Is orbital angular momentum efficiently transferred to spin degreeds of freedom at NICA energies?

Alejandro Ayala,

D. de la Cruz, L. A. Hernández, S. Hernández, J. Salinas,

arXiv:1909.00274 [hep-ph]

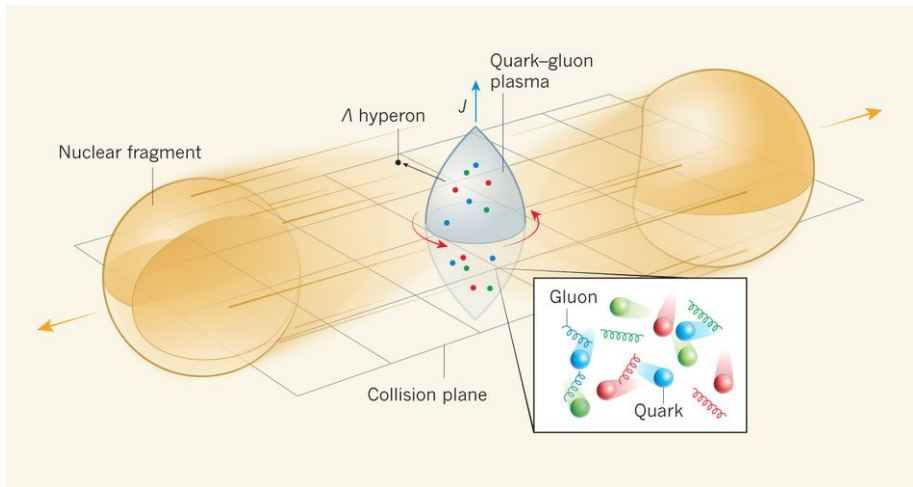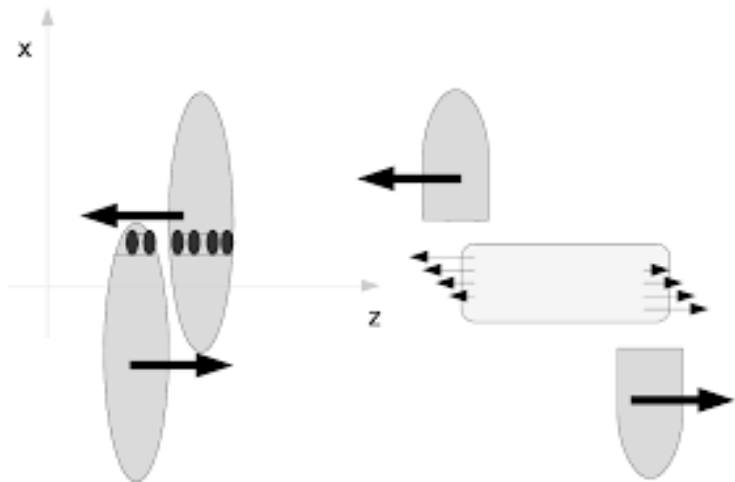Instituto de Ciencias Nucleares, UNAM

October, 2019

Instituto de
Ciencias
Nucleares
UNAM

Overview

1. Angular momentum and thermal vorticity in peripheral heavy-ion collisions

2. Λ polarization induced by thermal vorticity?

3. Microscopic approach to calculate vorticity and spin alignment relaxation time

4. Results: relaxation time for different beam energies, angular velocities and quark/antiquark chemical potential
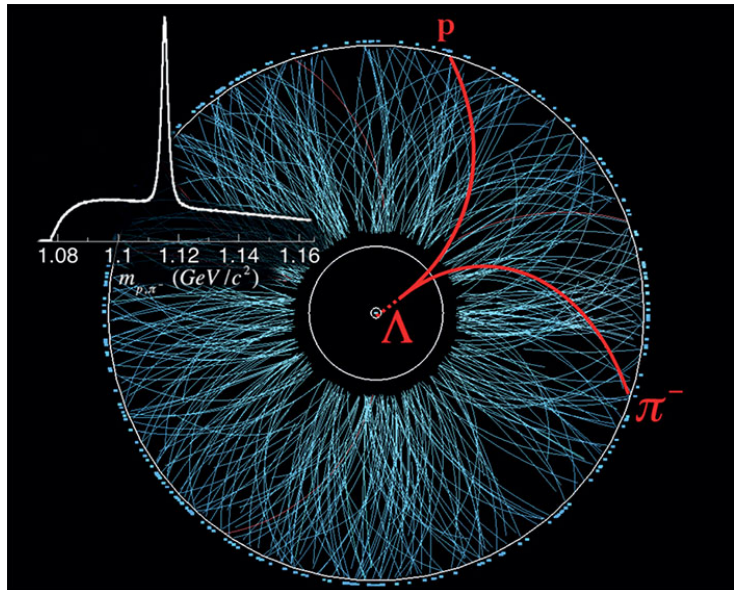
5. Conclusions

Consequences: Lambda polarization?

(a) reaction plane

target (η<0)

projectile (η>0)

participant zone
projectile spectators
target spectators

Thermal vorticity

$$\overline{\omega}_{\mu\nu} = \frac{1}{2}\left(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu\right)$$
$$\beta_\mu = u_\mu(x)/T(x)$$
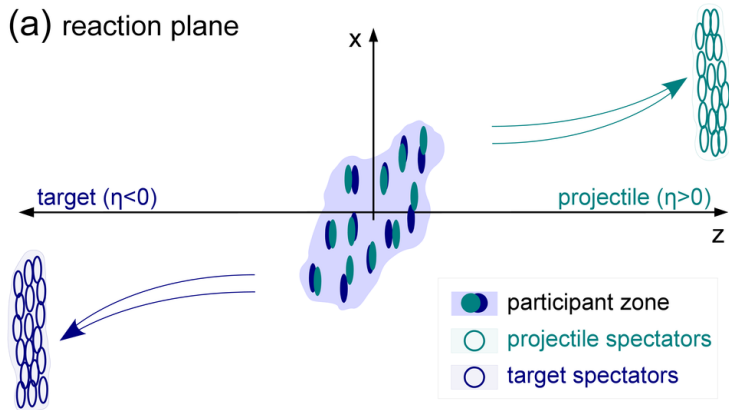
- For a quark spin directed along the $\hat{z}$-axis and a constant angular velocity ω and constant temperature T

$$\overline{\omega}_{\mu\nu}\sigma^{\mu\nu} \propto \omega/T$$

Thermal vorticity from hydro in transverse plane
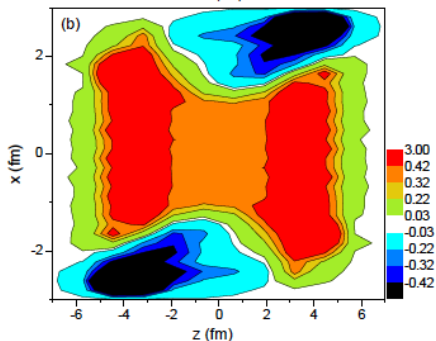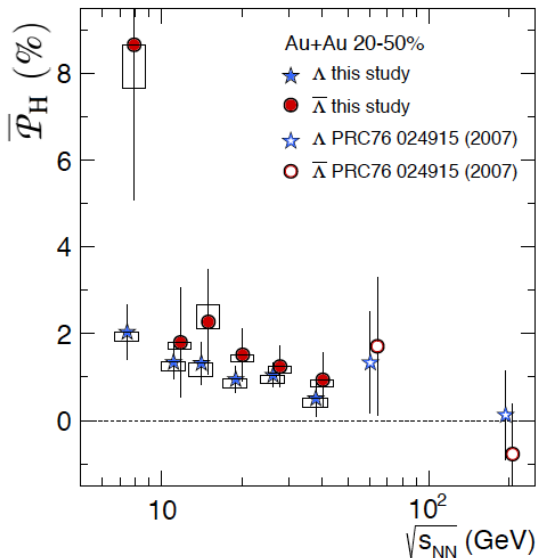
- Vorticity fluctuations/splin flip of s-quarks do not affect hyperon spin
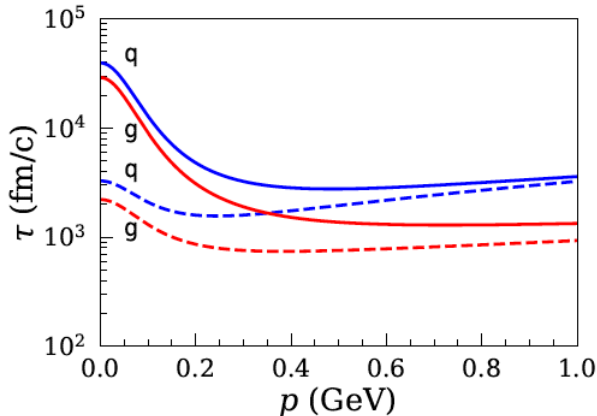
Quark self-energy with vorticity-spin interaction and an effective thermal and baryon density dependent gluon propagator
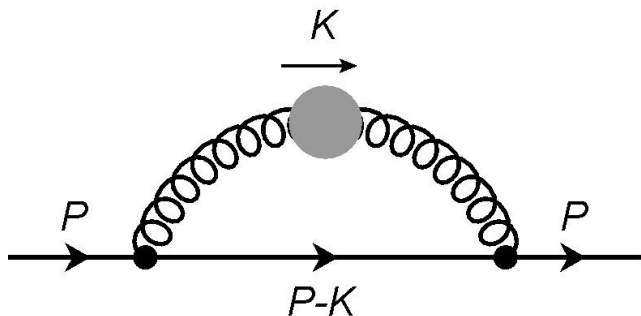
$$\Gamma(p_0) = \tilde{f}(p_0)\mathrm{Tr}\left[\gamma^0\mathrm{Im}\Sigma\right]$$

where $\tilde{f}(p_0)$ is the Fermi-Dirac distribution.

Quark self-energy

- The one-loop contribution to Σ is given explicitly by

$$\Sigma = T \sum_n \int \frac{d^3k}{(2\pi)^3} \lambda_a^\mu S(\slashed{P} - \slashed{K}) \lambda_b^{\nu*} G_{\mu\nu}^{ab}(K)$$

where S and $^*G$ are the quark and effective gluon propagators, respectively.

- The effective vertex is of the form

$$\lambda_a^\mu = g \frac{\sigma^{\alpha\beta}}{2} \overline{\omega}_{\alpha\beta} \gamma^\mu t_a$$

where $\sigma^{\alpha\beta}/2$, with $\sigma^{\alpha\beta} = \frac{i}{2}\left[\gamma^\alpha, \gamma^\beta\right]$ is the quark spin operator and t_a are the color matrices in the fundamental representation.

Propagators

- In a covariant gauge, the Hard Thermal Loop (HTL) approximation to the effective gluon propagator is given by

$$^*G_{\mu\nu}(K) = {}^*\Delta_L(K)P_{L\,\mu\nu} + {}^*\Delta_T(K)P_{T\,\mu\nu}$$

$P_{L,T\,\mu\nu}$ polarization tensors for longitudinal and transverse gluons.

-

$$
\begin{aligned}
{}^*\Delta_L(K)^{-1} &= K^2 + 2m^2\frac{K^2}{k^2}\left[1 - \left(\frac{i\omega_n}{k}\right)Q_0\left(\frac{i\omega_n}{k}\right)\right] \\
{}^*\Delta_T(K)^{-1} &= -K^2 - m^2\left(\frac{i\omega_n}{k}\right)\left[\left[1 - \left(\frac{i\omega_n}{k}\right)^2\right]\right. \\
&\times \left. Q_0\left(\frac{i\omega_n}{k}\right) + \left(\frac{i\omega_n}{k}\right)\right]
\end{aligned}
$$

where

$$Q_0(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

Propagators

- m is the gluon thermal mass given by

$$m^2 = \frac{1}{6}g^2 C_A T^2 + \frac{1}{12}g^2 C_F \left(T^2 + \frac{3}{\pi^2}\mu^2 \right)$$

where $C_A = 3$ and $C_F = 4/3$ are the Casimir factors for the adjoint and fundamental representations of $SU(3)$, respectively.

- The sum over Matsubara frequencies can be expressed as

$$S_{L,T} = T \sum_n {}^*\Delta_{L,T}(i\omega_n)\tilde{\Delta}_F(i(\omega_m - \omega_n))$$

- Introducing the spectral densities $\rho_{L,T}$ and $\tilde{\rho}$ for the gluon and fermion propagators, respectively, the imaginary part of S can thus be written as

$$
\begin{aligned}
\text{Im}\, S_{L,T} &= \pi \left(e^{(p_0-\mu)/T} + 1 \right) \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} f(k_0) \\
&\times \tilde{f}(p_0' - \mu)\delta(p_0 - k_0 - p_0')\rho_{L,T}(k_0)\tilde{\rho}(p_0')
\end{aligned}
$$

where $f(k_0)$ is the Bose-Einstein distribution.

Spectral densities

- The spectral densities $\rho_{L,T}(k_0, k)$ are obtained from the imaginary part of ${}^*\Delta_{L,T}(i\omega_n, k)$ after the analytic continuation $i\omega_n \to k_0 + i\epsilon$ and contain the discontinuities of the photon propagator across the real k_0-axis.

- Their support depends on the ratio $x = k_0/k$. For $|x| > 1$, $\rho_{L,T}$ have support on the (time-like) quasiparticle poles.

- For $|x| < 1$ their support coincides with the branch cut of $Q_0(x)$. On the other hand, the spectral density corresponding to a bare quark is given by

$$\tilde{\rho}(p_0') = 2\pi\epsilon(p_0')\delta(p_0'^2 - E_p^2)$$

where $E_p = |\vec{p} - \vec{k}|$.

Spectral densities

- The kinematical restrictions limit the integration over gluon energies to the space-like region $|x| < 1$.
- The part of the gluon spectral densities that contribute to the interaction rate are given by

$$\rho_L(k_0, k) = \frac{x}{1 - x^2} \frac{2\pi m^2 \theta(k^2 - k_0^2)}{\left[k^2 + 2m^2 \left(1 - (x/2) \ln |(1 + x)/(1 - x)|\right)\right]^2 + \left[\pi m^2 x\right]^2}$$

$$\rho_T(k_0, k) = \frac{\pi m^2 x (1 - x^2) \theta(k^2 - k_0^2)}{\left[k^2(1 - x^2) + m^2 \left(x^2 + (x/2)(1 - x^2) \ln \left|\frac{(1 + x)}{(1 - x)}\right|\right)\right]^2 + \left[\frac{\pi}{2} m^2 x (1 - x^2)\right]^2}$$

Interaction rate

- Collecting all the ingredients, the interaction rate for a quark with energy p_0 to align its spin with the thermal vorticity is given by

$$
\begin{aligned}
\Gamma(p_0) &= \frac{\alpha_s}{4\pi}\left(\frac{\omega}{T}\right)^2 \frac{C_F}{p_0} \int_0^\infty dk\, k \int_{-k}^k dk_0 \theta(2p_0 - k + k_0) \\
&\times (1 + f(k_0))\tilde{f}(p_0 + k_0 - \mu) \sum_{i=L,T} C_i(p_0, k_0, k)\rho_i(k_0, k)
\end{aligned}
$$

where the functions $C_{L,T}$ come from the contraction of the polarization tensors $P_{L,T\,\mu\nu}$ with the trace of the factors involving gamma matrices.

- For consistency of the approximation where we have considered massless quarks, we have also dropped terms proportional to the quark four-momentum components.

Interaction rate

- After implementing the kinematical restriction for the allowed values of the angle between the quark and gluon momenta, these functions are given explicitly by

$$
\begin{aligned}
C_T(p_0, k_0, k) &= 8k_0 \left(\frac{k^2 - 2k_0 p_0 - k_0^2}{2k p_0} \right)^2 \\
C_L(p_0, k_0, k) &= -8k_0 \left[\left(\frac{k^2 - 2k_0 p_0 - k_0^2}{2k p_0} \right)^2 - \frac{1}{2} \right]
\end{aligned}
$$

- The total interaction rate is obtained by integrating over the available phase space

$$
\Gamma = V \int \frac{d^3 p}{(2\pi)^3} \Gamma(p_0)
$$

where V is the volume of the overlap region in the collision and for massless quarks $p_0 = p$.

Interaction rate

- For collisions of symmetric systems of nuclei with radii R and a given impact parameter b, the interaction volume is

$$V = \frac{\pi}{3}(4R + b)(R - b/2)^2$$

- We use the expression for Γ to study the parametric dependence of the relaxation time for spin and vorticity alignment, defined as

$$\tau \equiv 1/\Gamma$$

- For the analysis we use $\alpha_s = 0.3$ and consider Au+Au collisions ($R = 7.27$ fm) at $\sqrt{s_{NN}} = 10$, 200 GeV for semicentral collisions with impact parameter of $b = 10$ fm, where the maximum angular momentum is expected to be imparted.
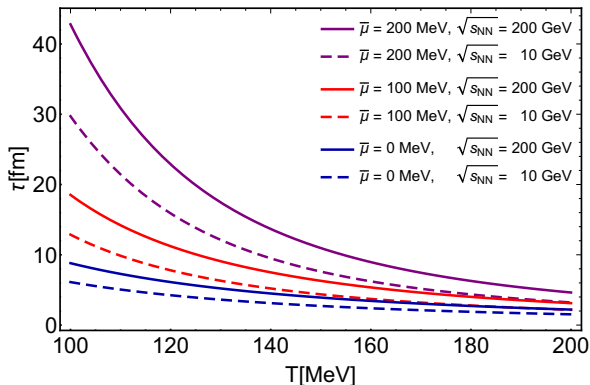
Relaxation time: Small ω scenario

- To estimate the magnitude of the angular velocity, we consider two extreme situations:
- First an scenario where the magnitude of ω competes against transverse expansion and only a fraction of order 10% of the angular momentum of the total participants is retained by the produced QGP in the interaction region.
- The estimated magnitude of ω is thus small and found to slowly decrease with the collision energy from about $\omega \sim 0.12$ fm^{-1} for $\sqrt{s_{NN}} \sim 10$ GeV to $\omega \sim 0.10$ fm^{-1} for $\sqrt{s_{NN}} \sim 200$ GeV.

Relaxation time for vorticity and *quark* spin alignment for two different beam energies for which $\omega \sim 0.10, 0.12$ fm^{-1} and different quark chemical potentials
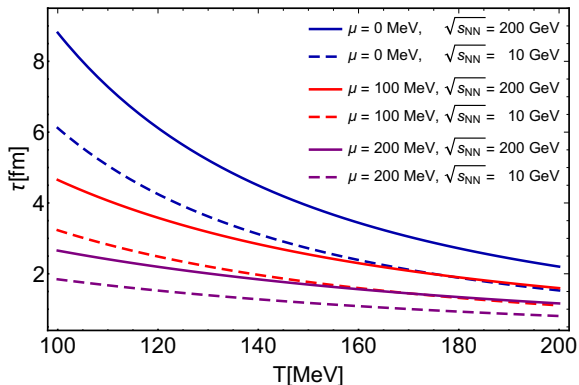
Relaxation time for vorticity and **antiquark** spin alignment for two different beam energies for which $\omega \sim 0.10, 0.12$ fm^{-1} and different quark chemical potentials

Relaxation time: Large ω scenario

- Next, a scenario where the total initial angular momentum of the participants is retained by the produced QGP withot transverse expansion and thus ω turns out to be large.
- We perform UrQMD simulations for Au+Au collisions. For both energies, 2×10^4 events were generated. The estimated angular velocity is computed non-relativistically as the average

$$\omega = \frac{1}{N} \sum_{i=1}^{N} v_i / r_i$$

where v_i and r_i are the initial velocity along the beam axis and distance to the normal to the reaction plane that bisects the overlap region, for each particle that takes part of the reaction at the beginning of the collision, respectively.

- In this case, ω is found to increase with the collision energy from about $\omega \sim 1.25$ fm^{-1} for $\sqrt{s_{NN}} = 10$ GeV to $\omega \sim 3.11$ fm^{-1} for $\sqrt{s_{NN}} = 200$ GeV.

Relaxation time for vorticity and **quark** spin alignment for two different beam energies for which $\omega \sim 1.25, 3.11$ fm^{-1} and different quark chemical potentials
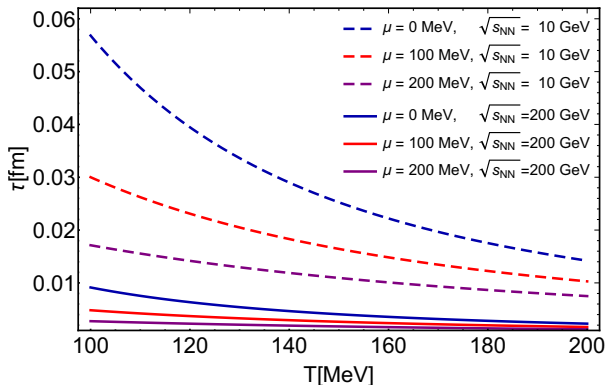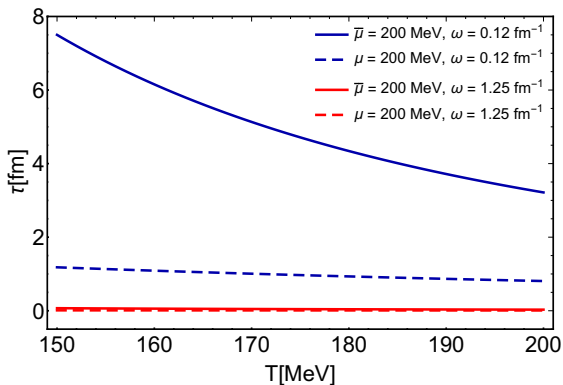
Relaxation time for vorticity and **antiquark** spin alignment for two different beam energies for which $\omega \sim 1.25, 3.11$ fm^{-1} and different quark chemical potentials
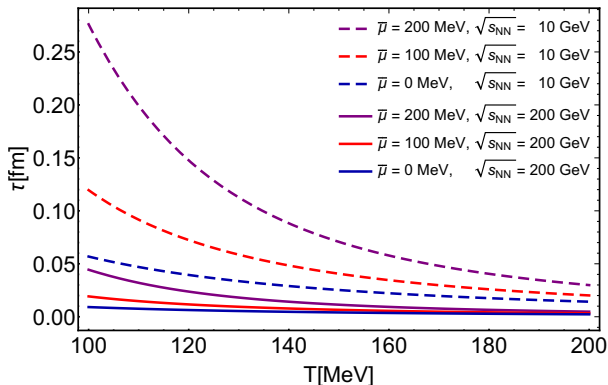
Comparison between the relaxation times for vorticity and **quark/antiquark** spin alignment for one NICA beam energy and two values for $\omega \sim 0.12$ and 1.25 fm^{-1} and one value of the quark chemical potential.

Conclusions

- Microscopic study to try estimate relaxation time for spin and vorticity alignment in relativistic heavy-ion collisions.
- When the collision energy goes mostly into angular velocity, alignment is efficient and the relaxation time lies well within the life-time of the system.
- When collision energy goes mostly into transverse expansion the alignment takes longer.
- Relaxation time decreases with T and with μ_B.
- Relaxation time is larger for antiquarks than for quarks.
- Masive quarks/antiquarks?
- Study how this alignment translates into hyperon polarization.