Measuring Fundamental Constants

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1. What do we mean by fundamental ?
2. Some dimensionless favourites
3. Constraints on their temporal variability from cosmology.
4. Constraints on their temporal variability using atomic clocks
5. Examples of models which lead to temporal variability
6. What use we can make of them ?

First Aion Workshop – Imperial College - Mar 26th 2019

Take your pick !

[Particle Data Group 2018]

1. Physical Constants

Table 1.1. Reviewed 2015 by P.J. Mohr and D.B. Newell (NIST). Mainly from the "CODATA Recommended Values of the Fundamental Physical Constants: 2014" by P.J. Mohr, D.B. Newell, and B.N. Taylor in arXiv:1507.07956 (2015) and RMP (to be submitted). The last set of constants (beginning with the Fermi coupling constant) comes from the Particle Data Group and is the only set updated for this 2018 edition. The figures in parentheses after the values give the 1-standard-deviation uncertainties in the last digits; the corresponding fractional uncertainties in parts per 10^9 (ppb) are given in the last column. This set of constants (aside from the last group) is recommended for international use by CODATA (the Committee on Data for Science and Technology). The full 2014 CODATA set of constants may be found at `http://physics.nist.gov/constants`. See also P.J. Mohr and D.B. Newell, "Resource Letter FC-1: The Physics of Fundamental Constants," Am. J. Phys. **78**, 338 (2010).

Quantity	Symbol, equation	Value	Uncertainty (ppb)
speed of light in vacuum	c	$299\ 792\ 458$ m s^{-1}	exact*
Planck constant	h	$6.626\ 070\ 040(81)\times10^{-34}$ J s	12
Planck constant, reduced	$\hbar \equiv h/2\pi$	$1.054\ 571\ 800(13)\times10^{-34}$ J s	12
		$= 6.582\ 119\ 514(40)\times10^{-22}$ MeV s	6.1
electron charge magnitude	e	$1.602\ 176\ 6208(98)\times10^{-19}$ C $= 4.803\ 204\ 673(30)\times10^{-10}$ esu	6.1, 6.1
conversion constant	$\hbar c$	$197.326\ 9788(12)$ MeV fm	6.1
conversion constant	$(\hbar c)^2$	$0.389\ 379\ 3656(48)$ GeV^2 mbarn	12
electron mass	m_e	$0.510\ 998\ 9461(31)$ MeV$/c^2 = 9.109\ 383\ 56(11)\times10^{-31}$ kg	6.2, 12
proton mass	m_p	$938.272\ 0813(58)$ MeV$/c^2 = 1.672\ 621\ 898(21)\times10^{-27}$ kg	6.2, 12
		$= 1.007\ 276\ 466\ 879(91)$ u $= 1836.152\ 673\ 89(17)\ m_e$	0.090, 0.095
deuteron mass	m_d	$1875.612\ 928(12)$ MeV$/c^2$	6.2
unified atomic mass unit (u)	(mass ^{12}C atom)/12 = (1 g)/(N_A mol)	$931.494\ 0954(57)$ MeV$/c^2 = 1.660\ 539\ 040(20)\times10^{-27}$ kg	6.2, 12
permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 817\ldots\times10^{-12}$ F m^{-1}	exact
permeability of free space	μ_0	$4\pi\times10^{-7}$ N A^{-2} $= 12.566\ 370\ 614\ldots\times10^{-7}$ N A^{-2}	exact
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297\ 352\ 5664(17)\times10^{-3} = 1/137.035\ 999\ 139(31)^\dagger$	0.23, 0.23
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.817\ 940\ 3227(19)\times10^{-15}$ m	0.68
(e^- Compton wavelength)$/2\pi$	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e\alpha^{-1}$	$3.861\ 592\ 6764(18)\times10^{-13}$ m	0.45
Bohr radius ($m_{\rm nucleus}=\infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	$0.529\ 177\ 210\ 67(12)\times10^{-10}$ m	0.23
wavelength of 1 eV$/c$ particle	$hc/(1\ \text{eV})$	$1.239\ 841\ 9739(76)\times10^{-6}$ m	6.1
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	$13.605\ 693\ 009(84)$ eV	6.1
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	$0.665\ 245\ 871\ 58(91)$ barn	1.4
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.788\ 381\ 8012(26)\times10^{-11}$ MeV T^{-1}	0.45
nuclear magneton	$\mu_N = e\hbar/2m_p$	$3.152\ 451\ 2550(15)\times10^{-14}$ MeV T^{-1}	0.46
electron cyclotron freq./field	$\omega^e_{\rm cycl}/B = e/m_e$	$1.758\ 820\ 024(11)\times10^{11}$ rad s^{-1} T^{-1}	6.2
proton cyclotron freq./field	$\omega^p_{\rm cycl}/B = e/m_p$	$9.578\ 833\ 226(59)\times10^{7}$ rad s^{-1} T^{-1}	6.2
gravitational constant‡	G_N	$6.674\ 08(31)\times10^{-11}$ m^3 kg^{-1} s^{-2}	4.7×10^4
		$= 6.708\ 61(31)\times10^{-39}\ \hbar c\ ({\rm GeV}/c^2)^{-2}$	4.7×10^4
standard gravitational accel.	g_N	$9.806\ 65$ m s^{-2}	exact
Avogadro constant	N_A	$6.022\ 140\ 857(74)\times10^{23}$ mol^{-1}	12
Boltzmann constant	k	$1.380\ 648\ 52(79)\times10^{-23}$ J K^{-1}	570
		$= 8.617\ 3303(50)\times10^{-5}$ eV K^{-1}	570
molar volume, ideal gas at STP	$N_A k(273.15\ \text{K})/(101\ 325\ \text{Pa})$	$22.413\ 962(13)\times10^{-3}$ m^3 mol^{-1}	570
Wien displacement law constant	$b = \lambda_{\rm max}T$	$2.897\ 7729(17)\times10^{-3}$ m K	570
Stefan-Boltzmann constant	$\sigma = \pi^2k^4/60\hbar^3c^2$	$5.670\ 367(13)\times10^{-8}$ W m^{-2} K^{-4}	2300
Fermi coupling constant**	$G_F/(\hbar c)^3$	$1.166\ 378\ 7(6)\times10^{-5}$ GeV^{-2}	510
weak-mixing angle	$\sin^2\hat\theta(M_Z)$ ($\overline{\rm MS}$)	$0.231\ 22(4)^{\dagger\dagger}$	1.7×10^5
$W^\pm$ boson mass	m_W	$80.379(12)$ GeV$/c^2$	1.5×10^5
Z^0 boson mass	m_Z	$91.1876(21)$ GeV$/c^2$	2.3×10^4
strong coupling constant	$\alpha_s(m_Z)$	$0.1181(11)$	9.3×10^6

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238$ $\quad e = 2.718\ 281\ 828\ 459\ 045\ 235$ $\quad \gamma = 0.577\ 215\ 664\ 901\ 532\ 861$

1 in ≡ 0.0254 m	1 G ≡ 10^{-4} T	1 eV = $1.602\ 176\ 6208(98)\times10^{-19}$ J	kT at 300 K = $[38.681\ 740(22)]^{-1}$ eV
1 Å ≡ 0.1 nm	1 dyne ≡ 10^{-5} N	1 eV$/c^2$ = $1.782\ 661\ 907(11)\times10^{-36}$ kg	0 °C ≡ 273.15 K
1 barn ≡ 10^{-28} m^2	1 erg ≡ 10^{-7} J	$2.997\ 924\ 58\times10^9$ esu = 1 C	1 atmosphere ≡ 760 Torr ≡ 101 325 Pa

* The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.
† At $Q^2 = 0$. At $Q^2 \approx m_W^2$ the value is $\sim 1/128$.
‡ Absolute lab measurements of G_N have been made only on scales of about 1 cm to 1 m.
** See the discussion in Sec. 10, "Electroweak model and constraints on new physics."
†† The corresponding $\sin^2\theta$ for the effective angle is 0.23155(4).

A few specifics ! [Particle Data Group 2018]

			Uncertainty (ppb)
Speed of light in vacuum:	c	$299792458\ \mathrm{ms}^{-1}$	exact
Planck constant:	h	$6.626070040(81)\times 10^{-34}\ \mathrm{J\ s}$	12
electron charge magnitude	e	$1.6021766208(98)\times 10^{-19}\ \mathrm{C}$	6.1
electron mass	m_{e}	$0.5109989461(31)\ \mathrm{MeV/c^2} = 9.10938356(11)\times 10^{-31}\ \mathrm{kg}$	$6.2, 12$
proton mass	m_{p}	$938.2720813(58)\ \mathrm{MeV/c^2} = 1.672621898(21)\times 10^{-27}\ \mathrm{kg}$	$6.2, 12$
fine structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$1/137.035999139(31)$	0.23
gravitational constant	G_N	$6.67408(31)\times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}} = 6.70861(31)\times 10^{-39}\ \hbar c\,(\mathrm{GeV/c^2})^{-2}$	$4.7\mathrm{x}10^4$
Fermi Coupling constant	$G_F/(\hbar c)^3$	$1.1663787(6)\times 10^{-5}\mathrm{GeV}^{-2}$	510
strong coupling constant	$\alpha_S(m_Z)$	$0.1181(11)$	9.3×10^6

I am going to interpret my brief today to consider constraints on the variation of fundamental constants as opposed to measuring their precise values as accurately as possible.

In particular I will concentrate on the temporal variation. But it raises a number of interesting questions. What constants should we think about and how should we interpret the results we obtain?

For example we often see constraints on $\dot{G}/G$, and there are papers considering $\dot{c}/c$ in the early universe and $\dot{e}/e$. Should we only consider variations of dimensionless quantities like α ?

Time is special !

Big Year for Metrology – New SI unit system

World Metrology Day – 20 May 2019

Quantity	Associated Constant	Unit	Uncertainty
Time	ν_{Cs}	second	10^{-18}
Distance	c	meter	10^{-9}
Current	e	ampere	10^{-9}
Mass	h	kilogram	10^{-8}
Temperature	k_B	kelvin	10^{-7}
Amount of Substance	N_A	mole	10^{-9}
Luminous Intensity	K_{cd}	candela	10^{-7}

Applied and Pure Science apps

NIST

[credit: Chris Oates, NIST, 2019]

Dirac 1937 - Large Number Hypothesis

"very large and very small dimensionless universal constants can not be pure mathematical numbers and should rather be considered as variable parameters characterising the state of the Universe"

He considered dimensionless couplings like

$$\alpha \equiv \frac{e^2}{\hbar c} \simeq \frac{1}{137.036} \text{ strength of emg int}$$

$$\alpha_G \equiv \frac{G m_p^2}{\hbar c} = \frac{m_p^2}{M_{\rm Pl}^2} \simeq 5.9 \times 10^{-39} \text{ strength of grav int}$$

$$\alpha_W \equiv \frac{G_F m_p^2 c}{\hbar^3} \simeq 1.03 \times 10^{-5} \text{ strength of weak int}$$

as well as:

$$\delta \equiv \frac{H_0 \hbar}{m_\mathrm{p} c^2} \simeq 10^{-42}, \quad \text{where } \mathrm{H}_0 \sim 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ Hubble parameter today}$$

Considering which of these parameters could vary in time he noticed the relative magnitude of the emg and grav interaction between a proton and electron is basically the same as the inverse number of times an electron has orbited around a proton during the age of the Universe:

$$\frac{\alpha_G}{\mu\alpha} = \frac{G m_\mathrm{p} m_\mathrm{e}}{e^2} \sim 3.7 \times 10^{-40}$$

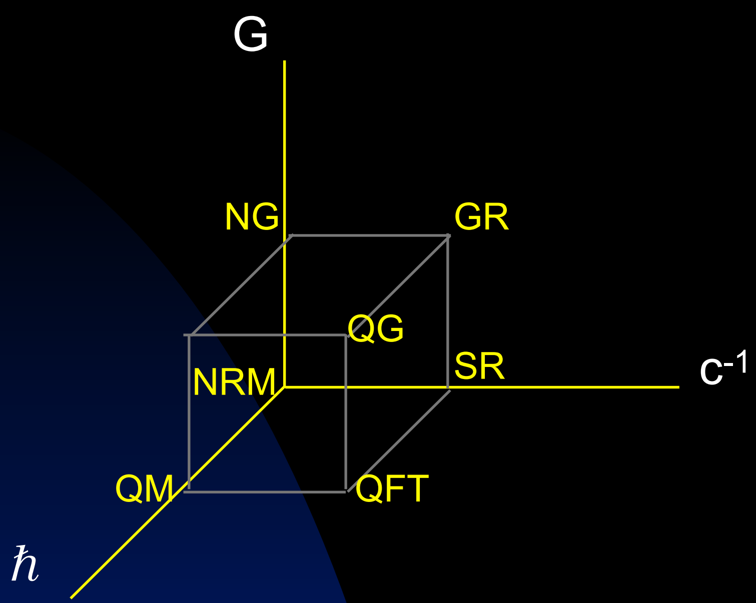
$$\frac{H_0 e^2}{m_\mathrm{e} c^3} = 4\pi\alpha\mu\delta \sim 2.4 \times 10^{-40}$$

$$\text{where } \mu \equiv \frac{m_\mathrm{p}}{m_\mathrm{e}} \sim 1836$$

As a result of this coincidence he speculated that

$$\delta \propto H_0 \text{ and } \alpha_G \text{ both vary } \propto 1/t$$

How many fundamental dimensionful constants are there ?



The Grand Cube of Theoretical Physics

[Gamov, Ivanenko & Landau 1928]

But see for example: [Duff, Okun & Veneziano, JHEP 2002]

Okun argues for 3 - G, $\hbar$ and c

Veneziano argues for 2 (within superstring theory) - c and λ_s where λ_s is a length satisfying $\lambda_s^{-2} = cT/\hbar$ where T is the string tension.

Duff argues for zero - saying the number of fundamental dimensionless quantities is important to know but the number of dimensionful quantities is arbitrary depending on the units. Hence why not choose zero.

Is this just semantics ? Maybe not when considering time variation of fundamental constants. [Duff 2002]

Davies et al argued that a BH can discriminate between two contending theories of varying α, one with varying c and the other with varying e

[Davies et al, Nature 2002]

Duff argued against this, saying using dimensional parameters is meaningless, they simply act to convert from one unit to another.

Given $\alpha = e^2/(\hbar c)$, and the claim of Webb et al (99) that it evolves with redshift, then which of these constants is const?

Davies et al claim that given BH thermodynamics, theories with decreasing c are different from (and preferred over) those with increasing e.

Entropy S of a non-rotating BH, mass M, charge Q

$$S = \frac{k\pi G}{\hbar c}(M + \sqrt{M^2 - Q^2/G})^2$$

Decreasing c increases S, but increasing e, hence Q decreases S

Hence Davies et al argue, BH can discriminate between two contending theories of varying α.

Duff: define dimensionless parameters s, μ and q

$$S = sk\pi, \quad M^2 = \mu^2 \hbar c/G \ \text{ and } \ Q^2 = q^2 \hbar c$$

the entropy becomes:

$$s = (\mu + \sqrt{\mu^2 - q^2})^2$$

Looks like the BH could in principle discriminate between contending theories with different variations of μ and q

$$s = (\mu + \sqrt{\mu^2 - q^2})^2$$

Now lets look how this appears in different units:

$$\text{Planck units}: \ \hbar = c = G = 1, \quad e^2 = \alpha, \quad M^2 = \mu^2$$

$$\text{Stoney units}: \ c = e = G = 1, \quad \hbar = 1/\alpha, \quad M^2 = \mu^2/\alpha$$

$$\text{Schrödinger units}: \ \hbar = e = G = 1, \quad c = 1/\alpha, \quad M^2 = \mu^2/\alpha$$

In all three units s is the same meaning assigning a change in α to a change in e (Planck), or a change in $\hbar$ (Stoney) or a change in c (Schrödinger) is a matter of units, not physics.

No experiment can claim changing c is better than changing e.

Observational constraints on Fundamental varying constants.

Experimental tests of the matter-gravity coupling.

The universality of the coupling between the metric $g_{\mu\nu}$ and standard model fields - Equivalence principle - predicts that the outcome of a local non-grav expt, referred to local standards does not depend on where, when and in which locally inertial frame the expt is performed.

It implies that local expts shouldn’t feel the cosmological evolution of the universe (“constants” should be constant), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance)

Observational constraints on Fundamental varying constants

1. Nuclear fission reactor phenomena at Okla, Gabon 1.8 billion years ago.

$$\frac{|\alpha_{\rm Oklo} - \alpha_0|}{\alpha_0} < 1.1 \times 10^{-8}$$

[Davis & Hamdan 2015]

2. Looking at: $X_q = m_q/\Lambda_{\rm QCD}$

determine strongest bound from shift of the ^{150}Sm resonance derived from Okla [Flambaum & Wiringa 2008]

$$\frac{|\delta X_q|}{X_q} < 4 \times 10^{-9}\ \text{1.8 billion yrs ago} \quad \text{or} \quad \frac{|\dot{X}_q|}{X_q} < 2.2 \times 10^{-18}\ \text{yr}^{-1}$$

3. Absorption lines in astronomical spectra give strong constraints on variability of α and $\mu = m_p/m_e$.

$$\frac{\Delta\alpha}{\alpha} = (1.2 \pm 1.7_{\rm stat} \pm 0.9_{\rm sys}) \times 10^{-6} \quad \text{at } z = 1.0 - 2.4$$

[Murphy et al 2016]

and

$$\frac{|\Delta\mu|}{|\mu|} < 4 \times 10^{-7} \ (95\% \ CL) \quad \text{at } z = 0.88582$$

[Kanekar et al 2015]

and

$$\frac{\Delta\alpha}{\alpha} = (3.6 \pm 3.7) \times 10^{-3} \quad \text{at } z = 1000$$

[Ade et al, Planck 2015]

Lots of attempts to constrain the variation

Object	z	$\Delta\alpha/\alpha$ (ppm)	Spectrographs	Reference
J0026−2857	1.02	3.5 ± 8.9	UVES	Murphy *et al.* (2016) [64]
J0058+0041	1.07	-1.4 ± 7.2	HIRES	Murphy *et al.* (2016) [64]
3 sources	1.08	4.3 ± 3.4	HIRES	Songaila & Cowie (2014) [67]
HS1549+1919	1.14	-7.5 ± 5.5	UVES/HIRES/HDS	Evans *et al.* (2014) [58]
HE0515−4414	1.15	-1.4 ± 0.9	UVES	Kotus *et al.* (2017) [65]
J1237+0106	1.31	-4.5 ± 8.7	HIRES	Murphy *et al.* (2016) [64]
HS1549+1919	1.34	-0.7 ± 6.6	UVES/HIRES/HDS	Evans *et al.* (2014) [58]
J0841+0312	1.34	3.0 ± 4.0	HIRES	Murphy *et al.* (2016) [64]
J0841+0312	1.34	5.7 ± 4.7	UVES	Murphy *et al.* (2016) [64]
J0108−0037	1.37	-8.4 ± 7.3	UVES	Murphy *et al.* (2016) [64]
HE0001−2340	1.58	-1.5 ± 2.6	UVES	Agafonova *et al.* (2011) [68]
J1029+1039	1.62	-1.7 ± 10.1	HIRES	Murphy *et al.* (2016) [64]
HE1104−1805	1.66	-4.7 ± 5.3	HIRES	Songaila & Cowie (2014) [67]
HE2217−2818	1.69	1.3 ± 2.6	UVES	Molaro *et al.* (2013) [56]
HS1946+7658	1.74	-7.9 ± 6.2	HIRES	Songaila & Cowie (2014) [67]
HS1549+1919	1.80	-6.4 ± 7.2	UVES/HIRES/HDS	Evans *et al.* (2014) [58]
Q1103−2645	1.84	3.5 ± 2.5	UVES	Bainbridge & Webb (2016) [66]
Q2206−1958	1.92	-4.6 ± 6.4	UVES	Murphy *et al.* (2016) [64]
Q1755+57	1.97	4.7 ± 4.7	HIRES	Murphy *et al.* (2016) [64]
PHL957	2.31	-0.7 ± 6.8	HIRES	Murphy *et al.* (2016) [64]
PHL957	2.31	-0.2 ± 12.9	UVES	Murphy *et al.* (2016) [64]

Object	z	$\Delta\mu/\mu$	Method	Reference
B0218+357	0.685	0.74 ± 0.89	$NH_3/HCO^+/HCN$	Murphy *et al.* (2008) [69]
B0218+357	0.685	-0.35 ± 0.12	$NH_3/CS/H_2CO$	Kanekar (2011) [70]
PKS1830−211	0.886	0.08 ± 0.47	NH_3/HC_3N	Henkel *et al.* (2009) [71]
PKS1830−211	0.886	-1.2 ± 4.5	CH_3NH_2	Ilyushin *et al.* (2012) [72]
PKS1830−211	0.886	-2.04 ± 0.74	NH_3	Muller *et al.* (2011) [73]
PKS1830−211	0.886	-0.10 ± 0.13	CH_3OH	Bagdonaite *et al.* (2013) [74]
J2123−005	2.059	8.5 ± 4.2	H_2/HD (VLT)	van Weerdenburg *et al.* (2013) [75]
J2123−005	2.059	5.6 ± 6.2	H_2/HD (Keck)	Malec *et al.* (2010) [76]
HE0027−1836	2.402	-7.6 ± 10.2	H_2	Rahmani *et al.* (2013) [57]
Q2348−011	2.426	-6.8 ± 27.8	H_2	Bagdonaite *et al.* (2012) [77]
Q0405−443	2.597	10.1 ± 6.2	H_2	King *et al.* (2008) [78]
J0643−504	2.659	7.4 ± 6.7	H_2	Albornoz-Vásquez *et al.* (2014) [79]
J1237+0648	2.688	-5.4 ± 7.5	H_2/HD	Daprà *et al.* (2015) [80]
Q0528−250	2.811	0.3 ± 3.7	H_2/HD	King *et al.* (2011) [81]
Q0347−383	3.025	2.1 ± 6.0	H_2	Wendt & Reimers (2008) [82]
J1443+2724	4.224	-9.5 ± 7.6	H_2	Bagdonaite *et al.* (2015) [83]

[Review - Martins 2017]

Spatial variation in $\Delta\alpha/\alpha$?

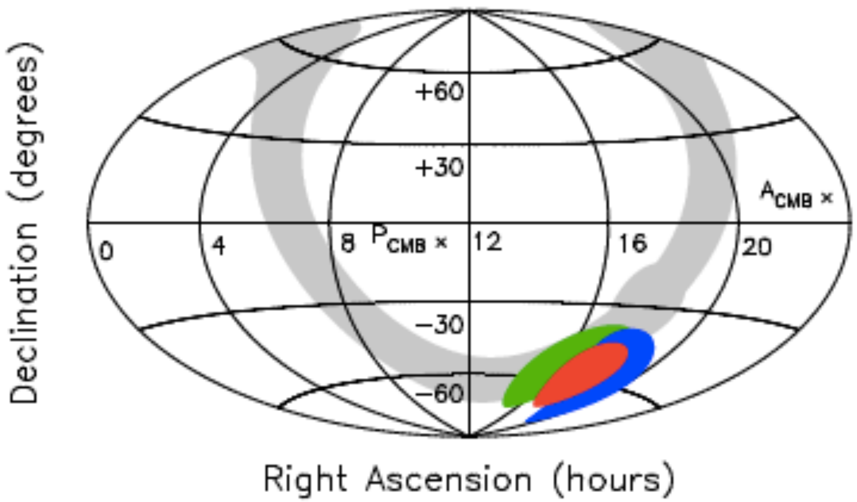


FIG. 1. All-sky plot in equatorial coordinates showing the independent Keck (green, leftmost) and VLT (blue, rightmost) best-fit dipoles, and the combined sample (red, centre), for the dipole model, $\Delta\alpha/\alpha = A\cos\Theta$, with $A = (1.02 \pm 0.21) \times 10^{-5}$. Approximate 1σ confidence contours are from the covariance matrix. The best-fit dipole is at right ascension 17.4 ± 0.9 hours, declination -58 ± 9 degrees and is statistically preferred over a monopole-only model at the 4.1σ level. For this model, a bootstrap analysis shows the chance-probability of the dipole aligments being as good or closer than observed is 6%. For a dipole+monopole model this increases to 14%. The cosmic microwave background dipole and antipole are illustrated for comparison.

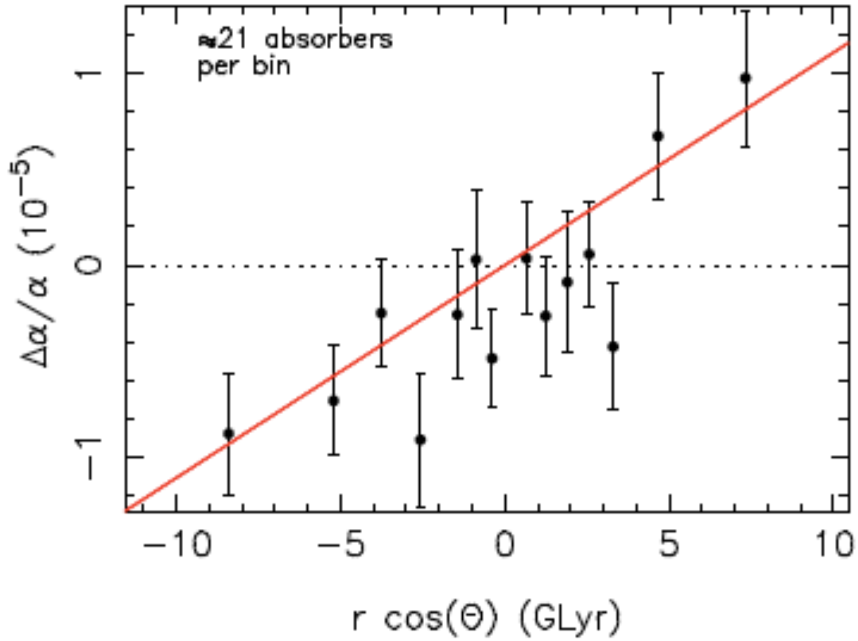
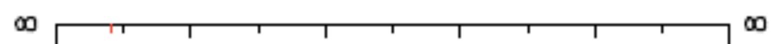


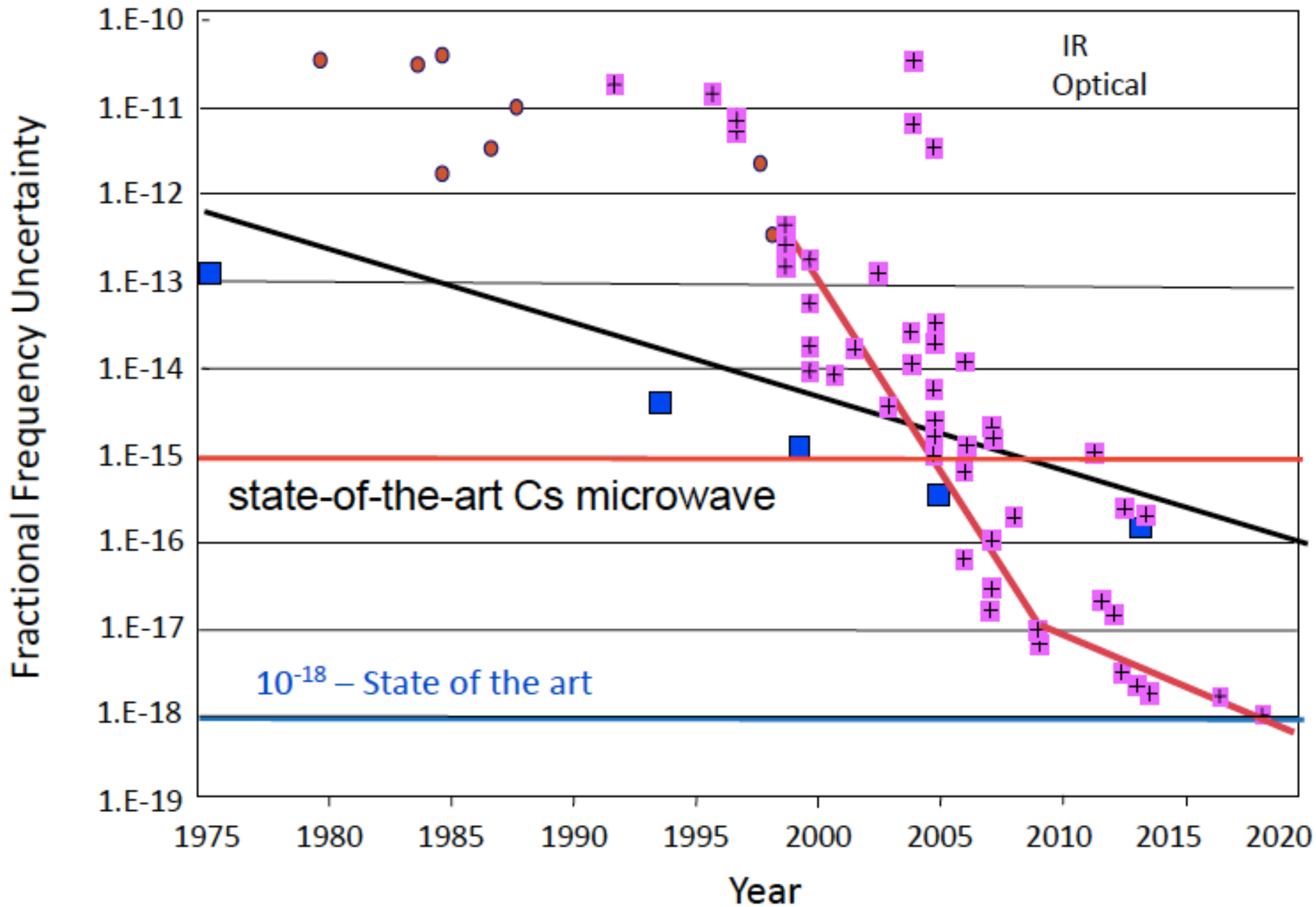
FIG. 3. $\Delta\alpha/\alpha$ vs $Ar\cos\Theta$ showing an apparent gradient in α along the best-fit dipole. The best-fit direction is at right ascension 17.5 ± 0.9 hours, declination -58 ± 9 degrees, for which $A = (1.1 \pm 0.25) \times 10^{-6}$ GLyr^{-1}. A spatial gradient is statistically preferred over a monopole-only model at the 4.2σ level. A cosmology with parameters $(H_0, \Omega_M, \Omega_\Lambda) = (70.5, 0.2736, 0.726)$ was used [18].



[Webb et al 2011]

Optical clocks offer hope of greater stability !

Atomic Clocks – recent results



NIST

[credit: Chris Oates, NIST, 2019]

4. Optical atomic clocks may transform the field. Constrain the present time variation of α, $\mu = m_p/m_e$ and X_q.

They measure energy difference between two atomic energy levels by relating it to the frequency of light. Create very stable frequency references, current best has $\delta\nu/\nu < 9.5 \times 10^{-19}$.[Brewer et al 2019]. Combining many clock systems:

$$\frac{\dot{\alpha}}{\alpha} = (-0.7 \pm 2.1) \times 10^{-17}/\text{yr}$$

and

$$\frac{\dot{\mu}}{\mu} = (0.2 \pm 1.1) \times 10^{-16}/\text{yr}$$

and

$$\frac{\dot{X}_q}{X_q} = (7.1 \pm 4.4) \times 10^{-15}/\text{yr}$$

[Godun et al 2014]

$$\frac{\dot{\mu}}{\mu} = (5.3 \pm 6.5) \times 10^{-17}/\text{yr}$$

[McGrew et al 2018]

5. Tests for isotropy of space - via quadrupolar shifts of nuclear energy levels. Null results interpreted as testing the fact that matter coupled to one and the same external metric to the 10^{-29} level [Smiciklas et al 2011]

Universal coupling to metric implies 2 (electrically neutral) test bodies dropped at same location and with same velocity in an ext grav field fall in the same way, indep of their masses and compositions

$$(\Delta a/a)_{\rm BeTi} = (0.3 \pm 1.8) \times 10^{-13}$$ [Wagner et al 2012]

$$(\Delta a/a)_{\rm EarthMoon} = (-0.8 \pm 1.3) \times 10^{-13}$$ [Williams et al 2012]

6. Tests for variation of G. Most bounds come from local measurements (Sun, solar system) or early times (Nucleosynthesis). Also, G is very poorly determined in contrast to α and μ.

		Uncertainty (ppb)
$\alpha = e^2/4\pi\epsilon_0\hbar c$	$1/137.035999139(31)$	0.23
G_N	$6.67408(31)\times10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2} = 6.70861(31)\times10^{-39}\ \hbar c\,(\mathrm{GeV/c}^2)^{-2}$	$4.7\mathrm{x}10^4$

Fascinating history of investigation:

(i) Paleontology : $T_\odot \propto G$, $L_\odot \propto T^7_\odot$, flux received on Earth has strong dependence on G. Under certain conditions some bacteria and organisms would not have developed 4.0 x 10^8 yrs ago. Temp on Earth in acceptable range only if

$$\frac{|\Delta G|}{G} < 0.1$$

[Teller 1948, Gamow 1967]

Similar bound to how varying G effects Earth Radius: [Dicke 1962]

(ii) Laser ranging and radar: separation of either moon, other planets or interplanetary probes. Important point, all analysis assumes only G variation affects clocks on Earth. First approach used Venus and Mercury as targets and compared time delay between the two planets with Cesium atomic clock.

$$\frac{|\dot{G}|}{G} < 4 \times 10^{-10}\ \mathrm{yr}^{-1}$$ [Shapiro 1972]

Measurement of freq shift or radio signal sent and received from Cassini

$$\frac{|\dot{G}|}{G} \leq 10^{-14}\ \mathrm{yr}^{-1}$$ [Bertotti 2003]

Lunar laser ranging using mirrors left by Apollo and Lunakod missions

$$\frac{|\dot{G}|}{G} = (4 \pm 9) \times 10^{-13}\ \mathrm{yr}^{-1}$$ [Williams et al 2004]

(iii) Pulsar constraints: observations of the period leads to strong constraints, but model depedent. For pulsar PSR 1913+16

$$\frac{|\dot{G}|}{G} = (4 \pm 5) \times 10^{-12}\ \mathrm{yr}^{-1}$$ [Kaspi et al 1994]

(iv) Nucleosynthesis: abundances depend on freeze out temperature, which in turn depend on fundamental constants. For example the expansion rate depends on G.

$$\frac{|\dot{G}|}{G} = (1.5 \pm 0.7) \times 10^{-12}\ \mathrm{yr}^{-1}$$ [Barrow 1978]

$$\frac{\Delta G}{G} = 0.01^{+0.2}_{-0.16}$$ [Copi et al 2004]

Nearly all bounds are the same up to a factor !

Modelling varying fundamental constants.

Promote the Lagrangian parameters to functions of a dynamical scalar field, hence have $\alpha(\phi)$, $G(\phi)$ etc..

$$\alpha_0 \equiv \alpha(\phi)|_{\phi=<\phi>}, \quad \text{where} \;\; <\phi> \;\; \text{is vev}$$

Common to have such moduli fields in string theory where they represent the size and shape of the extra compact dimensions in 4D eff description. Close to vev we have

$$\alpha(\phi) = \alpha_0 + \lambda\varphi/M_{\rm Pl}, \quad \text{where} \;\; \varphi = \phi - <\phi>$$

Leads to:

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha(\phi)-\alpha_0}{\alpha_0} = \frac{\lambda}{\alpha_0}\frac{\Delta\phi}{M_{\rm Pl}}$$

Typically $m_\phi \sim H_0 \sim 10^{-33}$eV, :

$$\frac{\Delta\alpha}{\alpha} < 10^{-5}, \quad \text{hence} \quad \frac{\lambda\Delta\phi}{M_{\rm Pl}} \sim 10^{-7} \quad \text{within} \quad \Delta t \sim H_0^{-1}$$

Basic idea - consider Jordan-Brans-Dicke

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-\gamma}\left[\phi\mathcal{R} + \frac{w_{\rm BD}(\phi)}{\phi}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} + 16\pi L_{\rm m}\right].$$

$w_{\rm BD} \to \infty$ recovers GR

[García-Berro et al 2007]

Weak field limit:

$$G = \phi^{-1}\frac{4+2w_{\rm BD}}{3+2w_{\rm BD}}$$

Low energy string action including loops

$$S = \int d^4x\sqrt{-\gamma}\left[B_g(\phi)\mathcal{R} - B_\phi(\phi)(\nabla\phi)^2 - \frac{1}{4}B_F(\phi)F^2 + \cdots\right]$$

$$B_i(\phi) = e^{-\phi} + a_0^{(i)} + a_1^{(i)}e^{\phi} + a_2^{(i)}e^{2\phi} + \cdots$$

in terms of string coupling $g_S^2 = e^{\phi} \to 0$

All fundamental constants related to Dilaton and its evolution

Extra dimensions:

$$ds^2 = -dt^2 + a^2(t) \sum_{i,j=1}^{3} \hat{\gamma}_{ij} dx^i dx^j + R^2(t) \sum_{m,n=4}^{d+3} \hat{\gamma}_{mn} dy^m dy^n$$

If consider constants in higher dimension as fundamental then they develop time dependence through R(t) coupling on compactification.

(i) Kaluza Klein:

$$S = \int d^{4+d}\hat{x} \sqrt{-\hat{\gamma}} \frac{1}{16\pi G_{(4+d)}} \mathcal{R}^{(4+d)}$$

compactly - identify components of metric with fields

$$S_{LO} = \int d^4x \sqrt{-\gamma} \left[\frac{1}{16\pi G} \mathcal{R}^{(4)} + \sum_i \frac{1}{4g_i^2} \mathrm{Tr} F^{(i)}_{\mu\nu} F^{(i)\mu\nu} \right]$$

gives eff 4D G and coupling consts

$$G = \frac{G_{(4+d)}}{V_d}, \qquad g_i^2 = \kappa_i \frac{G_{(4+d)}}{R^2 V_d},$$

where $V_d(t) \propto R^d(t)$

Note: variation of G and say α are linked through d and R(t)

[For a recent review see- Martins 2017]

Ex: Evolution of Fine Structure Constant

Olive and Pospelov; Barrow et al; Avelino et al; Sandvik et al

Non-trivial coupling to emg:

$$L_m = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu}$$

Bekenstein 82

Expand about current value of field:

$$B_F(\phi) = 1 + \zeta_F \phi + \frac{1}{2} \xi_F \phi^2$$

Eff fine structure const depends on value of field

$$\alpha(\phi) = \frac{e_0^2}{4\pi B_F(\phi)}$$

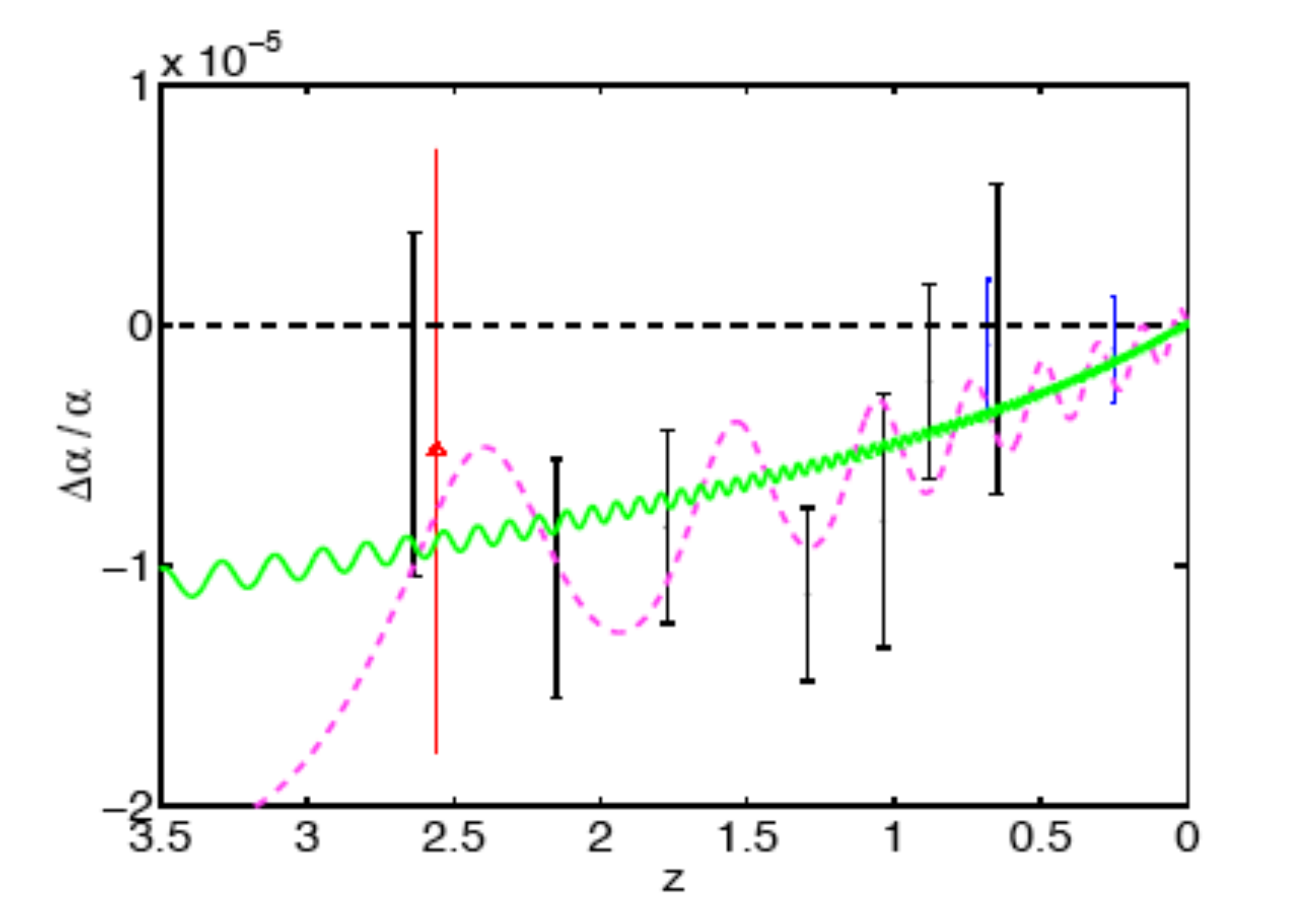
$$\frac{\Delta\alpha}{\alpha} = \zeta_F \phi + \frac{1}{2}(\xi_F - 2\zeta_F^2)\phi^2$$

Claim from analysing quasar absorption spectra:

$$\frac{\Delta\alpha}{\alpha}(z = 0.5 - 3.5) \approx 10^{-5}$$

Webb et al

$$V = V_0 e^{-\lambda \kappa \phi}$$



EC, Nunes & Pospelov

$\lambda = 100$ – solid

$\lambda = 10$ – dashed

A way of one day constraining the eqn of state?

Use the evolution to constrain models given the bounds:

Model		A	B	$\zeta \times 10^5$	$\dot{\alpha}/\alpha \times 10^{16}$	$\Delta\alpha/\alpha(z=0.14)$	$\Delta\alpha/\alpha(z=10^{10})$	osc
2EXP	a	10	–8	–20	8.6	-3.2×10^{-6}	1.6×10^{-3}	√
	b	15	0.1	3.9	4.0	-8×10^{-7}	-2.2×10^{-4}	×
	c	10	–0.5	3.9	–0.2	-7×10^{-8}	-3.2×10^{-4}	√
A-S	d	10	$0.9/A^2$	–50	–1.7	2×10^{-6}	3.5×10^{-3}	√
	e	6	$1.1/A^2$	4.5	1.2	-2×10^{-6}	-5.5×10^{-4}	×
	f	6	$0.985/A^2$	11.2	–1.4	-1×10^{-8}	-1.4×10^{-3}	√
	g	8.5	$0.93/A^2$	–30	–4.7	-1×10^{-8}	2.5×10^{-3}	√
SUGRA	h	0.5	–11	1.08	4.4	-9×10^{-7}	-0.3×10^{-4}	×
	i	0.5	–11	–0.85	4.5	-9×10^{-7}	-0.9×10^{-4}	×
	j	20	–2	25	10.7	-3×10^{-6}	4.4×10^{-4}	√
	k	2.2	–2	–1.7	–0.8	-2×10^{-8}	-9×10^{-5}	√

TABLE I: Approximate values for ζ, $\dot{\alpha}/\alpha(z=0)$ and $\Delta\alpha/\alpha$ for BBN ($z = 10^{10}$) and Oklo phenomenon ($z = 0.14$) epochs for several quintessence models. 2EXP: $V = V_0\left(e^{A\kappa\phi} + e^{B\kappa\phi}\right)$, Ref. [47]; A-S: $V = \kappa^{-4}e^{-A\kappa\phi}\left[(\kappa\phi - C)^2 + B\right]$, Ref. [48]; SUGRA: $V = V_0 \exp\left(A(\kappa\phi)^2\right)(\kappa\phi)^B$, Ref. [36]. In the Sugra model (h) the initial condition of the field is $(\kappa\phi_{\rm in})^2 \ll B/2A$ and in model (i) $(\kappa\phi_{\rm in})^2 \gg B/2A$. The models which have late time oscillations of the field have a tick in the last column. We have assumed $\Omega_\phi = 0.7$ at present.

EC, Nunes & Pospelov

[For a recent review see- Martins 2017]

Constraining varying constants from Planck 2018 - (+BAO+lensing+lowE)

$$\mathbf{H^2(z) = H_0^2 \left(\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right)}$$

(Expansion rate) -- H_0=67.66 ± 0.42 km/s/Mpc

(radiation) -- Ω_r = (8.5 ± 0.3) x 10^{-5} - (WMAP)

(baryons) -- $\Omega_b h^2$= 0.02242 ± 0.00014

(dark matter) -- $\Omega_c h^2$= 0.11933 ± 0.00091 —-(matter) - Ω_m = 0.3111 ± 0.0056

(curvature) -- Ω_k =0.0007 ± 0.0019

(dark energy) -- Ω_{de} = 0.6889 ± 0.0056 -- Implying univ accelerating today

(de eqn of state) -- 1+w = 0.028 ± 0.032 -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w' z/(1+z)$ then
w_0=-0.961 ±0.077 and w'=-0.28 ± 0.31 (68% CL)

What else is out there to find or constrain ?

Domain walls? - ultralight fields forming macroscopic objects. Use global positioning system as a -50,000km aperture `dark matter' detector to search for Domain Walls. [Roberts et al 2017]

Earth moves through galactic dark matter halo, interactions with domain walls cause a sequence of atomic clock perturbations that propagate through the satellite constellations at v~300km/s. Mining 16 yrs of data, the DW are hiding, but it improves the limits on certain quadratic scalar coupling of DW to standard model particles by many orders of mag.

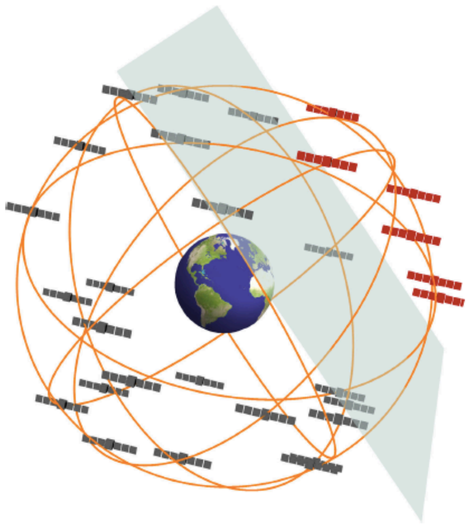


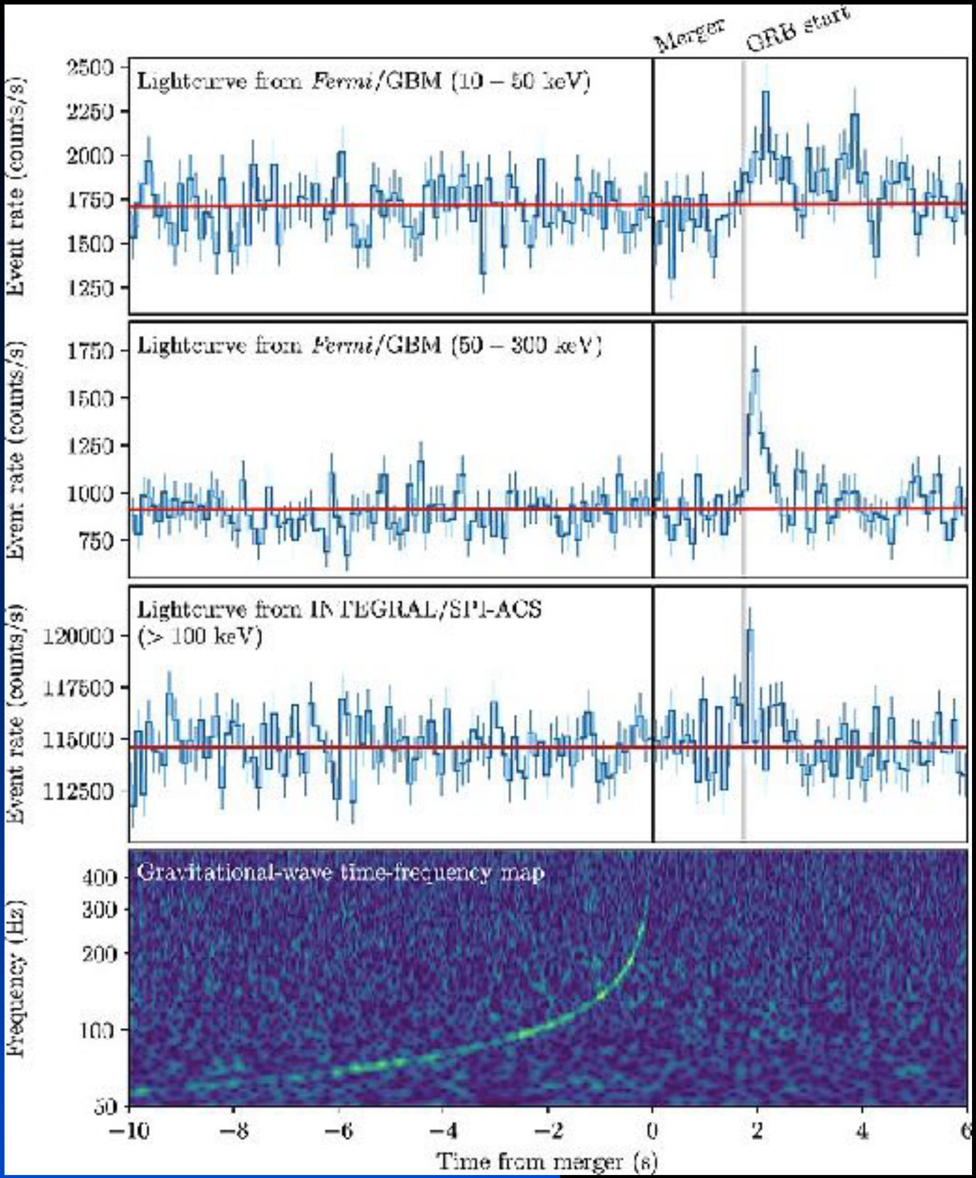
Fig. 1 Domain wall crossing. As a domain wall sweeps through the Global Positioning System constellation at galactic velocities, $v_g \sim 300\ \mathrm{km\,s^{-1}}$, it perturbs the atomic clocks on board the satellites causing a correlated propagation of glitches through the network. The red satellites have interacted with the domain wall, and exhibit a timing bias compared with the grey satellites. Image generated using Mathematica software[48]

Could be any ultra light field though seems to me!

Chameleon, dilaton, axion, symmetry coupled to matter - rapidly changing density.

Any condensate field, such as oscillon, QBall, axion stars, string !

The impact of the simultaneous detection of GWs and GRBs on Modified Gravity models !



Credit: LIGO-VIRGO Collaboration.

GW 170817 and GRB 170817A

speed of GW waves

$$c_T^2 = 1 + \alpha_T$$

$$\Delta t \simeq 1.7s$$

$$\rightarrow |\alpha_T| \leq 10^{-15}$$

Implication for scalar-tensor theories - [Horndeski (1974), Deffayet et al 2011]

Lagrangian couples field and curvature terms: $$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i$$

$$\mathcal{L}_2 = K \qquad \mathcal{L}_3 = -G_3 \Box\phi$$

$$\mathcal{L}_4 = G_4 R + G_{4,X}[(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi]$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}[(\nabla\phi)^3 - 3\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\Box\phi + 2\nabla^\nu\nabla_\mu\phi\nabla^\alpha\nabla_\nu\phi\nabla^\mu\nabla_\alpha\phi]$$

where $G_i = G_i(\phi, X)$ and $X = -\nabla^\mu\phi\nabla_\mu\phi/2$

Linearise theory and map to alpha parameter :

$$M_*^2 \alpha_T = 2X \left[2G_{4,X} - 2G_{5,\phi} - (\ddot{\phi} - H\dot{\phi})G_{5,X} \right]$$

$$M_*^2 = 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - H\dot{\phi}XG_{5,X})$$

Recall:

$$|\alpha_T| \leq 10^{-15}$$

Many authors assumed the following saying they held barring fine-tuned cancellation:

$$G_{4,X} = G_{5,\phi} = G_{5,X} = 0$$

This of course satisfies the bound meaning any model that satisfies those conditions (such as GR, f(R), Quintessence) is perfectly viable.

Creminelli & Vernizzi (2017), Baker et al (2017), Wang et el (2017), Sakstein & Jain (2017), Ezquiaga &Zumalacárregui (2017) — all same edition of PRL (2018)

Crucially though it does not imply that models that do not satisfy the assumptions are ruled out !

Copeland et al, PRL (2019)

Not had time to mention

Quantum corrections in compactified theories lead to corrections to Λ and G - Swampland effects.

CPT and EDM

Lorentz violation expts

Variation of more structure constants, linked say to QCD.

Links to Swampland and small field evolution ?

Conclusions

Exciting opportunity

How should we consider parameterising the fundamental constants we wish to examine ?

How many are there?

Which are the best ones to consider?

How to interpret the results ?

Impact from higher dimensions.

Can it influence recent discussions on the swampland.

Can we use it to actually rule out models.

The bounds are just getting tighter and tighter.