

- 2) absolute masses
and spinorial properties

- In the previous two lectures, we have neglected the ν spinorial properties.
- Such properties, usually irrelevant in oscillation phenomena, are important in other contexts, such as ν interactions, ν mass terms, $\nu/\bar{\nu}$ differences.
- We shall now discuss some of these issues, starting from the familiar Dirac equation.

Dirac representation (γ_0 -diagonal)

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \quad \vec{\gamma} = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix} \quad \vec{\gamma}^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \quad \vec{\sigma} = \text{Pauli matrices}$$

"PARTICLE" Solution:

$$\psi_P \sim \begin{bmatrix} \xi \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \xi \end{bmatrix} e^{-ip_\mu x^\mu}$$

$\xi, \eta = \text{Pauli spinors}$

"ANTIPARTICLE" Solution:

$$\psi_A \sim \begin{bmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \eta \\ \eta \end{bmatrix} e^{ip_\mu x^\mu}$$

States defined by \vec{p} and by helicity h ($\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} = \text{spin operator}$)

$$h = \frac{\vec{p} \cdot \vec{\sigma}}{|\vec{p}|} = \pm 1$$

Dirac repres. useful in nonrelativistic limit, as used in deriving $V = \sqrt{2} g_F N_e$.

Particle - antiparticle conjugation operator

$$\psi_{p,A} = \mathcal{C}(\psi_{A,p})$$

with :

$$\begin{aligned}\mathcal{C}(\psi) &= i\gamma^2\psi^* \\ &= i\gamma^2\gamma^0\bar{\psi}^\tau \\ &\stackrel{\text{def}}{=} C\bar{\psi}^\tau \\ &\stackrel{\text{def}}{=} \psi^c\end{aligned}$$

You may convince yourself that \mathcal{C} is indeed the C -conjugation operator by working out the following exercises :

Exercise 1 : Prove that $E(\psi_p) = \psi_A$

Exercise 2 : Prove that if ψ has an electric charge:

$$(i\gamma^\mu (\partial_\mu + i\mathbf{q} \cdot \mathbf{A}_\mu) - m) \psi = 0$$

then ψ^c has opposite charge:

$$(i\gamma^\mu (\partial_\mu - i\mathbf{q} \cdot \mathbf{A}_\mu) - m) \psi^c = 0$$

Hints: (1) Use $\hat{\sigma}_2 \hat{\sigma}^* = -\hat{\sigma}^* \hat{\sigma}_2$; set $\eta = -i\hat{\sigma}_2 \xi^*$

(2) Use $\hat{\sigma}_2 \gamma^\mu * = -\gamma^\mu * \hat{\sigma}_2$ after taking complex conjugate.

Weyl representation (γ_5 -diagonal, or "chiral")

$$\gamma^0 = \begin{bmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{bmatrix} \quad \vec{\gamma} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \quad \gamma^5 = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{bmatrix} \quad (*)$$

Chiral components:

$$\text{"RIGHT-HANDED": } \psi_R = \frac{1+\gamma_5}{2} \psi = \begin{bmatrix} \phi_R \\ 0 \end{bmatrix} \sim \begin{bmatrix} (\epsilon + m + \vec{\sigma} \cdot \vec{p}) \xi \\ 0 \end{bmatrix}$$

$$\text{"LEFT-HANDED": } \psi_L = \frac{1-\gamma_5}{2} \psi = \begin{bmatrix} 0 \\ \phi_L \end{bmatrix} \sim \begin{bmatrix} 0 \\ (\epsilon + m - \vec{\sigma} \cdot \vec{p}) \cdot \xi \end{bmatrix}$$

$$\rho_{R,L} = \frac{1 \pm \gamma_5}{2}$$

Obtained from Dirac basis via:
 $\psi \rightarrow T\psi ; \gamma^\mu \rightarrow T\gamma^\mu T^{-1} ; T = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbb{I} & \mathbb{I} \\ \mathbb{I} & -\mathbb{I} \end{bmatrix}$

The spinors ϕ_R and ϕ_L transform differently under a Lorentz boost in the direction \hat{v} ($|v| = 1$) with boost parameter u ($\beta = \gamma \hat{v} u$, $\gamma = \cosh u$) :

$$\phi'_{R,L} = e^{\pm \frac{m}{2} \hat{v} \cdot \vec{\sigma}} \phi_{R,L}$$

[While they behave in the same way under a rotation of angle ω around \hat{n} : $\phi'_{R,L} = e^{i \frac{\omega}{2} \hat{n} \cdot \vec{\sigma}} \phi_{R,L}$]

Exercise 3 : Dirac eq. from Lorentz boost

A Pauli spinor at rest (ξ) has no definite chirality. Under a Lorentz boost, it may behave in two possible ways: $\phi_{R,L} = e^{\pm \frac{m}{2} \hat{v} \cdot \vec{\sigma}} \xi$. Show that, by eliminating ξ in the previous eqs., one recovers the Dirac equation $(\not{p} - m)\psi = 0$ in Weyl representation:

$$\begin{bmatrix} -m & E + \vec{p} \cdot \vec{\sigma} \\ E - \vec{p} \cdot \vec{\sigma} & \phi_L \end{bmatrix} = \begin{bmatrix} 0 \\ \phi_R \end{bmatrix}$$

Exercise 3

First, let's reduce the exponential:

$$\begin{aligned}
 e^{\pm \frac{m}{2} \hat{v} \cdot \vec{\sigma}} &= \cosh \frac{m}{2} \pm \hat{v} \cdot \vec{\sigma} \sinh \frac{m}{2} \\
 &= \left(\frac{\cosh \frac{m+1}{2}}{2} \right)^{\frac{1}{2}} \pm \hat{v} \cdot \vec{\sigma} \left(\frac{\cosh \frac{m-1}{2}}{2} \right)^{\frac{1}{2}} \\
 &= \left(\frac{\gamma+1}{2} \right)^{\frac{1}{2}} \pm \hat{v} \cdot \vec{\sigma} \left(\frac{\gamma-1}{2} \right)^{\frac{1}{2}} \\
 &= \left(\frac{E+m}{2m} \right)^{\frac{1}{2}} \pm \hat{v} \cdot \vec{\sigma} \left(\frac{E-m}{2m} \right)^{\frac{1}{2}} \\
 &= \left[\frac{(E+m)^2}{2m(E+m)} \right]^{\frac{1}{2}} \pm \hat{v} \cdot \vec{\sigma} \left[\frac{(E^2-m^2)}{2m(E+m)} \right]^{\frac{1}{2}} \\
 &= \frac{E+m \pm \vec{p} \cdot \vec{\sigma}}{\sqrt{2m(E+m)}} \\
 &= (\vec{p} = p \hat{v})
 \end{aligned}$$

- Then: $\phi_{R,L} = \frac{E+m \pm \vec{p} \cdot \vec{\sigma}}{\sqrt{2m(E+m)}} \xi$ and $\xi = \frac{E+m \mp \vec{p} \cdot \vec{\sigma}}{\sqrt{2m(E+m)}} \phi_{R,L}$

- By eliminating ξ :

$$\begin{aligned}\phi_{R,L} &= \frac{(E+m \pm \vec{p} \cdot \vec{\sigma})^2}{2m(E+m)} \phi_{L,R} \\ &= \frac{E+\vec{p} \cdot \vec{\sigma}}{m} \phi_{L,R} \quad (\text{after simple algebra})\end{aligned}$$

- In matrix form:

$$\begin{bmatrix} -m & E+\vec{p} \cdot \vec{\sigma} \\ E-\vec{p} \cdot \vec{\sigma} & -m \end{bmatrix} \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} = 0$$

which is the Dirac equation in Weyl basis for the χ'_n 's :

$$(\vec{p}-m)\psi = 0$$

- Note that ϕ_R and ϕ_L decouple only for $m \rightarrow 0$.

Exercise 3

Helicity vs chirality

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From the previous exercise, for $\frac{m}{E} \ll 1$:

$$\begin{bmatrix} -\frac{m}{E} & 1+\rho \\ 1-\rho & -\frac{m}{E} \end{bmatrix} \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore, for:

$$m \equiv 0 \quad \rightarrow \quad \left\{ \begin{array}{l} \phi_R \text{ and } \phi_L \text{ decoupled ("Weyl spinors")} \\ \text{and eigenstates of } h: \quad h \phi_{R,L} = \pm \phi_{R,L} \end{array} \right.$$

$$E \gg m \neq 0 \quad \rightarrow \quad \left\{ \begin{array}{l} \phi_R \text{ and } \phi_L \text{ coupled at } \mathcal{O}(m/E); \\ \text{chirality } \neq \text{ helicity at } \mathcal{O}(m/E). \\ h \phi_{R,L} = \pm \phi_{R,L} + \mathcal{O}(m/E) \end{array} \right.$$

Weyl / Dirac / Majorana spinors

Let's consider a ν ($\bar{\nu}$) produced in LH (RH) state
in weak interactions :

$$\nu : \begin{array}{c} \rightarrow \\ \textcirclearrowleft \end{array} \rightarrow^L$$

$$\bar{\nu} : \begin{array}{c} \rightarrow \\ \textcirclearrowright \end{array} \rightarrow^R$$

If $m=0$, each of them evolves independently as a
2-component (Weyl) spinor with fixed helicity = chirality.

But, if $m \neq 0$, each can develop the opposite handedness at order m/E :

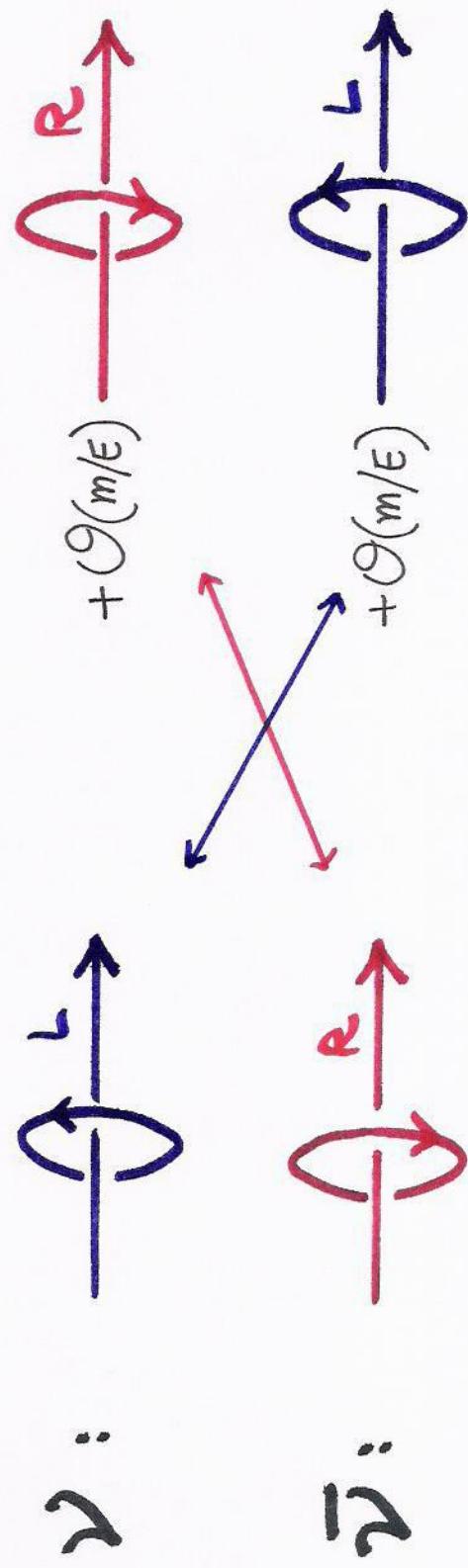
$$\nu : \quad \begin{array}{c} \text{---} \\ \text{---} \end{array} \xrightarrow{\text{L}} + \mathcal{O}(m/E)$$

$$\bar{\nu} : \quad \begin{array}{c} \text{---} \\ \text{---} \end{array} \xrightarrow{\text{R}} + \mathcal{O}(m/E)$$

If these 4 d.o.f. are independent : $\nu \neq \bar{\nu}$
 \rightarrow massive (**Dirac**) 4-spinor

This is the case for charged fermions.

But for neutral fermions, another option exists :



Two components might be identical !
 → no distinction between ν and $\bar{\nu}$ (up to an overall phase)

→ **massive (Majorana)** neutrino with 2 independent dof
 and $\nu = \bar{\nu}$

→ a very neutral particle (no charge of any kind).

Exercise 4 : $\nu/\bar{\nu}$ puzzle

- Define ν_e as the particle emitted in β^+ decay : $(A, z) \rightarrow (A, z-1) e^+ \nu_e$
- Define $\bar{\nu}_e$ as " in β^- " : $(A, z) \rightarrow (A, z+1) e^- \bar{\nu}_e$
- The following reactions have been observed :



- The following reactions, however, have not been observed :



Try to make sense of this fact for Majorana ν 's ($\nu_e = \bar{\nu}_e$).

Exercise 4

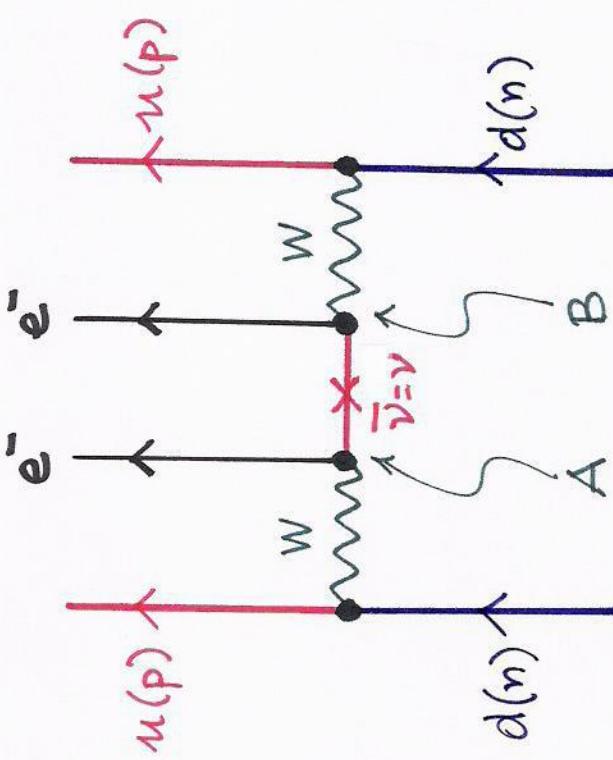
- If ν' 's are Dirac, then $\gamma_e \neq \bar{\nu}_e$, and one can attach a leptonic number to the doublets (γ_e, e^-) and $(\bar{\nu}_e, e^+)$, which is conserved in the observed reactions ($\Delta_L = 0$) and would be violated in the other two ($\Delta_L = 2$).
 - If ν' 's are Majorana, then $\gamma_e = \bar{\nu}_e$, and we are just naming:
 - " γ_e " = LH component of γ state
 - " $\bar{\nu}_e$ " = RH component of γ state
- The initial " γ_e " is LH, being produced in a weak (β^+) decay. While propagating, it remains dominantly LH, but can develop a small RH component (" $\bar{\nu}_e$ ") at $\mathcal{O}(m/\epsilon)$. Then also the reaction $\bar{\nu}_e + n \rightarrow p + e^-$ can take place in principle, but is so suppressed to be practically unobservable - lepton number violation ($\Delta_L = 2$) is allowed in principle, but suppressed at $\mathcal{O}(m/\epsilon)$ in practice.

Neutrinoless double beta decay

The only possibility to probe the possible Majorana nature of ν seems to be the rare (weak²!) $\nu\beta\beta$ decay :

$$(A, Z) \rightarrow (A, Z+2) + 2e^-$$

Intuitive picture :



- At point A, a $\bar{\nu}_e$ is emitted (RH)

- If $m \neq 0$ (not Weyl), it develops a LH component at $\mathcal{O}(m/E)$

- If $\bar{\nu}_e = \nu_e$, such component is a LH ν (not possible if Dirac)

- The ν_{eL} is absorbed in B

Initial state : $u(p) \bar{\nu}_e \rightarrow \Delta L = 2$
 Final state : $2 e^- \rightarrow \Delta L = 2$

Only possible for Majorana ν .

Effective Majorana mass

Amplitude of $\bar{\nu} \nu \beta \bar{\beta}$ decay:

- Depends on γ_e mixings U_{ei} with ν_i
- Is proportional to ν_i masses m_i (being a m/E effect)
- Depends on generalized Majorana condition $\bar{\nu}_i = \nu_i e^{i\phi_i}$

Probability:

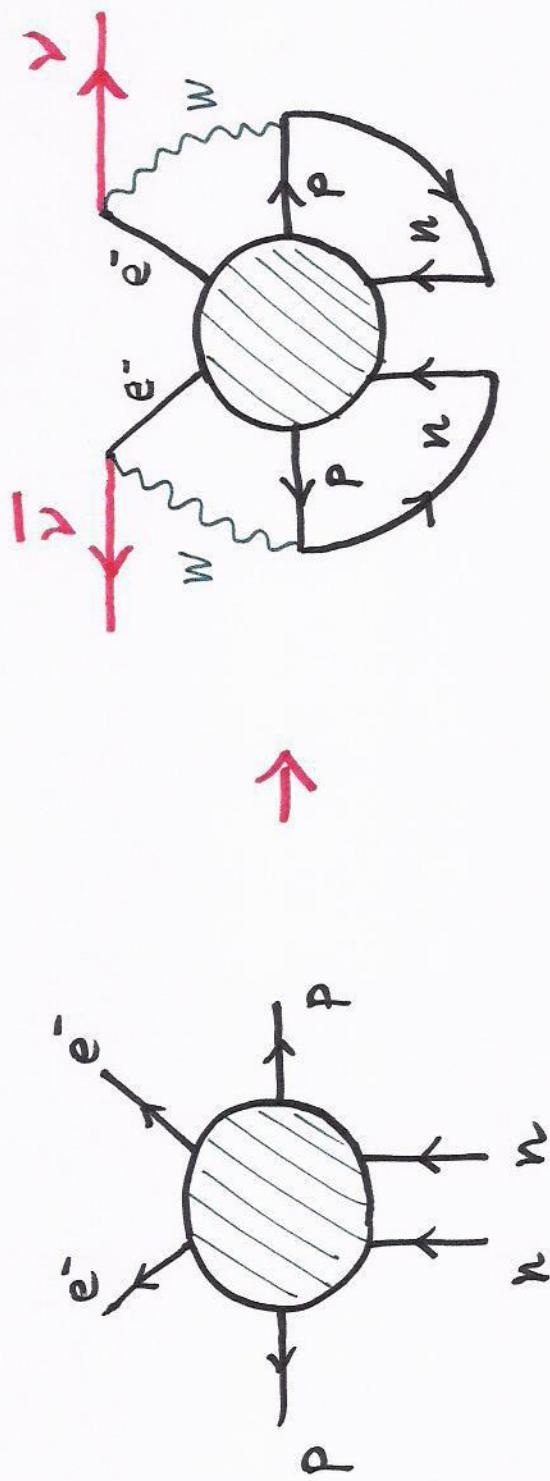
$$\left| \sum_{i=1}^3 \frac{e^- p}{U_{ei}} \bar{\nu}_i = \nu_i e^{i\phi_i} \right|^2 \propto \left| \sum_{i=1}^3 \frac{U_{ei}}{m_i} m_i e^{i\phi_i} \right|^2 = m_{\beta\beta}^2$$

$m_{\beta\beta}$ = effective Majorana mass

ϕ_i (Majorana phases) may induce cancellations.

Schechter-Valle theorem

- If ν Majorana $\rightarrow \bar{\nu}\beta\beta$ decay allowed.
- The reverse is also true, independently on the $\bar{\nu}\beta\beta$ underlying mechanism (e.g., new physics) :



Exercise 5

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For Majorana ν' 's, the mixing matrix is generalized as:

$$U \rightarrow U \cdot U_M$$

where $U_M = \text{diag}(1, e^{i\phi}, e^{i\phi'})$ contains two independent Majorana phases. [An overall common phase can be rotated away].

Show that U_M plays no role in oscillations
("Majorana / Dirac confusion")

Exercise 5

In the hamiltonian of ν oscillations (either in vacuum or in matter) the PMNS matrix U always appear in the form :

$$U \frac{m^2}{2\epsilon} U^+, \quad \text{with } m^2 = \text{diag}(m_1^2, m_2^2, m_3^2)$$

The replacement $U \rightarrow U U_M$ is ineffective :

$$U \frac{m^2}{2\epsilon} U^+ \rightarrow U U_M \frac{m^2}{2\epsilon} U_M^+ U^+ = U \frac{m^2}{2\epsilon} U^+$$

Thus, oscillations do not distinguish Dirac/Majorana ν 's.

Spinor-ology

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Majorana neutrinos can provide new mass terms with interesting properties, which are absent for charged (Dirac) leptons or quarks.

These mass terms require some familiarity with spinor properties.

Conventions :

When operations such as $P_{L,R}$, (\cdot) , and \mathcal{E}_j , are involved:
 $P_{L,R}$ acts before \mathcal{C} which acts before (\cdot)



$$\psi_{L,R}^c = (P_{L,R}\psi)^c = (\psi_{L,R})^c = P_{R,L}(\psi^c)$$

$$\bar{\psi}_{L,R} = \overline{(P_{L,R}\psi)} = \overline{(\psi_{L,R})} = \bar{\psi} P_{R,L}$$

$$\bar{\psi}^c = \overline{(\psi^c)}$$

$$\bar{\psi}_{L,R}^c = \overline{(P_{L,R}\psi)^c} = \overline{(\psi_{L,R})^c} = \overline{P_{R,L}(\psi^c)} = \bar{\psi}^c P_{L,R}$$

Exercise 6

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Under a generic Lorentz transformation (rotation $\vec{\omega} \oplus$ boost \vec{u}), chiral components behave as:

$$\phi_{R,L}' = e^{(i\vec{\omega} \pm \vec{u}) \cdot \frac{\vec{\sigma}}{2}} \phi_{R,L}$$

Show then that:

$$i\sigma_2 \phi_R^* \text{ is L.H.}$$

$$-i\sigma_2 \phi_L^* \text{ is R.H.}$$

- Hint: use $\sigma_2 \tilde{\sigma}_2^* = -\tilde{\sigma}_2^* \sigma_2$ and infinitesimal transf. ($e^{ix} \sim 1+ix$)

As a consequence, one can build a Dirac 4-spinor ψ from two RH spinors u and v :

$$\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix} = \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix}$$

(analogously, from two LH ones).

Exercise 7

Given the previous exercise, show that in Weyl basis:

$$\psi = \begin{bmatrix} v \\ i\sigma_2 v^* \end{bmatrix} \quad \psi_L = \begin{bmatrix} 0 \\ i\sigma_2 v^* \end{bmatrix} \quad \psi_R = \begin{bmatrix} u \\ 0 \end{bmatrix}$$

$$\bar{\psi} = [-iu^\tau \tilde{\sigma}_2, u^+] \quad \bar{\psi}_L = [-iu^\tau \tilde{\sigma}_2, 0] \quad \bar{\psi}_R = [0, u^+]$$

$$\psi^c = \begin{bmatrix} v \\ i\sigma_2 v^* \end{bmatrix} \quad \psi_L^c = \begin{bmatrix} v \\ 0 \end{bmatrix} \quad \psi_R^c = \begin{bmatrix} 0 \\ i\sigma_2 u^* \end{bmatrix}$$

$$\bar{\psi}^c = [-iu^\tau \tilde{\sigma}_2, v^+] \quad \bar{\psi}_L^c = [0, v^+] \quad \bar{\psi}_R^c = [-iu^\tau \tilde{\sigma}_2, 0]$$

Thus: Dirac $\nu \rightarrow \psi \neq \psi^c \rightarrow u \neq v \rightarrow \psi = \psi_R + \psi_L$

Major: $\nu \rightarrow \psi = \psi^c \rightarrow u = v \rightarrow \psi = \psi_R + \psi_R^c$
or $\psi = \psi_L + \psi_L^c$

Weyl / Dirac / Majorana Summary :

$m = 0$
Weyl

$$\psi = \psi_R$$

or $\psi = \psi_L$

simplest massless
case, 2 dof

$m \neq 0$
Majorana

$$\psi = \psi_R + \psi_R^c = \psi^c$$

or $\psi = \psi_L + \psi_L^c = \psi^c$

simplest massive
case, 2 dof

$m \neq 0$
Dirac

$$\psi = \psi_R + \psi_L \neq \psi^c$$

general massive
case, 4 dof

Dirac and Majorana mass terms (1 family)

- Dirac mass terms are of the form $m\bar{\psi}\psi$ (4 dof ψ)
- Majorana " " " " " $\frac{1}{2}m\bar{\psi}\psi$ (2 dof ψ)

Three possibilities:

$$\begin{array}{ll}
 \text{Dirac} & : \quad \psi = \psi_L + \psi_R \rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \\
 \text{Majorana (L)} & : \quad \psi = \psi_L + \psi_L^c \rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_L^c + \bar{\psi}_L^c\psi_L \\
 \text{Majorana (R)} & : \quad \psi = \psi_R + \psi_R^c \rightarrow \bar{\psi}\psi = \bar{\psi}_R\psi_R^c + \bar{\psi}_R^c\psi_R
 \end{array}$$

Most general mass term for one neutrino family:

$$m_D(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) + \frac{1}{2}m_L(\bar{\psi}_L\psi_L^c + \bar{\psi}_L^c\psi_L) + \frac{1}{2}m_R(\bar{\psi}_R\psi_R^c + \bar{\psi}_R^c\psi_R)$$

[Last two terms absent for charged fermions.]

- Previous mass term can be rewritten as :

$$\frac{1}{2} [\bar{\Psi}_L + \bar{\Psi}_L^c, \bar{\Psi}_R + \bar{\Psi}_R^c] \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \Psi_L + \Psi_L^c \\ \Psi_R + \Psi_R^c \end{bmatrix}$$

- Diagonalization provides fields with definite masses.
(If mass < 0, redefine field $\psi \rightarrow \gamma_5 \psi$ so that $m \rightarrow -m$)
- Since the basis fields $(\bar{\Psi}_L + \bar{\Psi}_L^c)$ and $(\bar{\Psi}_R + \bar{\Psi}_R^c)$ are Majorana, diagonalization will generally produce mass eigenvectors which are also Majorana
- **Exercise 8 :** Diagonalize $M = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$

Exercise 8

$$\mathbf{M} = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \quad T = \text{Tr } \mathbf{M} = m_L + m_R$$

$$D = \det \mathbf{M} = m_L m_R - m_D^2$$

Eigenvalues : $m_{\pm} = \frac{1}{2}(T \pm \sqrt{T^2 - 4D})$

Diagonalization angle : $\sin 2\theta = \frac{m_D}{\sqrt{T^2 - 4D}}$
 (not a mixing angle!)

$$\begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Eigenvectors

$$\begin{bmatrix} v'_1 & v'_2 \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} v'_1 \\ v'_2 \end{bmatrix} = \begin{bmatrix} v'_1 & v'_2 \end{bmatrix} \begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} \begin{bmatrix} v'_1 \\ v'_2 \end{bmatrix}$$

$$\begin{bmatrix} v'_1 \\ v'_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} v'_1 \\ v'_2 \end{bmatrix}$$

$$\cos 2\theta = \frac{m_L - m_R}{\sqrt{T^2 - 4D}}$$

The see-saw mechanism

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Many extensions of the Standard Model predict the existence of singlet neutrinos (ν_R).

E.g., in the 16 representation of $SO(10)$:

$$\begin{pmatrix} \nu_L & u_L & d_L & \nu_L \\ d_L & d_L & d_L & e_L \\ u_R & u_R & u_R & \nu_R \\ \nu_R & d_R & d_R & e_R \end{pmatrix}$$

→ can get a Majorana mass term $\sim m_R (\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c)$, where m_R is presumably a large mass scale characterizing the SM extension.

For $m \ll M$, diagonalization of $\begin{bmatrix} 0 & m \\ m & M \end{bmatrix}$ gives:
 $\gamma_L = m_R$

| Eigenvectors (or fields) | Eigenvalues (masses) |
|--|--------------------------|
| $\gamma_{\text{heavy}} \approx (\gamma_R + \gamma_R^c) + \frac{m}{M} (\gamma_L + \gamma_L^c)$ | M |
| $\gamma_{\text{light}} \approx -(\gamma_L + \gamma_L^c) + \frac{m}{M} (\gamma_R + \gamma_R^c)$ | $(-\frac{m^2}{M}) \ll m$ |

← See-saw

The light state is active (contains γ_L) and has a very small mass $\sim m^2/M$

Presumably:
 $m \sim \mathcal{O}(m_{\text{quarks}}, m_{\text{leptons}})$
 $M \sim \mathcal{O}(\Lambda_{\text{beyond SM}})$

Dirac and Majorana mass terms (more families)

In general we can have :

- 3 LH gauge doublets $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau$) "ACTIVE" in weak inter.
- N_s RH gauge singlets $\nu_{sR}^{'} (s=1, \dots, N_s)$ "STERILE" in weak int.

The most general mass matrix will have a block form

$$M = \begin{bmatrix} M_L & & M_D \\ & \ddots & \cdots \\ & M_D' & M_R \end{bmatrix} \quad \begin{aligned} M_L &= 3 \times 3 \\ M_D' &= 3 \times N_S \\ M_R &= N_S \times N_S \end{aligned}$$

After diagonalization:

- Generic eigenvectors (ν fields) will be Majorana
 - expect $O\nu\beta\beta$ decay allowed
 - expect Majorana phases *besides* of Dirac phases
- Active and sterile neutrinos can mix
 - expect see-saw suppression of such mixing, but... who knows?
 - Up to now precisely unitary in general
 - [no evidence of sterile ν so far, however]
- If see-saw at work, expect 3 light active neutrinos and N_S heavy neutrinos, mostly decoupled from each other. Can such heavy ν do something else for us?

Leptogenesis

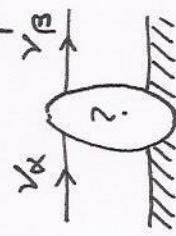
- CP violation at high energy scales might be responsible for different decay rates of heavy ν_R into charged leptons:
$$\Gamma(\nu_R \rightarrow l^+ + \dots) \neq \Gamma(\nu_R \rightarrow l^- + \dots)$$
- This difference might be at the origin of the matter-antimatter asymmetry in the early universe.

Discovery of Majorana neutrinos + CP violation in ν would make the see-saw + leptogenesis mechanisms more plausible.

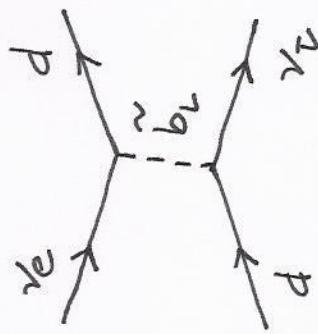
New interactions

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New physics beyond the SM might well be responsible also for new interactions, e.g., FCNC:



An example: SUSY \cancel{R}



→ May get important modifications of evolution hamiltonian:

$$H = \underbrace{H_{vac} + H_{mat}}_{\text{standard}} + \underbrace{H_{\text{new physics}}}_{\text{non-standard}}$$

Recap

- Neutrino nature (Dirac/ Majorana) difficult to explore , due to chirality of interactions and smallness of ν mass. But : $\text{O}\nu\beta\beta$ decay can probe it.
- Neutrino mass terms can be more general than in the quark sector, and are expected to be linked to physics beyond the SM
- Determination of ν masses is an important goal in fundamental physics.