Probing the QCD critical end point by multi-messenger observations of compact stars and heavy ion collisions

David E. Álvarez Castillo

*Joint Institute for Nuclear Research*

*Dubna, Russia*

NICA Days 2019

IVth MPD Collaboration Meeting

Warsaw University of Technology, Poland

October 24, 2019
Outline

• A brief introduction to the neutron star equation of state and its location within the QCD phase diagram.

• The compact star mass twins hypothesis.

• Astrophysics measurements of compact stars: multi-messenger astronomy & the GW170817 event.

• Astrophysical implications and perspectives.
Nuclear Matter

Compact Star Sequences  
(M-R ⇔ EoS)

- TOV Equations
- Equation of State (EoS)

\[ \frac{dp}{dr} = -\frac{(\varepsilon + p / c^2)G(m + 4\pi r^3 p / c^2)}{r^2(1 - 2Gm/rc^2)} \]

\[ \frac{dm}{dr} = 4\pi r^2 \varepsilon \]

\[ p(\varepsilon) \]

Lattimer,  
arXiv: 1305.3510
FIG. 6: Pressure region consistent with experimental flow data in SNM (dark shaded region). The light shaded region extrapolates this region to higher densities within an upper (UB) and lower border (LB).
Most massive neutron star ever detected strains the limits of physics

By Ashley Strickland, CNN

Updated 0050 GMT (0850 HKT) September 17, 2019

Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar


Despite its importance to our understanding of physics at supranuclear densities, the equation of state (EoS) of matter deep within neutron stars remains poorly understood. Millisecond pulsars (MSPs) are among the most useful astrophysical objects in the Universe for testing fundamental physics, and place some of the most stringent constraints on this high-density EoS. Pulsar timing—the process of accounting for every rotation of a pulsar over long time periods—can precisely measure a wide variety of physical phenomena, including those that allow the measurement of the masses of the components of a pulsar binary system. One of these, called relativistic Shapiro delay, can yield precise masses for both an MSP and its companion; however, it is only easily observed in a small subset of high-precision, highly inclined (nearly edge-on) binary pulsar systems. By combining data from the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) 12.5-yr data set with recent orbital-phase-specific observations using the Green Bank Telescope, we have measured the mass of the MSP J0740+6620 to be $2.14^{+0.07}_{-0.04} M_\odot$ (68.3% credibility interval; the 95.4% credibility interval is $2.14^{+0.08}_{-0.05} M_\odot$). It is highly likely to be the most massive neutron star yet observed, and serves as a strong constraint on the neutron star interior EoS.

Precise neutron star mass measurements are an effective way to constrain the EoS of the ultradense matter in neutron star interiors. Although radio pulsar timing cannot directly determine neutron star radii, the existence of pulsars with masses exceeding the maximum mass allowed by a given model can straightforwardly rule out that EoS.

In 2010, Demorest et al. reported the discovery of a $2 M_\odot$ MSP, J1614−2230 (ref. 1) (though the originally reported mass was $1.97^{+0.04}_{-0.04} M_\odot$, continued timing has led to a more precise mass measurement of $1.928^{+0.017}_{-0.016} M_\odot$ by Fonseca et al.2). This Shapiro-delay-enabled measurement disproved the plausibility of some hyperon, boson and free quark models in nuclear-density environments. In 2013, Antoniadis et al. used optical techniques in combination with pulsar timing to yield a mass measurement of $2.01^{+0.04}_{-0.04} M_\odot$ for the pulsar J0348+0432 (ref. 3). These two observational results (along with others) encouraged a reconsideration of the canonical $1.4 M_\odot$ neutron star. Gravitational-wave astrophysics has also begun to provide EoS constraints; for example, the Laser Interferometer Gravitational-Wave Observatory (LIGO) detection of a double neutron star merger constrains permissible EoSs, suggesting that the upper limit on neutron star mass is $2.17 M_\odot$ (90% credibility). Though the existence of extremely massive ($>2.4 M_\odot$) neutron stars has been suggested through optical spectroscopic
Massive Neutron Stars
PSR J1614-2230

A precise AND large mass measurement

Shapiro delay:
Massive Neutron Stars

PSR J0740+6620: M=2.17(+0.11, -0.10) Msun, at 68.3% credibility
Critical Endpoint in QCD
Compact Star Mass Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “third family of CS”.

- Measuring two disconnected populations of compact stars in the M-R diagram would represent the detection of a first order phase transition in compact star matter and thus the indirect proof for the existence of a critical endpoint (CEP) in the QCD phase diagram!

arxiv:1302.4732

\[ P_i(n) = \kappa_i n^{\Gamma_i} \]

\( i = 1 : \ n_1 \leq n \leq n_{12} \)
\( i = 2 : \ n_{12} \leq n \leq n_{23} \)
\( i = 3 : \ n \geq n_{23} \)

Here, 1st order PT in region 2:
\[ \Gamma_2 = 0 \quad \text{and} \quad P_2 = \kappa_2 = P_{\text{crit}} \]

\[ P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn} , \]
\[ \varepsilon(n)/n = \int dn \frac{P(n)}{n^2} = \int dn \kappa n^{\Gamma-2} = \frac{\kappa n^{\Gamma-1}}{\Gamma-1} + C , \]
\[ \mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \Gamma n^{\Gamma-1}}{\Gamma-1} + m_0 , \]

Seidov criterion for instability:
\[ \frac{\Delta \varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{3} \frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}} \]

Maxwell construction:
\[ n(\mu) = \left[ (\mu - m_0) \frac{\Gamma - 1}{\kappa \Gamma} \right]^{1/(\Gamma - 1)} \]
\[ P(\mu) = \kappa \left[ (\mu - m_0) \frac{\Gamma - 1}{\kappa \Gamma} \right]^{\Gamma/(\Gamma - 1)} \]
\[ P_1(\mu_{\text{crit}}) = P_3(\mu_{\text{crit}}) = P_{\text{crit}} \]
\[ \mu_{\text{crit}} = \mu_1(n_{12}) = \mu_3(n_{23}) \]
Compact Star Twins

Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
Multi-messenger Astronomy
GW170817: Neutron Star Merger

Anatomy of the GW signal

GNH3, $\bar{M} = 1.350 M_\odot$
Implications from GW170817

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral
What can we learn from the inspiral II

- Waveforms incl. finite-size effects are described by **tidal deformability** (how a star reacts on an external tidal field)
- Offer possibility to constrain EoS because tidal deformability depends on EoS

\[ \Lambda \equiv \frac{2}{3} k_2 \left( \frac{R}{M} \right)^5 \]

- Corresponding to \(~10\%\) error in radius \(R\) for nearby events (<100Mpc) (e.g. Read et al. 2013)
- Note: faithful templates to be constructed

  \( \frac{R}{M} \) compactness (EoS dependent)

  \( k_2 \) tidal love number (EoS dependent)
Computing the love number/tidal deformability

Extension of a standard TOV solver (i.e. numerically an integration of coupled ODEs):

Ansatz for the metric including a l=2 perturbation

\[
 ds^2 = -e^{2\Phi(r)} \left[ 1 + H(r)Y_{20}(\theta, \varphi) \right] dt^2 \\
 + e^{2\Lambda(r)} \left[ 1 - H(r)Y_{20}(\theta, \varphi) \right] dr^2 \\
 + r^2 \left[ 1 - K(r)Y_{20}(\theta, \varphi) \right] (d\theta^2 + \sin^2 \theta d\varphi^2)
\]

Following Hinderer et al. 2010

Integrate standard TOV system: And additional eqs. for perturbations:

\[
 e^{2\Lambda} = \left( 1 - \frac{2m_r}{r} \right)^{-1}, \\
 \frac{d\Phi}{dr} = -\frac{1}{\epsilon + p} \frac{dp}{dr}, \\
 \frac{dp}{dr} = -\left( \epsilon + p \right) \frac{m_r + 4\pi r^3 p}{r(r - 2m_r)}, \\
 \frac{dm_r}{dr} = 4\pi r^2 \epsilon.
\]

\[
 \frac{dH}{dr} = \beta \\
 \frac{d\beta}{dr} = 2 \left( 1 - \frac{2m_r}{r} \right)^{-1} H \left\{ -2\pi \left[ 5\epsilon + 9p + f(\epsilon + p) \right] \\
 + \frac{3}{r^2} + 2 \left( 1 - \frac{2m_r}{r} \right)^{-1} \left( \frac{m_r}{r^2} + 4\pi rp \right)^2 \right\} \\
 + \frac{2\beta}{r} \left( 1 - \frac{2m_r}{r} \right)^{-1} \left\{ -1 + \frac{m_r}{r} + 2\pi r^2 (\epsilon - p) \right\}.
\]

EoS to be provided \( \epsilon(p) \) \hspace{5cm} (K(r) given by H(r))

Note: Although multidimensional problem – computation in 1D since absorbed in Y20
Love number

\[ y = \frac{R \beta(R)}{H(R)} \]

\[ k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(y - 1) - y] \]
\[ \times \left\{ 2C[6 - 3y + 3C(5y - 8)] + 4C^3[13 - 11y + C(3y - 2) + 2C^2(1 + y)] + 3(1 - 2C)^2 [2 - y + 2C(y - 1)] \ln(1 - 2C) \right\}^{-1} \]

where \( C = \frac{M}{R} \) is the compactness of the star.
Implications from GW170817

Properties of the Binary Star Merger GW170817
Implications from GW170817

Vasileios Paschalidis, Kent Yagi, David Alvarez-Castillo, David B. Blaschke, Armen Sedrakian
Implications from GW170817

Vasileios Paschalidis, Kent Yagi, David Alvarez-Castillo, David B. Blaschke, Armen Sedrakian
Nonlocal NJL

In this model we introduce a doubly interpolated non-local NJL quark matter EoS as follows:

\[ P(\mu) = [f_<(\mu)P(\mu; \eta_-, B) + f_>(\mu)P(\mu; \eta_-, 0)]f_<(\mu) + f_>(\mu)P(\mu; \eta_+, 0), \]

where we have introduced two smooth switch-off functions, one that changes from one to zero at a lower chemical potential \( \mu_- \) with a width \( \Gamma_- \),

\[ f_< (\mu) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{\mu - \mu_-}{\Gamma_-} \right) \right], \]

and one that switches off at a higher chemical potential \( \mu_- \) with a width \( \Gamma_- \),

\[ f_\ll (\mu) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{\mu - \mu_- \ll}{\Gamma_- \ll} \right) \right], \]

whereby the corresponding switch-on functions are the complementary ones,

\[ f_>(\mu) = 1 - f_<(\mu), \quad f_\gg (\mu) = 1 - f_\ll (\mu). \]

The input EoS differ in their \( \eta \)-values, we have taken for those values at low (\( < \)) and high (\( > \)) chemical potentials \( \eta_- \) and \( \eta_+ \). Moreover, \( B \) represents a bag constant introduced to enforce confinement effects in the low chemical potential quark EoS.
Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Implications from GW170817
Nonlocal NJL

PSR J0030+0451
PSR J1614-2230
PSR J0348+0432

GW170817

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
GW170817 - Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
GW170817 - Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
GW170817 - Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
GW170817 - Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
GW170817 - Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
GW170817 - Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Nonlocal NJL

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Gravitational Wave Signals
First Order Phase Transitions

Massive Neutron Stars: Is there a concrete limit for the maximum mass?
Upper limit on the Maximum mass of static compact stars?
Universal relation for maximum mass increase upon rigid rotation

\[ \frac{M_{\text{crit}}}{M_{\text{TOV}}} = 1 + a_2 \left( \frac{j}{j_{\text{Kep}}} \right)^2 + a_4 \left( \frac{j}{j_{\text{Kep}}} \right)^4 \]

\[ M_{\text{max}} := M_{\text{crit}}(j = j_{\text{Kep}}) = (1 + a_2 + a_4) M_{\text{TOV}} \sim (1.203 \pm 0.022) M_{\text{TOV}} \]

“Universal” increase of maximum mass by 20% due to rigid rotation at maximum (critical) angular momentum

Breu & Rezzolla et al., MNRAS (2016)
Mixed phase effects (pasta phases)

High mass twin stars (ACB4)

Low mass twin stars (ACB5)
Speed of sound and causality

High mass twin stars
(ACB4)

Low mass twin stars
(ACB5)
Mass Radius Relations

ACB4

ACB5
Effect of Rotation and Mixed Phase

ACB4

ACB5

Maxwell construction

Critical value of Mixed-phase parameter

Full GR

Perturbative Approach $\sim \Omega$

Static (TOV)
Effect of Rotation and Mixed Phase

Maxwell construction

Critical value of Mixed-phase parameter
Universal Relationship for Rotating Compact Stars

\[ \alpha = \frac{M_{\text{rot}}}{M_{\text{max}}} \]

- **ACB4, \( \Delta_p = 4\% \)**
- **ACB5, \( \Delta_p = 2\% \)**

- \( M_{\text{max}} > 2.07 \, M_{\odot} \)
- \( M_{\text{max}} > 2.17 \, M_{\odot} \)

\[ \varepsilon_c \ [\text{MeV/fm}^3] \]

Graph showing the relationship between \( \alpha \) and \( \varepsilon_c \) with data points and shaded regions indicating mass limits.
Conclusions

• GW170817 favours softer EoS but also hybrid stars with strong phase order transitions.

• Future GW observations, NICER and SKA will soon result into stronger NS EoS constraints probing the mass twins hypothesis.

• Many possible astrophysical scenarios for mass twins could be confirmed implying a CEP in QCD.

• The mixed phase construction mimics the pasta phase in accordance with a full pasta calculation. This construction makes the approach more realistic and has advantages for numerical treatment of hybrid stars in general relativity.
Conclusions

- The conjecture of an upper limit on the maximum mass of nonrotating compact stars derived from GW170817 has been revisited. We find a criterion for the minimal central energy density in the maximum mass configuration that would correspond to the core of GW170817. The equation of state at high densities must be effectively soft, either as a relatively soft hadronic one or a hybrid one with a strong phase transition. The NICER radius measurement could be decisive

Gracias