

Finding resonances, unitarity, peculiarities and traps

Robert Kamiński

Institute of Nuclear Physics PAS, Kraków

NICA days, Warsaw X 2019

$\rho(770)$

$$I^G(J^{PC}) = 1^+(1^{- -})$$

A REVIEW GOES HERE – Check our WWW List of Reviews

 $\rho(770)$ MASS

We no longer list S-wave Breit-Wigner fits, or data with high com background.

NEUTRAL ONLY, e^+e^-

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMM
775.26 ± 0.25 OUR AVERAGE				
775.02 ± 0.35		1 LEES	12G BABR	e^+e^-
775.97 ± 0.46 ± 0.70	900k	2 AKHMETSHIN 07		e^+e^-
774.6 ± 0.4 ± 0.5	800k	3,4 ACHASOV 06	SND	e^+e^-
775.65 ± 0.64 ± 0.50	114k	5,6 AKHMETSHIN 04	CMD2	e^+e^-
775.9 ± 0.5 ± 0.5	1.98M	7 ALOISIO 03	KLOE	1.02 ϵ
775.8 ± 0.9 ± 2.0	500k	7 ACHASOV 02	SND	1.02 ϵ
775.9 ± 1.1		8 BARKOV 85	OLYA	e^+e^-
••• We do not use the following data for averages, fits, limits, etc. •••				
775.8 ± 0.5 ± 0.3	1.98M	9 ALOISIO 03	KLOE	1.02 ϵ
775.9 ± 0.6 ± 0.5	1.98M	10 ALOISIO 03	KLOE	1.02 ϵ
775.0 ± 0.6 ± 1.1	500k	11 ACHASOV 02	SND	1.02 ϵ
775.1 ± 0.7 ± 5.3		12 BENAYOUN 98	RVUE	e^+e^-
770.5 ± 1.9 ± 5.1		13 GARDNER 98	RVUE	0.28-1 π^+
764.1 ± 0.7		14 O'CONNELL 97	RVUE	e^+e^-
757.5 ± 1.5		15 BERNICHA 94	RVUE	e^+e^-
768 ± 1		16 GESHKEN... 89	RVUE	e^+e^-

CHARGED ONLY, τ DECAYS and e^+e^-

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COM
775.11 ± 0.34 OUR AVERAGE					
774.6 ± 0.2 ± 0.5	5.4M	17,18 FUJIKAWA 08	BELL	\pm	τ^-
775.5 ± 0.7		18,19 SCHAELE 05C	ALEP		τ^-
775.5 ± 0.5 ± 0.4	1.98M	7 ALOISIO 03	KLOE	1.02	π^-
775.1 ± 1.1 ± 0.5	87k	20,21 ANDERSON 00A	CLE2		τ^-
••• We do not use the following data for averages, fits, limits, etc. •••					
774.8 ± 0.6 ± 0.4	1.98M	10 ALOISIO 03	KLOE	-	1.02 π^-
776.3 ± 0.6 ± 0.7	1.98M	10 ALOISIO 03	KLOE	+	1.02 π^-
773.9 ± 2.0	± 0.3 -1.0	22 SANZ-CILLER003	RVUE		τ^-
774.5 ± 0.7 ± 1.5	500k	7 ACHASOV 02	SND	\pm	1.02 π^-
775.1 ± 0.5		23 PICH 01	RVUE		τ^-

MIXED CHARGES, OTHER REACTIONS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
763.0 ± 0.3 ± 1.2	600k	24 ABELE	99E	CBAR	0 ± 0.0 $\bar{p}p \rightarrow \pi^+\pi^-\pi^0$

CHARGED ONLY, HADROPRODUCED

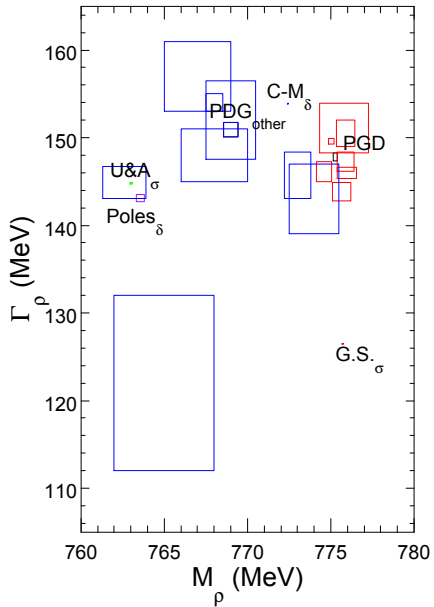
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
766.5 ± 1.1 OUR AVERAGE					
763.7 ± 3.2		ABELE 97	CBAR		$\bar{p}n \rightarrow \pi^-\pi^0\pi^0$
768 ± 9		AGUILAR... 91	EHS		400 pp
767 ± 3	2935	25 CAPRARO 87	SPEC	-	200 $\pi^-\pi^0\pi^0$ $\pi^-\pi^0\pi^0\pi^0$ $\pi^-\pi^0\pi^0\pi^0$ $\pi^-\pi^0\pi^0\pi^0$
761 ± 5	967	25 CAPRARO 87	SPEC	-	200 $\pi^-\pi^0\pi^0$ $\pi^-\pi^0\pi^0\pi^0$ $\pi^-\pi^0\pi^0\pi^0$
771 ± 4		HUSTON 86	SPEC	+	202 $\pi^+\pi^0\pi^0$ $\pi^+\pi^0\pi^0\pi^0$
766 ± 7	6500	26 BYERLY 73	OSPK	-	5 $\pi^-\pi^0$
766.8 ± 1.5	9650	27 PISUT 68	RVUE	-	1.7-3.2 $\pi^-\pi^0\pi^0$, $t < 10$
767 ± 6	900	25 EISNER 67	HBC	-	4.2 $\pi^-\pi^0\pi^0$, $t < 10$

NEUTRAL ONLY, PHOTOPRODUCED

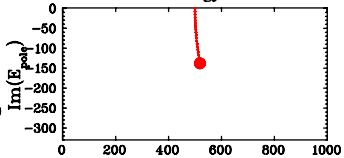
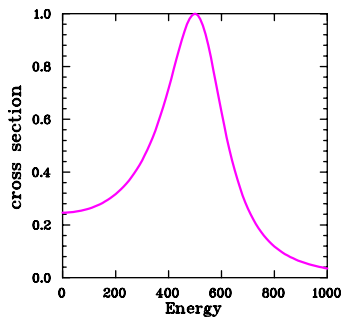
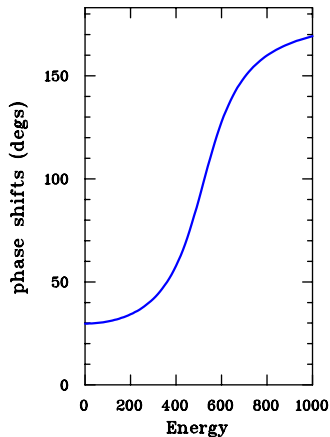
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
769.0 ± 1.0 OUR AVERAGE				
771 ± 2	± 2 -1	63.5k	28 ABRAMOWICZ12	ZEUS $e p \rightarrow e \pi^+ \pi^- p$
770 ± 2	± 1	79k	29 BREITWEG 98B	ZEUS 50-100 γp
767.6 ± 2.7		BARTALUCCI 78	CNTR	$\gamma p \rightarrow e^+e^-p$
775 ± 5		GLADDING 73	CNTR	2.9-4.7 γp
767 ± 4	1930	BALLAM 72	HBC	2.8 γp
770 ± 4	2430	BALLAM 72	HBC	4.7 γp
765 ± 10		ALVENSLEB... 70	CNTR	γA , $t < 0.01$
767.7 ± 1.9	140k	BIGGS 70	CNTR	<4.1 $\gamma C \rightarrow \pi^+\pi^-C$
765 ± 5	4000	ASBURY 67B	CNTR	$\gamma + Pb$
••• We do not use the following data for averages, fits, limits, etc. •••				
771 ± 2	79k	30 BREITWEG 98B	ZEUS	50-100 γp

NEUTRAL ONLY, OTHER REACTIONS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
769.0 ± 0.9 OUR AVERAGE					Error includes scale factor of 1.4. See the ideogram below.
765 ± 6		BERTIN 97C	OBLX		0.0 $\bar{p}p \rightarrow \pi^+\pi^-\pi^0$
773 ± 1.6		WEIDENAUER 93	ASTE		$\bar{p}p \rightarrow \pi^+\pi^-\pi^0$
762.6 ± 2.6		AGUILAR... 91	EHS		400 pp
770 ± 2		31 HEYN 81	RVUE		Pion form factor
768 ± 4	32,33	BOHACIK 80	RVUE	0	
769 ± 4	26	MUSALIMOV 80	ASTIC	0	$\pi^+\pi^-\pi^0$

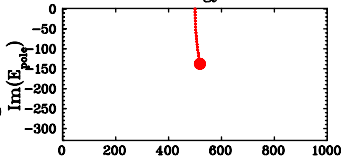
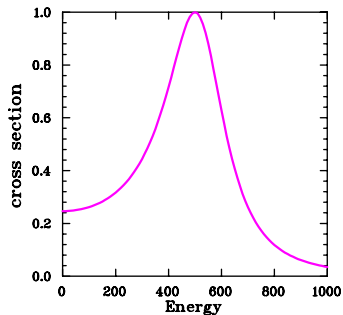
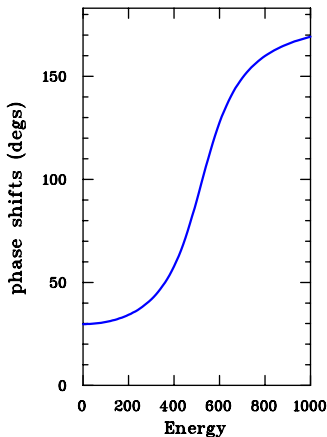


Resonances



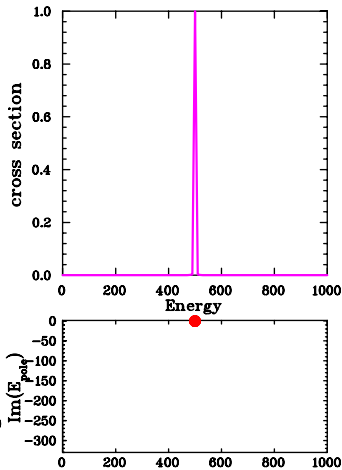
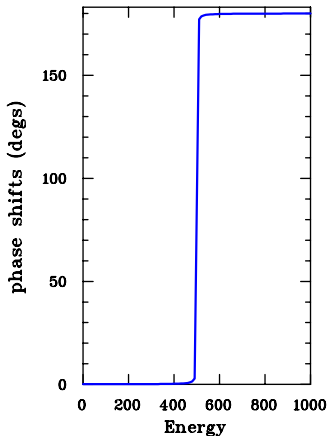
Resonances: Breit-Wigner (BW) approximation:

$$\text{Ampl} = \frac{C}{M_{BW} - E - i\Gamma_{BW}/2}, \quad \delta = \text{ArcTan}\left(\frac{\Gamma/2}{M_{BW} - E}\right), \quad \sigma = \frac{C^2}{(M_{BW} - E)^2 + \Gamma^2/4}$$



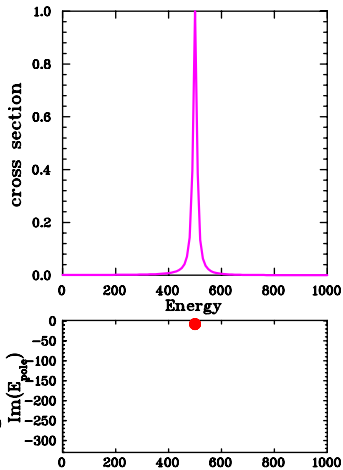
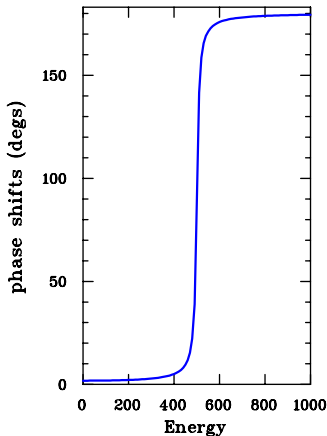
Resonances: exercise - Breit Wigner parameterization

$$\text{Ampl} = \frac{C}{M_{BW} - E - i\Gamma_{BW}/2}, \quad \delta = \text{ArcTan}\left(\frac{\Gamma/2}{M_{BW} - E}\right), \quad \sigma = \frac{C^2}{(M_{BW} - E)^2 + \Gamma^2/4}$$



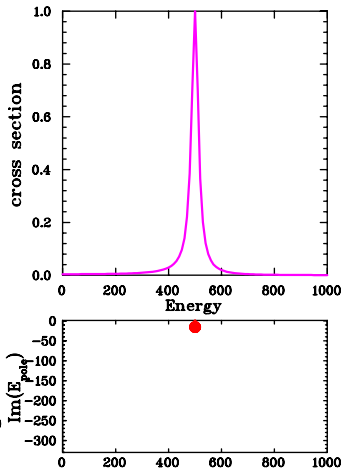
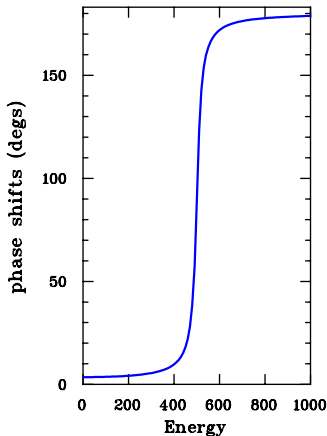
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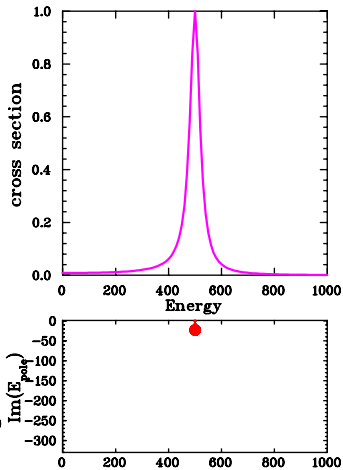
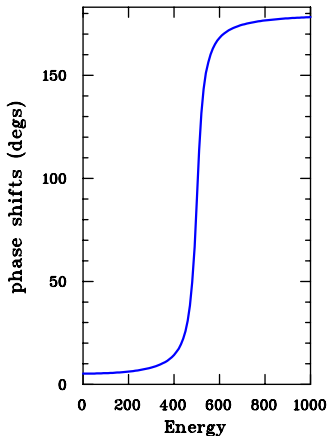
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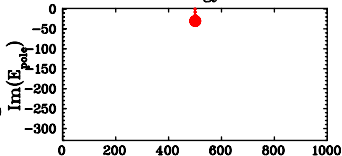
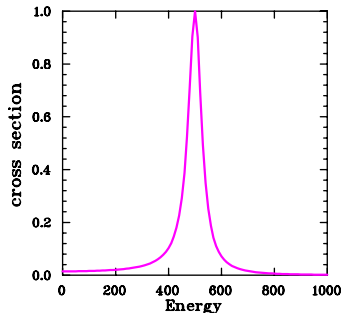
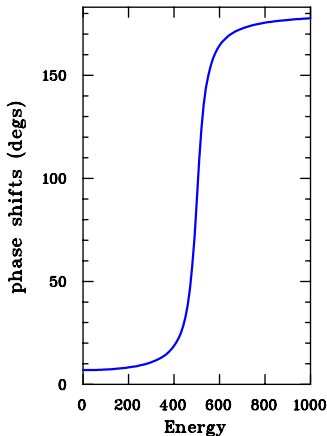
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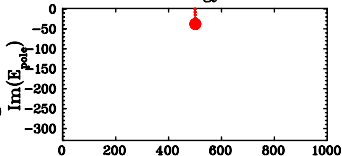
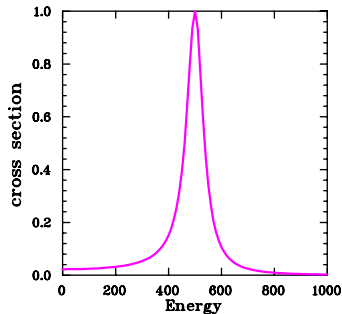
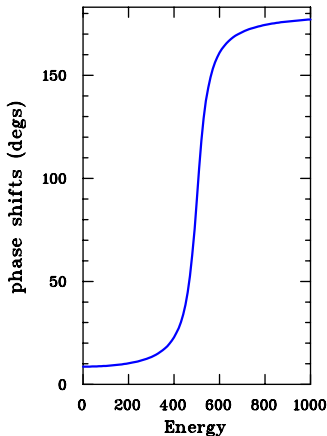
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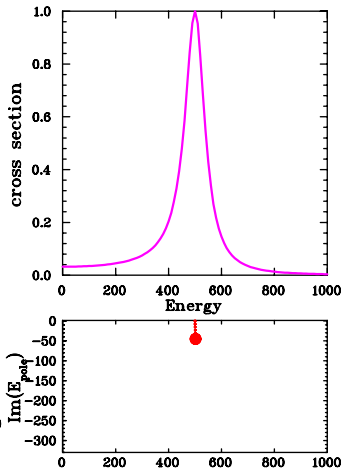
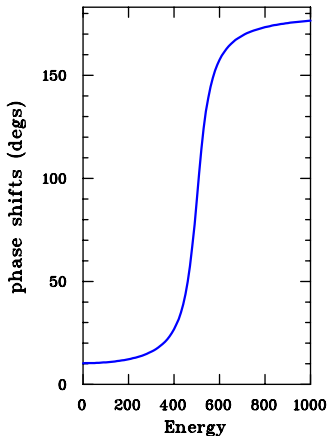
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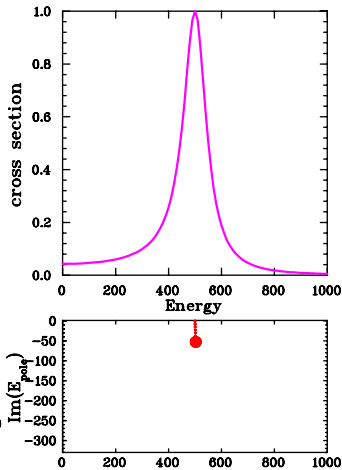
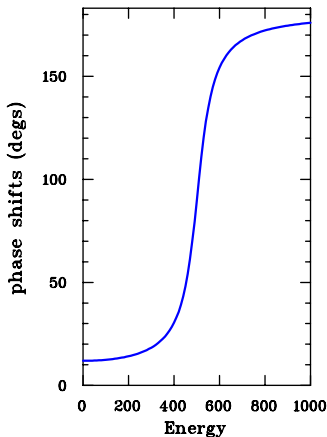
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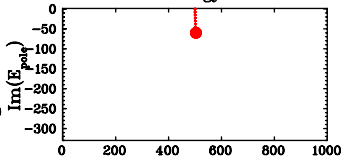
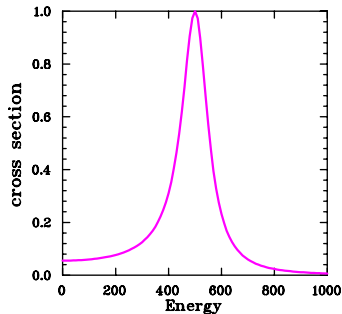
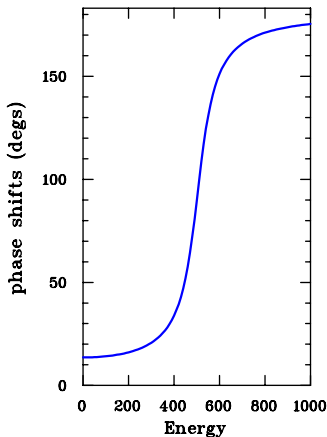
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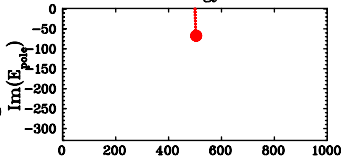
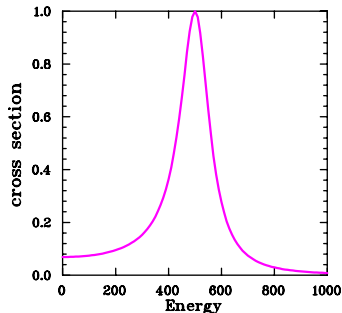
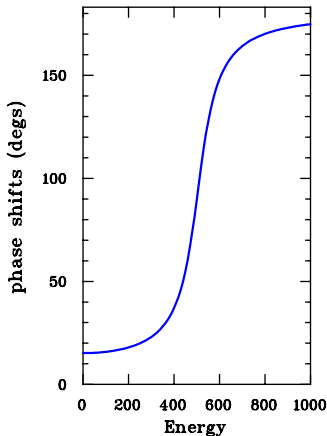
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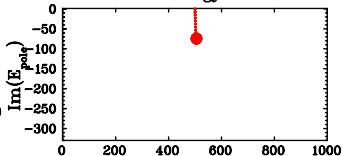
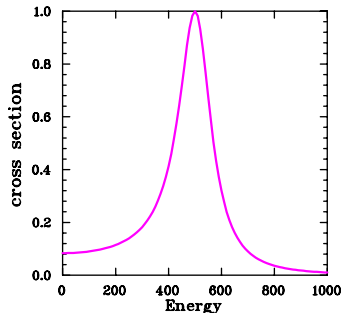
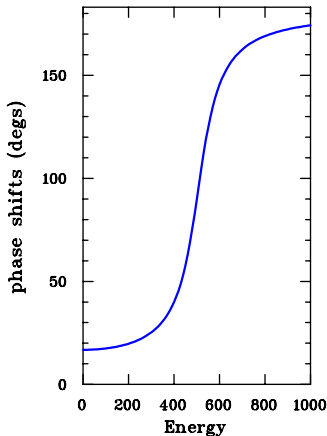
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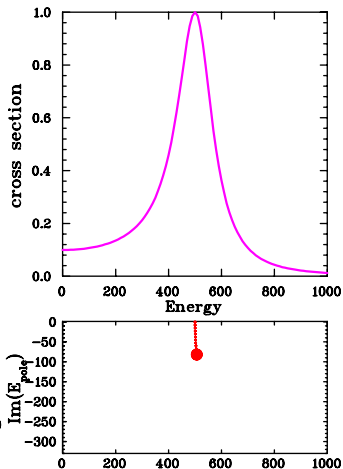
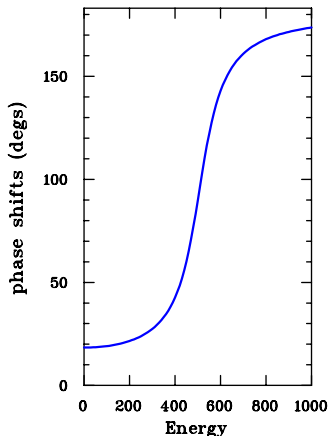
Resonances: exercise - Breit Wigner parameterization

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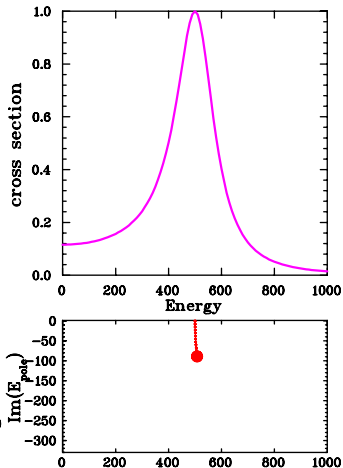
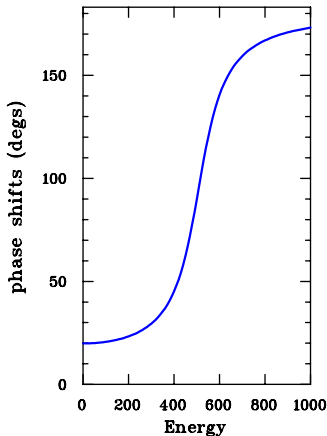
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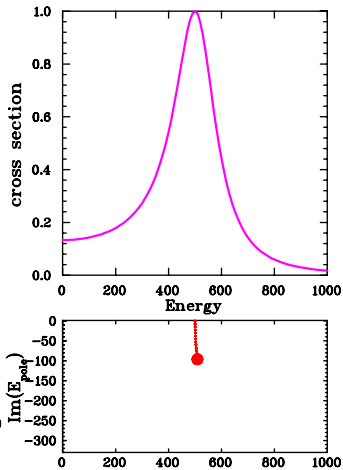
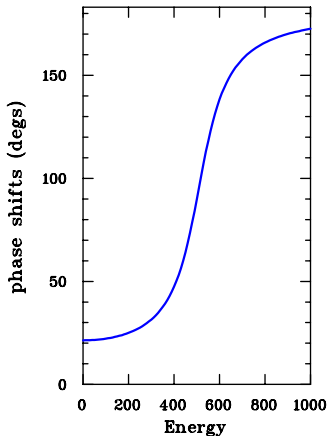
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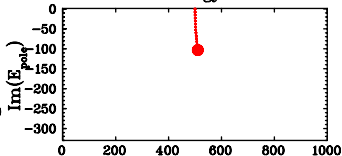
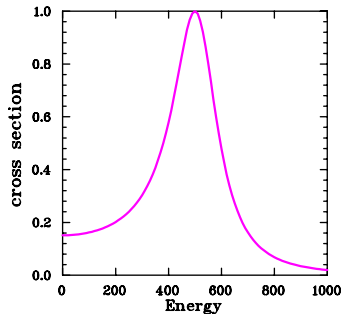
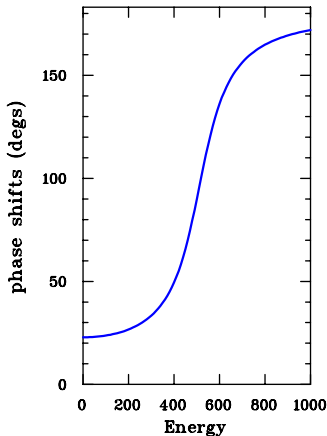
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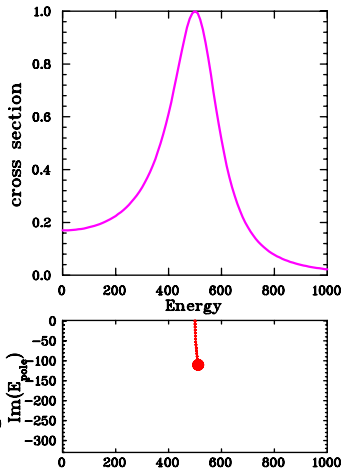
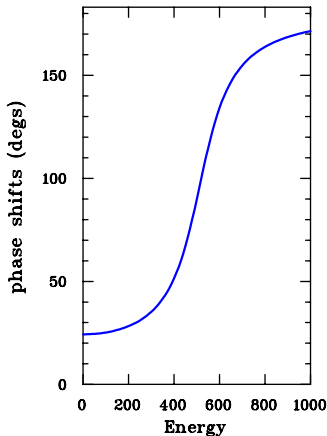
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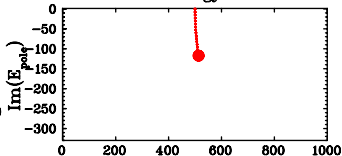
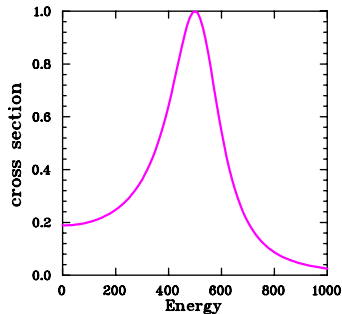
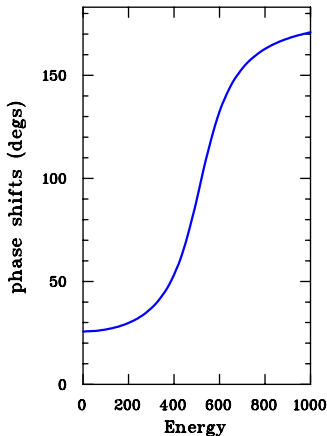
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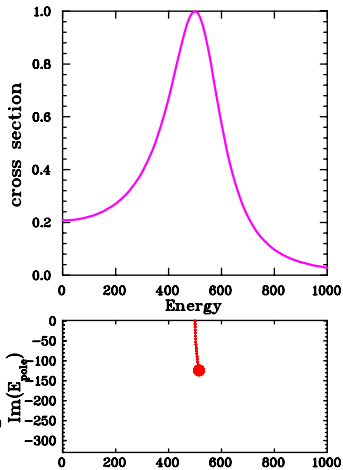
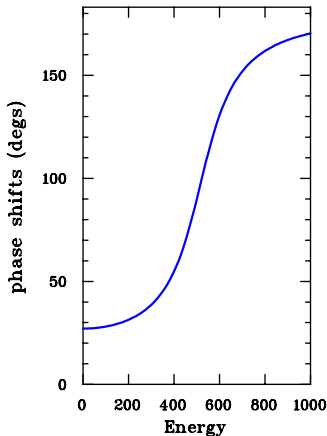
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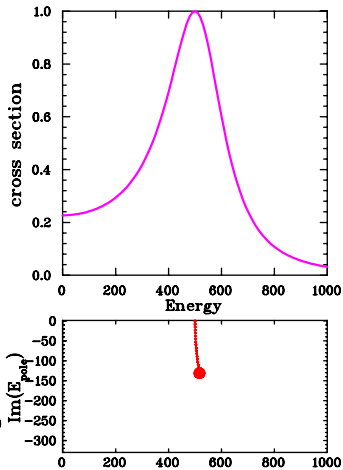
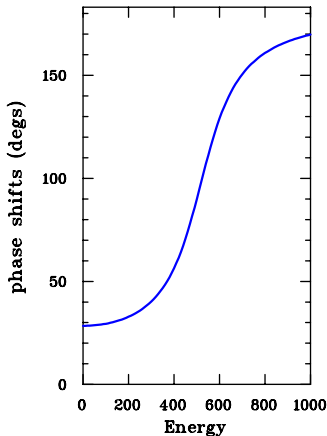
Resonances: exercise - Breit Wigner parameterization

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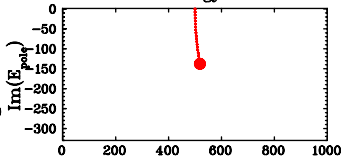
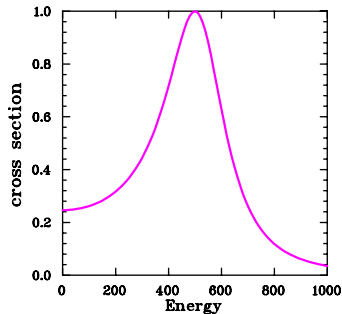
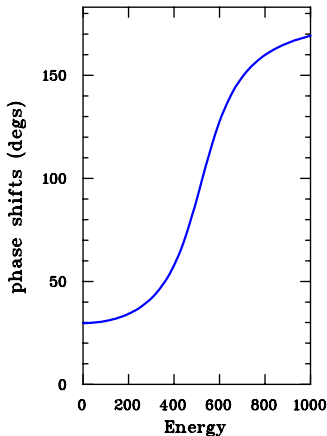
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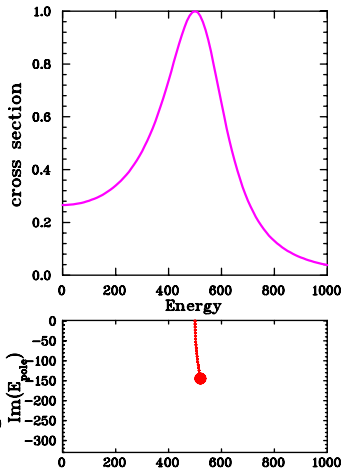
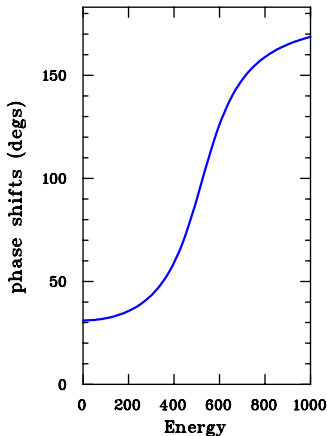
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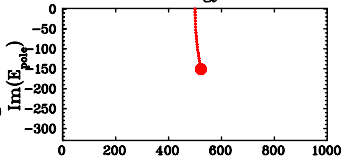
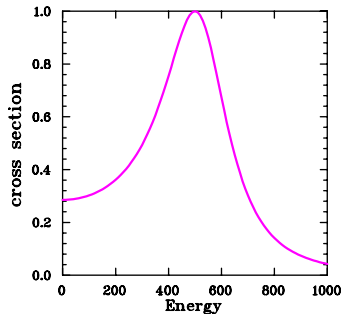
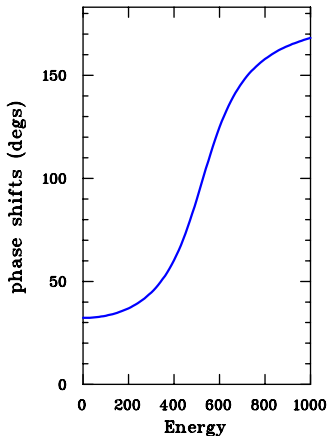
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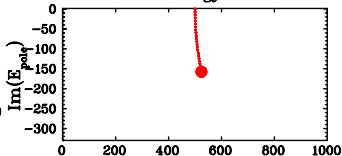
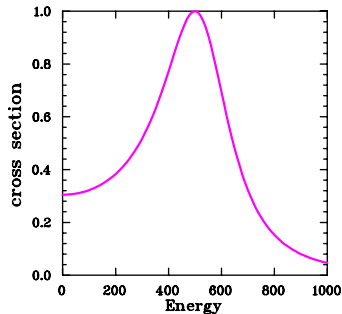
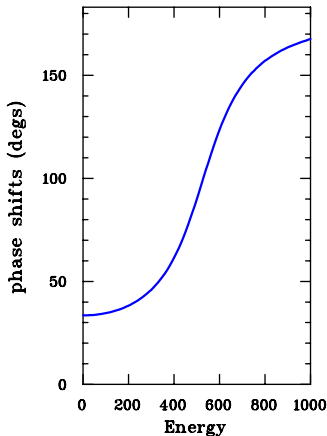
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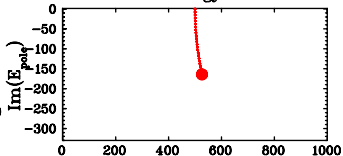
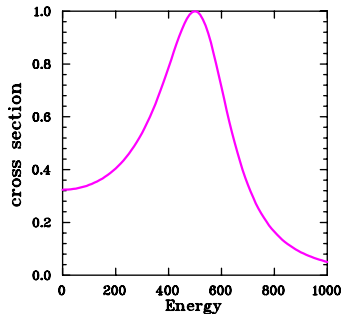
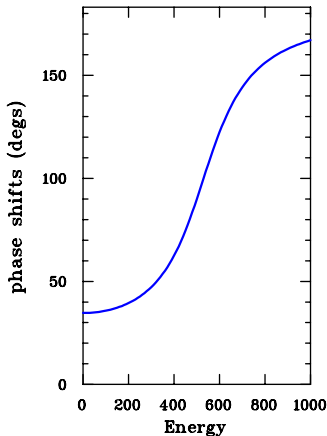
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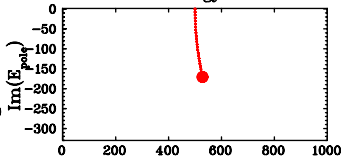
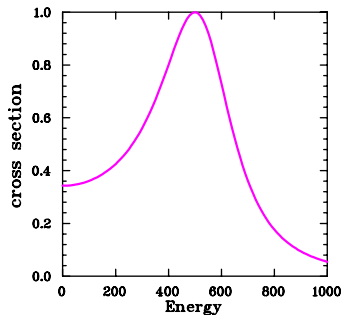
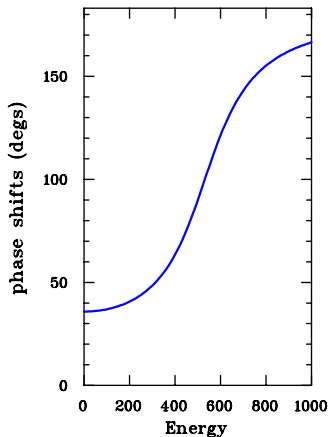
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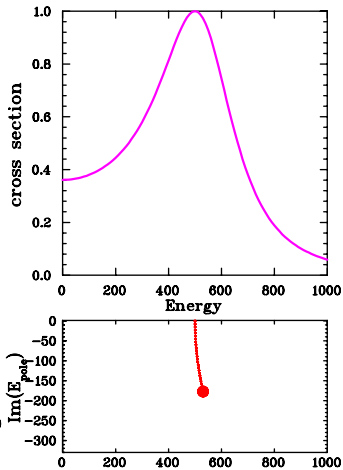
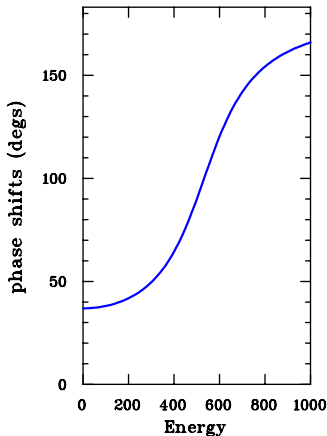
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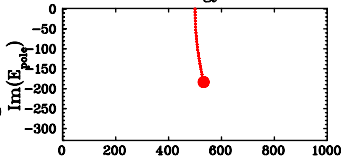
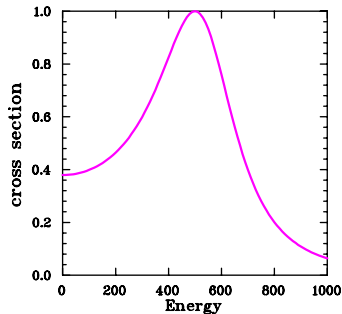
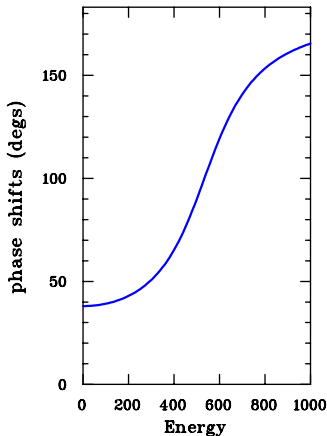
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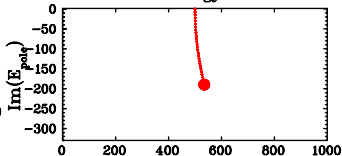
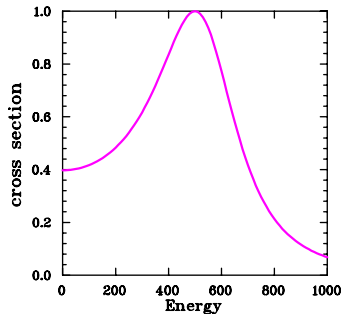
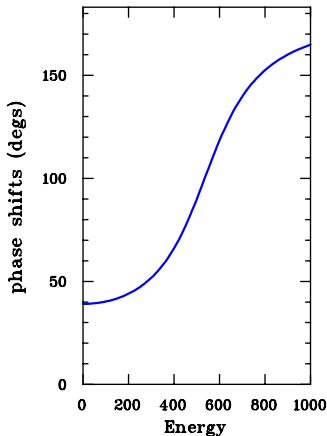
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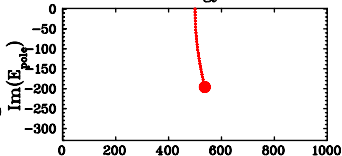
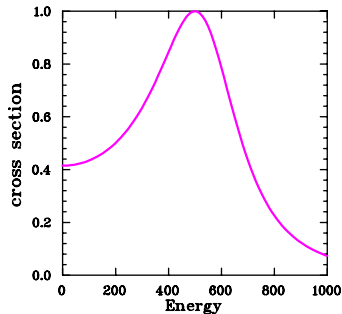
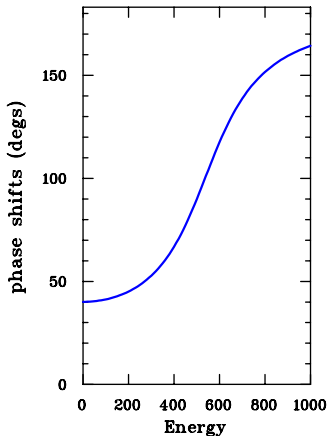
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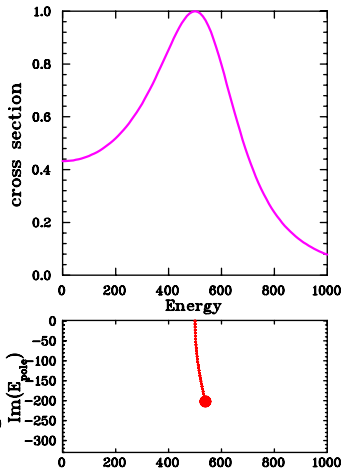
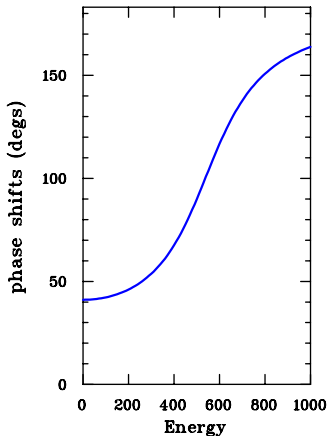
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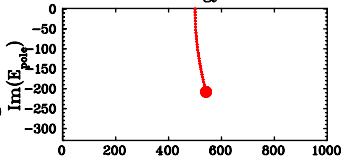
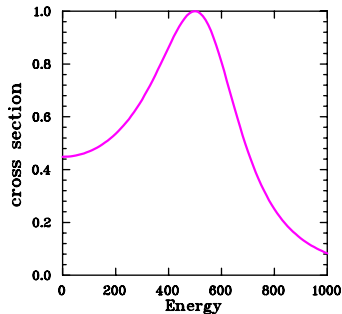
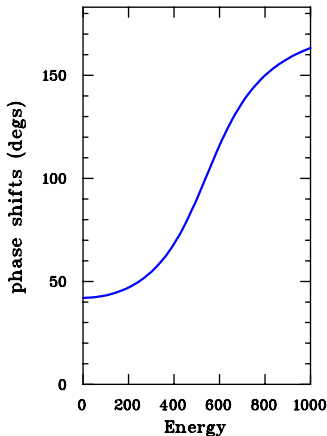
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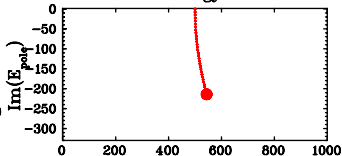
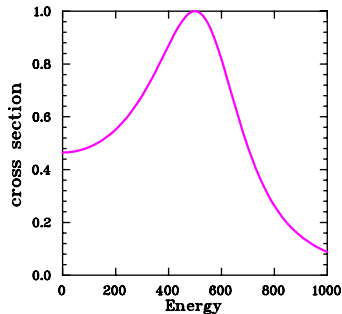
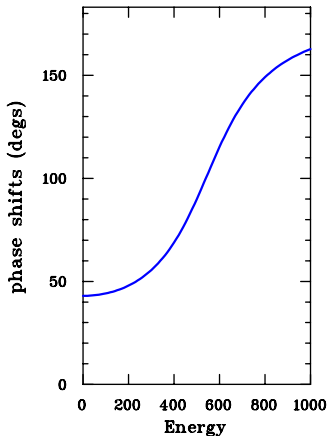
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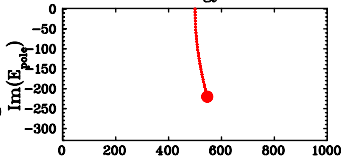
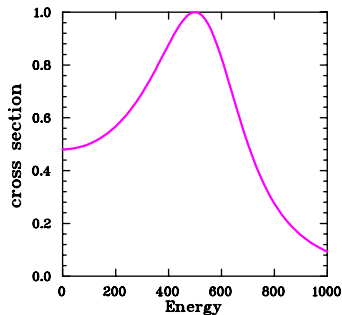
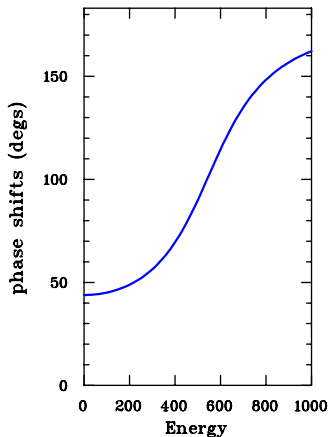
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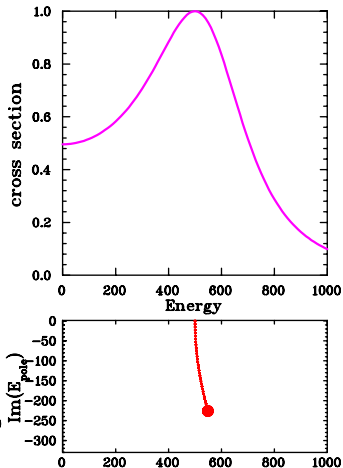
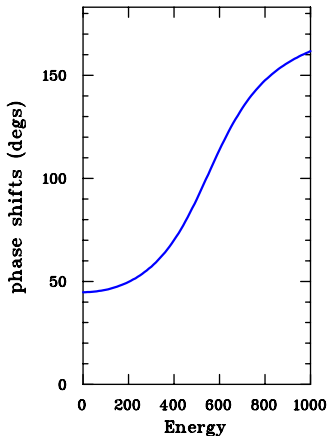
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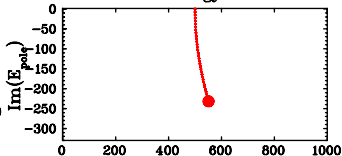
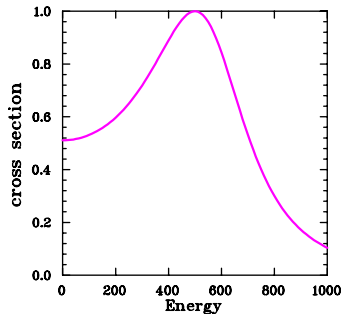
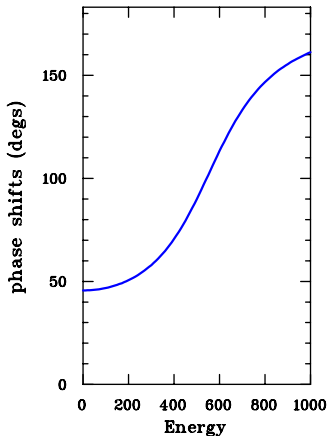
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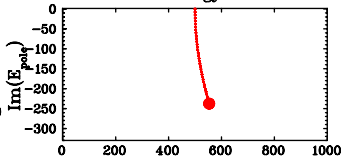
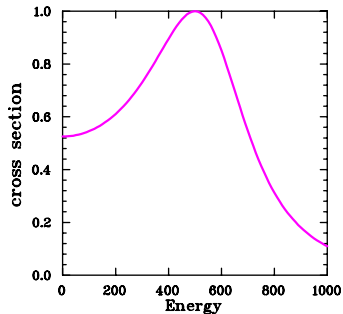
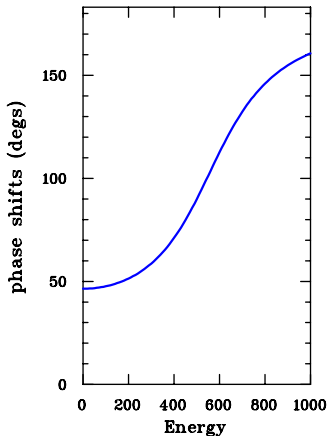
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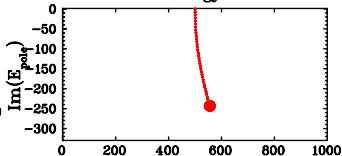
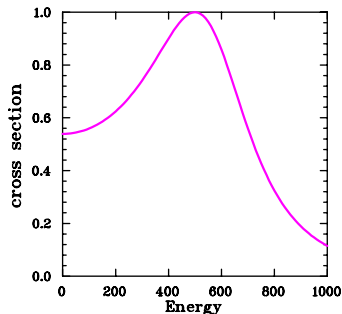
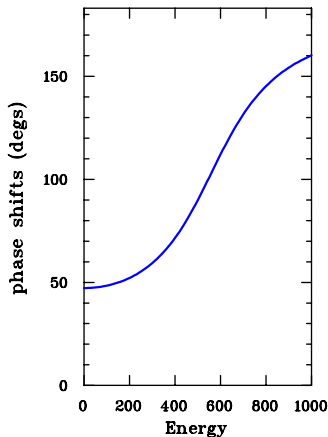
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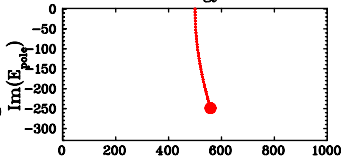
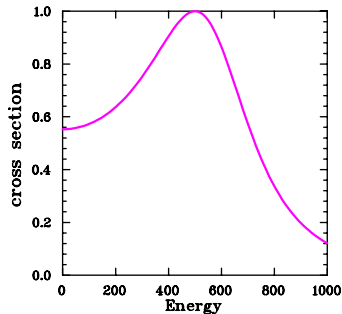
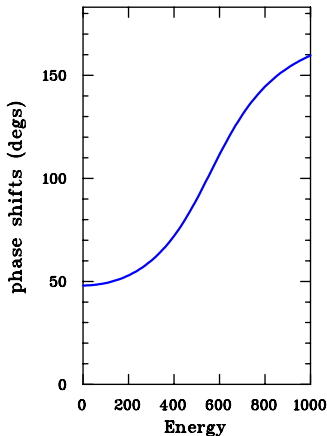
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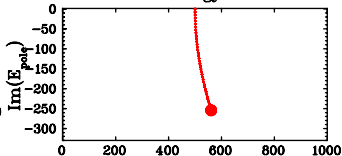
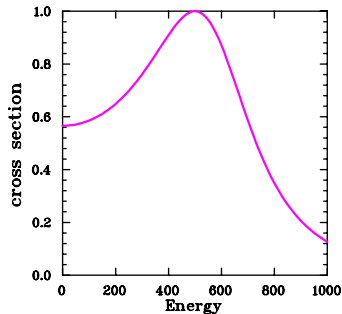
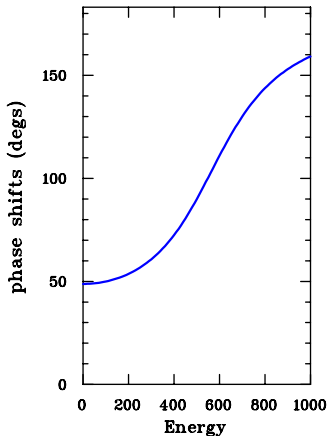
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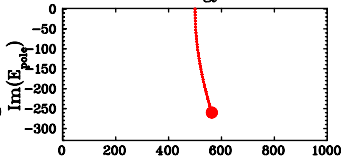
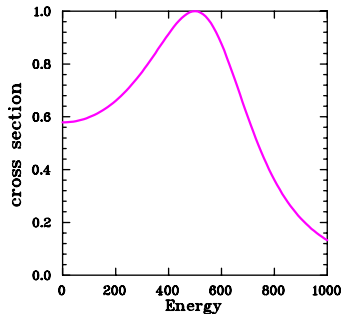
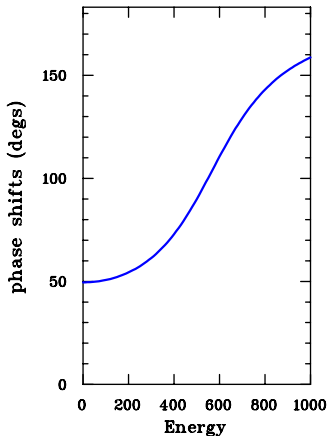
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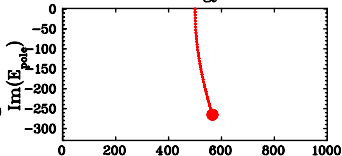
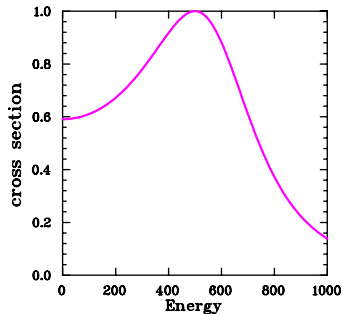
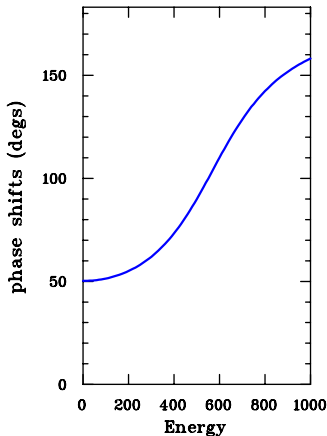
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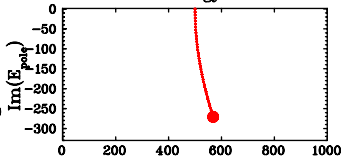
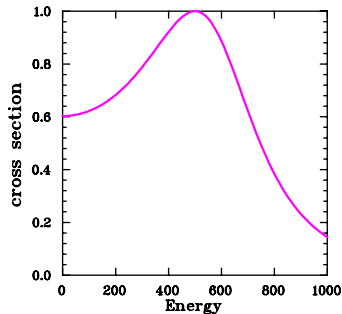
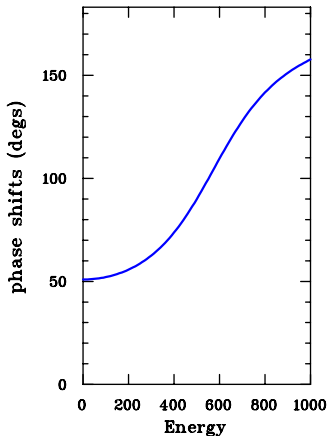
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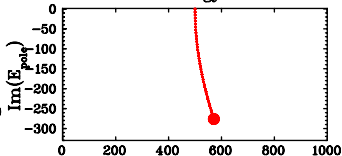
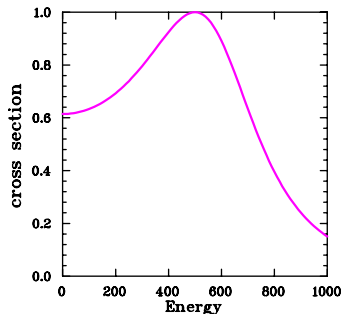
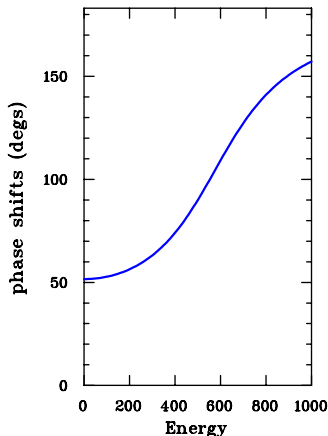
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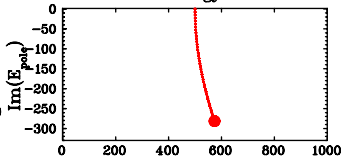
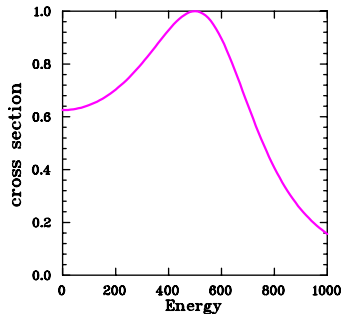
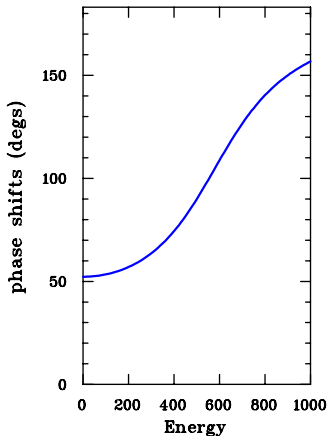
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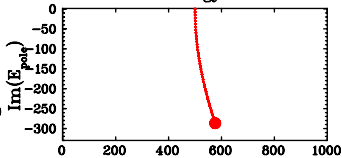
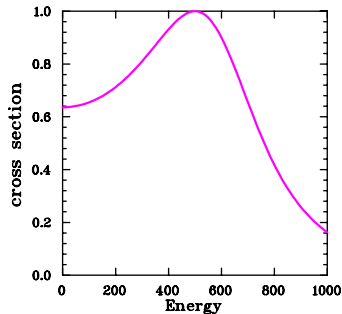
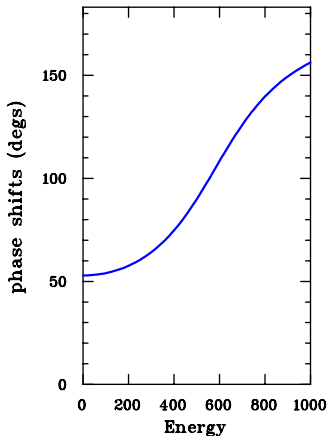
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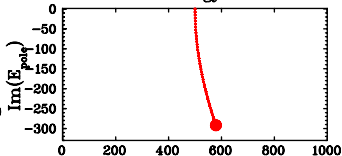
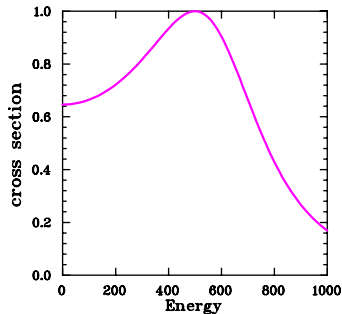
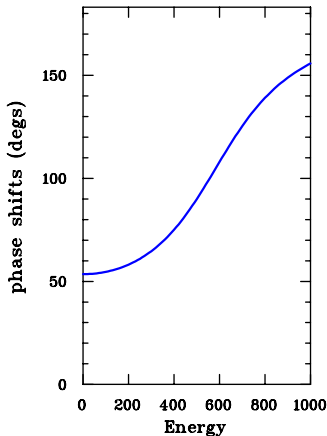
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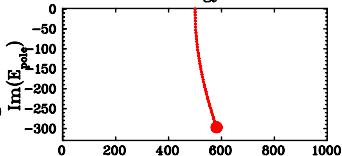
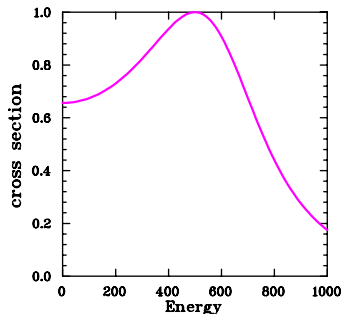
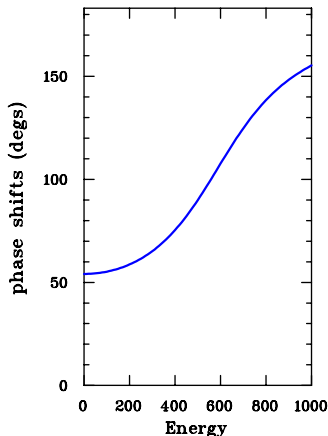
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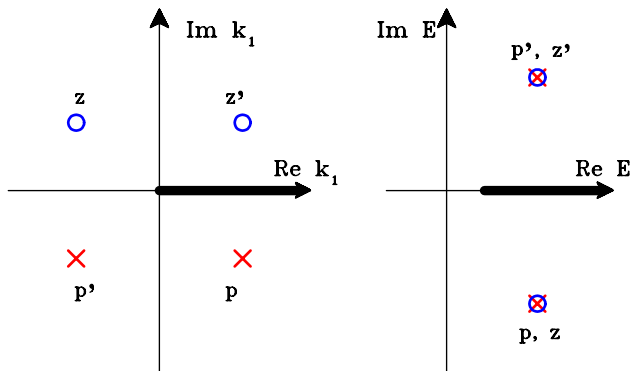
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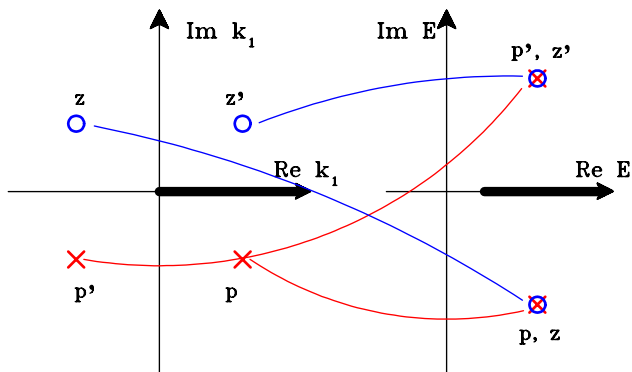
Complex momentum and energy space frame

$$E = 2\sqrt{(\pm k)^2 + m^2}$$



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Poles on which Riemann sheet?

- ▶ single resonance is seen as a bump of $\sigma(E)$ (or $N(E)$) around given $E = M_R$,
- ▶ resonance means something like source of increased particle emission,
- ▶ this bump can be always parametrized/approximated by function with poles in the complex momentum plane

Outgoing wave function of interacting particles: $\Phi \sim \frac{e^{ikr}}{r}$

At $k = a - ib$ it becomes $\Phi \sim \frac{e^{br}}{r}$

For resonances ($b > 0$) - exploding source of particles and
for bound states ($b < 0$) - implosion of particles.

One channel scattering

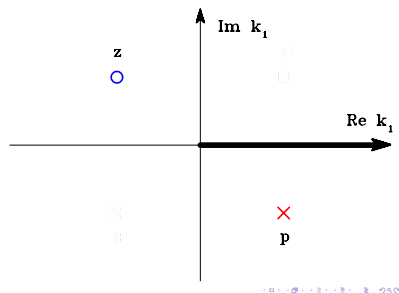
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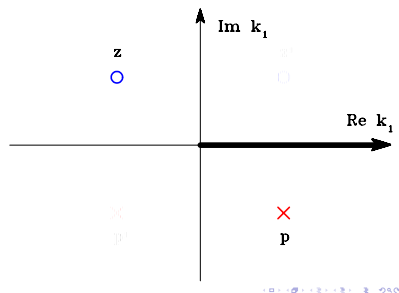
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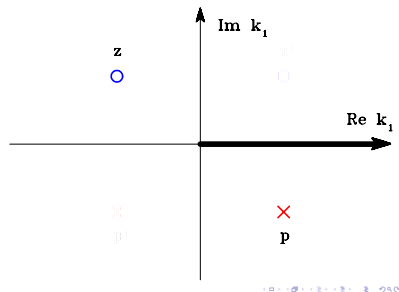
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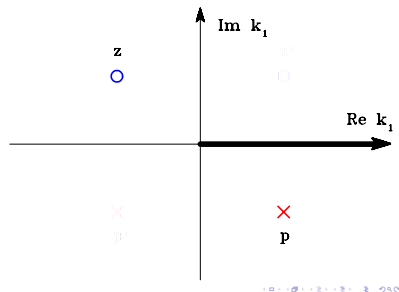
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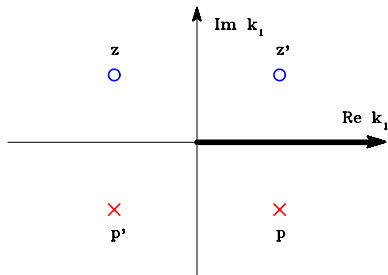
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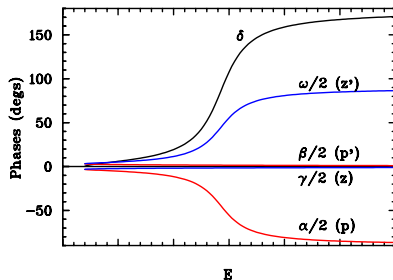
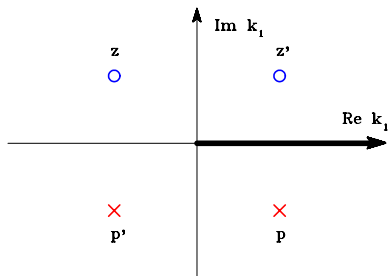
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- ▶ and $\delta = (-\alpha - \beta + \gamma + \omega)/2$

$$\text{angle} = \text{ArcTan}\left(\frac{-\text{Im}k_j}{k - \text{Re}k_j}\right)$$



Breit Wigner approximation

- ▶ $BW(E) = \frac{\Gamma/2k}{M_{BW} - E - i\Gamma/2}$
- ▶ $BW(E) = \frac{S_I(E) - 1}{2ik}$
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Pole and mass of a resonance

Let's imagine good fit of an amplitude to the data \rightarrow mass M_{BW} at $\delta = 90^\circ$

- ▶ $A_{S_{notU}}$ ($|S| \neq 1$): single pole at $k_j = a - ib$
then $\delta = \text{ArcTan}\left(\frac{-2bk}{k^2 - a^2 - b^2}\right)$ and $M_{BW} \neq 2\sqrt{a^2 + m^2}$
- ▶ A_{S_U} ($|S| = 1$): two symmetric poles at $k_j = a - ib$ and $k = -a - ib$
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Let's check it for $\rho(770)$:

$M_{BW} = 775.26 \pm 0.25$ MeV (PDG'2016),

$\Gamma = 149.1 \pm 0.8$ MeV (PDG'2016),

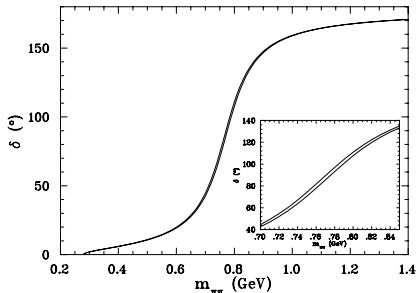
$2\sqrt{a^2 + m^2} < M_{BW}$ by ≈ 9 MeV !!!

Left (upper) line:

A_{BW} fitted to the data

Right (lower) line:

A_{S_U} with $2\sqrt{a^2 + m^2} = M_{BW}$



More resonances (but still one channel)

- ▶ **Isobar model:** adding amplitudes (even unitary ones):
 $T_{1,2} = T_1 + T_2 = \frac{S_1 - 1}{2ik} + \frac{S_2 - 1}{2ik} \rightarrow S_1 + S_2 = e^{2i\delta_1} + e^{2i\delta_2}$
of course $|S_1 + S_2| \neq 1$, violates unitarity!!!
- ▶ **Product of S matrices:** $|S_1 S_2| = 1$ in elastic case and $|S_1 S_2| < 1$ in inelastic case ($S = \eta e^{2i\delta}$)
For example $S_{1,2} = \frac{(-k - k_1)(-k + k_1^*)(-k - k_2)(-k + k_2^*)}{(k - k_1)(k + k_1^*)(k - k_2)(k + k_2^*)}$
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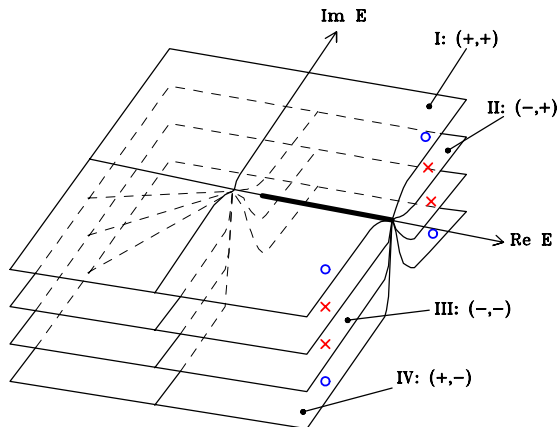
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1 pole $\longrightarrow 2^{n-1}$ poles due to $(\pm k)^2$ ambiguity and 2^n Riemann sheets

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- ▶ Displacement:

$$S_{11} = \frac{D_1(-k_1)D_2(k_2)}{D_1(k_1)D_2(k_2)} \text{ in decoupled case}$$

$$S = \frac{D_1(-k_1)D_2(k_2) + C(-k_1, k_2)}{D_1(k_1)D_2(k_2) + C(k_1, k_2)} \text{ in coupled case}$$

Multiplication and displacement of S matrix poles

► Multiplication:

1 pole $\longrightarrow 2^{n-1}$ poles due to $(\pm k)^2$ ambiguity and 2^n Riemann sheets

► Displacement:

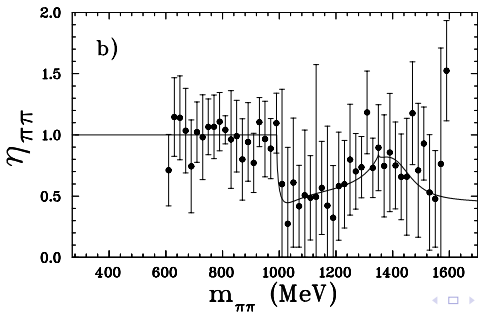
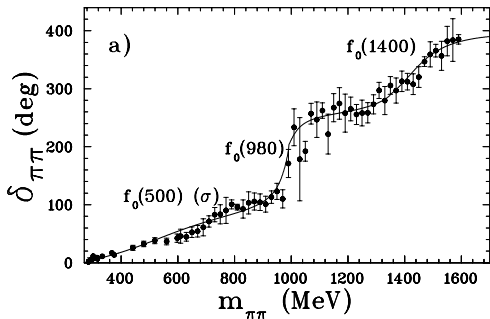
$$S_{11} = \frac{D_1(-k_1)D_2(k_2)}{D_1(k_1)D_2(k_2)} \text{ in decoupled case}$$

$$S = \frac{D_1(-k_1)D_2(k_2) + C(-k_1, k_2)}{D_1(k_1)D_2(k_2) + C(k_1, k_2)} \text{ in coupled case}$$

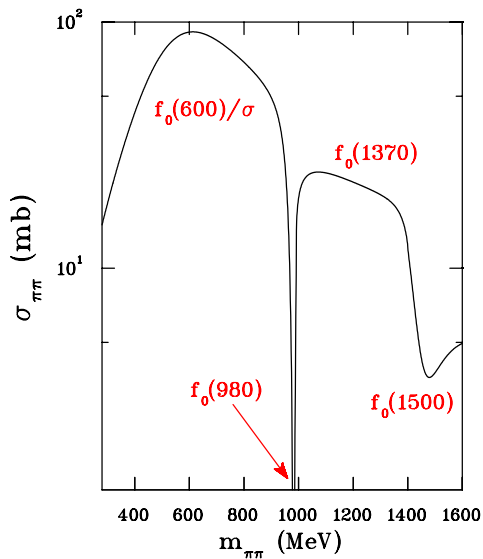
$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} = \begin{pmatrix} \frac{D(-k_1, k_2)}{D(k_1, k_2)} & S_{12} \\ S_{21} & \frac{D(k_1, -k_2)}{D(k_1, k_2)} \end{pmatrix}$$

$$\text{where } S_{12}^2 = S_{21}^2 = S_{11}S_{22} - \frac{D(-k_1, -k_2)}{D(k_1, k_2)}$$

Example for two channels: $J I = S 0$ wave



Puzzling (J/ψ) S_0 wave $\pi\pi$ cross section



Example for two channels: $Jl = S0$ wave

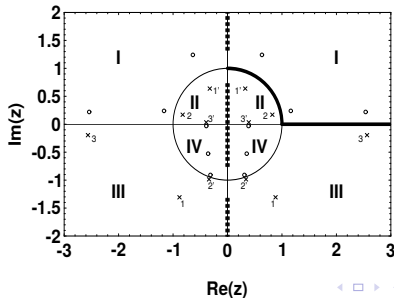
Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	(-, -) : III
1'	511.4	-230.6	(-, +) : II
2	982.0	-36.9	(-, +) : II
2'	432.4	-8.4	(-, -) : III
3	1431.7	-79.3	(-, -) : III
3'	1394.9	-120.6	(-, +) : II

Example for two channels: $JJ = S0$ wave

Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	(-, -) : III
1'	511.4	-230.6	(-, +) : II
2	982.0	-36.9	(-, +) : II
2'	432.4	-8.4	(-, -) : III
3	1431.7	-79.3	(-, -) : III
3'	1394.9	-120.6	(-, +) : II

$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}$$

Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierowego S_{11} macierzy rozpraszania dla dopasowania do zestawu D_{CKM} A. Gruba linia cięta oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\bar{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|\zeta| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.

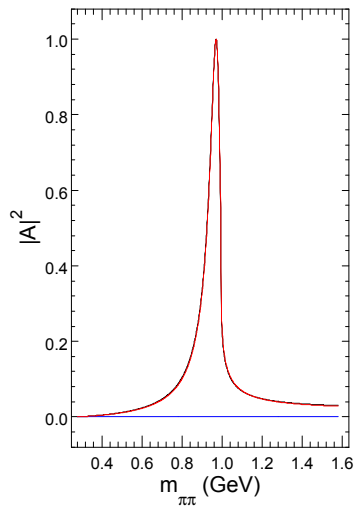
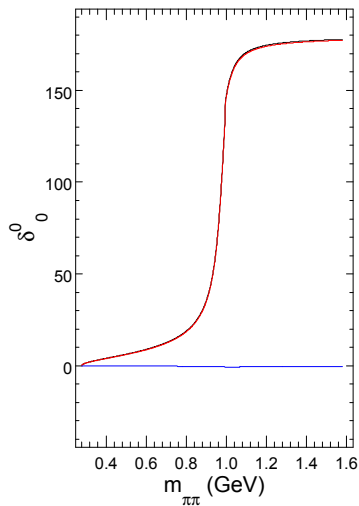


For two channels" TWO POLES 2 and 2' ($f_0(980)$)

2: 982.0-i*36.9

2': 432.4-i*8.4

both 2 and 2'

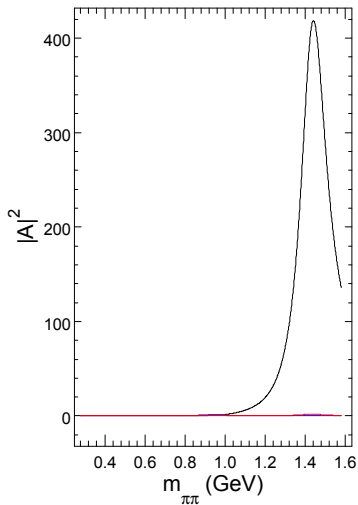
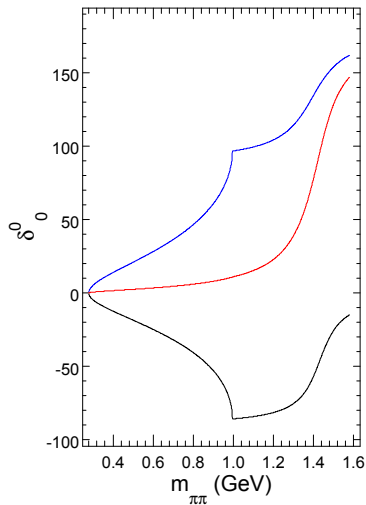


For two channels" TWO POLES 3 and 3' ($f_0(1400)$)

3: $1431.7-i*79.3$

3': $1394.9-i*120.6$

both 3 and 3'

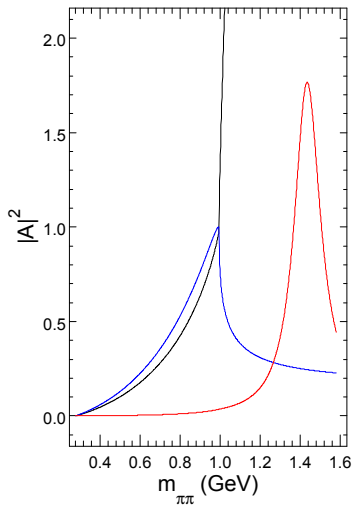
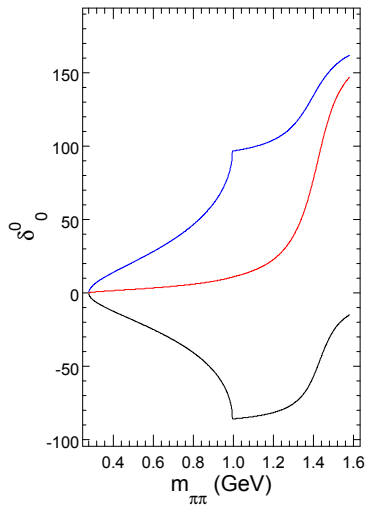


For two channels" TWO POLES 3 and 3' ($f_0(1400)$)

3: $1431.7-i*79.3$

3': $1394.9-i*120.6$

both 3 and 3'

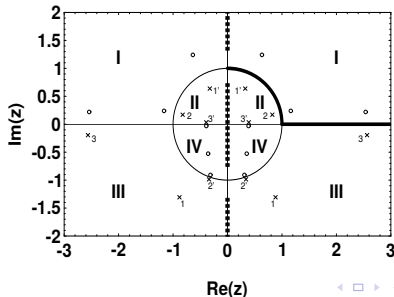


Example for two channels: $J0 = S0$ wave

Pole	$Re E_{pole}$ MeV	$Im E_{pole}$ MeV	R. sheet
1	639.6	-323.9	(-, -) : III
1'	511.4	-230.6	(-, +) : II
2	982.0	-36.9	(-, +) : II
2'	432.4	-8.4	(-, -) : III
3	1431.7	-79.3	(-, -) : III
3'	1394.9	-120.6	(-, +) : II

$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}$$

Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierzyowego S_{11} macierzy rozpraszania dla dopasowania do zestawu D_{CKM} A. Gruba linia ciągła oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\bar{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|\zeta| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.



More than 2 channels

- ▶ more poles and more Riemann sheets (2^n)
- ▶ no similar "z" variable

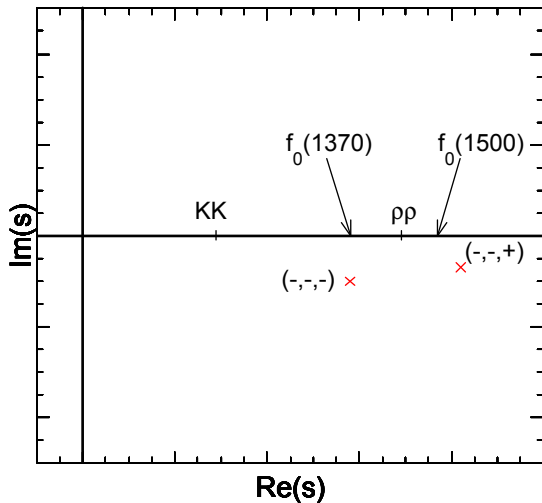
2^n Riemann sheets for n channels

channel	$C = 0$		$C = 1$		sign Imk_π, Imk_K, Imk_3	sheet
	ReE	ImE	ReE	ImE		
$\pi\pi$	658	-607	564	-279	$-, -, -$	VI
			518	-261	$-, +, +$	II
			211	0	$-, +, -$	VII
			532	-315	$-, -, +$	III
			235	0	$+, +, -$	VIII
$\pi\pi$	1346	-275	1405	-74	$-, -, -$	VI
			1445	-116	$-, +, +$	II
			1424	-94	$-, +, -$	VII
			1456	-47	$-, -, +$	III
$K\bar{K}$	881	-498	170	0	$+, -, -$	V
			159	0	$-, -, -$	VI
			418	-10	$-, -, +$	III
			1038	-204	$-, +, -$	VII
			988	-31	$-, +, +$	II
$\sigma\sigma$	118	-2227	4741	-4688	$-, -, -$	VI
			3687	-2875	$-, +, -$	VII
			3626	-3456	$+, -, -$	V
			3533	-579	$+, +, -$	VIII

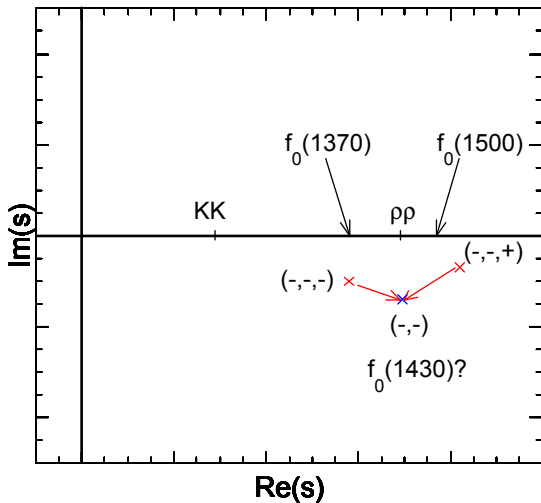
2^n Riemann sheets for n channels

channel	$C = 0$		$C = 1$		sign Imk_π, Imk_K, Imk_3	sheet
	ReE	ImE	ReE	ImE		
$\pi\pi$	658	-607	564	-279	$-, -, -$	VI
			518	-261	$-, +, +$	II $\leftarrow f_0(500)$
			211	0	$-, +, -$	VII
			532	-315	$-, -, +$	III
			235	0	$+, +, -$	VIII
$\pi\pi$	1346	-275	1405	-74	$-, -, -$	VI $\leftarrow f_0(1370)?$
			1445	-116	$-, +, +$	II
			1424	-94	$-, +, -$	VII
			1456	-47	$-, -, +$	III $\leftarrow f_0(1500)?$
$K\bar{K}$	881	-498	170	0	$+, -, -$	V
			159	0	$-, -, -$	VI
			418	-10	$-, -, +$	III
			1038	-204	$-, +, -$	VII
			988	-31	$-, +, +$	II $\leftarrow f_0(980)$
$\sigma\sigma$	118	-2227	4741	-4688	$-, -, -$	VI
			3687	-2875	$-, +, -$	VII
			3626	-3456	$+, -, -$	V
			3533	-579	$+, +, -$	VIII

$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 1$



$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 0$



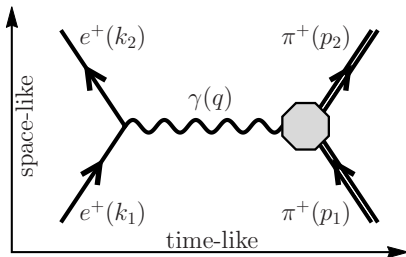
Pion electromagnetic form factor in the P wave

What are the correct $\rho(770)$ meson mass and width values?

PRD 96, 113004 (2017)

Erik Bartoš, Stanislav Dubnička, Andrej Liptaj Anna Zuzana Dubničkova and RK

$$\langle \pi^+(p_2) | J_\pi^\mu(0) | \pi^-(p_1) \rangle = e (p_1 + p_2)^\mu F_\pi(q^2)$$



Gounaris-Sakurai pion EM FF and $\rho(770)$

Gounaris-Sakurai pion electromagnetic form factor at the elastic region.

What are the correct $\rho(770)$ meson mass and width values?

Erik Bartoš, Stanislav Dubnička, Andrej Liptaj Anna Zuzana Dubničková and RK
PRD 96, 113004 (2017)

$$\sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3s} \beta_\pi^3(s) \left| F_\pi^{EM, l=1}(s) + \text{Re}^{i\phi} \frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega} \right|^2$$

where pion "velocity" $\beta_\pi(s) = \sqrt{\frac{s-4m_\pi^2}{s}}$, R - amplitude for $\rho - \omega$ interference (free parameter), phase $\phi = \text{ArcTan} \frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ is the $\delta_1^1 = \delta_\rho$ fixed at $s = m_\omega^2$

Fit to data for $\sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-)$

- M. Ablikin et al. (BESIII Collaboration), Phys. Lett. B 753, 629 (2016).
- J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 032013 (2012).

Gounaris-Sakurai pion electromagnetic form factor

G. J. Gounaris and J. J. Sakurai, *Phys. Rev. Lett.* 21, 244 (1968)

Assumption: $\frac{q^3}{\sqrt{s}} \text{Cotg}\delta_1^1(s) = a + bq^2 + h(s)q^2$ where $h(s) = \frac{2q}{\pi\sqrt{s}} \text{Log}\left(\frac{\sqrt{s}+2q}{2m_\rho}\right)$

Then $F_\pi^{GS}(s) = \frac{\sqrt{s}}{q^3} \frac{1}{\text{Cotg}\delta_1^1(s)-i}$ - no direct dependence on $\rho(770)$ parameters,
however taking into account two conditions:

▶ $\text{Cotg}\delta_1^1(s) \Big|_{s=m_\rho^2} = 0$ and

▶ $F_\pi^{BW}(s) = \frac{m_\rho^2}{m_\rho^2 - s - im_\rho\Gamma_\rho} \rightarrow \delta_1^1(s) = \text{ArcTan} \frac{m_\rho\Gamma_\rho}{m_\rho^2 - s} \rightarrow \frac{d\delta_1^1(s)}{ds} \Big|_{s=m_\rho^2} = \frac{1}{m_\rho\Gamma_\rho}$

$$a = \frac{4q_\rho^5}{m_\rho^2\Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2)$$

$$b = -\frac{4q_\rho^3}{m_\rho^2\Gamma_\rho} - 4q_\rho^4 h'(m_\rho^2) - h(m_\rho^2)$$

$$F_\pi^{GS}(s) = \frac{m_\rho^2 + m_\rho\Gamma_\rho \left[\frac{3m_\rho^2}{\pi q_\rho^2} \log\left(\frac{m_\rho + 2mq_\rho}{2m_\pi}\right) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\rho^2 m_\rho}{\pi q_\rho^3} \right]}{(m_\rho^2 - s) + \Gamma_\rho \frac{m_\rho}{q_\rho^3} \left[q^2 (h(s) - h(m_\rho^2)) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - s) \right] - im_\rho\Gamma_\rho \left(\frac{q}{q_\rho}\right)^3 \frac{m_\rho}{\sqrt{s}}}$$

Fit to unified BESIII-BABAR data at the elastic region using G-S model

- ▶ $\chi^2 = 40.6$ pdf
- ▶ $m_\rho = (775.73 \pm 0.10)$ MeV
- ▶ $\Gamma_\rho = (126.51 \pm 0.13)$ MeV
- ▶ PDG'2017:

$$m_\rho = 775.26 \pm 0.25 \text{ MeV (from } e^+e^- \rightarrow \pi^+\pi^-)$$

$$m_\rho = 769.0 \pm 0.9 \text{ MeV (other)}$$

$$\Gamma_\rho = 147.8 \pm 0.9 \text{ MeV (from } e^+e^- \rightarrow \pi^+\pi^-)$$

$$\Gamma_\rho = 150.9 \pm 1.7 \text{ MeV (other)}$$

in Ref. [6].

Now, the $\rho^0(770)$ meson parameters will be determined by an application of the U&A model of the pion EM FF to an optimal description of the same data on the total cross section of the $e^+e^- \rightarrow \pi^+\pi^-$ process.

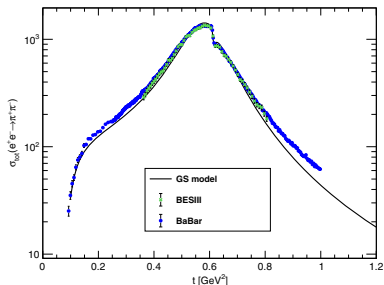


FIG. 2. Optimal description of the unified BESIII-BABAR data on $\sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-)$ at the elastic region by the pion EM FF G.-S. model.

Fit to unified BESIII-BABAR data at the elastic region using U&A model

- ▶ $F_{\pi}^{VDM}(s) = \frac{m_{\rho}^2 f_{\rho\pi\pi} / f_{\rho}}{m_{\rho}^2 - s}$
- ▶ $F_{\pi}^{VDM}(q) = \frac{(i-q_{\rho})(i+q_{\rho})}{(q-q_{\rho})(q+q_{\rho})} f_{\rho\pi\pi} / f_{\rho}$
- ▶ $F_{\pi}^{VDM}(q) = \frac{(q-q_Z)(i-q_P)(i-q_{\rho})(i+q_{\rho})}{(q-q_P)(i-q_Z)(q-q_{\rho})(q+q_{\rho})} f_{\rho\pi\pi} / f_{\rho}$
- ▶ $\chi^2 = 1.54 \text{ pdf}$
- ▶ $m_{\rho} = (763.03 \pm 0.14) \text{ MeV}$
- ▶ $\Gamma_{\rho} = (144.8 \pm 0.23) \text{ MeV}$

The optimal description of the recent data [1,2] on the total cross section of the $e^+e^- \rightarrow \pi^+\pi^-$ process at the region of the ρ meson resonance by (21) (see Fig. 3), achieved with $\chi^2/ndf = 1.5443$, gives the ρ meson mass and width

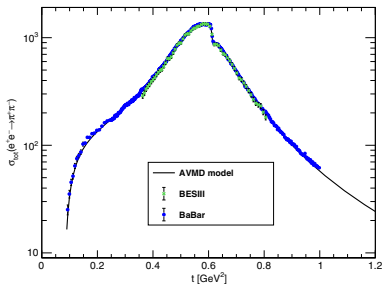
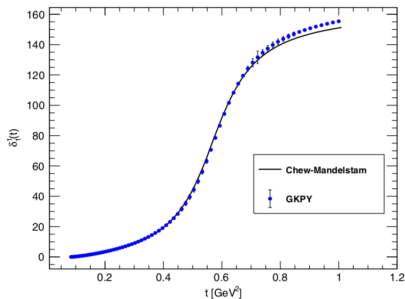


FIG. 3. Optimal description of the unified BESIII-BABAR data on $\sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-)$ at the elastic region by the pion EM FF U&A model.

FIG. P-wave effect

Fit to δ_1^1 data using Chew-Mandelstam type effective-range formula

- ▶ $\frac{q^3}{\sqrt{s}} \text{Cotg}\delta_1^1(s) = a + bq^2 + h(s)q^2$
where $h(s) = \frac{2q}{\pi\sqrt{s}} \text{Log}\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
- ▶ $a = 0.2860 \pm 0.0011 \text{ MeV}^2$,
 $b = -2.7025 \pm 0.0089$
- ▶ $\chi^2 = 2.45 \text{ pdf}$
- ▶ $m_\rho = (772.42 \pm 0.03) \text{ MeV}$
- ▶ $\Gamma_\rho = (153.85 \pm 0.11) \text{ MeV}$



$$\delta_1^1(q) = \arctg \frac{A_1 q + A_2 q^3 + A_3 q^5 + \dots}{1 + A_2 q^2 + A_4 q^4 + \dots}, \quad (26)$$

Fit to δ_1^1 data

► $F_\pi^{EM, I=1}(s) =$

$$P_n(s) \exp \left[\frac{s}{\pi} \int_4^\infty \frac{\delta_1^1(s')}{s'(s' - s)} ds' \right]$$

► $\delta_1^1(q) = \text{ArcTan} \frac{A_3 q^3 + A_5 q^5 + \dots}{1 + A_2 q^2 + A_4 q^4 + \dots}$

► $\equiv \text{Log} \frac{(q - q_2)(q - q_3)(q - q_4)(q - q_5)}{(q - q_2^*)(q - q_3^*)(q - q_4^*)(q - q_5^*)}$

► $\chi^2 = 0.024$ pdf

► $m_\rho = (763.56 \pm 0.51) \text{ MeV}$

► $\Gamma_\rho = (143.09 \pm 0.82) \text{ MeV}$

where $A_1 \equiv 0$ in order to secure the threshold behavior of $\delta_1^1(q)$. An optimal description of the GKPY phase shift $\delta_1^1(q)$ data is achieved (see Fig. 5) with $\chi^2/ndf = 0.0244$ and four nonzero coefficients A_2 , A_3 , A_4 , and A_5 .

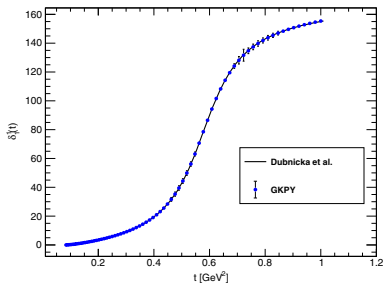


FIG. 5. Optimal description of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with model-independent parametrization (26).

Generalisation of Gounaris-Sakurai model to excited ρ mesons $\rho(1450)$ and $\rho(1700)$

- ▶
$$F_\pi = \frac{1}{1+\beta+\gamma} \left[F_{\rho(770)}^{GS}(s) \left(1 + \delta \frac{s}{m_\omega^2} BW_\omega(s) \right) + \beta F_{\rho(1450)}^{GS}(s) + \gamma F_{\rho(1700)}^{GS}(s) \right]$$
- ▶ $\chi^2 = 0.98 \text{ pdf}$

Generalisation of U&A model to excited ρ mesons

- ▶
$$F_\pi = \frac{\Pi(q-q_i)}{\Pi(q+q_i^*)}$$
- ▶ $\chi^2 = 1.84 \text{ pdf}$

ERIK BARTOŠ *et al.*

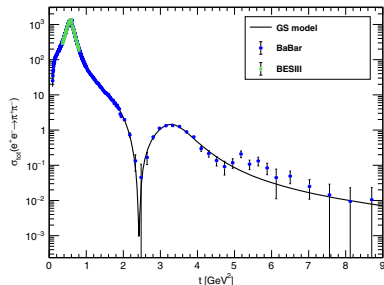


FIG. 6. Optimal description of the unified BESIII-BABAR complete data on $\sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-)$ by the generalized pion EM FF G.-S. model.

determined by the original pion EM FF G.-S. model (13) to be valid only at the elastic region.

A totally different situation is in a generalization of the U&A pion EM FF model. Here, the contribution of all three vector mesons is at an equal level. Only now, the effective inelastic threshold, which is left as a free parameter of the model, has to be taken into account explicitly. Therefore, instead of the q variable, the W variable

TABL
of BE
 e^+e^-
Analy

Param

m_ρ

$m_{\rho'}$

$m_{\rho''}$

Γ_ρ

$\Gamma_{\rho'}$

$\Gamma_{\rho''}$

χ^2/ndf

$W(t)$

is no

U&A

surf

Th

of all

Parameter	PDG MeV	G.S. MeV	U&A MeV
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2 pdf		0.98	1.84
		14 param.	11 param.

WHAT ARE THE CORRECT $\rho^0(770)$ MESON MASS ...

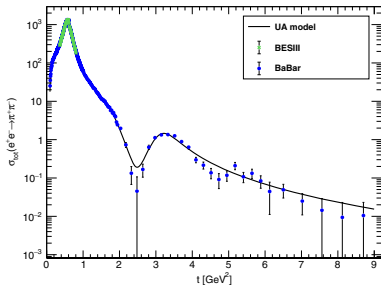


FIG. 7. Optimal description of the unified BESIII-BABAR complete data on $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ by the generalized pion

meson resonances. For this aim, totally the P-wave isovector mesons are included in the total cross-section. The UA model is exploited.

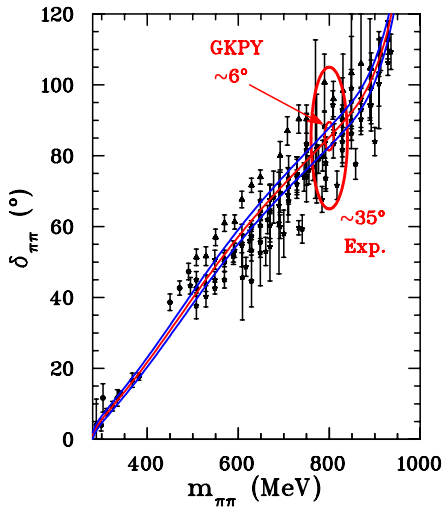
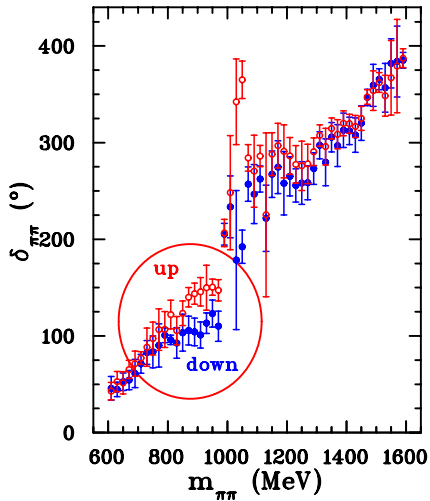
Just by a comparison of the ρ^0 meson parameters with the conclusion of the UA model, the most likely values are obtained.

We conclude that the $\rho^0(770)$ meson parameters are listed in Table II, i.e. $m_\rho = 774.81 \pm 0.01$ MeV, $\Gamma_\rho = 144.28 \pm 0.01$ MeV. These values are in good agreement with the UA model results.

We would like to thank the BESIII and BABAR collaborations for providing the data. This work is supported by the National Natural Science Foundation of China (11575001, 11575002, 11575003, 11575004, 11575005, 11575006, 11575007, 11575008, 11575009, 11575010, 11575011, 11575012, 11575013, 11575014, 11575015, 11575016, 11575017, 11575018, 11575019, 11575020, 11575021, 11575022, 11575023, 11575024, 11575025, 11575026, 11575027, 11575028, 11575029, 11575030, 11575031, 11575032, 11575033, 11575034, 11575035, 11575036, 11575037, 11575038, 11575039, 11575040, 11575041, 11575042, 11575043, 11575044, 11575045, 11575046, 11575047, 11575048, 11575049, 11575050, 11575051, 11575052, 11575053, 11575054, 11575055, 11575056, 11575057, 11575058, 11575059, 11575060, 11575061, 11575062, 11575063, 11575064, 11575065, 11575066, 11575067, 11575068, 11575069, 11575070, 11575071, 11575072, 11575073, 11575074, 11575075, 11575076, 11575077, 11575078, 11575079, 11575080, 11575081, 11575082, 11575083, 11575084, 11575085, 11575086, 11575087, 11575088, 11575089, 11575090, 11575091, 11575092, 11575093, 11575094, 11575095, 11575096, 11575097, 11575098, 11575099, 11575100, 11575101, 11575102, 11575103, 11575104, 11575105, 11575106, 11575107, 11575108, 11575109, 11575110, 11575111, 11575112, 11575113, 11575114, 11575115, 11575116, 11575117, 11575118, 11575119, 11575120, 11575121, 11575122, 11575123, 11575124, 11575125, 11575126, 11575127, 11575128, 11575129, 11575130, 11575131, 11575132, 11575133, 11575134, 11575135, 11575136, 11575137, 11575138, 11575139, 11575140, 11575141, 11575142, 11575143, 11575144, 11575145, 11575146, 11575147, 11575148, 11575149, 11575150, 11575151, 11575152, 11575153, 11575154, 11575155, 11575156, 11575157, 11575158, 11575159, 11575160, 11575161, 11575162, 11575163, 11575164, 11575165, 11575166, 11575167, 11575168, 11575169, 11575170, 11575171, 11575172, 11575173, 11575174, 11575175, 11575176, 11575177, 11575178, 11575179, 11575180, 11575181, 11575182, 11575183, 11575184, 11575185, 11575186, 11575187, 11575188, 11575189, 11575190, 11575191, 11575192, 11575193, 11575194, 11575195, 11575196, 11575197, 11575198, 11575199, 11575200).

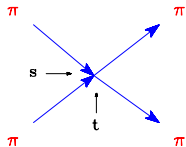
Experimental data for the $\pi\pi$ in the S0 wave (J1)

In PWA (CERN-Munich group'74) $A(s, t) \sim \text{Cos}(\theta_S - \theta_P)$



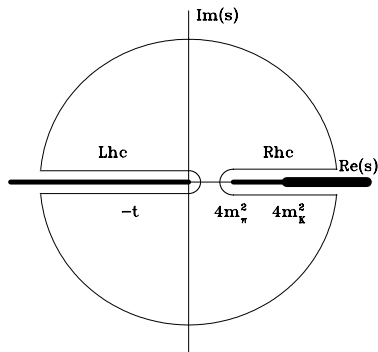
Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory \longleftrightarrow experiment

crossing symmetry:



$$\rightarrow \vec{T}_s(s, t) = \hat{C}_{st} \vec{T}_t(t, s)$$

$\vec{T}(s, t)$ + crossing symmetry \rightarrow dispersion relations for $4m_\pi^2 < s < \sim (1150 \text{ MeV})^2$



Once subtracted DR:

$$\begin{aligned} \text{Re } \vec{F}(s, t) &= \text{Re } \vec{F}(s_0, t) + \frac{s - s_0}{\pi} \\ &\times \left[\int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im } \vec{F}(s', t)}{(s' - s_0)(s' - s)} \right. \\ &\left. + \int_{-t}^{-\infty} ds' \frac{\text{Im } \vec{F}(s', t)}{(s' - s_0)(s' - s)} \right] \end{aligned}$$

GKPY equations and $\pi\pi$ amplitudes

partial waves: Jl

experiment

F1

D2

S0

D0

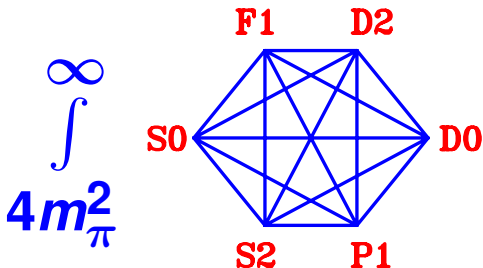
S2

P1

GKPY equations and $\pi\pi$ amplitudes

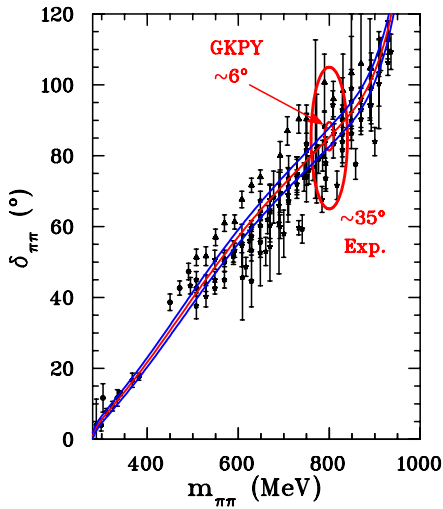
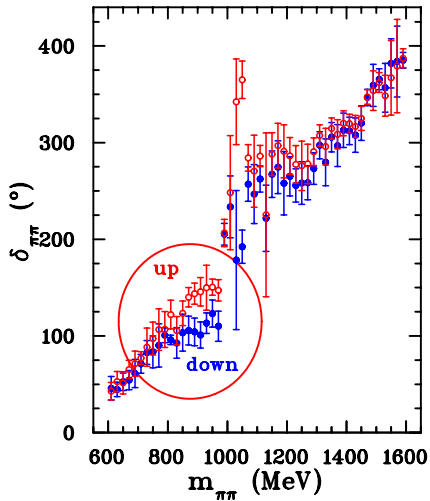
partial waves: Jl

experiment + theory (GKPY)



Experimental data for the $\pi\pi$ in the S0 wave (J1)

In PWA (CERN-Munich group'74) $A(s, t) \sim \text{Cos}(\theta_S - \theta_P)$



precise determination of $f_0(500)$ (σ) meson and threshold parameters

$f_0(500)$ (σ)

► **PDG 2010:**

$M = 400 - 1200$ MeV

$\Gamma = 2 \times (250 - 500)$ MeV

► **PDG 2012:**

$M = 400 - 550$ MeV

$\Gamma = 2 \times (200 - 350)$ MeV

► **GKPY:**

$E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV

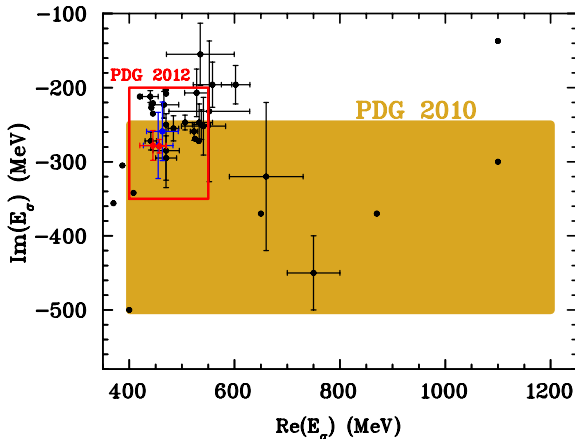
threshold parameters, e.g. a_0^0 :

► **ChPT + Roy eqs (Bern group):**

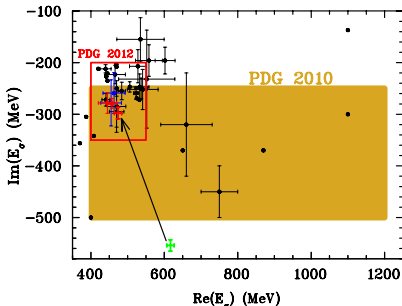
$0.220 \pm 0.005 m_\pi^{-1}$

► **GKPY:**

$0.220 \pm 0.008 m_\pi^{-1}$



what forces GKPY eqs to pull up-left the sigma pole?



Two things: **trigonometry** and **crossing symmetry algebra** lead to narrower and lighter σ .

Modified $\pi\pi$ amplitude with σ pole PRD 90, 116005 (2014) P. Bydzovský, 1, R. Kamiński, V. Nazari

Nothing more and nothing instead of it is needed.

Conclusions and what to do?

Conclusions:

- ▶ there is significant difference between parameters (especially mass) of a resonance using different methods of parameterisation of amplitudes,
- ▶ amplitudes should be unitary and analytic
- ▶ we should always determine what definition we use,
- ▶ the determination of the most significant poles in amplitudes becomes difficult for $n > 2$,

What to do?

- ▶ never use isobar model!
- ▶ anyway be carefull! = check the 2nd point of conclusions!
- ▶ possible parameterizations: made for wide energy range including many resonances we are interested in,
- ▶ please use only parameterization based on S - or K -matrix approach!
- ▶ possible degrees of freedom: poles themselves! mesons
- ▶ on $\pi\pi$, $K\bar{K}$... amplitudes in all partial waves up to 1.8 GeV ask RK - they are done
- ▶ on other channels ask RK - they may be done

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Conclusions on scalar resonances

Experimental situation in the 0^+0^{++} sector is finally clear (although without new experiments!),

- ▶ $f_0(500)$ (σ) - loosely bound $\pi\pi$ system or/and $2q2\bar{q}$ state,
- ▶ $f_0(980)$ - dynamically generated $2q2\bar{q}$ state or/and $K\bar{K}$ quasi bound state,
- ▶ $f_0(1370)$ - ordinary $q\bar{q}$ state,
- ▶ $f_0(1500)$ - ordinary $q\bar{q}$ state + glueball,
- ▶ $f_0(1720)$ - glueball

Probably $K_0^*(800)$ (κ) is also dynamical effect of coupling of $K_0^*(1430)$