)$$

And the spatial overlap contribution is now independent on the choice of a particular ion

$$\bar{\sigma} \approx \sigma_{\max}\frac{\Gamma\pi}{2}\frac{\lambda_0^2}{hc\Delta\lambda\gamma_i(1+\beta_i\cos\theta)}$$

While the spatio-temporal term does (see previous slide)

The single-ion interaction probablity thus reads $p_{max}(1-\exp(-2\mathcal{L}\sigma))/2$

This probability is sampled over the iokn bunch population to estimate via Monte-Carlo an ion-beam averaged excitation probaibilty

Dynamical simulation

$$E\left[n+1\right] = E\left[n\right] + V_{RF}(Z - N_e)\cos\phi\left[n\right],$$
$$\phi\left[n+1\right] = \phi\left[n\right] + 2\pi\eta/\beta_i^2\Delta E\left[n+1\right]/E_i.$$

Generate 200 ions by propagating 1ion at E[0]=Espread, phi[0]=0

Thus generate the 1-sigma ellipse of the ion beam. All these ions with E[n,0] are equiprobable

Each ion with E[n,m] (n=ion number, m=turn number) radiates $\bar{E} = \tilde{N}_X/n_I E_X/2$
If and only if E[n,m]>Emin and E[n,m]<Emax where Emin and Emax are driven by the laser bandwidth

Results were shown yesterday, i will not reproduce them here !

Outlook

This average modeling have been corss-checked by Alexey, seems consistent with Monte-Carlo approaches

Possible improvements:

- the effect of laser chirping (needs much effort, forget about it for now)
- Gaussian laser spectrum (can be more easily done)

Backup from yesterday's: 4-mirror cavity