

Stochastic Inflation beyond slow-roll

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S.I. powerful tool

- calc. prob. distribution for fields during inflation, $P(\varphi, t)$, e.g. vev for curvaton
- calc. non-linear primordial density perturb. $P(N | \varphi) \rightarrow$ stochastic SN (Vennin & Starobinsky)
where $N = \int H dt$, $\frac{\delta P}{P} \sim S = SN$

Requirements for S.I.

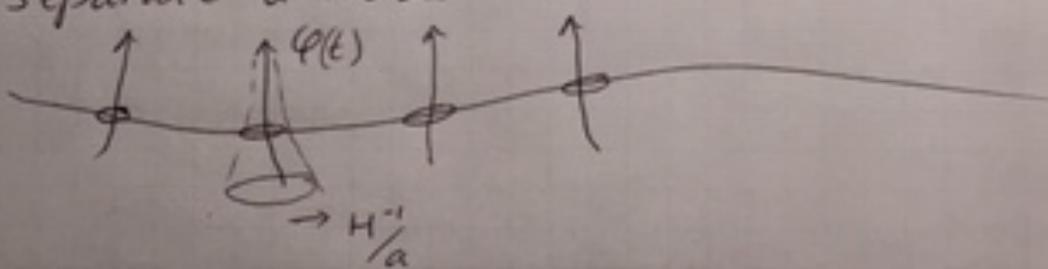
① quantum \rightarrow classical transition

qu. oscillator \rightarrow over-damped, squeezed state
sub \rightarrow super-Hubble, ~~H~~ $H \gg k/a$

② inhomog. field $\varphi(t, \vec{x})$ on super-H scales

described by local FRW cosmology, $\varphi(t)|_{\vec{x}}$

"separate universes"



③ stochastic noise from qu. field

in spatially-flat gauge; $\dot{Q} = \delta\varphi + \frac{\dot{\varphi}}{H} u$

- difficult to prove in absence of full
non-perturb. qu. gravity theory

- consistency check:
linear perturb. about FRW cosmology

$$\underline{\text{S.I.}} : \text{split } \varphi(t, \vec{x}) = \bar{\varphi}_c(t) \Big|_{\vec{x}} + \hat{\varphi}_s(t, \vec{x})$$

$\bar{\varphi}$ coarse-grained,
classical

$$\text{short-wavelengths } \hat{\varphi}_s = \int d^3k W\left(\frac{k}{k_{cg}}\right) \hat{a}_k \varphi_k(t) e^{ik\vec{x}}$$

$\hat{\varphi}$ coarse-graining, k_{cg}
scale = aH

long-wavelengths

- obey separate universe + stoch. kick

$$\frac{d}{dN} \bar{\varphi} = \bar{\pi} + \hat{\Sigma}_{\varphi} \quad \left. \right\}$$

$$\frac{d}{dN} \bar{\pi} = -(3 - \epsilon_1) \bar{\pi} - \frac{V'}{H^2} + \hat{\Sigma}_{\pi} \quad \left. \right\}$$

$$\text{where } \epsilon_1 = -\frac{\dot{H}}{H^2}$$

$$H^2 = \frac{8\pi G}{3} \left(V + \frac{1}{2} \dot{\varphi}^2 \right)$$

time-dependence of $k_{cg} = aH$

$$\Rightarrow \langle \xi_{\varphi}^2 \rangle = \frac{1}{6\pi^2} \frac{d k_{cg}^3}{dN} |\varphi_{k_{cg}}|^2 \cdot \delta(N_i - N_f)$$

$$\underline{\text{slow-roll}} : \epsilon_1 \ll 1, \left| \frac{d}{dN} (\ln \bar{\pi}) \right| \ll 1$$

$$\Rightarrow \bar{\pi} \approx -\frac{V'}{3H^2}$$

$$\frac{d}{dN} (\bar{\varphi}) = -\frac{V'}{3H^2} + \hat{\Sigma}_{\varphi}$$

$$\text{and } \langle \xi_{\varphi}^2 \rangle \sim \left(\frac{H}{2\pi} \right)^2$$

Separate universes beyond S.R.

require (A) full field perturbations in full perturbed metric
 obey same field eqns at leading order in gradient expansion
 as (B) perturbed FRW cosmology

$$(A) \quad \varphi(t, \vec{x}) = \bar{\varphi}(t) + \delta\varphi(t, \vec{x})$$

$$ds^2 = -(1+2A)dt^2 + 2\cancel{B}_{ij}^{(0)} dx^i dt$$

$$+ a^2 [(1-2\psi) \delta_{ij} + 2E_{ij}] dx^i dx^j$$

then K.G eqn:

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + \left(\frac{k^2}{a^2} + V''\right)\delta\varphi = O(A, \psi, \delta\varphi)$$

self-gravity

e.g. in spatially-flat gauge, $\psi = 0$,

use energy + m.m. constraints to eliminate A

$$\Rightarrow \ddot{\delta\varphi}_k + 3H\dot{\delta\varphi}_k + \left(\frac{k^2}{a^2} + V''\right)\delta\varphi_k$$

$$= \frac{8\pi G}{3} \frac{d}{dt} \left(\frac{a^3 \dot{\varphi}^2}{H} \right) \delta\varphi_k$$

$$(B) \quad H^2 = \frac{8\pi G}{3} \left(V + \frac{1}{2} \dot{\varphi}^2 \right)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0$$

let $\varphi \rightarrow \varphi + \delta\varphi$, $H \rightarrow H + \delta H$, $dt \rightarrow (1+A)dt$

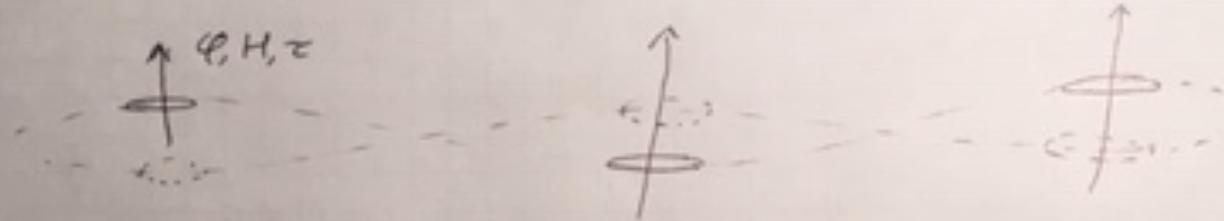
$$KG: \frac{1}{1+A} \frac{d}{dt} \left(\frac{1}{1+A} \frac{d\varphi}{dt} \right) + 3(H + \delta H) \frac{1}{1+A} \frac{d\varphi}{dt} + (V + V''\delta\varphi + \dots) = 0$$

$$\Rightarrow \ddot{\delta\varphi} + 3H\dot{\delta\varphi} + V''\delta\varphi = 2A\ddot{\varphi} + \dot{A}\dot{\varphi} + 3H\dot{\varphi}A - 3\dot{\varphi}\delta H$$

$$= G(A, \delta H) = \text{self-gravity}$$

where Friedmann constraint: $2H\delta H = \frac{8\pi G}{3} (V'\delta\varphi + \dot{\varphi}\delta\varphi - \dot{\varphi}^2)$

what about A ? perturbed lapse f_0 in each sep. uni.
unobservable locally
proper time : $d\tau = (1+A) dt$



to compare local field with perturbed full field in pert. spacetime
need to specify global time-slicing ("gauge")
which constrains specifies lapse function

e.g. (i) choose spatially-flat gauge ($\mathcal{A}=0$)

$$\text{Momentum constraint: } A = 4\pi G \frac{\dot{\varphi}}{H} \delta\varphi$$

substitute into K.G. eqns. in sep. uni: ⑧

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + V''\delta\varphi = \left(\frac{8\pi G}{3} \frac{d}{dt} \left(\frac{a^3 \dot{\varphi}}{H} \right) \right) \delta\varphi$$

matches exactly full K.G. eqn except $\frac{k^2}{a^2} \delta\varphi$

(vanishes on large-scales)

(ii) choose uniform- N gauge ($\delta N=0 \Rightarrow \mathcal{A} - \frac{1}{2} \nabla^i E = 0$)

$$\text{energy constraint } A = -4\pi G \delta\rho + \frac{1}{3H^2} \frac{\nabla^2}{a^2} \Psi$$

substitute into K.G.

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + V''\delta\varphi = G(A, 3H) + D$$

$$\text{where difference } D = \frac{\nabla^2}{a^2} \left(\delta\varphi + \frac{\dot{\varphi}}{H} \Psi \right) = \frac{\nabla^2}{a^2} (Q)$$

$$\rightarrow 0$$

in large-scale limit

Finally

Stochastic noise in uniform- N slicing

transform	spatially-flat ($\nabla^2 E = 0$)	\rightarrow	uniform N ($\nabla^2 E = 0$)
t		\rightarrow	$\tilde{t} = t - \alpha$
φ^0		\rightarrow	$\tilde{\varphi} = \varphi^0 - H\alpha$
$\delta\varphi = Q$		\rightarrow	$\tilde{\delta\varphi} = Q + \frac{\dot{\varphi}}{H}\alpha$

where from energy+mtm constraints

$$3H\dot{\alpha} + \left(\frac{3}{2}(1+3w)H^2 + \frac{\nabla^2}{\alpha^2} \right)\alpha \propto \delta P_{\text{rad}}$$

$\Rightarrow \alpha = 0$ for adiabatic perturbations

e.g. single-field slow-roll

corrections to noise

(i) slow-roll : $\tilde{\delta\varphi} = Q \left[1 - \frac{\epsilon_{1+}}{6} \left(\frac{k}{aH} \right)^2 + \dots \right]$

(ii) ultra-slow-roll : $\tilde{\delta\varphi} = Q \left[1 + \frac{\epsilon_{1+}}{3} \left(\frac{k}{aH} \right)^6 + \dots \right]$

in both cases $\tilde{\delta\varphi} \rightarrow Q$ for $\frac{k}{aH} \rightarrow 0$

Conclusions

- separate universe approach works beyond slow-roll
 - neglects spatial gradients , $\frac{k^2}{a^2} \delta \phi$ in spatially-flat
 - does not require slow-roll
 - does not fix local time coordinate
 - comparison to full perturbation theory in specific gauge requires extra constraint e.g.(i) spatially-flat mto constraint fixes lapse $A = 4\pi G \frac{Q}{R} \delta \phi$
(in slow-roll lapse unperturbed in spat.flat)
(ii) uniform- N , use energy const. to fix lapse
- applications to stochastic inflation
 - linear equations for qu. vacuum on small scales
 - sources classical stochastic noise on large scales
 - uses separate universe approach
(non-linear FRW equations) for coarse-grained fr.
 - using ~~the~~ e-folds as time coordinate
 \Rightarrow uniform ^{integrated/coord.} - $\dot{\phi}$ expansion gauge " N "
 $\Rightarrow \nabla^2 - \frac{1}{3} \nabla^2 E_0 = 0$
coincides with spatially-flat gauge for adiabatic density perturb.
e.g. slow-roll inflation