Ioannis Dalianis, Natl. Tech. U. Athens

Properties of the primordial power spectrum for PBH production

17/5/2019, Warszawa

Workshop on PBHs and inflation

Based on the works:

1/ ID, Alex Kehagias & George Tringas, JCAP 1901 (2019) 037

2/ ID, arXiv:1812.09807

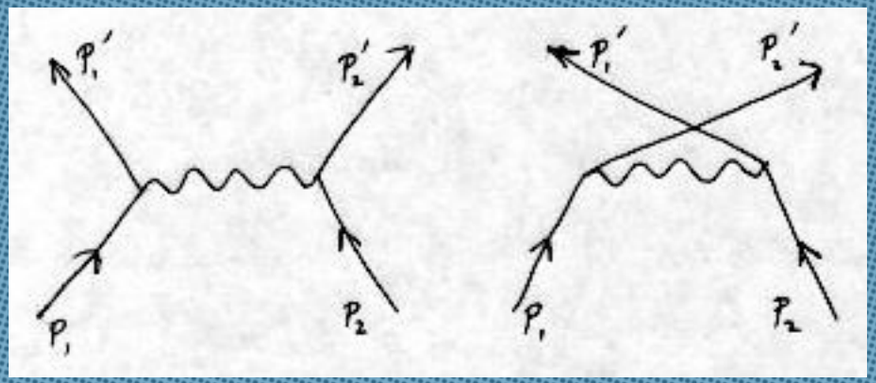
3/ ID & George Tringas, arXiv:1905.01741

Outline of the talk

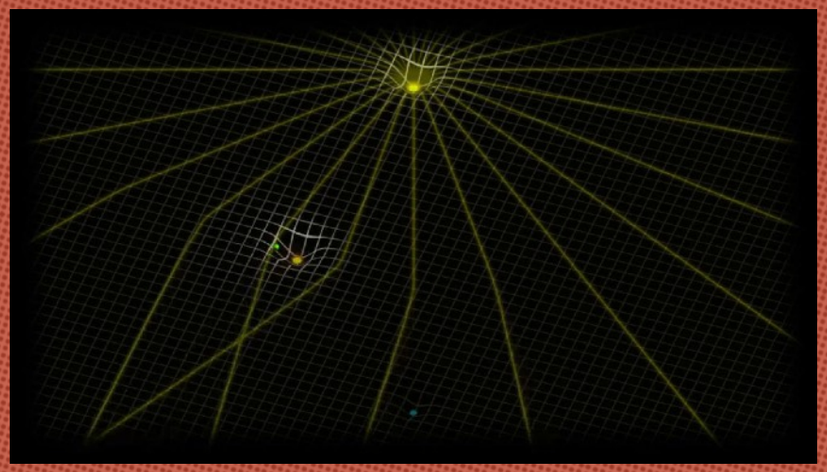
- The motivation to investigate the PBHs scenario
- Observational constraints on the PBHs
- What we learn about the early universe from the cosmology of the PBHs
- Inflationary models that predict PBHs
- PBH remnants as dark matter
- Runaway inflationary models

Dark Matter Scenarios

It is a particle



It is an object



Dark Matter Scenarios

It is a particle:

LSP
ALPS
Asymmetric
Exotic Neutrinos
...

It is an object:

MACHOS
Black Holes
...

Dark Matter Scenarios

It is a particle:

LSP
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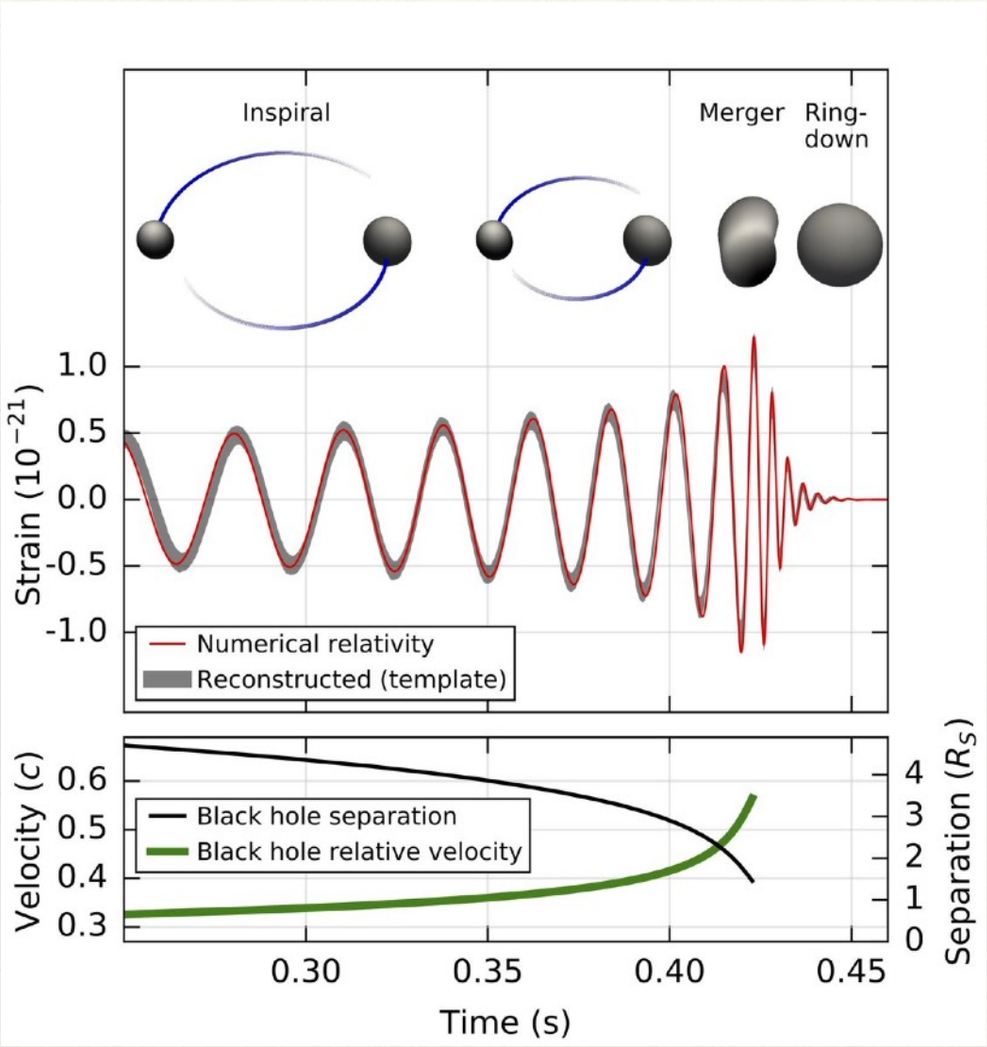
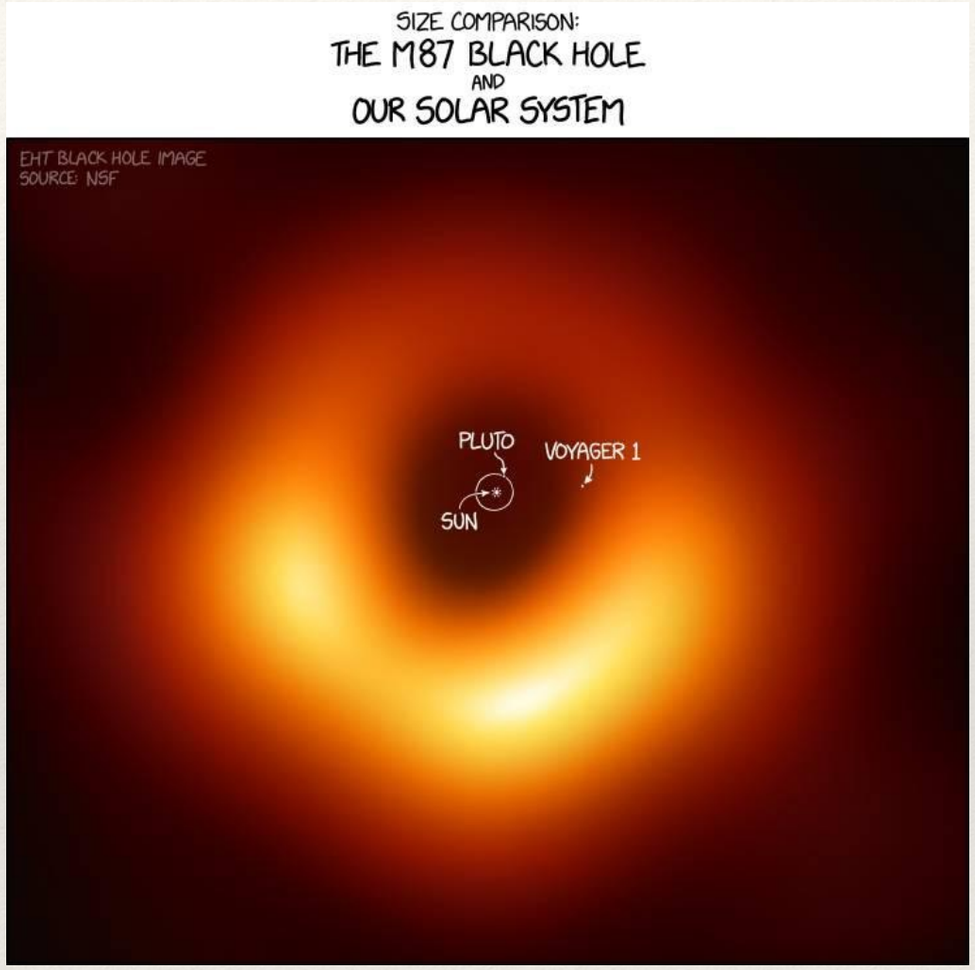
It is an object:

MACHOs
~~Stellar Black Holes~~
Primordial Black Holes
...

Black Holes (2019) : YES





Of primordial origin ?

Basics of PBHs

- PBHs form from the collapse of large-amplitude inhomogeneities.

- B. J. Carr and S. W. Hawking (1974). P. Meszaros, (1974), B. J. Carr (1975), I. D. Novikov, A.G. Polnarev, A. A. Starobinsky and Ya. B. Zeldovich, (1979)

- In order to decouple from the background expansion it has to be GM/R ~ 1, for a region of mass M over a scale R.

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$$R_J \sim \sqrt{w} R_H \lesssim R_{\rm max} \lesssim R_H \Rightarrow$$

$$w \simeq \delta_c \lesssim \delta_H \lesssim \delta_{\rm max} \simeq 1$$

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- However there is some ambiguity in the choice of the Jeans radius

  - Harrada, Yoo, Kohri (2013),

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- The formation criterion: the sound crossing time over the radius of the over-density be longer than the free fall time from the maximum expansion to complete collapse

$$R_J = a_{\max} \sin\left(\frac{\pi\sqrt{w}}{1+3w}\right) \rightarrow \delta_c^{UH} = \sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right)$$

$$\delta_c^{com} = \frac{3(1+w)}{5+3w}\sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right)$$

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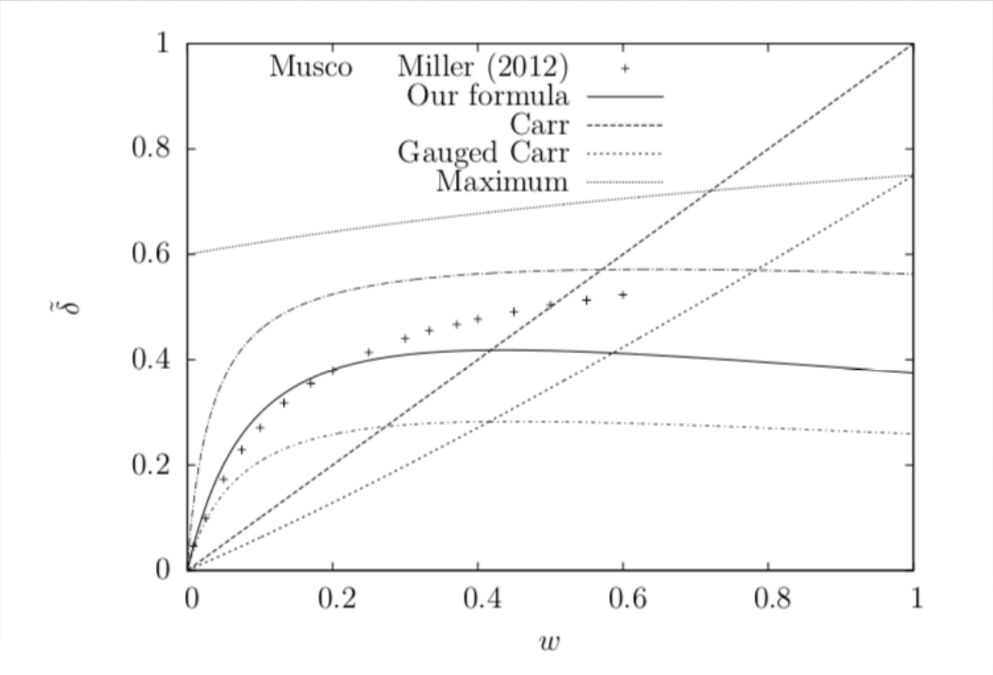

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- ***Large primordial inhomogeneities can be achieved if the power spectrum is enhanced at a scale $R^{-1} \sim k$, characteristic of the PBH mass, by many orders of magnitude.***

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- Large wavenumbers yield light PBH which if they have mass $M < 10^{15}$ g evaporate at timescales less than the age of the universe.

S. W. Hawking, (1974,1975)

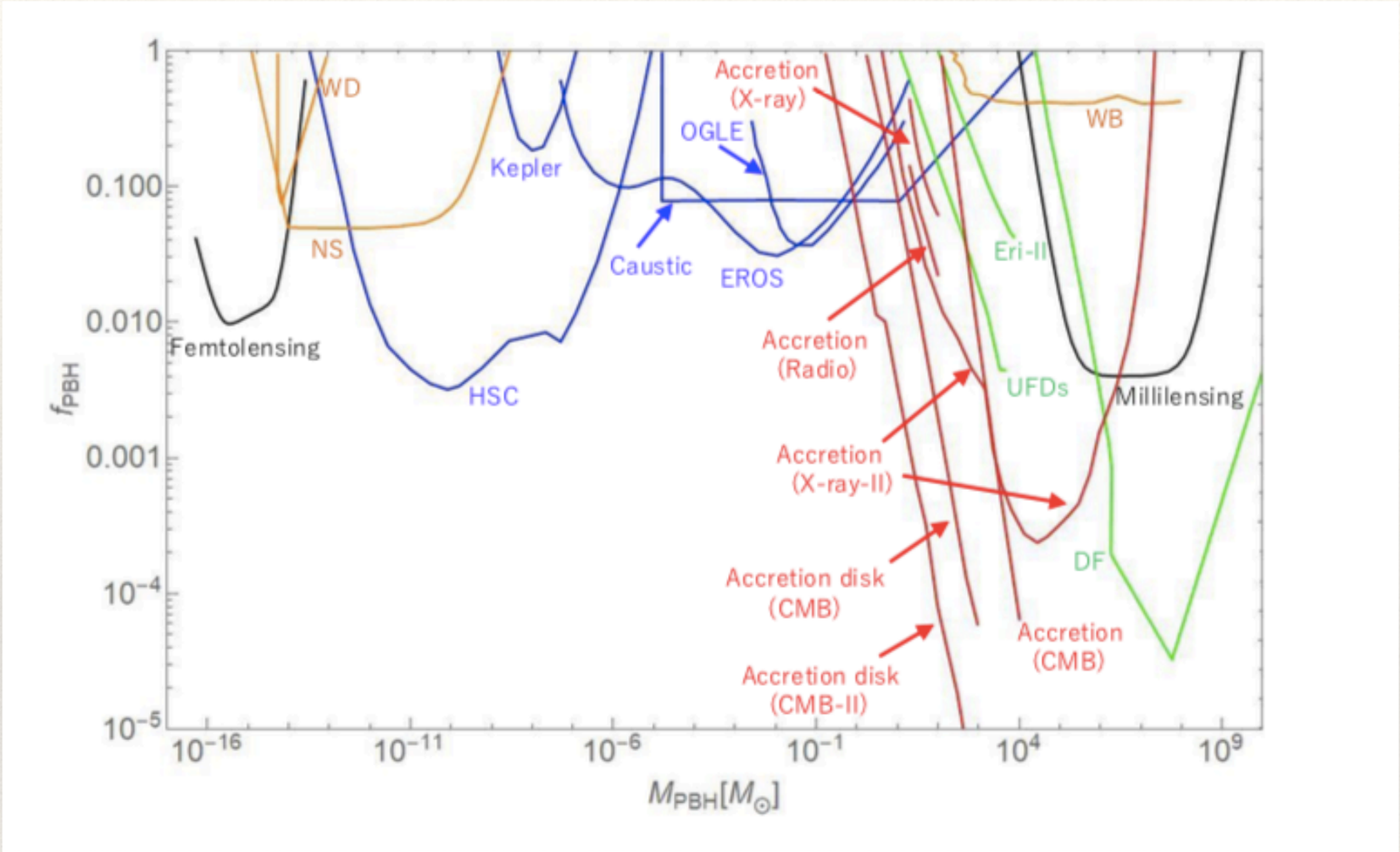
- PBHs with $M > 10^{15}$ g would still survive today and would be dynamically cold component of the dark matter in galactic structures.

Extra Motivation:
PBH scenarios can be tested observationally !

PBHs make their presence manifest due to :

- Accretion of CMB photons
- Ultra Faint Dwarf Galaxies
- Lensing effects
- White Dwarfs
- Hawking radiation
- Gravitational Waves

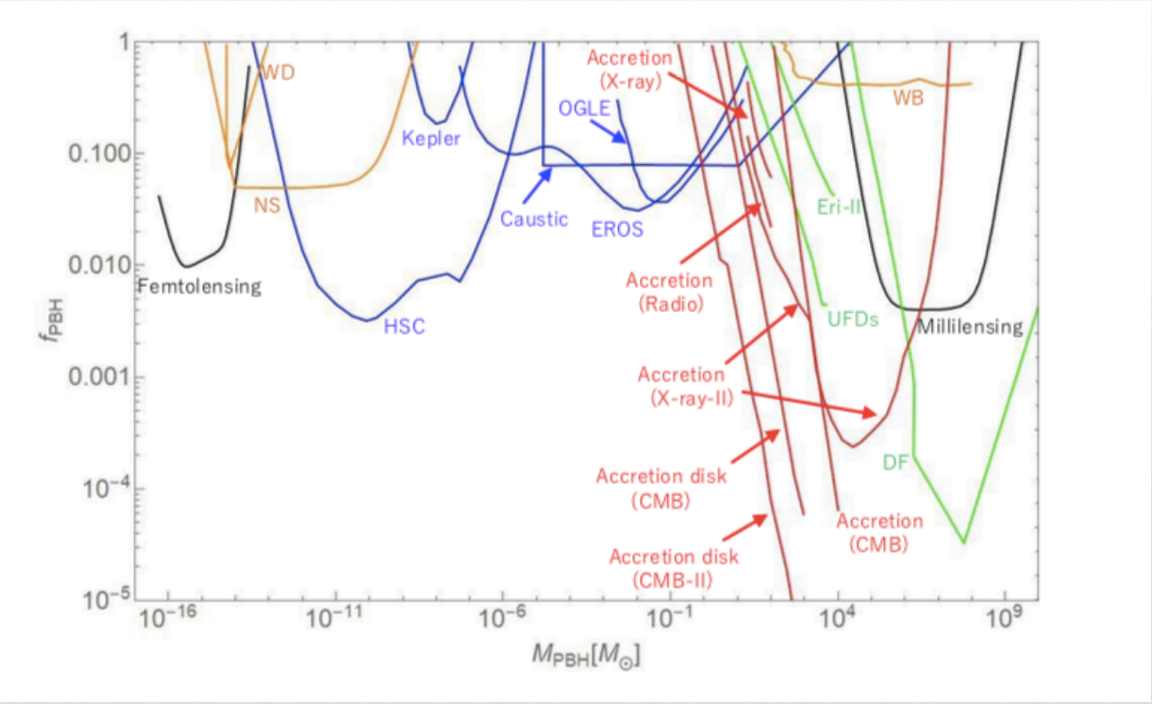
Observational constraints

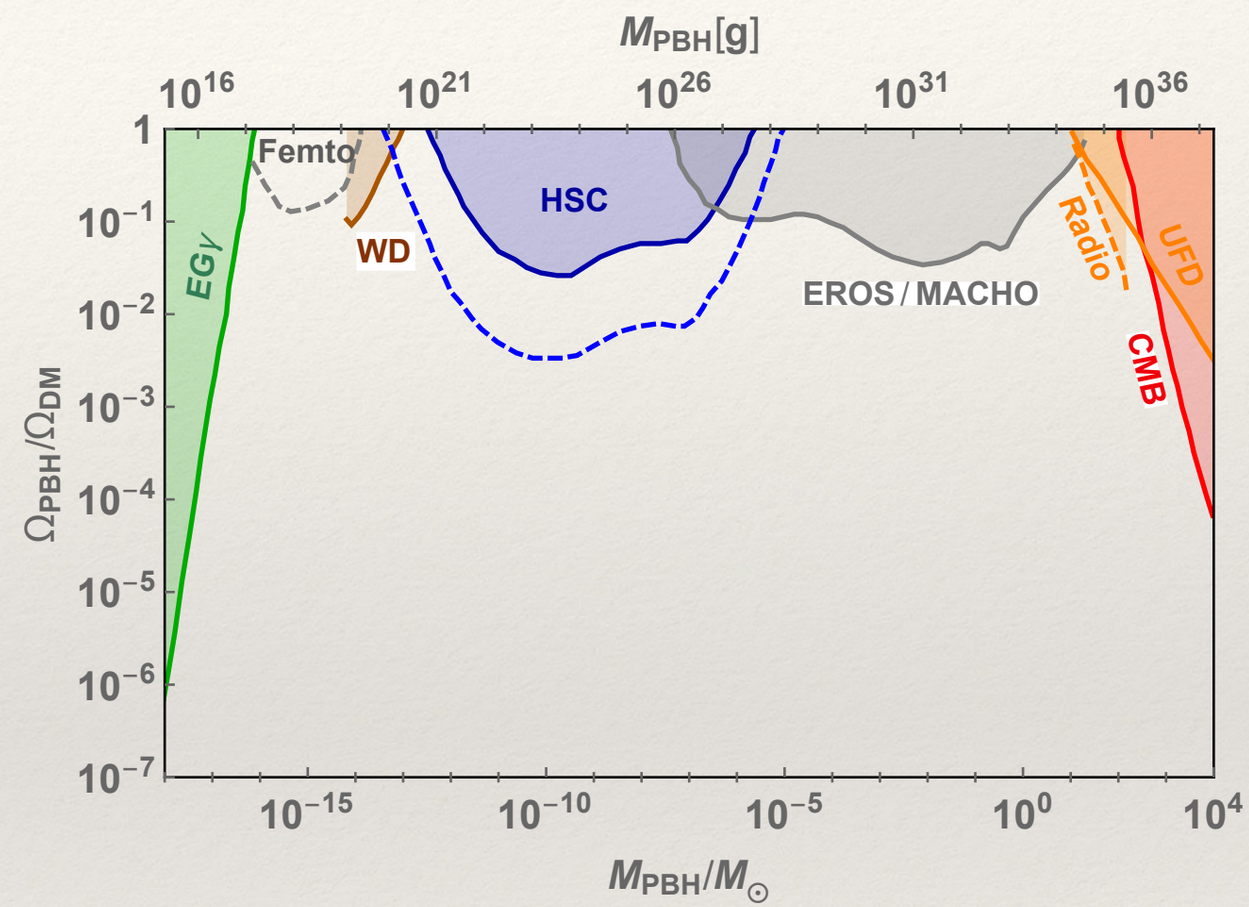


M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2018)

Upper limit on fPBH = $\frac{\Omega_{PBH}}{\Omega_{DM}}$ for various PBH mass (assuming monochromatic mass function)

Updated observational constraints

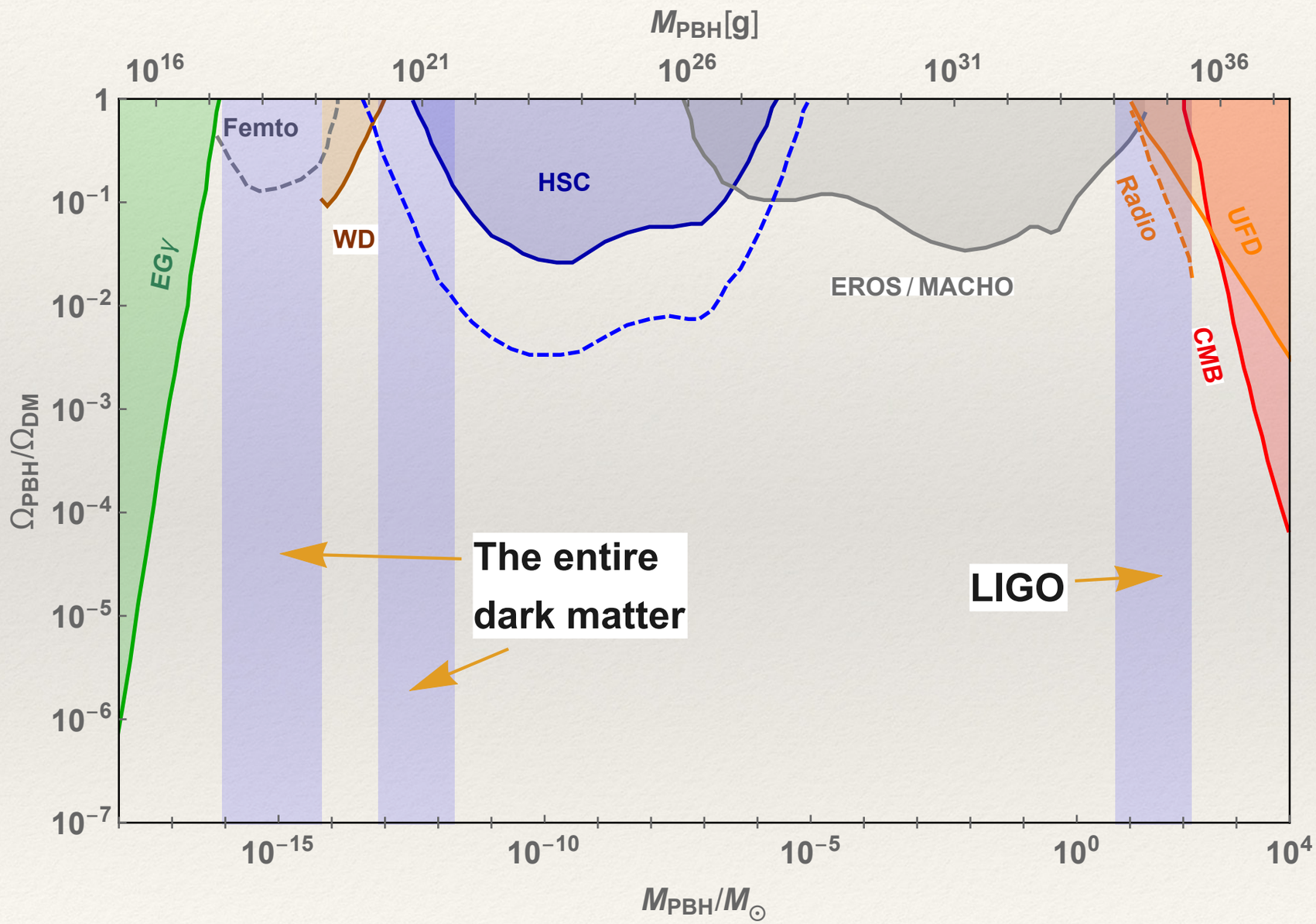




A. Katz, J. Kopp, S. Sibiryakov and W. Xue (2018)

Upper limit on fPBH = $\frac{\Omega_{PBH}}{\Omega_{DM}}$ for various PBH mass (assuming monochromatic mass function)

Motivated PBH mass windows



From the power spectrum to the PBH abundance

Radiation Domination

$$f_{PBH}(M) \simeq \left(\frac{\beta(M)}{10^{-14}}\right)\left(\frac{\gamma}{0.12}\right)^{3/2}\left(\frac{M}{M_{\rm sun}}\right)^{-1/2}$$

$$\beta(M) = \int_{\delta_c} d\delta \frac{1}{\sqrt{2\pi\sigma^2(k)}} e^{-\frac{\delta^2}{2\sigma^2(k)}}$$

$$\sigma^2(k) \simeq \left(\frac{4}{9}\right)^2 \mathscr{P}_{\mathscr{R}}(k)$$

Matter Domination

$$f_{PBH} \simeq \gamma \left(\frac{\beta(M)}{10^{-19}}\right)\left(\frac{T_{\rm rh}}{10^{10}\,{\rm GeV}}\right)$$

$$\beta(M) = 0.056\,\sigma^5(k)$$

$$\beta(M) = 2\times 10^{-6}\sigma^2(k)\, e^{-0.143\frac{I^{4/3}}{\sigma^{2/3}(k)}}$$

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If the reheating temperature is *not maximal*, PBH formation during matter era has to be taken into account

How large inhomogeneities are generated in the early universe?

The most attractive mechanism to generate PBHs is inflation.

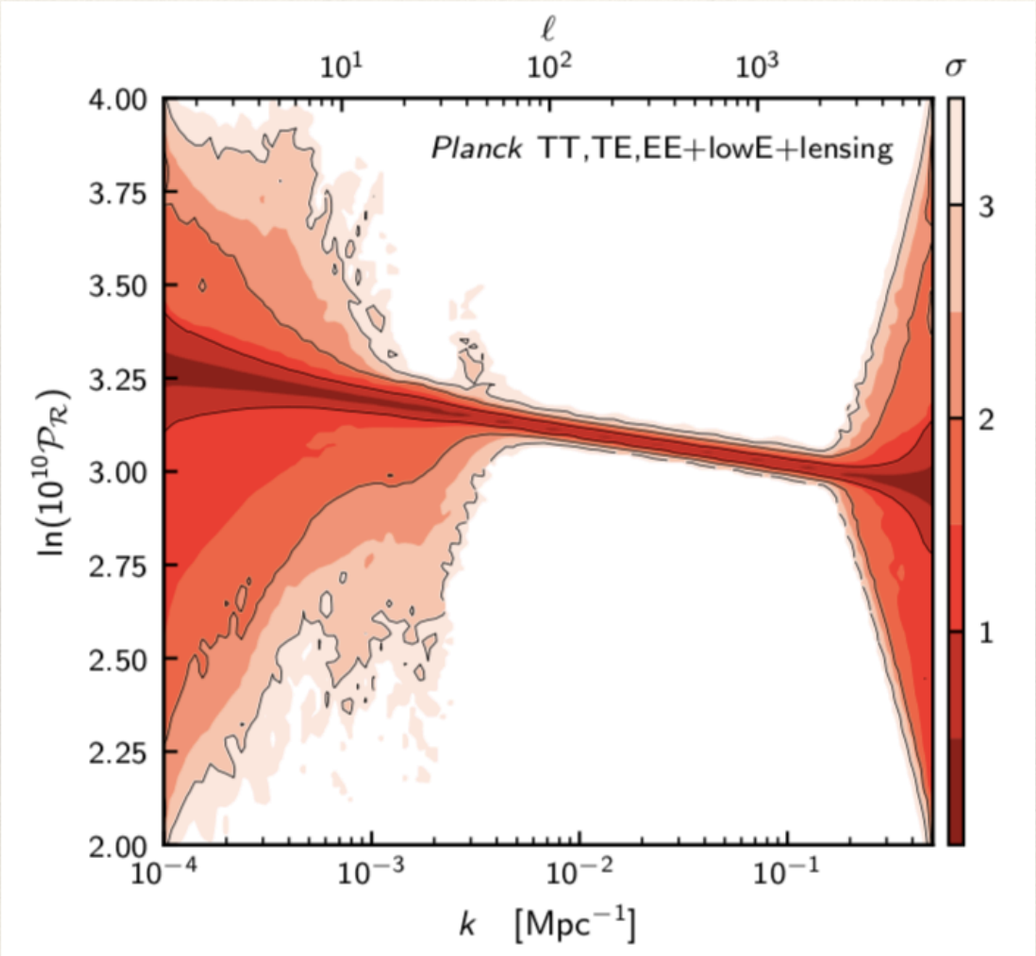
Due to the natural generation of large scale perturbations from quantum fluctuations, inflation is the dominant paradigm that cosmologists follow to explain the origin of the large scale structure and has been, so far, successfully tested by the CMB precision measurements.

Inflation does not seed large scale perturbations only,
it seeds perturbations in all scales.

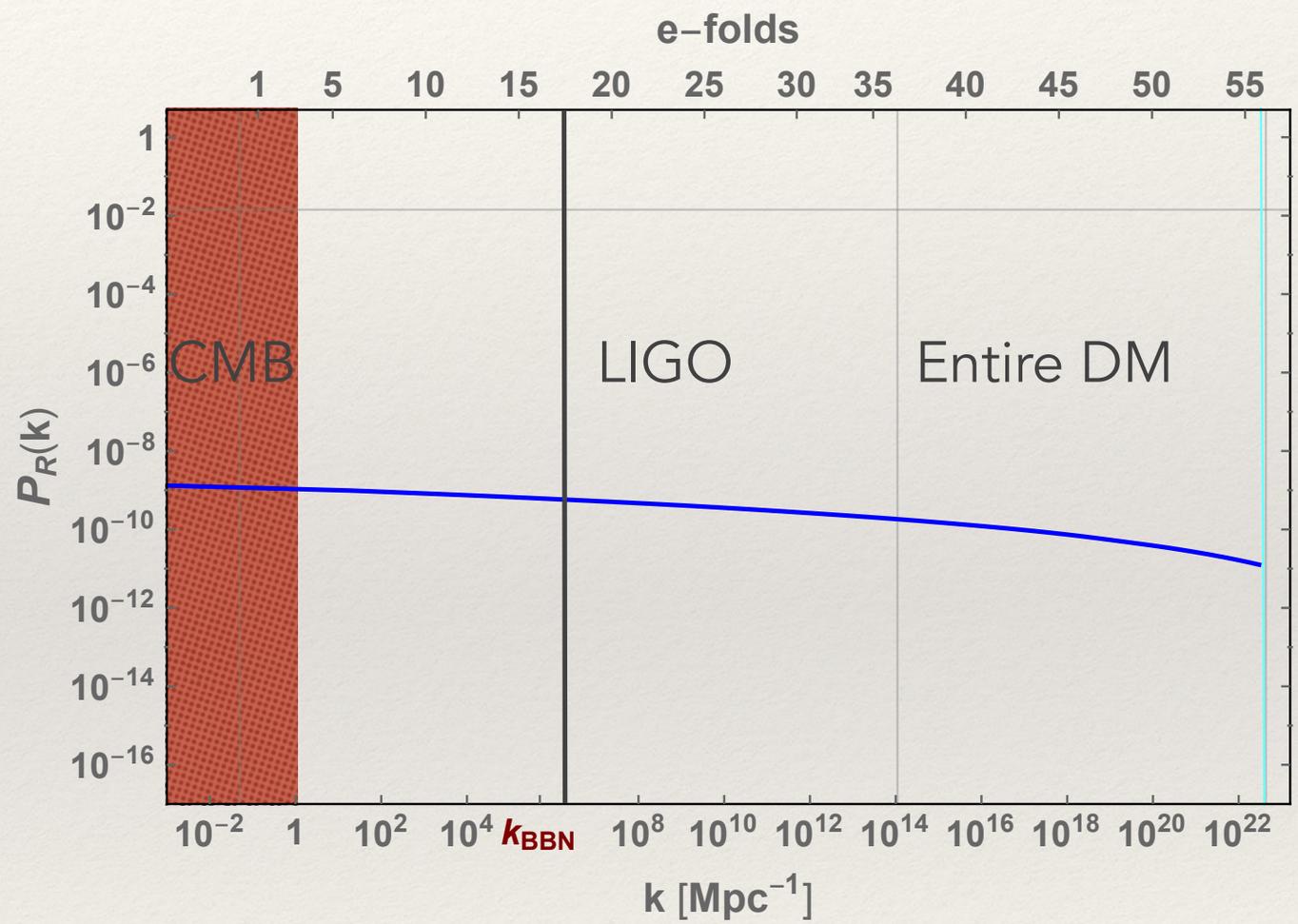
Hence, PBHs can form if perturbations strong enough to collapse are produced in scales $k^{-1} \ll k_{\rm cmb}^{-1}$ characteristic of the PBH mass.

The power spectrum of the curvature perturbations

Planck 2018



The PS of a "conventional" inflationary model:

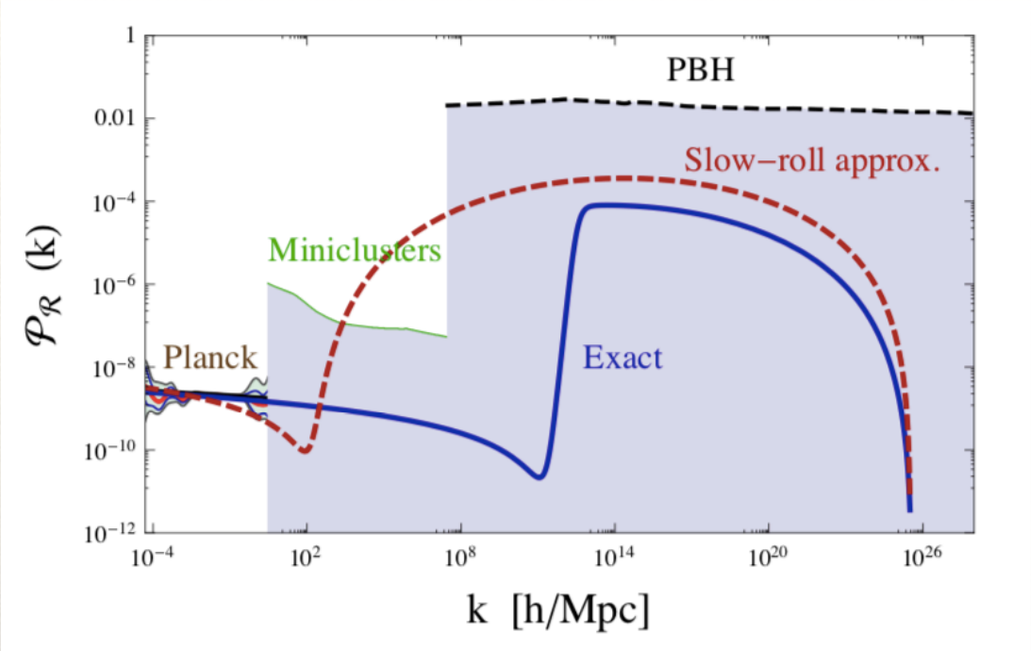


Q: How many PBHs in our observable universe?

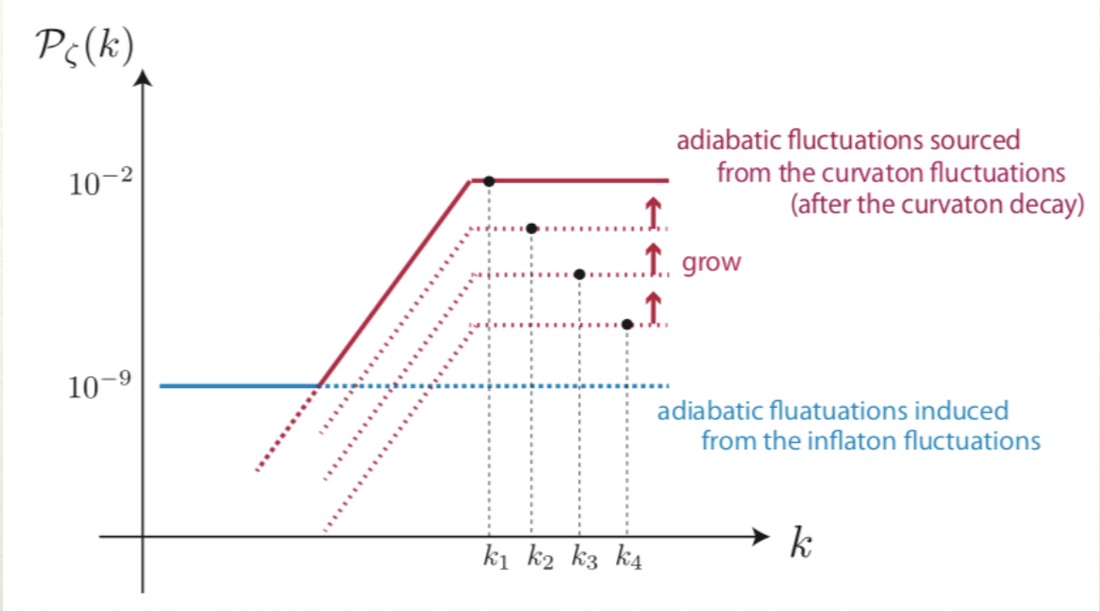
A: According to the PS there is the probability factor : $6 \times 10^{-38,180,513}$

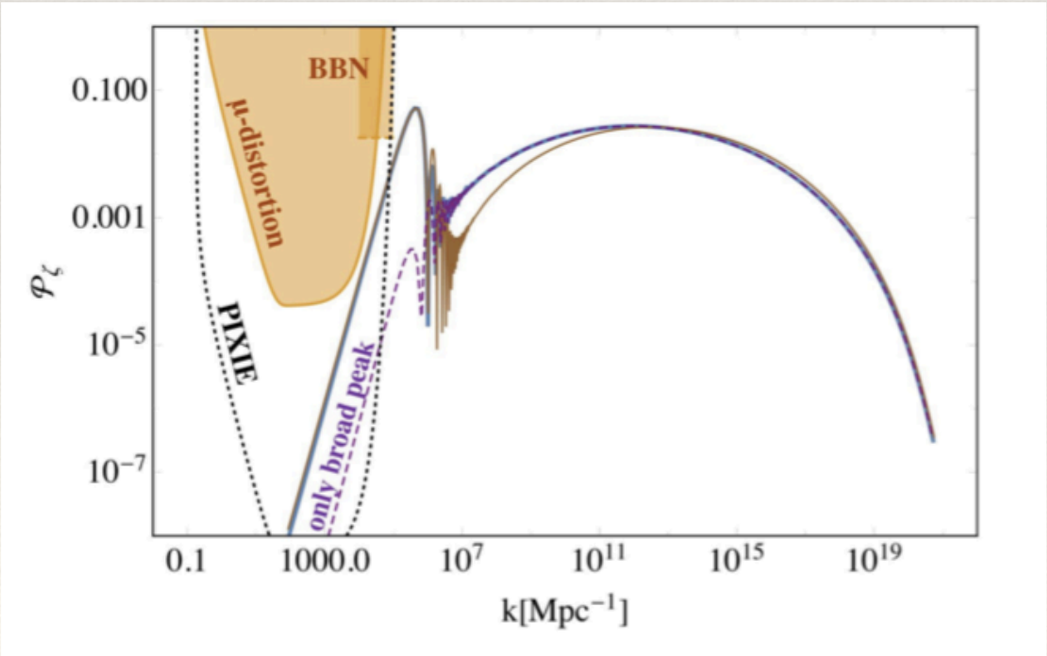
Examples of power spectra from, arXiv:1702.03901, arXiv:1711.06129, arXiv:1801.05235, arXiv:1709.05565

Bellido et.al

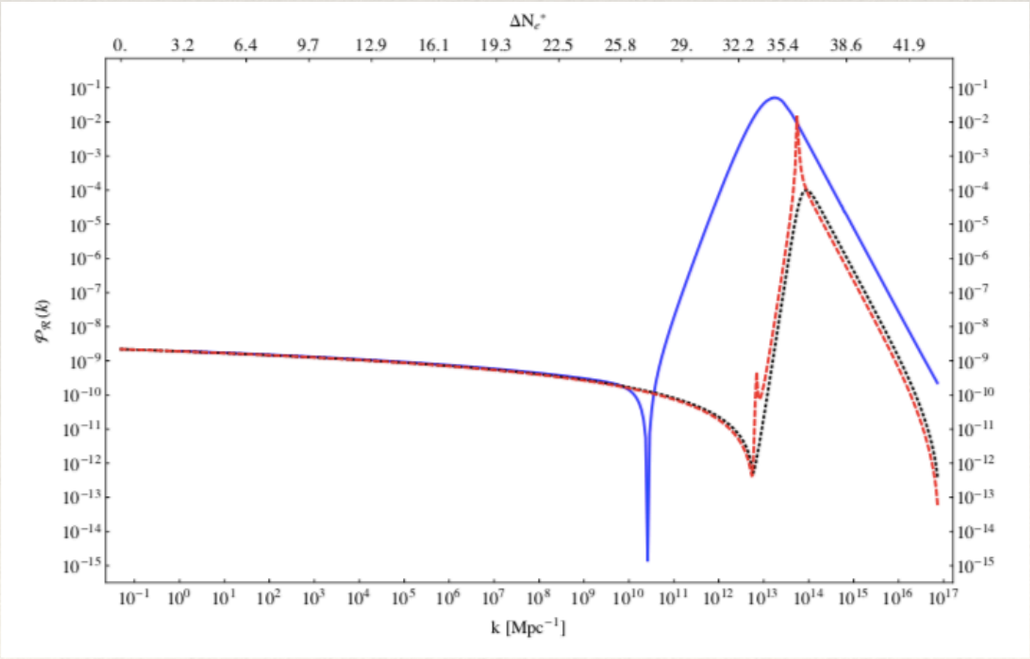


Sasaki et.al



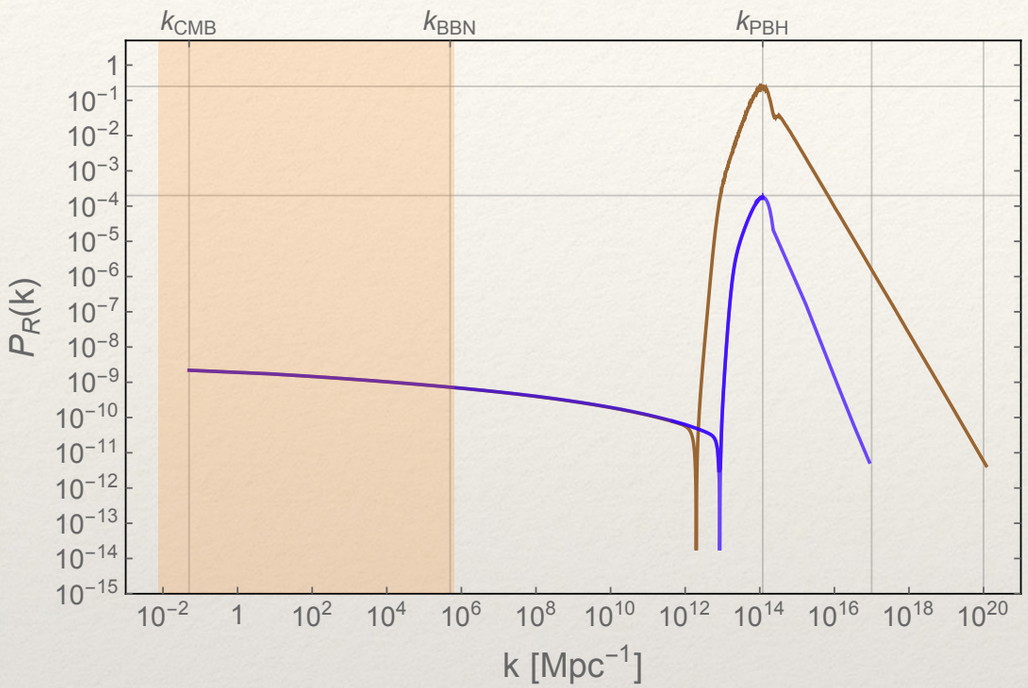


Yanagida et.al



Ballesteros et.al

The power spectrum of the curvature perturbations

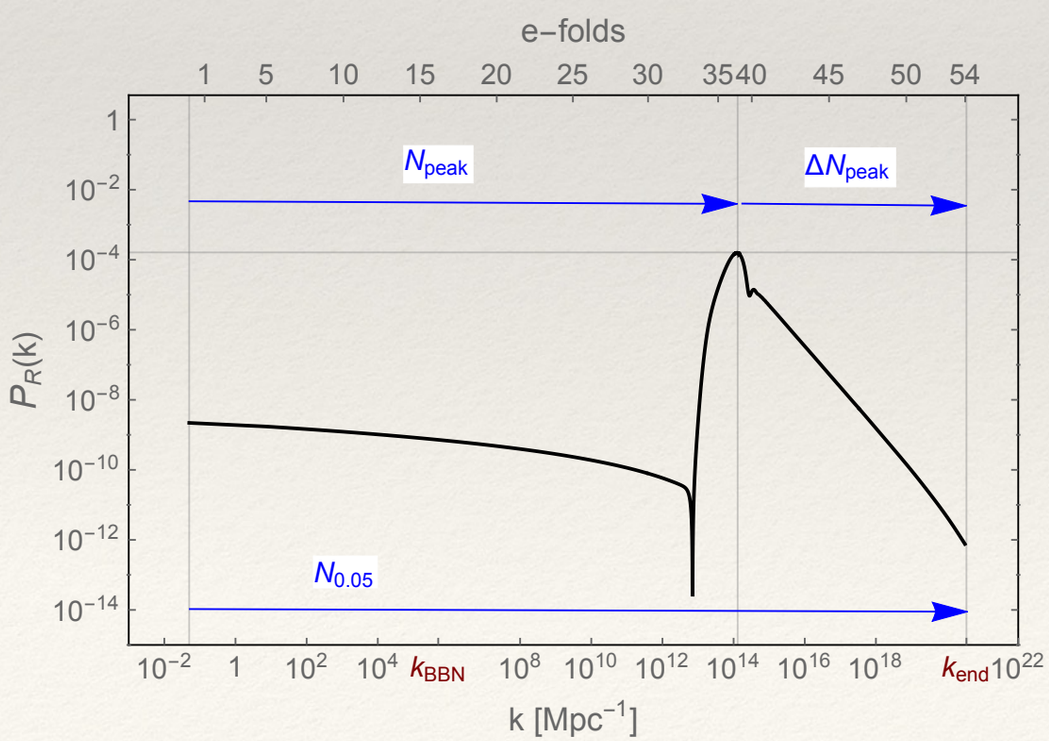


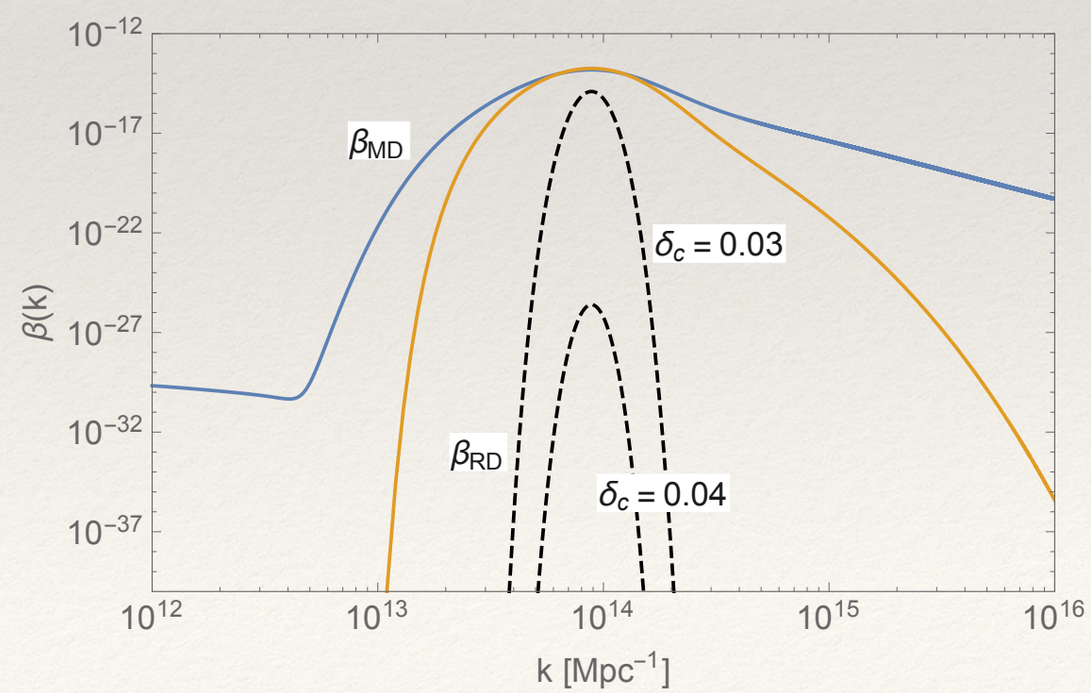
The power spectrum for the curvature perturbation and the reheating temperature.

The corresponding mass fraction that collapses into a black hole for matter and radiation domination background for the power spectrum.
The upper solid curve gives the β when spin effects are neglected and the lower when spin is important.

Extreme δc values for β, depicted with dashed curves, were chosen to make the distributions visible.

$$f_{PBH}^{(RD)} \propto \sqrt{\mathcal{P}_{\mathcal{R}}}\, e^{-\delta_c^2/\mathcal{P}_{\mathcal{R}}}, \qquad f_{PBH}^{(MD)} \propto \mathcal{P}_{\mathcal{R}}^{5/2}.$$





Q: Any shape for the Power Spectrum is acceptable?

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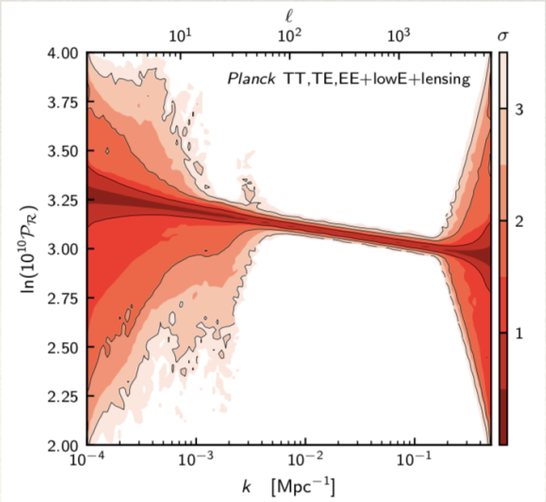
A: No, for two reasons:

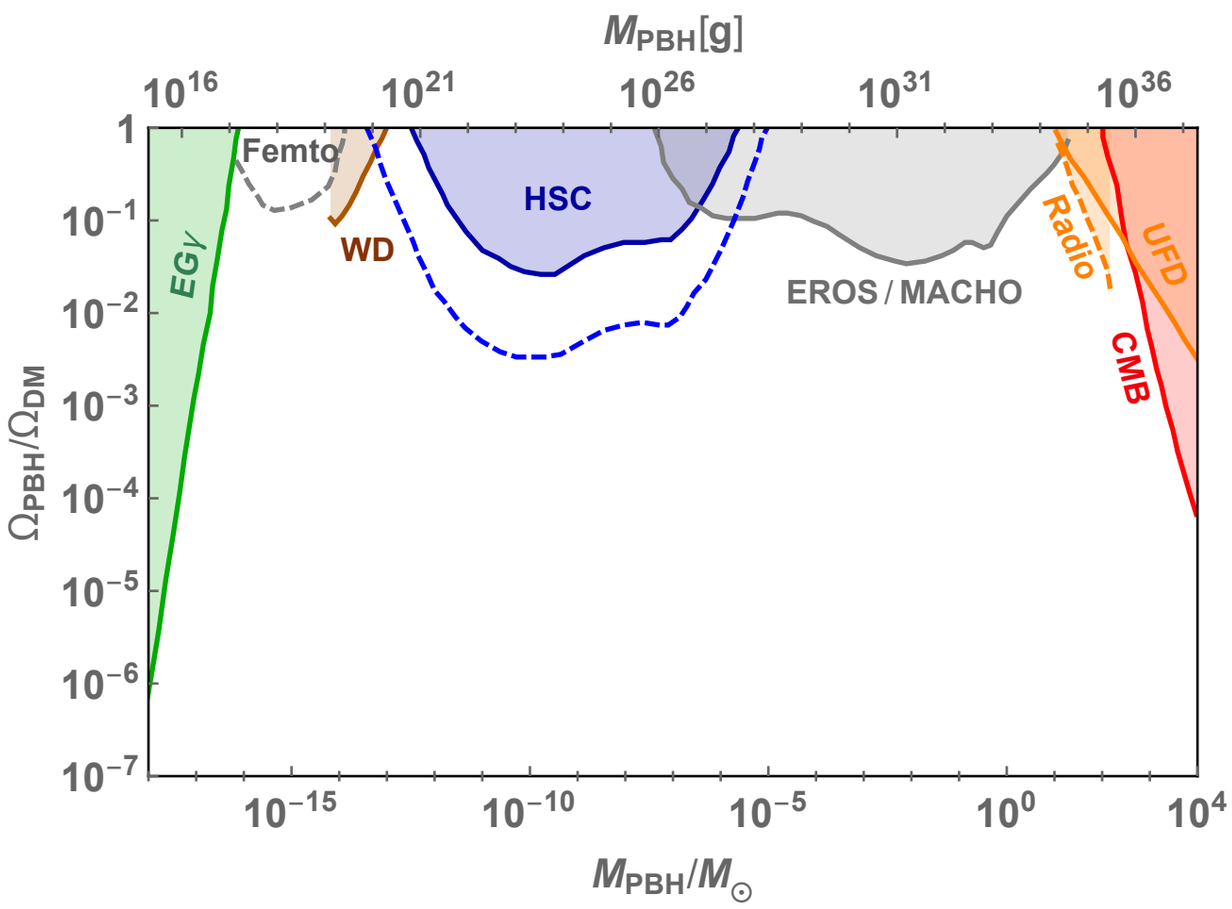
1. The spectral index value
2. The ultra light PBHs evaporate fast

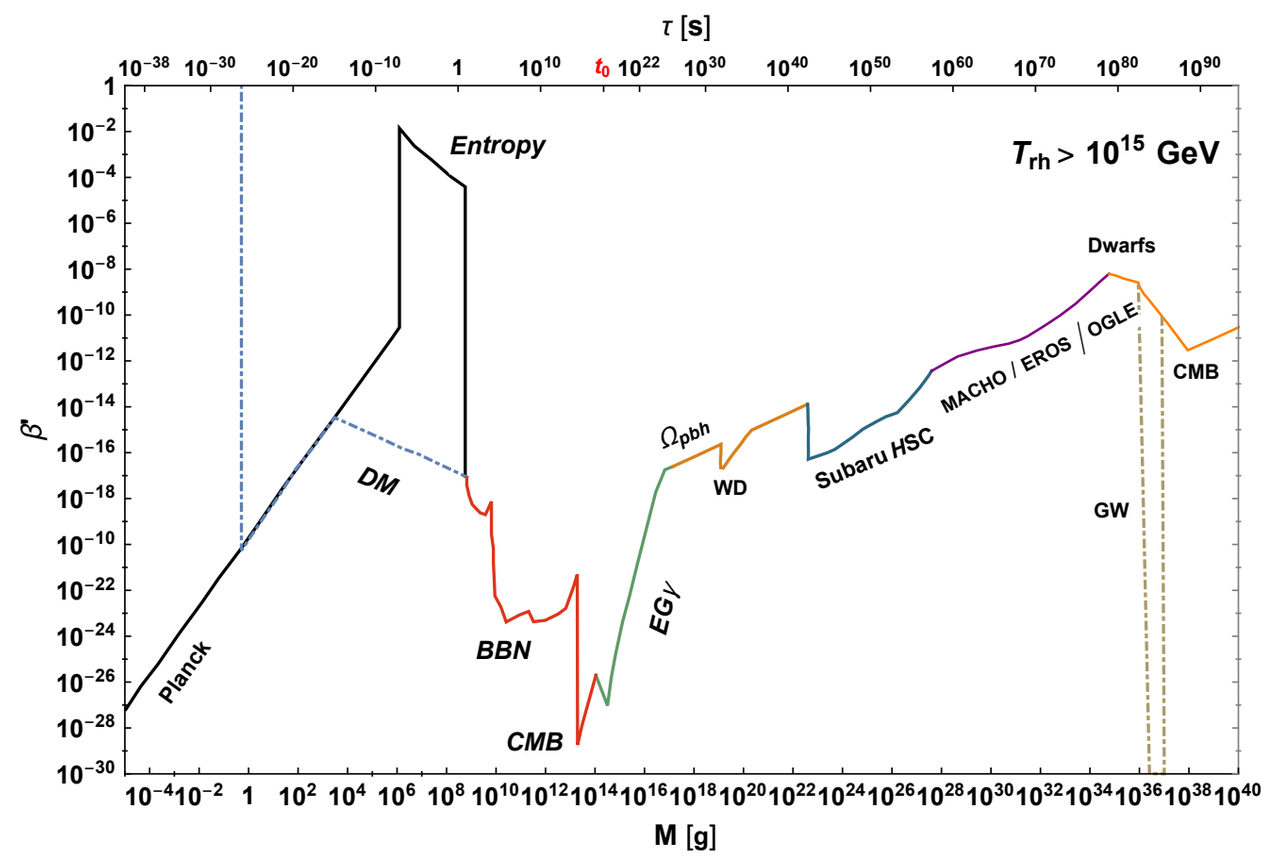


$$n_s = 0.9649 \pm 0.0042$$

$$t_{\rm evap} = 407\tilde{f}(M)\left(\frac{M}{10^{10}\rm g}\right)^3 \rm s$$

Power spectrum constraints at all scales

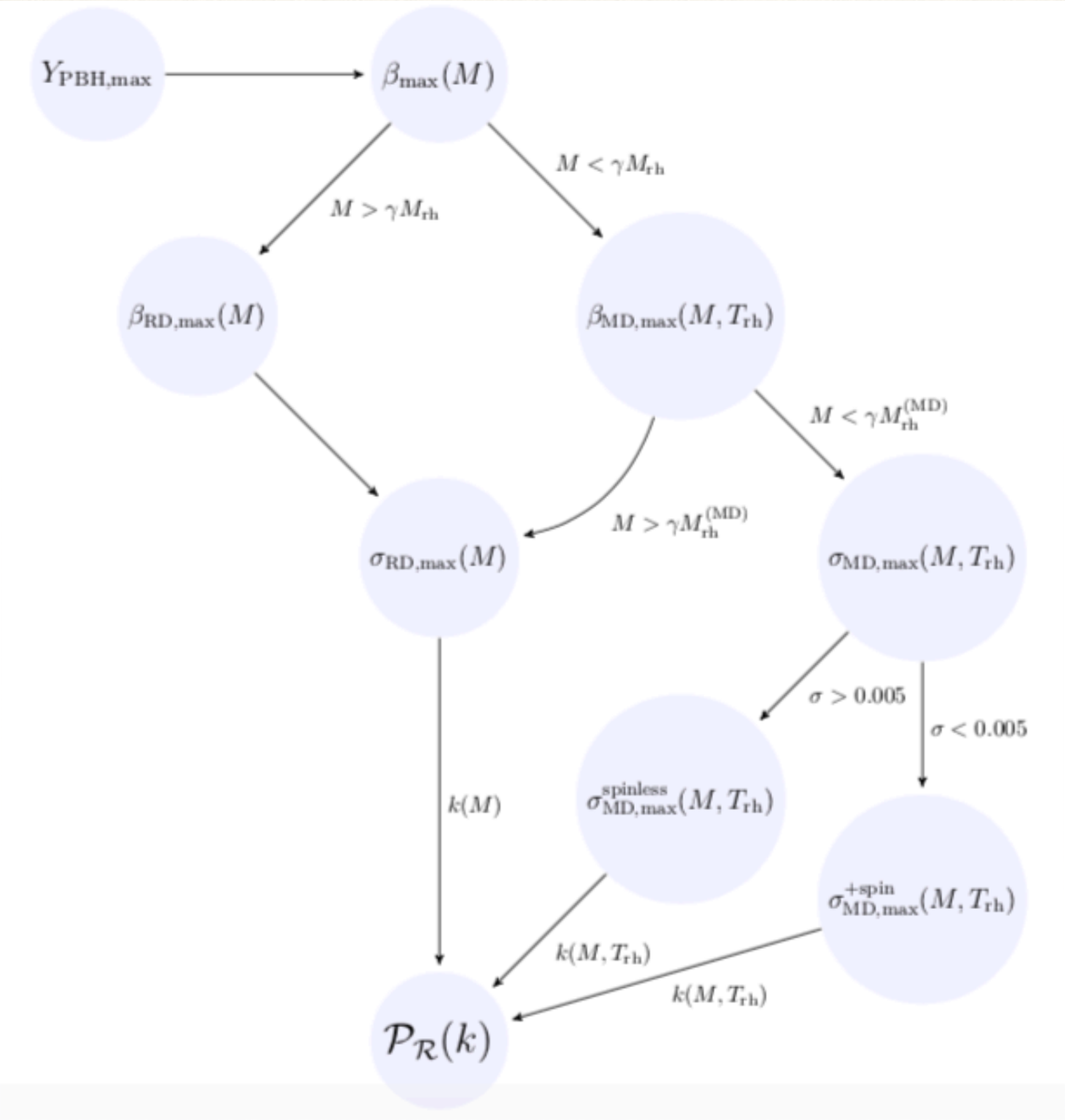




B.J.Carr, K. Kohri, Y. Sendouda and J.Yokoyama (2010)

The fractional abundance of PBHs (**f**) and the mass fraction of the universe that collapsed (β)

Transforming the PBH abundance constraints into power spectrum constraints



ID, arXiv:1812.09807

The steps one has to follow to derive upper bounds for the spectrum of the comoving curvature perturbations

Transforming the PBH abundance constraints into power spectrum constraints

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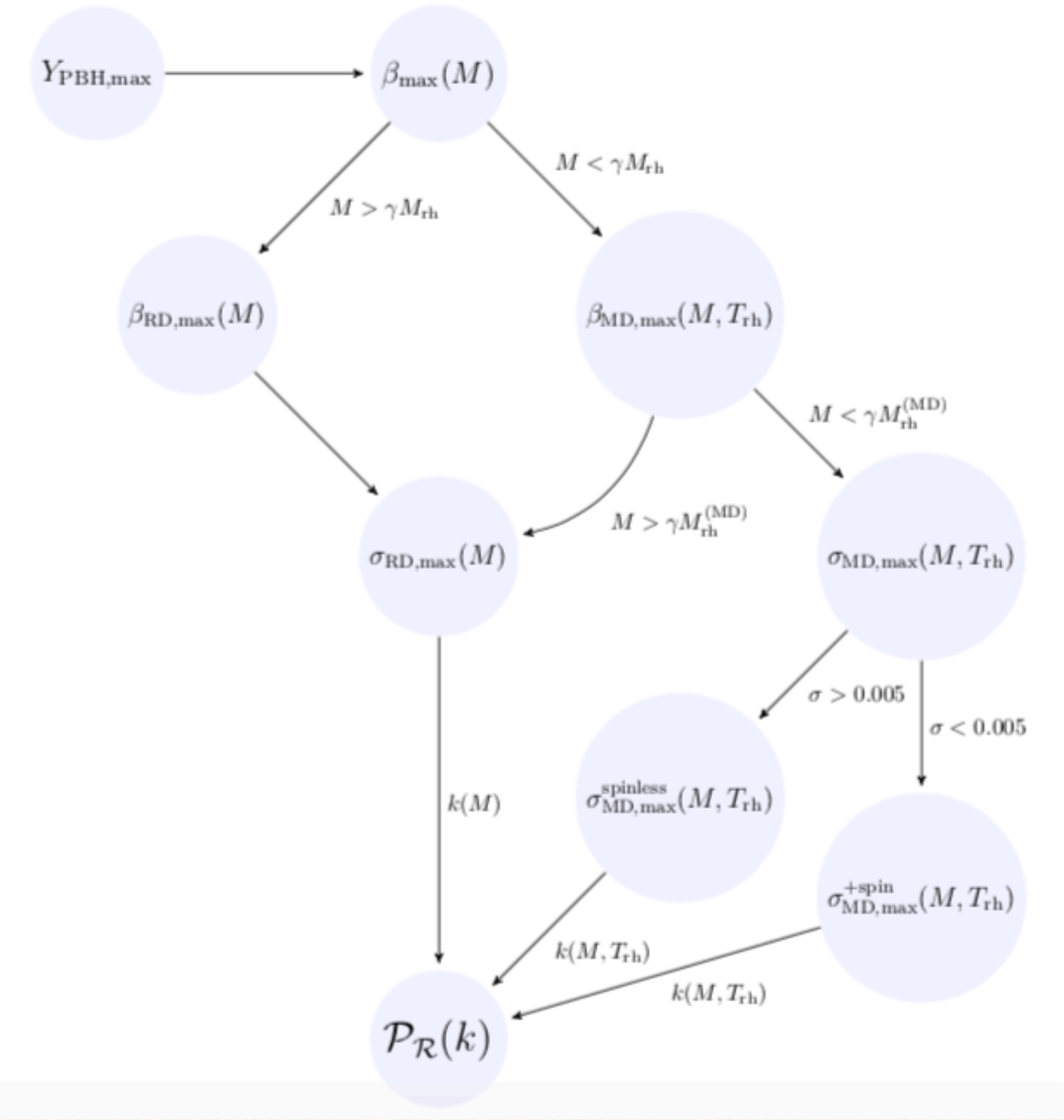

- The knowledge of the β can constrain the PS only if one assumes a model for the PBH formation.
- I assume spherical symmetric Gaussian primordial perturbations and that the PBHs form on the high σ-tail according to the Press-Schechter formalism
- I follow the monochromatic mass spectrum approximation and assume a one-to-one correspondence between the scale of perturbation and the mass of PBHs.
- I do not consider possible impacts on the power spectrum from non-Gaussianities and diffusion

$$\sigma^2(k) = \left(\frac{4}{9}\right)^2 \int \frac{dq}{q} W^2(qk^{-1})(qk^{-1})^4 \mathscr{P}_{\mathscr{R}}(q)\,,$$

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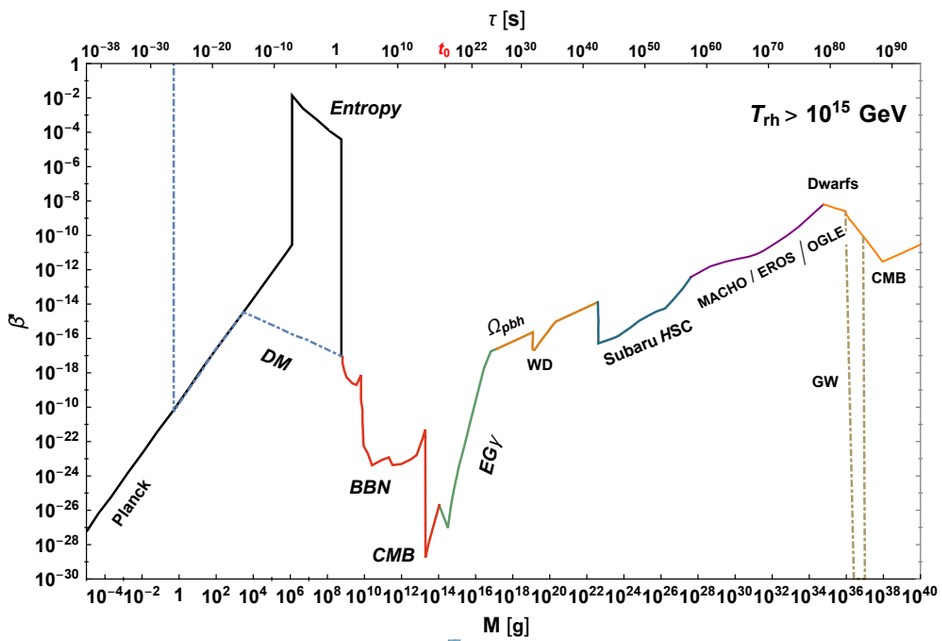
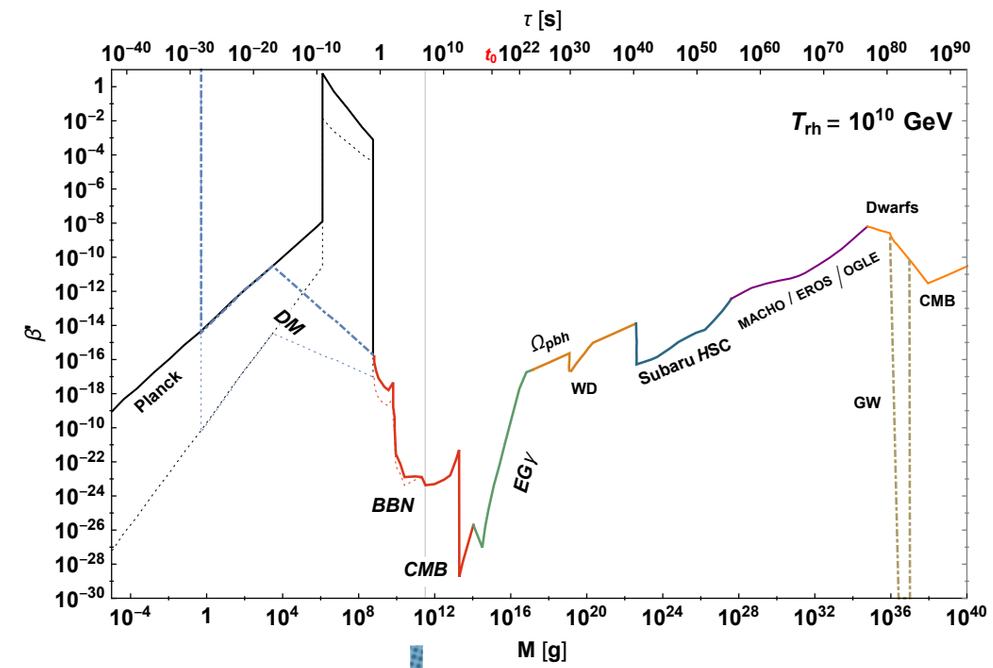
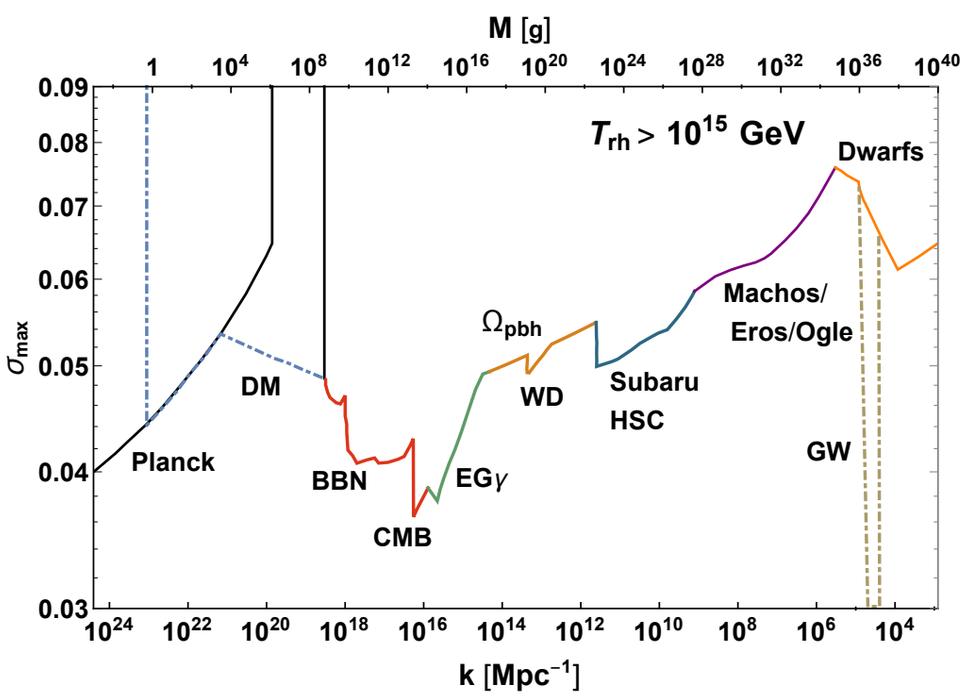
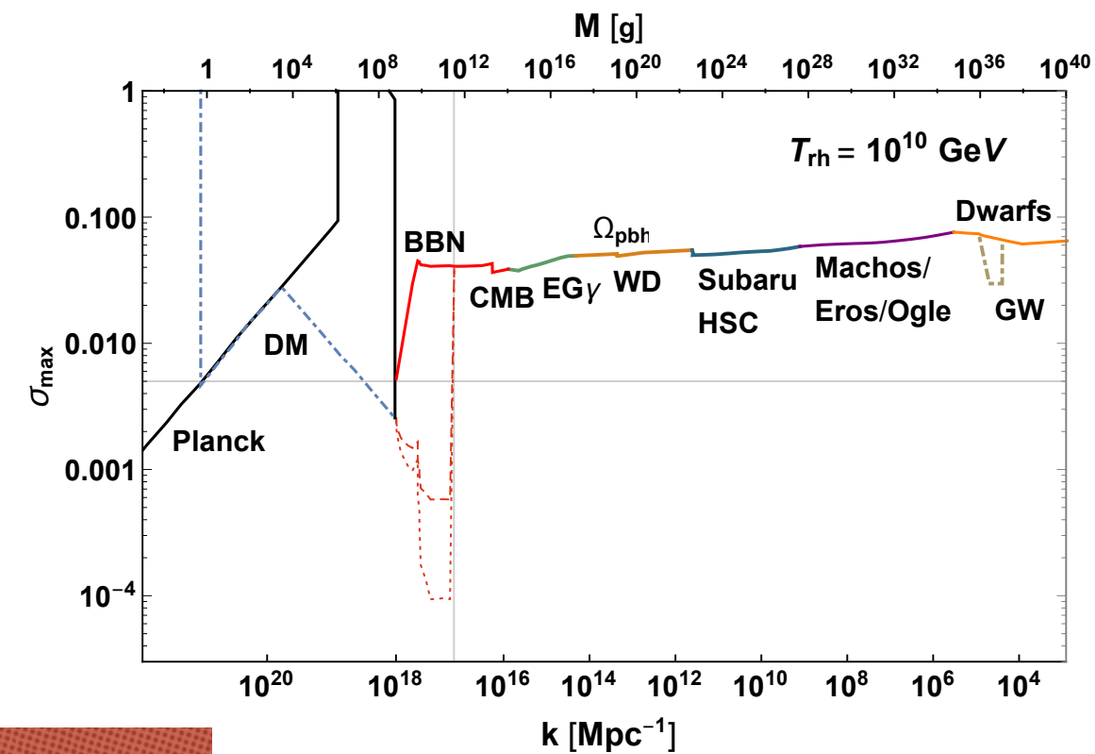




$T = 10^{10} GeV$





From the PBH abundance bounds to PS bounds

Power spectrum constraints at all scales

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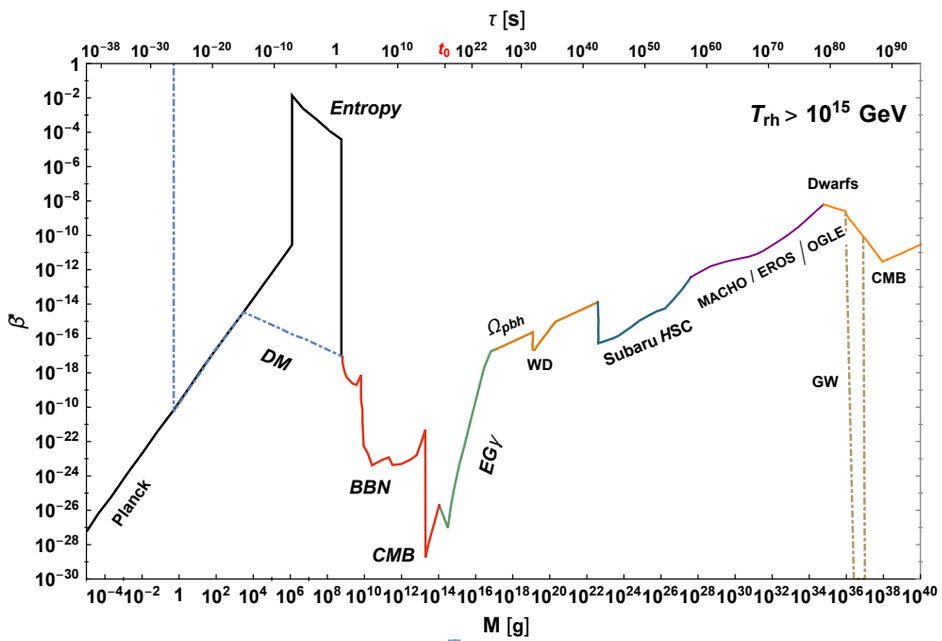

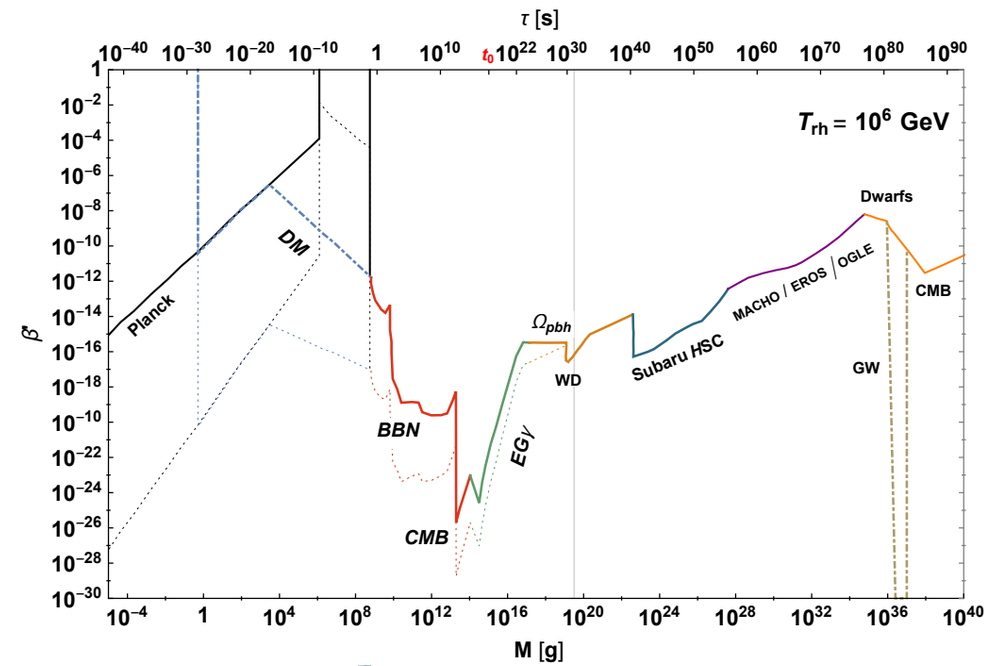

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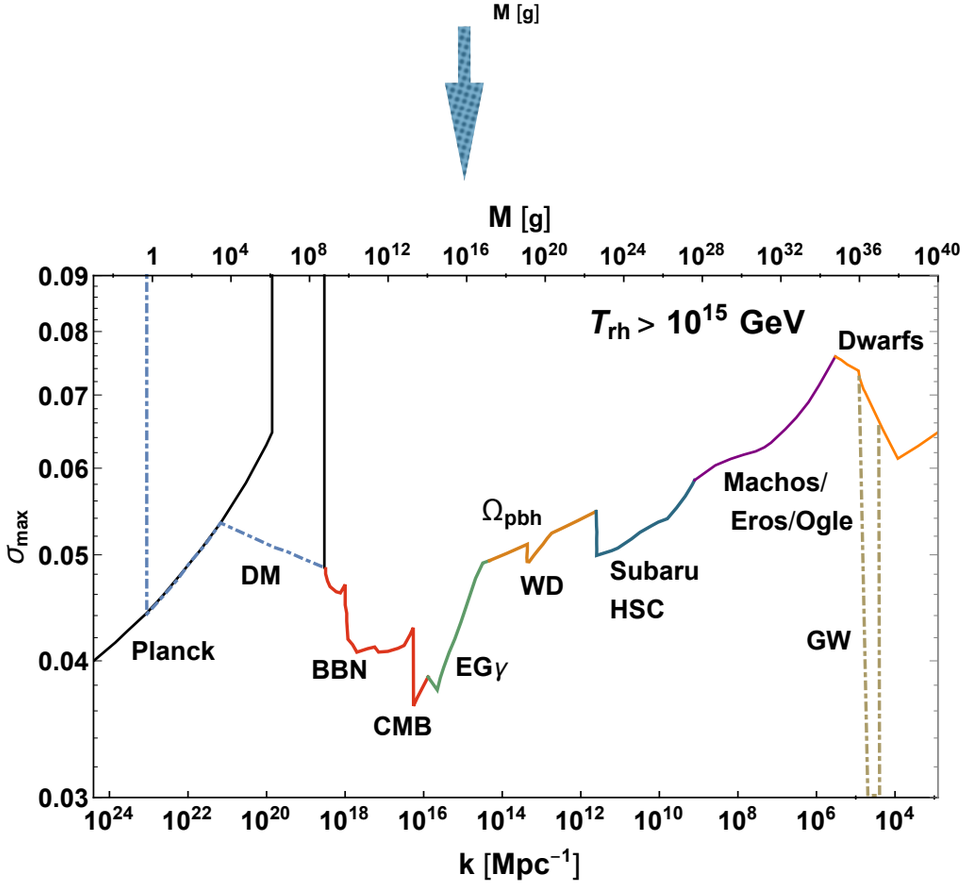

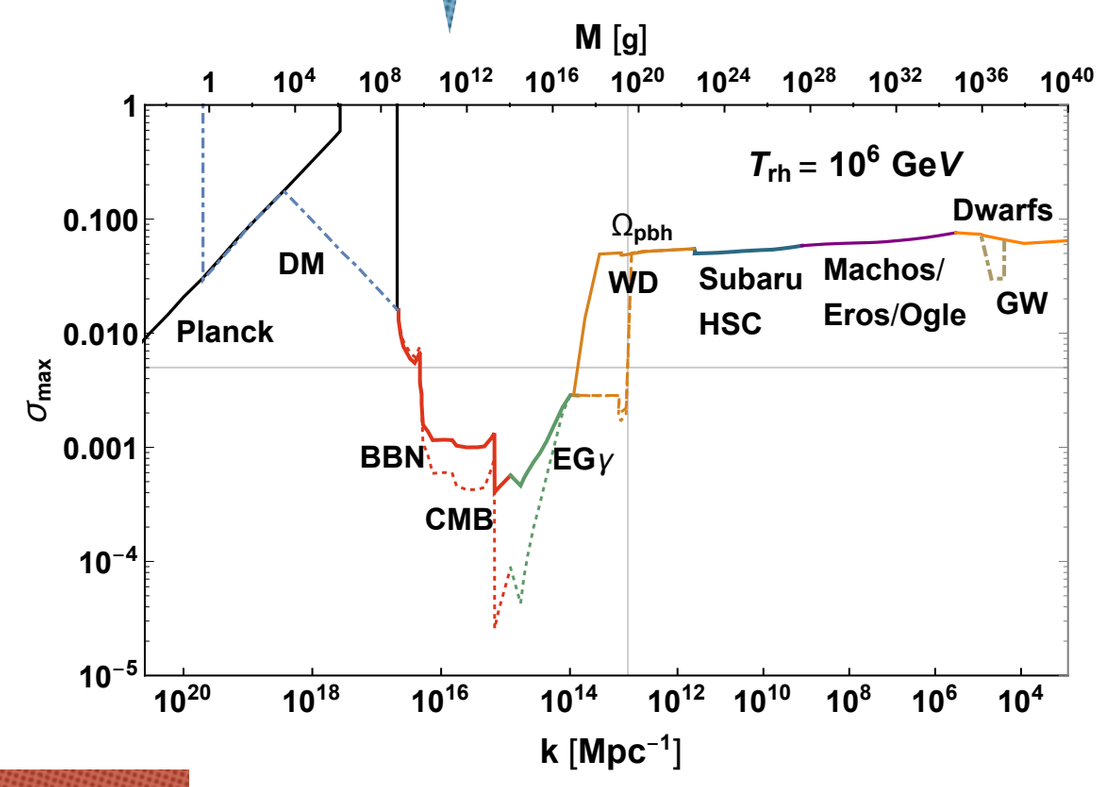

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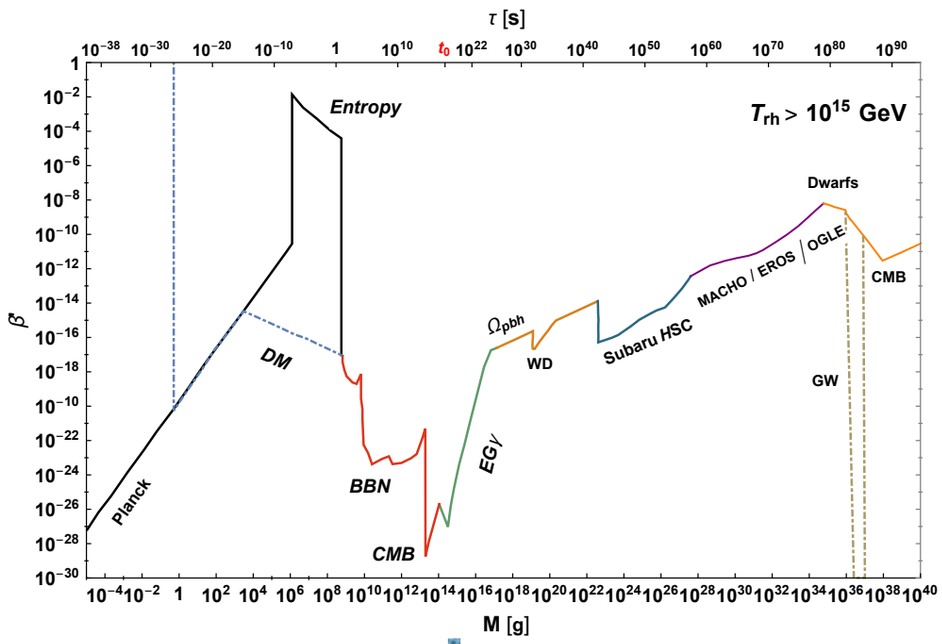
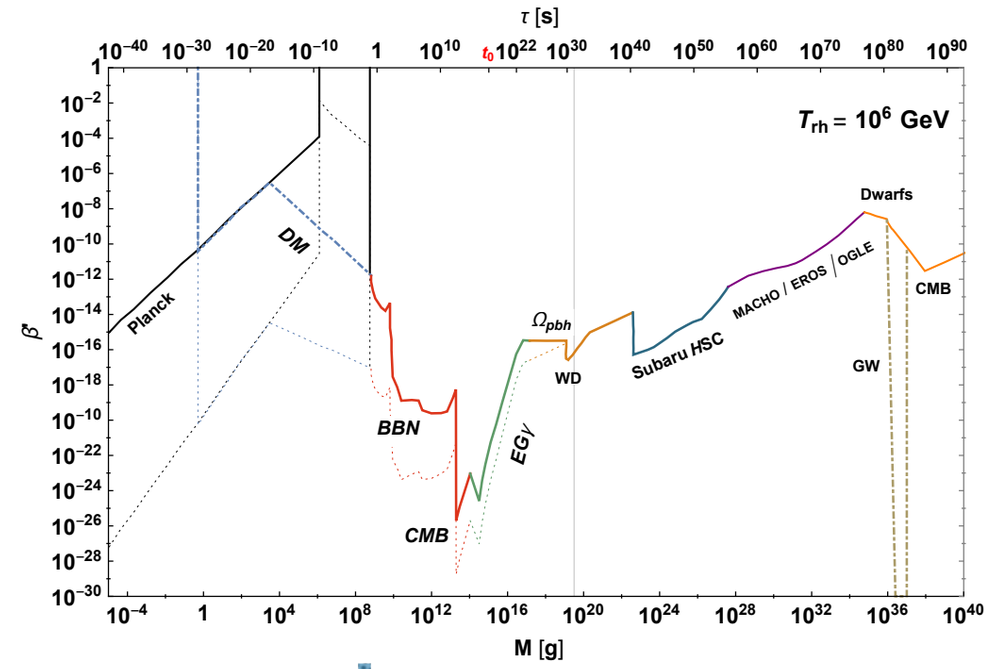
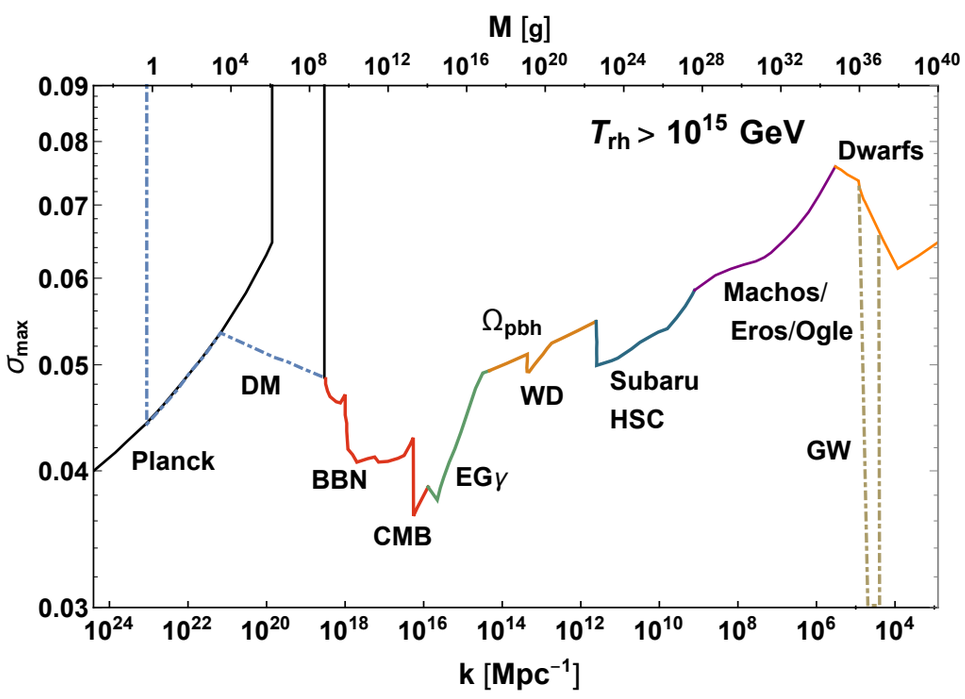
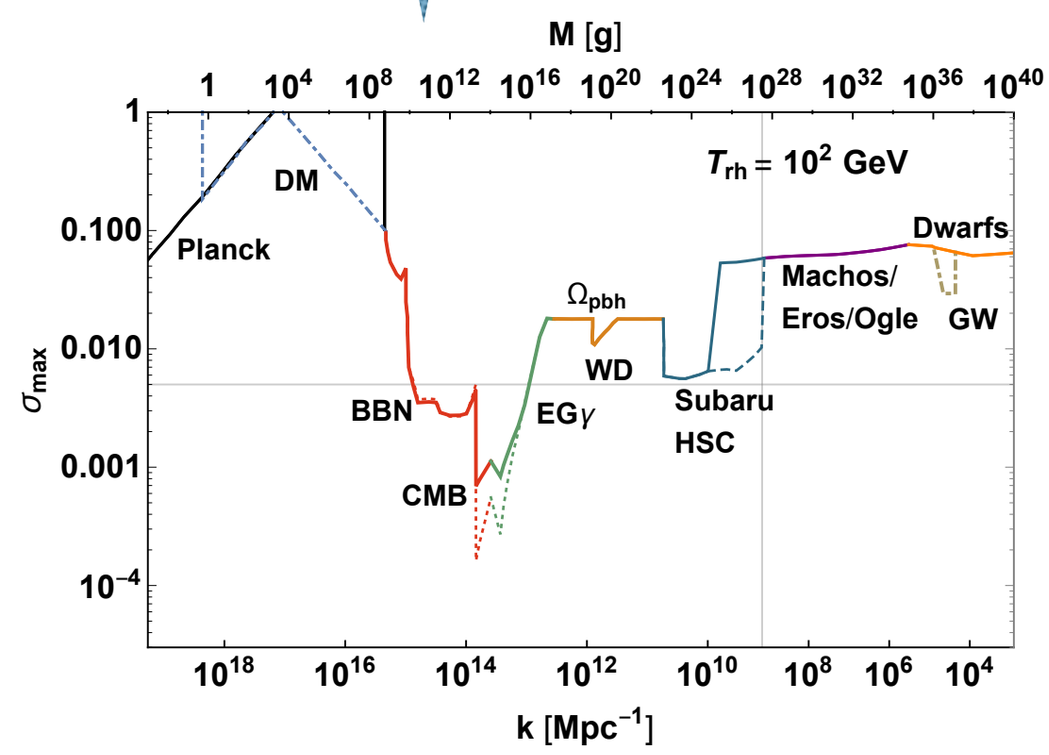




$T = 10^2 GeV$





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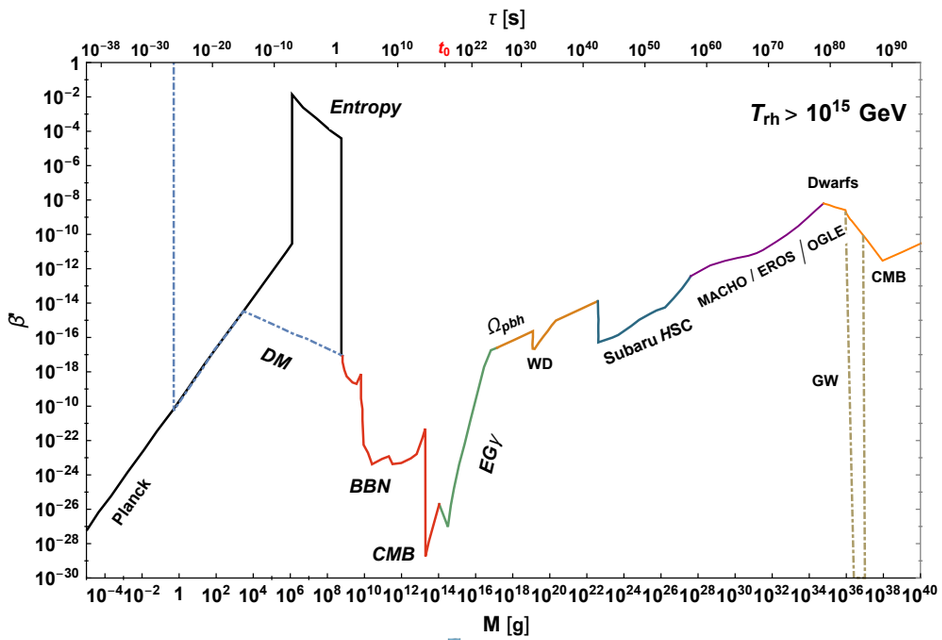
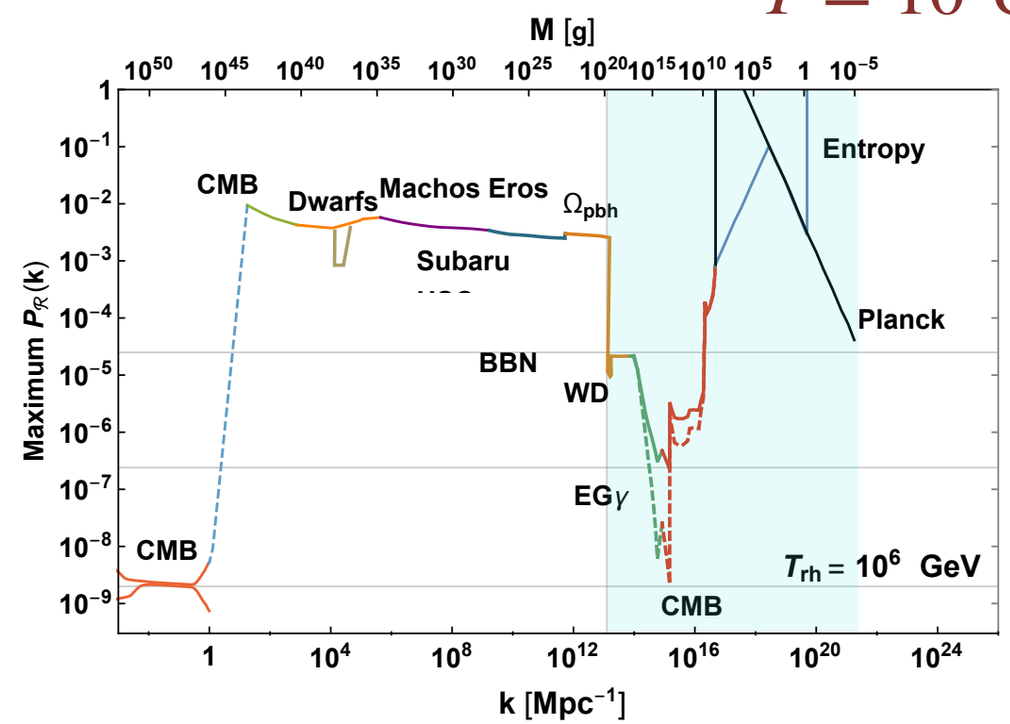
Power spectrum constraints at all scales

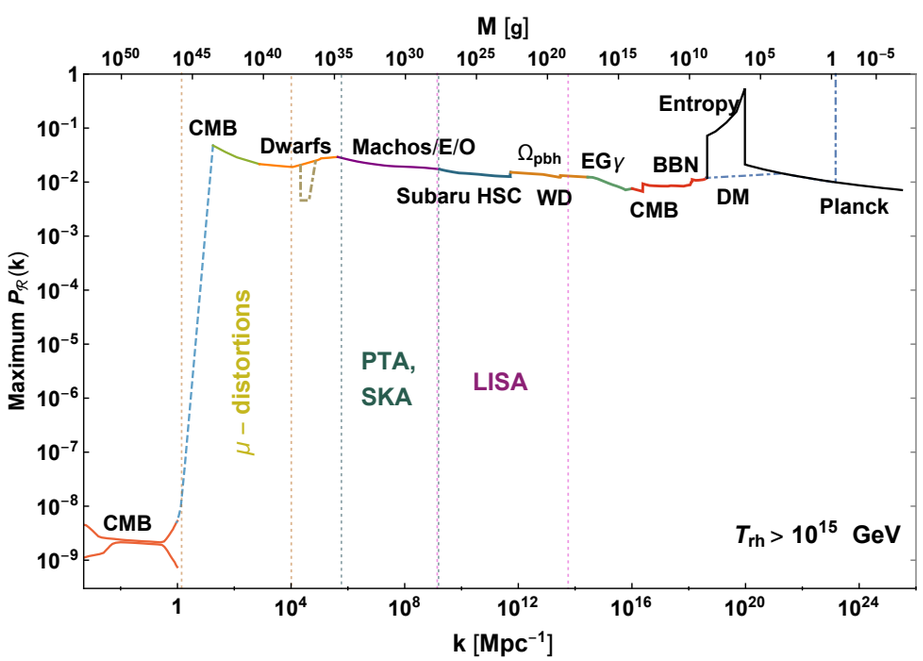
ID, arXiv:1812.09807

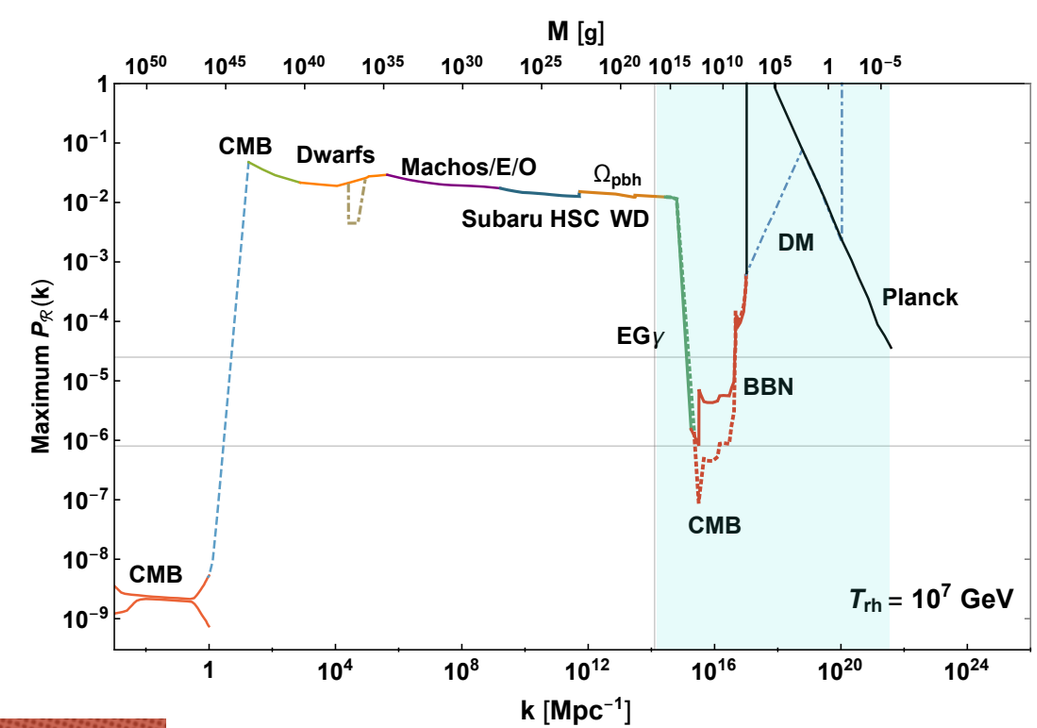


$T = 10^6 GeV$



$T = 10^7 GeV$

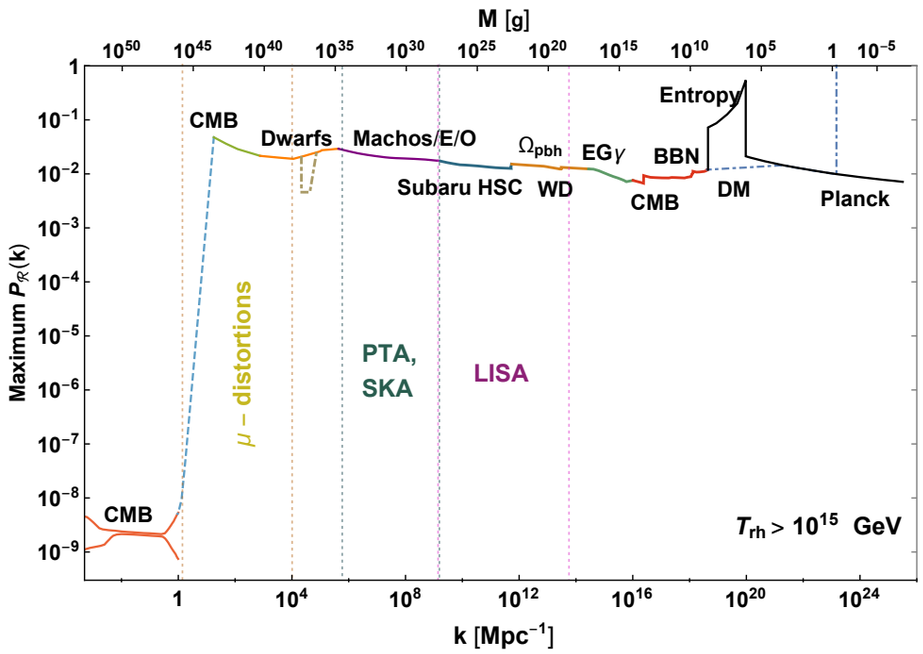




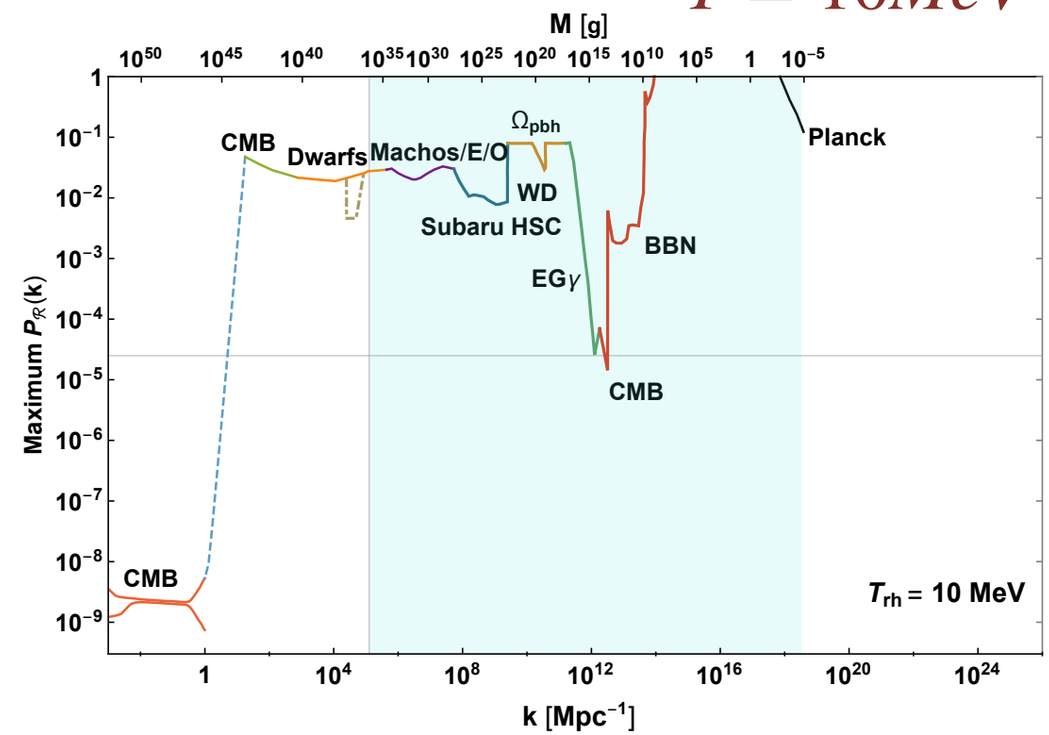
From the PBH abundance bounds to PS bounds

Power spectrum constraints at all scales

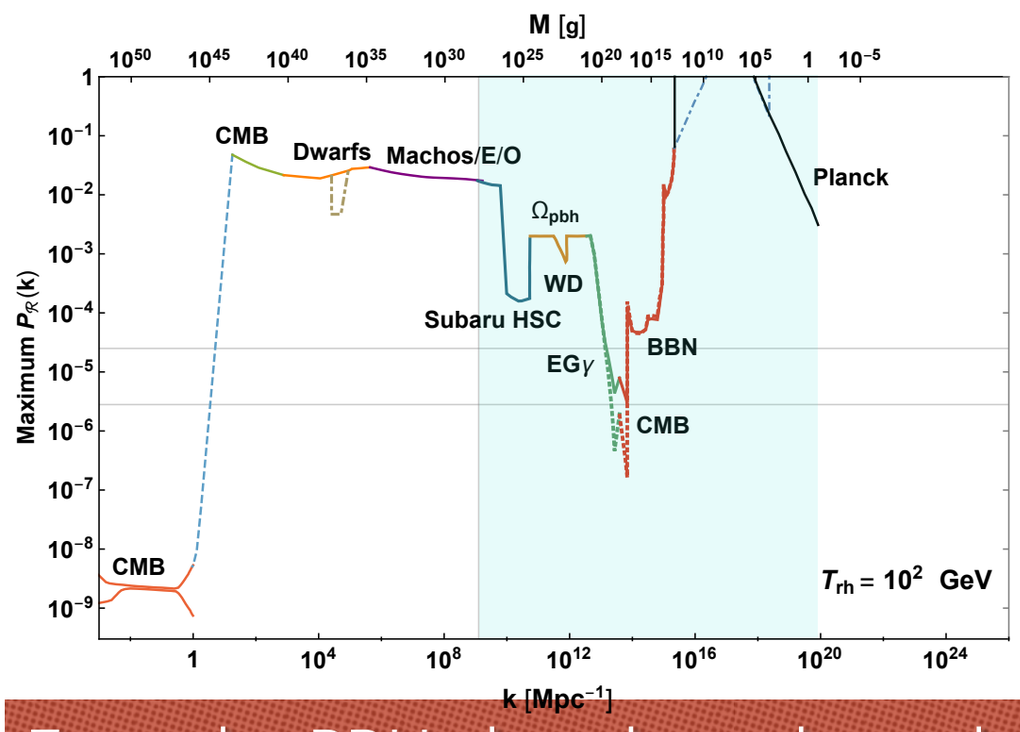
ID, arXiv:1812.09807



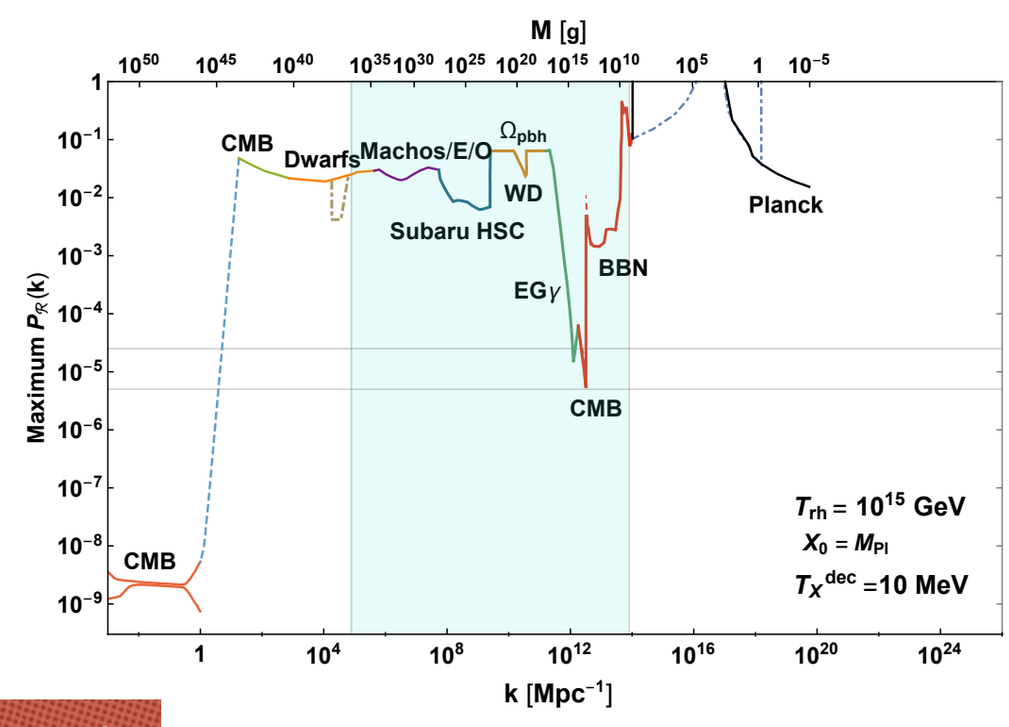
$T = 10MeV$



$T = 10^2 GeV$



Moduli case



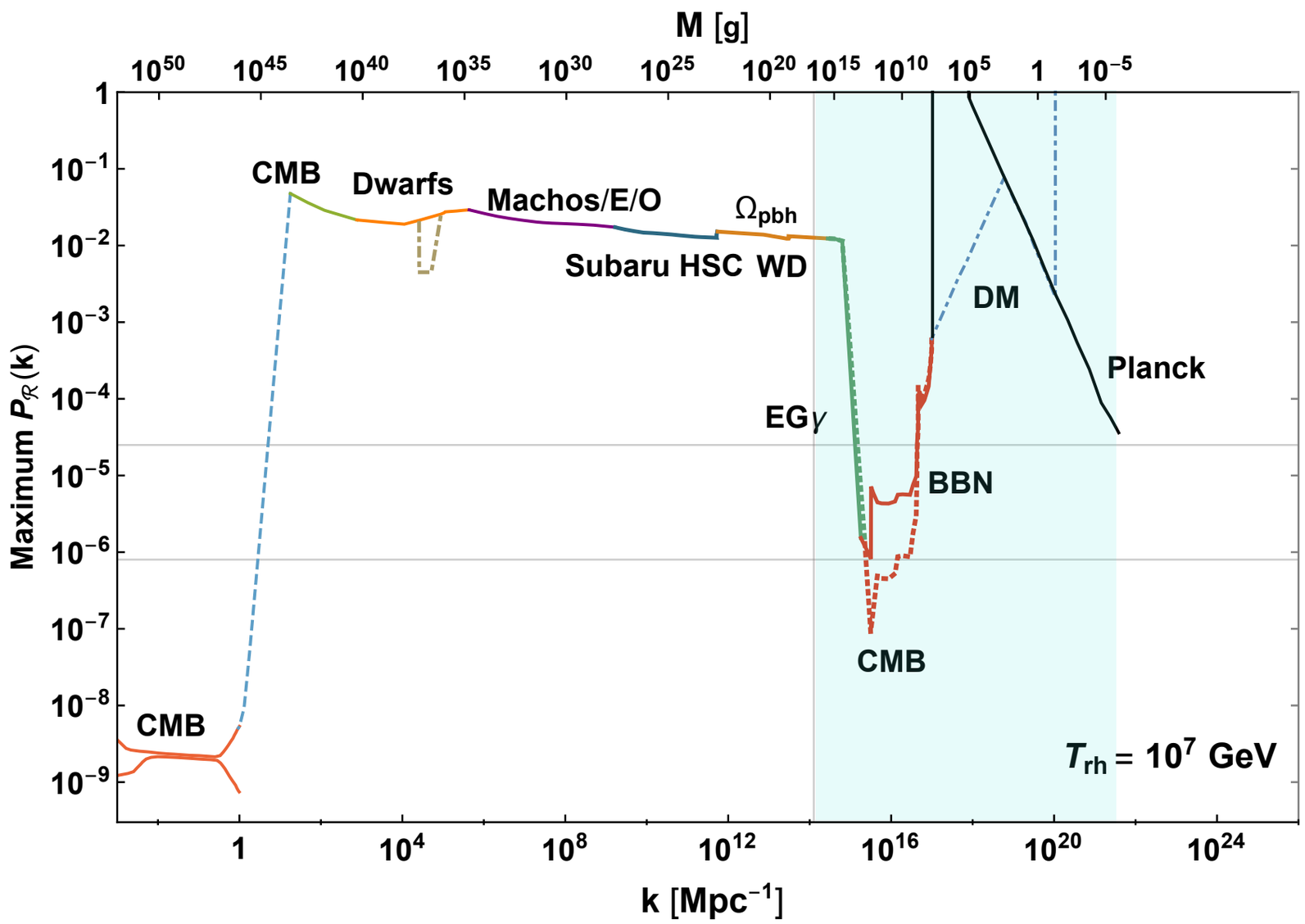
From the PBH abundance bounds to PS bounds

Power spectrum constraints at all scales

ID, arXiv:1812.09807

A specific example

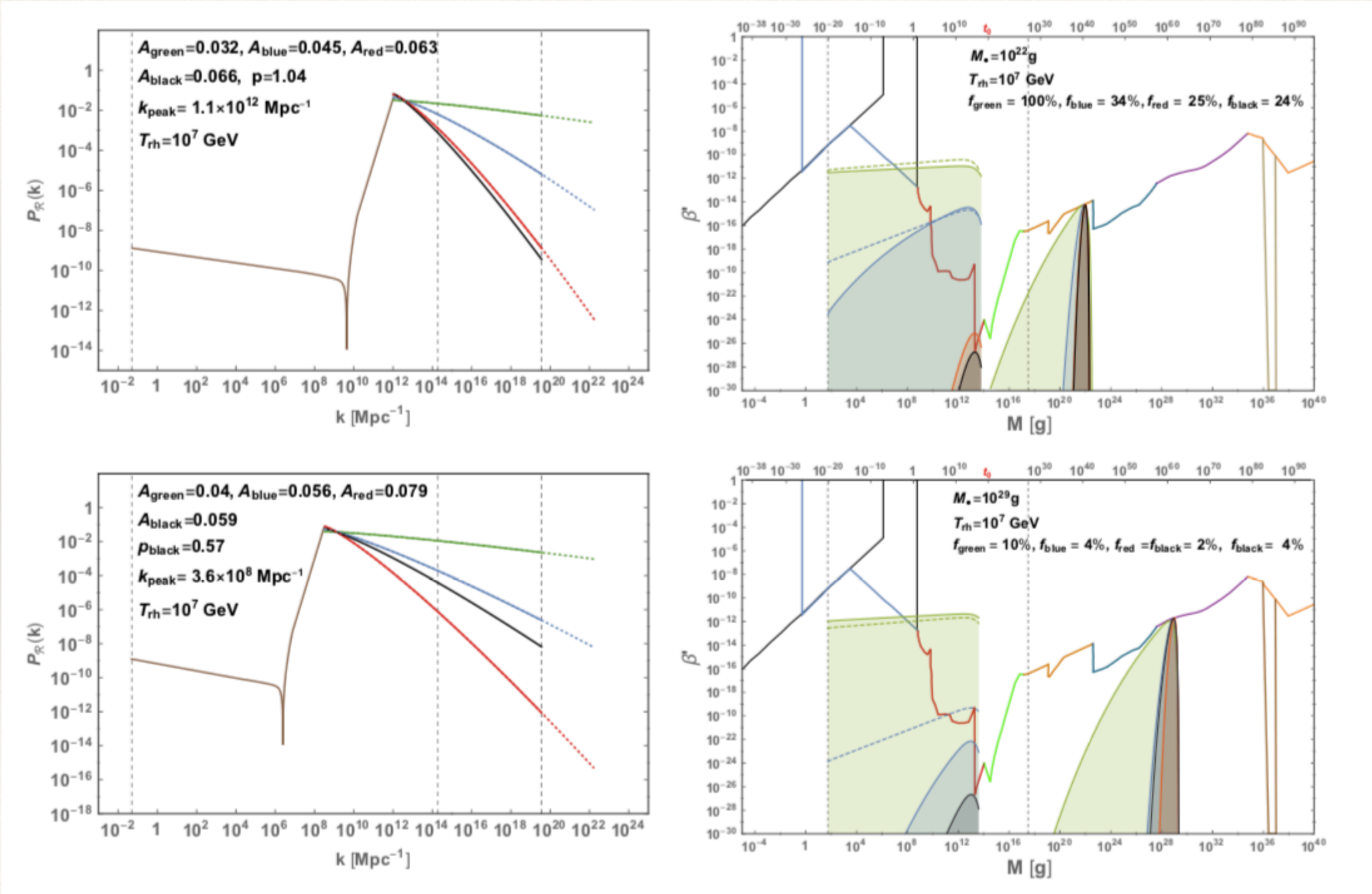
$T = 10^7 GeV$



To be more explicit: The power spectrum has to be particularly narrow

$$\mathscr{P}_{\mathscr{R}}(k \geq k_{peak}) = A_{max}\left(k/k_{peak}\right)^{-p}, \quad p = 0.1, 0.5, 1$$

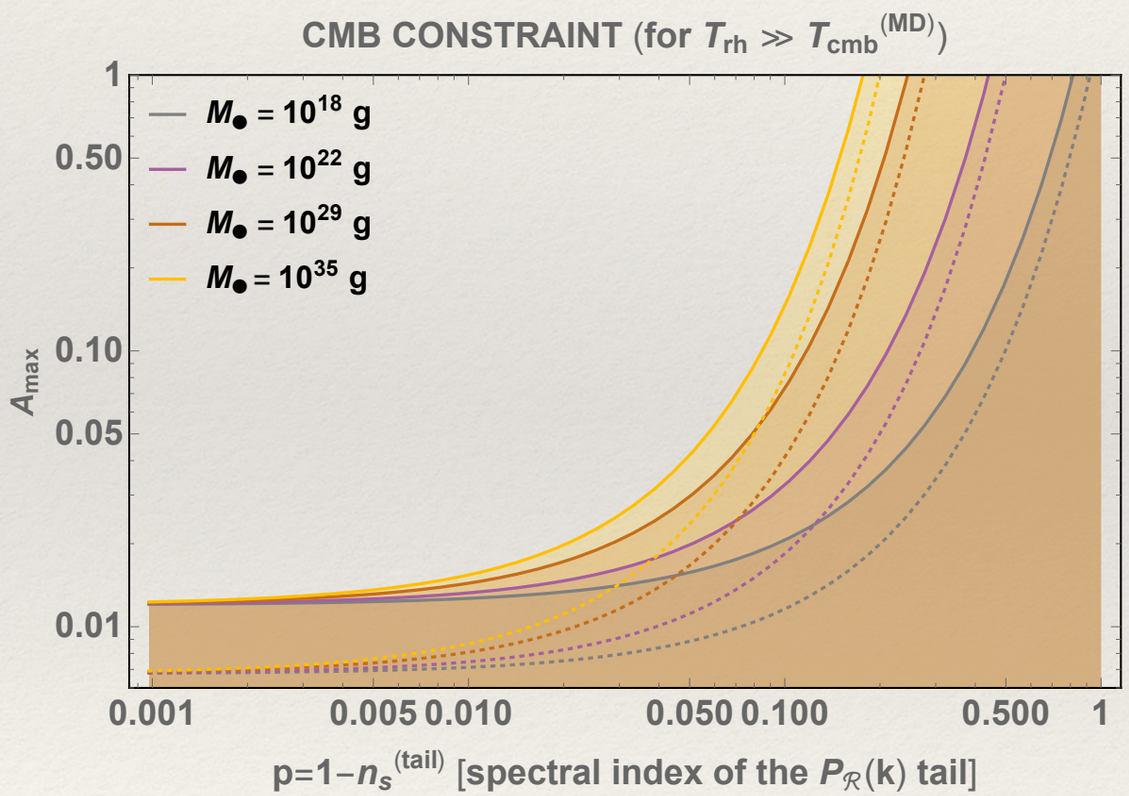
ID, arXiv:1812.09807

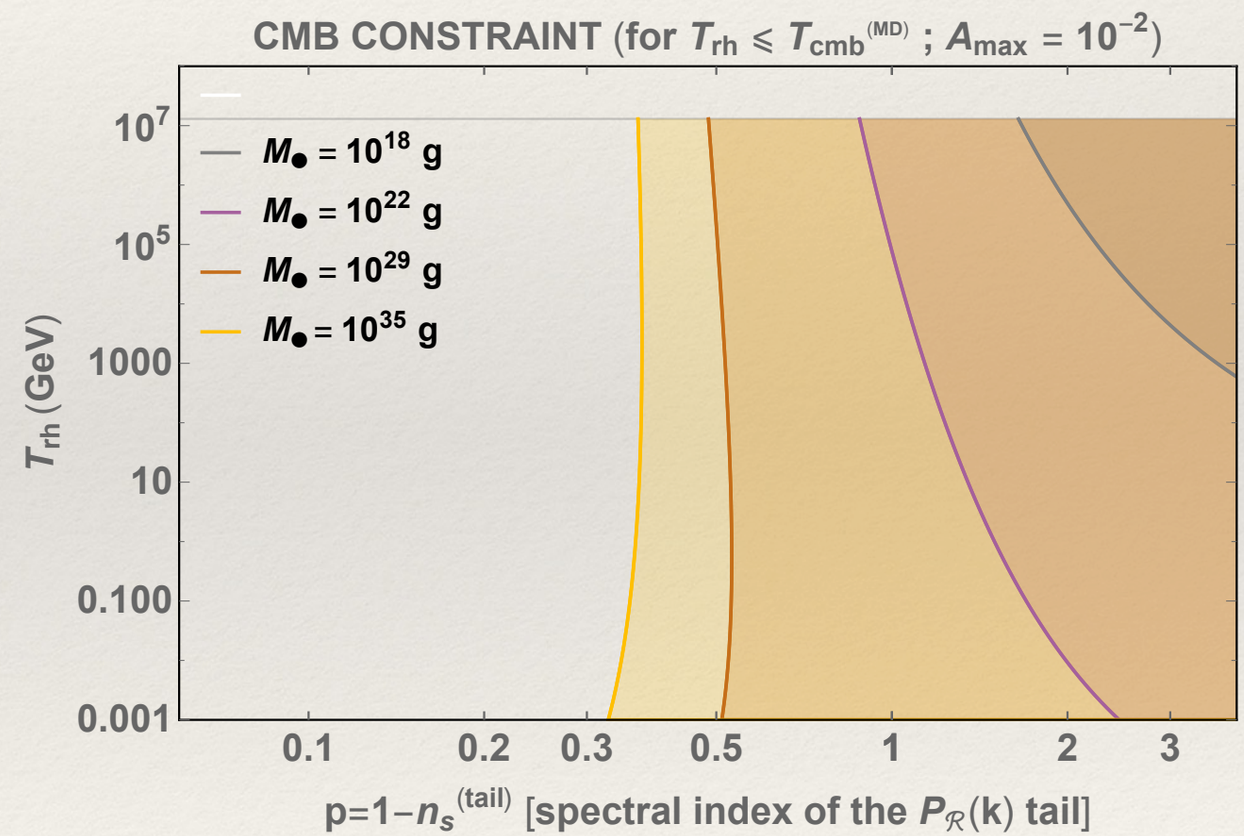


To be more explicit: Constraints on the tail of the PS

$$\mathscr{P}_{\mathscr{R}}(k \geq k_{peak}) = A_{max}\left(k/k_{peak}\right)^{-p}, \quad p = 0.1, 0.5, 1$$

ID, arXiv:1812.09807



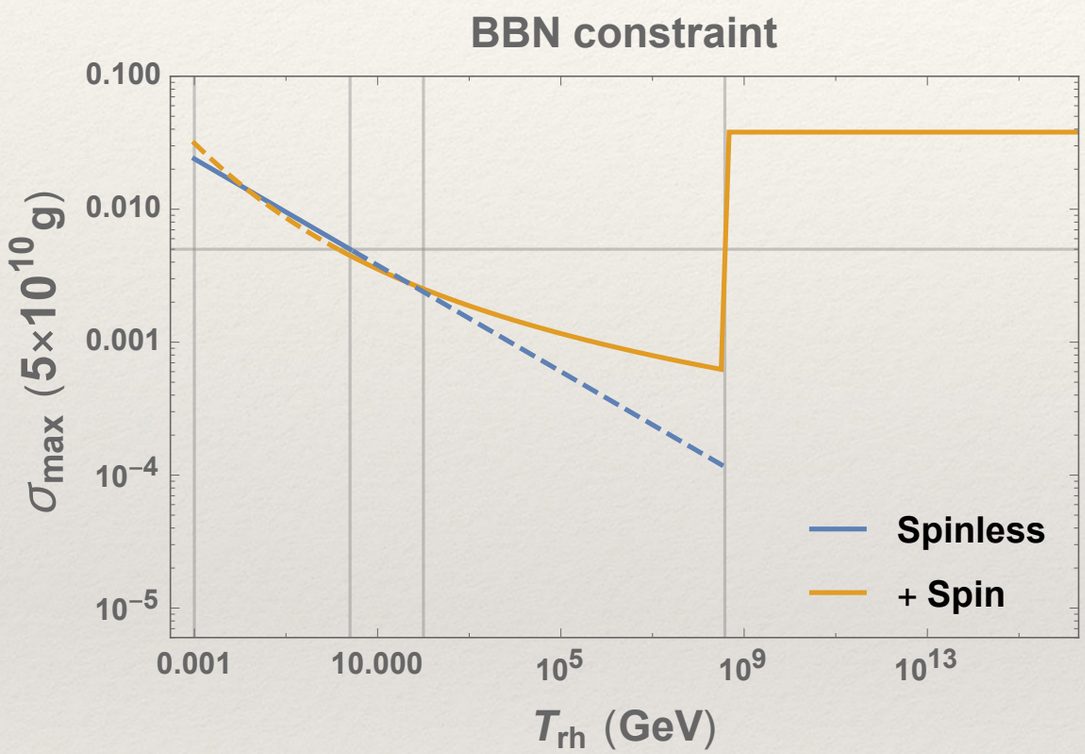


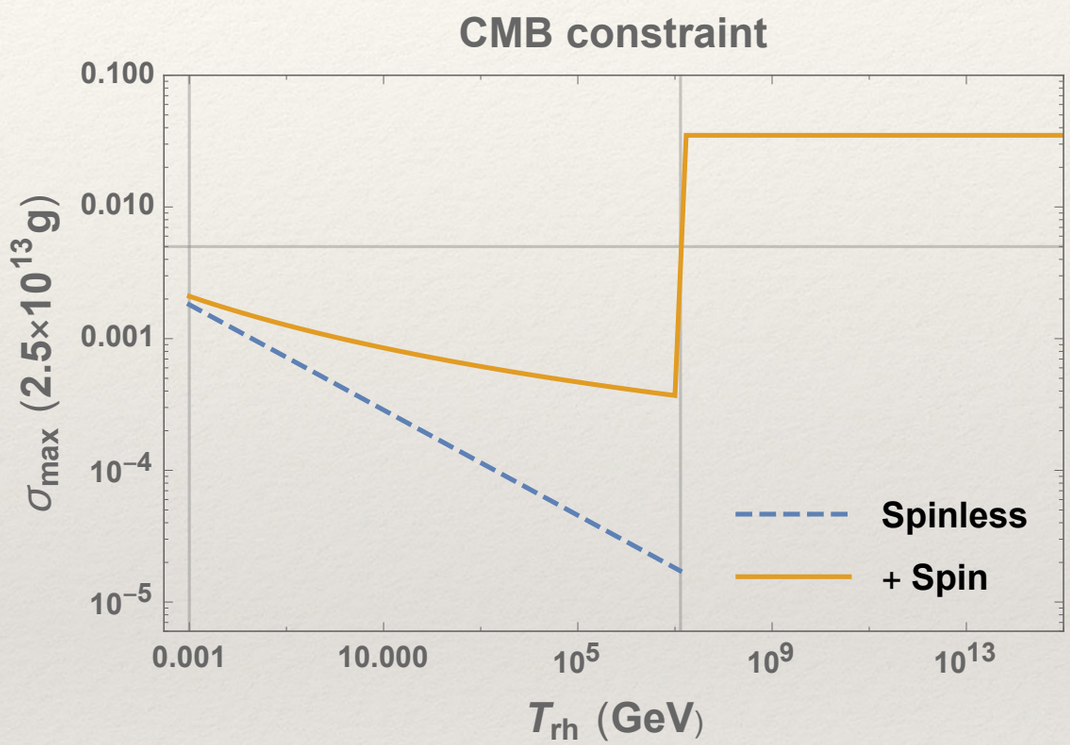
$$T_{bbn}^{(MD)} = 3.9 \times 10^8 GeV \qquad \text{and} \qquad T_{cmb}^{(MD)} = 1.3 \times 10^7 GeV$$

ID, arXiv:1812.09807

A particular constraint for the variance of the density perturbations is obtained

$$T_{\rm rh} < T_{bbn}^{(MD)} \equiv \left(\frac{2}{5}\mathcal{J}\right)^{1/2} T_{bbn}\,\sigma^{1/2}(M_{bbn}) \qquad \text{and} \qquad T_{\rm rh} < T_{cmb}^{(MD)} \equiv \left(\frac{2}{5}\mathcal{J}\right)^{1/2} T_{cmb}\,\sigma^{1/2}(M_{cmb})$$



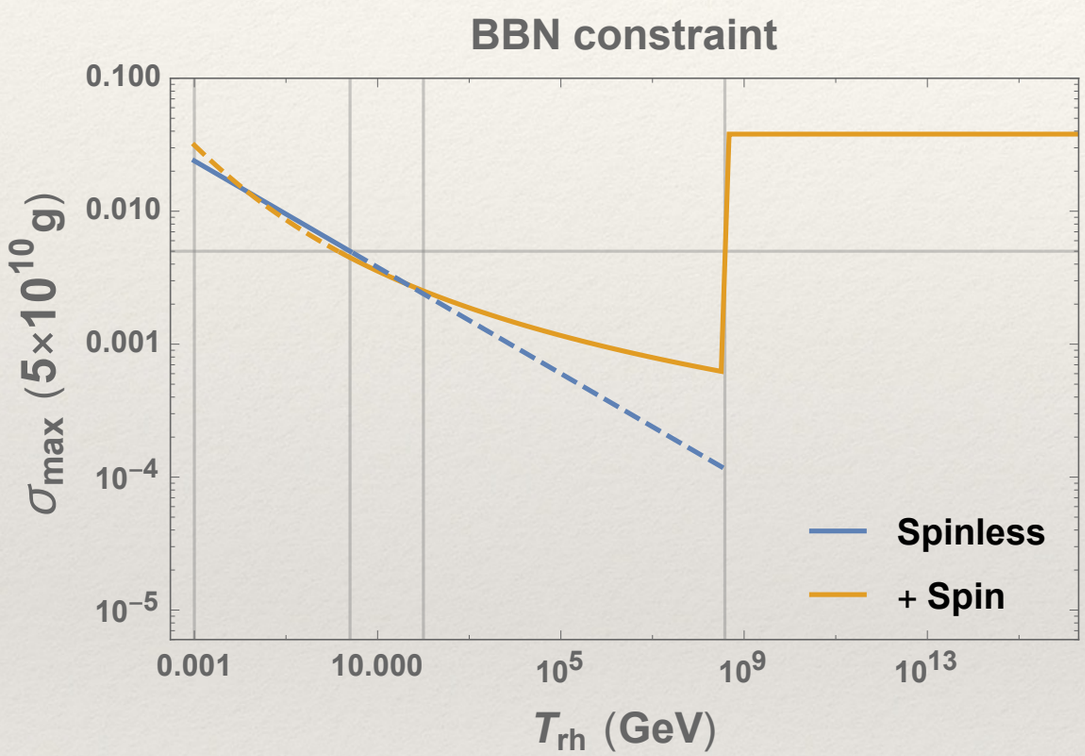


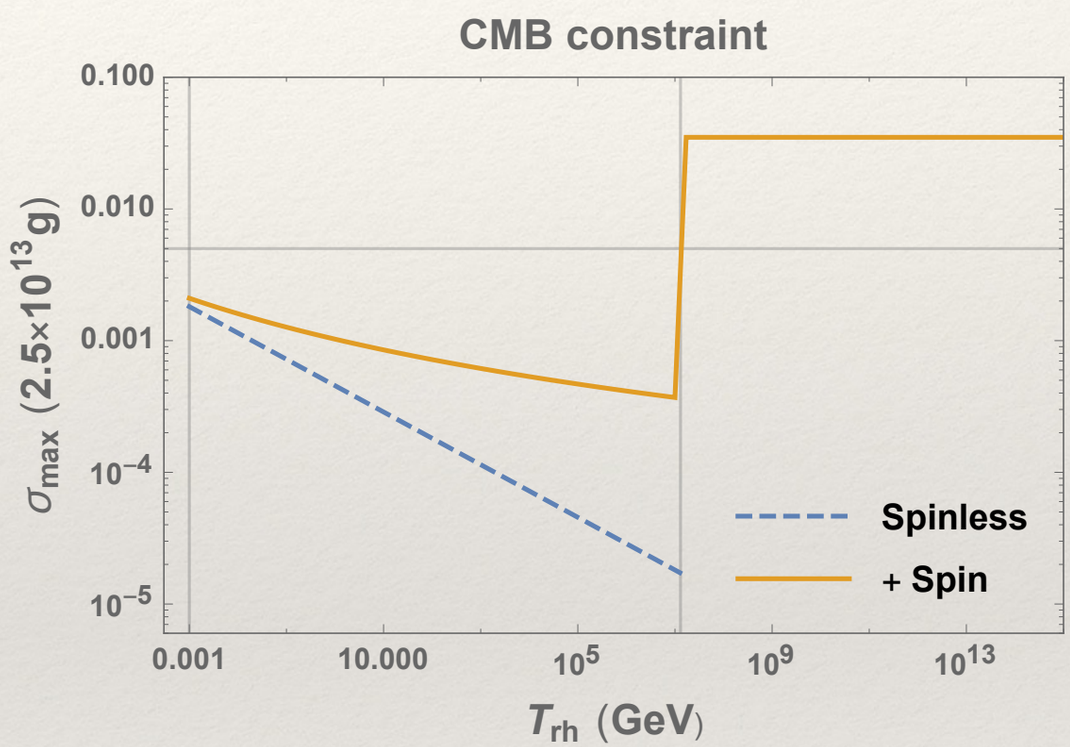
$$\sigma(5\times10^{10}\text{g})\Big|_{+\text{spin}} \lesssim \text{Exp}\left[-5.22 + 0.196\,\ln\frac{T_{\rm rh}}{\text{GeV}} + 6.8\times10^{-3}\left(\ln\frac{T_{\rm rh}}{\text{GeV}}\right)^2 - 1.2\times10^{-4}\left(\ln\frac{T_{\rm rh}}{\text{GeV}}\right)^3\right] \qquad \text{(BBN)}$$

$$\sigma(2.5\times10^{13}\text{g})\Big|_{+\text{spin}} \lesssim \text{Exp}\left[-6.88 - 0.087\,\ln\frac{T_{\rm rh}}{\text{GeV}} + 2\times10^{-3}\left(\ln\frac{T_{\rm rh}}{\text{GeV}}\right)^2 - 3\times10^{-5}\left(\ln\frac{T_{\rm rh}}{\text{GeV}}\right)^3\right] \qquad \text{(CMB)}$$

ID, arXiv:1812.09807

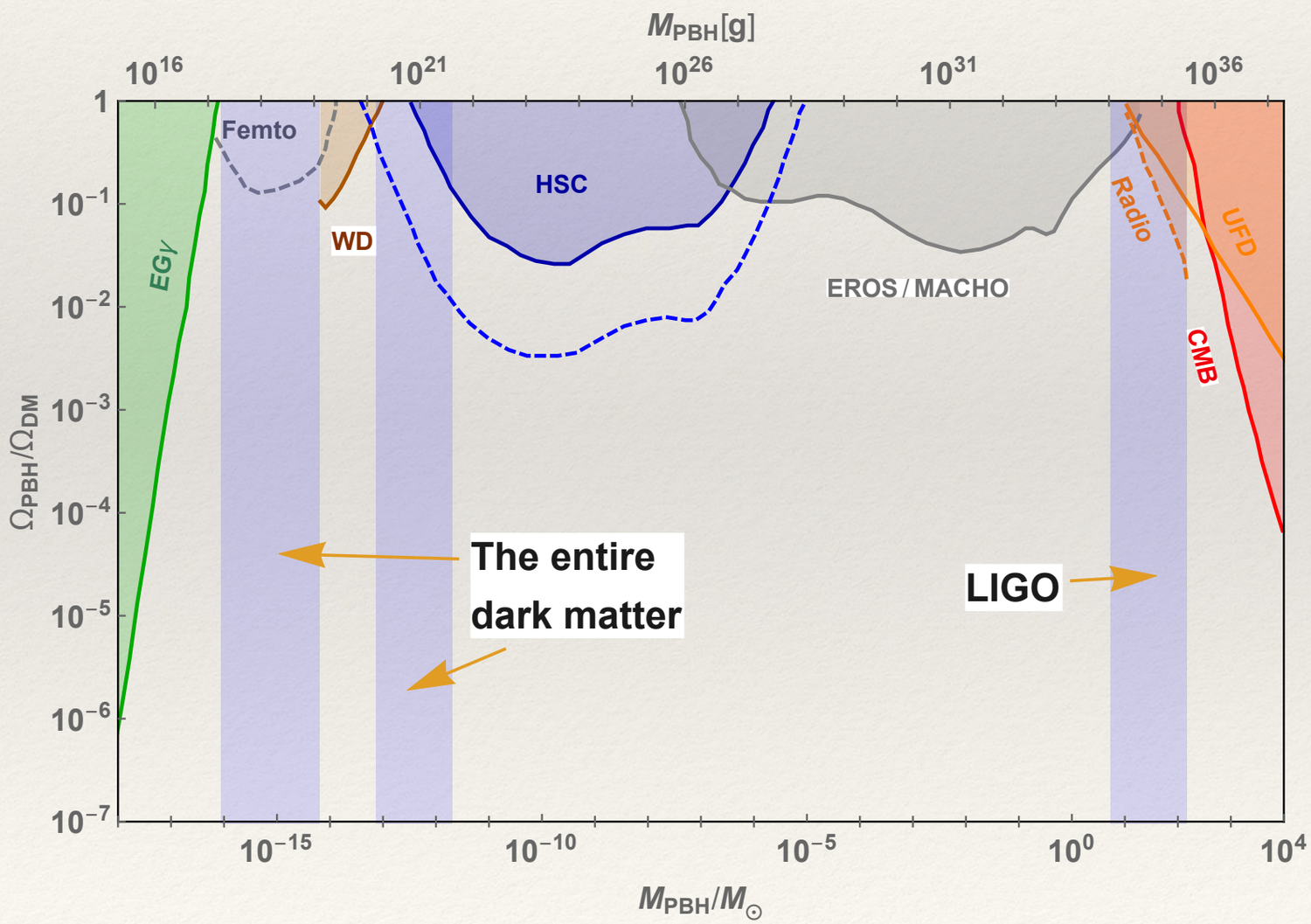
A particular constraint for the variance of the density perturbations is obtained





This constraint has to be taken into account for every inflationary model

Building inflationary models that predict PBHs

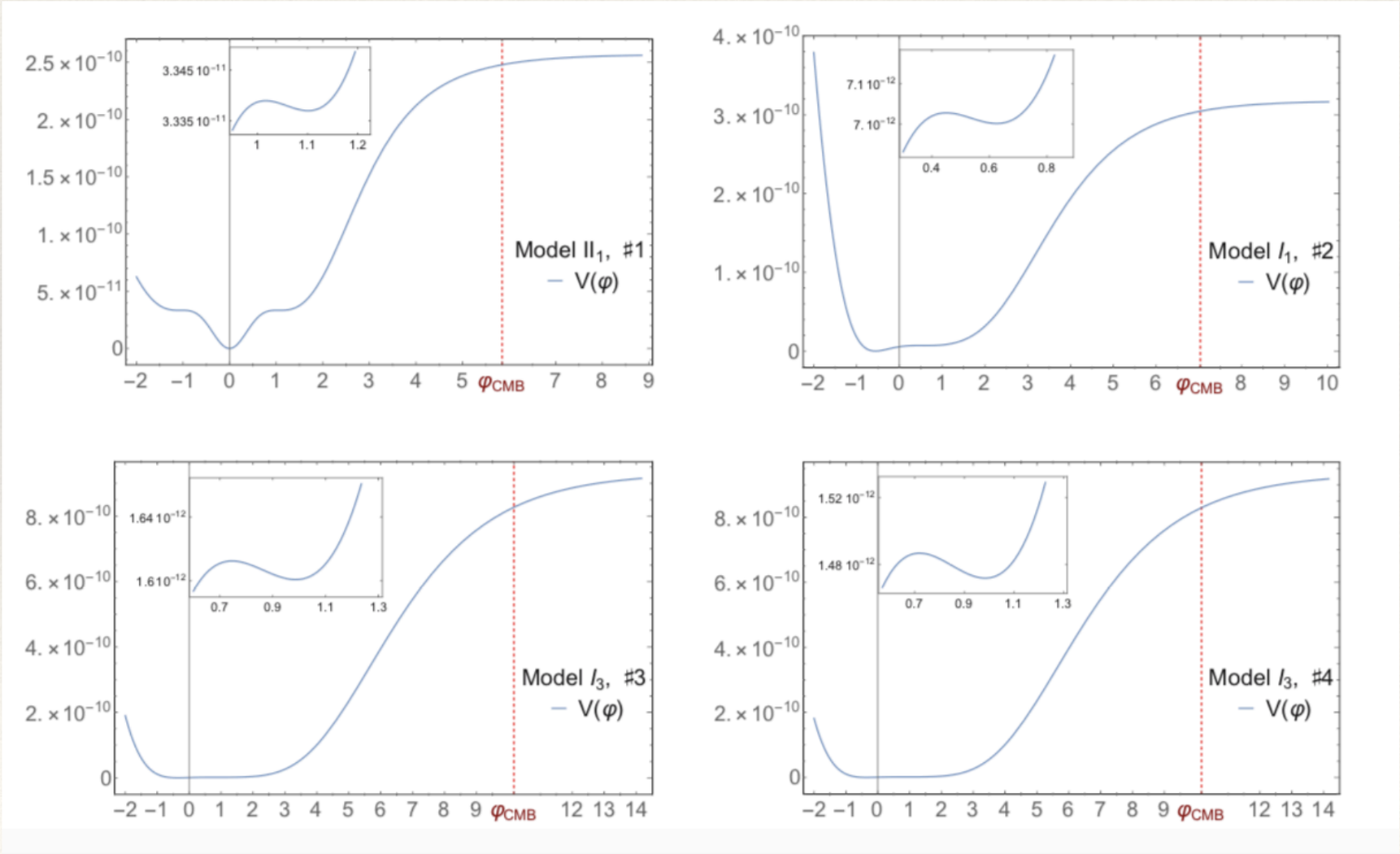


ID + Kehagias + Tringas, JCAP 1901 (2019) 037

Superconformal α-Attractor inflationary models

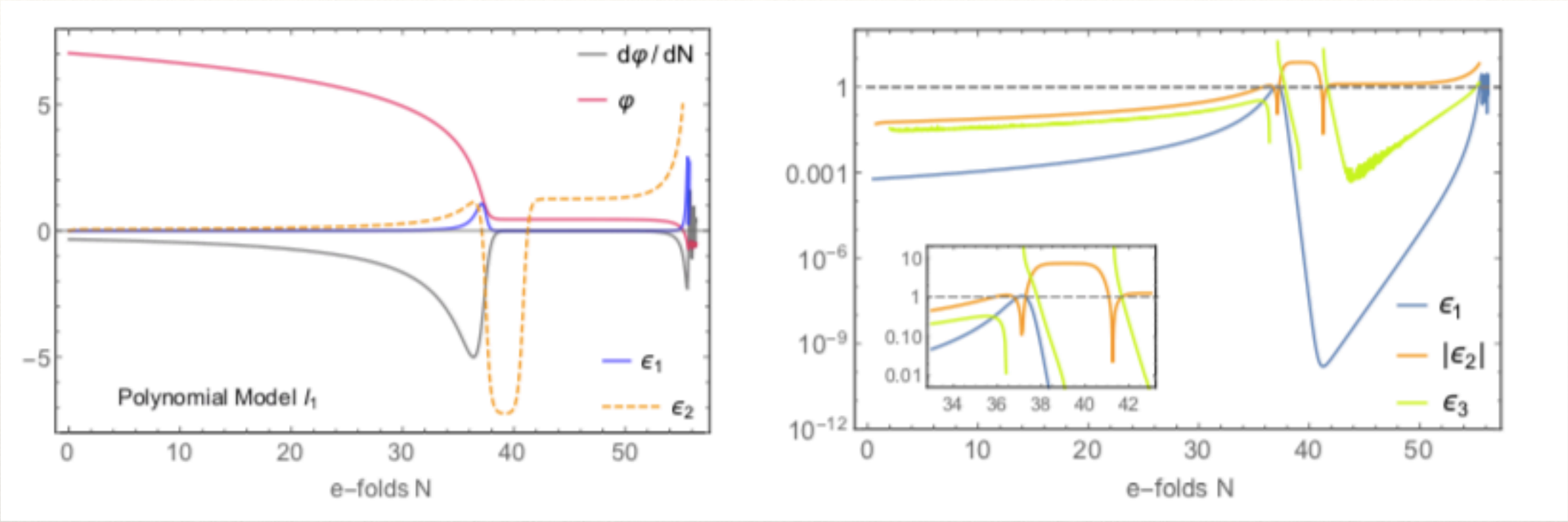
$$V(\varphi) = V_0 \left\{ c_0 + c_1 \tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right) + c_2 \tanh^2\left(\frac{\varphi}{\sqrt{6\alpha}}\right) + c_3 \tanh^3\left(\frac{\varphi}{\sqrt{6\alpha}}\right) \right\}^2$$

$$V(\varphi) = V_0 \left[\tanh(\varphi/\sqrt{6}) + A \sin\left(\tanh(\varphi/\sqrt{6})/\theta \right) \right]^2$$



ID + Kehagias + Tringas, JCAP 1901 (2019) 037

Superconformal α-Attractor inflationary models



$$S_{(2)} = \frac{1}{2}\int d^4x\sqrt{-g}\,a^3\frac{\dot{\varphi}^2}{H^2}\left[\dot{\mathscr{R}}^2 - \frac{(\partial_i\mathscr{R})^2}{a^2}\right]$$

The Mukhanov-Sasaki equation has to be solved numerically

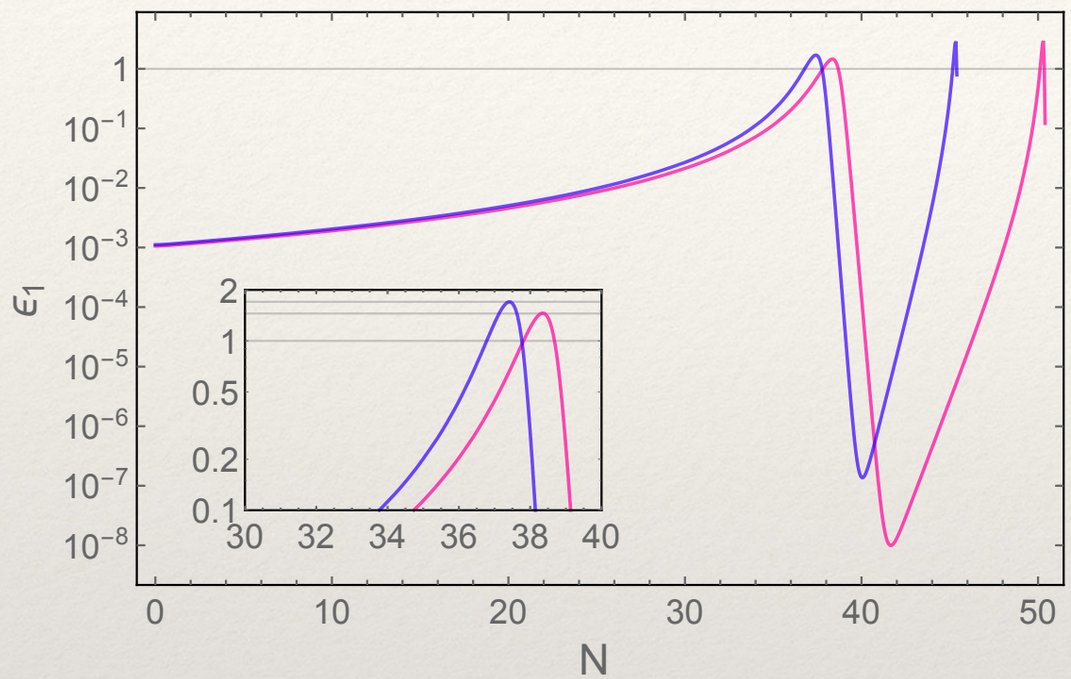
$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0$$

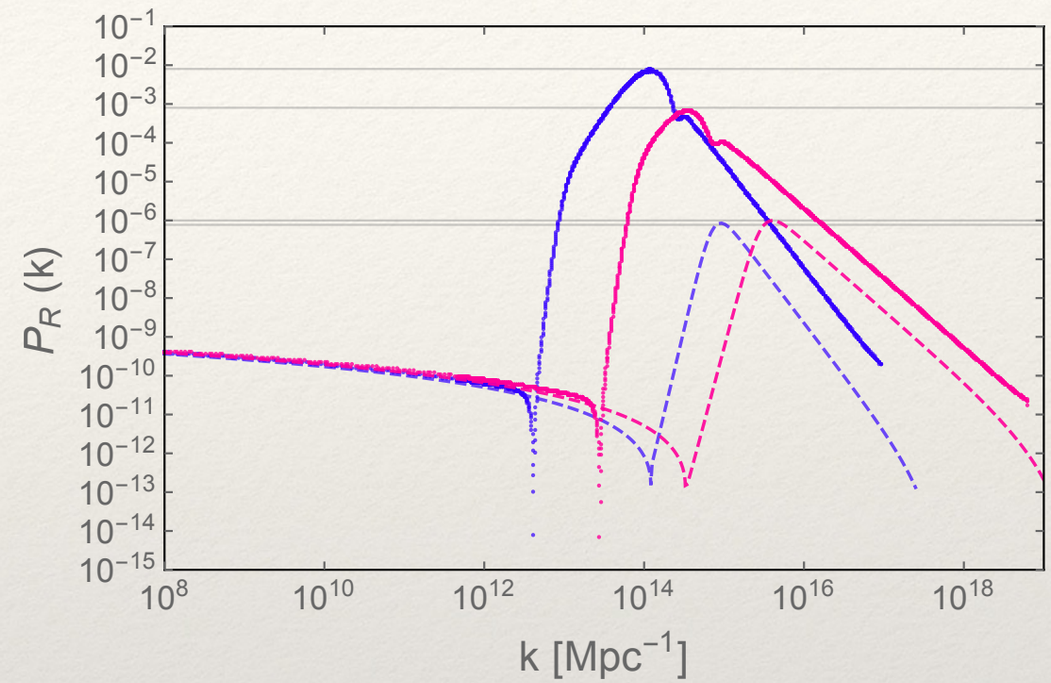
$$\frac{z''}{z} = (aH)^2\left[2 - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3\right]$$

$$\mathscr{P}_{\mathscr{R}} = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2}\Bigg|_{k\ll aH}$$

ID + Kehagias + Tringas, JCAP 1901 (2019) 037

Superconformal α-Attractor inflationary models





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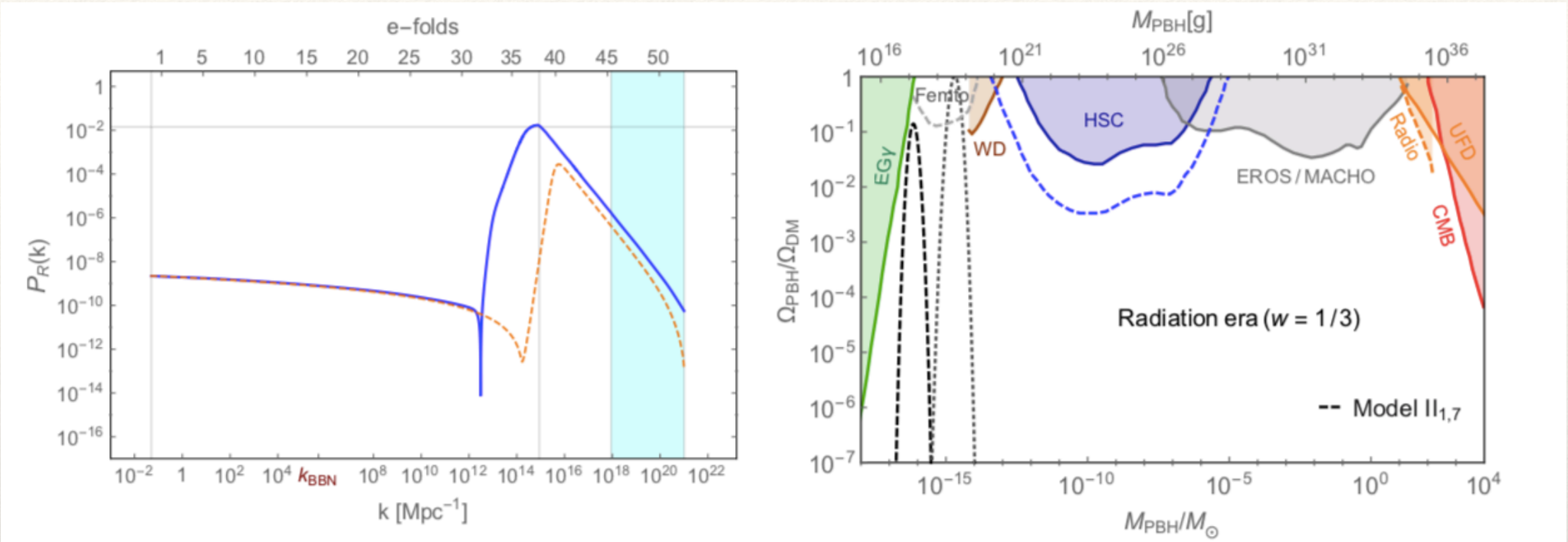
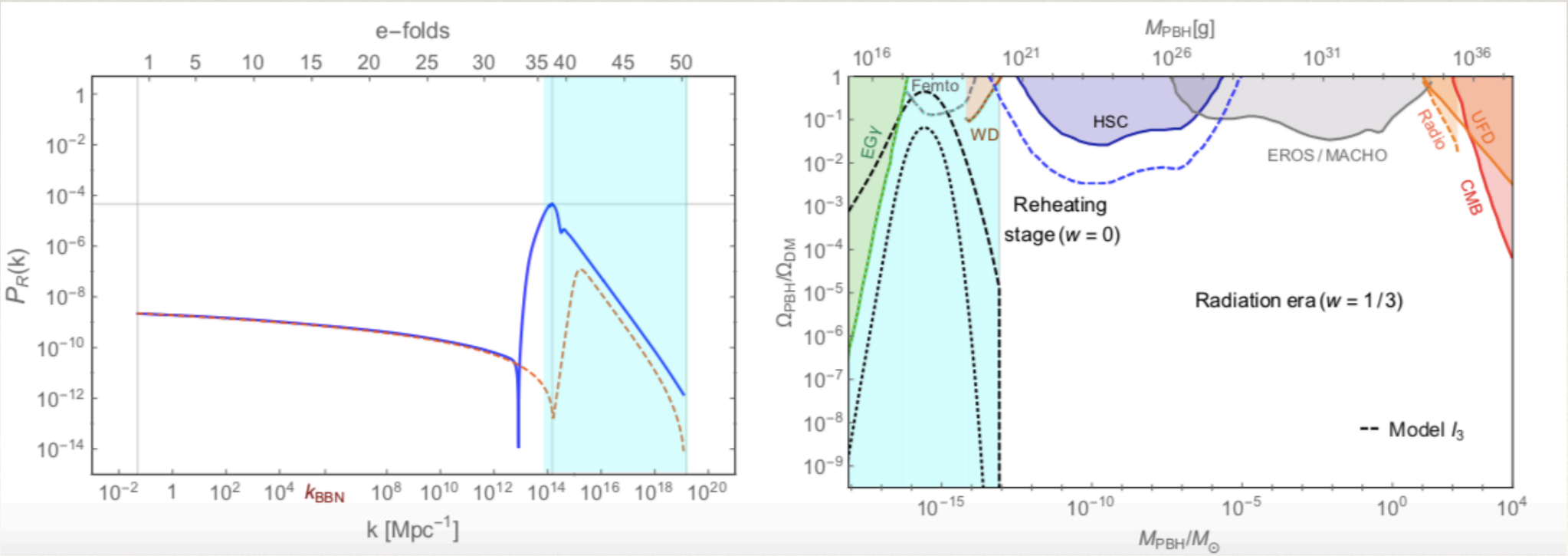
$$\mathscr{P}_{\mathscr{R}} = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2}\Bigg|_{k\ll aH}$$

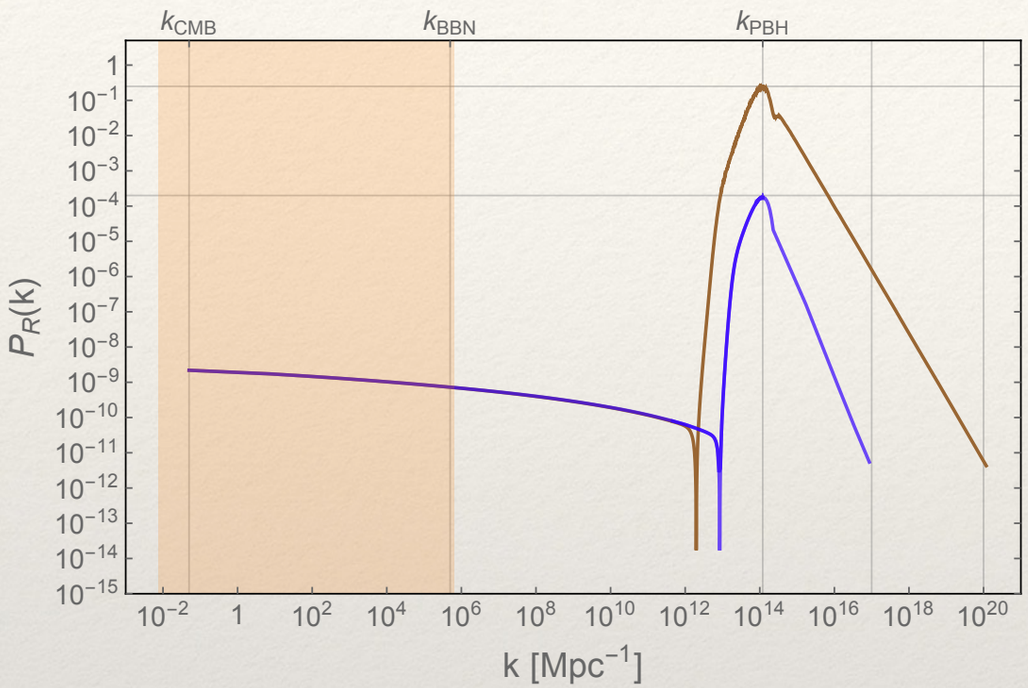
The predicted PBH abundance from α-Attractor inflationary models

ID + Kehagias + Tringas, JCAP 1901 (2019) 037





The power spectrum of the curvature perturbations

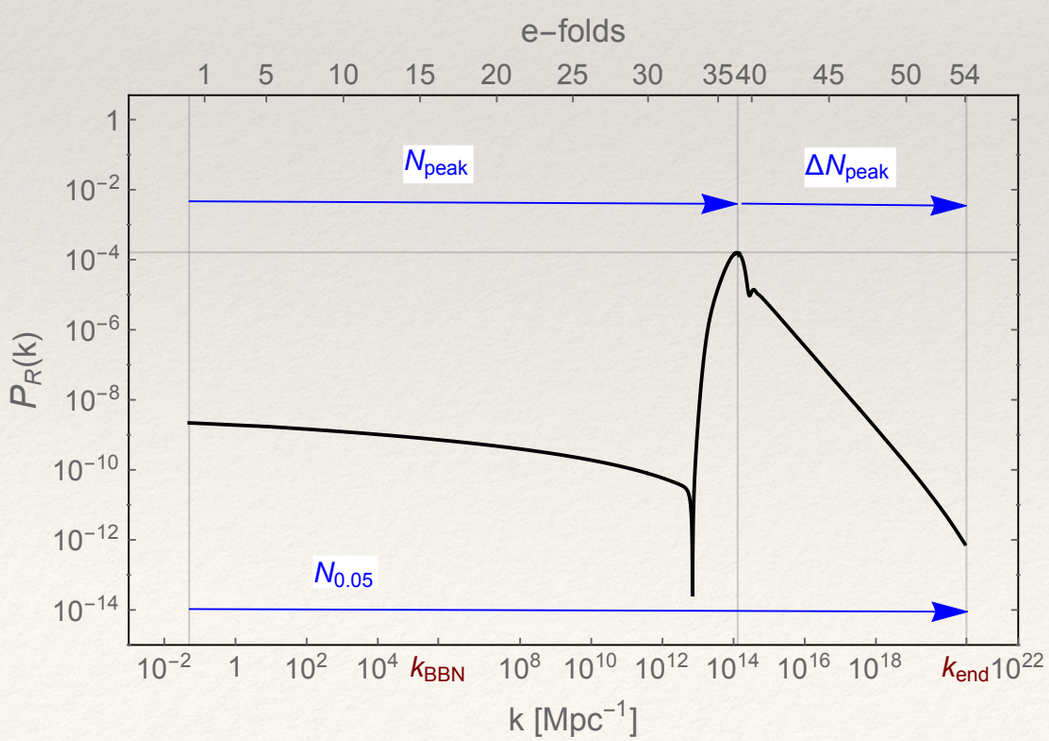


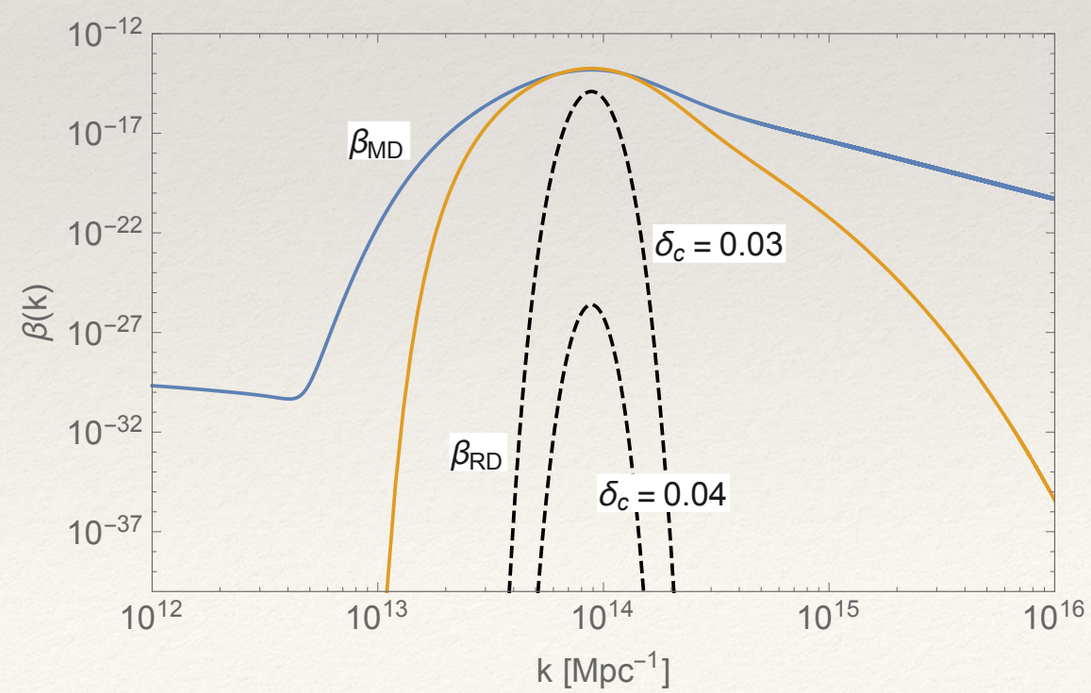
The power spectrum for the curvature perturbation and the reheating temperature.

The corresponding mass fraction that collapses into a black hole for matter and radiation domination background for the power spectrum.
The upper solid curve gives the β when spin effects are neglected and the lower when spin is important.

Extreme δc values for β, depicted with dashed curves, were chosen to make the distributions visible.

$$f_{PBH}^{(RD)} \propto \sqrt{\mathcal{P}_{\mathcal{R}}}\, e^{-\delta_c^2/\mathcal{P}_{\mathcal{R}}}, \qquad f_{PBH}^{(MD)} \propto \mathcal{P}_{\mathcal{R}}^{5/2}.$$





Q: What if the Power Spectrum peak is at the very end?

PBH remnants

J.D. Barrow, E.J. Copeland and A.R. Liddle (1992)

B.J. Carr, J.H. Gilbert and J.E. Lidsey (1994)

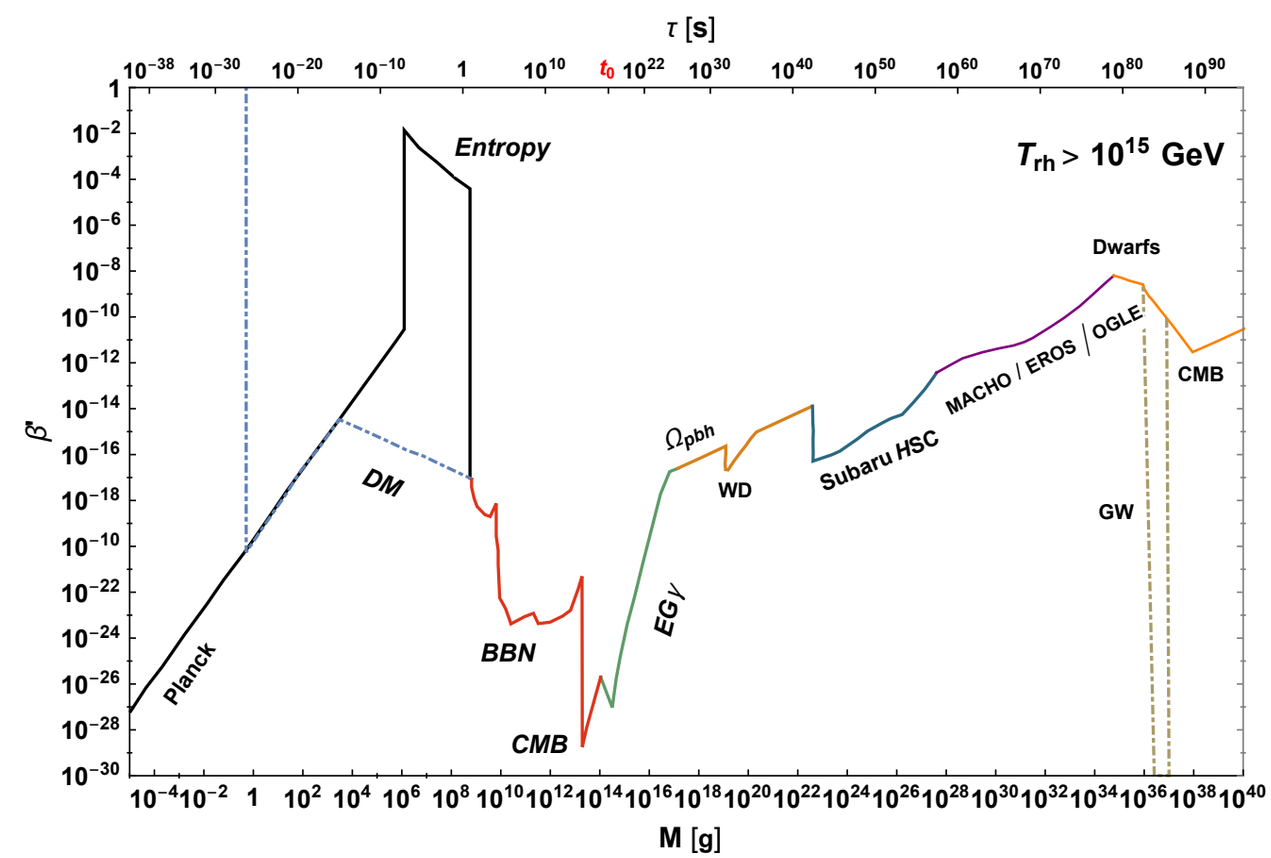
There are several theoretical reasons to anticipate that black holes do not evaporate completely but leave behind a stable mass state. The Hawking radiation is derived by treating matter fields quantum mechanically, while treating the space-time metric classically.

There are arguments based on :

- The information loss paradox
- Black holes with quantum hair
- A generalized uncertainty principle
- Extra spatial dimensions
- Higher order corrections to the action of general relativity

See Chen et. al arXiv:1412.8366 for a review

$$M_{rem} = \kappa\, m_{Pl}$$



PBH remnants cosmology

ID + Tringas (2019)

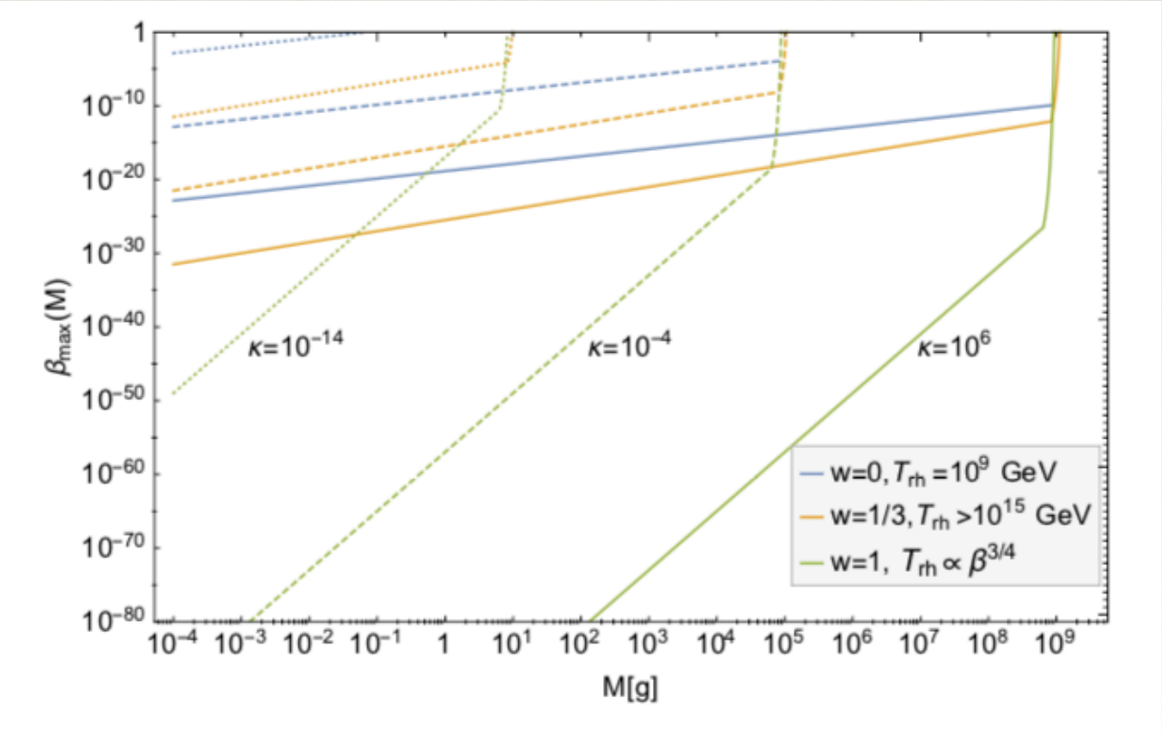
- PBH remnants can be a significant fraction of the dark matter in the universe only if $M \lesssim \kappa^{2/5} 10^6 g$

- Possible range for the PBH remnants masses $10^{-24}\,\mathrm{g} < M_{\mathrm{rem}} \ll 10^8\,\mathrm{g}$

$$f_{rem}(M) \simeq \kappa \left(\frac{\beta}{10^{-12}}\right) \left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}} \left(\frac{M}{10^5\mathrm{g}}\right)^{-3/2}$$

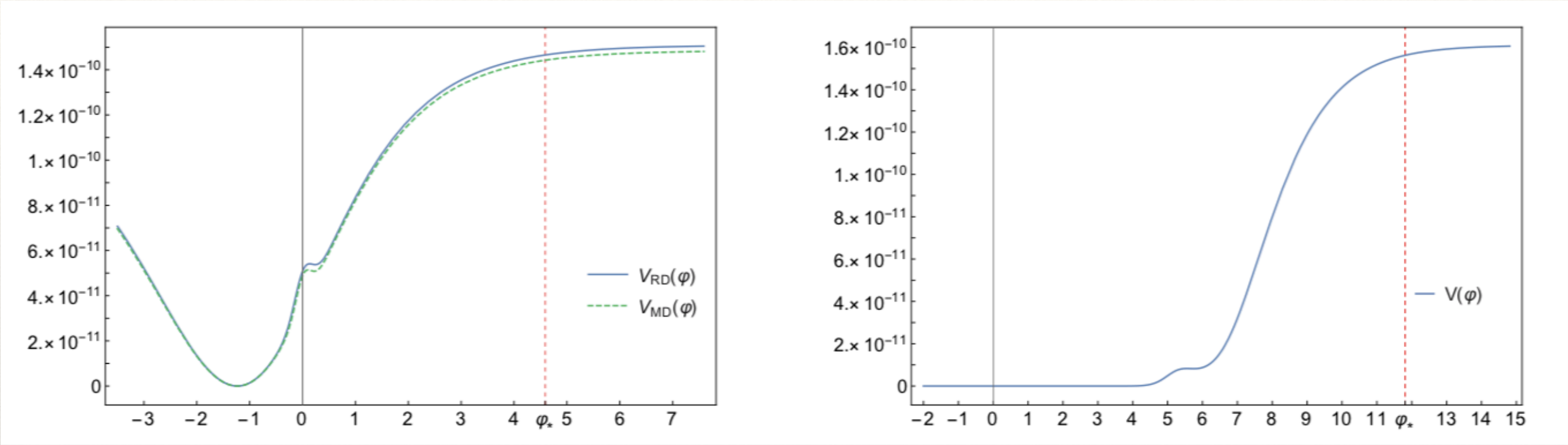
$$f_{rem}(M, M_{rh}) \simeq 3\,\kappa\gamma \left(\frac{\beta}{10^{-9}}\right) \left(\frac{M_{rh}}{10^{10}\mathrm{g}}\right)^{-1/2} \left(\frac{M}{10^5\mathrm{g}}\right)^{-1}$$

$$f_{rem}(M) \simeq 4\,\kappa\sqrt{\gamma} \left(\frac{\beta}{10^{-32}}\right)^{1/4} \left(\frac{M}{10^5\mathrm{g}}\right)^{-2}$$



Potentials for PBH remnants dark matter

ID + Tringas (2019)

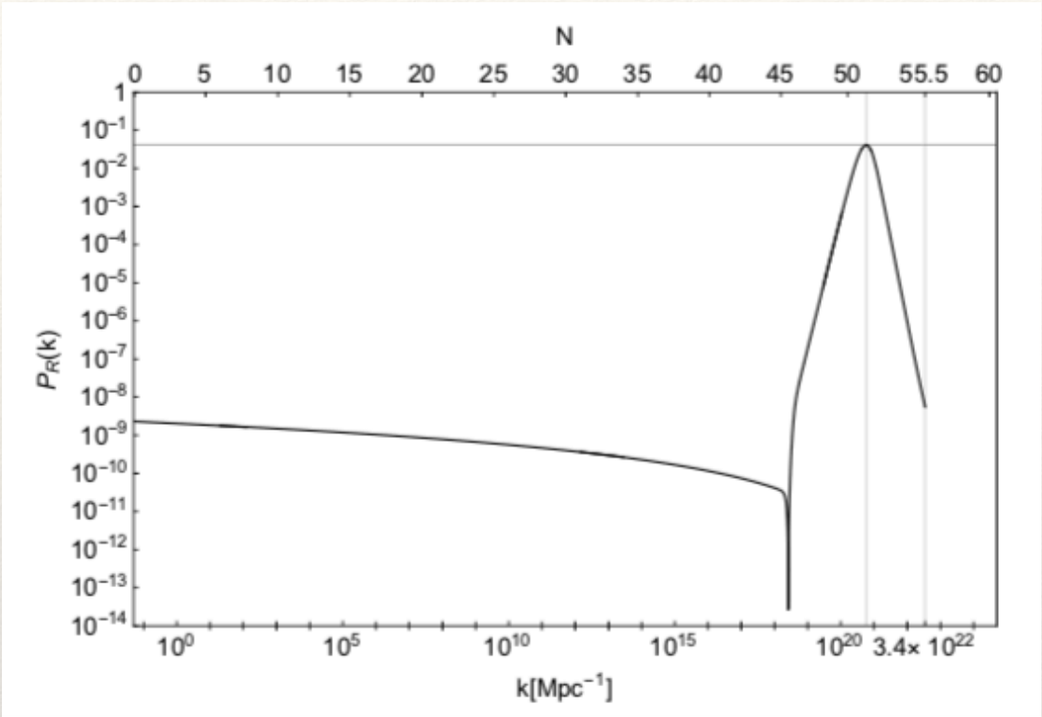


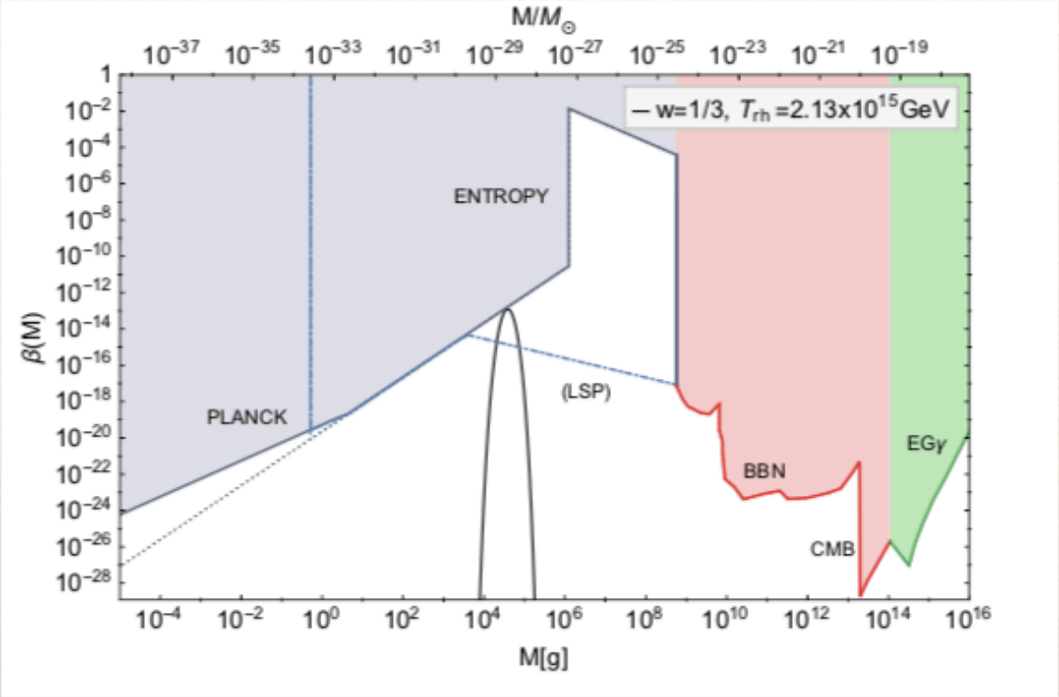
Oscillatory: $$V(\varphi) = f_0^2 \left(c_0 + c_1 e^{\lambda_1 \tanh \varphi/\sqrt{6}} + c_2 e^{\lambda_2 (\tanh \varphi/\sqrt{6})^2} \right)^2$$

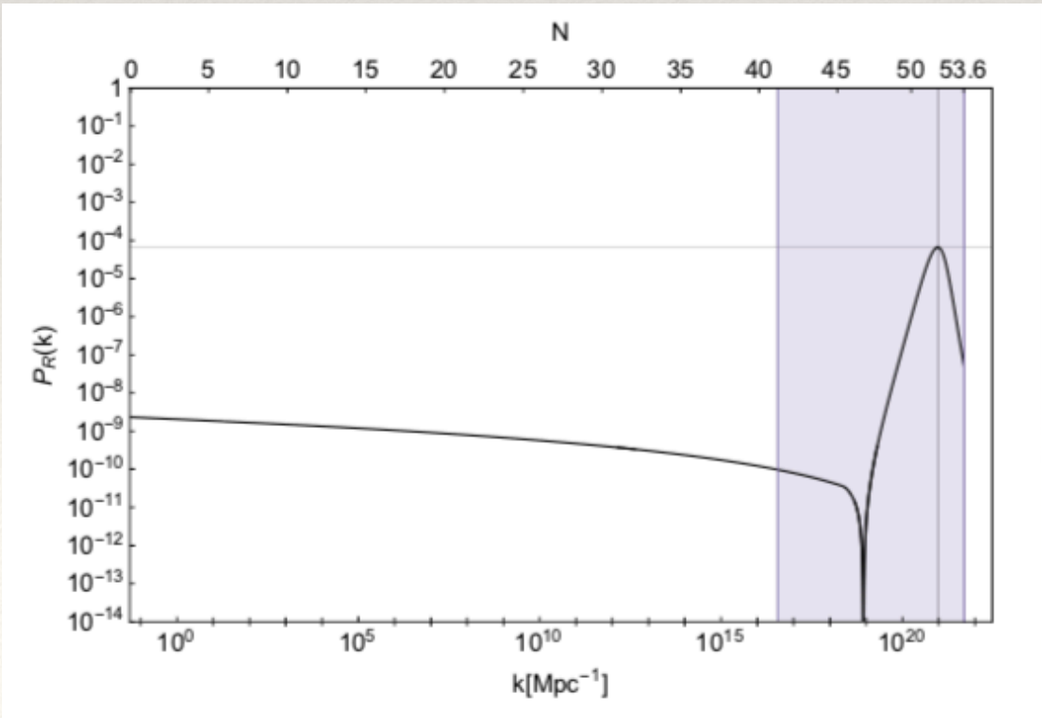
Runaway: $$V(\varphi) = f_0^2 \left[c_0 + c_1 e^{\lambda_1 \tanh \varphi/\sqrt{6}} + c_2 e^{\lambda_2 \left(\tanh(\varphi/\sqrt{6}) - \tanh(\varphi_P/\sqrt{6}) \right)} \right]^2$$

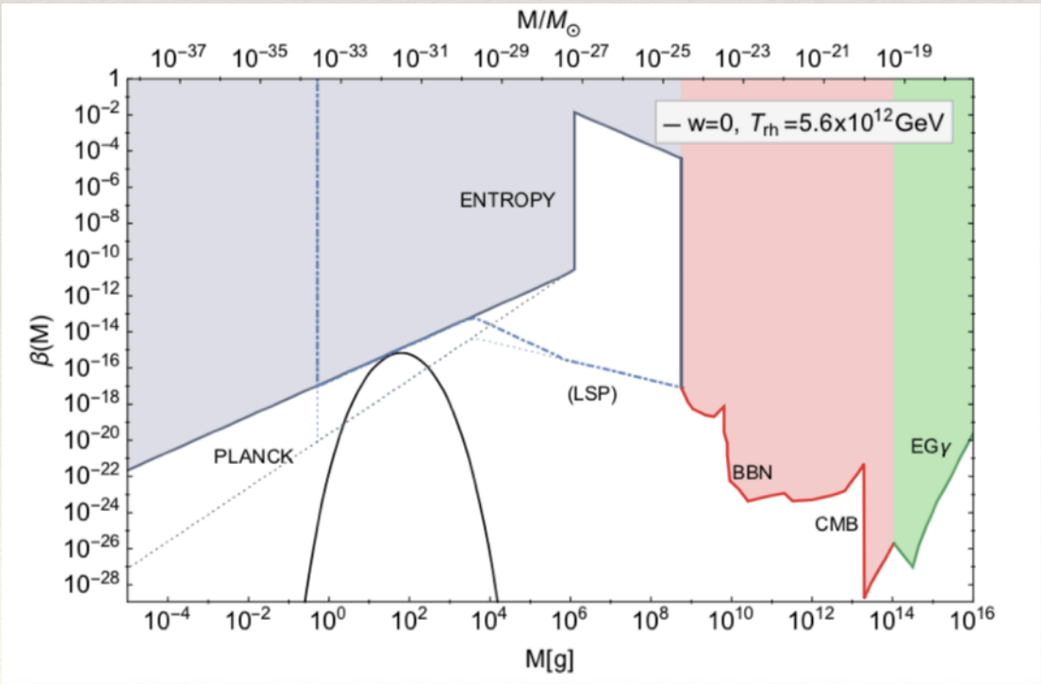
PS for PBH remnants dark matter

ID + Tringas (2019)



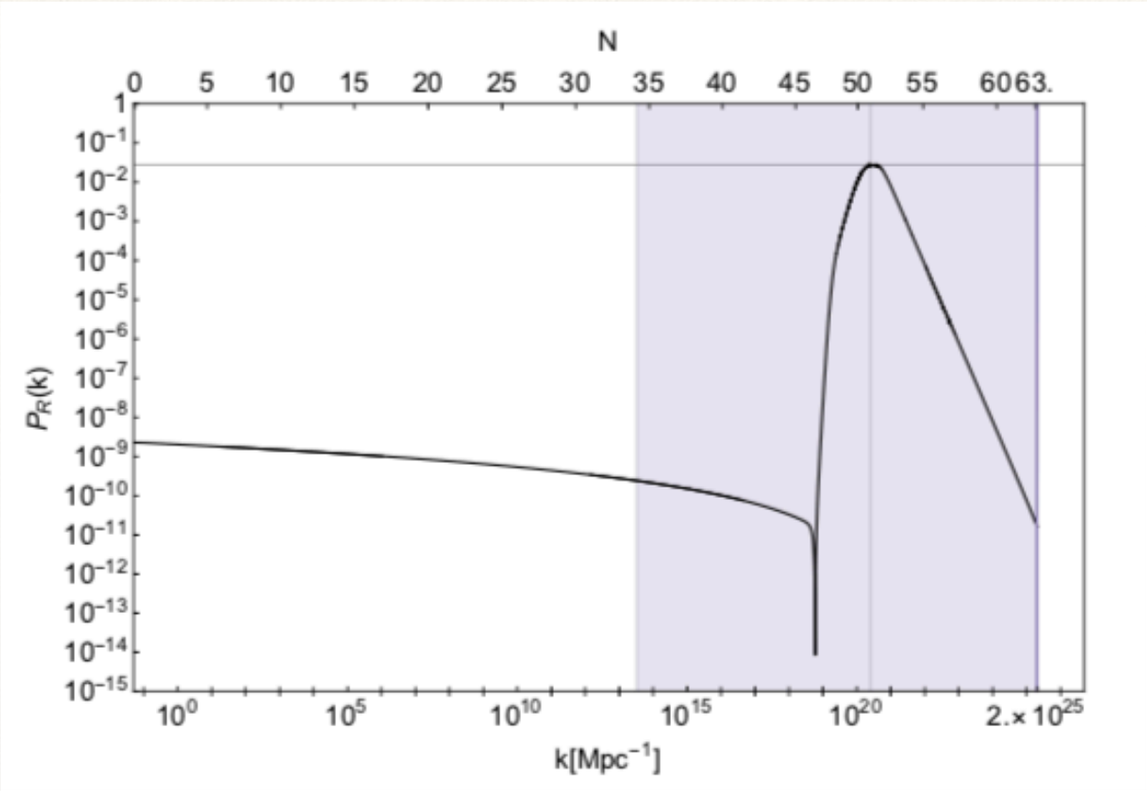


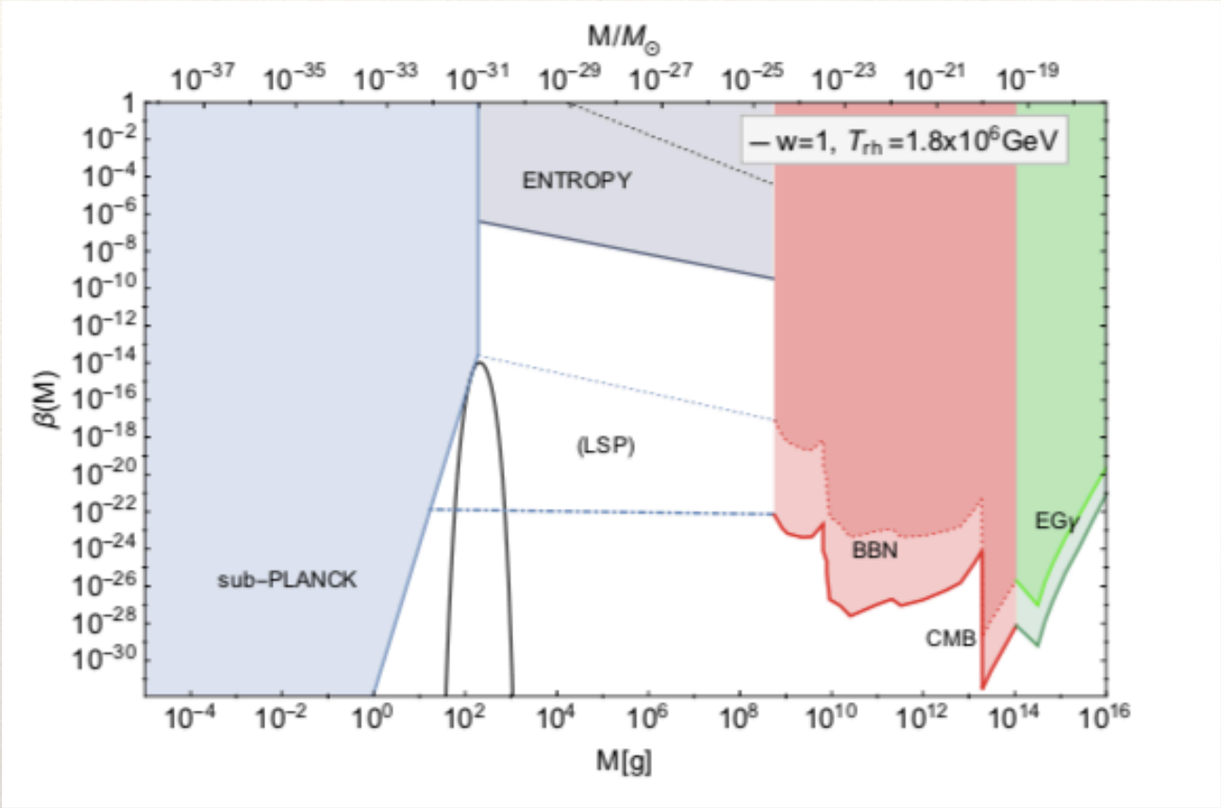




PBH remnants, Runaway model

ID + Tringas (2019)



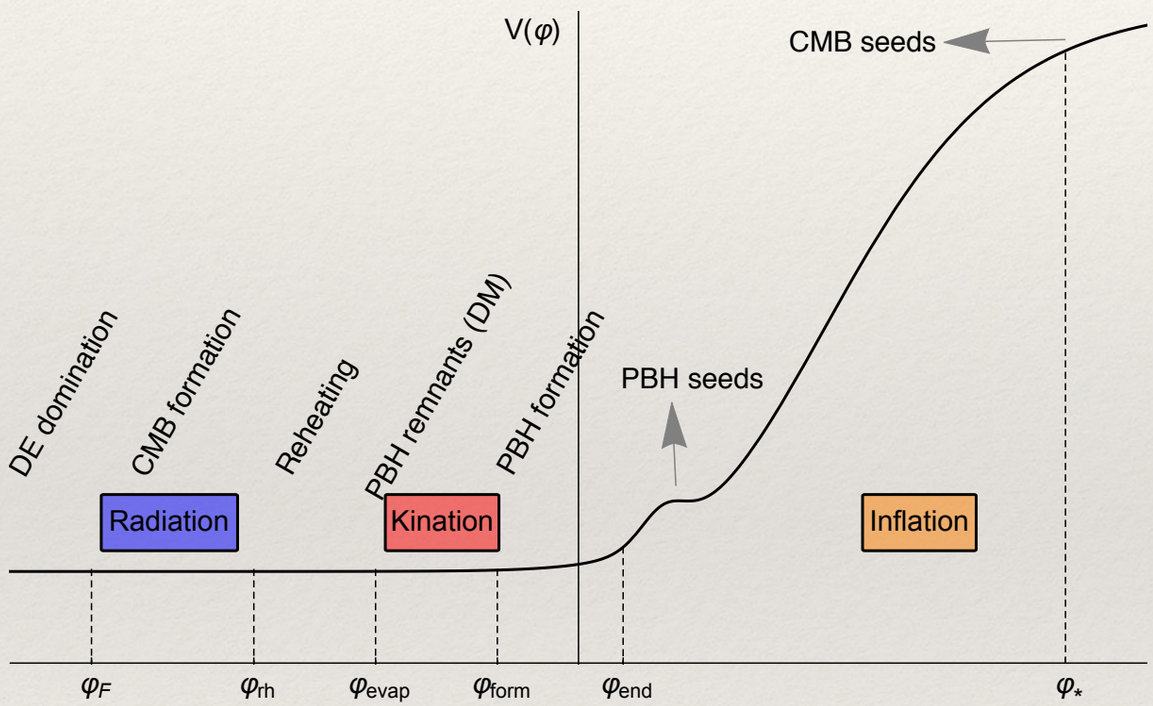


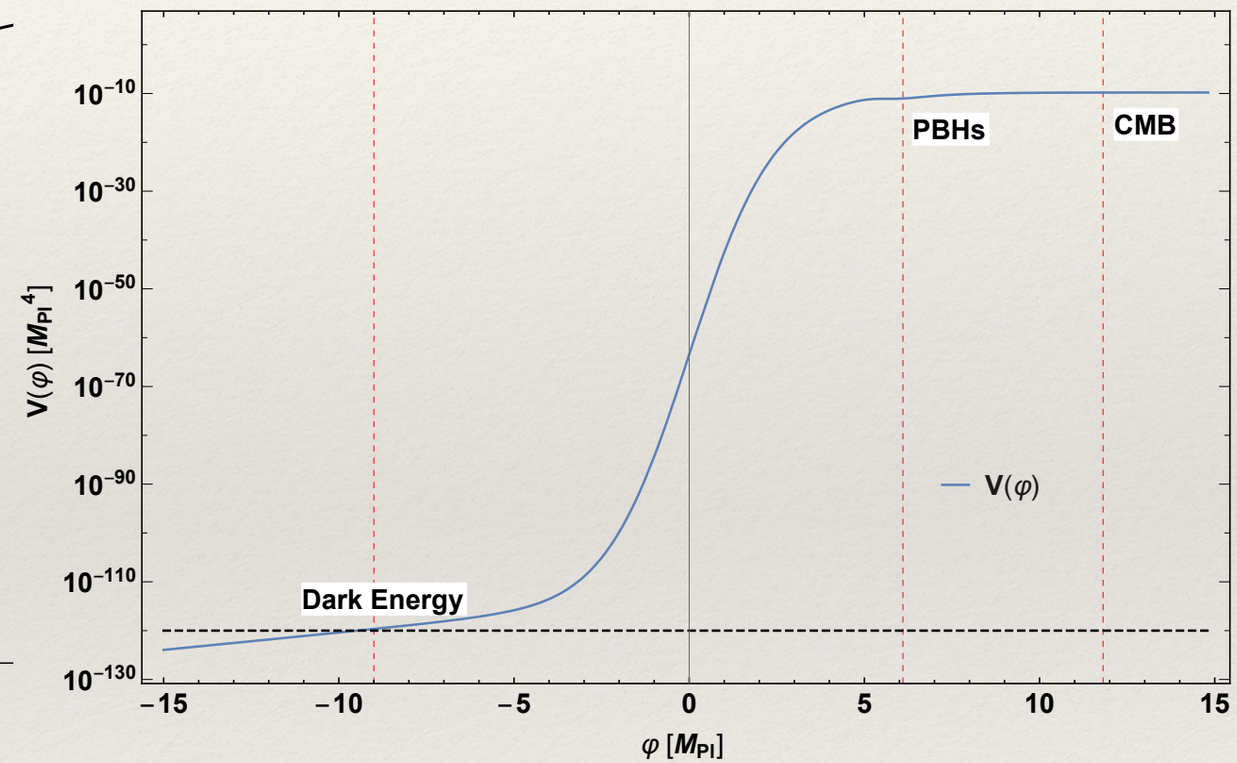
GW constraints:

$$T_{\rm rh} \equiv 6.3\,{\rm MeV}\left(\frac{\beta}{10^{-28}}\right)^{3/4}\gamma^{3/2}g_*^{-1/2}$$

$$\frac{\kappa}{10^{-10}} \lesssim 8.5\,\gamma^{-5/2}\left(\frac{H_{end}}{10^{-6}\,M_{Pl}}\right)^{-10/3}\left(\frac{g_*}{106.75}\right)^{-5/6}.$$

Runaway inflationary models + PBHs





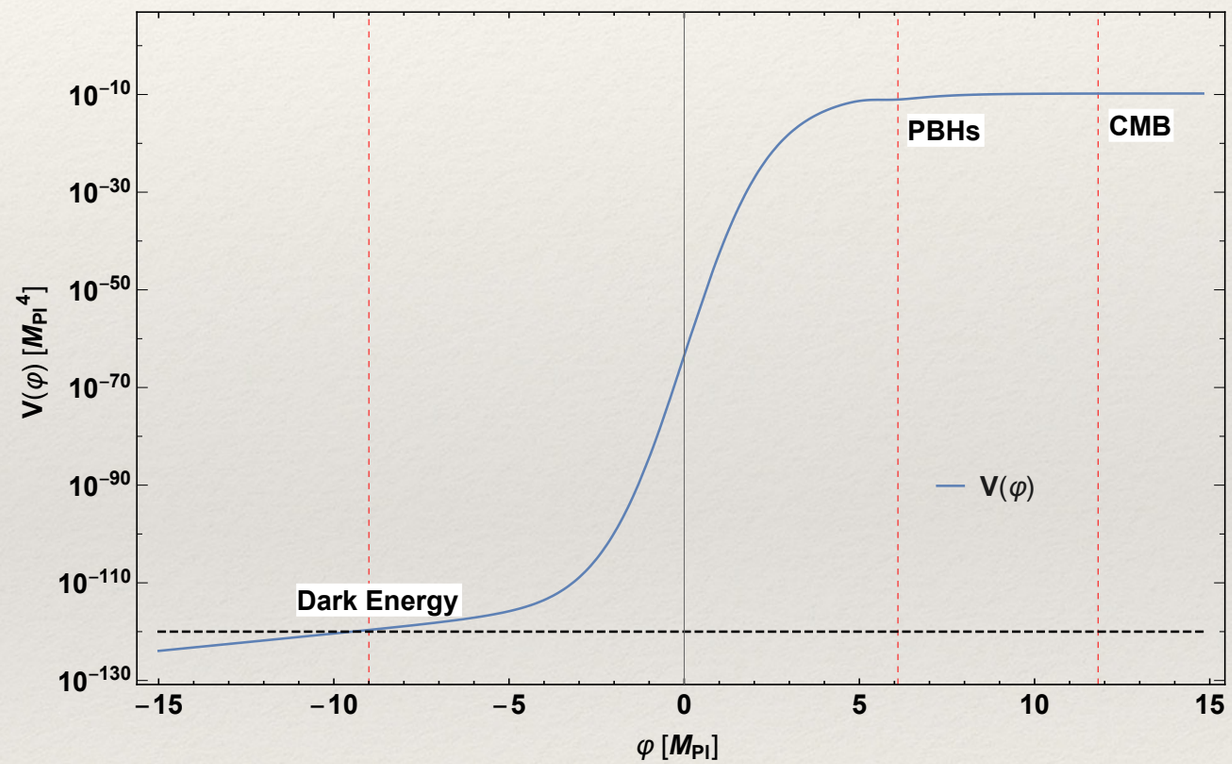
A runaway inflationary model introduced that produces PBHs that explain the dark matter with their evaporation remnants, reheats the Universe and implements a wCDM late time cosmology.

Runaway inflationary models + PBHs

$$\varphi_F \simeq \varphi_{\rm end} - \sqrt{\frac{2}{3}}\left(\sqrt{2} - \frac{3}{2}\ln\Omega_{\rm rad}(t_{\rm evap}) + \ln\left(\frac{t_{\rm evap}}{t_{\rm end}}\right)\right) M_{Pl}$$

$$\varphi_F \simeq \varphi_{\rm end} - \sqrt{\frac{2}{3}}\left[19 + 13\ln(M/10^5{\rm g}) + 4\ln(1/\kappa)\right] M_{Pl}$$

$$\frac{\rho_{\rm inf}}{\rho_0} \simeq \frac{V(\varphi \gg 1)}{V(\varphi_F)} \sim \frac{e^{2\lambda_1}}{e^{-2\lambda_1}} \sim 10^{108}$$



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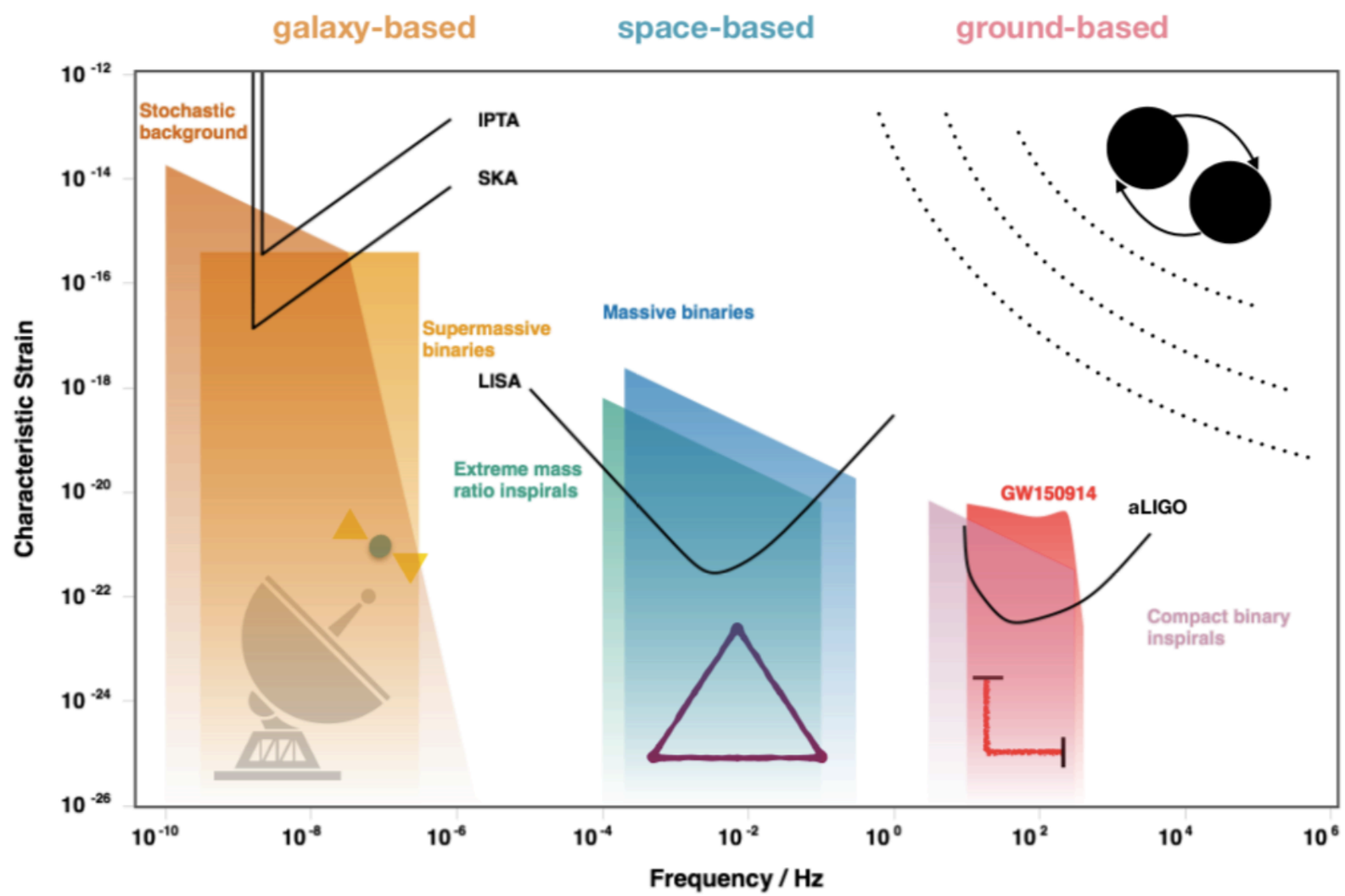
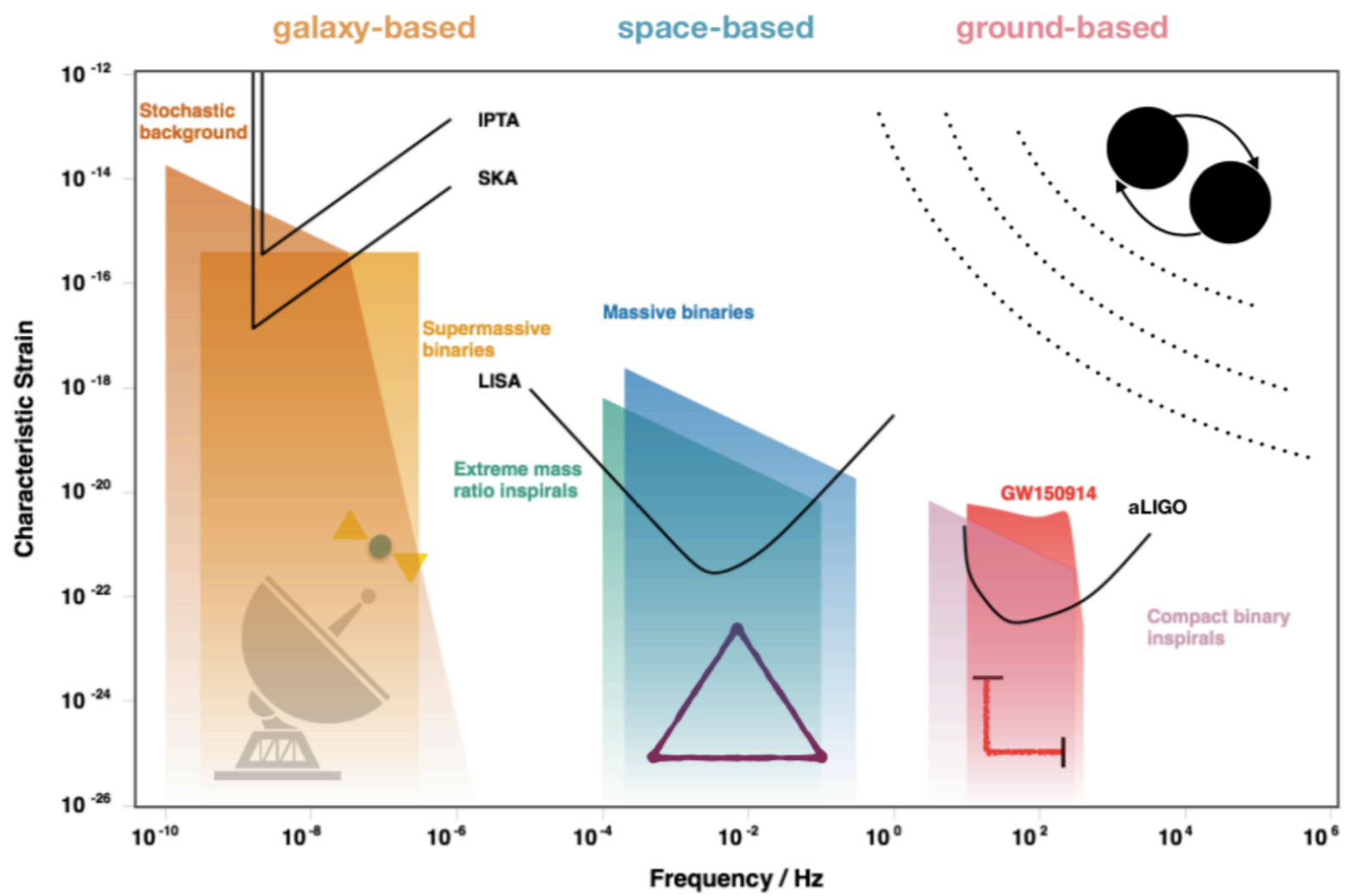
Stochastic Gravitational Waves



Current and future Gravitational Detection Experiments can significantly constrain the PBH dark matter scenario

Stochastic Gravitational Waves



Current and future Gravitational Detection Experiments can significantly constrain the PBH dark matter scenario

Thank you!