HIC simulations with different EoS models

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Outline

I. Motivation
II. Mixed phase construction for cold and dense nuclear matter
III. Bayesian analysis for extracting properties of the nuclear equation of state from observational data
IV. HIC Simulations
V. Conclusions
Motivation: What if we have twin NS?

- Does hybrid neutron star exist?
- Does NS twins exist?
- Does CEP exist on QCD phase diagram?
Neutron star mass-radius relation

Seidov criterion for instability:

\[
\frac{\Delta \varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{2} \frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}}
\]

Finite-size effects in mixed phase

Coulomb interaction vs Surface tension
Tends to break up the like-charged regions into smaller ones
Requires minimization of the surface

Pasta Structures

The surface tension $\sigma$ is unknown and used as free parameter.

Mimicking the Pasta phase.

Schematic representation of the interpolation function $P_M(\mu)$, it has to go through three points: $P_H(\mu_H)$, $P_c + \Delta P$ and $P_Q(\mu_Q)$. 
The Interpolation Method

\[ P_M(\mu) = \sum_{q=1}^{N} \alpha_q (\mu - \mu_c)^q + (1 + \Delta_P) P_c \]

where \( \Delta_P \) is a free parameter representing additional pressure of the mixed phase at \( \mu_c \).

\[ P_H(\mu_H) = P_M(\mu_H) \]
\[ \frac{\partial^q}{\partial \mu^q} P_H(\mu_H) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_H) \]
\[ P_Q(\mu_Q) = P_M(\mu_Q) \]
\[ \frac{\partial^q}{\partial \mu^q} P_Q(\mu_Q) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_Q) \]

where \( q = 1, 2, \ldots, k \). All \( N + 2 \) parameters \( (\mu_H, \mu_Q \) and \( \alpha_q \), for \( q = 1, \ldots, N \) can be found by solving the above system of equations, leaving one parameter \( (\Delta_P) \) as a free one.

The Interpolation Method

The squared speed vs chemical potential given by the interpolation with $k = 1$ (upper left) $k = 2$ (upper right) and $k = 3$ (right).

The results of pasta mimicking

![Graphs showing the relationship between pressure (P) and density (ρ) for different pasta mimicking percentages.](image)
Robustness of third family solutions
Dependence on surface tension
Dependence on surface tension

\[ \sigma = 0 \text{ MeV/fm}^2 \]

\[ \sigma = 40 \text{ MeV/fm}^2 \]

\[ \sigma = 80 \text{ MeV/fm}^2 \]

\[ \sigma = 0 \text{ MeV/fm}^2 \]

\[ \sigma = 16 \text{ MeV/fm}^2 \]

\[ \sigma = 32 \text{ MeV/fm}^2 \]
Dependence on surface tension

\[
\sigma_c = d (P_c - P_0) + \sigma_0:
\]

\[
d = 0.45 \pm 0.02 \text{ fm},
\]

\[
P_0 = 40 \text{ MeV/fm}^3,
\]

and \[
\sigma_0 = 31.6 \pm 1.19 \text{ MeV/fm}^2
\]

\[
\Delta_P(\sigma) = \Delta_P(0) S(\sigma/\sigma_c; \beta): \quad \bar{\beta} = 0.64
\]

\[
S(x; \beta) = e^{-x}(1 - x^\beta)\theta(1 - x)
\]

Maslov, Yasutake, Blaschke, Ayriyan, Grigorian, Maruyama, Tatsumi, Voskresensky. PRC100, 025802 (2019)
Bayesian method

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.
One of the most frequent case is analysis of probable values of model parameters.

**Bayes' theorem:**

\[
p(H_1 | D, I) = \frac{p(D | H_1, I) p(H_1 | I)}{p(D | I)}
\]

- **Prior:** knowledge before experiment (logically)
- **Likelihood:** Probability for data if the hypothesis was true
- **Posterior:** Probability that the hypothesis is true given the data
- **Evidence:** normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!
Bayesian method

Formulation of set of models (set of hypothesis):
\[ \pi_i \text{ here } i = 0 \ldots N - 1 \]

Finding the \textit{a priori} probabilities of the models:
\[ P(\pi_i) = \frac{1}{N} \text{ for } \forall i = 0 \ldots N - 1 \]

Calculating the conditional probabilities of the events:
\[ P(E | \overrightarrow{\pi} _i) = \prod_\alpha P(E_\alpha | \overrightarrow{\pi} _i), \]
where \( \alpha \) is the index of the observational constraints.

Calculating the \textit{a posteriori} probabilities of the models:
\[ P(\overrightarrow{\pi} _i | E) = \frac{P(E | \overrightarrow{\pi} _i)P(\overrightarrow{\pi} _i)}{\sum_{j=0}^{N-1} P(E | \overrightarrow{\pi} _j)P(\overrightarrow{\pi} _j)} \]
Model EoS for Hybrid NS

DD2 with excluded volume plus color superconducting two-flavor quark matter, described within a nonlocal covariant chiral quark model.

\[ P(\mu) = P(\mu; \eta(\mu), B(\mu)) = -\Omega^{MFA}(\eta(\mu)) - B(\mu) \]

\[ \Omega^{MFA} = \frac{\tilde{\sigma}^2}{2G_S} + \frac{\tilde{\Delta}^2}{2H} - \frac{\tilde{\omega}^2}{2G_V} - \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \ln \det [ S^{-1}(\tilde{\sigma}, \tilde{\Delta}, \tilde{\omega}, \mu_{fc}) ] \]

\[
\frac{d\Omega^{MFA}}{d\tilde{\Delta}} = 0, \quad \frac{d\Omega^{MFA}}{d\tilde{\sigma}} = 0, \quad \frac{d\Omega^{MFA}}{d\tilde{\omega}} = 0.
\]

The parameter set for EoS model

The set of parameters of models could be represented in the parameter space with introduction of the vector of parameters, each vector is one fixed model from considered types of EoS model and transition construction:

$$\vec{\pi}_i = \left\{ \mu_{<}(j), \Delta P(k) \right\},$$

where $i = 0..N - 1$ and $i = N_2 \times j + k$ and $j = 0..N_1 - 1$, $k = 0..N_2 - 1$ and $N_1$ and $N_2$ are number of values of model parameters $\mu_<$ and $\Delta P$ correspondingly.
Stabile NS Configurations

![Graph showing stable NS configurations with mass and radius as parameters.]

- **PRS J0740+6620**
- **GW170817 M1**
- **GW170817 M2**

mass $[M_\odot]$: 0.0 to 2.5
radius $[\text{km}]$: 9 to 15

Legend:
- $\mu_c = 1058$
- $\mu_c = 1081$
- $\mu_c = 1104$
- $\mu_c = 1127$
- $\mu_c = 1150$
Likelihood of a EoS model for the mass constraint

\[ P(E_M | \pi_i) = \Phi(M_i, \mu_A, \sigma_A) \]

Here \( M_i \) is maximum mass of the given by \( \pi_i \), and \( \mu_A = 2.14 \, M_\odot \)
and \( \sigma_A = 0.105 \, M_\odot \) is the mass measurement of PSR J0740+6620

\[ 2.14 \pm 0.10 \, M_\odot \] [Cromartie et al., arXiv:1904.06759 (2019)].

Note, that here we replace previously used mass measurement for two
solar mass pulsar J0348+0432 \[ 2.01_{-0.04}^{+0.04} \, M_\odot \] [Antoniadis et al., Science 340, 6131 (2013)].
$\Lambda_1 - \Lambda_2$ diagram and observational constraints
Likelihood for the $\Lambda_1$-$\Lambda_2$ constraint

\[
P(E_{GW} | \tau_i) = \int_{l_{22}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \int_{l_{23}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau \\
+ \int_{l_{32}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \int_{l_{33}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau,
\]

where $l_{ps}$ are the length of the line at $\Lambda_1$–$\Lambda_2$, the indecies $p$ and $s$ determine to which family of compact stars the GW170817 components belong. The parameter $\tau$ is, for instance, central density of a star.

Credit: Abbott et al. PRL121 (2018)
Observational estimation for tilde deformability

The PDF $\beta(\Lambda_1, \Lambda_2)$ has been reconstructed by the method Gaussian kernel density estimation with $\Lambda_1-\Lambda_2$ data given at LIGO web-page https://dcc.ligo.org/LIGO-P1800115/public.
Likelihood of a model for the fictitious $M$-$R$ constraint

13.84 $^{+1.18}_{-1.25}$

Preliminary

$M_{NS}$ ($M_\odot$)

$R_{NS}$ (km)

1.44 $^{+0.17}_{-0.18}$

PSR J0030+0451

[Guillot. Talk at the Workshop “NSs and their environments”, (April 8, 2019)]
Likelihood of a model for the fictitious $M$-$R$ constraint

Fictitious measurement of mass and radius of PSR J0030+0451
The fictitious $M-R$ constraint

![Graph showing mass-radius constraints for different objects such as MSP J0740+6620, GW170817 M1, GW170817 M2, and PSR J0030+0451.](image)
Likelihood fictitious measurements

The fictitious $M-R$ measurement has been implemented, inspired by the preliminary results of NICER observation of $M-R$ of the PSR J0030+0451.

$$P(E_{MR} | \pi_i) = \int_{l_2} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho) d\tau + \int_{l_3} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho) d\tau,$$

where $\mu_R = 13.84$, $\sigma_R = 1.2276$, $\mu_M = 1.44$, $\sigma_M = 0.18$, and the correlation parameter $\rho = 0.9566$, winch corresponds to $8^\circ$ of the ellipse rotation. $l_2$ and $l_3$ are length of the lines at $M-R$ diagram of the second and third families correspondingly.
The total likelihood and posteriori probability of the model parameters

The full likelihood for the given $\pi_i$ can be calculated as a product of all likelihoods, since the considered constraints are independent of each other

$$P(E \mid \pi_i) = \prod_{m} P(E_m \mid \pi_i).$$

where $m$ is index of the constraints.

The posterior distribution of models on parameter diagram is given by Bayes’ theorem

$$P(\pi_i \mid E) = \frac{P(E \mid \pi_i) P(\pi_i)}{\sum_{j=0}^{N-1} P(E \mid \pi_j) P(\pi_j)},$$

where $P(\pi_j)$ is a prior distribution of a models taken to be uniform: $P(\pi_j) = 1/N$. 
Results with and without fictitious measurements

Conclusions

The mixed phase interpolation method is very simple and well describes quark-hadron pasta phase for any give possible surface tension value.

The third family survives against pasta phase for the considered EoS models.

$\Lambda_1 - \Lambda_2$ relation from GW170817 favours softer EoS and hybrid stars with strong phase order transitions (even with no third family due to the mixed phase).

The region $\Lambda_2 < \Lambda_1$ has physical meaning in case of low-mass twins, when heavier companion belongs to the second family and the lighter one to the third family.

If NICER will approve the “fictitious radius measurement” it will support the low-mass twin stars around $1.4 \, M_\odot$ for considered models.
Simulations of Heavy Ion Collisions

Parameter optimization

Bayesian analysis

Data analysis

Initial state

hydrodynamic evolution

Particulization

Event simulation at MPD detector

MPD root

Physical analysis of simulated data

Development of EoS models

Parametrized EoS

3-fluid hydro

adapt the procedure from existing hybrid model

GEANT, MPD
Multifluid Dynamic of Heavy Ion Collisions

Au+Au collision at $\sqrt{s_{NN}} = 6.4$ GeV (Elab = 20A GeV) with impact parameter $b = 6$ fm

Rapidity Distribution & Curvature at Midrapidity

A reduced curvature of the spectrum at midrapidity

$$C_y = \left( \frac{d^3N}{dy^3} \right)_{y=y_{cm}} / \left( \frac{dN}{dy} \right)_{y=y_{cm}} = \left( \frac{y_{cm}}{w_s} \right)^2 \left( \sinh^2 y_s - w_s \cosh y_s \right)$$


Hadron Ratios of Midrapidity

Directed Flow

Summary

- **Varying EoS models**
  - Formulation and solution of the optimization problem for definition of free hydrodynamic parameters
  - Development of hybrid EoS model construction

- **Bayesian analysis for finding the best model parameter regions**
  - Heavy ion simulation with different EoS models (parameters)
  - Collecting suitable experimental data around NICA energy range
  - Formulation and performing the Bayesian analysis

- **Simulation of heavy ion collision at MPD detector**
  - HIC simulation with the best models
  - Physical analysis of the simulated data within MPDroot
  - Comparing the results with the various experimental data
References


The results of pasta effects

Third family robust against $\Delta P$ up to around 5%!
The realistic hadron and quark matter models

The hadron EoS model KVOR with modification of stiffness


The quark EoS model SFM with available volume fraction parameter

Robustness of third family solutions

Ayriyan, Bastian, Blaschke, Grigorian, Maslov, Voskresensky. PRC 97, 045802 (2018)
Lambda-Lambda diagram: Hybrid EoS
NS – NS merging

Secondary mass
Primary mass

\( \mu_\xi = 1150, \Delta_p = 0 \% \)
Hadron - Hybrid

Secondary mass
Primary mass

$\mu_\phi = 1150, \Delta_p = 0\%$
Hybrid - Hybrid
The same phenomena were found in Montana, Tolos, Hanuske, Rezzolla.

PRD99, 103009 (2019) for polytropic models

More interesting results have been achieved by Prof. Armen Sedrakian for triplet of compact stars produced by the forth family.

The region $\Lambda_2 < \Lambda_1$ was called unphysical at Abbott et al. PRL121 (2018).
Mass constraint

Flows

Fourier transformation of azimuthal particle distribution in momentum space yields coefficients of different order.

\[ v_n = \left< \cos n \cdot \phi \right> \]

\[ \phi = \text{atan} \frac{p_y}{p_x} \]

- \( v_1 \): “directed flow”
- \( v_2 \): “elliptic flow”

Credit: M. Oldenburg