QCD-axion and more: Theory

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1. Introduction
2. “Invisible” axion
3. QCD phase transition
4. Hierarchy together with “invisible axion”

1. Introduction

With the gauge symmetry as the only symmetry at low energy,
chiral fields are the only light fields.

My original contributions were to be found in the future. Some of my examples are

arXiv:1703.10925: PRD96 (2017) 055033

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$$Q=\frac{1}{2}:\ \ell_i=\begin{pmatrix}E_i\\N_i\end{pmatrix}_{\frac{+1}{2}},\qquad \begin{matrix}E^c_{i,-1}\\N^c_{i,0}\end{matrix}\qquad (i=1,2,3),$$

$$Q=-\frac{3}{2}:\ \mathcal{L}=\begin{pmatrix}\mathcal{E}\\\mathcal{F}\end{pmatrix}_{\frac{-3}{2}},\qquad \begin{matrix}\mathcal{E}^c_{,+1}\\\mathcal{F}^c_{,+2}\end{matrix},$$

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So, there is a good reason that these particles will be discovered at low energy. First, by kinetic mixing!!

Weak-Interaction Singlet and Strong *CP* Invariance

Jihn E. Kim
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 16 February 1979)

Strong *CP* invariance is *automatically* preserved by a spontaneously broken chiral $U(1)_4$ symmetry. A weak-interaction singlet heavy quark Q, a new scalar meson σ^0, and *a very light axion* are predicted. Phenomenological implications are also included.

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The new scalar σ^0.—By the spontaneous symmetry breaking of $U(1)_A$, σ will be split into a scalar boson σ^0 of mass $(2\mu_0)^{1/2}$ and an axion a. This σ^0 is *not* a Higgs meson, because it does not break the gauge symmetry, but the phenomenology of it is similar to the Higgs because of its coupling to quark as m_Q/v'. If this scalar mass is $\gtrsim 2m_Q$, we will see spectacular final state of stable particles such as $(Q\bar{u})$ and $(\bar{Q}u)$. If its mass is $\lesssim 2m_Q$, the effective interaction through loops $(c/v')F_{\mu\nu}{}^a F^{a\mu\nu}\sigma^0$, with numerical constant c, will describe the decay $\sigma^0 \to$ ordinary hadrons. The order of magnitude of its lifetime is $\tau(\sigma^0) \approx \tau(\pi^0)(v'/f_\pi)^2(m_{\pi^0}/m_{\sigma^0})^3 \approx 2\times10^{-10}$ sec for $v'\approx 10^5$ GeV and $m_{\sigma^0}\approx 10$ GeV. This kind of particle can be identified as a jet in pp high-energy collisions, 1st BSM

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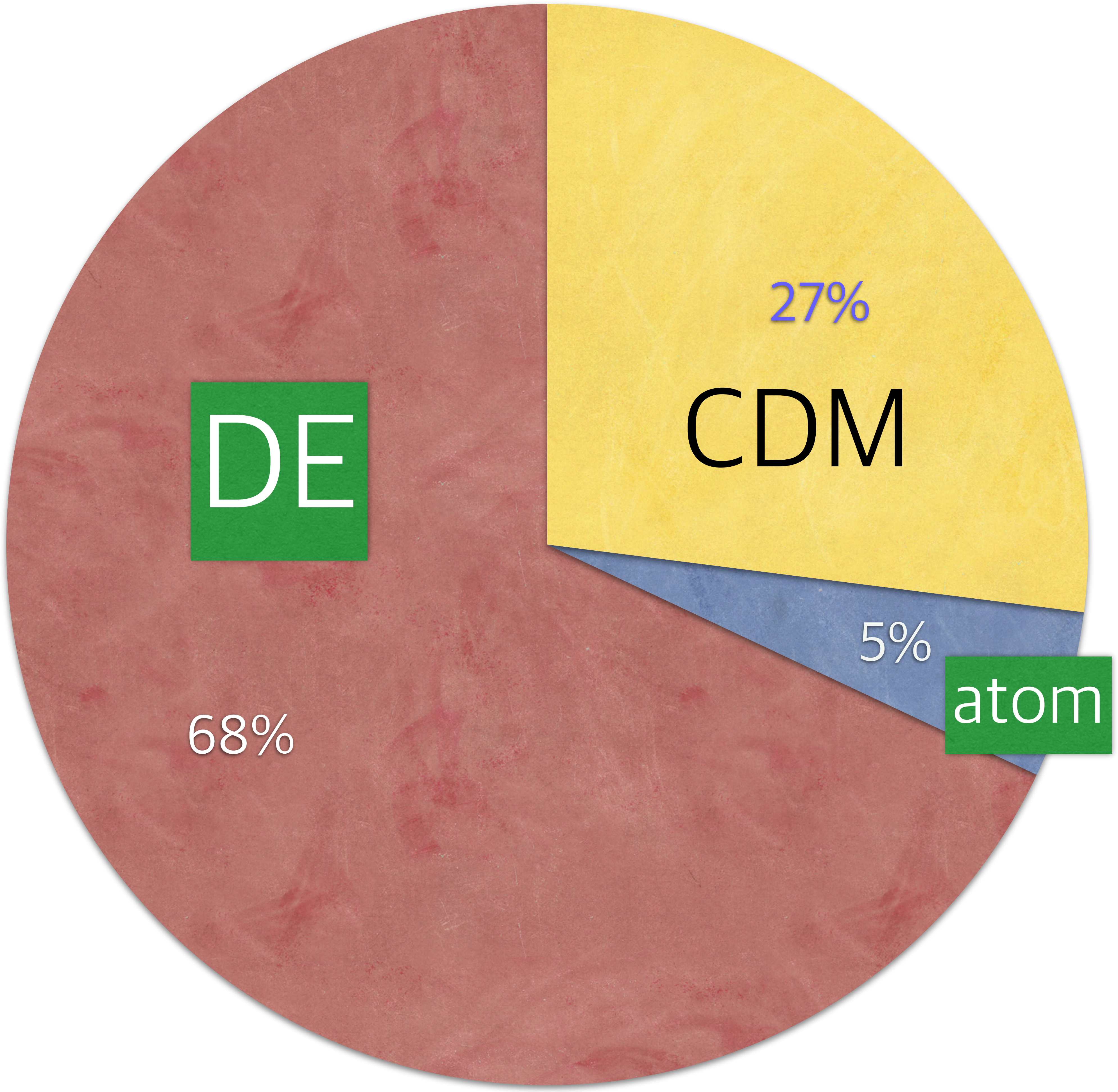
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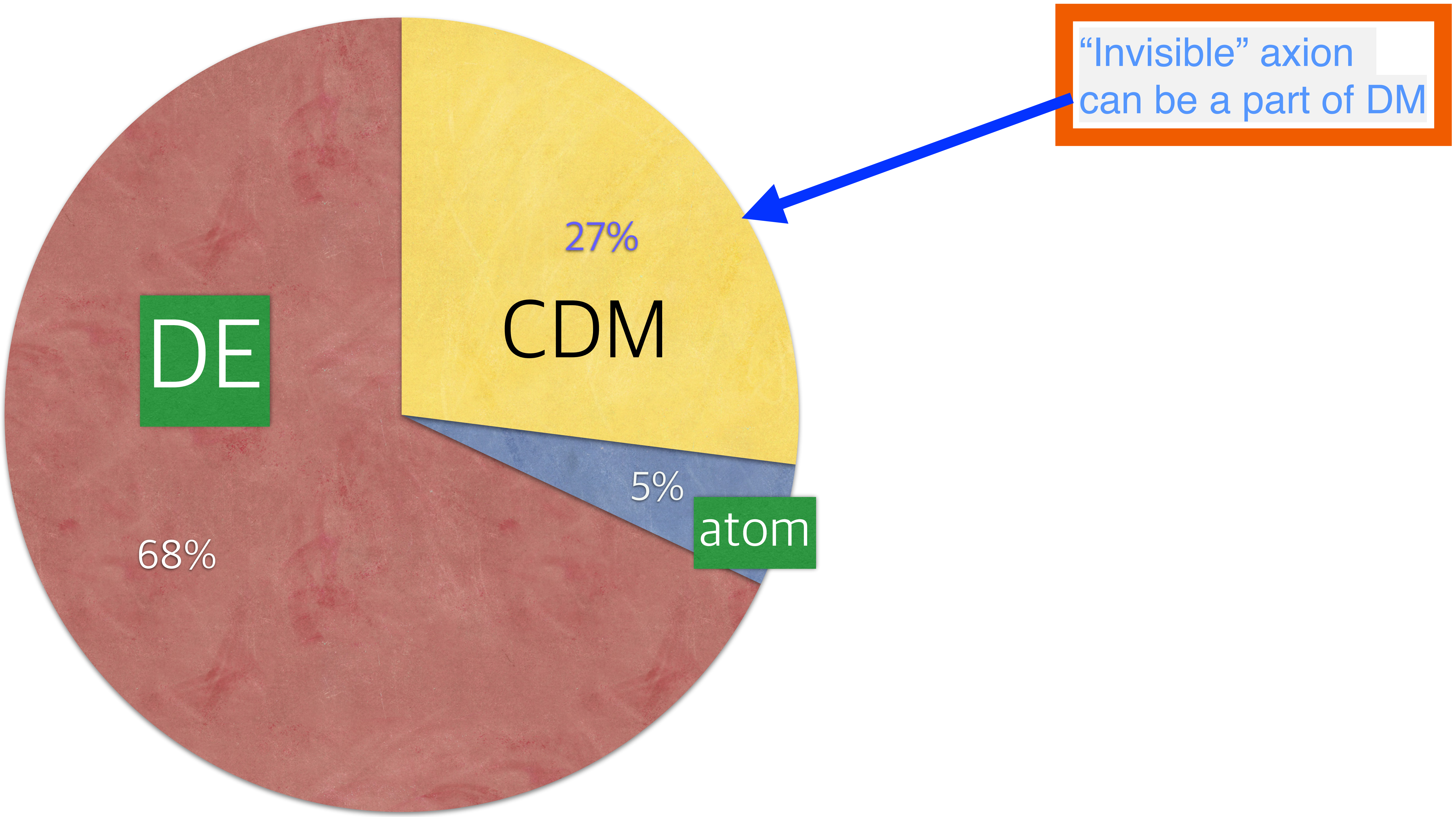
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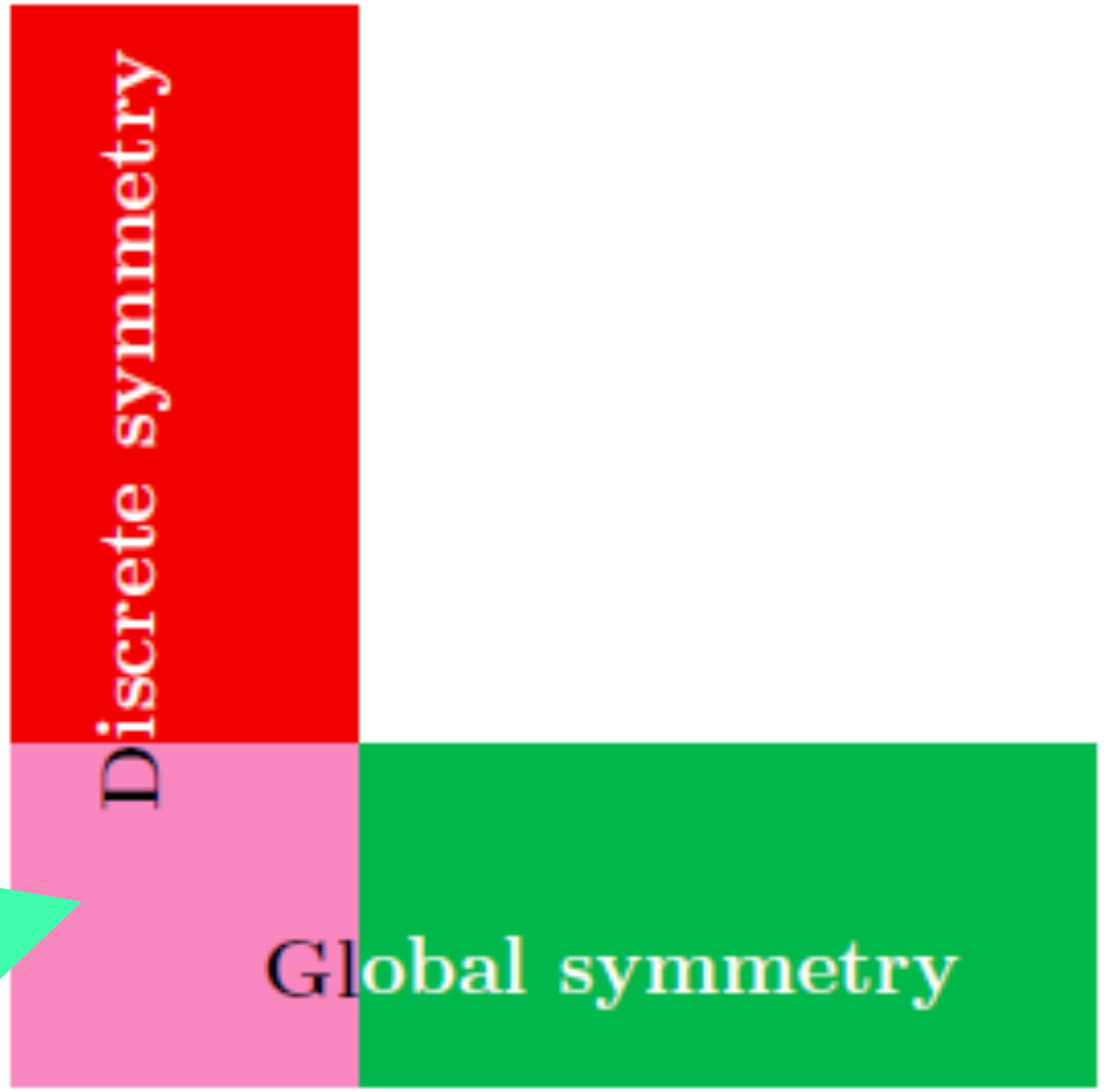
“Invisible” axion
can be a part of DM


Discrete symmetry
Gl

Keep only the leading terms



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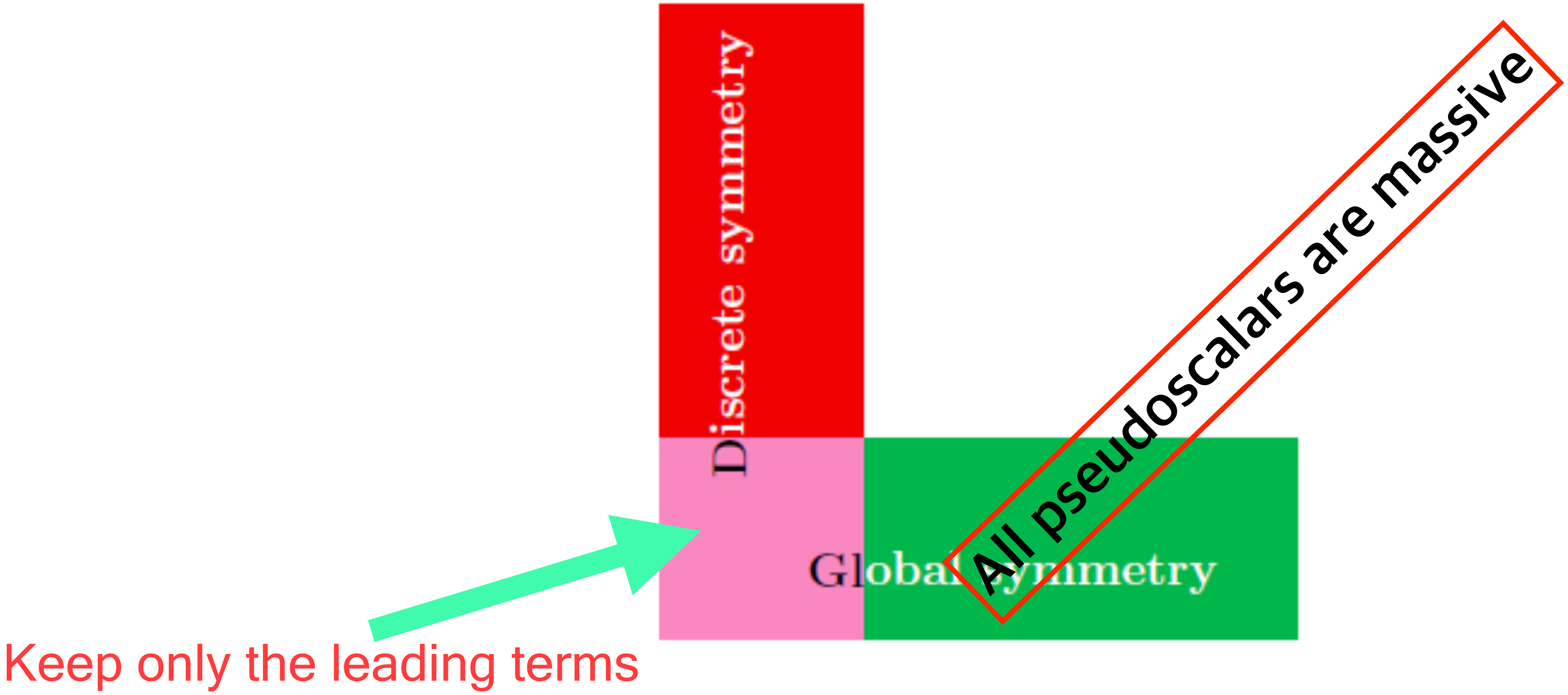
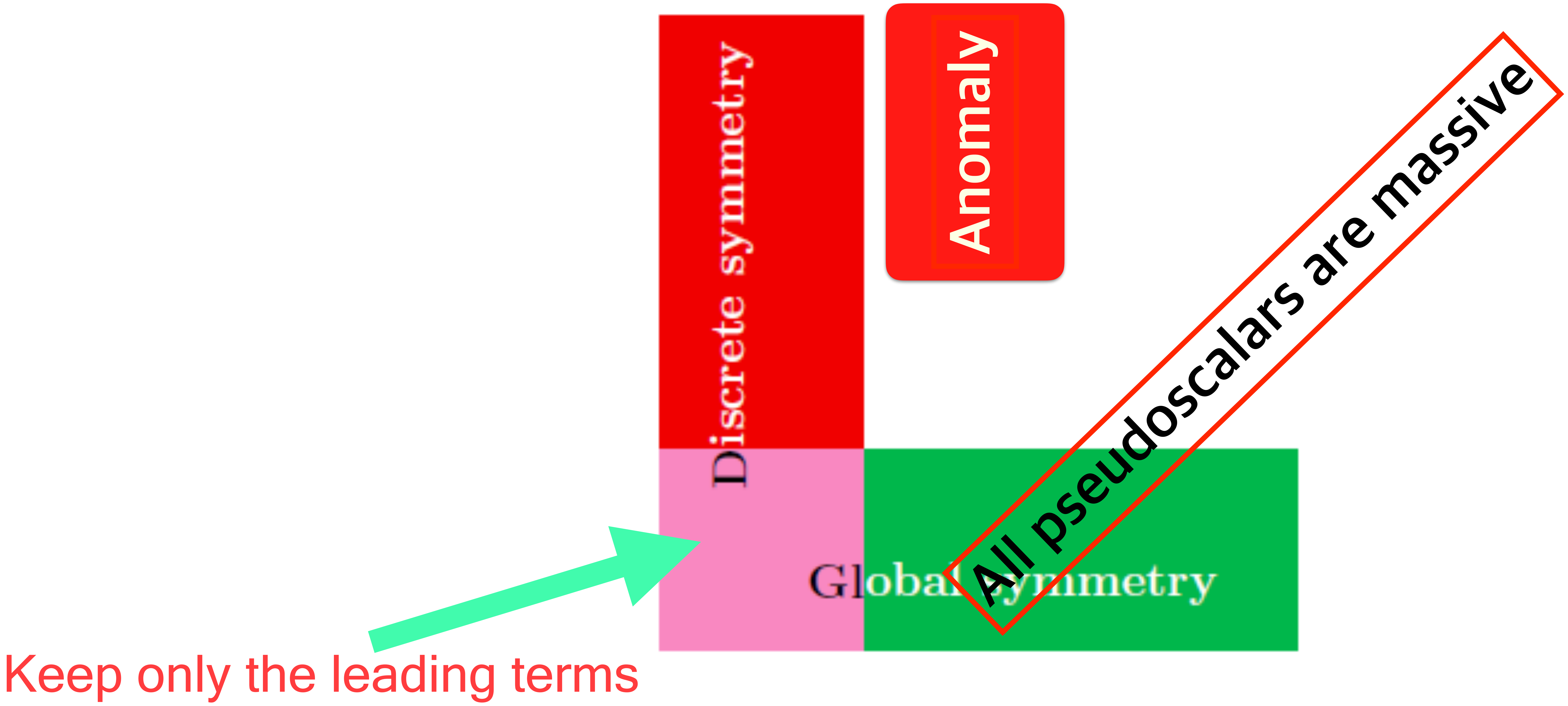

Discrete symmetry
Global symmetry
All pseudoscalars are massive
Keep only the leading terms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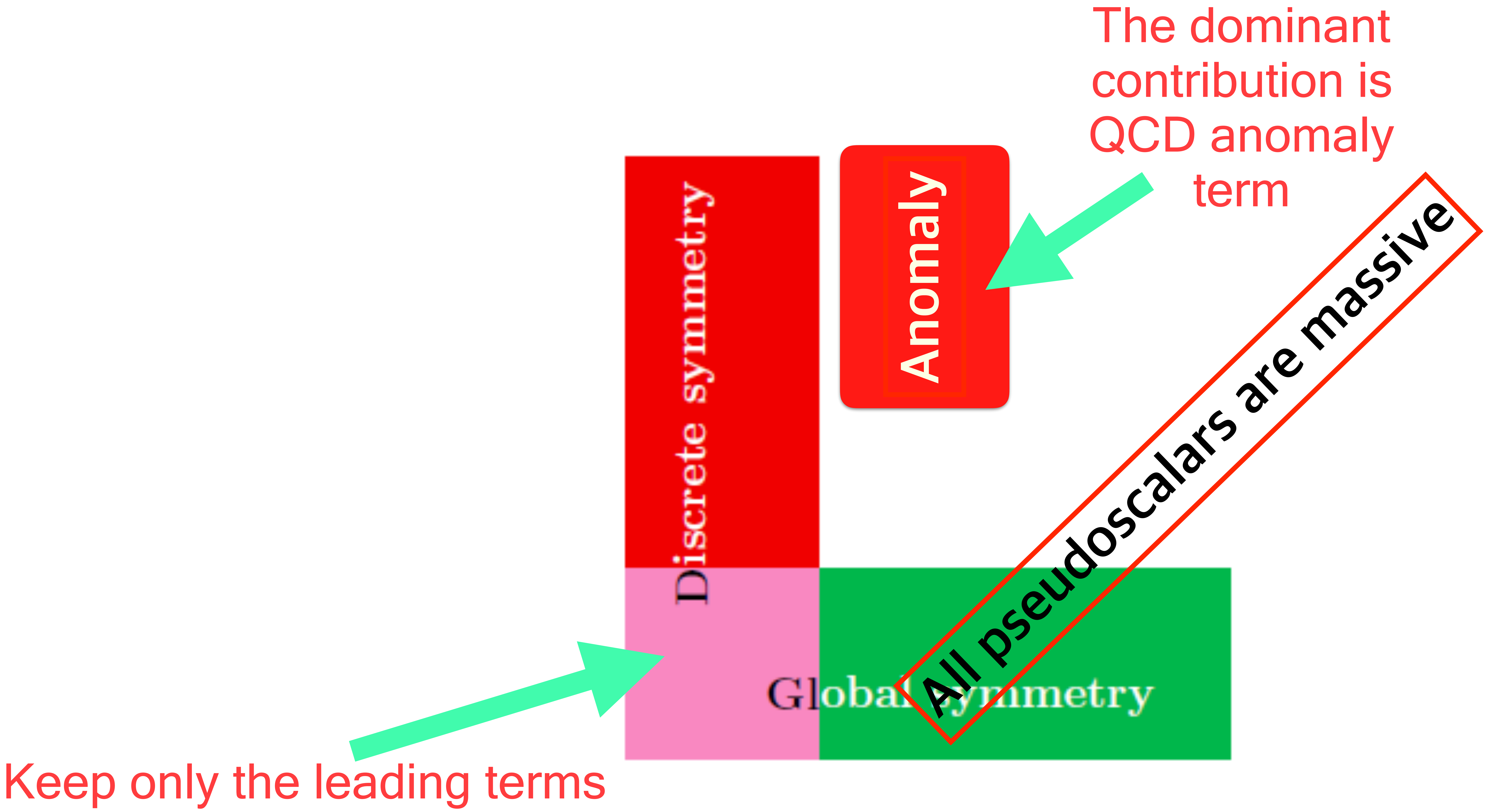
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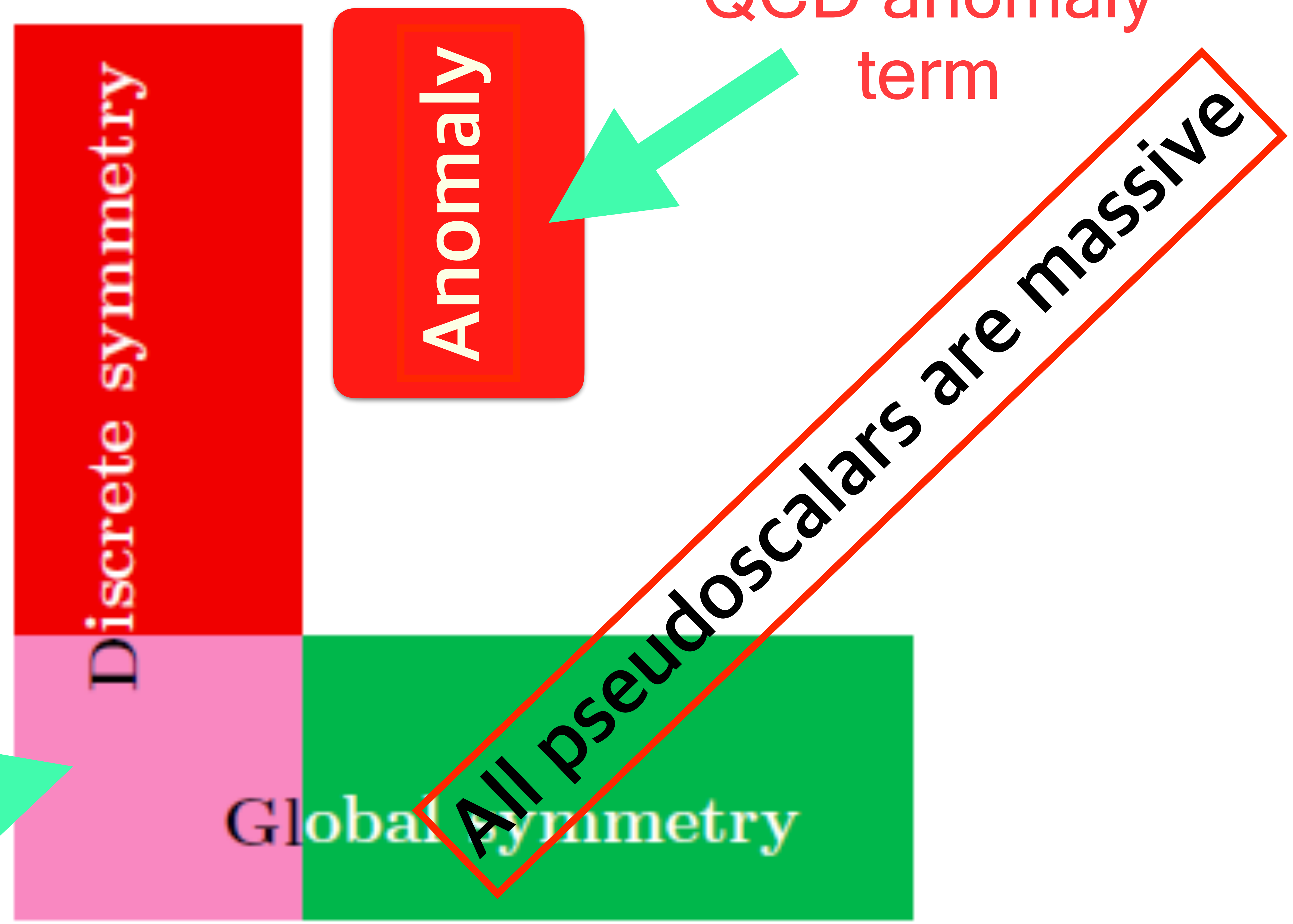

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$U(1)_{anom}$ as the symmetry for the “invisible” axion (family symmetry?)

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JEK, Kyae, Nam, 1703.05345 [Eur. Phys. J. C77 (2017) 847] ,
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Unbroken $X = Q_{global} - Q_{gauge}$

$$\phi \to e^{i\alpha(x)Q_{\text{gauge}}} e^{i\beta Q_{\text{global}}} \phi$$

the α direction becomes the longitudinal mode of heavy gauge boson. The above transformation can be rewritten as

$$\phi \to e^{i(\alpha(x)+\beta)Q_{\text{gauge}}} e^{i\beta(Q_{\text{global}}-Q_{\text{gauge}})} \phi$$

Redefining the local direction as $\alpha'(x) = \alpha(x) + \beta$, we obtain the transformation

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So, the gauge boson becomes heavy and there remains the x-independent transformation parameter beta. The corresponding charge is a combination:

$X=Q_{global}-Q_{gauge}$

The MI axion

$$H_{\mu\nu\rho} = M_{MI}\epsilon_{\mu\nu\rho\sigma}\,\partial^{\sigma} a_{MI}.$$

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It is the 't Hooft mechanism working in the string theory. So, the continuous direction a_{MI} –> a_{MI} + (constant) survives as a global symmetry at low energy:

"Invisible" axion!! appearing at $10^{10\text{-}11}$ GeV scale when the global symmetry is broken.

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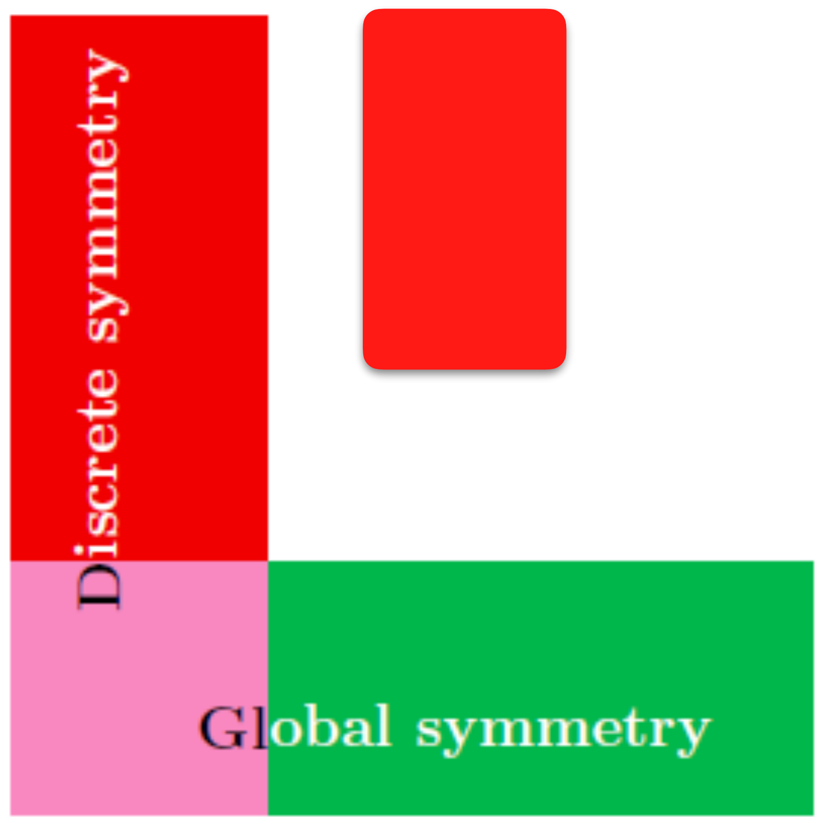

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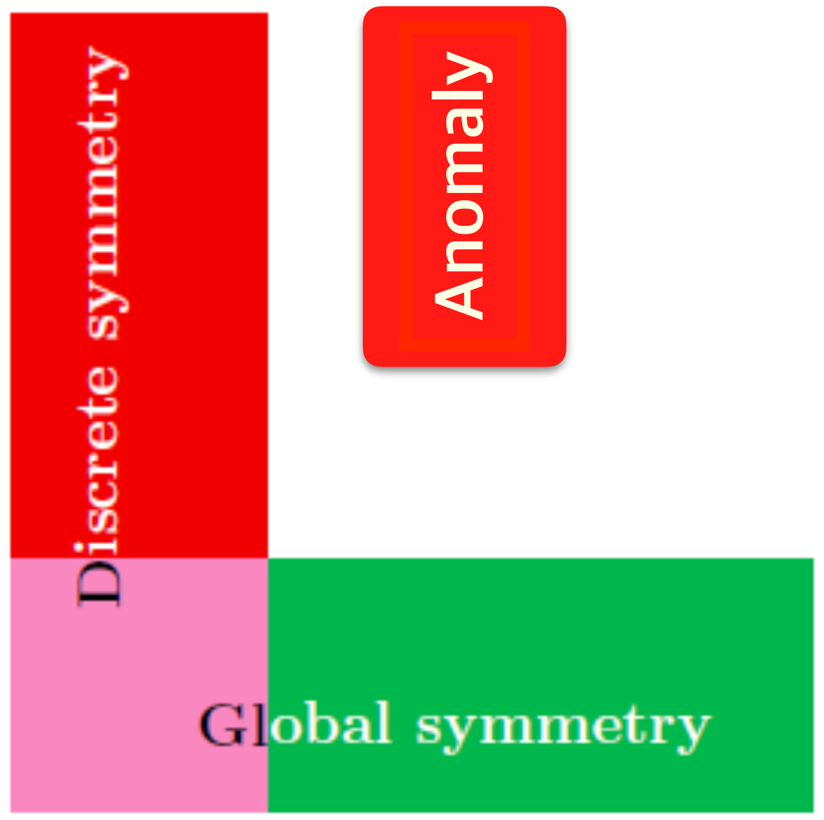

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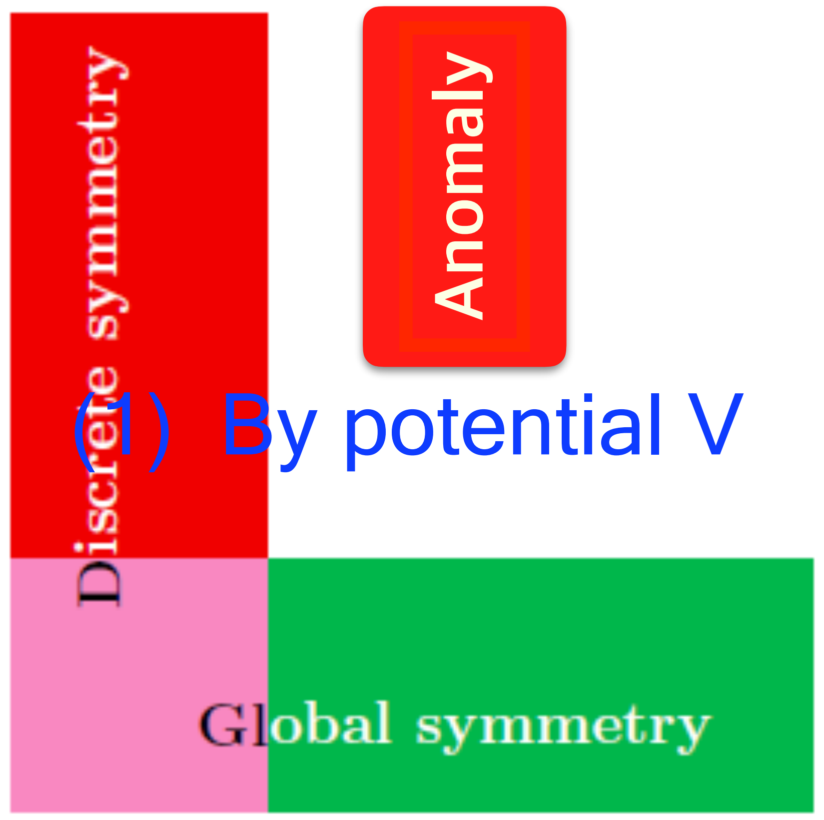
Discrete symmetry
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(1) By potential V
Global symmetry

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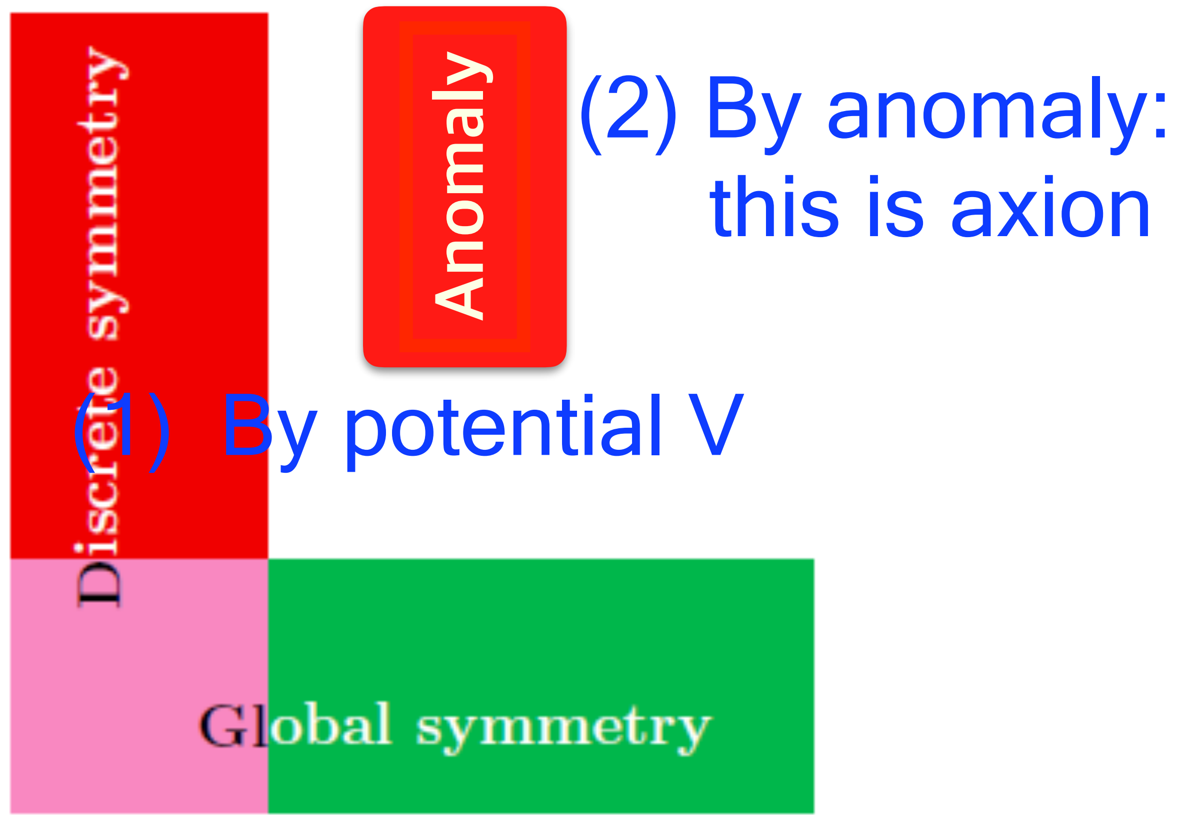

Discrete symmetry
Anomaly
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this is axion
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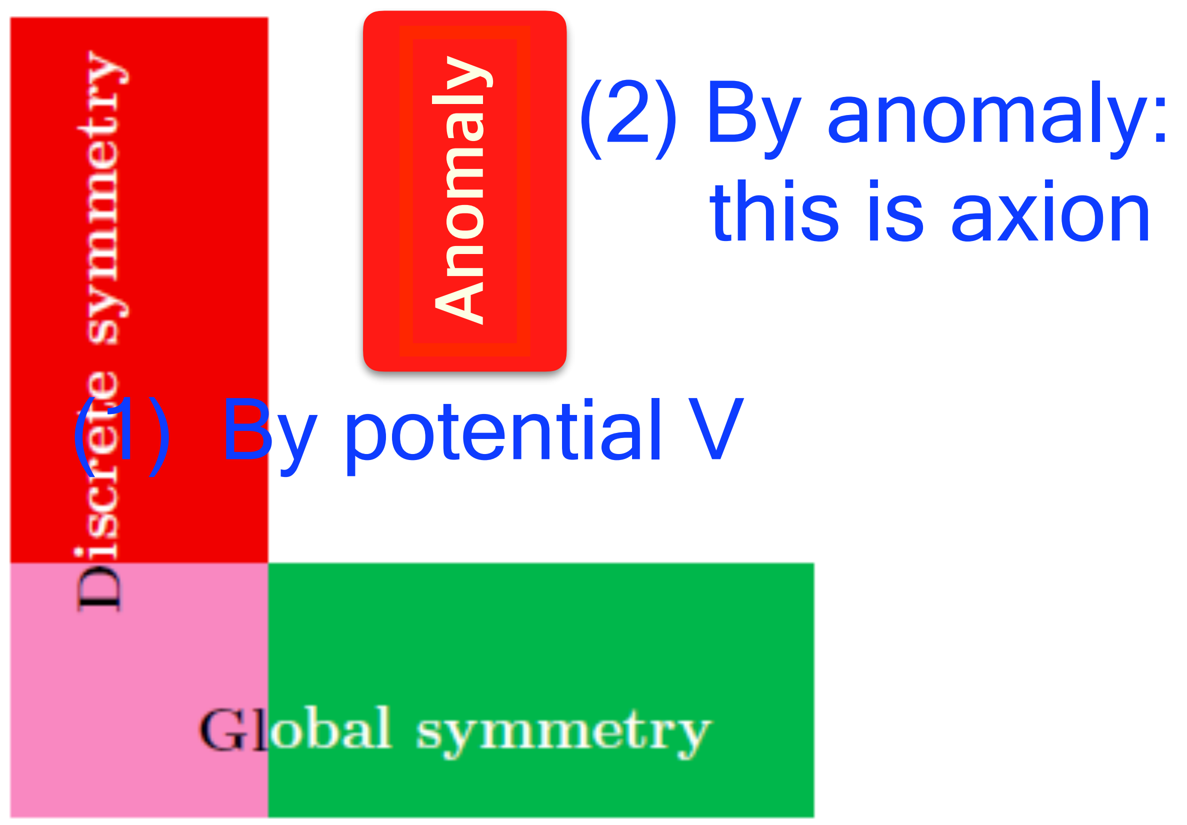

Peccei+Quinn (1977): Chiral symmetry broken ONLY by anomaly

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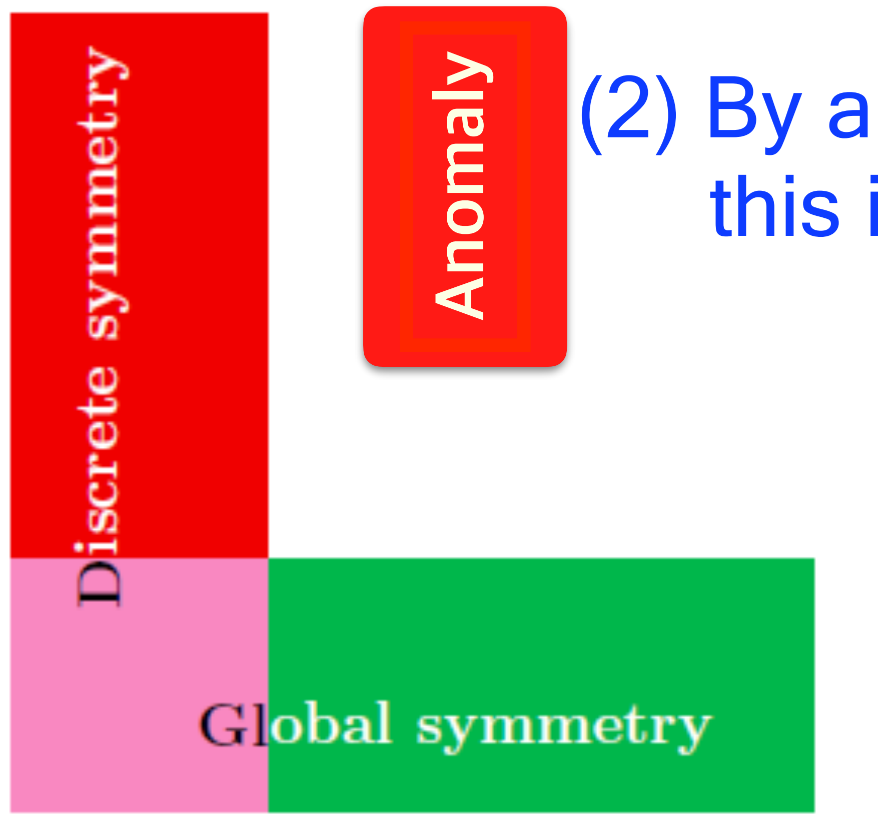

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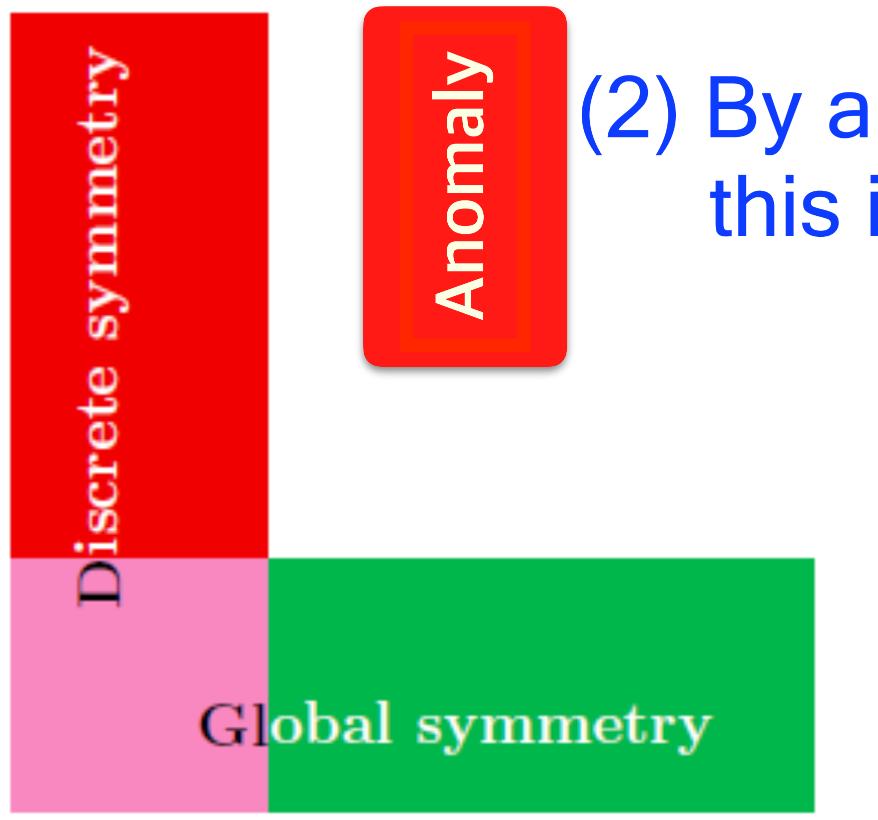

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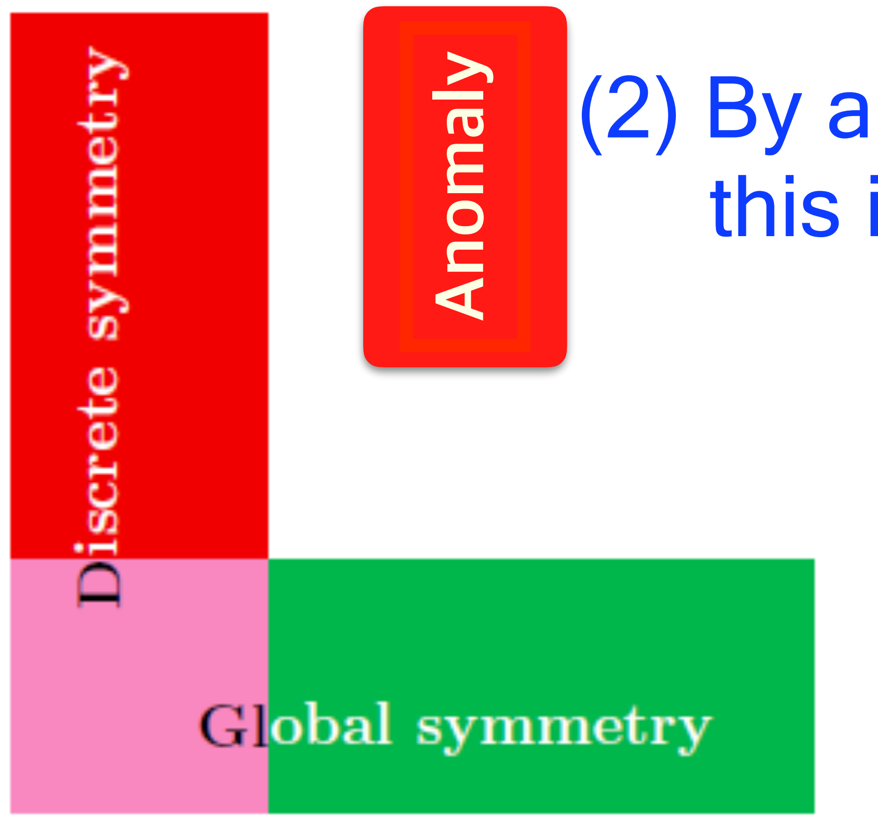

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Sikivie(1983): Haloscope cavity detector

PQ symmetry

Where is the identification of shifting-theta symmetry?

What have WW done? potential.

Non-Abelian gauge theory

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Nonlinear terms in the field strength in non-Abelian gauge groups enable localized field configuration with the identification of S3 in the group space to S3 in the Euclidian space. Instanton solution

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$$A_\mu = \mathrm{i}\, f(r)\, g^{-1}(x)\, \partial_\mu g(x)$$

$$g(x) = \frac{x_4 - \mathrm{i} x_a \sigma_a}{r} = -\frac{\mathrm{i}}{r} x_\mu \sigma_\mu$$

$$g(x)^{-1} = \frac{\mathrm{i}}{r} x_\mu \bar{\sigma}_\mu$$

$$r^2 = \sum_{i=1}^{4} x_i^2$$

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Pontryagin index which is a topological number

$$q = \frac{1}{16\pi^2} \int \mathrm{d}^4 x \,\mathrm{Tr}\, F_{\mu\nu} \tilde{F}_{\mu\nu} = \frac{1}{32\pi^2} \int \mathrm{d}^4 x\, F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}\,.$$

Since the gauge field configuration has many q values, we construct a gauge invariant instanton configuration: theta vacuum. [Callan-Dashen-Gross, Jackiw-Rebbi (1976)]

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$$\begin{aligned}
\langle \theta' | e^{-Ht} | \theta \rangle &= \sum_{n'} \sum_{n} e^{-i(n'\theta' - n\theta)} \langle n' | e^{-Ht} | n \rangle \\
&= \sum_{n'=-\infty}^{\infty} e^{-in'(\theta'-\theta)} \sum_{q=-\infty}^{\infty} \int [dA_\mu]_q \exp\left\{ -iq\theta - \int d^4x \, \mathscr{L} \right\}
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This is the CP violating theta term. It is due to gluons of QCD and QCD has the built-in CP violation.

But, there is no hint that CP is violated in strong interactions. Flavor singlet operator gives neutron electric dipole moment of order 10^{-15} ecm, and experimental bound is 2.1×10^{-26} ecm [C A Baker et al, PRL97 (2006) 131801]. Theta must be less than 10^{-10}. This is a fine-tuning problem.

θ (Strong CP)

O(1) — O(1)

O(10^{-3}) GUT axion

θ (Strong CP)

$O(1)$ — $O(1)$

ULA and quint axion by extremely small V

$O(10^{-3})$ GUT axion

Solutions of strong CP problem

We start from the observation that CP is anyway violated at the weak interaction level. Start with theta=0 but introduce CP violation to see if it generate the next order of theta less than 10^{-10}. This is calculable models, and CP symmetry at the Lagrangian can be the underlying symmetry. The weak CP violation must be of "spontaneous" a la T D Lee. If we treat couplings given below some scale, say 10^{10} GeV, as the given Yukawa couplings, the KM form is the allowed one. However, the Nelson-Barr types cannot escape the high order problem because one can calculate theta at higher orders.

Can we make theta unphysical ?

In QM, the phase of wave function A is unobservable since observables are the probability A*A.

What can be the required symmetry for the theta term?

As the phase symmetry in QM, if there is a shift symmetry of theta, the value of theta is unobservable.

Then, look at what can change theta. The chiral transformation of colored fermions, i.e. the phase of a quark(s). For the corresponding current,

$$\partial^{\mu} J^{5}_{\mu} = \frac{2}{32\pi^2} G^{a}_{\mu\nu} \tilde{G}^{a\,\mu\nu} + m_q\, \bar{q}\, i\gamma_5\, q$$

The theta term is changed by a chiral transformation of some quark field,

$$\theta \to \theta - 2\,\alpha$$

For it to be an exact symmetry, the quark must be massless,

$$m_q = 0$$

It is basically the Peccei-Quinn symmetry. But the traditional PQ symmetry involves more.

The PQ mechanism consists of two parts:

(1) The shift symmetry-QCD-QCD anomaly should be present.

(2) The shift direction must be properly embedded as a fundamental pseudo scalar field or by a composite pseudoscalar field.

In the PQWW axion, the theta direction is one out of two pseudoscalars because the scheme must break a gauge symmetry also to give Z boson mass. The axion direction is orthogonal to that.

The simplest case is not involving breaking a gauge symmetry. Only break the PQ global symmetry. ——->
To introduce the color anomaly only, a heavy quark Q is introduced: the so-called KSVZ model (It is one example, housing axion a in a complex BSM field singlet S [JEK(1979)].)

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	Q_L ,	Q_R ,	S
SU(3)xSU(2)xU(1)	(3,1,Ne).	(3,1,Ne).	(1,1,0)
PQ charge	1	–1	2

The KSVZ line in most cross-section vs. m_a line is for neutral one N=0. Spontaneous breaking of PQ creates a.

Lagrangians with a PQ symmetry

KSVZ

$$\mathcal{L}_Q = -f\,\overline{Q}_L Q_R\,\sigma + \text{h.c.}$$

Renormalizable couplings

DFSZ

$$V = \frac{\lambda}{4}(\sigma^*\sigma)^2 - \frac{\mu^2}{2}\sigma^*\sigma + \lambda_1\sigma^2 H_u H_d + \cdots$$

To have $\langle H \rangle / f_a = 10^{-8}$, one needs a fine-tuning of order 10^{-16}.

SUSY

$$\begin{array}{cccc} PQ: & -1\,-1 & 1 & 1 \\ & \dfrac{H_u H_d}{M} & \sigma & \sigma \end{array}$$

The PQ symmetry is used to allow the electroweak scale mu [K-Nilles (1984)].

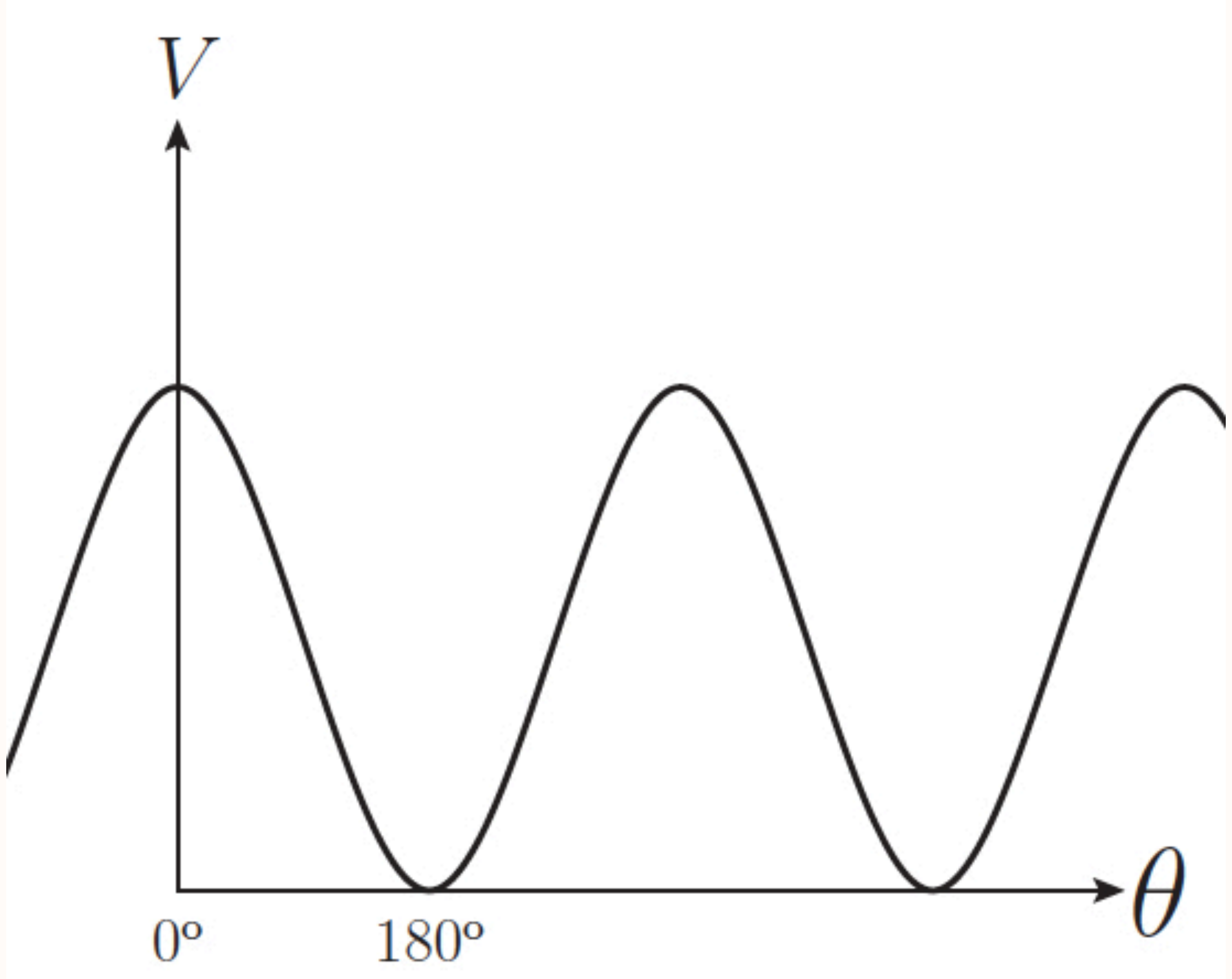
If a global U(1) symmetry is spontaneously broken, the minimum is at 0 or 180°.

$$V = V_{\rm sym} + m^3\sigma + +m^{*3}\sigma*$$
$$= \sqrt{2}\,|m|^3 v\ \cos(\alpha + c)$$

$$\sigma = \frac{v}{\sqrt{2}}e^{i\alpha}, \quad m^3 = |m|^3 e^{ic}$$

What Vafa and Witten showed is that 0 is the minimum for axion (breaking is only by anomaly without breaking from V).

If we include pseudo scalars from spontaneously broken U(1), not by the anomaly but by V, then 180° can be a minimum depending on parameters. I suggest to classify the inflationary scenario of this type as



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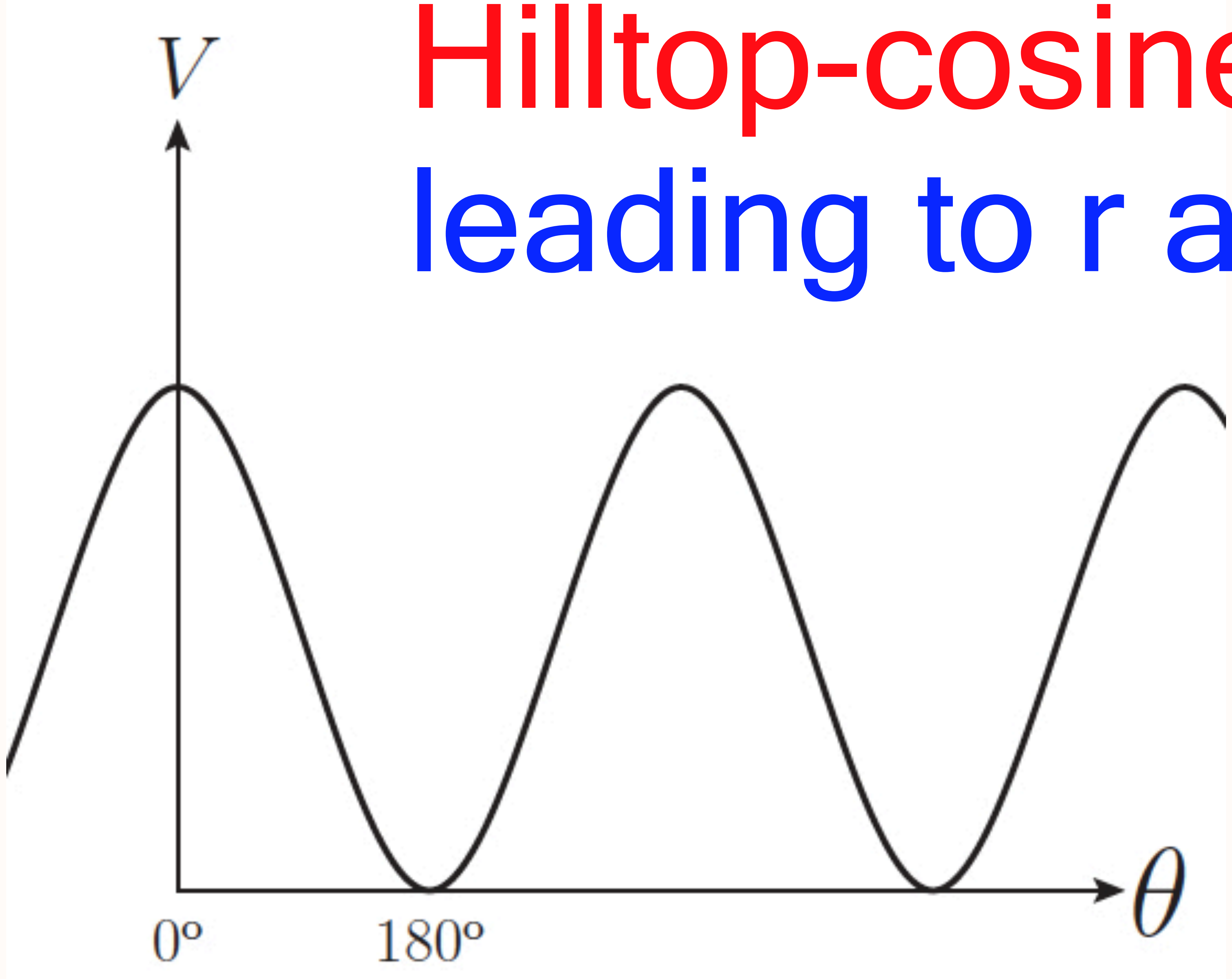
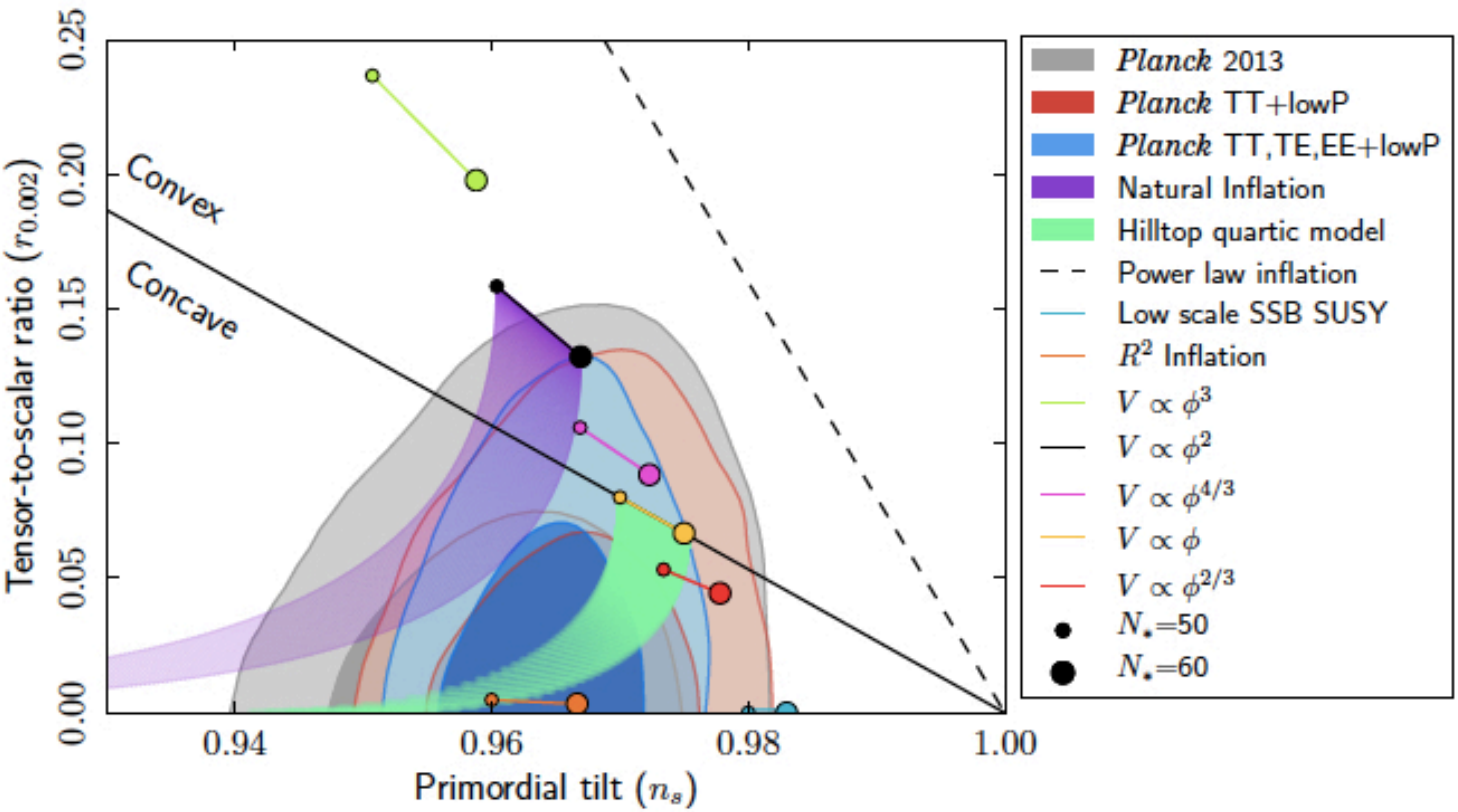
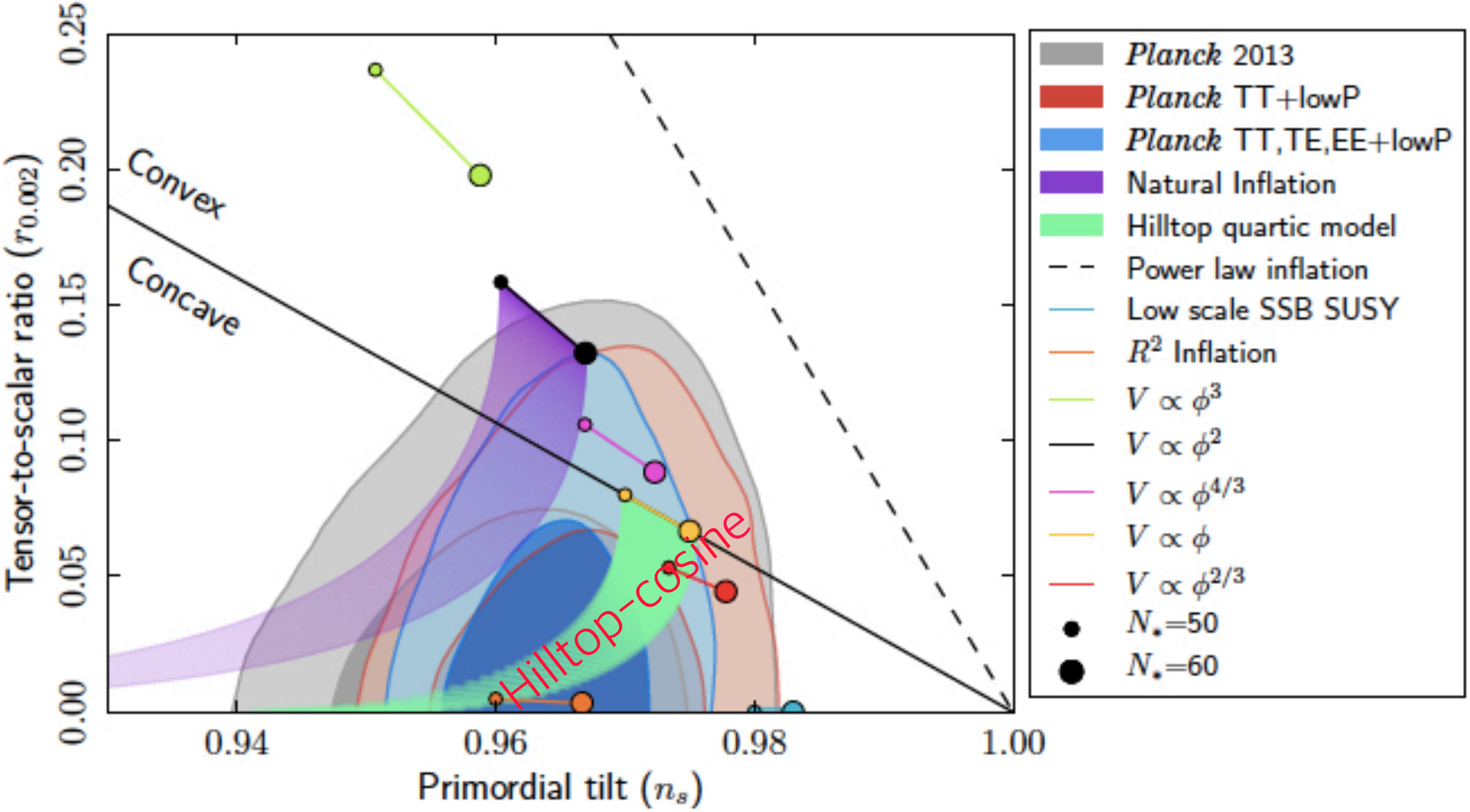


Hilltop-cosine inflation, leading to r always close 0.



Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Planck 2013
Planck TT+lowP
Planck TT,TE,EE+lowP
Natural Inflation
Hilltop quartic model
Power law inflation
Low scale SSB SUSY
R^2 Inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60


Tensor-to-scalar ratio ($r_{0.002}$)
0.00
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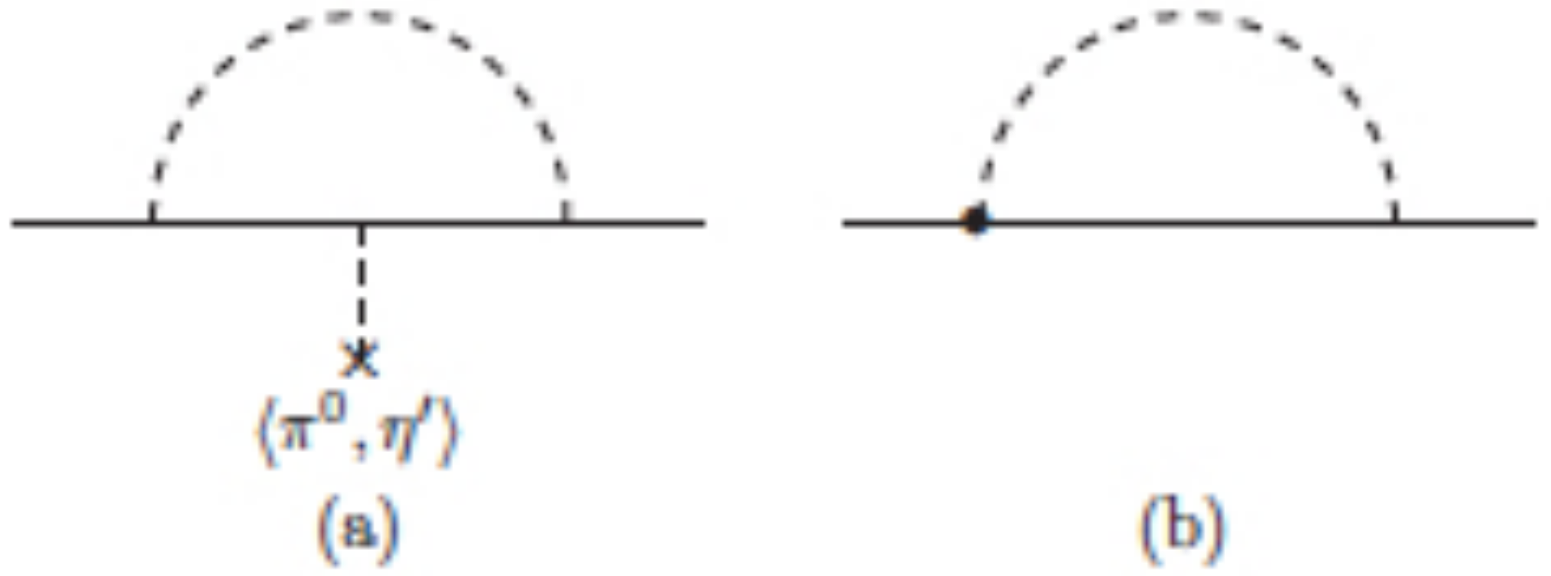


FIG. 4. Loop corrections for $\bar{n}n$-meson coupling. Insertion of the CP violation effect by VEVs of π^0 and η' in (a). They can be transferred to one vertex shown as a bullet in (b). With this bullet, CP violation is present because of a mismatch between the CP-conserving RHS vertex and CP-violating LHS vertex.

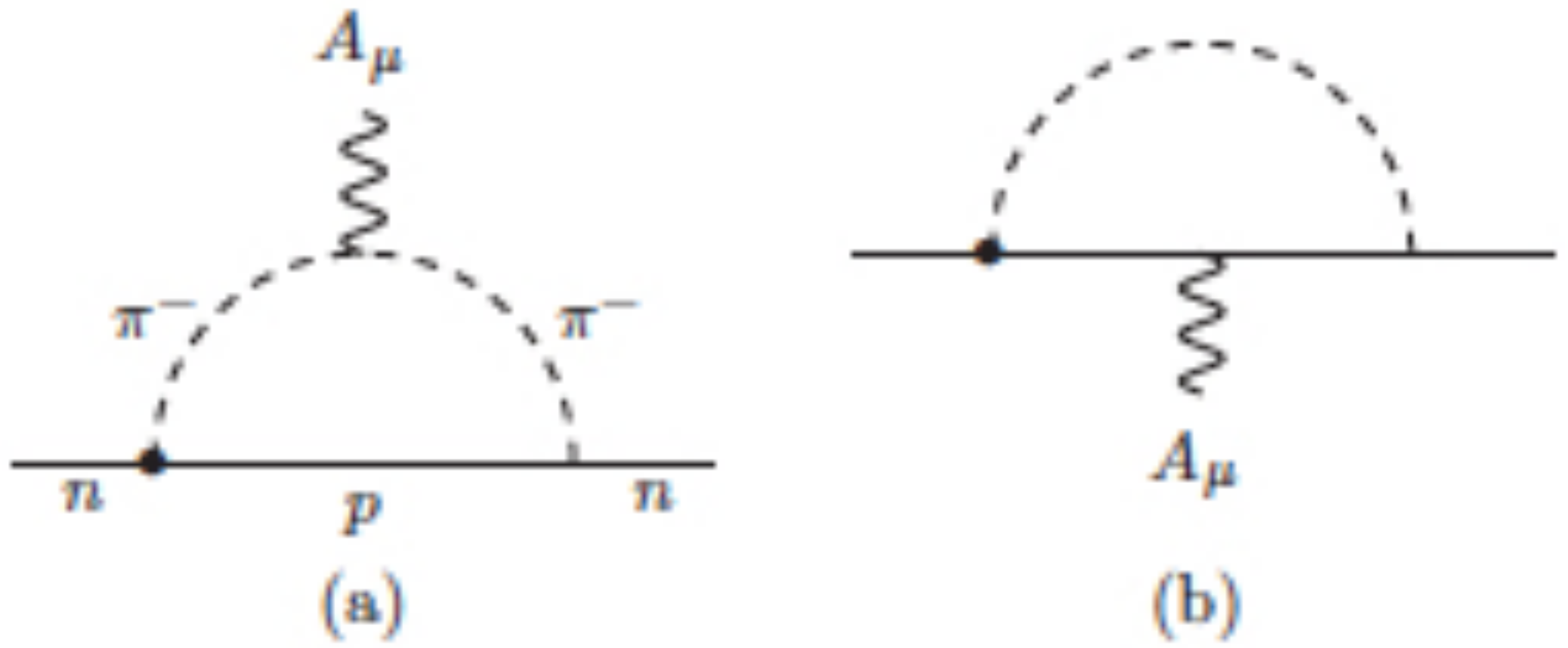


FIG. 5. Diagrams contributing to the NEDM with the bullet representing the CP violation effect. (a) is the physically observable contribution.

Crewther et al. (1979)

$$
\begin{aligned}
-V = {} & m_u v^3 \cos(\theta_\pi + \theta_{\eta'}) + m_d v^3 \cos(-\theta_\pi + \theta_{\eta'}) \\
& + \frac{v^9}{K^5} \cos[2\theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta] \\
& + m_u \frac{\Lambda_u^2 v^6}{K^5} \cos[-\theta_\pi + \theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta] \\
& + m_d \frac{\Lambda_d^2 v^6}{K^5} \cos[\theta_\pi + \theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta],
\end{aligned}
$$

$$
\overline{g_{\pi NN}} = -\bar{\theta} \frac{2(m_\Xi - m_\Sigma) m_u m_d}{f_\pi (m_u + m_d)(2m_s - m_u - m_d)} \approx -0.023\bar{\theta},
$$

$$
\overline{g_{\pi NN}} = -\bar{\theta} \frac{Z}{(1+Z)} \simeq -\frac{\bar{\theta}}{3},
$$

Kim-Carosi missed a factor: The pion neutron coupling

$$\frac{d_n}{e} = \frac{g_{\pi nn}\overline{g_{\pi nn}}}{4\pi^2 m_n} \ln\left(\frac{m_n}{m_\pi}\right) = \frac{2g^2_{\pi nn}}{4\pi^2 m_n}\frac{\bar\theta}{3} \ln\left(\frac{m_n}{m_\pi}\right)$$

$$= \frac{3.60}{m_n}\bar\theta = \bar\theta \quad 0.76 \times 10^{-13}\,\mathrm{cm}$$

$$\rightarrow \quad \bar\theta \lesssim 2.8 \times 10^{-13}$$

3. QCD phase transition

3. QCD phase transition

JEK+S Kim, 1805.08153

$$\text{Before}\begin{cases}\rho = \dfrac{\pi^2}{30}\, g_*^i T^4\\[2mm] s = \dfrac{2\pi^2}{45}\, g_*^i T^3\end{cases}$$

$$\text{After}\begin{cases}\rho = \dfrac{\pi^2}{30}\, g_*^f T^4\\[2mm] s = \dfrac{2\pi^2}{45}\, g_*^f T^3\end{cases}$$

$$g_*^i = 51.25,\ \ g_*^f = 17.25.$$

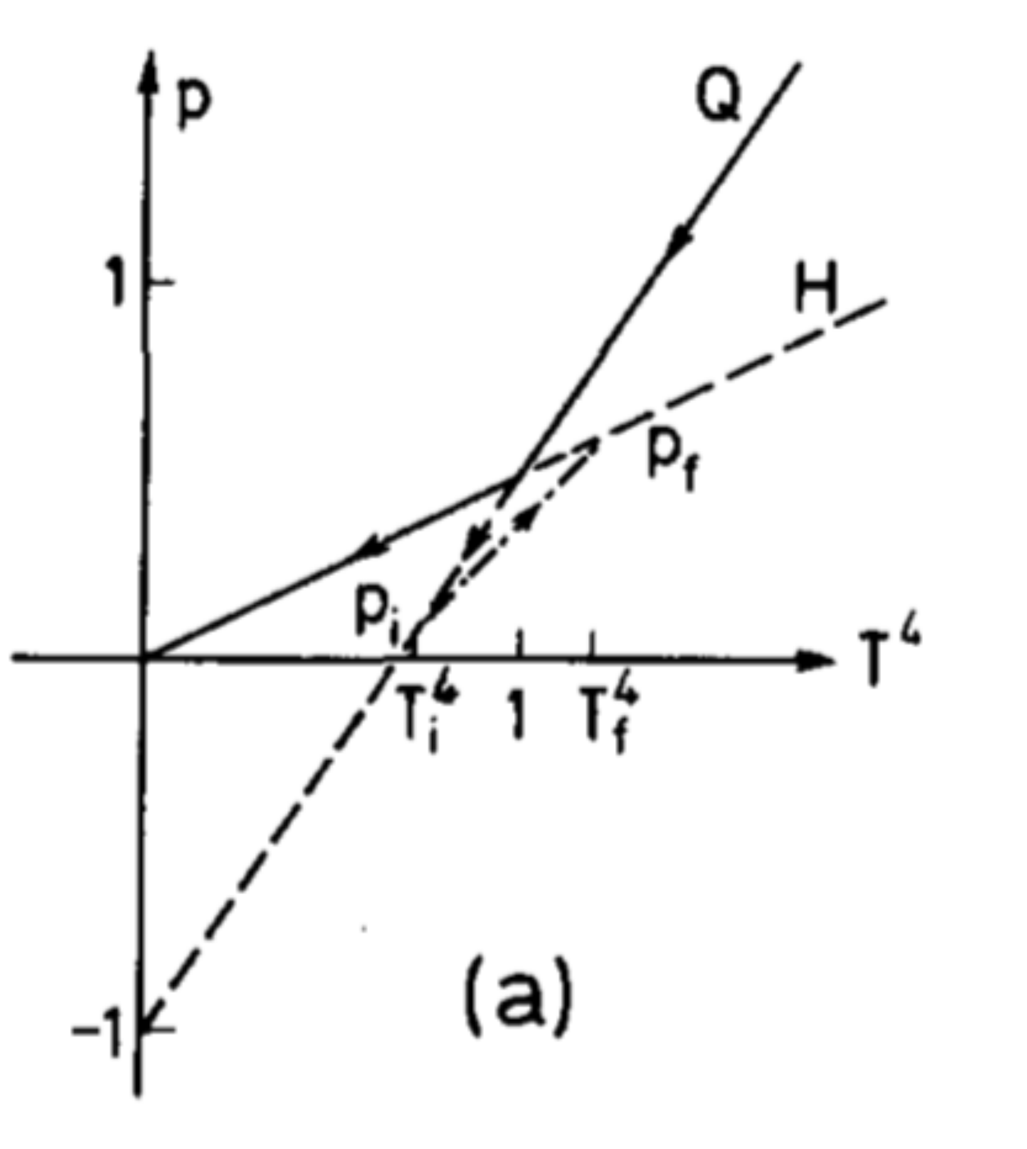
$$\text{Before}\begin{cases}\rho = \frac{\pi^2}{30}\, g_*^i T^4 \\ s = \frac{2\pi^2}{45}\, g_*^i T^3\end{cases}$$

$$\text{After}\begin{cases}\rho = \frac{\pi^2}{30}\, g_*^f T^4 \\ s = \frac{2\pi^2}{45}\, g_*^f T^3\end{cases}$$

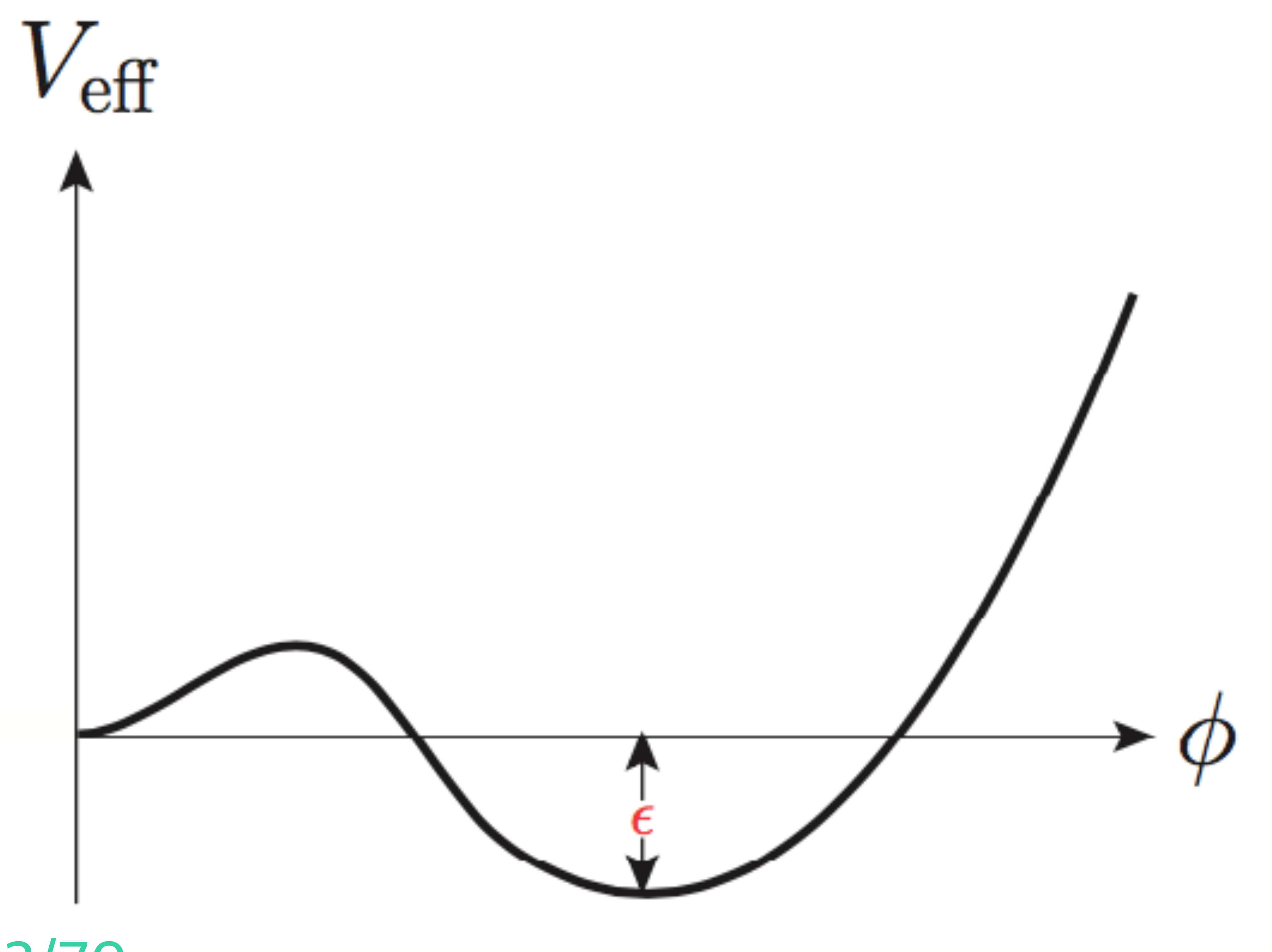
$$g_*^i = 51.25,\ \ g_*^f = 17.25.$$

37, 3, for hadrons only

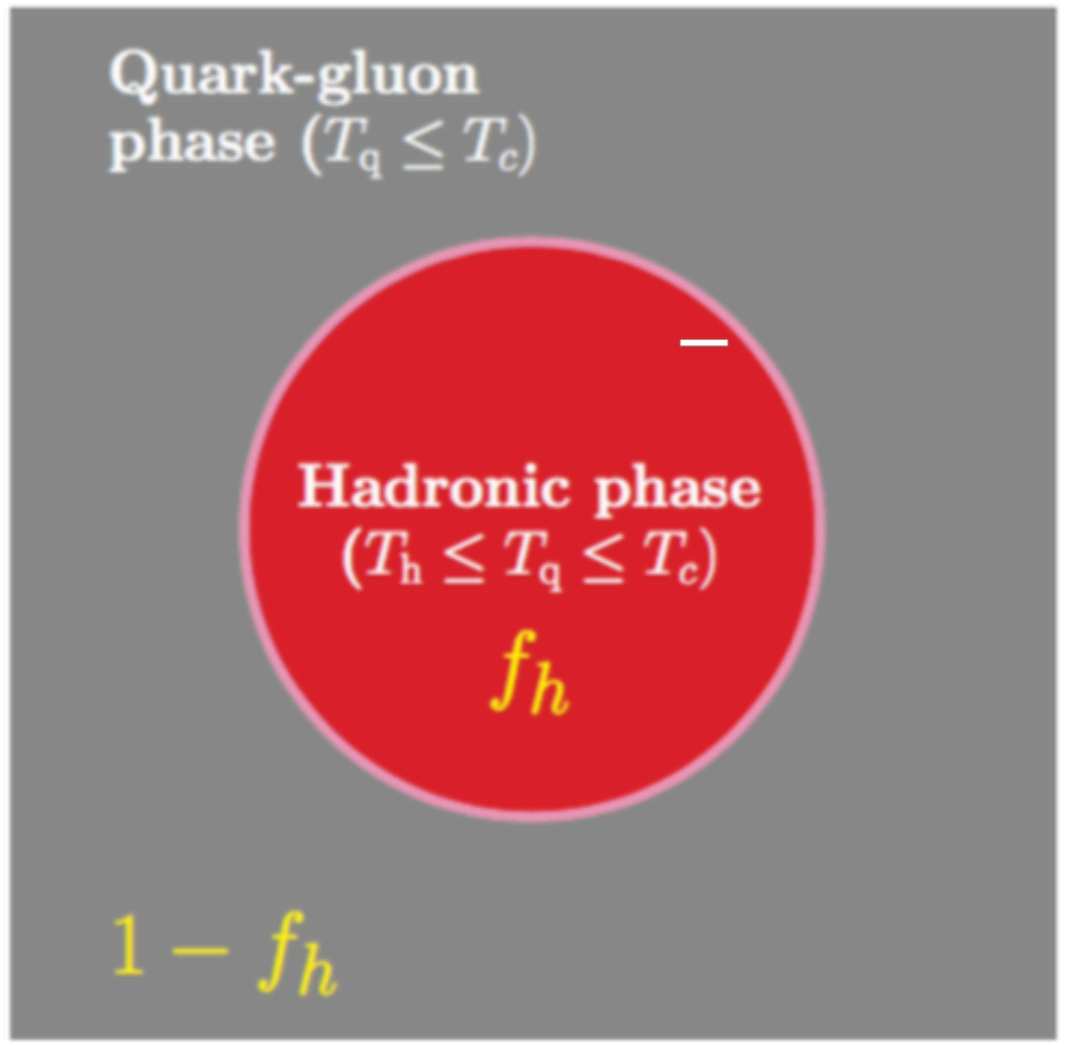
DeGrand and callaborators studied QCD phase transition and axion since 1984 with MIT bag model. This calculation has a complicated behavio



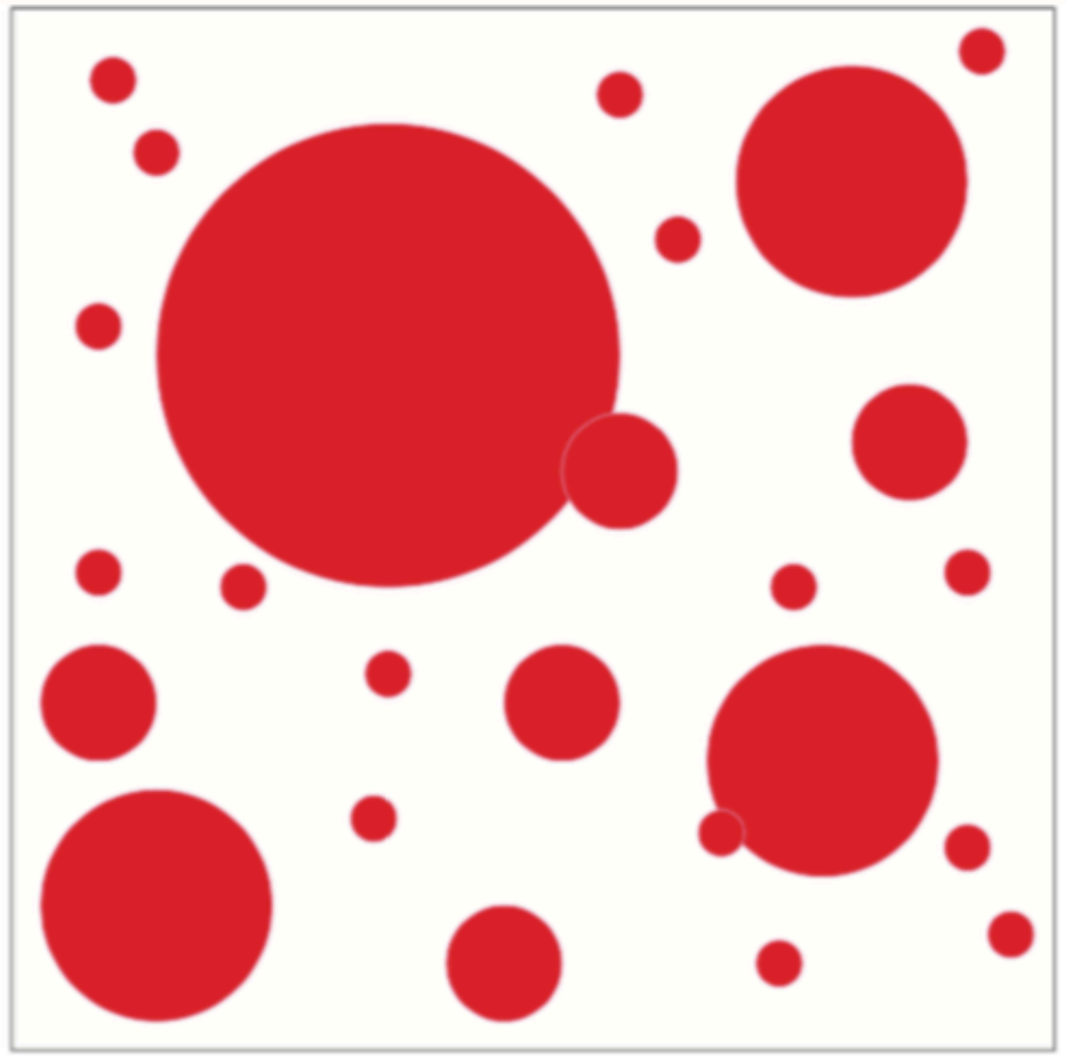
Kolb and Turner studied with a phenomenological Lagrangian with ϵ parameter.

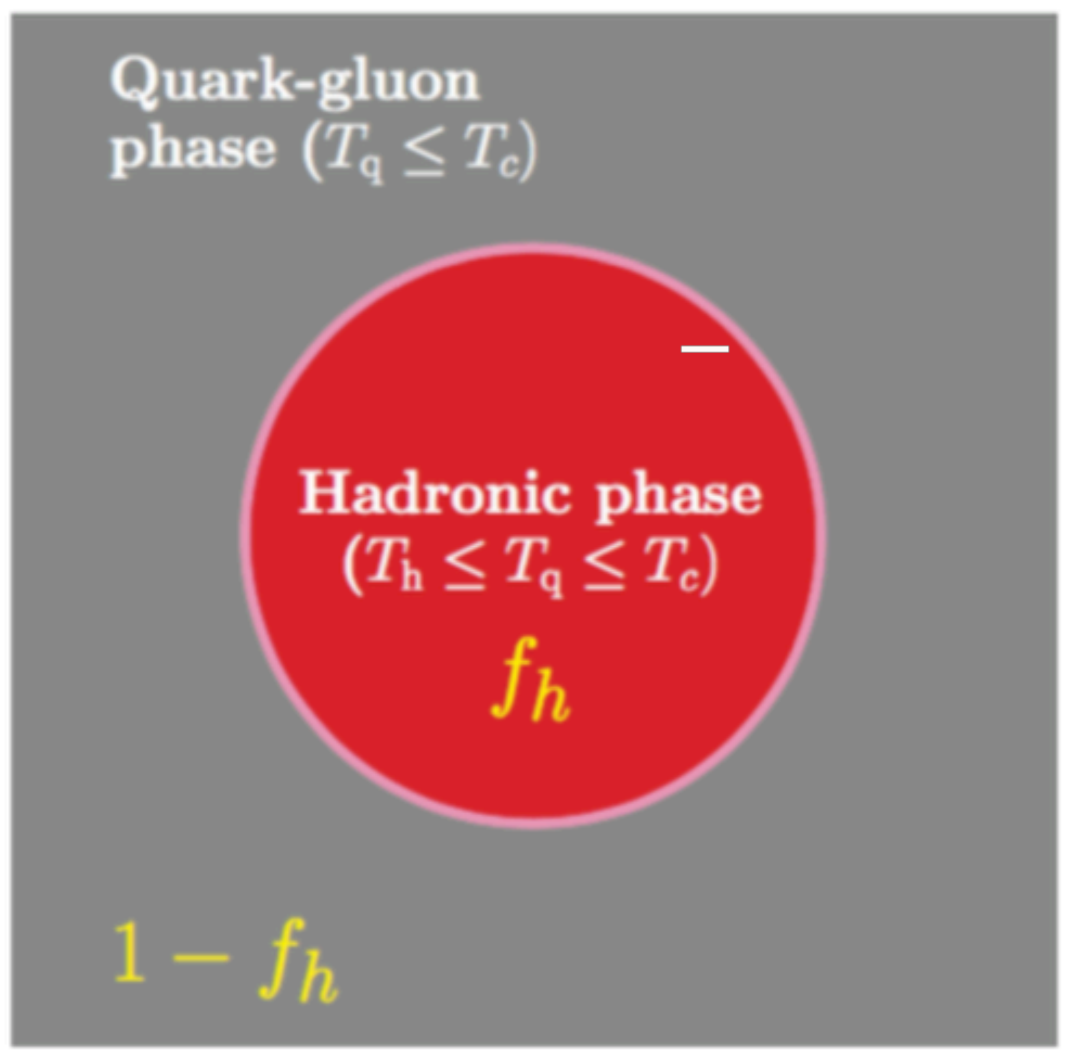


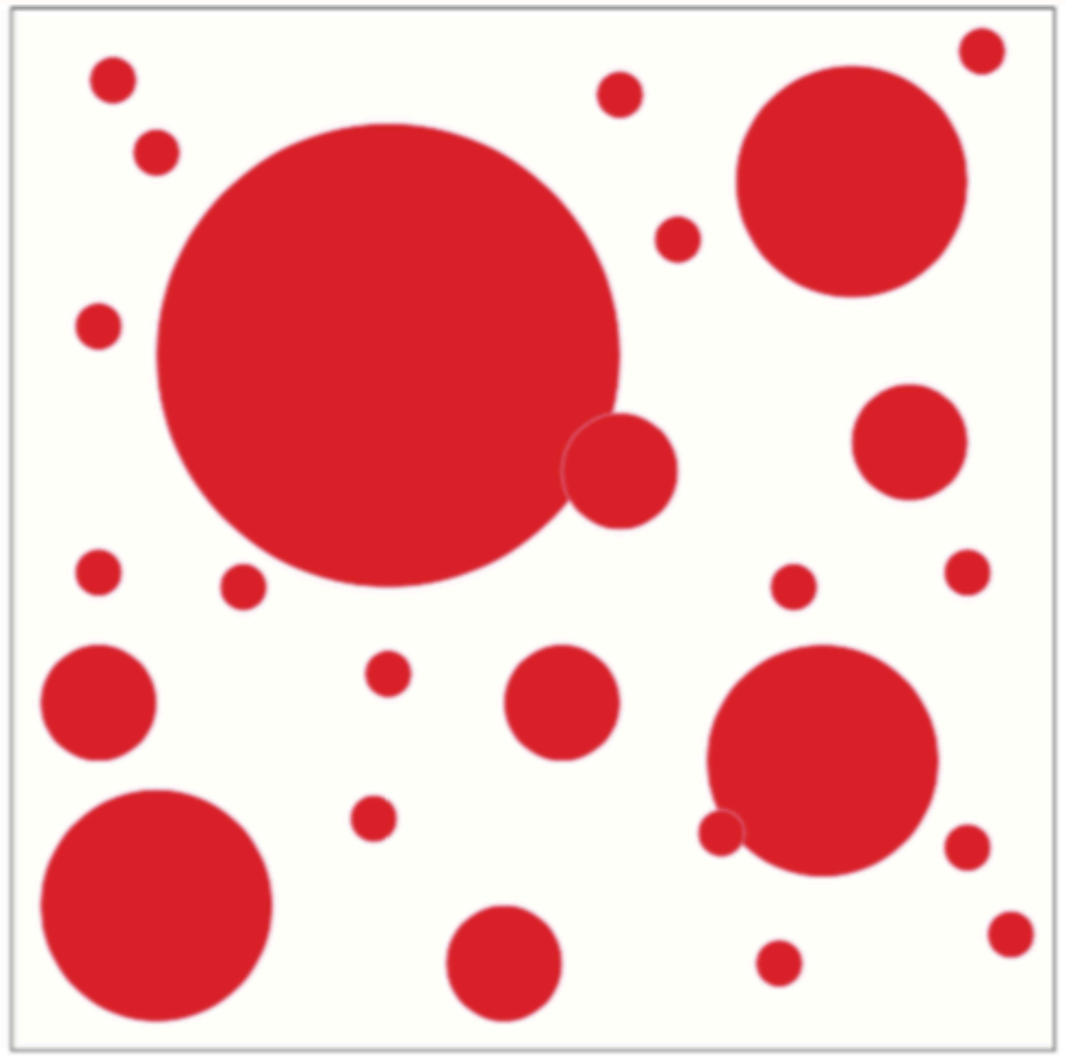
We calculate the phase transition from the first principles.



Quark-gluon phase ($T_q \le T_c$)
Hadronic phase ($T_h \le T_q \le T_c$)
f_h
$1 - f_h$







Here, we study the following two parameter differential equation on the fraction of h-phase in the evolving universe.

$$\frac{df_h}{dt} = \alpha(1 - f_h) + \frac{3}{(1 + Cf_h(1 - f_h))(t + R_i)} f_h$$

It is possible to calculate it because the critical temperature is pinned down now at 165+-5 MeV in the lattice community.

- Knowledge on the axion mass is important.
- Susceptibility χ is important.

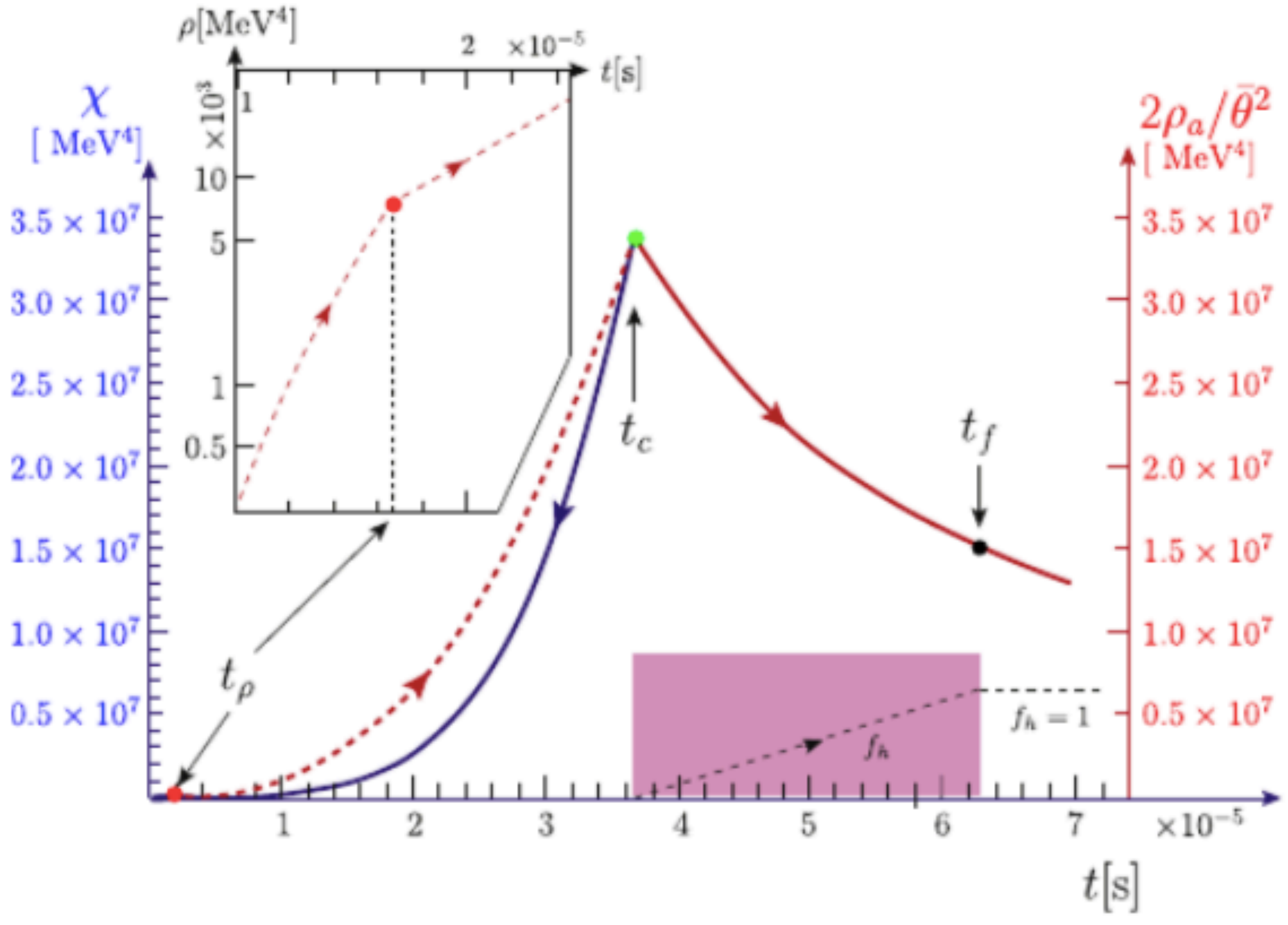
Quark and gluon phase with $\Lambda_{\rm QCD}$:

$$f_a^2 m_a^2 = \frac{(\sin^2\bar\theta/\bar\theta^2)}{2Z\cos\bar\theta + 1 + Z^2}\, m_u^2 \Lambda_{\rm QCD}^2 \left(\frac{1}{2}\bar\theta^2\right),$$

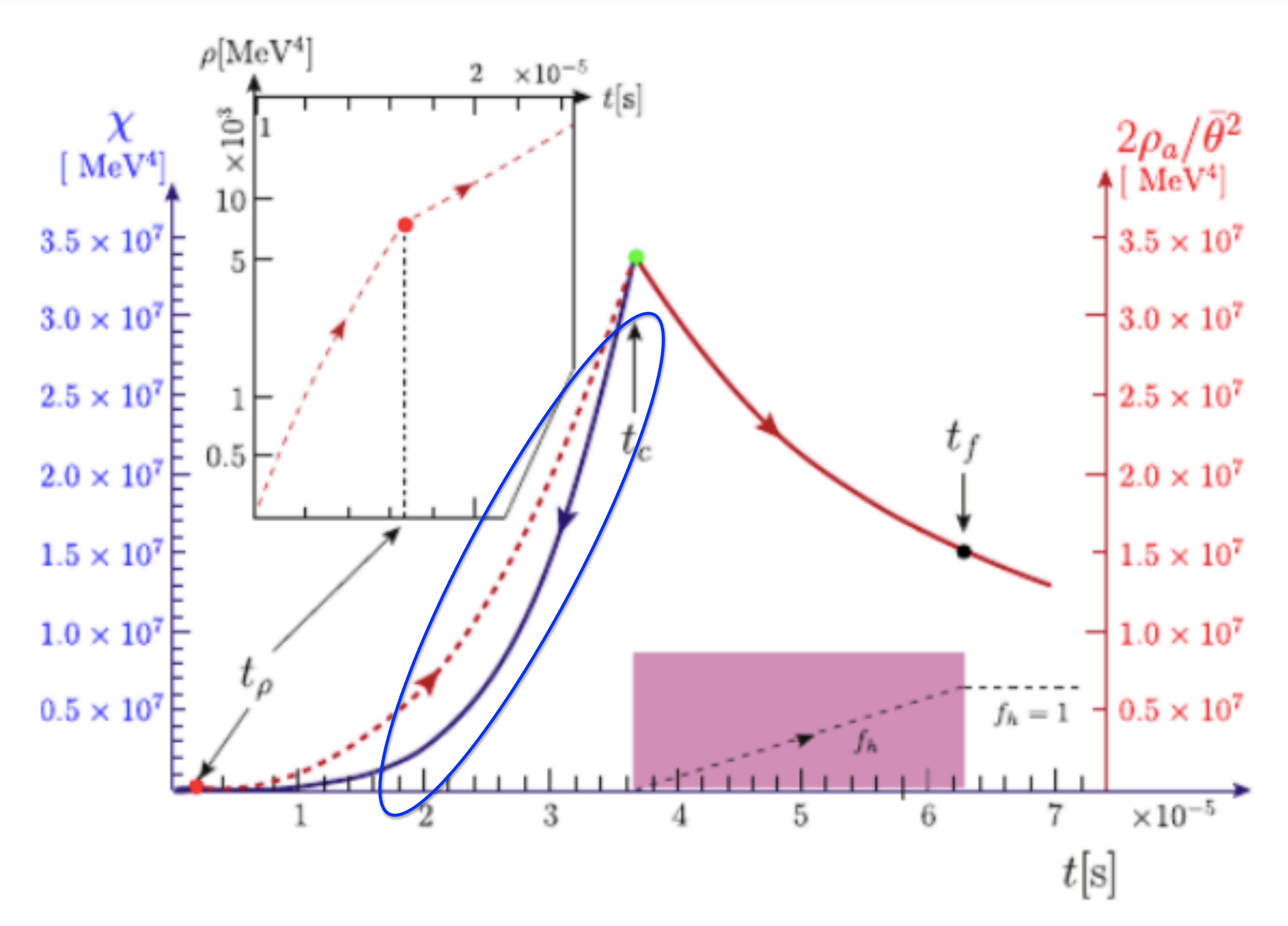
Hadronic phase in terms of $f_{\pi^0}^2 m_{\pi^0}^2$:

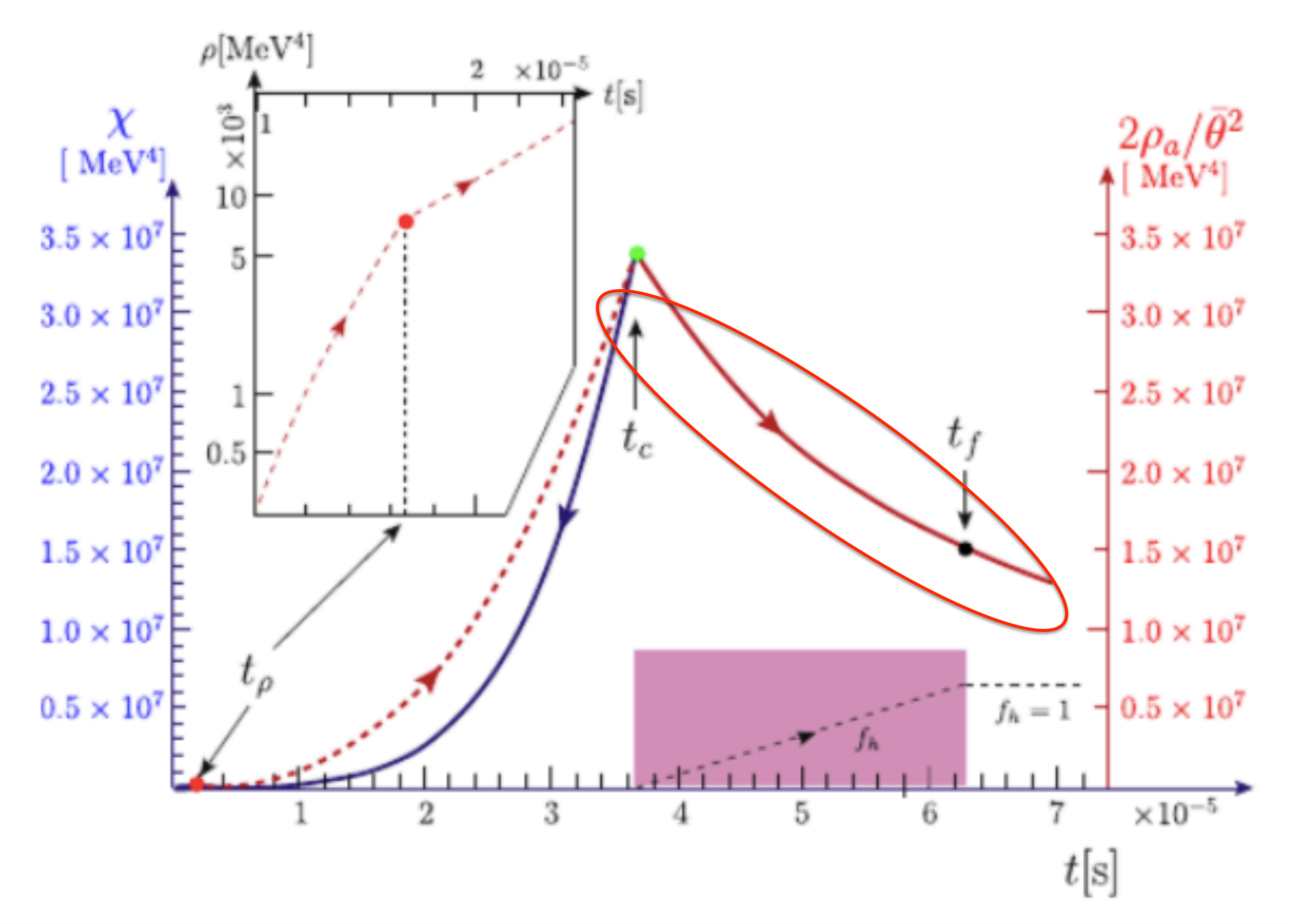
$$f_a^2 m_a^2 = \frac{Z(\sin^2\bar\theta/\bar\theta^2)}{2Z\cos\bar\theta + 1 + Z^2}\, f_{\pi^0}^2 m_{\pi^0}^2 \left(\frac{1}{2}\bar\theta^2\right),$$

Lattice susceptibility χ : $f_a^2 m_a^2 = \chi \left(\frac{1}{2}\bar\theta^2\right),$



χ [MeV^4]
3.5×10^7
3.0×10^7
2.5×10^7
2.0×10^7
1.5×10^7
1.0×10^7
0.5×10^7
ρ[MeV^4]
2 $\times 10^{-5}$
t[s]
$\times 10^3$
1
10
5
1
0.5
t_ρ
t_c
t_f
f_h
$f_h = 1$
$2\rho_a/\bar\theta^2$ [MeV^4]
1 2 3 4 5 6 7 $\times 10^{-5}$
t[s]


χ
[MeV^4]
$2\rho_a/\bar{\theta}^2$
[MeV^4]
ρ[MeV^4]
t[s]
$\times 10^{-5}$
$\times 10^3$
t_ρ
t_c
t_f
f_h
$f_h = 1$
3.5×10^7
3.0×10^7
2.5×10^7
2.0×10^7
1.5×10^7
1.0×10^7
0.5×10^7



χ [MeV^4]
3.5×10^7
3.0×10^7
2.5×10^7
2.0×10^7
1.5×10^7
1.0×10^7
0.5×10^7
ρ[MeV^4]
$\times 10^3$
2
$\times 10^{-5}$
t[s]
1
10
5
1
0.5
t_ρ
t_c
t_f
$f_h = 1$
f_h
$2\rho_a/\bar{\theta}^2$ [MeV^4]
3.5×10^7
3.0×10^7
2.5×10^7
2.0×10^7
1.5×10^7
1.0×10^7
0.5×10^7
1
2
3
4
5
6
7
$\times 10^{-5}$
t[s]

There are two aspects in this study: (1) the strong interaction, (2) axion energy density evolution in the evolving Universe.

At and below Tc, the quark-gluon phase and the hadronic phase co-exist. So, at Tc we know what is the energy density in the q&g-phase. And pressure is just 1/3 of it. So, we know the pressure of h-phase since the pressures are the same during the 1st order phase transition.

Now, at Tc the pion pressure is known. So, it is known below Tc also.

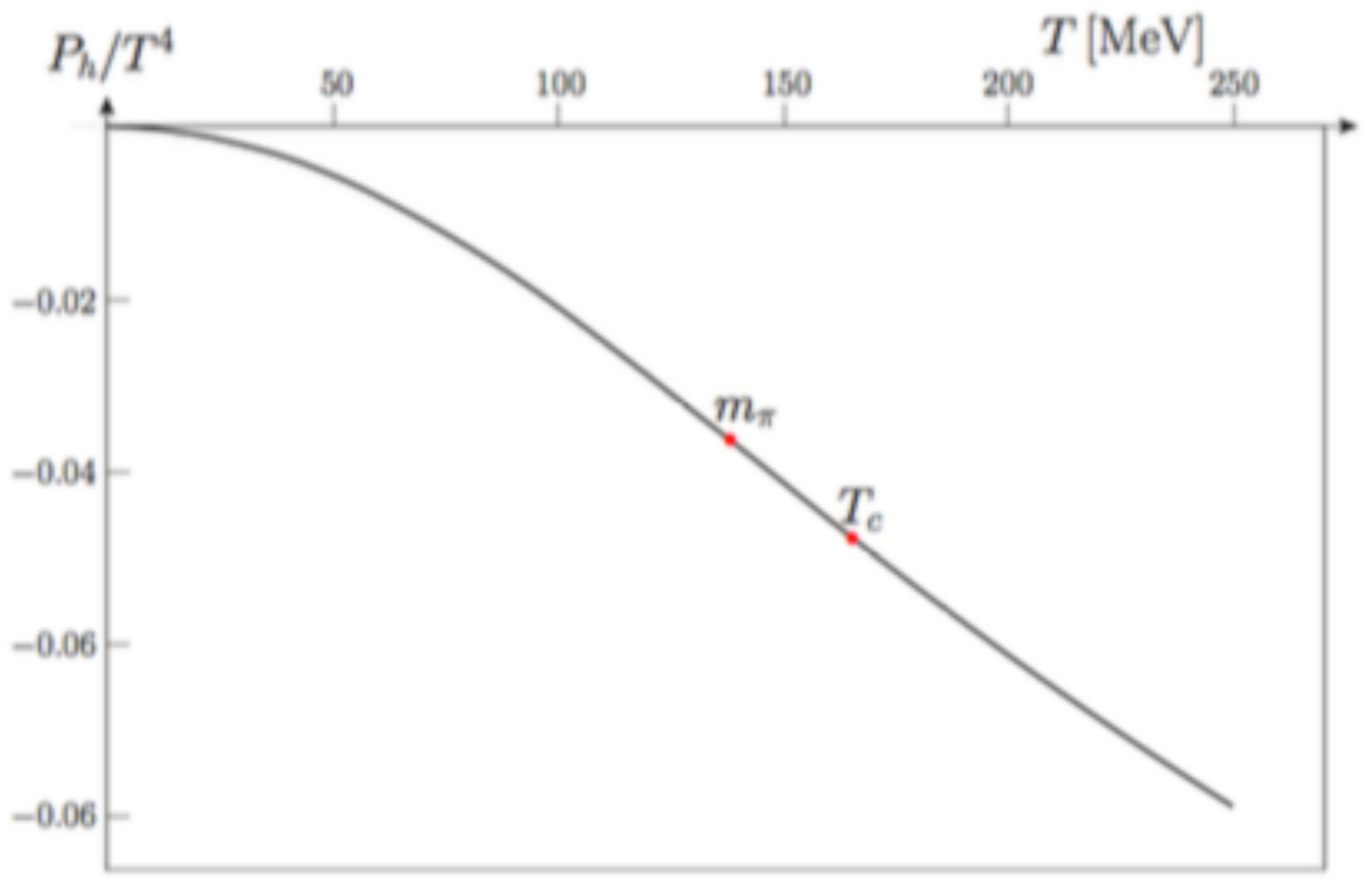


FIG. 3: P_h versus T.

We used Eq. (8.55) of Huang's book, "Statistical Mechanics", in realtivistic form.

$$dU = dQ - PdV + \mu dN,$$
$$dA = -SdT - PdV + \mu dN,$$
$$dG = -SdT + VdP + \mu dN,$$

$$dU = dQ - PdV + \mu dN,$$
$$dA = -SdT - PdV + \mu dN,$$
$$dG = -SdT + VdP + \mu dN,$$

Used in the1st law

$$
\begin{aligned}
dU &= dQ - PdV + \mu dN, \\
dA &= -SdT - PdV + \mu dN, \\
dG &= -SdT + VdP + \mu dN,
\end{aligned}
$$

Used in the1st law

Used in the evolveing Univ.

$$dU = dQ - PdV + \mu dN,$$ Used in the1st law

$$dA = -SdT - PdV + \mu dN,$$ Used in the evolveing Univ.

$$dG = -SdT + VdP + \mu dN,$$

During the 1st order(cross-over) phase transition, the Gibbs free energy is conserved. At the same temperature and pressure. We know P of massless quarks and gluons at temperatures T, 1/3 of energy density.
Now, we have to know P of massive pions at and below Tc.

- In the expanding Universe, the free energy is conserved,

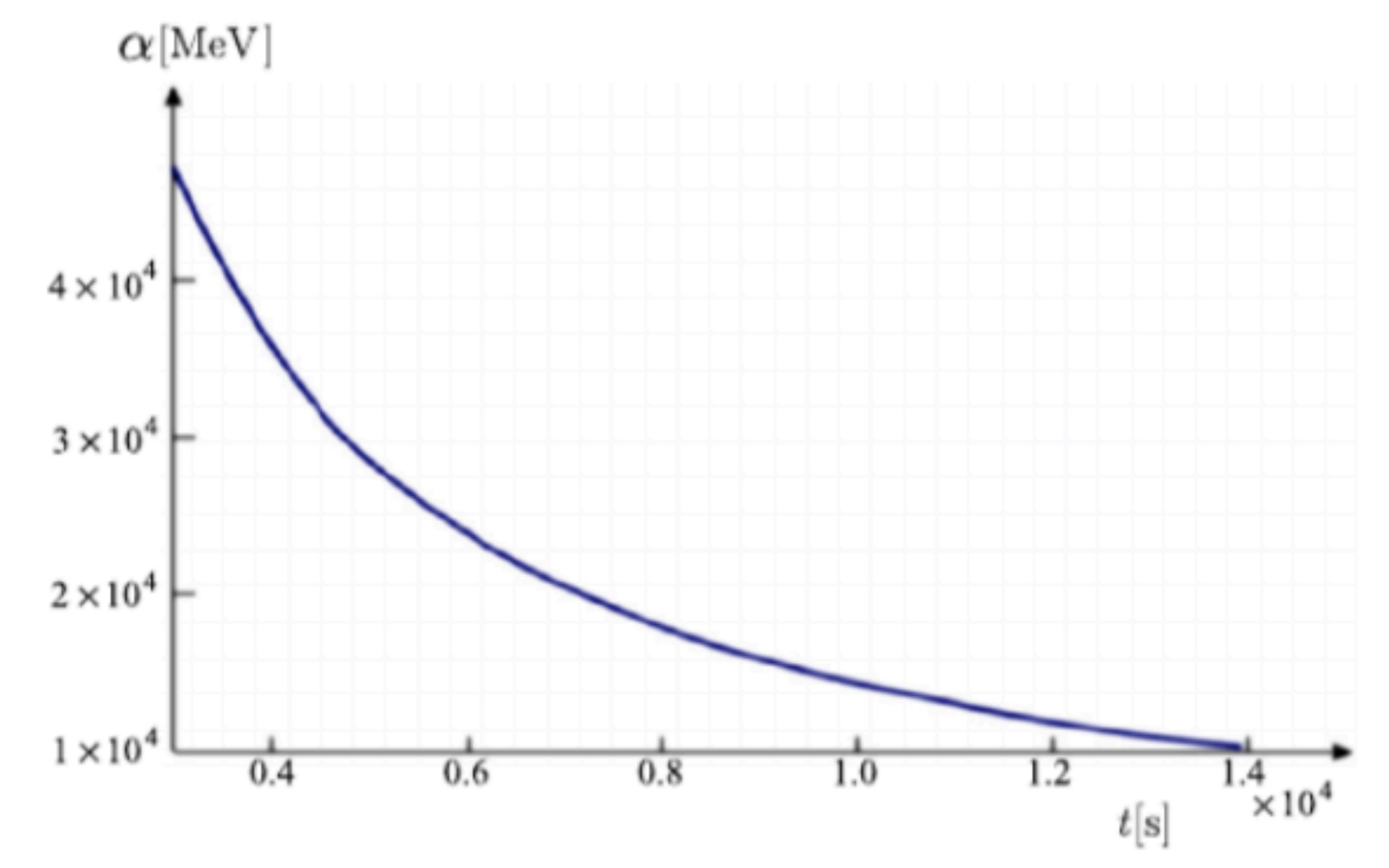
$$(-SdT - PdV + \mu dN)_q + (-SdT - PdV + \mu dN)_h = 0.$$

Using $dV_q = -dV_h$,

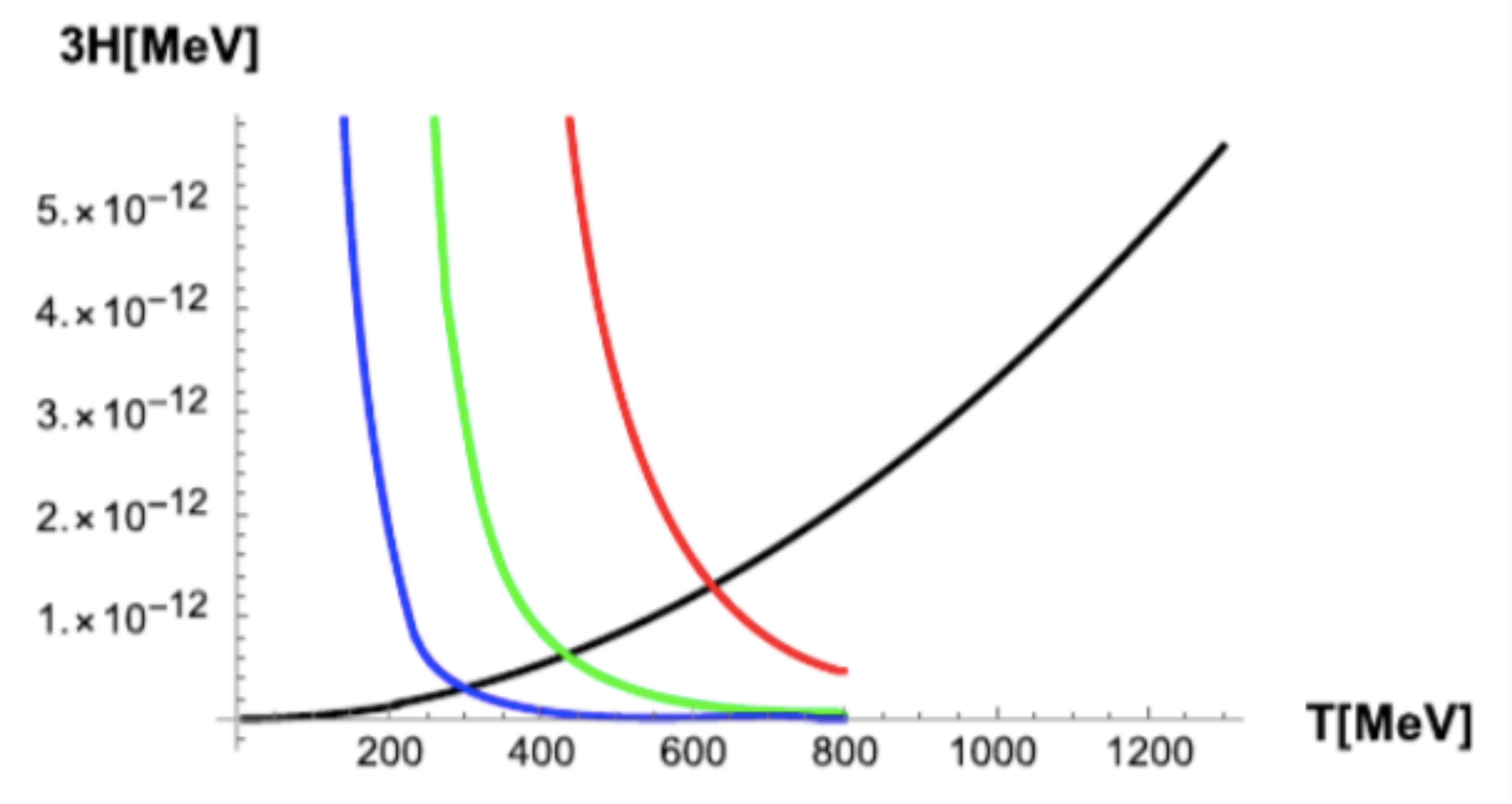
$$(P_h - P_q)dV_h = (S_q - S_h)dT + \mu_h dN_h - \mu_q dN_q = (S_q - S_h)dT.$$

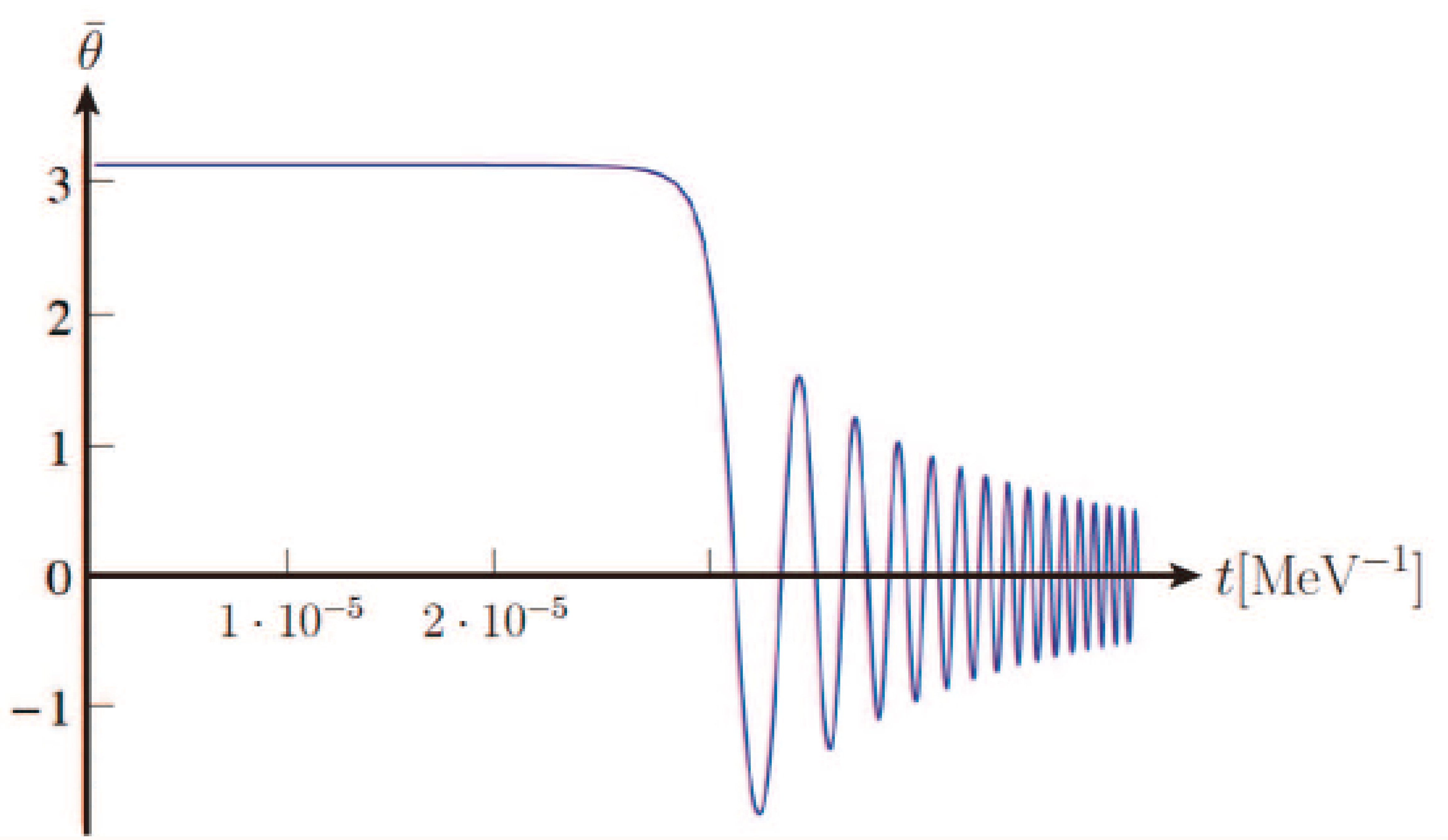
$$\frac{1}{V}\frac{dV_h}{dt} = \frac{(S_q - S_h)}{(P_h - P_q)}\frac{dT}{dt}.$$

$$\alpha(T) = \frac{(S_q - S_h)}{(P_h - P_q)}\frac{dT}{dt} \approx \frac{-37\pi^2}{45(P_h - P_q)}\frac{T^6}{\text{MeV}}, \text{ with } T^2 t_{[s]} \simeq \text{MeV}.$$



α [MeV]
4×10^4
3×10^4
2×10^4
1×10^4
0.4
0.6
0.8
1.0
1.2
1.4
×10^4
t [s]


3H[MeV]
5.×10−12
4.×10−12
3.×10−12
2.×10−12
1.×10−12
200
400
600
800
1000
1200
T[MeV]



Show movie for m_a independence

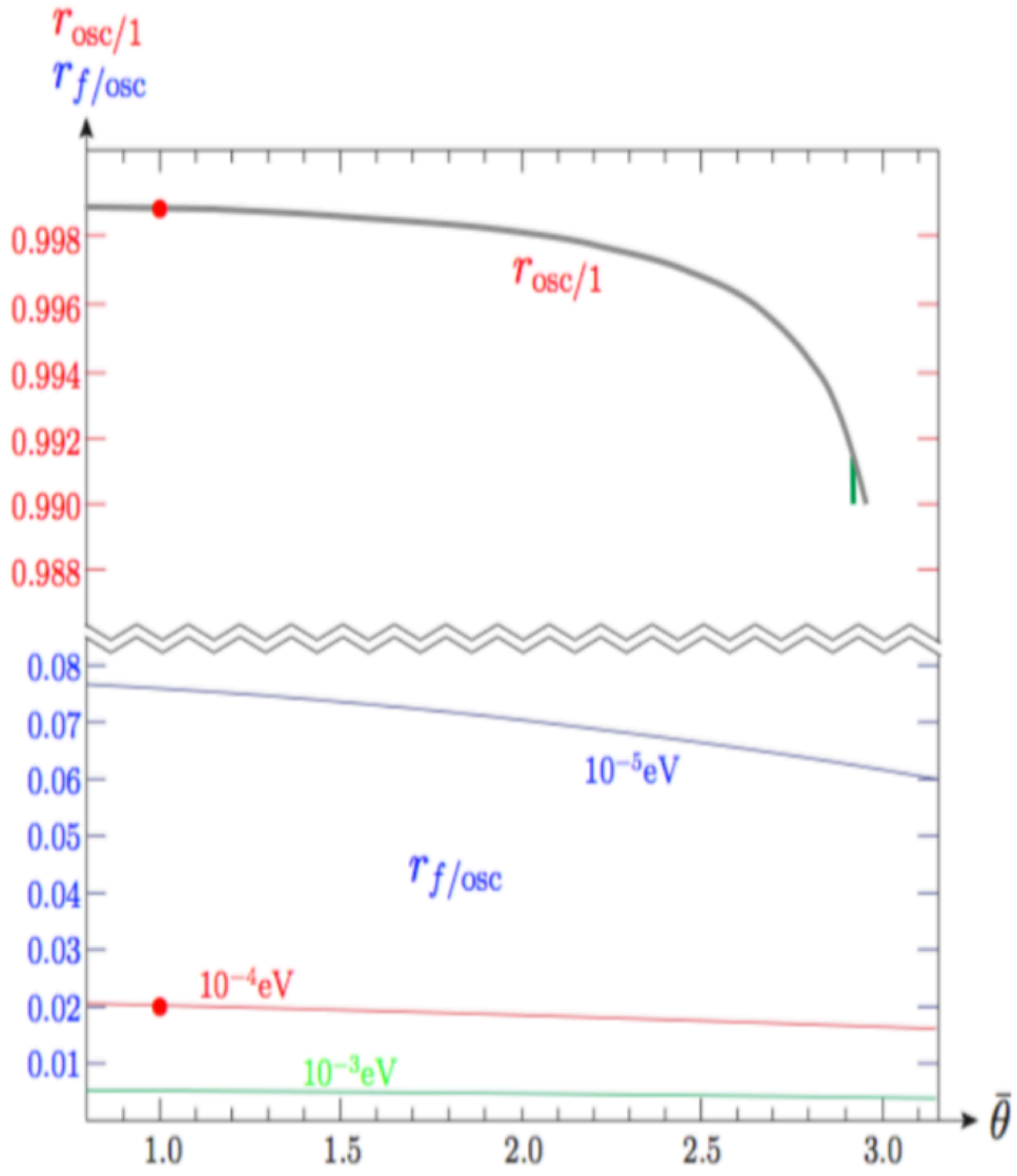
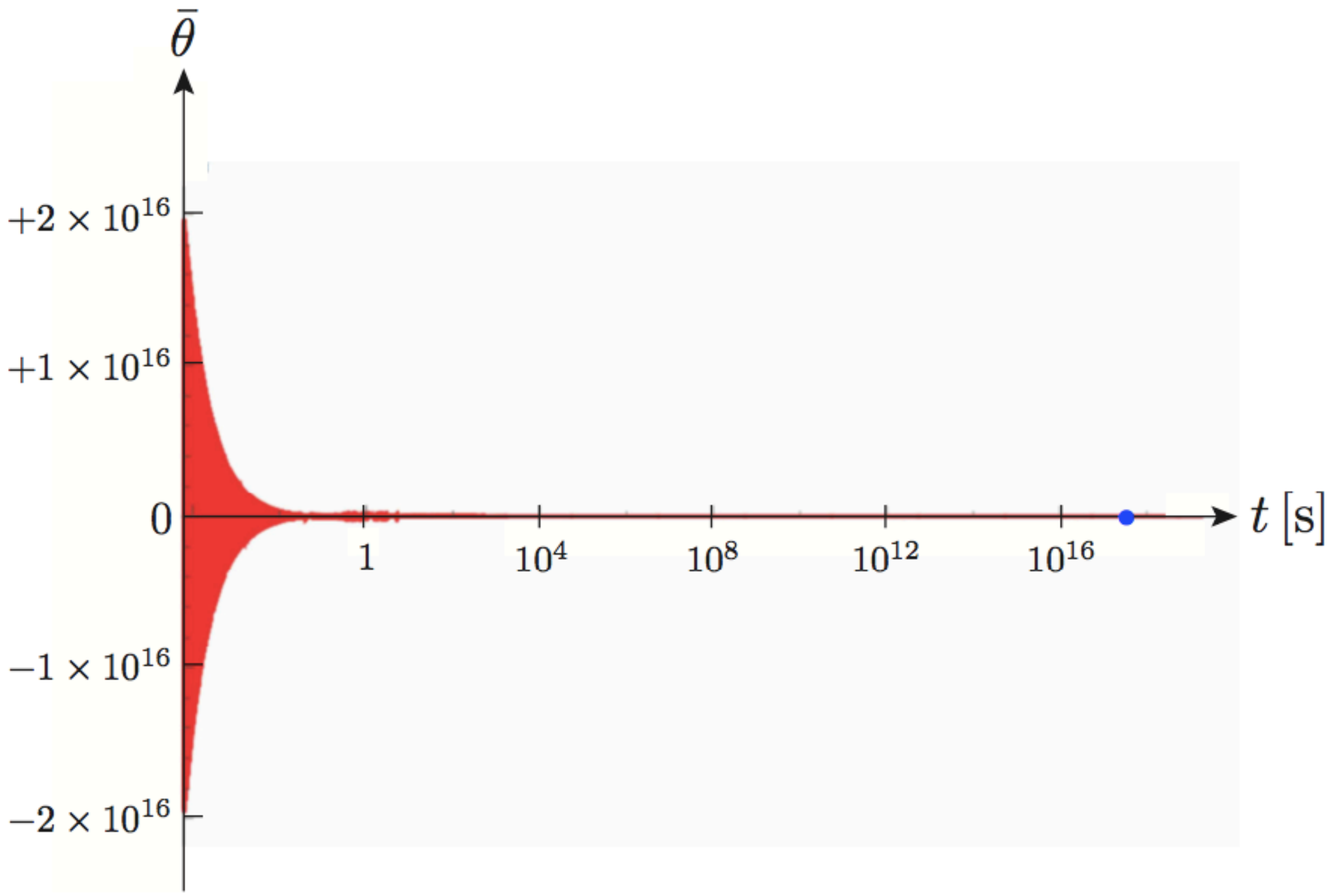
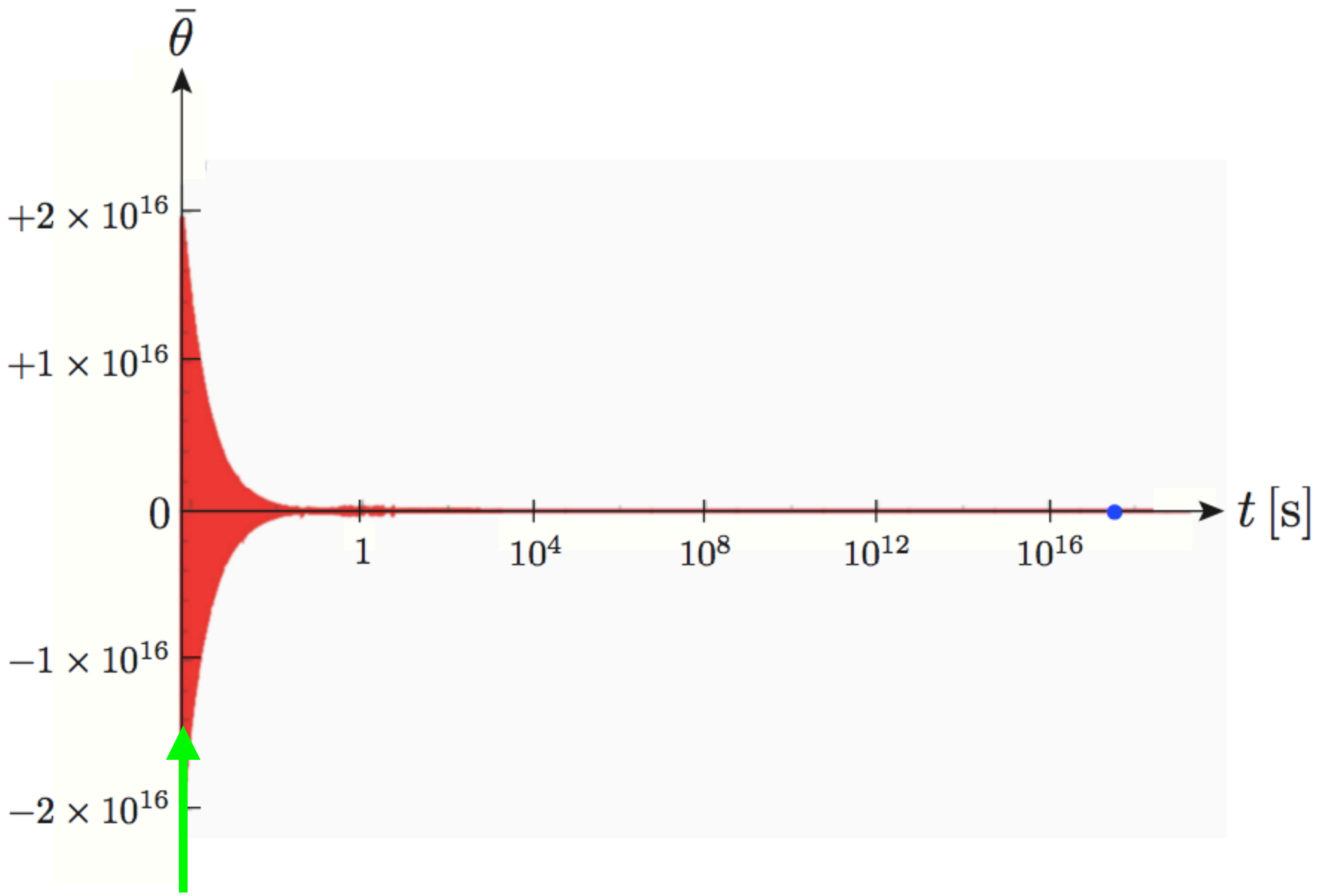


FIG. 7: The ratios $r_{\mathrm{osc}/1} \equiv \bar\theta_{\mathrm{osc}}/\bar\theta_1$ and $r_{f/\mathrm{osc}} \equiv \bar\theta_f/\bar\theta_{\mathrm{osc}}$ as functions of $\bar\theta_1$ for three $m_a(0)(= 10^{-3}\,\mathrm{eV}(\mathrm{green}), 10^{-4}\,\mathrm{eV}(\mathrm{red}),$ $10^{-5}\,\mathrm{eV}(\mathrm{blue})$. In the upper figure, these curves are almost overlapping (shown as gray) except the green for a large $\bar\theta_1$. [See also Supplement.] t_{osc} is the time of the 1st oscillation after which the harmonic motion is a good description. Different T_1's are used for different $m_a(0)$, as presented in Fig. 4.

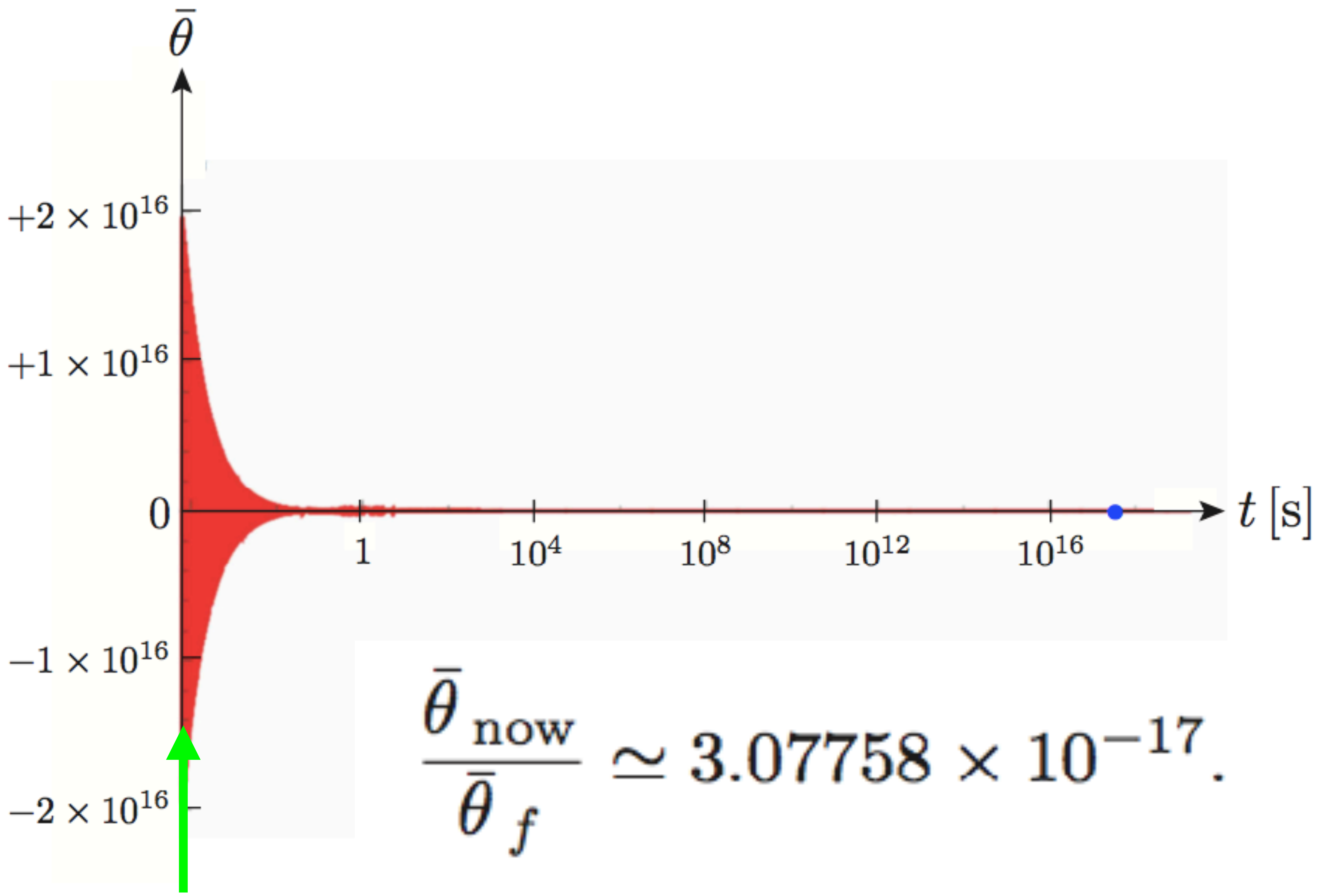
$$\bar\theta_{\mathrm{now}} \simeq \bar\theta_1 \cdot r_{f/1} \cdot \left(\frac{\bar\theta_{\mathrm{now}}}{\bar\theta_f}\right)$$

$$r_{f/1} \simeq 0.02 \left(\frac{m_a}{10^{-4}\,\mathrm{eV}}\right)^{-0.591\pm0.008}$$

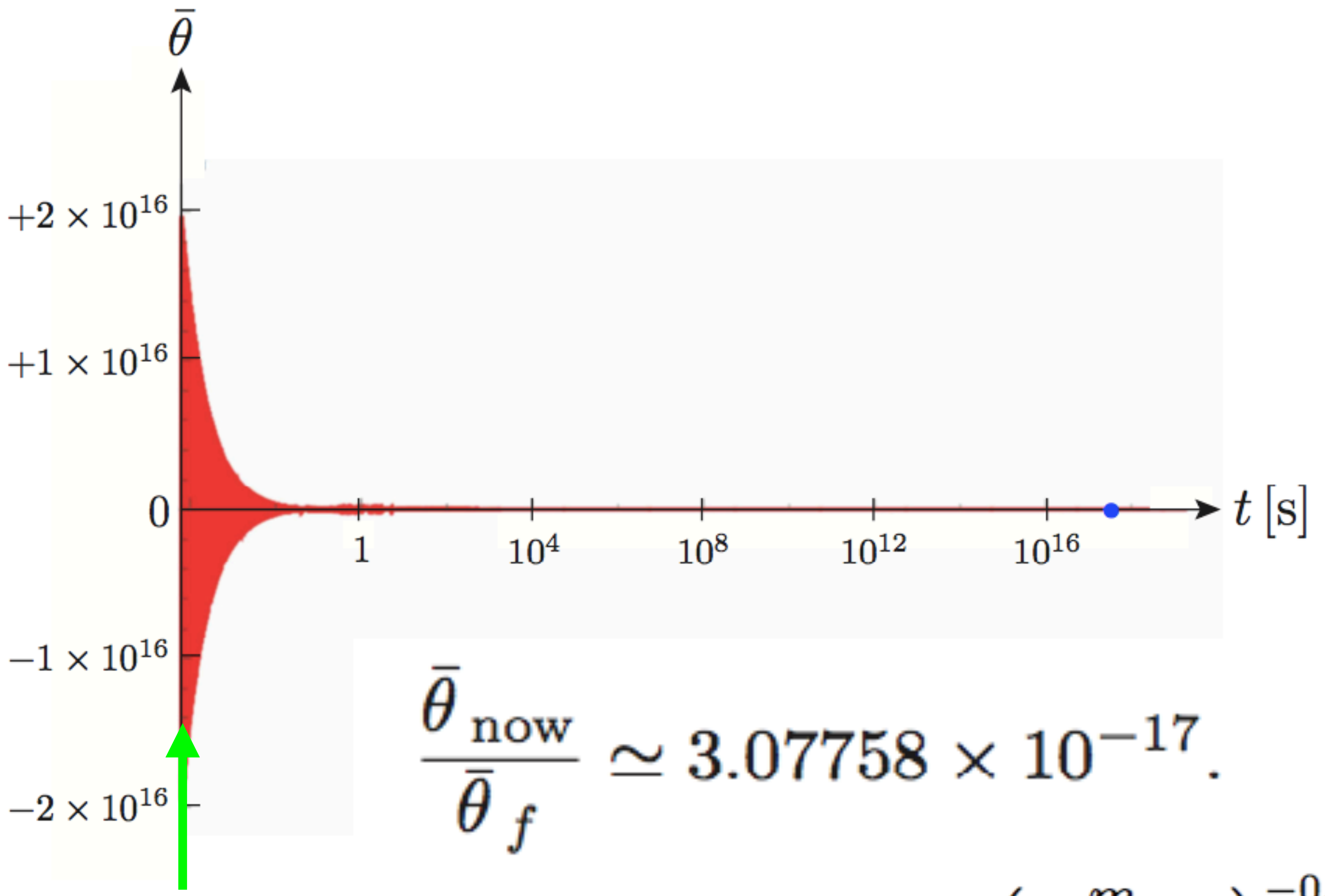

$\bar{\theta}$
$+2 \times 10^{16}$
$+1 \times 10^{16}$
0
-1×10^{16}
-2×10^{16}
1
10^4
10^8
10^{12}
10^{16}
t [s]



$\bar{\theta}$
$+2 \times 10^{16}$
$+1 \times 10^{16}$
0
-1×10^{16}
-2×10^{16}
1
10^4
10^8
10^{12}
10^{16}
t [s]



$$\frac{\bar{\theta}_{\rm now}}{\bar{\theta}_f} \simeq 3.07758\times10^{-17}.$$



$$\frac{\bar\theta_{\rm now}}{\bar\theta_f} \simeq 3.07758\times10^{-17}.$$

$$r_{f/1} \simeq 0.02\left(\frac{m_a}{10^{-4}\,{\rm eV}}\right)^{-0.591\pm0.008}$$

From t_f to t_{now}: (JEK, S. Kim, Nam, 1803.03517)

$$3.07758\text{x}10^{-17}$$

We calculated a new number F_{now}.

The final factor is

$$\bar{\theta}_{\rm now} \simeq \bar{\theta}_1 \cdot r_{f/1} \cdot \left(\frac{\bar{\theta}_{\rm now}}{\bar{\theta}_f}\right)$$

From t_f to t_{now}: (JEK, S. Kim, Nam, 1803.03517)

$$3.07758\times10^{-17}$$

We calculated a new number F_{now}.

The final factor is

$$\bar\theta_{\rm now} \simeq \bar\theta_1 \cdot r_{f/1} \cdot \left(\frac{\bar\theta_{\rm now}}{\bar\theta_f}\right) = 0.62\times10^{-18}\,\bar\theta_1$$

4. Hierarchy together with "invisible" axion

4. Hierarchy together with "invisible" axion

JEK+Kyae, 1904.07371

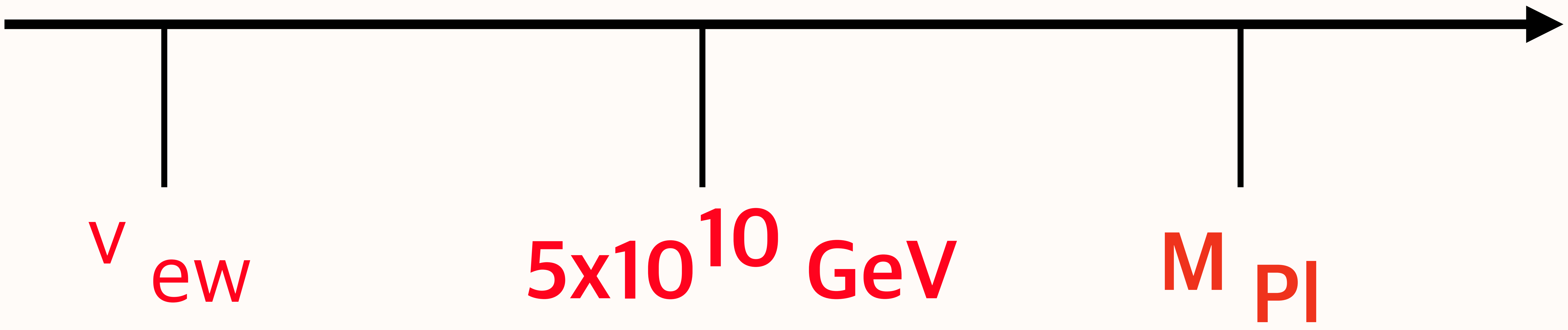
4. Hierarchy together with "invisible" axion

May be relates to —nepLES

JEK+Kyae, 1904.07371

Chirality is the one.

Chirality ensures small scales.



Common scale for f_a and source of SUSY breaking

Mass scales:

Planck mass 2.44×10^{18} GeV

Next scale defines physics disciplines

Particle physics 246 GeV

Strong Interaction 300 MeV

Nuclear physics 7 MeV

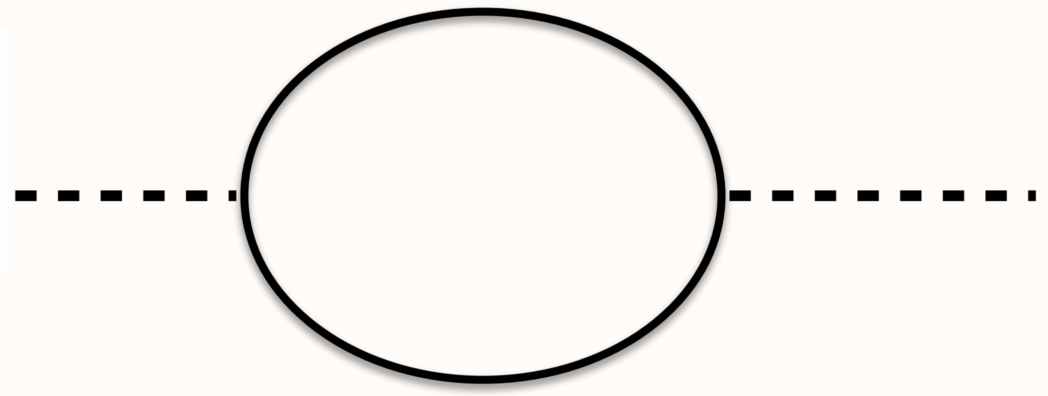
Atomic physics 1 eV

Condensed matter phys 10^{-3} eV

Mass hierarchy:

(Planck mass)/(EW scale) 10^{16}
(GUT mass)/(EW scale) 10^{14}

$$V = -M^2\Sigma^*\Sigma - v_{\rm ew}^2 H^\dagger H + \cdots$$



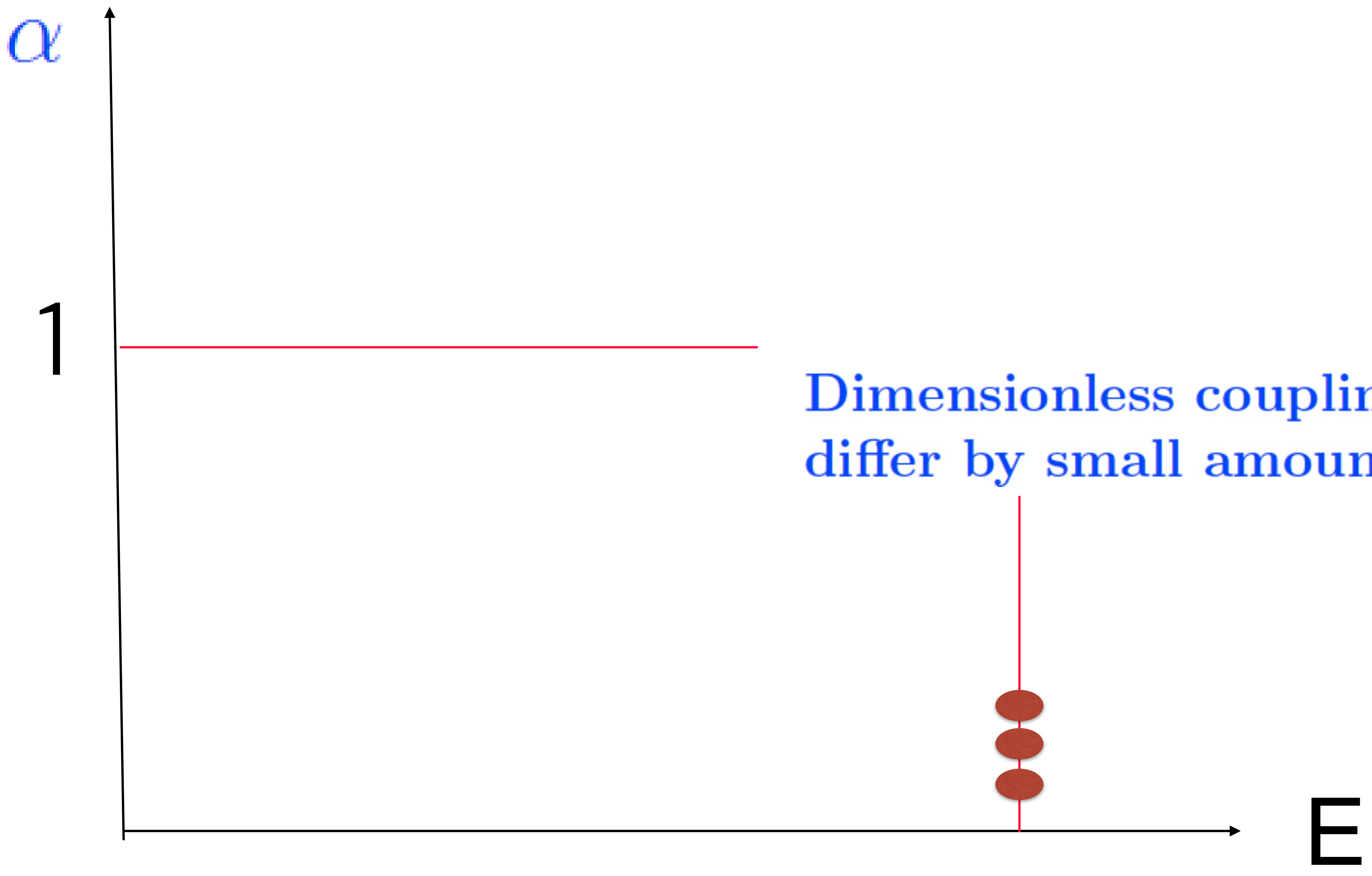
In the potential V, the scalar $(\text{mass})^2$ parameters have the ratio of 10^{28}.

Why is there such a large ratio of parameters? Including loop corrections?

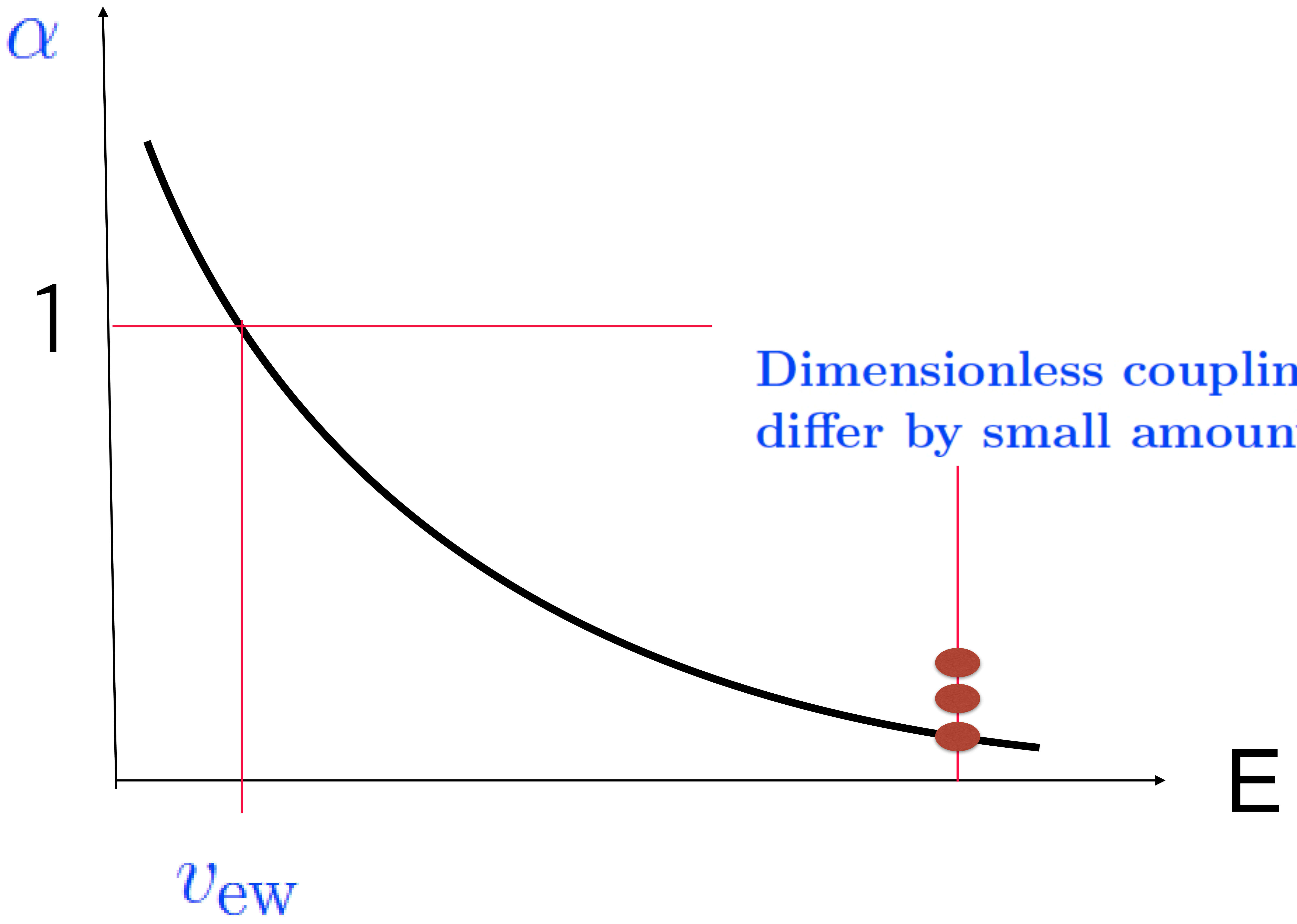
TeV is the cutoff.

This was pointed out by S.Weinberg after the GUT models were proposed. The GUT models must have parameters such that the Higgs mechanism breaks both SU(5) and SU(2)xU(1) SM.

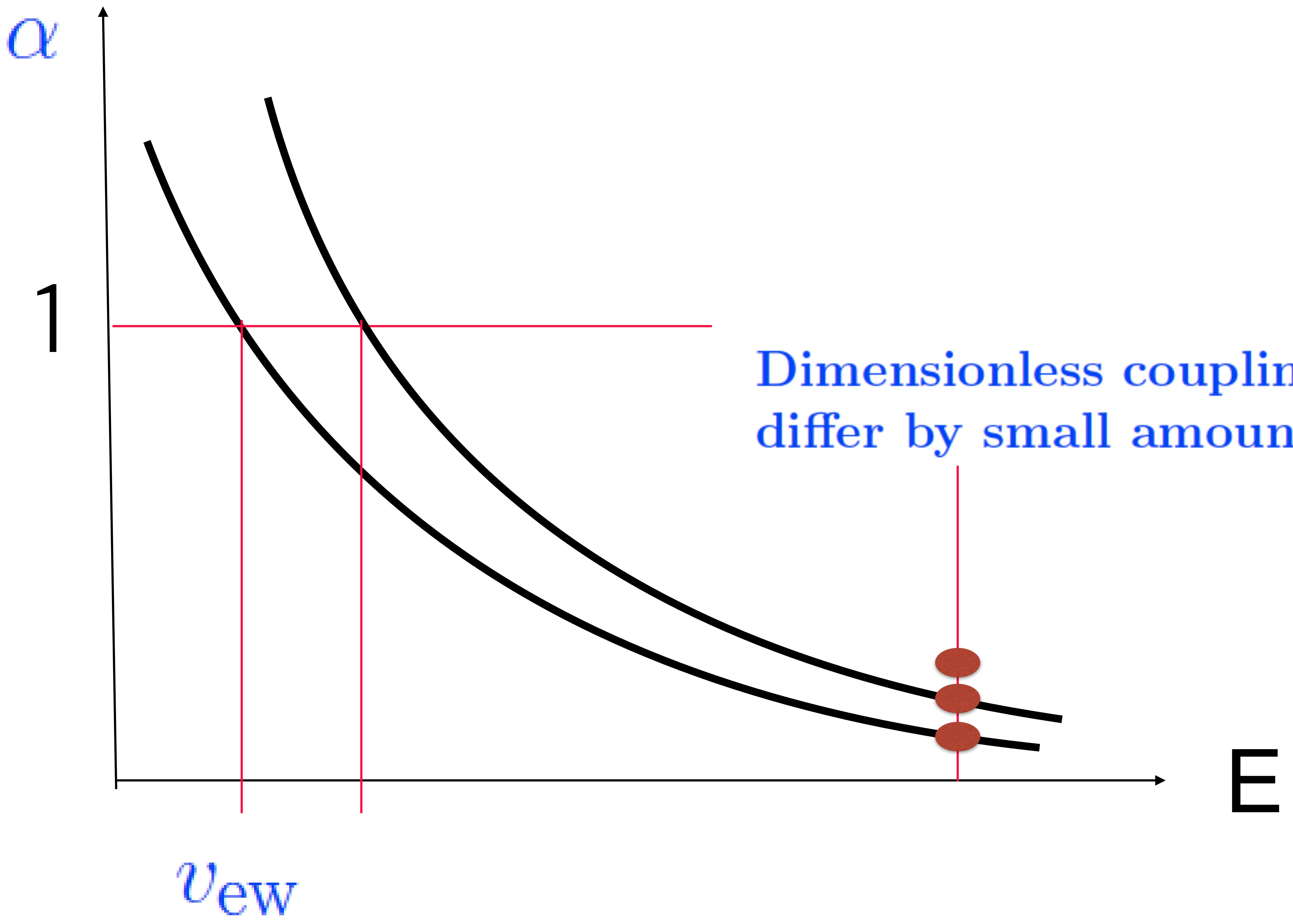
An exponential hierarchy obtained by dimensional transmutation.

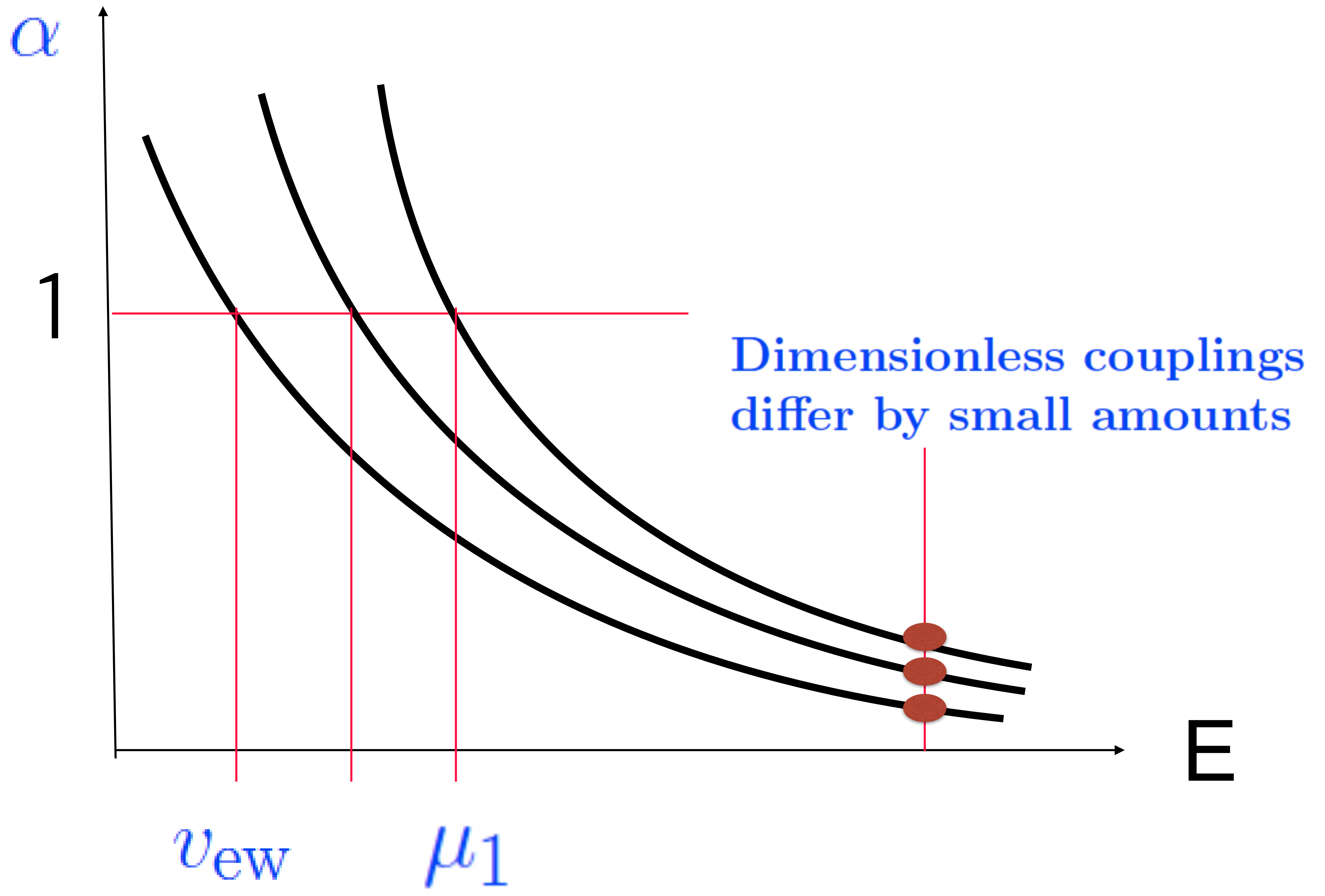


α
1
Dimensionless couplings
differ by small amounts
E



α
1
Dimensionless couplings differ by small amounts
E
$v_{\rm ew}$


α
1
Dimensionless couplings
differ by small amounts
E
$v_{\rm ew}$



α
1
Dimensionless couplings
differ by small amounts
E
$v_{\rm ew}$
μ_1

1st confining force:

Technicolor confines at 3 TeV: Susskind and Weinberg 1979.

exponential 3 TeV= $M_{GUT} \times e^{-40}$.

Dimensional transmutation, e.g. 300 MeV

But it failed in flavor physics, by extended technicolor, through S and T parameter constraints.

Yukawa couplings are definitely needed: scalars are needed definitely.

SUSY idea: 1981~
Supergravity phenomenology: 1983~
Supersymmetry: LSP added for DM candidate: 1984~

SUSY breaking needed: Needed for SM partners ~(TeV)^2,

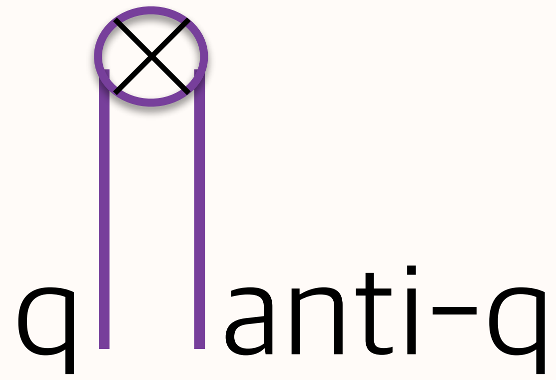
Source of SUSY breaking: 10^{13} GeV, by Gaugino condensation

L = SUSY terms + SUSY breaking soft terms of O(TeV^2)

Gaugino condensation by R=0 singlet:
Nilles(1982),
Derendinger-Ibanes-Nilles(85),
Dine-Rohm-Seiberg-Witten(85)

Condensation of q and anti-q in QCD:

$(300\ \mathrm{MeV})^3$



Similarly, gauginos in SUSY theory may condense: Gaugino condensation.

$(10^{13}\mathrm{GeV})^3$

Then, scalar $(\mathrm{mass})^2$ parameters in the SM feel this breaking by gravity effects: $(10^{13}\mathrm{GeV})^3$ / $(\text{Planck mass})^2$ ~ TeV

How come, "Is there such source of 10^{13}GeV confining force?

Is there really "gaugino condensation"?

DIN, DRSW.

SQCD before, SGUT now

SQCD before, SGUT now

SUSY SU(N) gauge theory with L-handed q and R-handed q.

SU(N_c) gauge group

SU(N_f)xSU(N_f) flavor group (global)

Introducing a vector-like representation.

Studied extensively by Seiberg and his collaborators, and many more. These focussed on duality and not obtained SUSY breaking from the gauge theory.

Anomaly free theories.

fundamentals: [1] one contra-variant index,
[2] two contra-variant index, etc.

SU(3): only quarks or anti-quarks
SU(4): only quarks or anti-quarks plus [2]=self-dual (removed)

SU(5): the smallest gauge group to have a chiral representation, [2] + [4] which is anomaly free. Due to Georgi's criteria, this is the simplest example.

$$[2] = \Psi^{\alpha\beta} \;:\; \Psi^{\alpha\beta} = -\Psi^{\beta\alpha}$$

$$[4] = \psi^{\beta\gamma\delta\rho} \;:\; \bar{\psi}_\alpha = [\bar{1}]$$

$$\Psi^{\alpha\beta} \oplus \bar{\psi}_\alpha : \text{Anomaly-free Georgi-Glashow model}$$

Meurice-Veneziano considered this SUSY one-family GG model, and suggested a possibility of dynamical SUSY breaking.

$$[2] = \Psi^{\alpha\beta} \; : \; \Psi^{\alpha\beta} = -\Psi^{\beta\alpha}$$

$[4] = \psi$

$\Psi^{\alpha\beta} \oplus$

Meurice-Ver ... nd

suggested a

In conclusion, we have found that in theories with chiral fermions the presence of several currents which do not fall into a vector/axial vector classification brings about strong constraints on SUSY vacua. These constraints, once coupled to reliable small-size instanton effects, lead in certain cases to a contradiction. Even the SQCD way out of a vacuum at infinity appears to be blocked leaving us with the only possibility of a non-supersymmetric ground state.

In the near future further calculations should not fail to provide a complete systematics of the circumstances under which spontaneous SUSY breaking take place.

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Our model (took 36 years):

Generation of M_I

JEK+Kyae: 1904.07371 "A model for dynamical SUSY breaking "

SU(5) representation:

[2] + [1] + 2[4] which is anomaly free.

$$\bar{\Psi}^{\alpha\beta} \oplus \bar{\psi}_1^{\alpha} \oplus 2\cdot\psi_{2\alpha}$$

$$(\overline{\mathbf{10}},\mathbf{1}) \oplus (\overline{\mathbf{5}},\mathbf{1}) \oplus (\mathbf{5},\mathbf{2})$$

$$(\mathrm{SU}(5)_{\rm gauge}, \mathrm{SU}(2)_{\rm global})$$

Now we can construct superpotential terms,

$$W_0 \ni \frac{1}{4}\bar{\Psi}^{\alpha\beta}\psi^i_{2\alpha}\psi^j_{2\beta}\epsilon_{ij},\ \ \bar{\psi}^\alpha_1\psi^i_{2\alpha}D_{1i},\ \ \frac{1}{5!}\bar{\Psi}^{\alpha\beta}\bar{\Psi}^{\gamma\delta}\bar{\psi}^\epsilon_1\,\epsilon_{\alpha\beta\gamma\delta\epsilon},$$

This is not possible with Meurice-Veneziano. In ours, one U(1) remaining.

$$\begin{aligned}
&U(1)_{\bar{\Psi}} + 2U(1)_{\psi_2} = 0,\\
&U(1)_{\bar{\psi}_1} + U(1)_{\psi_2} + U(1)_{D_1} = 0,\\
&2U(1)_{\bar{\Psi}} + U(1)_{\bar{\psi}_1} = 0.
\end{aligned}$$

$\mathrm{SU(5)_{gauge}}-$singlet chial fields,

$$\phi = \frac{1}{5!}\bar{\Psi}^{\alpha\beta}\bar{\Psi}^{\gamma\delta}\bar{\psi}^\epsilon_1\,\epsilon_{\alpha\beta\gamma\delta\epsilon},\ \ \Phi_i = \bar{\psi}^\alpha_1\psi_{2\,\alpha\,i}.$$

$U(1)_{global}$-$SU(2)_{gauge}$-$SU(2)_{gauge}$ anomaly below conf. scale

As in axion physics theta term is considered by triangle loops.

$$\sim \frac{\theta}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}$$

Confinement of SU(2) leads to this anomaly, due to instanton calculus, even if we integrate out the SU(2) charged fermions. If we consider infinite spacetime, gauged SU(2) is like global SU(2). So, we satisfy

$U(1)_{global}$-$SU(2)_{global}$-$SU(2)_{global}$ anomaly

For U(1), we do not have the instanton argument, and there is no need to match

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$U(1)_{gauge}$

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$U(1)_{global}$-$U(1)_{global}$-$U(1)_{global}$ anomaly

$\boxed{U(1)_{gauge}}$ $\boxed{U(1)_{gauge}}$

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Even if we consider it, we know that it is a total derivative.

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$$\sim \frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Even if we consider it, we know that it is a total derivative.

$$\begin{aligned}
&\propto \theta \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\rho A_\sigma - \partial_\sigma A_\rho) = \theta\epsilon_{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma) \\
&= \partial_\rho[\theta\epsilon_{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(A_\sigma)] - [\partial_\rho(\theta\epsilon_{\mu\nu\rho\sigma}\partial_\mu A_\nu)](A_\sigma) \\
&= \partial_\rho[\theta\epsilon_{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(A_\sigma)] - [\theta\epsilon_{\mu\nu\rho\sigma}\partial_\rho\partial_\mu A_\nu](A_\sigma) \\
&= \partial_\rho[\theta\epsilon_{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(A_\sigma)]
\end{aligned}$$

	$2\ell(R_{SU(5)})$	$SU(2)$	$U(1)_{\bar\Psi}$	$U(1)_{\bar\psi_1}$	$U(1)_{\psi_2}$	$U(1)_{D_1}$	$U(1)_{AF}$	$U(1)_R$	dimension
ϑ	0	0	0	0	0	0	0	$+1$	$\frac{1}{2}$
$\bar\Psi\sim(\overline{\mathbf{10}},\mathbf{1})$	–	$\mathbf{1}$	$+1$	0	0	0	-1	$+1$	1
fermion	$+3$	$\mathbf{1}$	$+1$	0	0	0	-1	0	–
$\bar\psi_1\sim(\overline{\mathbf{5}},\mathbf{1})$	–	$\mathbf{1}$	0	$+1$	0	0	$+2$	0	1
fermion	$+1$	$\mathbf{1}$	0	$+1$	0	0	$+2$	-1	–
$\psi_2\sim(\mathbf{5},\mathbf{2})$	–	$\mathbf{2}$	0	0	$+1$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	1
fermion	$+1\times 2$	$\mathbf{2}$	0	0	$+1$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	–
$D\sim(\mathbf{1},\mathbf{2})$	–	$\mathbf{2}$	0	0	0	$+1$	$-\frac{5}{2}$	$+\frac{3}{2}$	1
fermion	$+1\times 2$	$\mathbf{2}$	0	0	0	$+1$	$-\frac{5}{2}$	$+\frac{1}{2}$	–
$W^a\sim\lambda^a$	0	–	0	0	0	0	0	$+1$	$\frac{3}{2}$
Λ^b		–	–	–	–	–	–	$\frac{2b}{3}$	b
ϕ	–	$\mathbf{1}$	–	–	–	–	-5	$+2$	1
fermion	–	$\mathbf{1}$	–	–	–	–	-5	$+1$	–
Φ_i	–	$\mathbf{2}$	–	–	–	–	$+\frac{5}{2}$	$+\frac{1}{2}$	1
fermion	–	$\mathbf{2}$	–	–	–	–	$+\frac{5}{2}$	$-\frac{1}{2}$	–
S	–	$\mathbf{1}$	0	0	0	0	0	$+2$	1
fermion	–	$\mathbf{1}$	0	0	0	0	0	$+1$	–
$D^i\sim(\mathbf{1},\mathbf{2})$	–	$\mathbf{2}$	–	–	–	$+1$	$-\frac{5}{2}$	$+\frac{3}{2}$	1
fermion	–	$\mathbf{2}$	–	–	–	$+1$	$-\frac{5}{2}$	$+\frac{1}{2}$	–

The superpotential consistent with $SU(2)_{global} \times U(1)_{global}$ is

$$W = M^2\phi + \frac{N_c(N_c^2-1)}{32\pi^2}\mu_0^2 S\left(1 - a\log\frac{\Lambda^3}{S\mu_0^2}\right) + bM\Phi_i D^i,$$

$$\frac{\partial W}{\partial \phi} = 0: \;\; M^2 = 0,$$

$$\frac{\partial W}{\partial \Phi_i} = 0: \;\; D^i = 0,$$

$$\frac{\partial W}{\partial D^i} = 0: \;\; \Phi_i = 0,$$

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Gaugino condensation

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Gaugino condensation

SUSY is broken by the 'O Raifeartaigh mechanism!!!
This is shown here for the first time.
So, we have a solution for the gauge hierarchy problem.

$$\frac{\lambda_0}{5!}\,\bar{\Psi}^{\alpha\beta}\,\bar{\Psi}^{\gamma\delta}\,\bar{\psi}_1^{\epsilon}\,\epsilon_{\alpha\beta\gamma\delta\epsilon} \to \lambda_0\mu_0^2\,\phi$$

$\boxed{\text{At SU(5)' level}}$

$\boxed{M^2}$

$$\frac{\lambda_0}{5!}\,\bar{\Psi}^{\alpha\beta}\,\bar{\Psi}^{\gamma\delta}\,\bar{\psi}_1^{\epsilon}\,\epsilon_{\alpha\beta\gamma\delta\epsilon} \to \lambda_0\mu_0^2\,\phi$$

At SU(5)' level

M^2

If lambda0 is nonzero, M^2 is nonzero

Common scale for SUSY breaking and f_a

Common scale for SUSY breaking and f_a

So, if the hidden SU(5)' confines at 10^{13} GeV - 5×10^{10} GeV, the SUSY breaking scale for SM partners is above 1 TeV.

In particular, the lower end 5×10^{10} - 10^{11} GeV is particularly interesting because it is the anticipated axion scale, which is the most difficult region for axion search.

The SU(5)' confinement provides this region because of the scalar condensation, rather than gaugino condensation.

In our case, the confinement scale by singlet composite scalar is somewhere between 5×10^{10} GeV—10^{12} GeV. Not at 10^{13} GeV.

With this, M_{SUSY} can be raised to the scale of the little hierarchy. The super partner scale at a TeV needs $a^{1/2} \times 5 \times 10^{10}$ GeV for the confinement scale. 6 TeV needs 10^{11} GeV confinement scale.

5. Conclusion

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1. Axion theory discussed.
2. “Invisible” axion from string is commented.
3. How this intermediate scale is realized is proved by a SU(5)’ confining force.
4. This can be used for the little hierarchy with fine-tuning of order less than 100.

Randall: To discover one, one should be an expert in model building [APS Denver, April, 2019]