Quantum effects in axion dark matter

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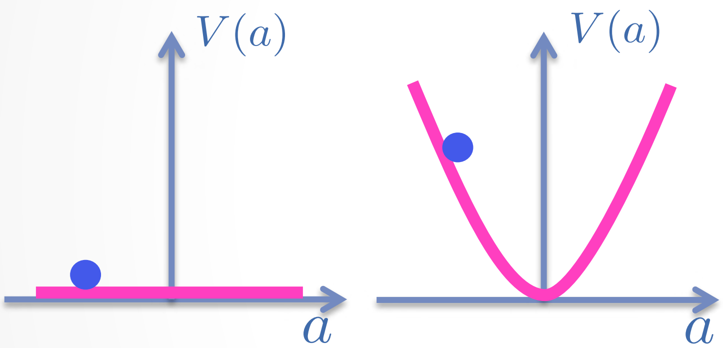
Outline

- Motivation
- Duration of classicality
- Method to calculate duration of classicality in quantum field theory
- Application of method
  - Self-gravitating homogeneous field
  - Inhomogeneous field with repulsive contact interactions
- Conclusions

Axion Dark Matter Production

- Non perturbative QCD effects ‘turn on’ the axion mass
- Vacuum realignment



$T \geq 1$ GeV $\qquad$ $T \leq 1$ GeV

$$n_a(t) = \frac{f_a^2}{2t_1}\left(\frac{R(t_1)}{R(t)}\right)^3$$

$$\delta p(t) = t_1^{-1}\left(\frac{R(t_1)}{R(t)}\right)$$

$$m_a(t_1)\, t_1 = 1$$

$$t_1 \simeq 10^{-7}\ \mathrm{s}\left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{1/3}$$

$$\mathcal{N} \sim \frac{(2\pi)^3 n_a}{\frac{4\pi}{3}(\delta p)^3} \sim 10^{60}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{8/3}$$

Axion Dark Matter

- Axions from vacuum realignment

$$\mathcal{N} \sim 10^{60} \overset{?}{\Rightarrow} \quad \text{classical field theory}$$

- Is this true in the presence of interactions? For how long?

- Axion interactions
  - Quartic self-interactions $V_a \simeq \lambda\, a^4$
  - Gravitational self-interactions
  - Number changing interactions

- Are these important? Relaxation rate vs. Hubble rate

$$\Gamma(t) \overset{?}{>} H(t)$$

Bose-Einstein Condensation

- Thermalizing interaction: gravity

Sikivie, Yang, Phys. Rev. Lett. 103 (2009) 111301

$$\Gamma(t) > H(t) \qquad \text{for} \qquad T \lesssim 500 \text{ eV} \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{1/2}$$

- Necessary and sufficient conditions for Bose-Einstein condensation
  1. Large number of identical Bosons
  2. Conservation of the number of particles
  3. High degeneracy $\mathcal{N} > \mathcal{O}(1)$
  4. Effective thermalizing interaction

- Quantum mechanics $\Rightarrow$ Bose-Einstein distribution $\Rightarrow$ BEC

- Can we use classical field theory instead if $\mathcal{N} \gg 1$?

Duration of Classicality

$$H = \sum_j \omega_j a_j^\dagger a_j + \sum_{jklm} \frac{1}{4} \Lambda_{jk}^{lm} \; a_j^\dagger a_k^\dagger a_l a_m$$

Quantum theory

$$[a_j, a_k^\dagger] = \delta_{jk}$$

$$i\dot{a}_j = \omega_j a_j + \sum_{klm} \frac{1}{2} \Lambda_{jk}^{lm} \; a_k^\dagger a_l a_m$$

$$\mathcal{N}_j(t) = a_j^\dagger(t) a_j(t)$$

$$\langle \mathcal{N}_j(t) \rangle = \langle \Psi | \mathcal{N}_j(t) | \Psi \rangle$$

Classical analogue

$$i\dot{A}_j = \omega_j A_j + \sum_{klm} \frac{1}{2} \Lambda_{jk}^{lm} \; A_k^* A_l A_m$$

$$N_j(t) = A_j^*(t) A_j(t)$$

Duration of Classicality

Quantum theory

$\langle \mathcal{N}_j(0) \rangle$

$\langle \mathcal{N}_j(t) \rangle$

τ

$$\langle \mathcal{N}_j \rangle = \frac{1}{\mathrm{e}^{\frac{1}{T}(\varepsilon_j - \mu)} - 1}$$

(+BEC)

t_{cl}?

Classical theory

$N_j(0) \sim \langle \mathcal{N}_j(0) \rangle$

$N_j(t)$

$$N_j = \frac{T}{\varepsilon_j - \mu}$$

Gravitational Self-Interactions

- Non-relativistic limit

$$H_G = -\frac{G}{2}\int d^3r\ d^3r'\ \frac{:\psi^\dagger(\vec{r},t)\psi(\vec{r},t)\psi^\dagger(\vec{r}\,',t)\psi(\vec{r}\,',t):}{|\vec{r}-\vec{r}\,'|}$$

- Heisenberg equations of motion

$$\begin{aligned} i\partial_t\psi(\vec{r},t) &= -\tfrac{1}{2m}\nabla^2\psi(\vec{r},t) + m\varphi(\vec{r},t)\psi(\vec{r},t) \\ \nabla^2\varphi(\vec{r},t) &= 4\pi G m \psi^\dagger(\vec{r},t)\psi(\vec{r},t) \end{aligned}$$

- Schrödinger-Poisson equations (classical)

$$\begin{aligned} i\partial_t\Psi(\vec{r},t) &= -\tfrac{1}{2m}\nabla^2\Psi(\vec{r},t) + m\Phi(\vec{r},t)\Psi(\vec{r},t) \\ \nabla^2\Phi(\vec{r},t) &= 4\pi G m|\Psi(\vec{r},t)|^2 \end{aligned}$$

S. Chackrabarty, S. Enomoto, Y. Han, P. Sikivie, E. T., PR D97 (2018) no. 4, 043531

Method to get t_{cl} in QFT

Step 1: Expand field over ONC mode functions

$$\psi(\vec{r},t) = \sum_{\vec{\alpha}} u^{\vec{\alpha}}(\vec{r},t) a_{\vec{\alpha}}(t) \qquad \Psi(\vec{r},t) = \sum_{\vec{\alpha}} u^{\vec{\alpha}}(\vec{r},t) A_{\vec{\alpha}}(t)$$

$$u^{\vec{\alpha}}(\vec{r},t) = \frac{1}{\sqrt{N}} \Psi(\vec{r},t)\ \mathrm{e}^{i\vec{\alpha}\cdot\vec{\chi}(\vec{r},t)}$$

where $\Psi(\vec{r},t)$ solves the classical field equations

- $\Psi(\vec{r},t)$ defines a fluid with number density and velocity

$$\begin{aligned} n(\vec{r},t) &= \Psi^*(\vec{r},t)\Psi(\vec{r},t) \\ \vec{v}(\vec{r},t) &= \frac{1}{2imn}(\Psi^*\vec{\nabla}\Psi - \Psi\vec{\nabla}\Psi^*) \end{aligned}$$

- Choose $\vec{\chi}$ to be comoving coordinates, such that the density in $\vec{\chi}$ space is constant

- The classical evolution is trivial $\qquad A_{\vec{\alpha}}(t) = \sqrt{N}\delta^{\vec{0}}_{\vec{\alpha}}$

Method to get $t_{\rm cl}$ in QFT

- Deviation from classical theory as $\vec{\alpha} \neq \vec{0}$ modes get occupied quantum mechanically

Step 2: calculate coefficients of equations of motion

$$i\partial_t a_{\vec{\alpha}} = \sum_{\vec{\beta}} M_{\vec{\alpha}}^{\vec{\beta}}\ a_{\vec{\beta}} + \sum_{\vec{\beta}\vec{\gamma}\vec{\delta}} \frac{1}{2}\Lambda_{\vec{\alpha}\vec{\beta}}^{\vec{\gamma}\vec{\delta}}\ a^\dagger_{\vec{\beta}} a_{\vec{\gamma}} a_{\vec{\delta}}$$

$$\begin{aligned} M_{\vec{\alpha}\vec{\beta}}(t) &= \int_V d^3r\ u^{\vec{\alpha}}(\vec{r},t)^*(-i\partial_t - \frac{1}{2m}\nabla^2)u^{\vec{\beta}}(\vec{r},t) \\ \Lambda_{\vec{\alpha}\vec{\beta}}^{\vec{\gamma}\vec{\delta}}(t) &= -Gm^2 \int_V d^3r \int_V d^3r' \frac{1}{|\vec{r}-\vec{r}\,'|} \times \\ &\quad \times\ u^{\vec{\alpha}}(\vec{r},t)^* u^{\vec{\beta}}(\vec{r}\,',t)^*\ (u^{\vec{\gamma}}(\vec{r},t)u^{\vec{\delta}}(\vec{r}\,',t) + u^{\vec{\gamma}}(\vec{r}\,',t)u^{\vec{\delta}}(\vec{x},t)) \end{aligned}$$

Step 3: Bogoliubov ansatz

- Large occupation of $\vec{\alpha} = \vec{0}$

$$\begin{aligned} a_{\vec{0}} &\rightarrow \sqrt{N} \\ a_{\vec{\alpha}} &\rightarrow b_{\vec{\alpha}}, \quad \text{for} \quad \vec{\alpha} \neq \vec{0} \end{aligned}$$

Method to get $t_{\rm cl}$ in QFT

Step 4: expand equations for $b_{\vec{\alpha}}$ in powers of $1/\sqrt{N}$ and keep first order terms only. Solve.

$$i\partial_t b_{\vec{\alpha}} = \sum_{\vec{\beta}\neq\vec{0}} \left[\left(M_{\vec{\alpha}}^{\vec{\beta}} + \Lambda_{\vec{\alpha}\vec{0}}^{\vec{\beta}\vec{0}} \right) N \; b_{\vec{\beta}} + \frac{1}{2} \Lambda_{\vec{\alpha}\vec{\beta}}^{\vec{0}\vec{0}} \; N \; b_{\vec{\beta}}^{\dagger} \right]$$

Step 5: duration of classicality

- Expectation value $\langle \mathcal{N}_{\vec{\alpha}}(t) \rangle = \langle 0 | b_{\vec{\alpha}}^{\dagger}(t) b_{\vec{\alpha}}(t) | 0 \rangle$ for $\vec{\alpha} \neq \vec{0}$
- Evaporation $$N_{ev}(t) = \sum_{\vec{\alpha}\neq\vec{0}} \langle \mathcal{N}_{\vec{\alpha}}(t) \rangle$$
- Duration of classicality

$$N_{ev}(t_{\rm cl}) \sim N$$

Homogeneous State

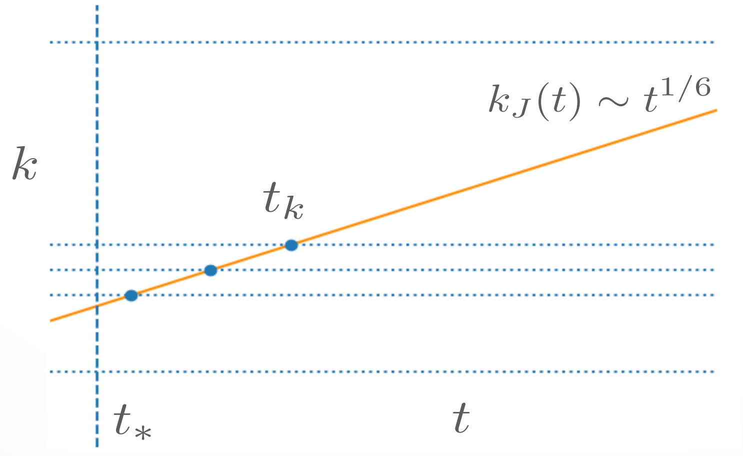
- Classical solution

$$\Psi_0(\vec{r},t) = \sqrt{n_0(t)}\ \mathrm{e}^{\frac{i}{2}mH(t)r^2} \qquad \Phi_0(\vec{r},t) = \frac{2\pi G m}{3} n_0(t) r^2$$

- Mode functions

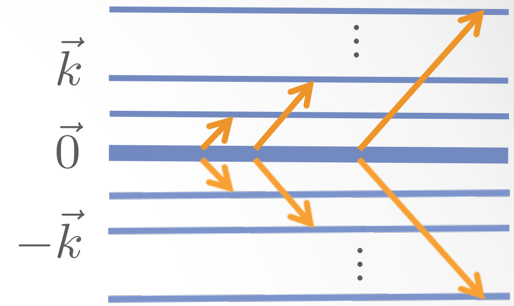
$$u^{\vec{k}}(\vec{r},t) = \sqrt{\frac{n_0(t)}{N}}\ \mathrm{e}^{\frac{i}{2}mH(t)r^2 + i\frac{\vec{k}\cdot\vec{r}}{R(t)}}$$

- Jeans wavenumber $k_J(t) = (16\pi G m^3 n(t))^{1/4} R(t)$



Homogeneous State

- Unstable modes $k < k_J(t)$



$$\langle \mathcal{N}_{\vec{k}}(t) \rangle = \frac{1}{10} \left(\frac{t}{t_*} \right)^2 \left(\frac{t_*}{t_k} \right)^{1/3} \qquad t \gg t_k > t_*$$

$$\langle \mathcal{N}_{\vec{k}}(t) \rangle = \frac{1}{10} \left(\frac{t}{t_k} \right)^2 \qquad t \gg t_* > t_k$$

- Evaporation of the homogenous state

$$N_{ev}(t) = \sum_{\vec{k}} \langle \mathcal{N}_{\vec{k}}(t) \rangle \sim 0.221 N G m^2 \sqrt{m t_*} \left(\frac{t}{t_*} \right)^2$$

- Duration of classicality

$$t_{\mathrm{cl}} \sim t_* \frac{1}{(G m^2 \sqrt{m t_*})^{1/2}}$$

Contact Self-Interactions

- Gross-Pitaevskii equation

$$i\partial_t \Psi(\vec{x},t) = -\frac{1}{2m}\nabla^2\Psi(\vec{x},t) + \frac{\lambda}{8m^2}\Psi^*(\vec{x},t)\Psi(\vec{x},t)\Psi(\vec{x},t)$$

- Homogenous case, $\lambda > 0$

$$\Psi_0(\vec{x},t) = \sqrt{n_0}\,\mathrm{e}^{-i\delta\omega t} \qquad \delta\omega = \frac{\lambda n_0}{8m^2}$$

- Duration of classicality

$$t_{\rm cl} = \infty$$

A. Arza, S. Chackrabarty, S. Enomoto, Y. Han, E. T., arXiv:1907.04941

One-mode perturbation

- Linearize

$$\Psi(\vec{x},t) = \Psi_0(t) + \Psi_1(\vec{x},t)$$

- Number density and velocity

$$\begin{aligned} n(\vec{x},t) &= n_0 + n_1(\vec{x},t) \\ \vec{v}(\vec{x},t) &= \vec{v}_1(\vec{x},t) \end{aligned}$$

- Classical solution: one plane wave

$$\begin{aligned} n_1(\vec{x},0) &= \delta n \ \cos(\omega_p t - \vec{p}\cdot\vec{x}) \\ \vec{v}_1(\vec{x},0) &= \delta\vec{v} \ \cos(\omega_p t - \vec{p}\cdot\vec{x}) \end{aligned}$$

$$\omega_p = \sqrt{\frac{p^2}{2m}\left(\frac{p^2}{2m} + 2\delta\omega\right)}$$

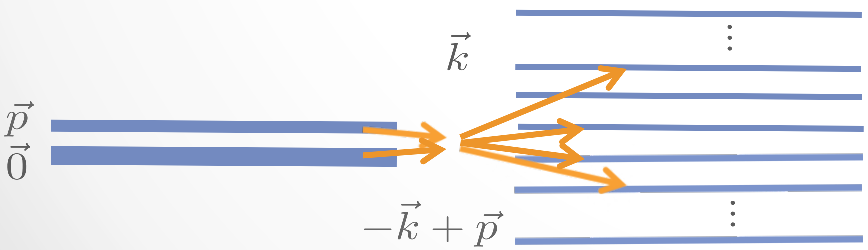
One-mode perturbation

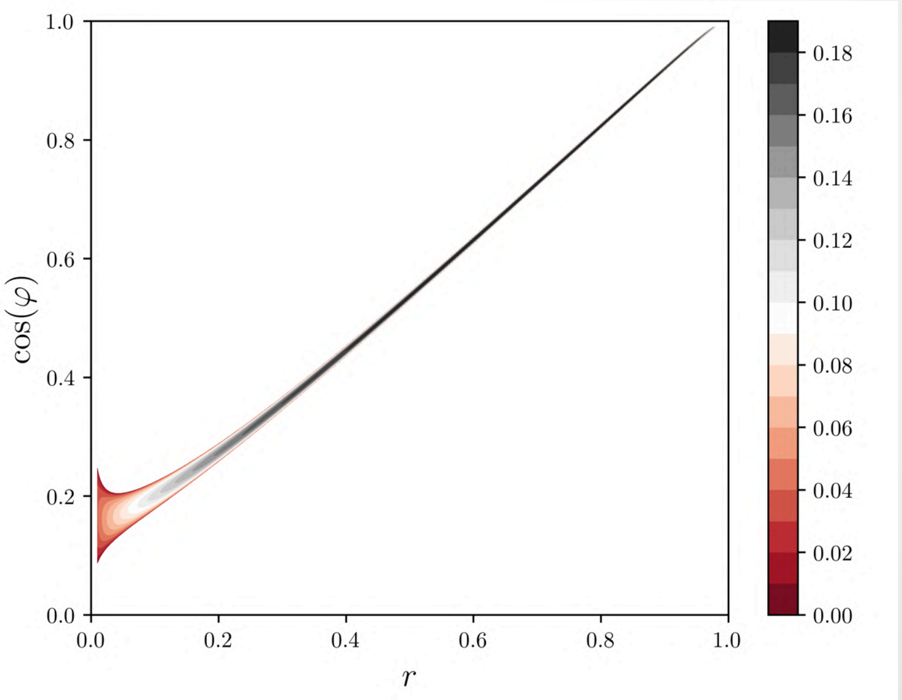
- Quantum behavior: occupation of modes grows exponentially within a resonance region

$$\gamma = \sqrt{\left(\delta\omega \frac{\delta n}{n} Q^{\vec{p}}_{\vec{k}}\right)^2 - \frac{1}{4}\epsilon^2_{\vec{k}\vec{p}}}$$

$$\epsilon_{\vec{k}\vec{p}} = \omega_k - \omega_p + \omega_{|\vec{k}-\vec{p}|}$$

- γ imaginary: stable modes
- γ real: unstable modes





One-mode perturbation

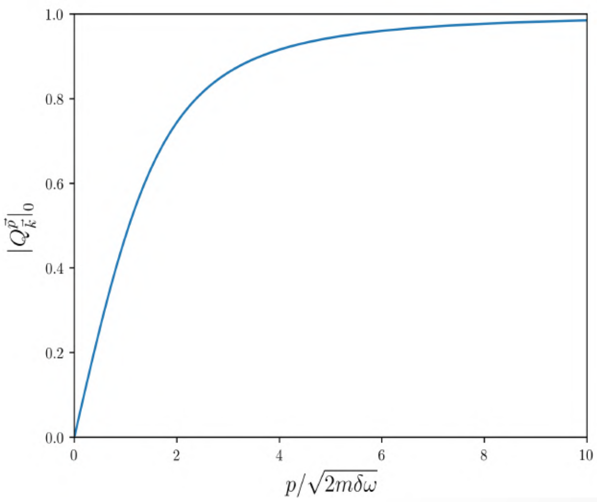
- Evaporation of the classical state

$$N_{ev}(t) = \sum_{\vec{k}} \langle \mathcal{N}_{\vec{k}}(t) \rangle \simeq V \int_{\gamma>0} \frac{d^3k}{(2\pi)^3} \, \langle \mathcal{N}_{\vec{k}}(t) \rangle$$

- Take integral

$$N_{ev} \approx \mathrm{e}^{2\gamma_0 t}$$

$$\gamma_0 = \delta\omega \frac{\delta n}{n} \, |Q_{\vec{k}}^{\vec{p}}|_0$$



- Duration of classicality

$$t_{\rm cl} \approx \frac{1}{\delta\omega \frac{\delta n}{n} \, |Q_{\vec{k}}^{\vec{p}}|_0}$$

Conclusions

- Quantum mechanics is needed to describe quantum fields over timescales of order the relaxation time
- Axion dark matter can relax through gravitational self-interaction
- Quantify timescale of departure from classical field theory for a self-gravitating homogeneous field and an inhomogeneous field with repulsive contact interactions
- Duration of classicality becomes shorter when there are inhomogeneities? Eg. Repulsive contact self-interactions:

Homogeneous

$$t_{\rm cl} = \infty$$

Homogeneous + perturbation

$$t_{\rm cl} \propto \frac{1}{\delta\omega \frac{\delta n}{n_0}}$$

- Self-gravitating inhomogeneous field? Growth of perturbations in axion dark matter?