HIC simulations with different EoS models

Hovik Grigorian

JINR LIT (Dubna, Russia),
AANL CP&IT
Yerevan State University, (Yerevan, Armenia)

NICA-days 2019 : associated with the **IVth MPD Collaboration Meeting**

24 October, Warsaw

The research was carried out under financial support of the Russian Science Foundation (project #17-12-01427)

Collaborators

Alexander Ayriyan (LIT & AANL)

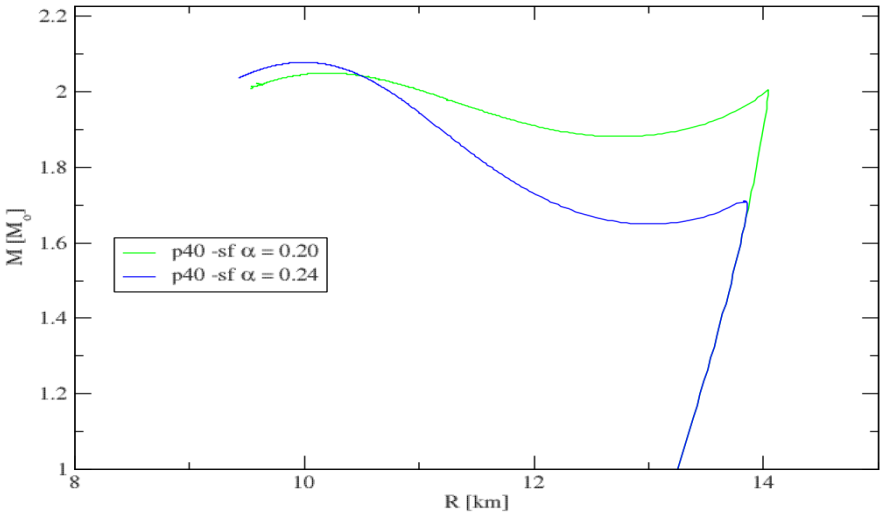
Arpine Piloyan (AANL)
David Blaschke (UWr & JINR & MEPhI)
Dmitry Voskresensky (MEPhI & JINR)
Konstantin Maslov (MEPhI & JINR)
David-Edwin Alvarez-Castillo (JINR)
Nobutoshi Yasutake (Chiba Institute of Technology)
Vahagn Abgaryan (JINR & AANL)
Noshad Khosravi Largani (UWr)

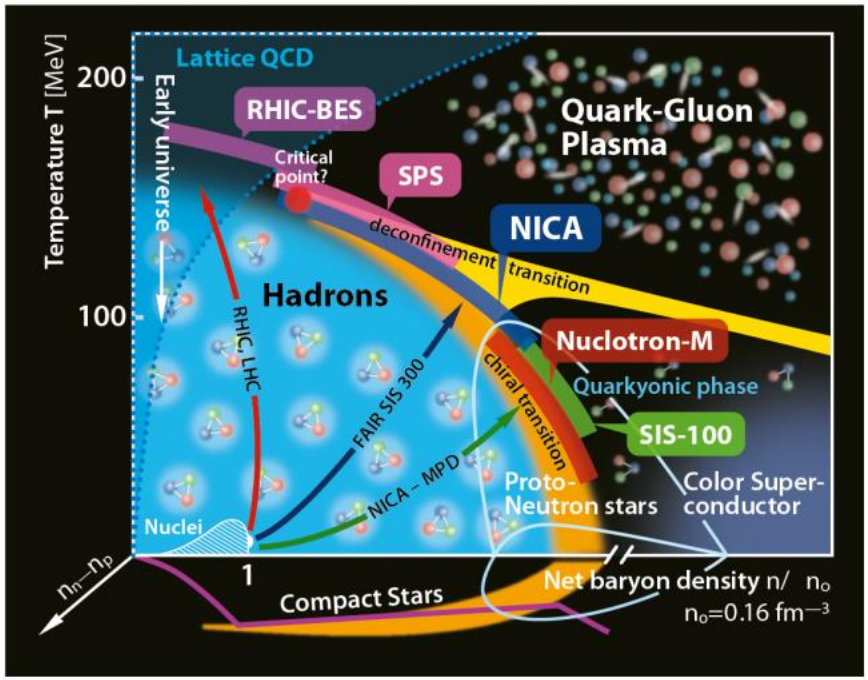
Mixed Phase Constraction & Bayesian analysis for seletion of the best EoS models

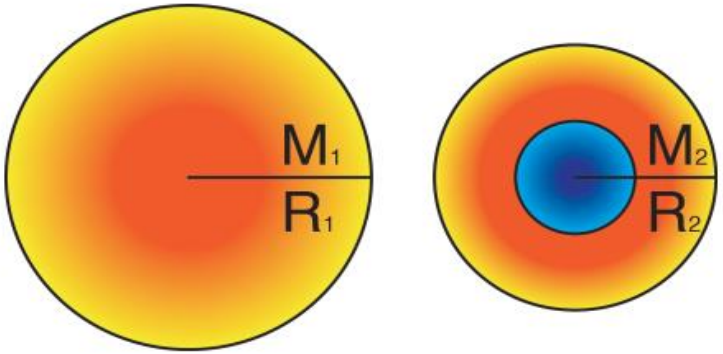
Outline

I. Motivation
II. Mixed phase construction for cold and dense nuclear matter
III. Bayesian analysis for extracting properties of the nuclear equation of state from observational data
IV. HIC Simulations
V. Conclusions

Motivation : What if we have twin NS ?

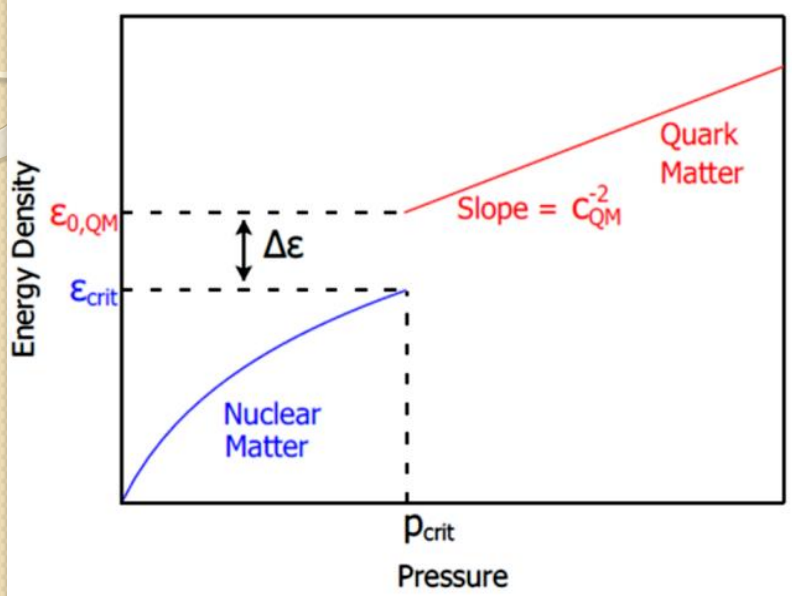


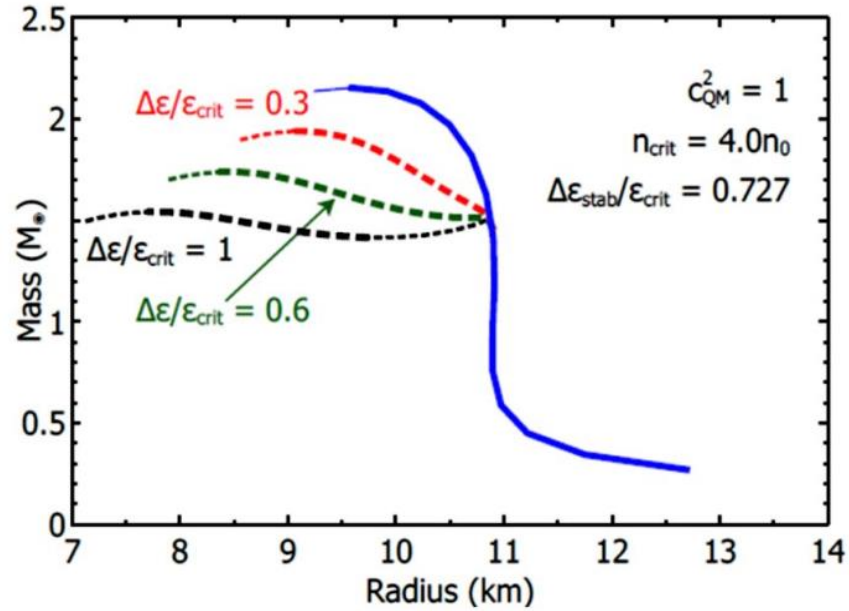




- Does hybrid neutron star exist?
- Does NS twins exist?
- Does CEP exist on QCD phase diagram?

Neutron star mass-radius relation





Seidov criterion for instability:

$$\frac{\Delta\varepsilon}{\varepsilon_{crit}} \geq \frac{1}{2} + \frac{3}{2}\frac{P_{crit}}{\varepsilon_{crit}}$$

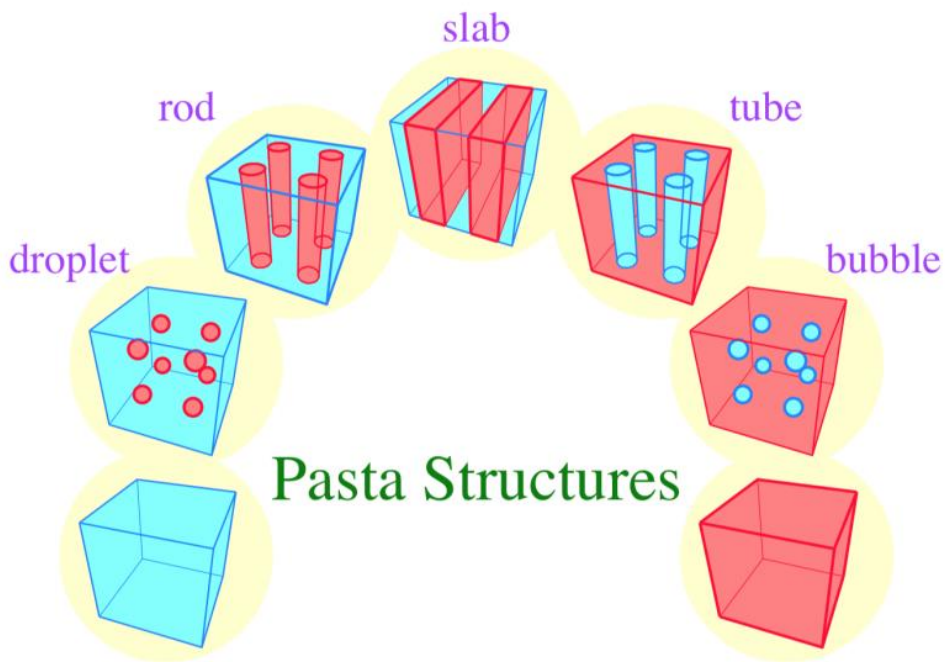
Credit: Mark G. Alford, Sophia Han, and Madappa Prakash. Phys. Rev. D 88, 083013 (2013)

Finite-size effects in mixed phase

Coulomb interaction vs **Surface tension**

Tends to break up the like-charged regions into smaller ones

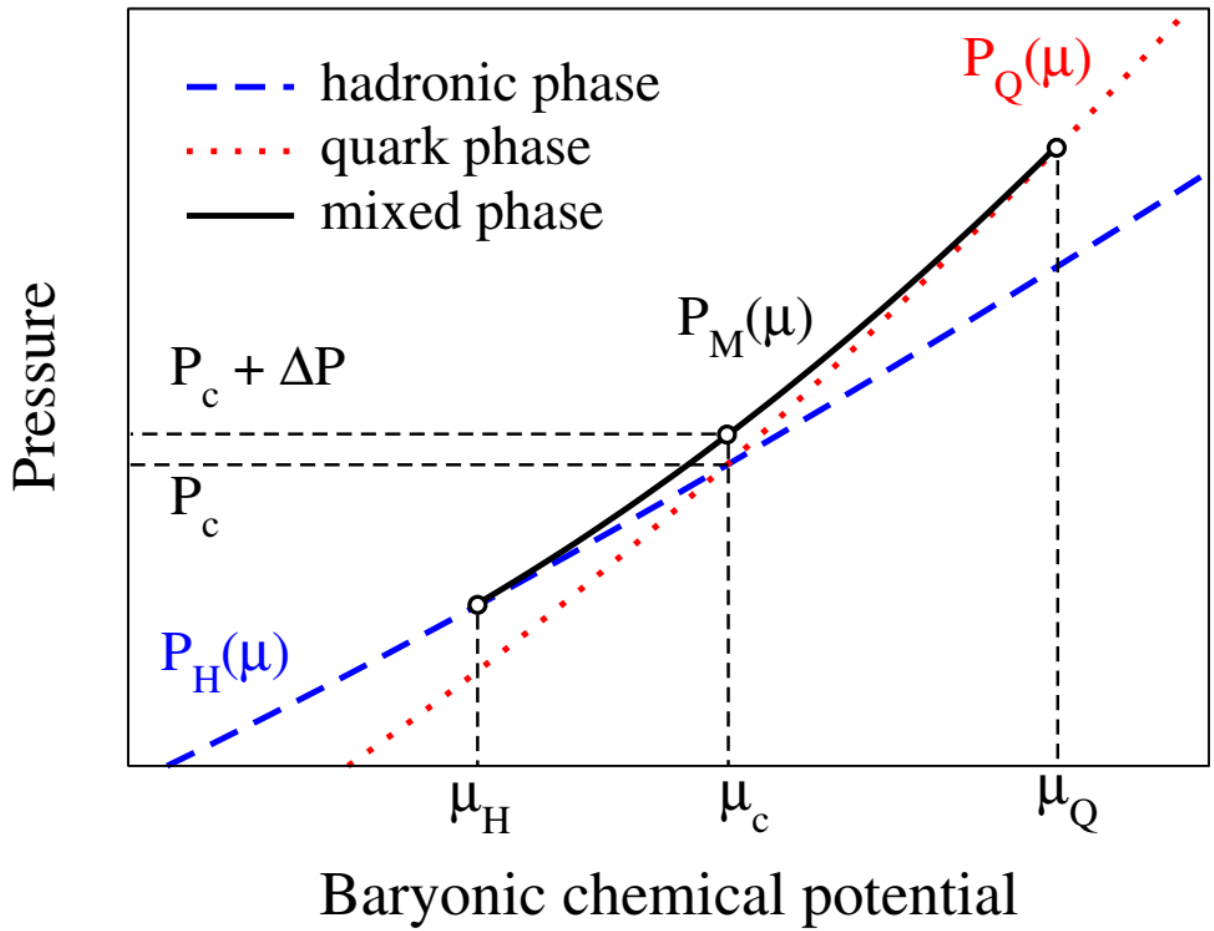
Requires minimization of the surface



The surface tension σ is unknown and used as free parameter.

Yasutake, Maruyama, Tatsumi, Phys. Rev. D80 (2009) 123009

Mimicking the Pasta phase.



Schematic representation of the interpolation function $P_M(\mu)$, it has to go though three points: $P_H(\mu_H)$, $P_c + \Delta P$ and $P_Q(\mu_Q)$.

The Interpolation Method

$$P_M(\mu) = \sum_{q=1}^{N} \alpha_q (\mu - \mu_c)^q + (1 + \Delta_P) P_c$$

where Δ_P is a free parameter representing additional pressure of the mixed phase at μ_c.

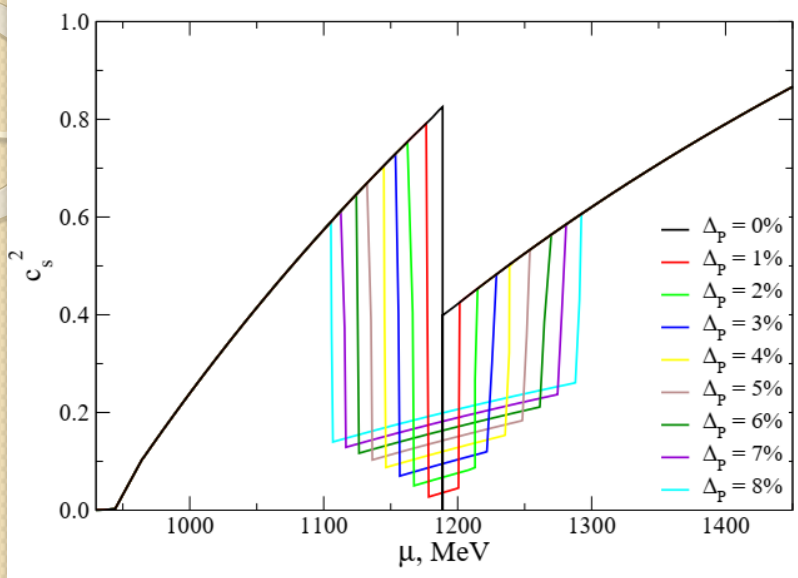
$$P_H(\mu_H) = P_M(\mu_H) \qquad P_Q(\mu_Q) = P_M(\mu_Q)$$

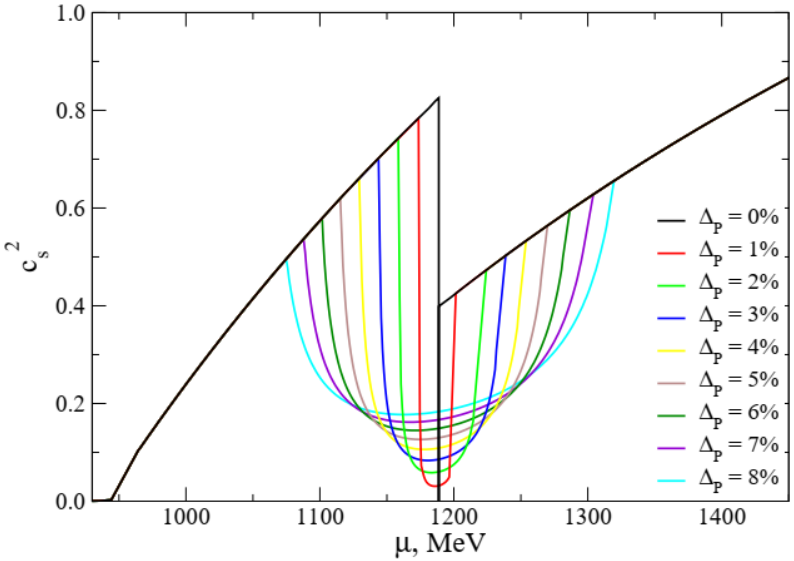
$$\frac{\partial^q}{\partial \mu^q} P_H(\mu_H) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_H) \qquad \frac{\partial^q}{\partial \mu^q} P_Q(\mu_Q) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_Q)$$

where $q = 1, 2, \ldots, k$. All $N + 2$ parameters (μ_H, μ_Q and α_q, for $q = 1, \ldots, N$) can be found by solving the above system of equations, leaving one parameter (ΔP) as a free one.

Ayriyan and Grigorian, *EPJ Web Conf.* **173**, 03003 (2018)
Abgaryan, Alvarez-Castillo, Ayriyan et al. *Universe* **4(9)**, 94 (2018)

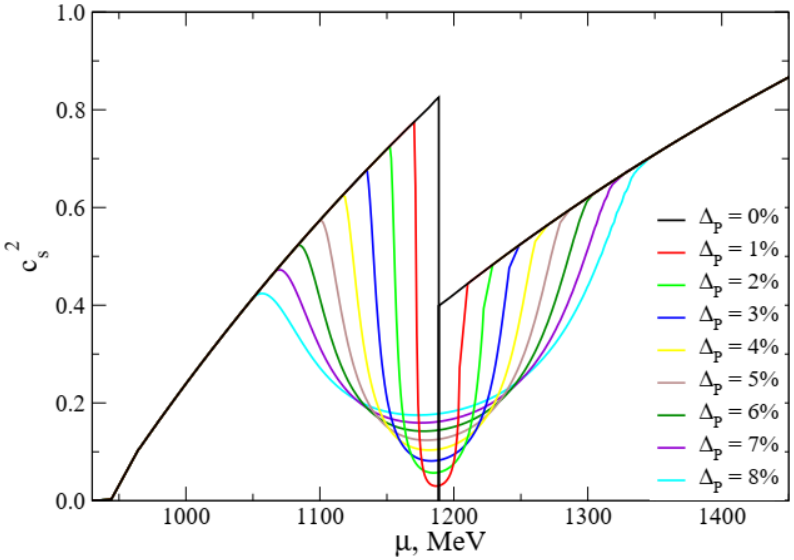
The Interpolation Method



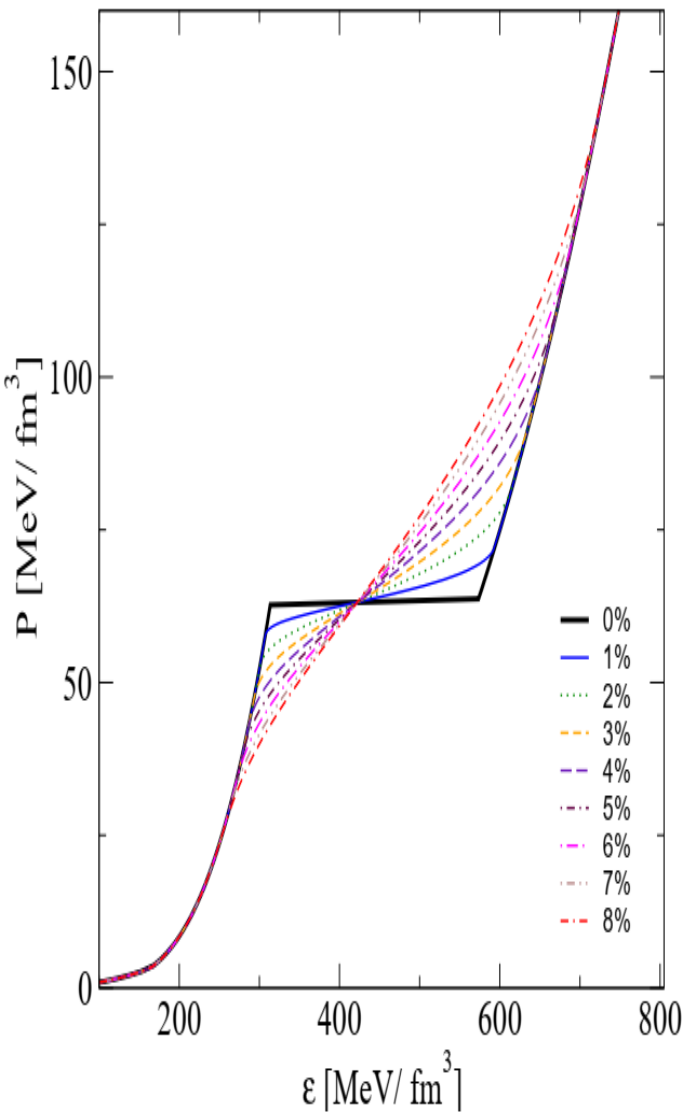


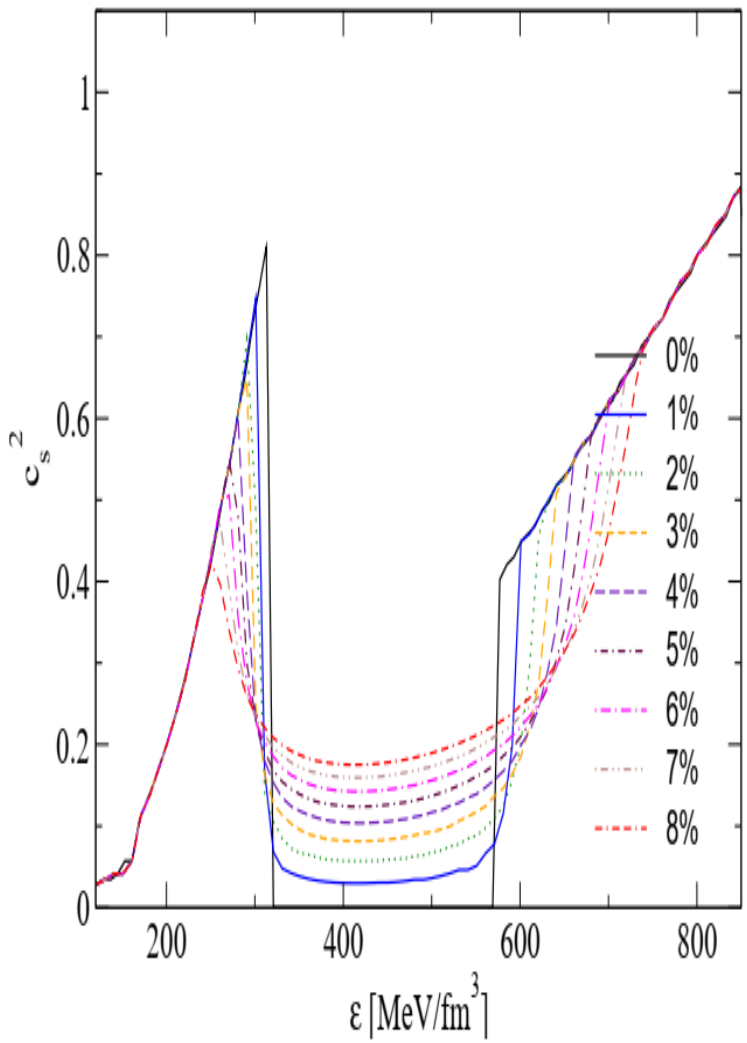
The squared speed vs chemical potential given by the interpolation with $k = 1$ (upper left) $k = 2$ (upper right) and $k = 3$ (right).

Abgaryan, Alvarez-Castillo, Ayriyan, Blaschke and Grigorian. Universe 4(9) (2018), 94

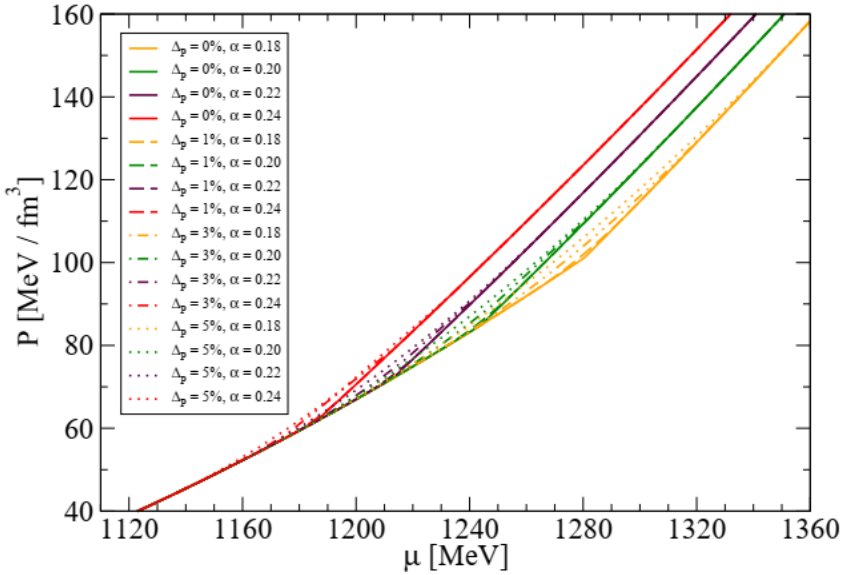


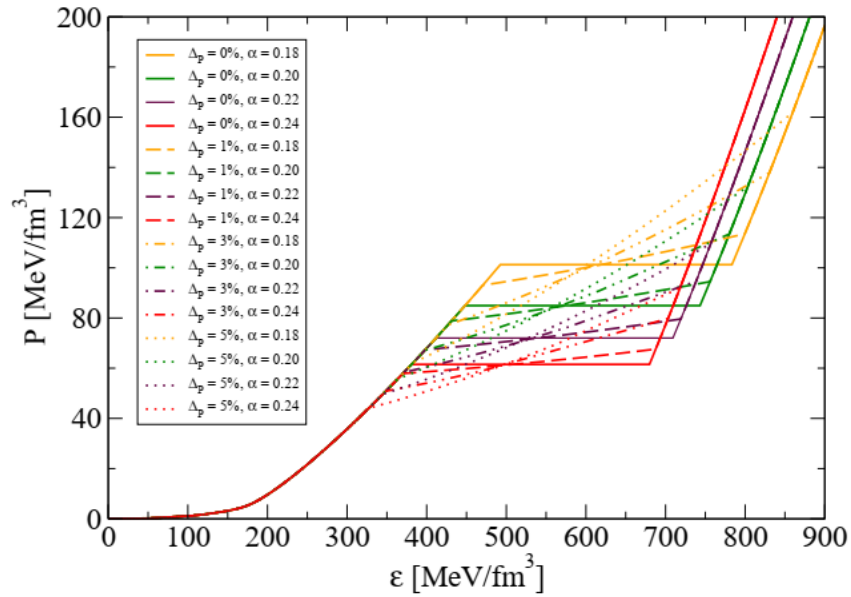
The results of pasta mimicking

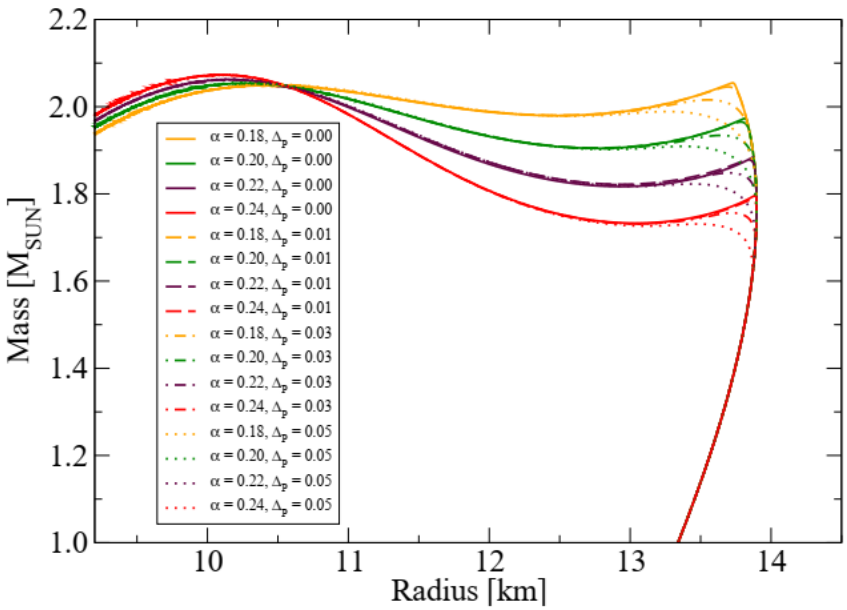


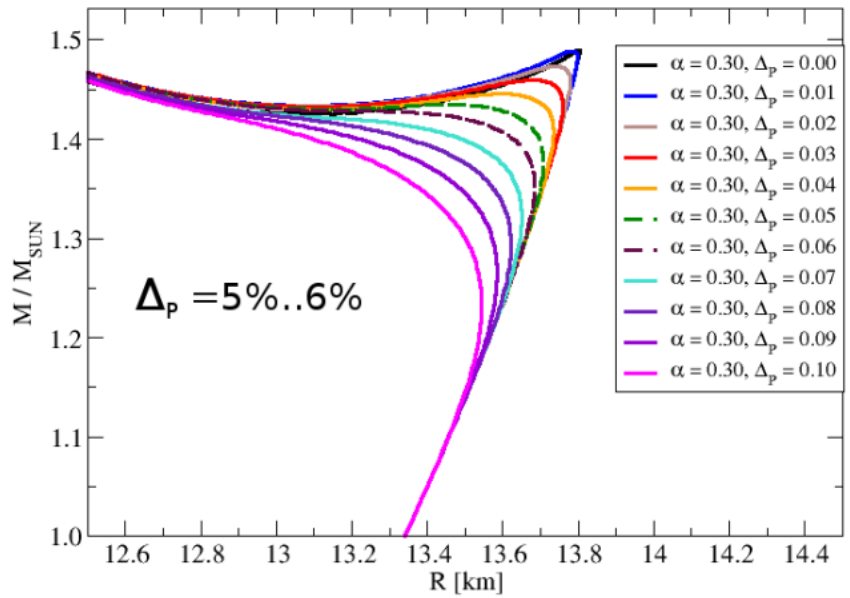


Robustness of third family solutions

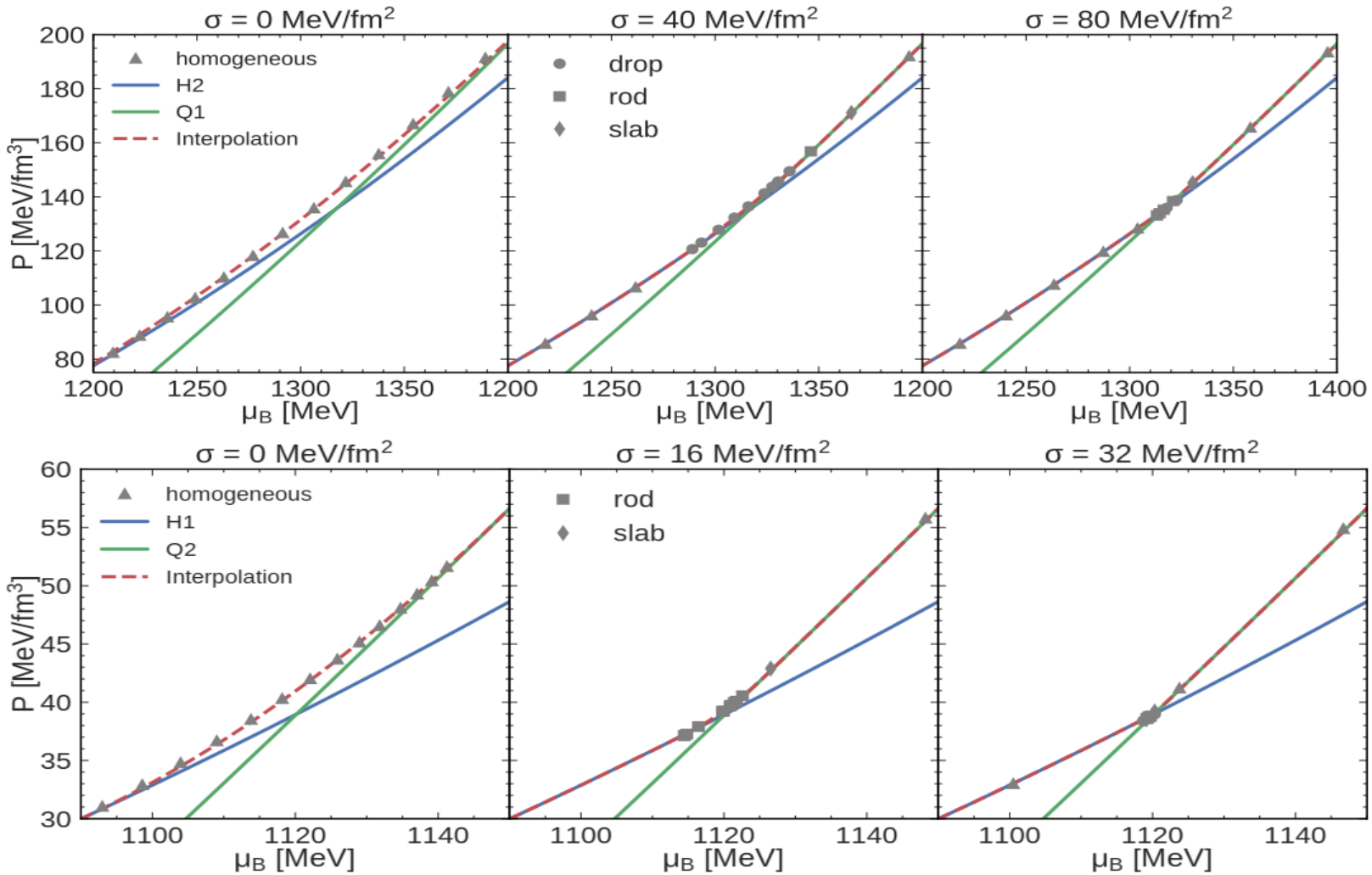




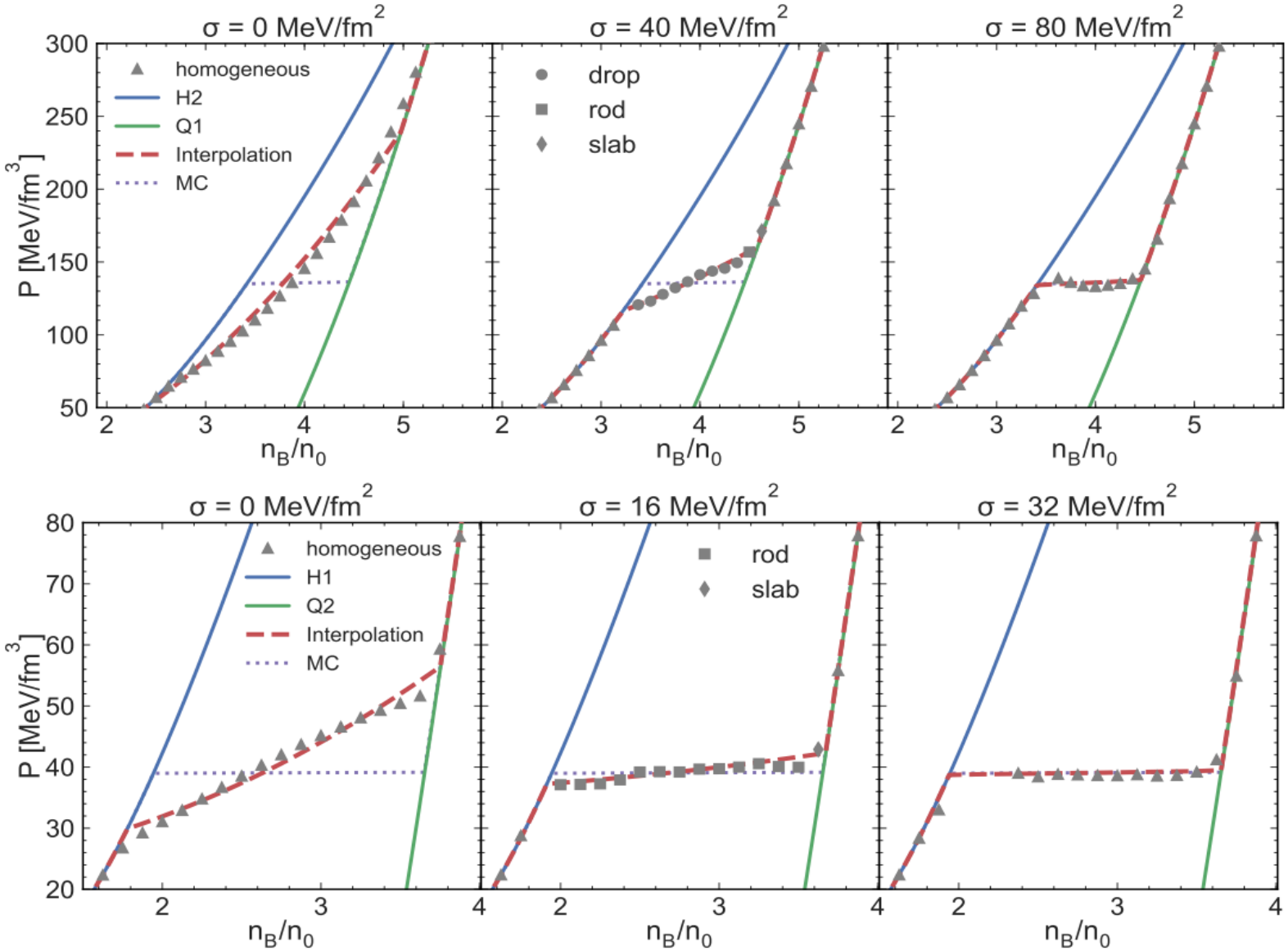




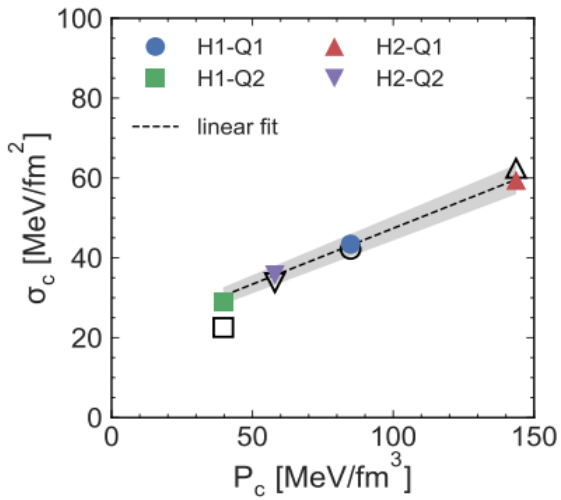
Dependence on surface tension

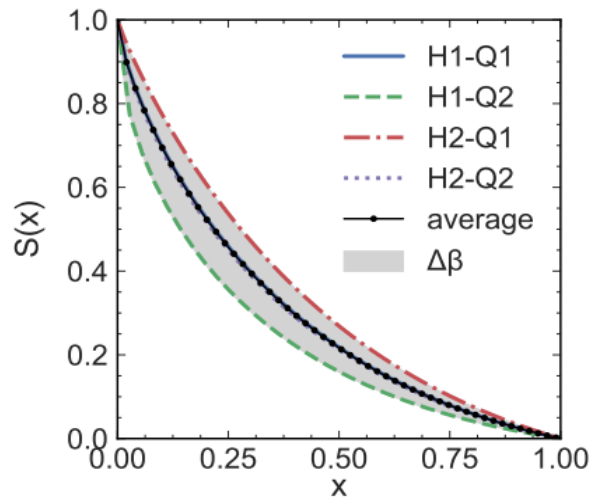


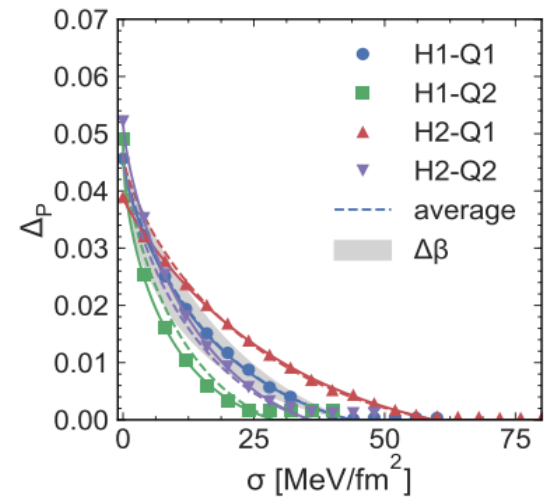
Dependence on surface tension



Dependence on surface tension







$\sigma_c = d\,(P_c - P_0) + \sigma_0$: $d = 0.45 \pm 0.02$ fm, $P_0 = 40$ MeV/fm^3, and $\sigma_0 = 31.6 \pm 1.19$ MeV/fm^2

$\Delta_P(\sigma) = \Delta_P(0) S(\sigma/\sigma_c; \beta)$: $\overline{\beta} = 0.64$

$S(x;\beta) = e^{-x}(1 - x^{\beta})\theta(1 - x)$

Maslov, Yasutake, Blaschke, Ayriyan, Grigorian, Maruyama, Tatsumi, Voskresensky. PRC100, 025802 (2019)

Bayesian method

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.

One of the most frequent case is analysis of probable values of model parameters.

Bayes' theorem:

$$\underbrace{p(H_1 \mid D, I)}_{\text{Posterior}} = \frac{\overbrace{p(D \mid H_1, I)}^{\text{Likelihood}}\,\overbrace{p(H_1 \mid I)}^{\text{Prior}}}{\underbrace{p(D \mid I)}_{\text{Evidence}}}$$

Prior: knowledge before experiment (logically)
Likelihood: Probability for data if the hypothesis was true
Posterior: Probability that the hypothesis is true given the data
Evidence: normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!

Bayesian method

Formulation of set of models (set of hypothesis):
π_i here $i = 0..N-1$

$\downarrow$

Finding the *a priori* probabilities of the models:
$P(\pi_i) = 1/N \quad \text{for} \quad \forall i = 0..N-1$

$\downarrow$

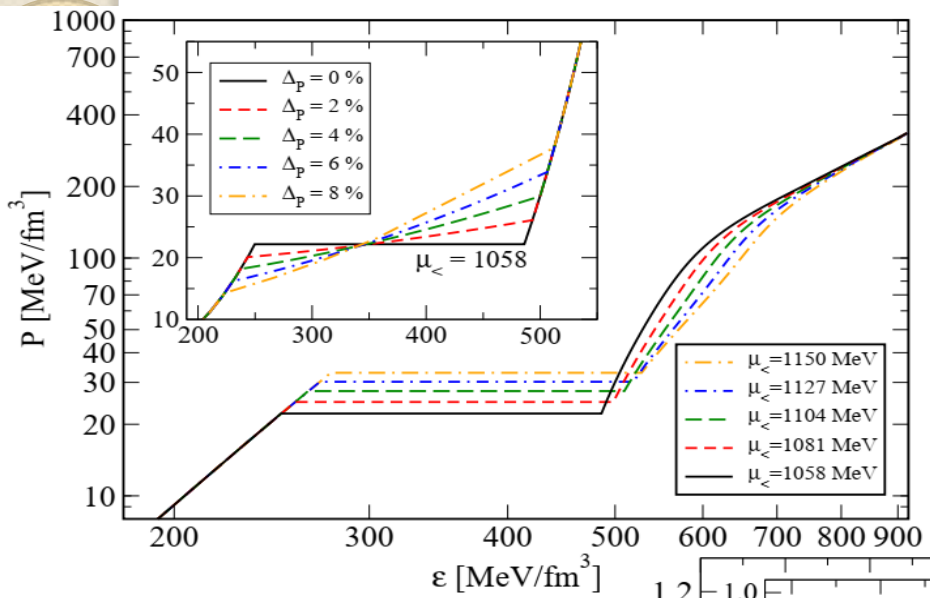
Calculating the coditional probabilities of the events:
$P(E|\vec{\pi}_i) = \prod_\alpha P(E_\alpha|\vec{\pi}_i)$,
where α is the index of the observational constraints.

$\downarrow$

Calculating the *a posteriori* probabilities of the models:

$$P(\vec{\pi}_i|E) = \frac{P(E|\vec{\pi}_i)P(\vec{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\vec{\pi}_j)P(\vec{\pi}_j)}$$

Model EoS for Hybrid NS

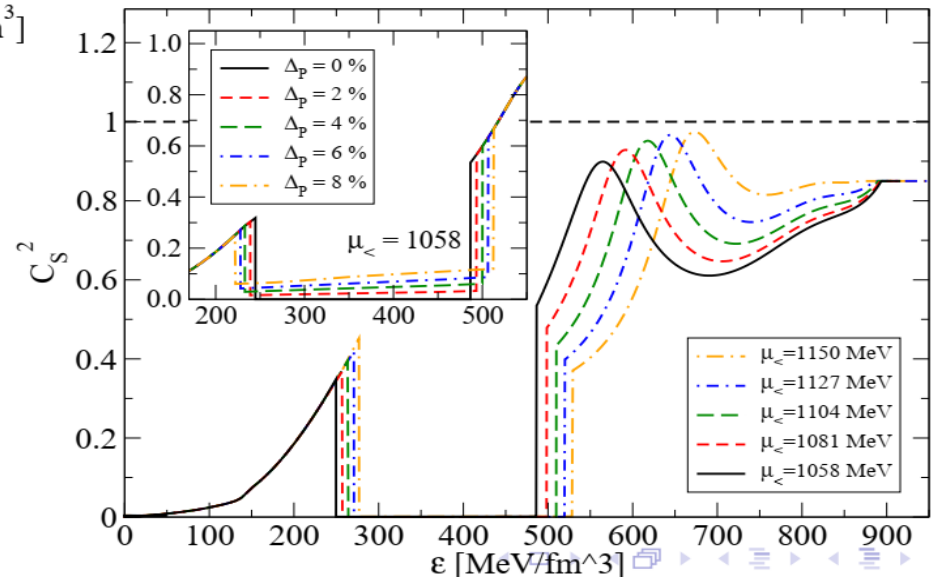


DD2 with excluded volume plus color superconducting two-flavor quark matter, described within a nonlocal covariant chiral quark model.

$$P(\mu) = P(\mu; \eta(\mu), B(\mu)) = -\Omega^{MFA}(\eta(\mu)) - B(\mu)$$

$$\Omega^{MFA} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\bar{\Delta}^2}{2H} - \frac{\bar{\omega}^2}{2G_V} - \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \ln\det\left[S^{-1}(\bar{\sigma}, \bar{\Delta}, \bar{\omega}, \mu_{fc})\right]$$

$$\frac{d\Omega^{MFA}}{d\bar{\Delta}} = 0\,, \quad \frac{d\Omega^{MFA}}{d\bar{\sigma}} = 0\,, \quad \frac{d\Omega^{MFA}}{d\bar{\omega}} = 0\,.$$



Alvarez-Castillo, Blaschke, Grunfeld, and Pagura, PRD99, 063010 (2019)

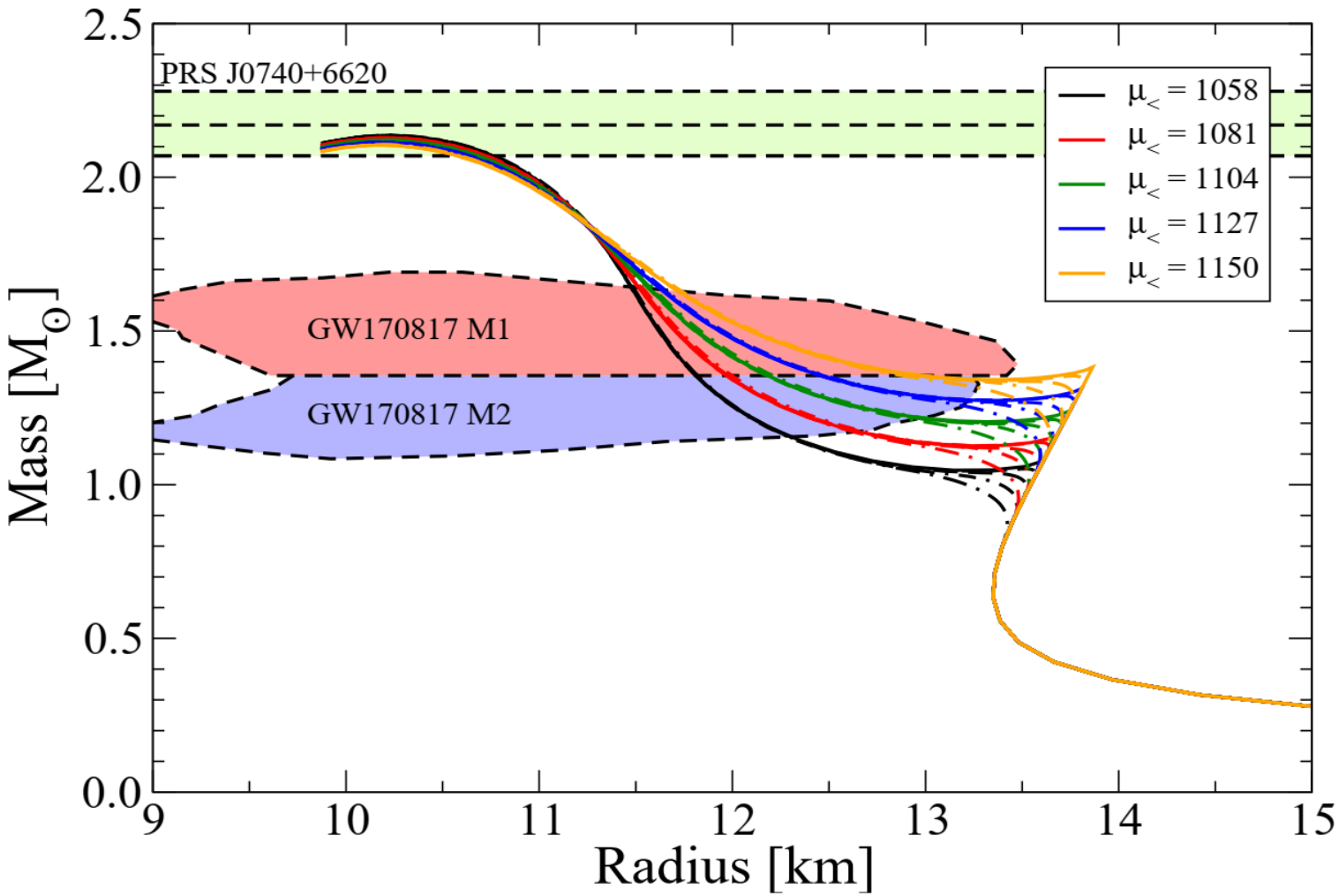
The parameter set for EoS model

The set of parameters of models could be represented in the parameter space with introduction of the vector of parameters, each vector is one fixed model from considered types of EoS model and transition construction:

$$\vec{\pi}_i = \left\{ \mu_{<(j)}, \Delta_{P(k)} \right\},$$

where $i = 0..N-1$ and $i = N_2 \times j + k$ and $j = 0..N_1 - 1$, $k = 0..N_2 - 1$ and N_1 and N_2 are number of values of model parameters $\mu_<$ and Δ_P correspondingly.

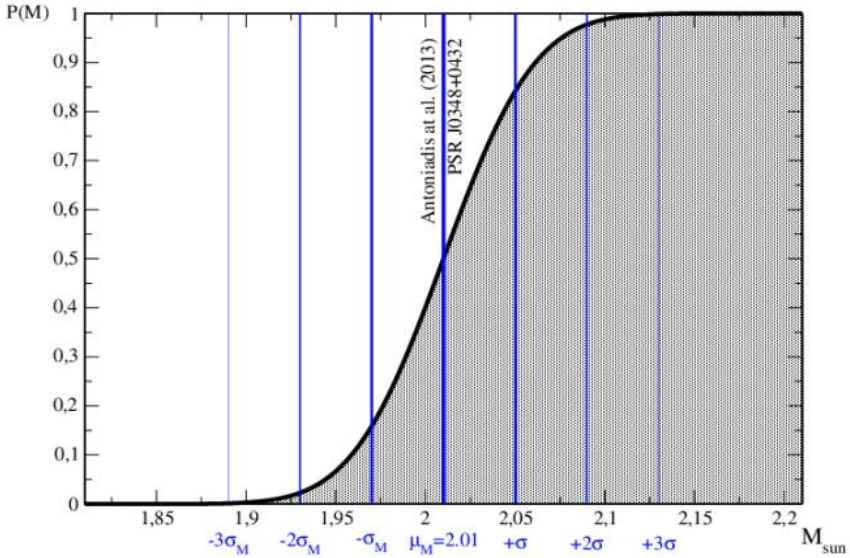
Stabile NS Configurations



Likelihood of a EoS model for the mass constraint

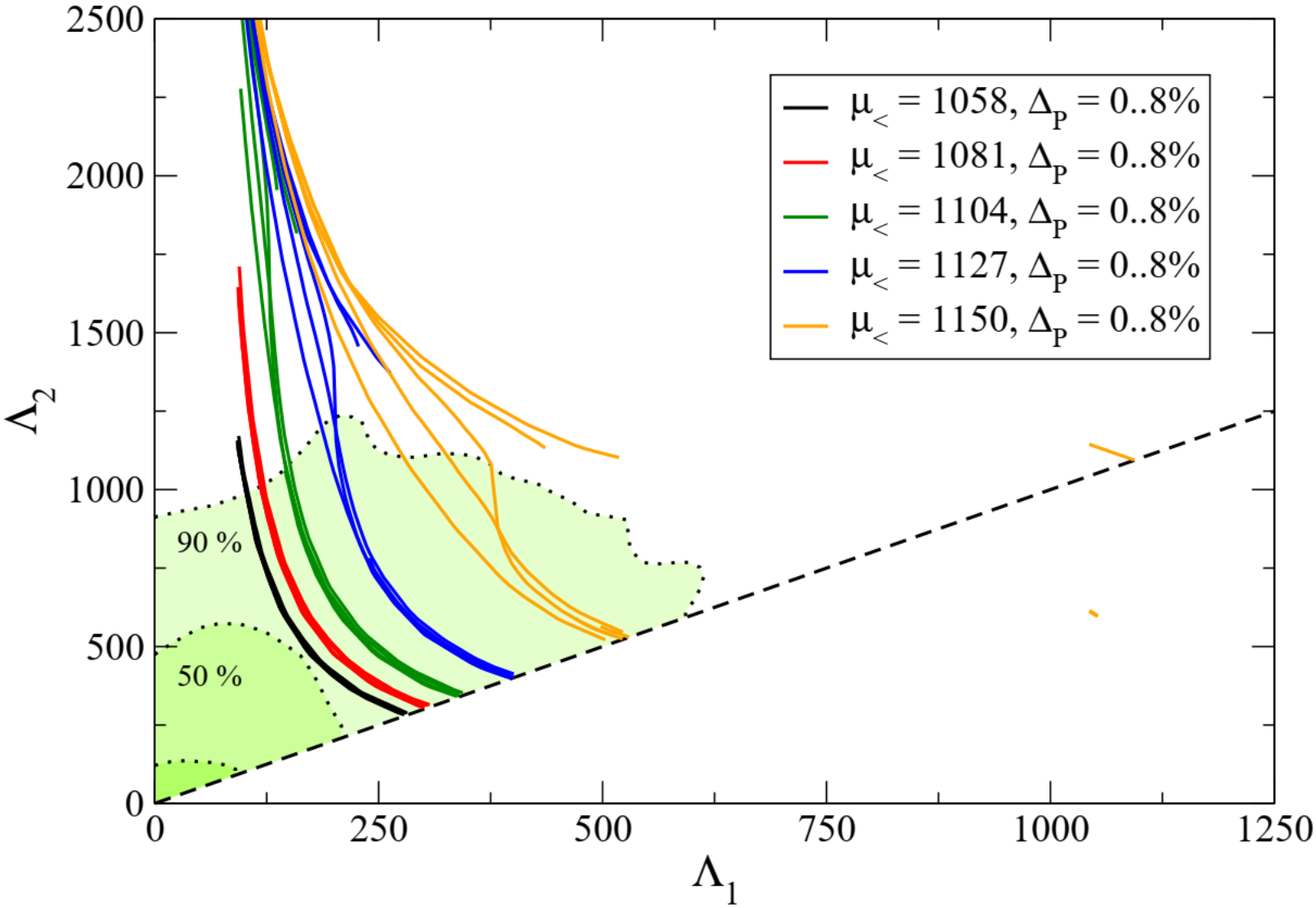
$$P\left(E_M \,|\pi_i\right) = \Phi(M_i, \mu_A, \sigma_A)$$

here M_i is maximum mass of the given by π_i, and $\mu_A =$ **2.14** $\mathrm{M}_\odot$ and $\sigma_A = 0.105\ \mathrm{M}_\odot$ is the mass measurement of PSR J0740+6620 **2.14** $^{+0.10}_{-0.09}$ $\mathrm{M}_\odot$ [Cromartie *et al.*, arXiv:1904.06759 (2019)].



Note, that here we replace previously used mass measurement for two solar mass pulsar J0348+0432 $2.01^{+0.04}_{-0.04}$ $\mathrm{M}_\odot$ [Antoniadis *et al.*, Science **340**, 6131 (2013)].

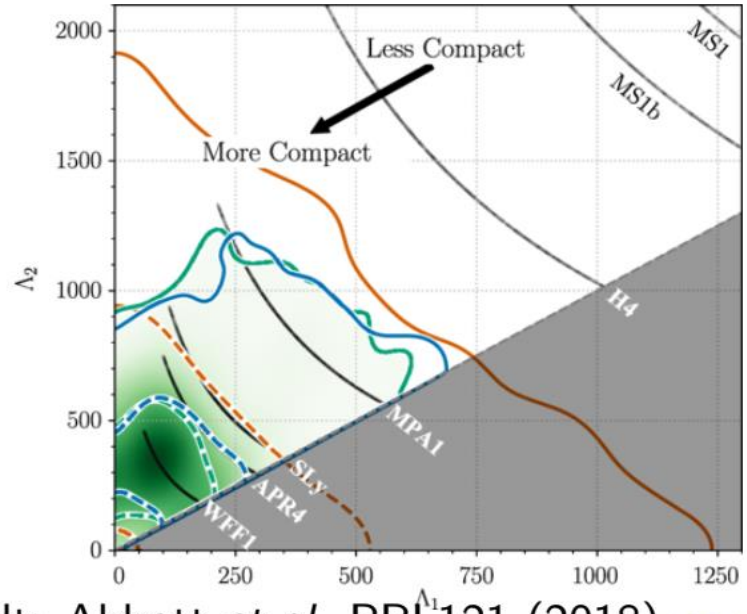
Λ_1- Λ_2 diagram and observational constraints



Likelihood for the Λ1-Λ2 constraint

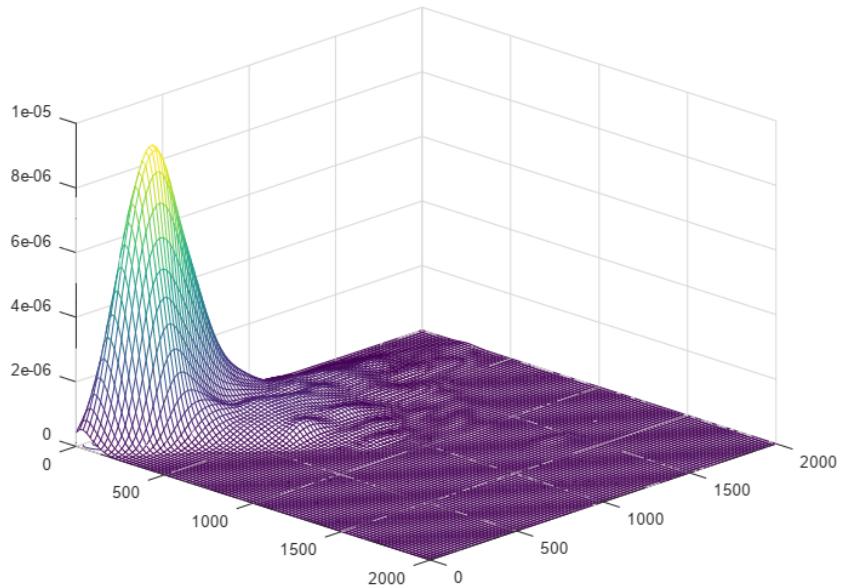
$$P\left(E_{GW}\,|\pi_i\right) = \int_{l_{22}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{23}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau$$
$$+ \int_{l_{32}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{33}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau,$$

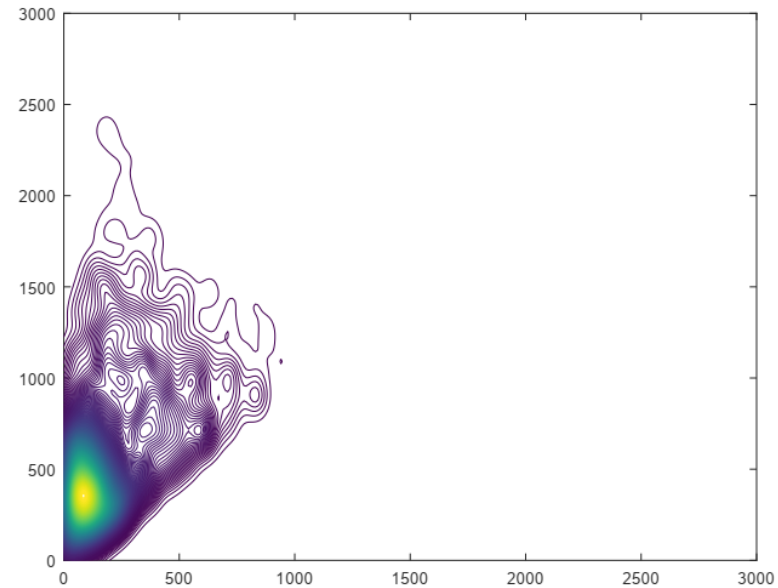
where l_{ps} are the length of the line at Λ_1–Λ_2, the indecies p and s determine to which family of compact stars the GW170817 components belong. The parameter τ is, for instance, central density of a star.



Credt: Abbott *et al.* PRL121 (2018)

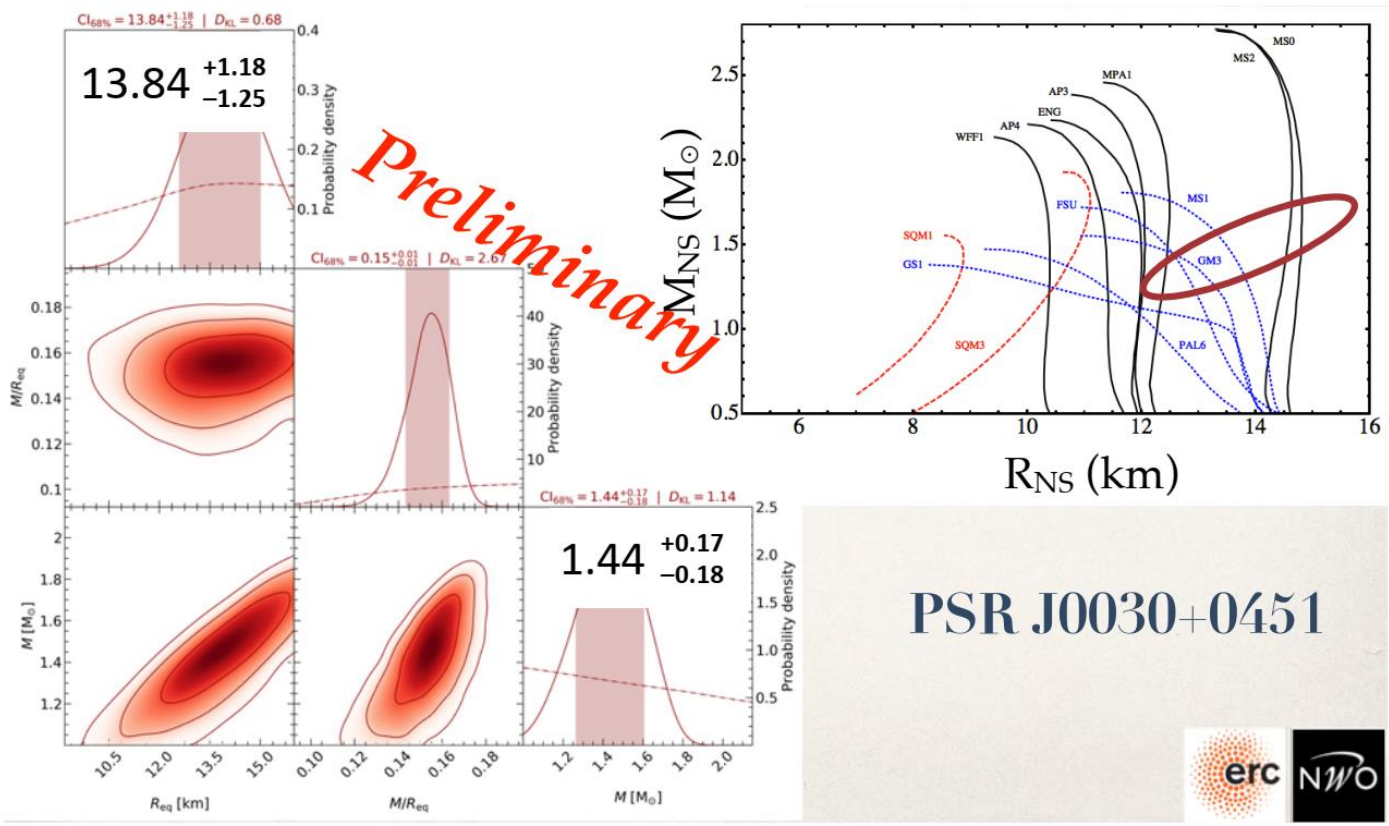
Observational estimation for tilde deformability





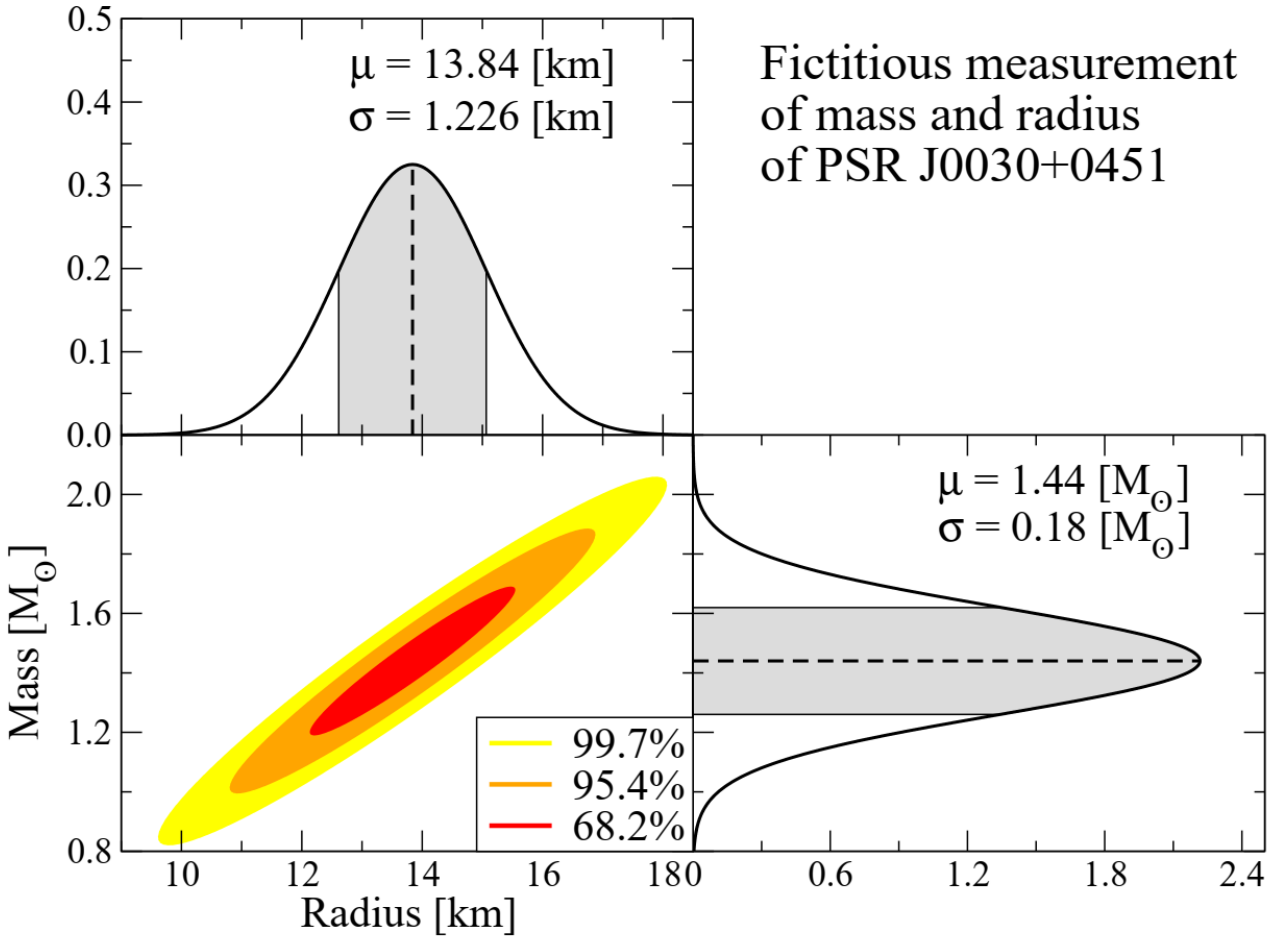
The PDF $\beta(\Lambda_1, \Lambda_2)$ has been reconstructed by the method Gaussian kernel density estimation with Λ_1–Λ_2 data given at LIGO web-page https://dcc.ligo.org/LIGO-P1800115/public.

Likelihood of a model for the fictitious *M-R* constraint

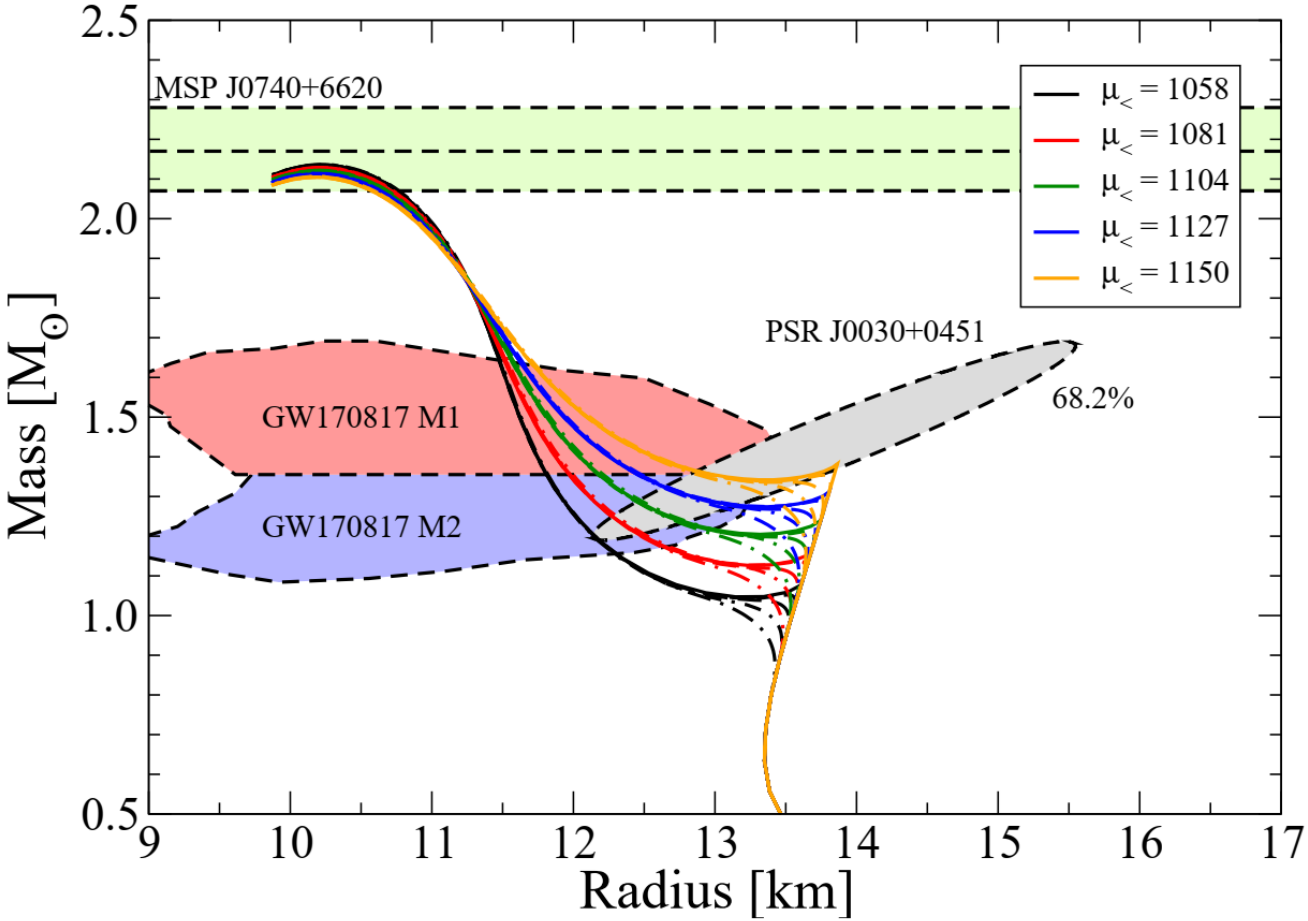


[Guillot. Talk at the Workshop "NSs and their environments", (April 8, 2019)]

Likelihood of a model for the fictitious *M-R* constraint



The fictitious *M-R* constraint



Likelihood fictitious measurements

The fictitious M-R measurement has been implemented, inspired by the preliminary results of NICER observation of M-R of the PSR J0030+0451.

$$P\left(E_{MR}\,|\pi_i\right) = \int_{l_2} \mathcal{N}(\mu_R,\, \sigma_R,\, \mu_M,\, \sigma_M, \rho) d\tau + \int_{l_3} \mathcal{N}(\mu_R,\, \sigma_R,\, \mu_M,\, \sigma_M, \rho) d\tau,$$

where $\mu_R = 13.84$, $\sigma_R = 1.2276$, $\mu_M = 1.44$, $\sigma_M = 0.18$, and the correlation parameter $\rho = 0.9566$, winch corresponds to 8° of th ellipse rotation. l_2 and l_3 are length of the lines at M-R diagram of the second and third families correspondingly.

The total likelihood and posteriori probability of the model parameters

The full likelihood for the given π_i can be calculated as a product of all likelihoods, since the considered constraints are independent of each other

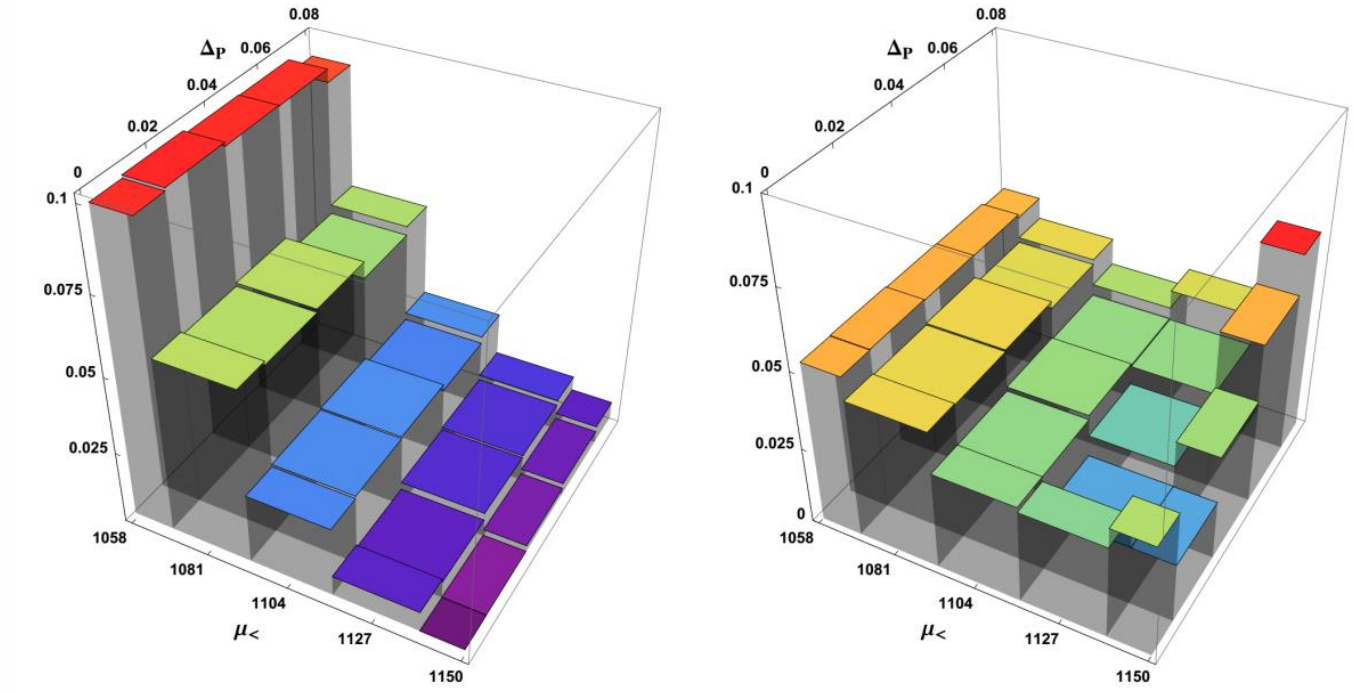
$$P\left(E\,|\,\vec{\pi}_i\right) = \prod_m P\left(E_m\,|\,\vec{\pi}_i\right).$$

where m is index of the constraints.
The posterior distribution of models on parameter diagram is given by Bayes' theorem

$$P\left(\vec{\pi}_i\,|\,E\right) = \frac{P\left(E\,|\,\vec{\pi}_i\right) P\left(\vec{\pi}_i\right)}{\sum_{j=0}^{N-1} P\left(E\,|\,\vec{\pi}_j\right) P\left(\vec{\pi}_j\right)},$$

where $P\left(\vec{\pi}_j\right)$ is a prior distribution of a models taken to be uniform: $P\left(\vec{\pi}_j\right) = 1/N$.

Results with and without fictitious measurements



Ayriyan, Alvarez-Castillo, Blaschke, Grigorian. In preparation.

Conclusions

The mixed phase interpolation method is very simple and well describes quark-hadron pasta phase for any give possible surface tension value.

The third family survives against pasta phase for the considered EoS models.

$\Lambda_1 - \Lambda_2$ relation from GW170817 favours softer EoS and hybrid stars with strong phase order transitions (even with no third family due to the mixed phase).

The region $\Lambda_2 < \Lambda_1$ has physical meaning in case of low-mass twins, when heavier companion belongs to the second family and the lighter one to the third family.

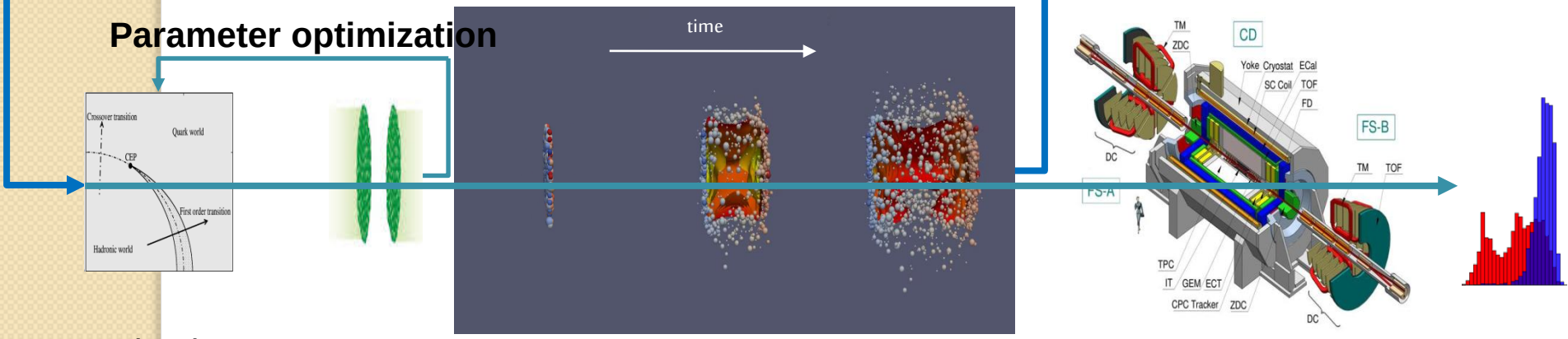
If NICER will approve the "fictitious radius measurement" it will support the low-mass twin stars around 1.4 $M_\odot$ for considered models.

Simulations of Heavy Ion Collisions

Bayesian analysis

Data analysis

Parameter optimization



Parametrized EoS → Initial state → hydrodynamic evolution → Particulization → Event simulation at MPD detector → MPD root

Development of EoS models

3-fluid hydro

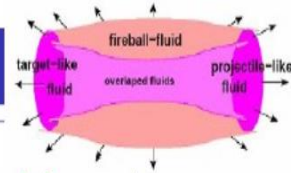
adapt the procedure from existing hybrid model

GEANT, MPD

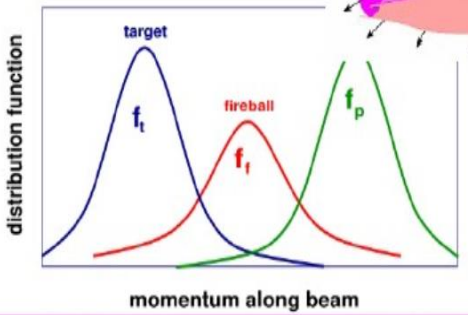
Physical analysis of simulated data

Multifluid Dynamic of Heavy Ion Collisions

3-Fluid Dynamics



Produced particles populate mid-rapidity
⇒ fireball fluid



Target-like fluid: $\partial_\mu J_t^\mu = 0$ $\quad$ $\partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$

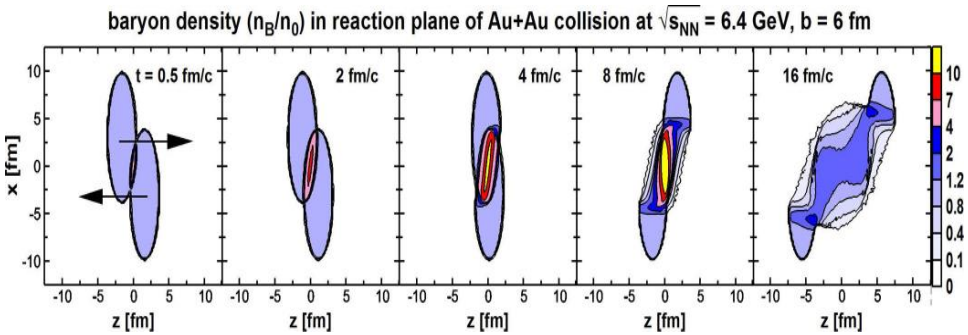
Leading particles carry bar. charge $\quad$ exchange/emission

Projectile-like fluid: $\partial_\mu J_p^\mu = 0,$ $\quad$ $\partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$

Fireball fluid: $J_f^\mu = 0,$ $\quad$ $\partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$

Baryon-free fluid $\quad$ Source term $\quad$ Exchange

The source term is delayed due to a formation time $\tau \sim 1$ fm/c



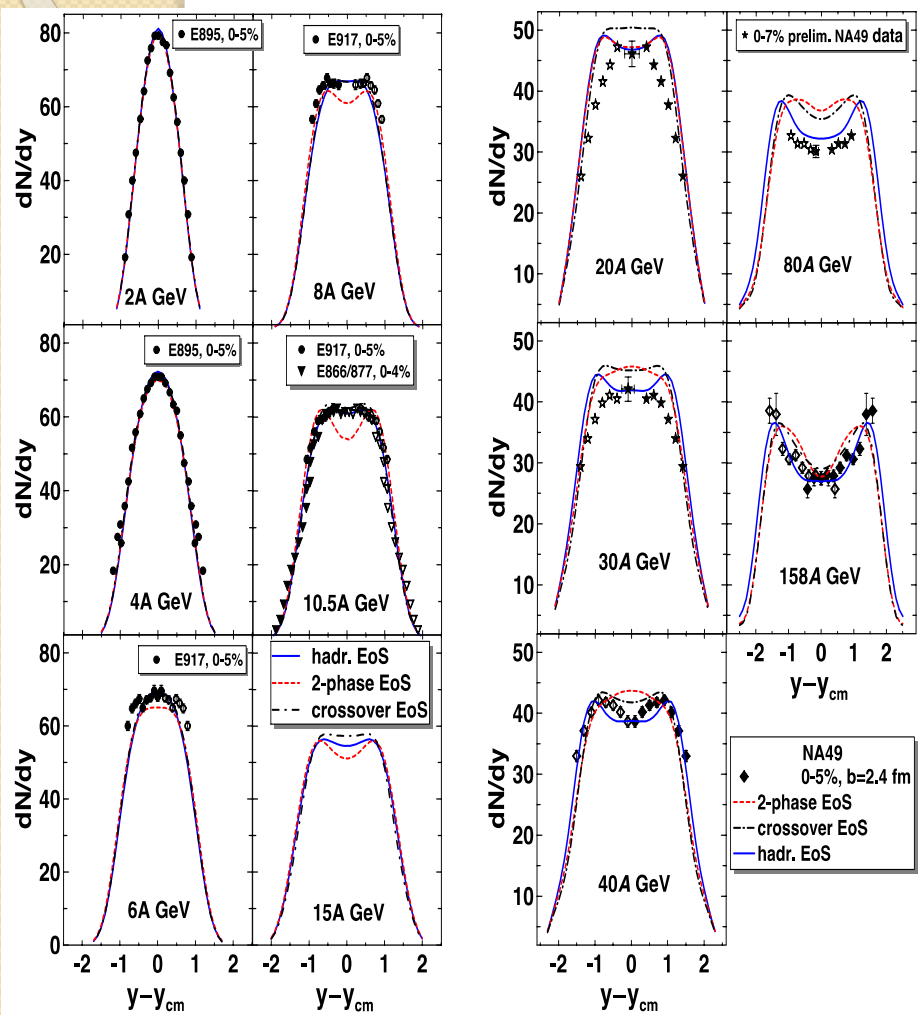
Au+Au collision at $\sqrt{s_{NN}}$ = 6.4 GeV (Elab = 20A GeV) with impact parameter b = 6 fm

Total energy-momentum conservation

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

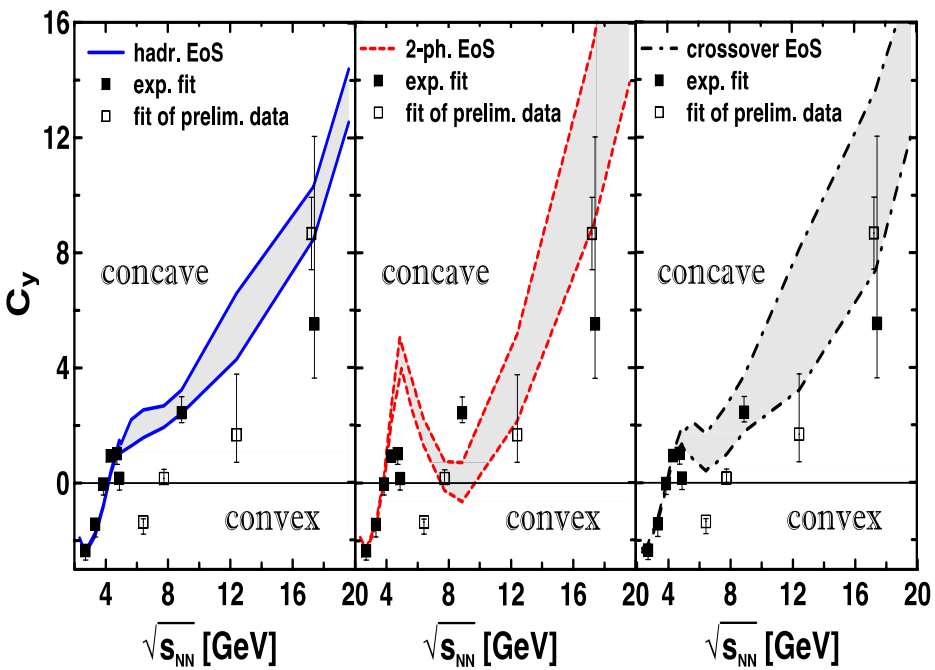
Yu. B. Ivanov, V. N. Russkikh, V. D. Toneev. Fluid Dynamics of Heavy Ion Collisions, http://mfd.jinr.ru

Rapidity Distribution & Curvature at Midrapidity



A reduced curvature of the spectrum at midrapidity

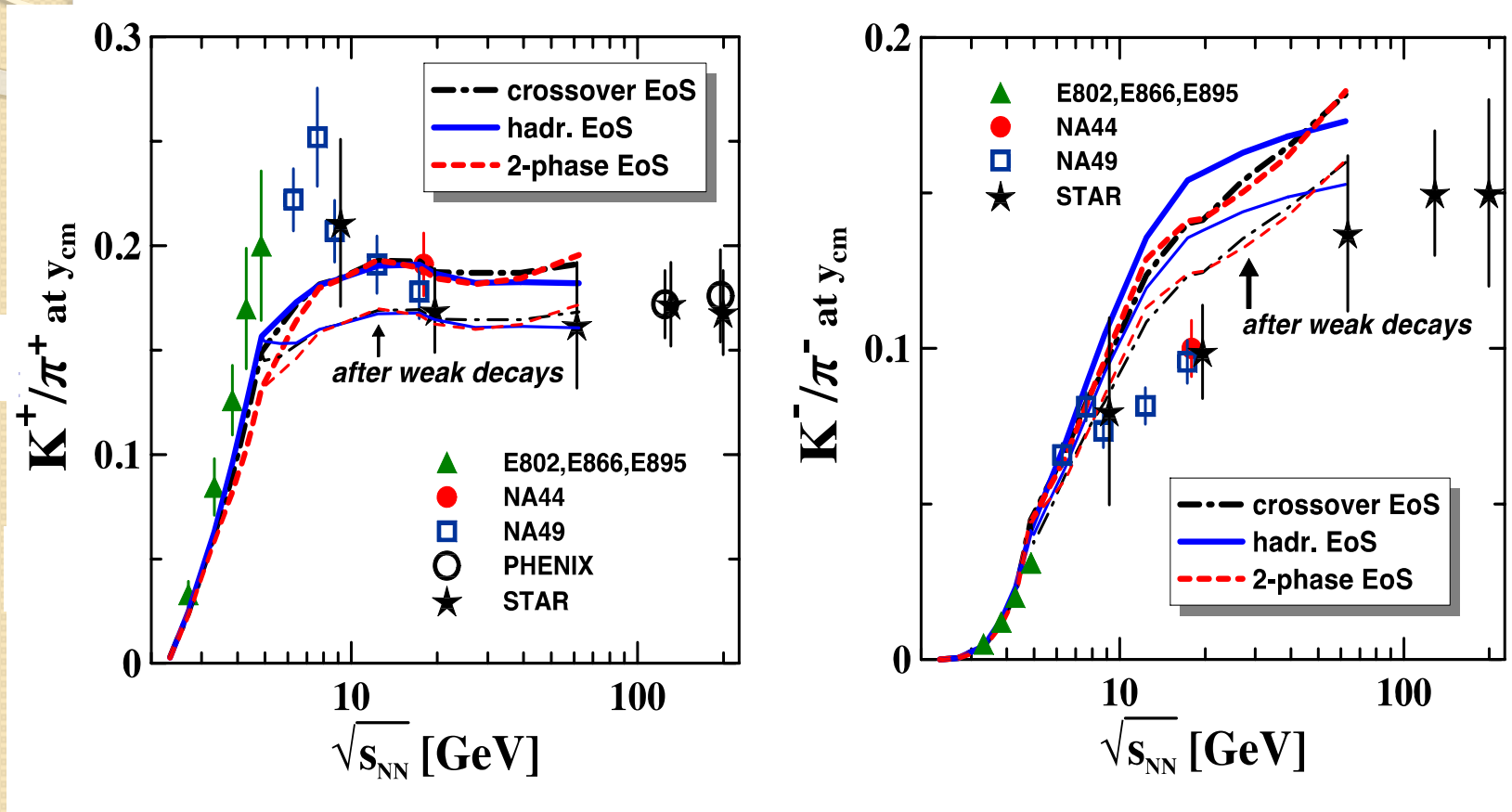
$$C_y = \left(y_{cm}^3 \frac{d^3N}{dy^3}\right)_{y=y_{cm}} \Big/ \left(y_{cm}\frac{dN}{dy}\right)_{y=y_{cm}} = (y_{cm}/w_s)^2\left(\sinh^2 y_s - w_s \cosh y_s\right)$$



Yu. B. Ivanov. Phys. Lett. B 690(4

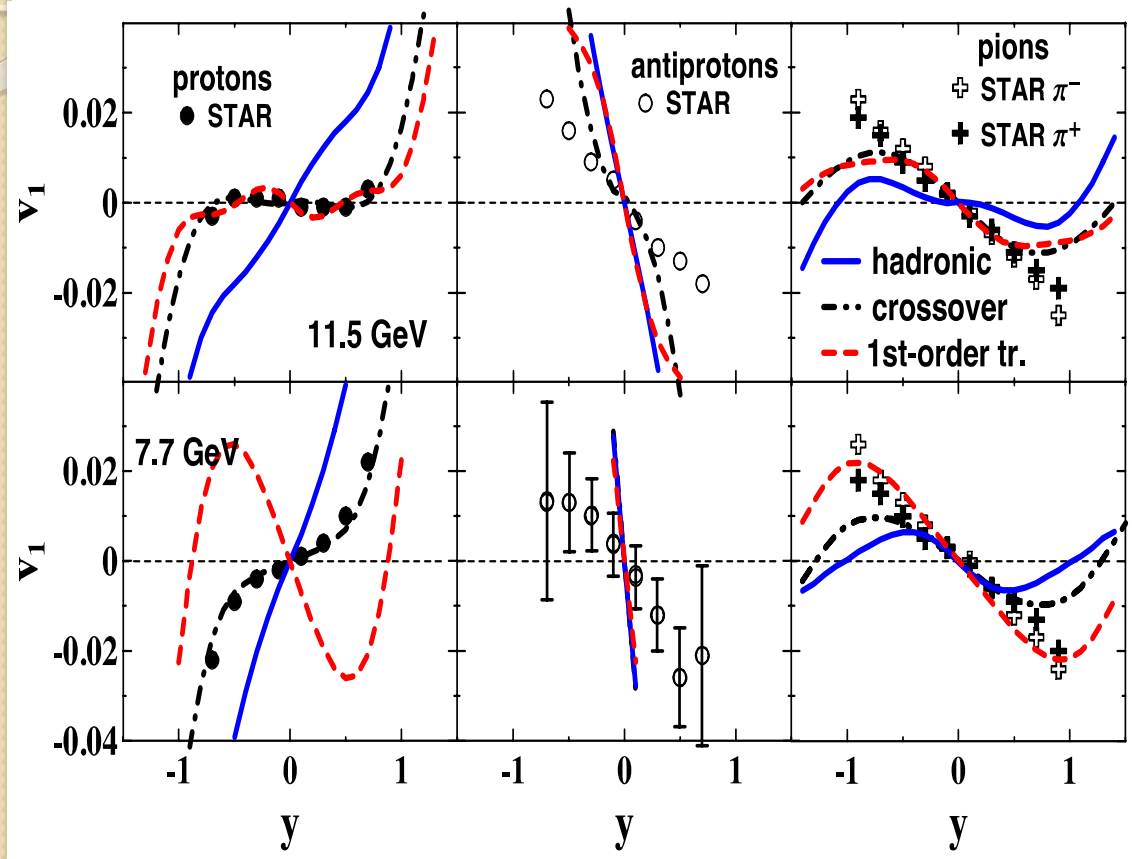
P. Batyuk et al. Phys Rev C94, 0449

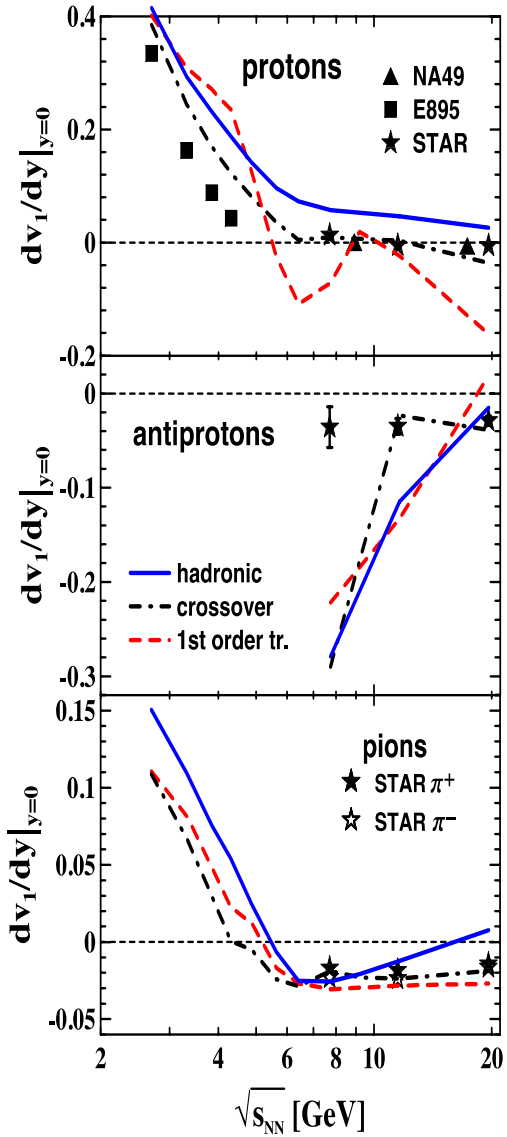
Hadron Ratios of Midrapidity



P. Batyuk et al. Phys Rev C94, 044917 (2016)

Directed Flow





P. Batyuk et al. Phys Rev C94, 044917 (2016)

Summary

- Varying EoS models
  - Formulation and solution of the optimization problem for definition of free hydrodynamic parameters
  - Development of hybrid EoS model construction
- Bayesian analysis for finding the best model parameter regions
  - Heavy ion simulation with different EoS models (parameters)
  - Collecting suitable experimental data around NICA energy range
  - Formulation and performing the Bayesian analysis
- Simulation of heavy ion collision at MPD detector
  - HIC simulation with the best models
  - Physical analysis of the simulated data within MPDroot
  - Comparing the results with the various experimental data

A. ALIKHANYAN NATIONAL LABORATORY

References

A. Ayriyan, D. Alvarez-Castillo, D. Blaschke and H. Grigorian, *Bayesian Analysis for Extracting Properties of the Nuclear Equation of State from Observational Data including Tidal Deformability from GW170817*, **Universe** 5, 61 (2019), doi 10.3390/universe5020061

K. Maslov, N. Yasutake, A. Ayriyan, D. Blaschke, H. Grigorian, T. Maruyama, T. Tatsumi, D. N. Voskresensky. *Hybrid equation of state with pasta phases, and third family of compact stars* **Physical Review C** 100, 025802 (2019), doi 10.1103/PhysRevC.100.025802

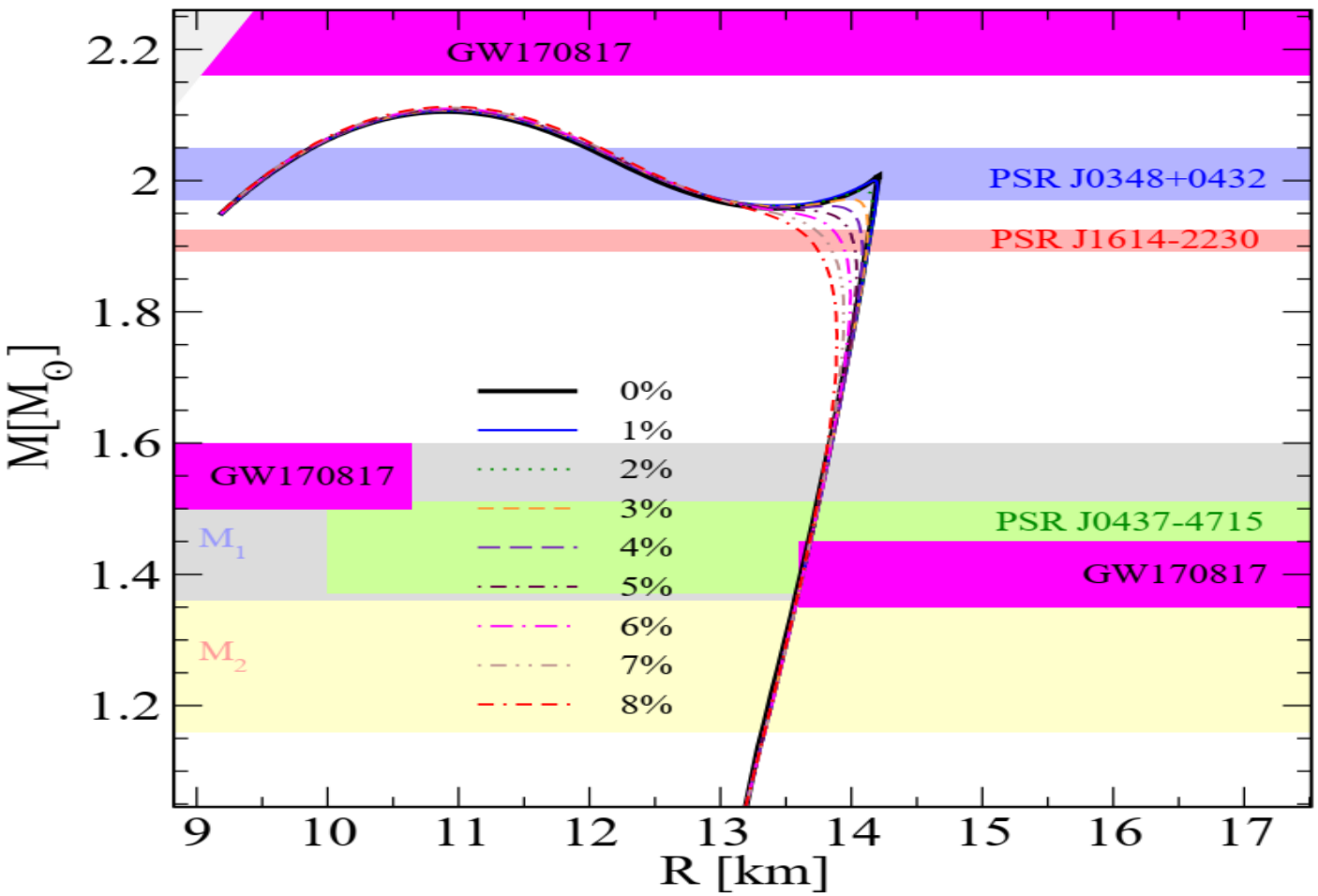
V. Abgaryan, D. Alvarez-Castillo, A. Ayriyan, D. Blaschke and H. Grigorian. *Two Novel Approaches to the Hadron-Quark Mixed Phase in Compact Stars.* **Universe** 4(9), 94 (2018), doi 10.3390/universe4090094

A. Ayriyan, N.-U. Bastian, D. Blaschke, H. Grigorian, K. Maslov, and D. N. Voskresensky. *Robustness of third family solutions for hybrid stars against mixed phase effects.* **Physical Review C** 97, 045802 (2018), doi 10.1103/PhysRevC.97.045802

A. Ayriyan and H. Grigorian. *Model of the Phase Transition Mimicking the Pasta Phase in Cold and Dense Quark-Hadron Matter.* **European Physical Journal WoC** 173, 03003 (2018), doi 10.1051/epjconf/201817303003

D. Alvarez-Castillo, A. Ayriyan, S. Benic, D. Blaschke, H. Grigorian and S. Typel, *New class of hybrid EoS and Bayesian M-R data analysis*, **European Physical Journal A** 52, 69 (2016), doi 10.1140/epja/i2016-16069-2

The results of pasta effects

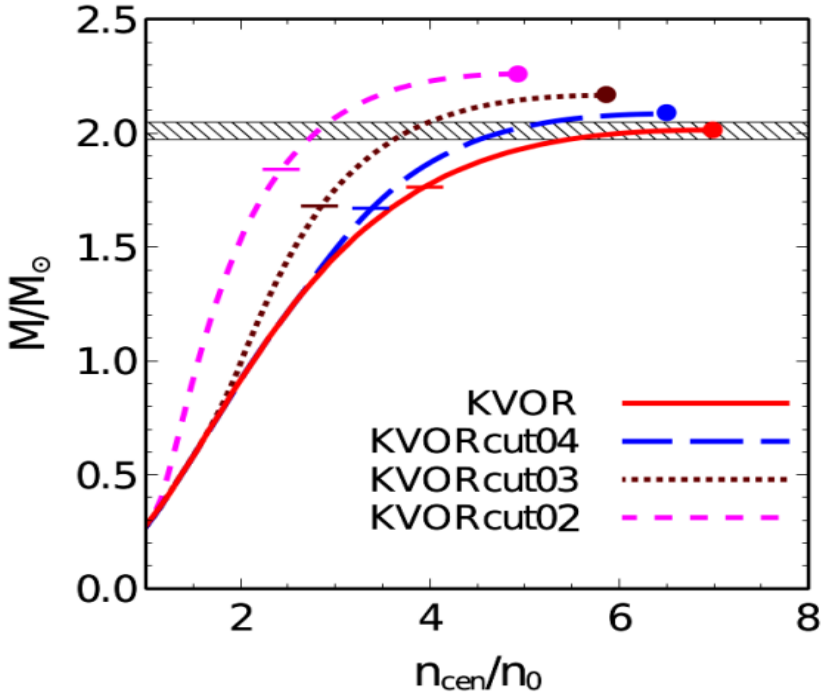


Third family robust against Δ_P up to around 5%!

Abgaryan, Alvarez-Castillo, Ayriyan et al. Universe 4(9), 94 (2018)

The realistic hadron and quark matter models

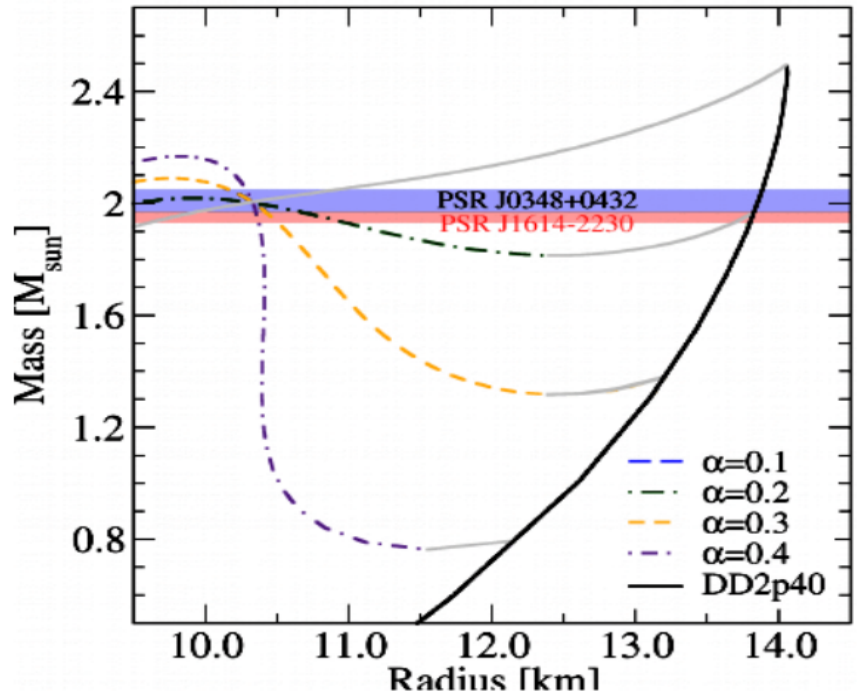
The hadron EoS model KVOR with modification of stiffness



Maslov, Kolomeitsev, Voskresensky, Nucl.Phys. A950 (2016)

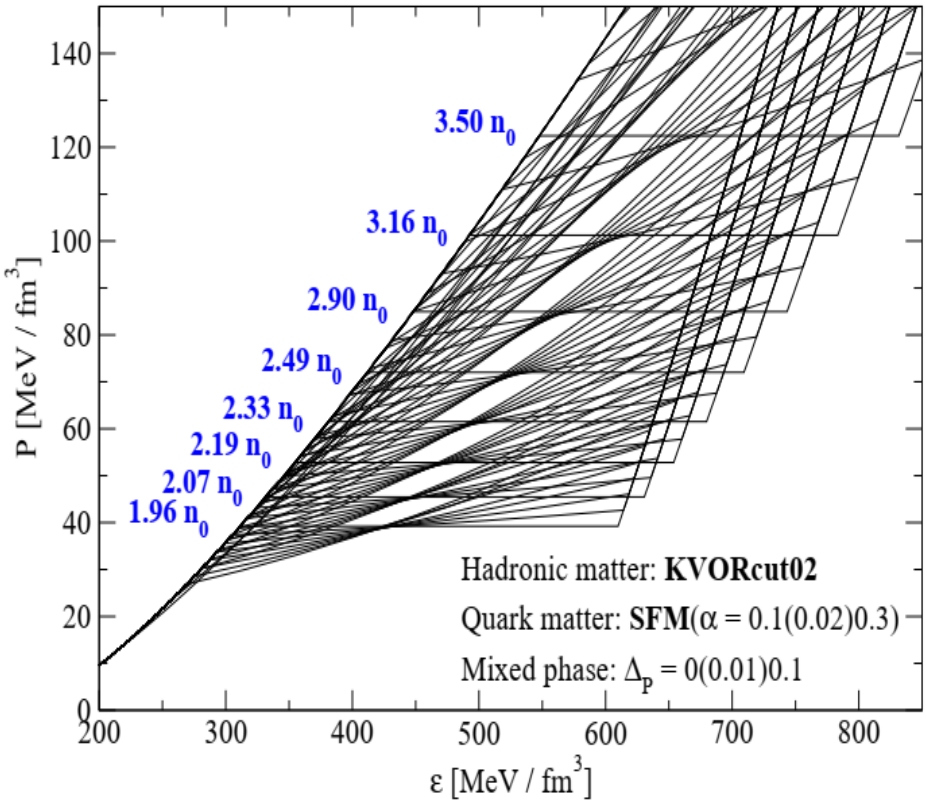
Kolomeitsev & Voskresensky, Nuc. Phys. A 759 (2005)

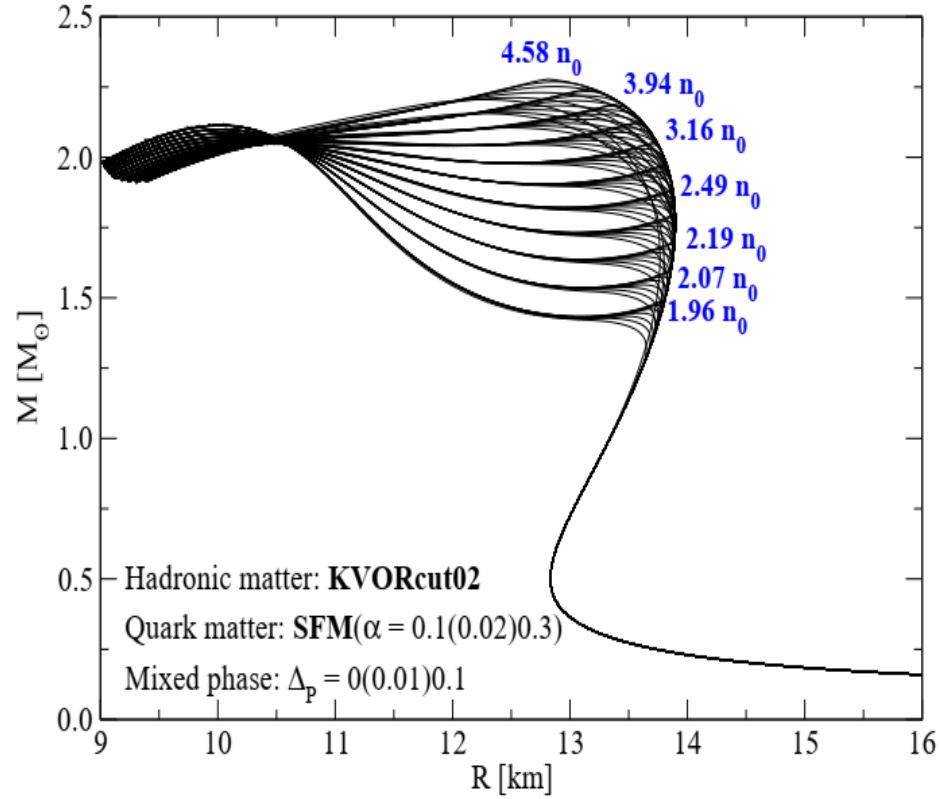
The quark EoS model SFM with available volume fraction parameter



Kaltenborn, Bastian, Blaschke, Phys. Rev. D 96, 056024 (2017)

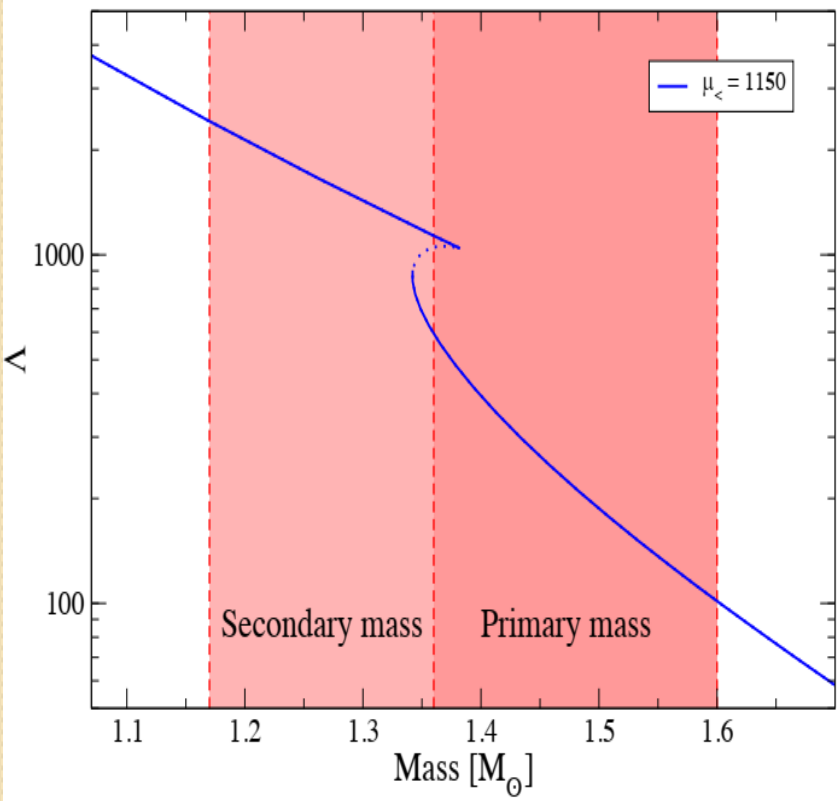
Robustness of third family solutions

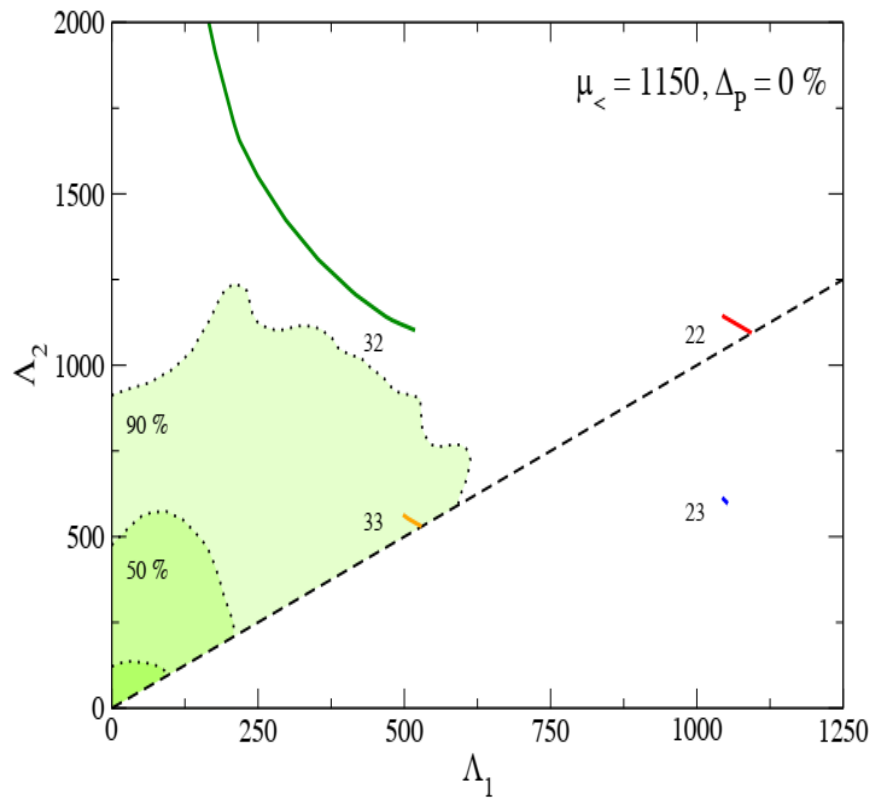




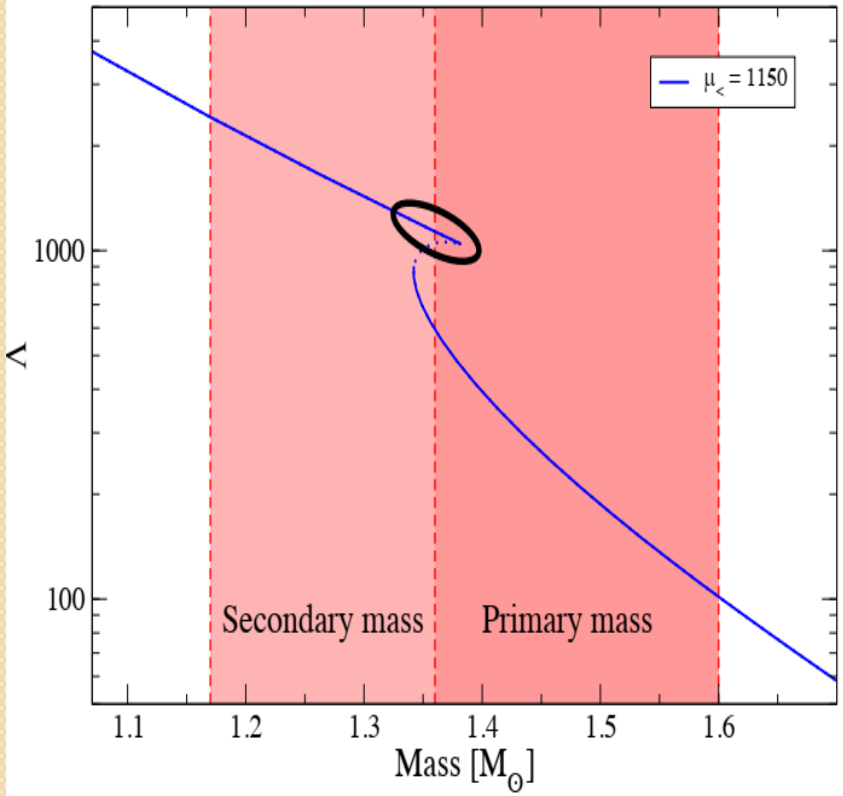
Ayriyan, Bastian, Blaschke, Grigorian, Maslov, Voskresensky. PRC 97, 045802 (2018)

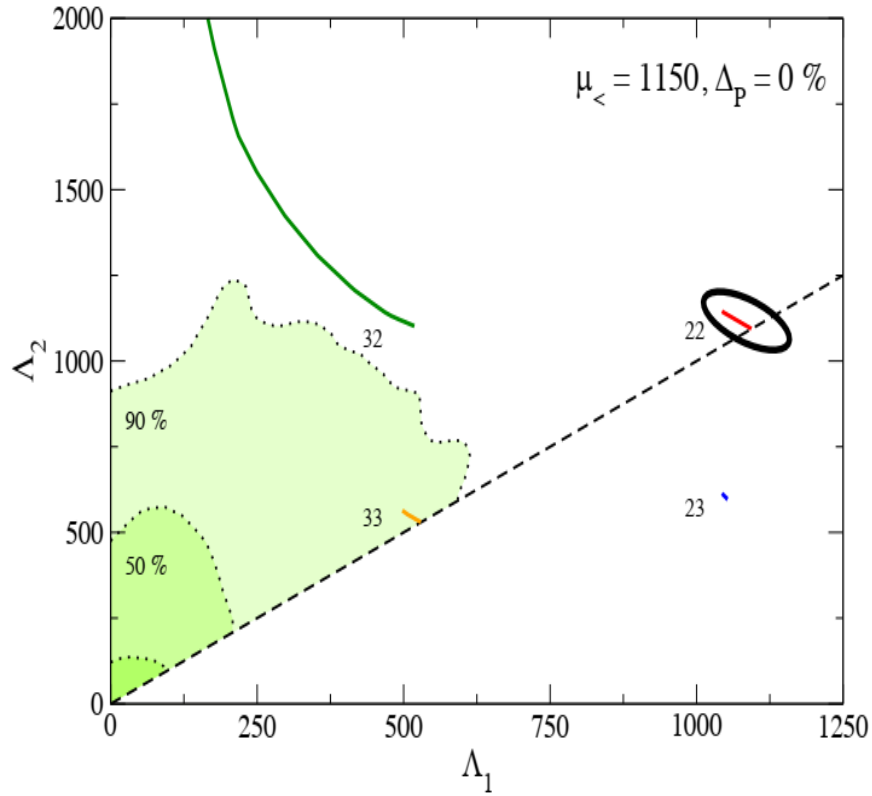
Lambda-Lambda diagram: Hybrid EoS
NS – NS merging



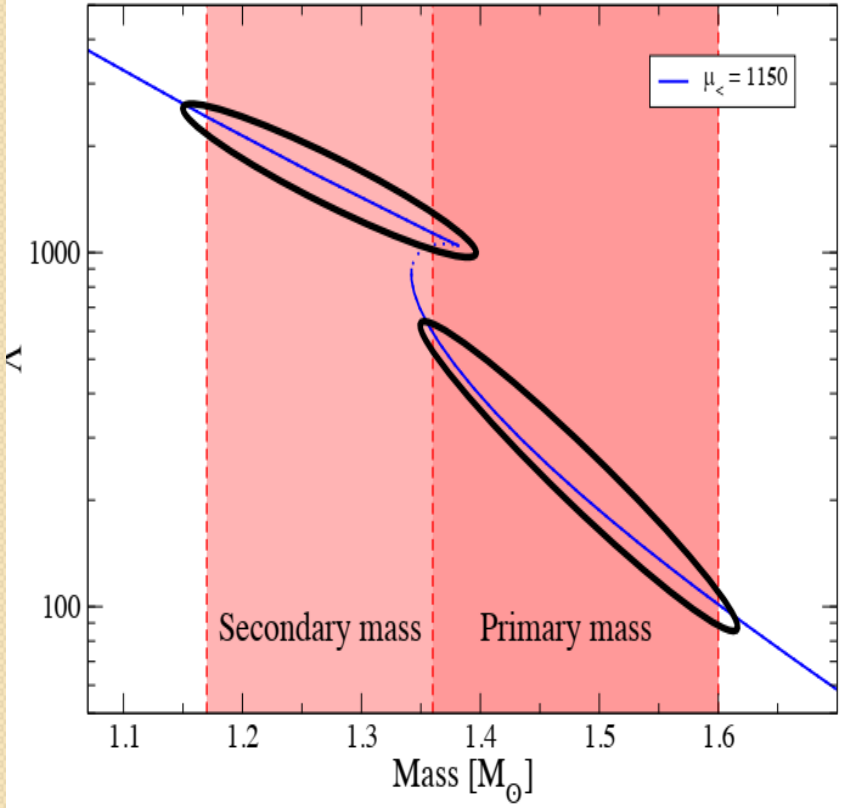


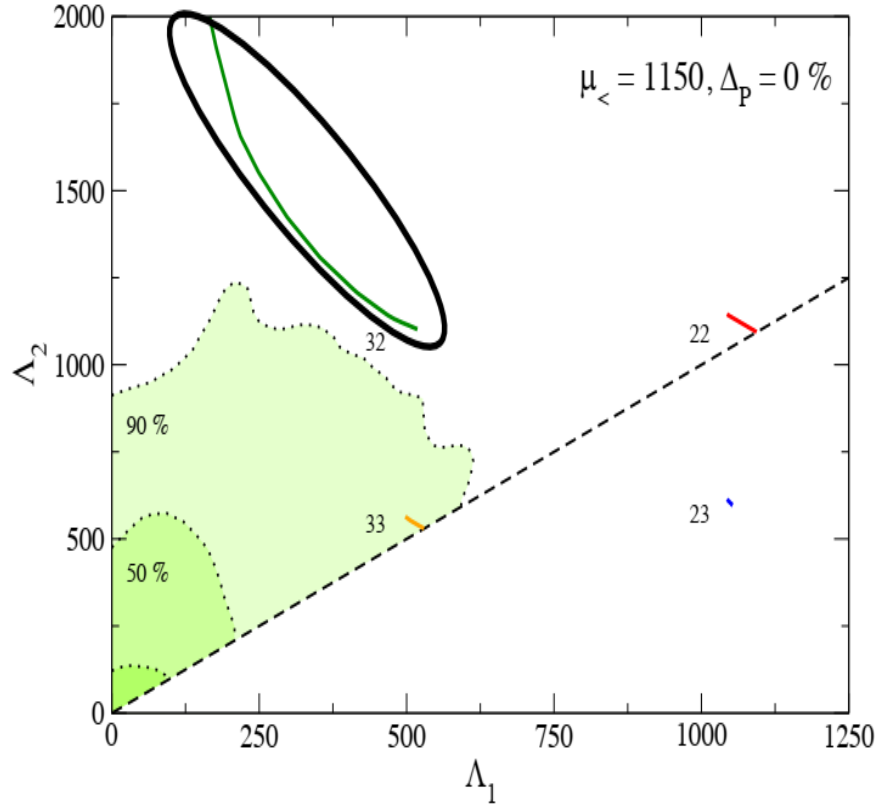
Hadron - Hadron



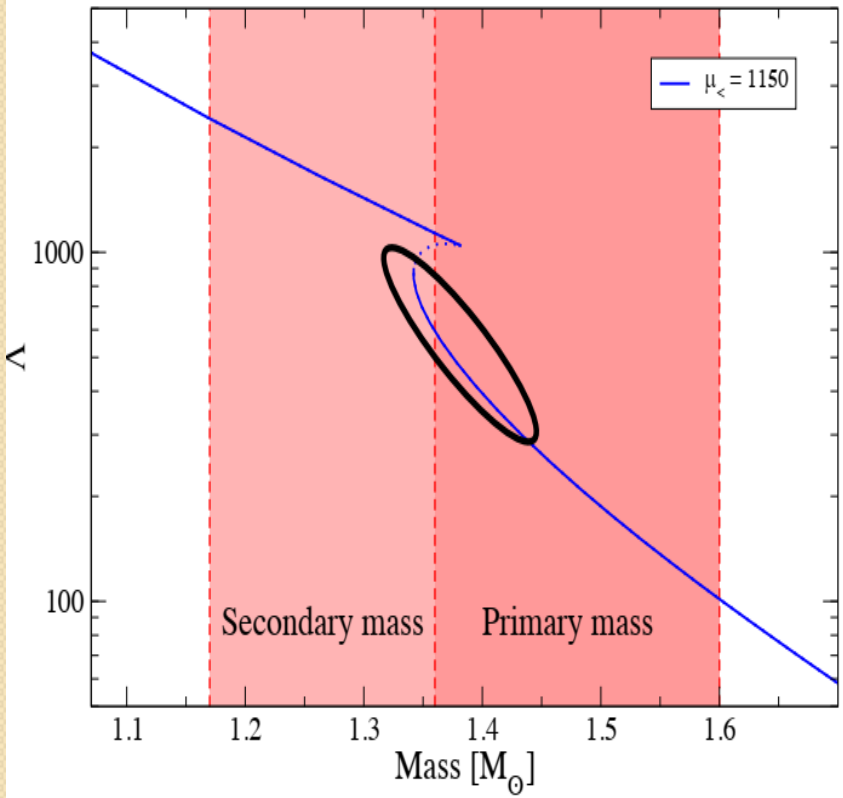


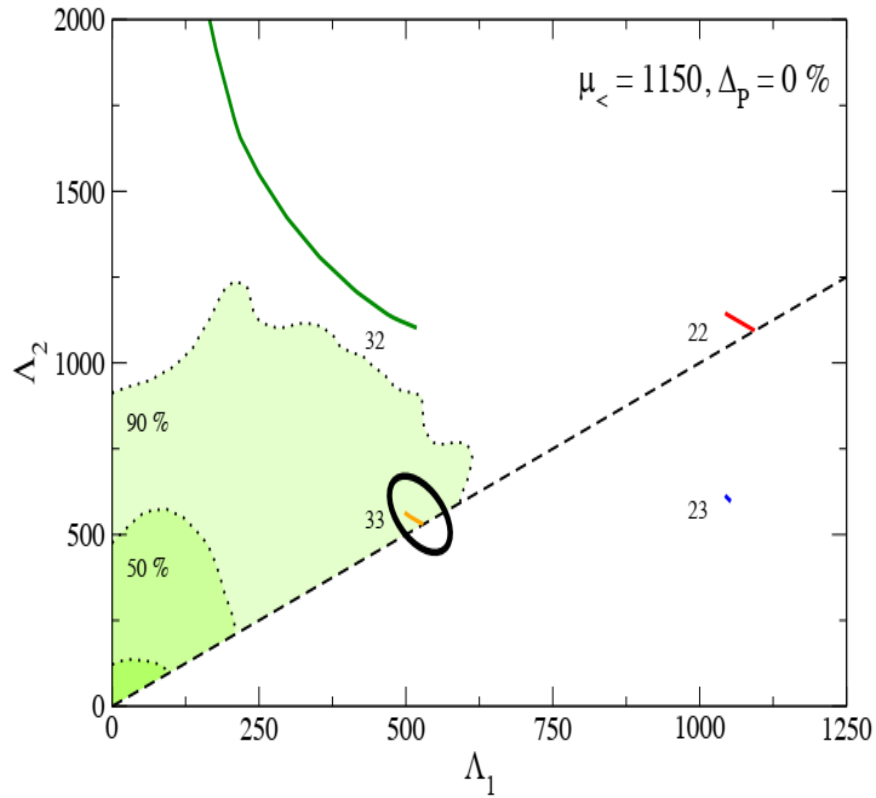
Hadron - Hybrid



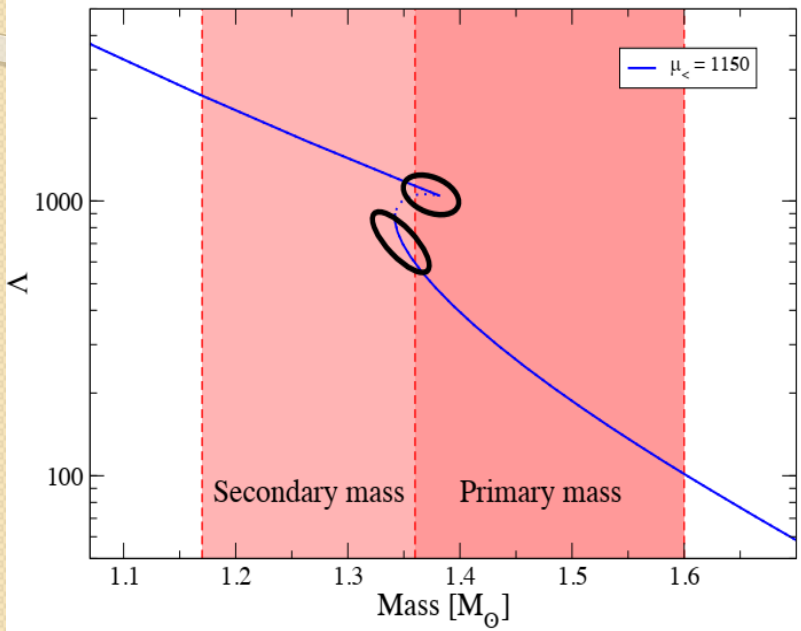


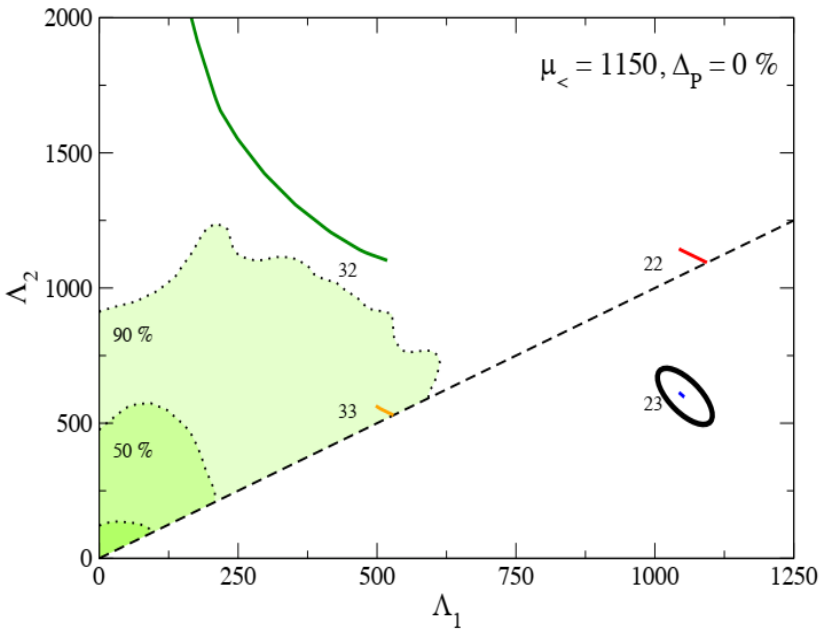
Hybrid - Hybrid





Hybrid - Hadron



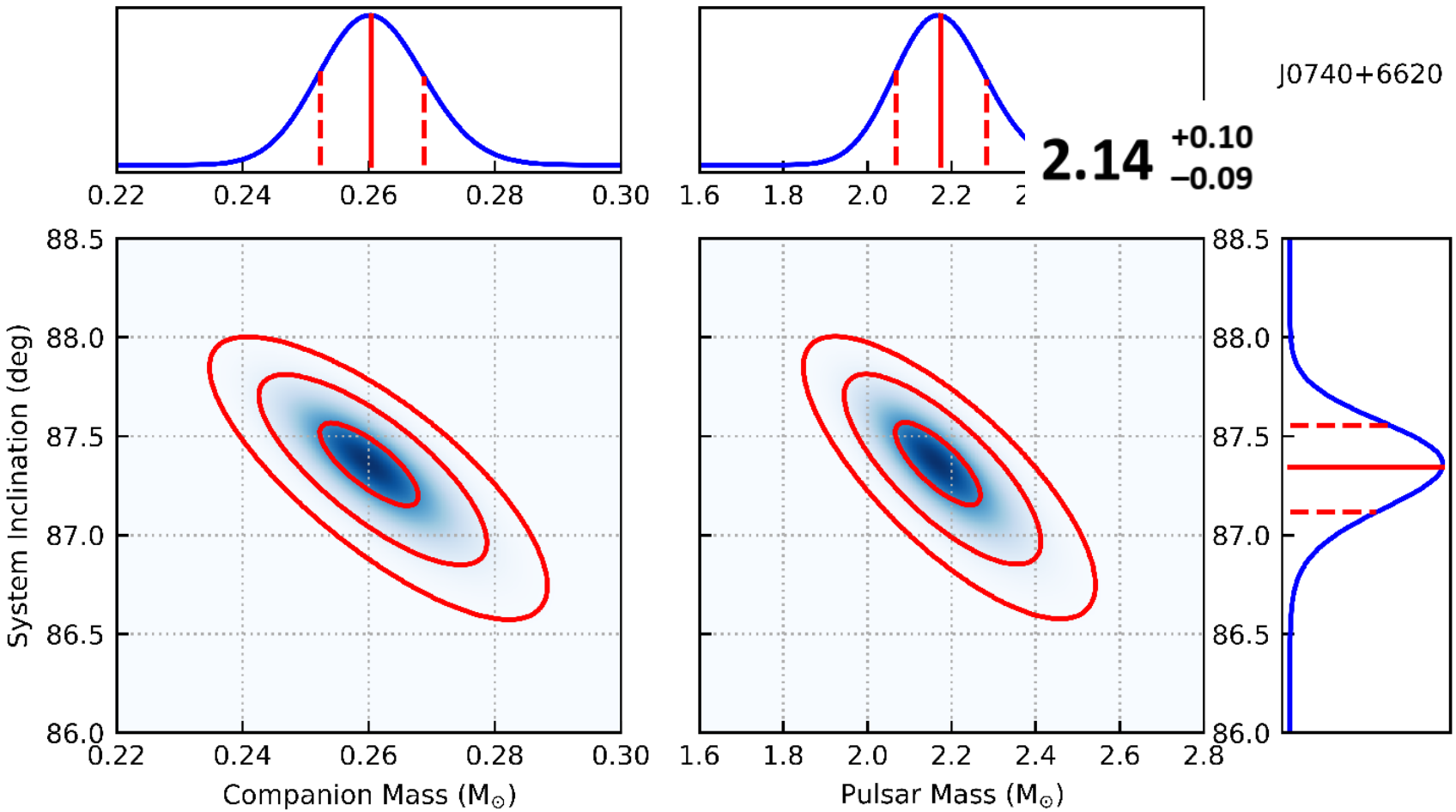


The same phenomena were found in Montana, Tolos, Hanauske, Rezzolla. PRD99, 103009 (2019) for politropic models

More interesting results have been achieved by Prof. Armen Sedrakian for triplet of compact stars produced by the forth family.

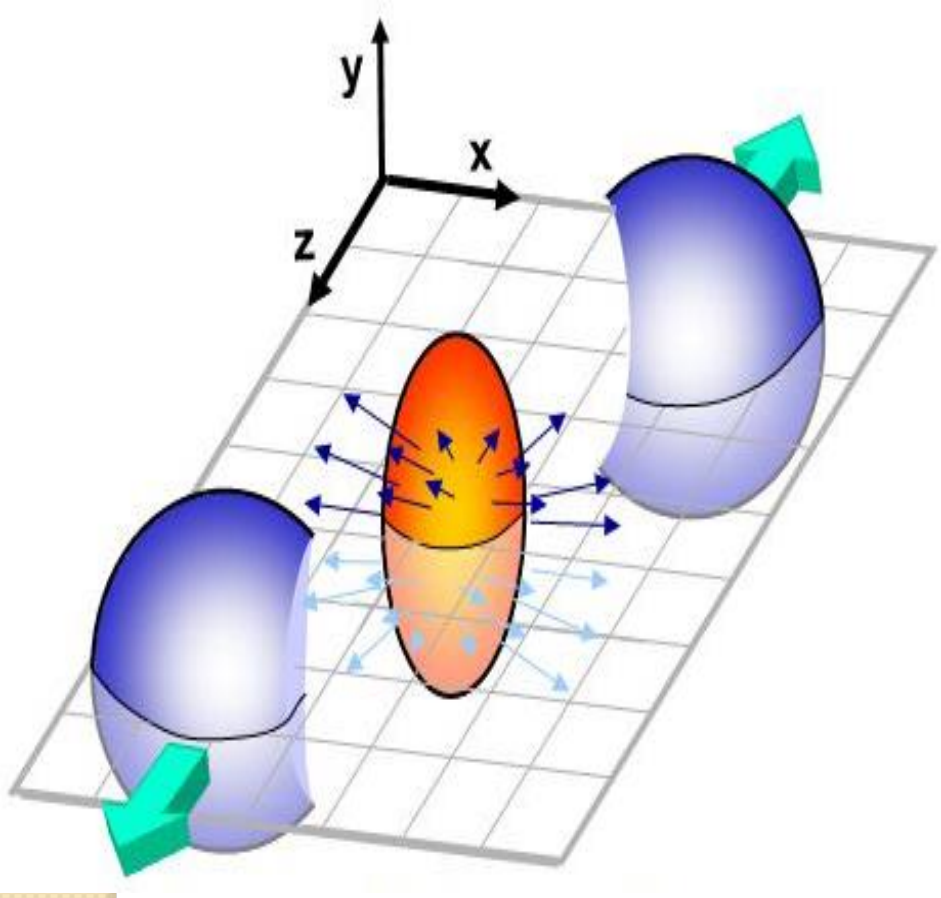
The region $\Lambda_2 < \Lambda_1$ was called unphysical at Abbott *et al.* PRL121 (2018).

Mass constraint



Cromartie et. al. Nature Astronomy (2019), doi: 10.1038/s41550-019-0880-2

Flows



Credit: M. Oldenburg

Fourier transformation of azimuthal
particle distribution in momentum
Space yields coefficients of different order

$$v_n = \langle \cos n \cdot \phi \rangle$$

$$\phi = \operatorname{atan} \frac{p_y}{p_x}$$

- v_1: "directed flow"
- v_2: "elliptic flow"