Relic neutrino detection through angular correlations in inverse β-decay

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Relic neutrinos

Produced in the hot and dense early Universe and decoupled from cosmic plasma at $t \sim 1$ sec after the BB ($T_\nu \sim 2$ MeV). Cooled down in the course of Universe expansion. At present have nearly Fermi-Dirac spectrum with

$$T_{\nu 0} \simeq 1.945\,\mathrm{K} \simeq 1.676 \times 10^{-4}\,\mathrm{eV}$$

and are in mass eigenstates (flavour eigenstates have lost their coherence). Average number density per mass eigenstate and per spin degree of freedom:

$$n_{\nu 0} = \frac{3\zeta(3)}{4\pi^2} T_{\nu 0}^3 \simeq 56\ \mathrm{cm}^{-3}.$$

Average momentum:

$$\langle p_{\nu 0} \rangle \simeq 3.151 T_{\nu 0} \simeq 5.314 \times 10^{-4}\,\mathrm{eV}.$$

In the standard 3ν picture: at least 2 out of 3 relic neutrino species are non-relativistic now.

Relic neutrinos

Should carry very important information about the Universe and its evolution!

- Are relic neutrino still there (are they stable enough against decay and annihilation)?
- What is their composition? Are there sterile components?
- What are their energy distributions? Are there contributions from decays of heavy relics? Are there non-thermal components?
- What are velocity and spin distributions of relic νs? Are there CνB anisotropies (like in CMB)? Gravitational clustering?

A wealth of information (probably much more than in CMB!)
May also shed light on Dirac vs. Majorana neutrino nature.

But: Because of very low energies and very weak interactions, extremely difficult to detect

How to detect them?

Several suggestions so far.

I. Coherent detection through mechanical effects.

- Order $\sim G_F$ forces – coherent effects (Opher, 1974; Lewis, 1980). Predicted measurable effects due to neutrino reflection or refraction.

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- Order $\sim G_F$ angular momentum transfer from CνB to polarized electrons due to "neutrino wind" (Stodolsky, 1975).

  Requires non-zero lepton asymmetry in CνB ($N_\nu \neq N_{\bar{\nu}}$); proportional to the peculiar velocity of the solar system ($\sim 10^{-3}$) $\Rightarrow$ Correct but too small to be observable (Langacker, Leveille and Sheiman, 1983; Duda et al., 2001).

How to detect them? – contd.

II. Order $\sim G_F^2$ coherent effects.

- Scattering on targets loosely filled with small (~ 1 mm) pellets. Employs macroscopic size of CνB de Broglie wavelength ($\lambda_D \simeq 2.4$ mm) (Zeldovich & Khlopov, 1981; Shvartsman, Braginsky, Gershtein, Zeldovich & Khlopov, 1982; Duda, Gelmini & Nussinov, 2001).

  Predicted acceleration:

$$a \sim (10^{-34} - 10^{-28})\ \mathrm{cm/s^2}$$

  Smallest acceleration currently measured: $a \simeq 5 \times 10^{-14}\ \mathrm{cm/s^2}$.

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- Coherent radiative ν scattering on electrons in conductors ($\nu e \to \nu e \gamma$) (Loeb & Starkman, 1990). Predicted small but observable effects.

  But: missed a factor $(E_\nu/\omega_p)^4 \sim 10^{-20}$ which makes the effect unobservable (S. Bahcall & A. Gould, 1991).

How to detect them? – contd.

III. Resonant annihilation of UHE CR neutrinos on relic neutrinos of opposite helicity into Z^0 boson (Weiler, 1982):

$$\nu_{\rm CR} + \nu_{{\rm C}\nu{\rm B}} \to Z^0 \to \ldots$$

Resonance energy:

$$E_{\nu_{{\rm CR}i}} = \frac{M_Z^2}{2m_{\nu i}} \simeq 4.2\times 10^{22}\ {\rm eV}\left(\frac{0.1\ {\rm eV}}{m_{\nu i}}\right).$$

$\Rightarrow$ Absorption dips in the spectra of UHE neutrinos; production of particles above the GZK cutoff.

But: No sources of such UHE neutrinos known.

Relic neutrino capture on β-decaying nuclei

β-decay:

$$\diamondsuit \quad A(Z,N) \;\rightarrow\; A(Z+1,N-1)+e^-+\bar{\nu}_e$$

Neutrino capture on β-decaying nuclei (inverse β-decay):

$$\diamondsuit \quad \nu_e + A(Z,N) \;\rightarrow\; A(Z+1,N-1)+e^-$$

Exothermic reaction (no energy threshold). Neutrinos of $E_\nu = 0$ can be captured with finite rate:

$$\lim_{v_\nu \to 0} v_\nu \sigma_\nu = const. \neq 0\,.$$

Can be used for detection of relic neutrinos! (Weinberg, 1962).

Relic neutrino capture on β-decaying nuclei

Main problem: electrons form the accompanying usual β-decay are *much* more abundant. For β decays with $E_0 - \delta \le E_e \le E_0 \quad (\delta \ll E_0)$:

$$\frac{\Gamma_c}{\Gamma_d} \simeq 6\pi^2 \frac{n_\nu}{\delta^3} \simeq \frac{n_\nu}{56\ \mathrm{cm}^{-3}} \frac{2.54 \times 10^{-11}}{[\delta\,(\mathrm{eV})]^3}\,,$$

independently of the Q_β-value $(Q_\beta = E_0 - m_e)$.

Weinberg's (1962) assumptions: massless neutrinos; CνB is degenerate with large chemical potential μ_ν: $\xi_\nu = \mu_\nu / T_\nu \gtrsim 10^5$.

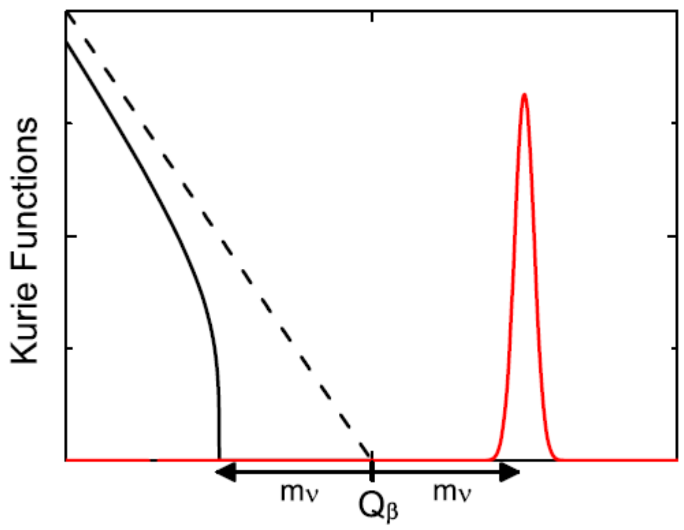
$\Rightarrow$ Energies of electrons from relic ν capture exceed E_0 by up to μ_ν ($\gtrsim 10\ \mathrm{eV}$), can be distinguished for moderate energy resolution.

But: Current constraints $|\xi_\nu| \lesssim 0.07$ rule this out.

Relic neutrino capture on β-decaying nuclei

Taking into account $m_\nu \neq 0$: a gap $\sim 2m_\nu$ between the spectra of electrons from ν capture and β-decay

(Cocco, Mangano & Messina, 2007, 2009).

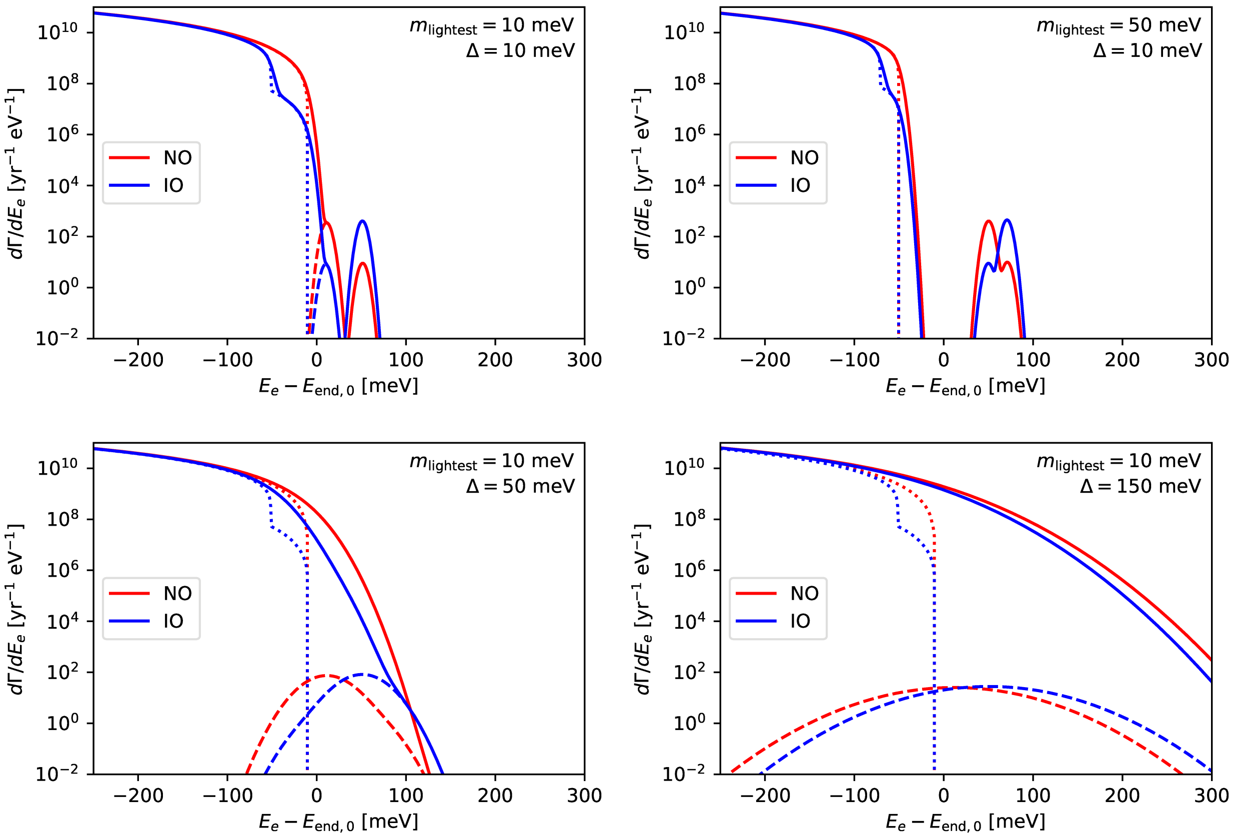


(From Y.-F. Li, 1504.03966)

The only currently known potentially feasible approach!

♢ Main problem: Requires an extremely high energy resolution $\Delta \lesssim m_\nu$.

PTOLEMY: $\nu + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-$



(From Betti et al., 1902.05508)

PTOLEMY: 100 g of ^{3}H; $\mathcal{O}(10)$ events/yr. Aim at $\Delta = 0.05$ eV.

<u>But:</u> For normal mass ordering and $m_1 \lesssim 0.01$ eV even ~ 10 times smaller energy resolution may be necessary! (N.B: the weight of ν_e in ν_3 is $|U_{e3}|^2 \simeq 2 \times 10^{-2}$).

A different approach

The idea: Use time dependence of angular correlations to distinguish ν capture from β-decay (EA, arXiv:1905.10207)

β-processes exhibit a number of angular correlations. An example: correlation between the spin of the parent nucleus $\vec{s}_N$ and electron direction $\vec{v}_e$ in expts. with polarized targets ($\not{P}$).

Similarly, angular correlation between $\vec{s}_N$ and $\vec{v}_\nu$.

How can this be used?

Time-varying angular correlations

Solar system moves w.r.t. CνB rest frame with velocity $-\vec{u}$ $(u \sim 10^{-3})$
$\Rightarrow$ neutrino wind: relic νs have a preferred arrival direction $\vec{u}$.

Angular correlation:

$$\Gamma_c \propto (1 + \beta \vec{u} \cdot \vec{s}_N)\,.$$

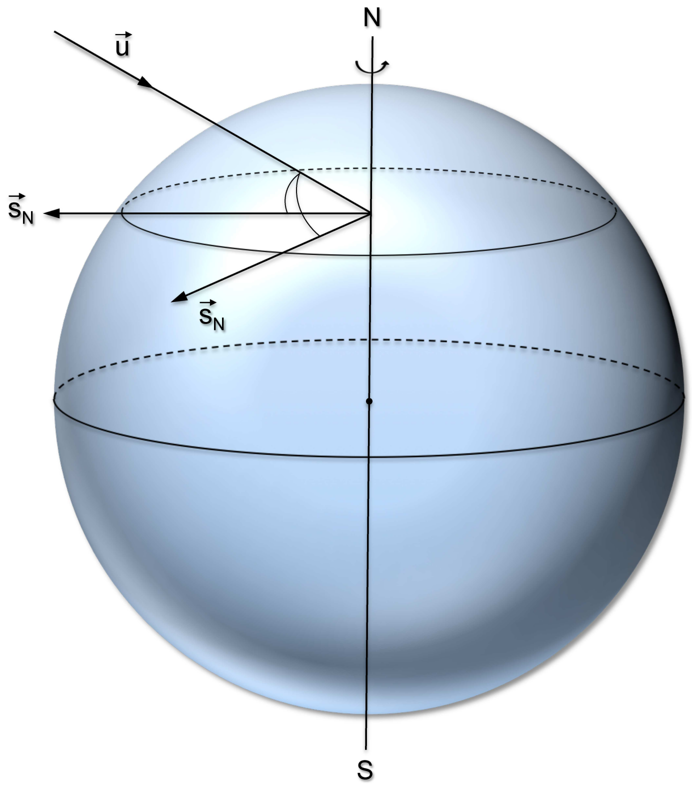
For a polarized nuclear target with fixed direction of polarization $\vec{s}_N$ in the lab frame:

Because of the Earth's rotation, the angle between $\vec{u}$ and $\vec{s}_N$ will change during the day. Periodic variations with period $T_0 \simeq 23h\ 56m\ 4s$ (sidereal day).

$\Rightarrow$ Time dependence of the total relic neutrino capture rate.

CνB caprture on polarized tritium (including time variations) considerd by Lisanti, Safdi & Tully (2014) within the approach based on the separation of spectra.

Earth rotation effect



$$\vec{u}\cdot\vec{s}_N = u[\cos\theta_u \cos\theta_N + \sin\theta_u \sin\theta_N \cos(\phi_u(t) - \phi_N)]$$

$$\phi_u(t) = (2\pi/T_0)t + \phi_0 \qquad (T_0 \simeq 24\ h).$$

Time-varying angular correlations

What if the target nuclei are not polarized?

- Measure polarization of daughter nuclei with $J_f \neq 0$ (e.g. measuring circular polarization of de-excitation γ-rays for transitions to A_f^*, as in the Goldhaber, Grodzins and Sunyar experiment).

- Make use of β–ν angular correlations $\Rightarrow$ Time dependent forward-backward asymmetry of electrons w.r.t. a fixed direction in the lab frame in expts. with unpolarized targets.

  Can work even for pure Fermi $0^\pi \to 0^\pi$ transitions!

  (But β-ν correlation may be strongly suppressed for mixed Fermi–Gamow-Teller β-transitions, such as the $1/2^+ \to 1/2^+$ transition $\nu_e+{}^3$H$\to{}^3$He$+e^-$).

CνB detection in inverse β-decay

Neutrino detection in the inverse β^- decay process

$$\nu_j(q) + A_i(p) \to A_f(p') + e^-(k)\,,$$

CνB neutrinos: Produced as chiral states; now are in helicity states (in CνB rest frame)

Produced:		Now:
left-chiral	$\longrightarrow$	left-helical
right-chiral	$\longrightarrow$	right-helical

Dirac neutrinos: right-helical states are $\bar{\nu}$. Cannot be captured in β^--processes, but can be detected through inverse β^+-decay.

Majorana neutrinos: for non-rel. νs both left-helical and right-helical states can participate in inverse β^--processes through their left-chirality components $\Rightarrow$ $\Gamma_c^{\rm Maj} \simeq 2\Gamma_c^{\rm Dir}$ (Long, Lunardini & Sabancilar, 2014).

CνB detection in inverse β-decay

Helicity is not Lorentz-invariant $\Rightarrow$ relic neutrinos that are in helicity eigenstates in CνB rest frame may not have definite helicity in the lab frame. $\Rightarrow$ Consider ν capture process for arbitrary direction of ν spin.

For pure Gamow-Teller transition $1^\pi \to 0^\pi$ (e.g. ^{64}Co$\to$^{64}Ni, ^{80}Br$\to$^{80}Kr)

$$\diamondsuit \quad v_j d\sigma_j = \frac{G_\beta^2}{2}|U_{ej}|^2 \frac{1}{(2\pi)^2}\frac{(\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu})}{4E_e E_j}|M_{\rm GT}|^2 F(Z,E_e)E_e\sqrt{E_e^2-m_e^2}\,d\Omega_e\,.$$

Energy conservation:

$$E_e = E_0 + E_j\,,$$

E_0: is total energy release in the corresponding β^--decay; $E_0 = Q_\beta + m_e$.
ε_μ: polarization 4-vector of the parent nucleus.

$$X^{\mu\nu} = [\bar{u}_e(k)\gamma^\mu(1-\gamma_5)u_j(q)][\bar{u}_j(q)\gamma^\nu(1-\gamma_5)u_e(q)]\,.$$

CνB detection in inverse β-decay

Define

$$A^\mu \equiv k^\mu - m_e S_e^\mu\,, \qquad\qquad B^\mu \equiv q^\mu - m_j S_j^\mu\,,$$

S_e^μ and S_j^μ – electron and neutrino spin 4-vectors:

$$S_e^\mu = \left(\frac{\vec{k}\cdot\vec{s}_e}{m_e},\; \vec{s}_e + \frac{(\vec{k}\cdot\vec{s}_e)\vec{k}}{m_e(E_e+m_e)}\right),$$

$$S_j^\mu = \left(\frac{\vec{q}\cdot\vec{s}_j}{m_j},\; \vec{s}_j + \frac{(\vec{q}\cdot\vec{s}_j)\vec{q}}{m_j(E_j+m_j)}\right).$$

$\vec{s}_e$ and $\vec{s}_j$ – unit vectors in the direction of the e and ν spin in their rest frames.
The squared amplitude:

$$\boxed{\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 2\big(A^0 - \vec{A}\cdot\vec{s}_N\big)\big(B^0 + \vec{B}\cdot\vec{s}_N\big)\,.}$$

($\vec{s}_N$ – unit vector in the direction of spin of the parent nucleus).

Unpolarized nuclear targets

Polarization of large targets – a difficult task.
Can one use angular correlations in expts. with unpolarized targets?

- Make use of polarization of daughter nuclei with $J_f \neq 0$ (e.g. in expts. of Goldhaber, Grodzins and Sunyar type; for $0^\pi \to 1^\pi$ replace $\vec{s}_N \to -\vec{s}_N$).
- More practical approach: measure β–ν angular correlations (or correl. between the direction of incoming ν and the spin of e).

Averaging over the polarizations of the parent nucleus:

$$\frac{1}{3}\sum_\lambda \varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda)X^{\mu\nu} = 2\left[A^0B^0 - \frac{1}{3}\vec{A}\cdot\vec{B}\,\right].$$

[The same result can be obtained by taking $\int (d\Omega_{\vec{s}_N}/4\pi)\varepsilon_\mu\varepsilon^*_\nu X^{\mu\nu}$].

Electron asymmetry

β–ν correlation:

$$\frac{d\sigma}{d\Omega_e} = const.(1+\alpha\vec{u}\cdot\vec{v_e})\,,$$

Leads to time dependent forward–backward asymmetry of electron emission with respect to a fixed direction $\vec{\xi}$ in the lab frame.
Time dependence: maximized when $\vec{\xi}\perp$ to the Earth's rotation axis.

In the geocentric spherical coordinates:

$$\vec{u} = u\big(\cos\phi_u(t)\sin\theta_u\,,\sin\phi_u(t)\sin\theta_u\,,\cos\theta_u\big)\,,$$

$$\vec{v_e} = v_e\big(\cos\phi_e\sin\theta_e\,,\sin\phi_e\sin\theta_e\,,\cos\theta_e\big)\,,$$

$$\vec{\xi} = \big(\cos\phi_\xi\,,\sin\phi_\xi\,,0\big)\,,$$

where

$$\phi_u(t) = \frac{2\pi}{T_0}t+\phi_0\,,$$

$T_0 \simeq 24\,\mathrm{h}$ is the sidereal day.

$$\vec{u}\cdot\vec{v}_e = uv_e\Big\{\cos\theta_u\cos\theta_e + \sin\theta_u\sin\theta_e\cos[\phi_u(t)-\phi_e]\Big\}\,.$$

Forward-backward asymmetry of the electron emission with respect to $\vec{\xi}$:

$$\diamondsuit \quad \mathcal{A} \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\frac{1}{2}[\sigma_\uparrow + \sigma_\downarrow]} = \alpha u v_e \sin\theta_u \cos[\phi_u(t)-\phi_\xi]\,.$$

Electron spin asymmetry with respect to a fixed direction in the lab frame can be considered similarly.

Time dependence of the signal

For polarized targets – time variation of the total signal rate:

$$\Gamma_d \propto 1 + \beta \vec{u}\cdot\vec{s}_N \,.$$

For unpolarized targets: time-varying angular correlation

$$\frac{d\Gamma_d}{d\Omega_e} \propto 1 + \alpha \vec{u}\cdot\vec{v}_e \,.$$

(and similarly $\vec{u}\cdot\vec{s}_e$ correlation). $\Rightarrow$ Forward-backward anisotropy w.r.t. a fixed direction in lab frame.

Coeffs. α, β: depend on v_j and on neutrino nature (Dir./Maj.); typically $\mathcal{O}(1)$.

<u>But:</u> $u = \mathcal{O}(10^{-3})$.

The problem: how to separate a small periodic signal from large fluctuating backgrounds. Very challenging!

Signal from noise separation

Main background: electrons from competing β decay of target nuclei.

For extraction of periodic signal, main problem comes from large statistical fluctuations of the background (due to quantum nature of the underlying processes) and not directly from the high average background level.

The problem of separation of signals from noisy backgrounds: Routinely encountered in astronomy, acoustics, radiodetection, engineering, medical applications such as electrocardiography, magnetic resonance imaging, etc. – well studied. Enormous literature exists!

S-from-N separation: Especially efficient when signal has a known periodicity.

This work: A simple approach to signal-from-noise separation based on the Fourier analysis and filtering in the frequency domain.

Signal from noise separation

Consider

$$f(t) = s(t) + n(t)$$

$s(t)$ – periodic signal of zero average, $n(t)$ – "noise" due to statistical fluctuations of the background (constant average background subtracted). By definition $\overline{s(t)} = \overline{n(t)} = 0 \quad \Rightarrow$ signal and noise strenghts characterized by their variances $\overline{s(t)^2}$ and $\overline{n^2(t)}$. Take signal in the form

$$s(t) = A_0 \sin(\omega_0 t + \varphi)$$

with period $T_0 = 2\pi/\omega_0$ and constant A_0 and φ. Signal autocorrelation function:

$$R_{ss}(\tau) = \overline{s(t)s(t+\tau)} \equiv \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)s(t+\tau)dt\,,$$

and similarly for noise.

N.B.: the signal and noise variances satisfy $\overline{s(t)^2} = R_{ss}(0)$, $\overline{n^2(t)} = R_{nn}(0)$.

Power spectra

The signal and noise power spectra $S(\omega)$ and $N(\omega)$:

$$S(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{-i\omega\tau} R_{ss}(\tau)\,, \qquad N(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{-i\omega\tau} R_{nn}(\tau)\,.$$

For sinusoidal signal:

$$R_{ss}(\tau) = \frac{A_0^2}{2} \cos \omega_0 \tau\,.$$

Signal power spectrum:

$$\diamondsuit \quad S(\omega) = \frac{\pi}{2} A_0^2 \{\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\}\,.$$

Noise $n(t)$ is due to statistical fluctuations of background $\Rightarrow$ white noise:

$$\diamondsuit \quad N(\omega) = N_0 = const.$$

(flat power spectrum). The autocorrelation function :

$$R_{nn}(\tau) = N_0 \delta(\tau).$$

$N_0 =$ (number of bckgr. events)/(unit time).

$$R_{ss}(\tau) = \frac{A_0^2}{2} \cos \omega_0 \tau \,.$$

$$R_{nn}(\tau) = N_0 \delta(\tau)$$

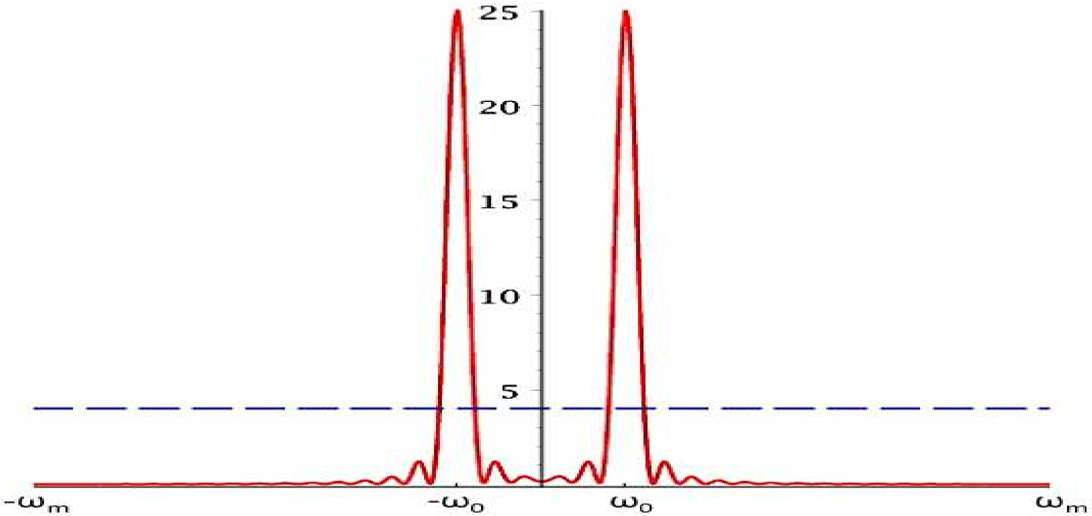
In this approach: formally possible to separate arbitrarily weak periodic signal from noise [Consider their autocorrelation functions at any $\tau \neq 0$ for which $R_{ss}(\tau)$ does not vanish]. But: this would only be the case for infinite T.

Also: The total noise power obtained by integrating the flat power spectrum over the whole freq.cy interval $(-\infty, \infty)$ is infinite. The resolution: any realistic measurement actually is characterized by a finite interval of frequencies.

Realistic case: finite total observation time T and finite measurement bandwidth. The signal power spectrum for finite T:

$$S(\omega) = \frac{A_0^2}{T}\left\{\frac{\sin^2\left[(\omega-\omega_0)\frac{T}{2}\right]}{(\omega-\omega_0)^2} + \frac{\sin^2\left[(\omega+\omega_0)\frac{T}{2}\right]}{(\omega+\omega_0)^2}\right\}.$$

Peaks at $\omega = \pm\omega_0$ of width $\sim 4\pi/T$ (interval $[-2\pi/T,\, 2\pi/T]$ around the center of each peak contains about 90% of its strength).



The bandwidth of the experiment: $\Omega = [-\omega_m,\ \omega_m]$.

If time binning (with bin length Δt) is involved: ω_m related to the Nyquist linear frequency $f_{\rm N} \equiv (2\Delta t)^{-1}$ by

$$\omega_m = 2\pi f_{\rm N} = \frac{\pi}{\Delta t}\,.$$

[ω_m is the highest available frequency because Δt is the shortest time interval spanned].

In expts. allowing real-time detection:

$$\Delta t \;\to\; \text{(mean time interval between two consecutive events)}\,,$$

That is,

$$1/\Delta t \;=\; \text{(mean bckgr. event rate)} \;=\; N_0\,,$$

$$\diamondsuit \quad \omega_m \;=\; \pi N_0\,.$$

$$\overline{s(t)^2} = \int_\Omega \frac{d\omega}{2\pi} S(\omega)\,, \qquad \overline{n(t)^2} = \int_\Omega \frac{d\omega}{2\pi} N(\omega)\,.$$

For the noise $(N(\omega) = N_0 = const.)$:

$$\overline{n(t)^2} \;=\; \int_{-\omega_m}^{\omega_m} \frac{d\omega}{2\pi} N_0 \;=\; N_0^2\,.$$

Without noise suppression: S-to-N ratio $\rho_0 \;\equiv\; \overline{s(t)^2}/\overline{n(t)^2}$ is

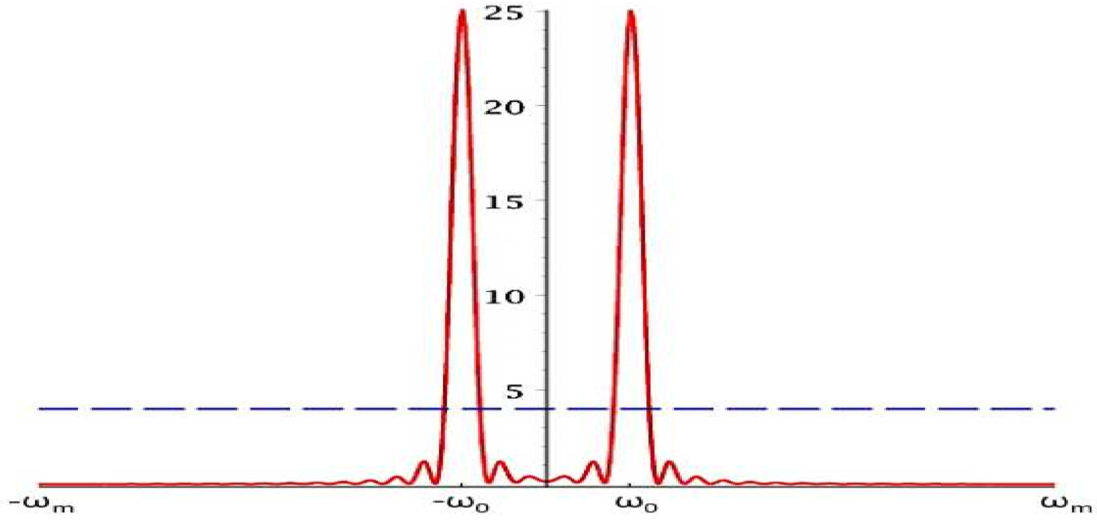
$$\rho_0 = \frac{\int_\Omega \frac{d\omega}{2\pi} S(\omega)}{\int_\Omega \frac{d\omega}{2\pi} N(\omega)} \;=\; \frac{A_0^2/2}{N_0^2}\,.$$

N.B.: $(A_0/\sqrt{2})T$ is the r.m.s. number of signal events, $N_0 T$ is the total number of bckgr. events.

- Very small quantity ($(\mathcal{S}/\mathcal{B})^2$)
- Does not depend on the total running time of the expt. T
- Does not depend on the detector mass

Filtering in frequency domain

Passband filtering: make use of the fact that the signal is concentrated in the narrow intervals in the frequency domain whereas the noise power is constant per unit bandwidth.



To improve the signal-to-noise ratio: integrate the signal and noise power spectra over the intervals $\Delta\omega = [-2\pi/T,\, 2\pi/T]$ around the points $\omega = \pm\omega_0$.

Noise suppression

Improved signal-to-noise ratio:

$$\diamondsuit \qquad \rho = \frac{\int_{\Delta\omega} \frac{d\omega}{2\pi} S(\omega)}{\int_{\Delta\omega} \frac{d\omega}{2\pi} N(\omega)}\,.$$

We have:

$$\int_{\Delta\omega} \frac{d\omega}{2\pi} S(\omega) \;\simeq\; \frac{A_0^2}{4\pi} \int_{-\pi}^{\pi} \frac{\sin^2 x}{x^2} dx \;\simeq\; \frac{A_0^2}{4} \cdot 0.9028\,,$$

$$\int_{\Delta\omega} \frac{d\omega}{2\pi} N(\omega) \simeq \frac{2N_0}{T}\,.$$

$$\Rightarrow \qquad \rho \simeq \frac{0.9028}{8}\,\frac{A_0^2 T}{N_0}\,.$$

Coincides (up to about a factor of $\sim$1/2) with the ratio of the peak value of the signal power to the (flat) value of the noise power.

Improved S-to-N ratio

Compare with the original ("un-filtered") S-to-N ratio ρ_0:

$$\rho_0 = \frac{A_0^2/2}{N_0^2} \qquad \longrightarrow \qquad \rho = \frac{0.9028}{8}\frac{A_0^2 T}{N_0}\,.$$

Improvement factor: $0.225 N_0 T$ ($\gg 1$). [$N_0 T$ is the total # of bckgr. events].

The improved S-to-N ratio:

- Increases linearly with the running time of the experiment T
- Increases linearly with the mass of the detector M ($A_0 \propto M$, $N_0 \propto M$).

Alternatively, compare ρ with $\mathcal{S}/\mathcal{B}$ rather than with un-filtered ρ_0:

$$\rho \simeq \frac{(A_0/\sqrt{2})T}{N_0 T}\cdot\left[0.225(A_0 T/\sqrt{2})\right].$$

First factor: the $\mathcal{S}/\mathcal{B}$ ratio; [...] – enhancement factor, increases with signal statistics.

Noise reduction by simple frequency-domain filtering: greatly improves S-to-N, still may not be enough unless the total number of signal events is very large!

$\Rightarrow$ More sophisticated methods of noise reduction may be necessary.
E.g. measure noise power spectrum in freq.cy regions where the signal is practically absent (away from $\omega = \pm\omega_0$) to predict its value in the regions of ω where the signal is concentrated and cancel (subtract) it there (Samsing, 2015).

But: The cancellation will be incomplete because noise spectrum is in reality only approximately flat.

Other methods can be used, e.g. studying cross-correlation of the data with test functions in the form of the expected time-dependent signal – similarly to approach used in gravitational wave detection.

$\Rightarrow$ Extensive signal and noise simulations in the conditions of a particular experiment will be necessary.

Conclusions

- New approach for CνB detection proposed.
- Based on neutrino capture one beta-decaying nuclei (as suggested by Weinberg and by Cocco et al.), but uses a different method of discrimination between the relic ν capture and the usual β-decay.
- Employs periodic time dependence of angular correlations in the CνB signal due to the peculiar motion of the solar system and rotation of the Earth about its axis. Does not depend crucially on energy resolution.
- Angular correlation coeffs. found for the special case of pure $1^\pi \to 0^\pi$ allowed G.-T. transitions. Advantage of pure transitions (F. and G.-T.): correlation coeffs. do not depend on nuclear matrix elements.
- For polarized nuclear targets: time variation of the total ν capture rate.
- For unpolarized nuclear targets: time-dependent correlation between preferred direction of neutrino arrival and electron direction.

Conclusions – contd.

- Main challenge: Separation of weak periodic signal from large fluctuating background.
- Nuclids with larger Q_β-values may be preferable (lead to larger absolute detection rates).
- Selection of suitable nuclides should take into account
  - lifetime and availability
  - type of β transition
  - (neutrino capture)/(β decay) rate ratio
  - feasibility of target polarization (for expts. w/ polarized nuclei).

    $\ldots$

◇ Extensive simulations of signal and background will be necessary to clarify if the proposed CνB detection method is viable and competitive with the approach based on the separation of the spectra.

Backup slides

Order $\sim G_F$ angular momentum transfer

CνB-induced torque due to coherent order $\sim G_F$ interaction with particles a ($a = e,\ p,\ n$) in anisotropic (e.g. spin-polarized) targets (Stodolsky, 1975).

$$\vec{\tau} = \langle\dot{\vec{\sigma}}\rangle = \frac{\Delta E}{2}(\hat{\vec{v}}_a \times \langle\vec{\sigma}\rangle)\,,$$

$$\Delta E = 2\sqrt{2}G_F u N_a^{\rm tot}\sum_i (n_{\nu_i} - n_{\bar{\nu}_i})g_A^{ai}K(p, m_i) \qquad (u = u_{\rm Earth}),$$

$$K(p,m_i) = \frac{(E+m_i)}{4E}\left[1+\frac{p}{E+m_i}\right]^2 \simeq \begin{cases} 1, & p \gg m_i, \\ \frac{1}{2}, & p \ll m_i. \end{cases}$$

For transversely polarized electrons: spin precession around $\vec{v}_e$ with $\omega = \dot{\phi} = \Delta E$. Assuming $\Delta n_\nu / n_\nu \sim 1$ (actually excluded by the data), for a single electron:

$$\Delta E \sim 10^{-38}\ {\rm eV} \sim 10^{-23}\ \frac{\rm rad}{\rm s}.$$

Could be enhanced by $\sim 10^{27}$ in a $\sim 1\,t$ ferromagnet. $\Rightarrow$ $\Delta E \sim 10^{-11}\ {\rm eV}$ – still many orders of magn. smaller than the magnet interaction with any feasible magn. field (Langacker et al., 1983).

Elastic scattering of relic neutrinos $(\sim G_F^2)$

For relativistic relic neutrinos (with $\langle p_{\nu 0}\rangle \simeq 3.15 T_{\nu 0}$):

$$\sigma_{\nu N}^{\mathrm{D,M}} \simeq \frac{G_F^2 E_\nu^2}{\pi} \simeq 5\times 10^{-63}\ \mathrm{cm}^2\,.$$

For non-relativistic neutrinos:

$$\sigma_{\nu N}^{\mathrm{D}} \simeq \frac{G_F^2 m_\nu^2}{\pi} \simeq 10^{-56}(m_\nu/\mathrm{eV})^2\ \mathrm{cm}^2\,,$$

$$\sigma_{\nu N}^{\mathrm{M}} \simeq \frac{G_F^2 p_\nu^2}{\pi} \simeq 5\times 10^{-63}\ \mathrm{cm}^2\,.$$

Coherent neutrino scattering

The force acting on a target pellet of size $r \lesssim \lambda_D (\simeq 2\mathrm{mm})$:

$$F \simeq (\sigma_{\nu N} n_\nu u)\Delta p \cdot k_L^2 N_0^2$$

$$N_0 = \frac{\rho}{A m_p}\frac{4}{3}\pi r^3, \quad \Delta p \simeq \begin{cases} m_\nu u, & non-rel., \\ \langle p_\nu\rangle u, & rel. \end{cases}, \quad k_L = \begin{cases} 2Z - N, & \nu_e, \\ -N, & \nu_{\mu,\tau} \end{cases}.$$

Acceleration:

$$a \simeq \frac{F}{N_0 A m_p} \sim (10^{-34} - 10^{-28})\ \mathrm{cm/s^2}$$

Smallest acceleration currently measured: $a \simeq 5 \times 10^{-14}\ \mathrm{cm/s^2}$.

CνB detection in inverse β-decay

For electron and neutrino in definite spin states:

$$u_e(k)\bar{u}_e(k) = \frac{1}{2}(\not{k} + m_e)(1 + \gamma_5 \not{S}_e)\,,$$

$$u_j(q)\bar{u}_j(q) = \frac{1}{2}(\not{q} + m_j)(1 + \gamma_5 \not{S}_j)\,.$$

S_e^μ and S_j^μ – electron and neutrino spin 4-vectors:

$$S_e^\mu = \left(\frac{\vec{k}\cdot\vec{s}_e}{m_e},\ \vec{s}_e + \frac{(\vec{k}\cdot\vec{s}_e)\vec{k}}{m_e(E_e + m_e)}\right),$$

$$S_j^\mu = \left(\frac{\vec{q}\cdot\vec{s}_j}{m_j},\ \vec{s}_j + \frac{(\vec{q}\cdot\vec{s}_j)\vec{q}}{m_j(E_j + m_j)}\right).$$

$\vec{s}_e$ and $\vec{s}_j$: unit vectors in the direction of the electron and neutrino spin in their rest frames.

CνB detection in inverse β-decay

If neutrinos are in helicity eigenstates and $\vec{s}_j \parallel \vec{s}_N$, then $\vec{B}\cdot\vec{s}_N = -B^0 \quad \Rightarrow$ the transition amplitude vanishes [in agreement with angular momentum conservation (initial state with total angular momentum $J + 1/2$ cannot decay into a nucleus with spin $J - 1$ and electron)].

Notation:

$$K_e \equiv 1 - \frac{E_e}{E_e + m_e}\vec{v}_e\cdot\vec{s}_e\,, \qquad\qquad K_j \equiv 1 - \frac{E_j}{E_j + m_j}\vec{v}_j\cdot\vec{s}_j\,,$$

$$\Downarrow$$

$$A^\mu = E_e\Big(1 - \vec{v}_e\cdot\vec{s}_e,\; K_e\vec{v}_e - \frac{m_e}{E_e}\vec{s}_e\Big)\,,$$

$$B^\mu = E_j\Big(1 - \vec{v}_j\cdot\vec{s}_j,\; K_j\vec{v}_j - \frac{m_j}{E_j}\vec{s}_j\Big)\,.$$

$\Longrightarrow$

$$\diamondsuit \quad \varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 2E_e E_j \Big[(1 - \vec{v}_e \cdot \vec{s}_e) - K_e (\vec{v}_e \cdot \vec{s}_N) + \frac{m_e}{E_e} (\vec{s}_e \cdot \vec{s}_N) \Big] \\ \times \Big[(1 - \vec{v}_j \cdot \vec{s}_j) + K_j (\vec{v}_j \cdot \vec{s}_N) - \frac{m_j}{E_j} (\vec{s}_j \cdot \vec{s}_N) \Big].$$

$$\diamondsuit \quad \varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 2E_e E_j \Big[(1 - \vec{v}_e \cdot \vec{s}_e) - K_e (\vec{v}_e \cdot \vec{s}_N) + \frac{m_e}{E_e} (\vec{s}_e \cdot \vec{s}_N) \Big] \times \Big[(1 - \vec{v}_j \cdot \vec{s}_j) + K_j (\vec{v}_j \cdot \vec{s}_N) - \frac{m_j}{E_j} (\vec{s}_j \cdot \vec{s}_N) \Big].$$

Depends on 5 vectors – $\vec{v}_e$, $\vec{v}_j$, $\vec{s}_e$, $\vec{s}_j$ and $\vec{s}_N$ $\Rightarrow$ one can form 10 dot-products:

$$\diamondsuit \quad \varepsilon_\mu \varepsilon^*_\nu X^{\mu\nu} = 2E_e E_j \Big[(1 - \vec{v}_e \cdot \vec{s}_e) - K_e(\vec{v}_e \cdot \vec{s}_N) + \frac{m_e}{E_e}(\vec{s}_e \cdot \vec{s}_N)\Big] \times \Big[(1 - \vec{v}_j \cdot \vec{s}_j) + K_j(\vec{v}_j \cdot \vec{s}_N) - \frac{m_j}{E_j}(\vec{s}_j \cdot \vec{s}_N)\Big].$$

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$$\vec{v}_e \cdot \vec{s}_e \,, \quad \vec{v}_e \cdot \vec{s}_N \,, \quad \vec{s}_e \cdot \vec{s}_N \,, \quad \vec{v}_j \cdot \vec{s}_j \,,$$

$$\vec{v}_e \cdot \vec{v}_j \,, \quad \vec{v}_e \cdot \vec{s}_j \,, \quad \vec{v}_j \cdot \vec{s}_e \,, \quad \vec{s}_e \cdot \vec{s}_j \,, \quad \vec{v}_j \cdot \vec{s}_N \,, \quad \vec{s}_j \cdot \vec{s}_N \,.$$

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$$\times \Big[(1 - \vec{v}_j \cdot \vec{s}_j) + K_j(\vec{v}_j \cdot \vec{s}_N) - \frac{m_j}{E_j}(\vec{s}_j \cdot \vec{s}_N) \Big].$$

Depends on 5 vectors – $\vec{v}_e$, $\vec{v}_j$, $\vec{s}_e$, $\vec{s}_j$ and $\vec{s}_N$ $\Rightarrow$ one can form 10 dot-products:

$$\vec{v}_e \cdot \vec{s}_e\,, \quad \vec{v}_e \cdot \vec{s}_N\,, \quad \vec{s}_e \cdot \vec{s}_N\,, \quad \vec{v}_j \cdot \vec{s}_j\,,$$
$$\vec{v}_e \cdot \vec{v}_j\,, \quad \vec{v}_e \cdot \vec{s}_j\,, \quad \vec{v}_j \cdot \vec{s}_e\,, \quad \vec{s}_e \cdot \vec{s}_j\,, \quad \vec{v}_j \cdot \vec{s}_N\,, \quad \vec{s}_j \cdot \vec{s}_N\,.$$

$\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu}$ contains only six of them [terms $\vec{v}_e \cdot \vec{v}_j$, $\vec{v}_e \cdot \vec{s}_j$, $\vec{v}_j \cdot \vec{s}_e$, $\vec{s}_e \cdot \vec{s}_j$ are not there.

$$\diamondsuit \quad \varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 2E_e E_j \Big[(1 - \vec{v}_e \cdot \vec{s}_e) - K_e(\vec{v}_e \cdot \vec{s}_N) + \frac{m_e}{E_e}(\vec{s}_e \cdot \vec{s}_N)\Big] \times \Big[(1 - \vec{v}_j \cdot \vec{s}_j) + K_j(\vec{v}_j \cdot \vec{s}_N) - \frac{m_j}{E_j}(\vec{s}_j \cdot \vec{s}_N)\Big].$$

Depends on 5 vectors – $\vec{v}_e$, $\vec{v}_j$, $\vec{s}_e$, $\vec{s}_j$ and $\vec{s}_N$ $\Rightarrow$ one can form 10 dot-products:

$$\vec{v}_e \cdot \vec{s}_e\,, \quad \vec{v}_e \cdot \vec{s}_N\,, \quad \vec{s}_e \cdot \vec{s}_N\,, \quad \vec{v}_j \cdot \vec{s}_j\,,$$

$$\vec{v}_e \cdot \vec{v}_j\,, \quad \vec{v}_e \cdot \vec{s}_j\,, \quad \vec{v}_j \cdot \vec{s}_e\,, \quad \vec{s}_e \cdot \vec{s}_j\,, \quad \vec{v}_j \cdot \vec{s}_N\,, \quad \vec{s}_j \cdot \vec{s}_N\,.$$

$\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu}$ contains only six of them [terms $\vec{v}_e \cdot \vec{v}_j$, $\vec{v}_e \cdot \vec{s}_j$, $\vec{v}_j \cdot \vec{s}_e$, $\vec{s}_e \cdot \vec{s}_j$ are not there.

Terms $\vec{v}_e \cdot \vec{s}_e$, $\vec{v}_e \cdot \vec{s}_N$, $\vec{s}_e \cdot \vec{s}_N$: do not depend on the neutrino variables.

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$$\times\Big[(1-\vec{v}_j\cdot\vec{s}_j) + K_j(\vec{v}_j\cdot\vec{s}_N) - \frac{m_j}{E_j}(\vec{s}_j\cdot\vec{s}_N)\Big].$$

Depends on 5 vectors – $\vec{v}_e$, $\vec{v}_j$, $\vec{s}_e$, $\vec{s}_j$ and $\vec{s}_N$ $\Rightarrow$ one can form 10 dot-products:

$$\vec{v}_e\cdot\vec{s}_e\,,\quad \vec{v}_e\cdot\vec{s}_N\,,\quad \vec{s}_e\cdot\vec{s}_N\,,\quad \vec{v}_j\cdot\vec{s}_j\,,$$
$$\vec{v}_e\cdot\vec{v}_j\,,\quad \vec{v}_e\cdot\vec{s}_j\,,\quad \vec{v}_j\cdot\vec{s}_e\,,\quad \vec{s}_e\cdot\vec{s}_j\,,\quad \vec{v}_j\cdot\vec{s}_N\,,\quad \vec{s}_j\cdot\vec{s}_N\,.$$

$\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu}$ contains only six of them [terms $\vec{v}_e\cdot\vec{v}_j$, $\vec{v}_e\cdot\vec{s}_j$, $\vec{v}_j\cdot\vec{s}_e$, $\vec{s}_e\cdot\vec{s}_j$ are not there.

Terms $\vec{v}_e\cdot\vec{s}_e$, $\vec{v}_e\cdot\vec{s}_N$, $\vec{s}_e\cdot\vec{s}_N$: do not depend on the neutrino variables.

Term $\vec{v}_j\cdot\vec{s}_j$: not useful [expts. on helicity measurements cannot distinguish between absorption of a L-helical neutrino and production of a R-helical (anti)neutrino]. But: $\vec{v}_j\cdot\vec{s}_j$ affects the total capture rate and may be important for studying Dirac vs. Majorana ν nature].

Terms relevant for CνB capture on polarized targets: $\vec{v}_j \cdot \vec{s}_N$, $\vec{s}_j \cdot \vec{s}_N$ – do not depend on $\vec{v}_e$ and $\vec{s}_e$. Summing over s_e and averaging over the directions of $\vec{v}_e$:

$$\diamondsuit \quad \int \frac{d\Omega_e}{4\pi} \sum_{s_e} \varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 4E_e E_j \Big\{ (1 - \vec{v}_j \cdot \vec{s}_j) + \Big(K_j \vec{v}_j - \frac{m_j}{E_j} \vec{s}_j \Big) \cdot \vec{s}_N \Big\}.$$

[N.B.: For helical neutrinos $\Rightarrow$ $4E_e E_j (1 - \lambda_j v_j)(1 - \vec{s}_j \cdot \vec{s}_N)$.]

For fixed direction of $\vec{s}_N$ in the Earth frame – will vary with time due to variations of the directions of $\vec{v}_j$ and $\vec{s}_j$.

$\Rightarrow$ In expts. with polarized nuclear targets time variations of the CνB signal can be studied by merely measuring the total rate of electron production.

Measuring the angular and/or spin distributions of electrons will be useful for experiments with unpolarized nuclei.

Unpolarized targets

When the direction of the spin of the produced electrons is observed while the directions of their momenta are not:

$$\int \frac{d\Omega_e}{4\pi}\,\frac{1}{3}\sum_{\lambda}\varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda)X^{\mu\nu} = 2E_e\Big\{B^0+\frac{1}{9}\Big(1+2\frac{m_e}{E_e}\Big)(\vec{s}_e\cdot\vec{B})\Big\}$$

$$= 2E_eE_j\Big\{1-\vec{v}_j\cdot\vec{s}_j+\frac{1}{9}\Big(1+2\frac{m_e}{E_e}\Big)\Big[\vec{v}_j\cdot\vec{s}_e-\frac{E_j}{E_j+m_j}(\vec{v}_j\cdot\vec{s}_j)(\vec{v}_j\cdot\vec{s}_e)-\frac{m_j}{E_j}\vec{s}_e\cdot\vec{s}_j\Big]\Big\}.$$

When neither the spin state nor the direction of the produced electron is observed:

$$\int \frac{d\Omega_e}{4\pi}\,\frac{1}{3}\sum_{\lambda,s_e}\varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda)X^{\mu\nu} = 4E_eE_j\left(1-\vec{v}_j\cdot\vec{s}_j\right),$$

– the usual inclusive squared amplitude for neutrino capture on unpolarized nuclei.

β–ν angular correlation

If the spin state of the produced electron is not measured, sum over s_e:

$$\frac{1}{3}\sum_{\lambda,s_e}\varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda)X^{\mu\nu} = 4E_e\big(B^0 - \frac{1}{3}\vec{B}\cdot\vec{v}_e\big)\,.$$

In a more detailed form:

$$\Rightarrow \quad 4E_eE_j\Big[1-\vec{v}_j\cdot\vec{s}_j-\frac{1}{3}\vec{v}_e\cdot\vec{v}_j+\frac{1}{3}\frac{E_j}{E_j+m_j}(\vec{v}_j\cdot\vec{s}_j)(\vec{v}_e\cdot\vec{v}_j)+\frac{1}{3}\frac{m_j}{E_j}\vec{v}_e\cdot\vec{s}_j\Big].$$

Describes anisotropies of electron emission with respect to the directions of $\vec{v}_j$ and $\vec{s}_j$ which change with time in the lab frame.

For helical neutrinos:

$$\Rightarrow \quad 4E_eE_j(1-\lambda_j v_j)\Big\{1+\frac{1}{3}\frac{\lambda_j}{v_j}\vec{v}_e\cdot\vec{v}_j\Big\}.$$

In the relativistic limit:

$$\Rightarrow \quad \begin{cases} 8E_eE_j\Big\{1-\frac{1}{3}\vec{v}_e\cdot\vec{v}_j\Big\}, & \lambda_j=-1, \\ 0\,, & \lambda_j=+1 \end{cases} \qquad \Rightarrow \qquad a=-\frac{1}{3}.$$

Solar ν fluxes from the Sudbury Neutrino Observatory SNO

Question to think about:

The lightest nucleus, deuterium, consisting of one proton and one neutron has spin 1

Why do the SNO measurements confirm S(d) = 1 ?

$1000\,t\ D_2O$

9500 8“ PMTs

$5300\,t\ H_2O$

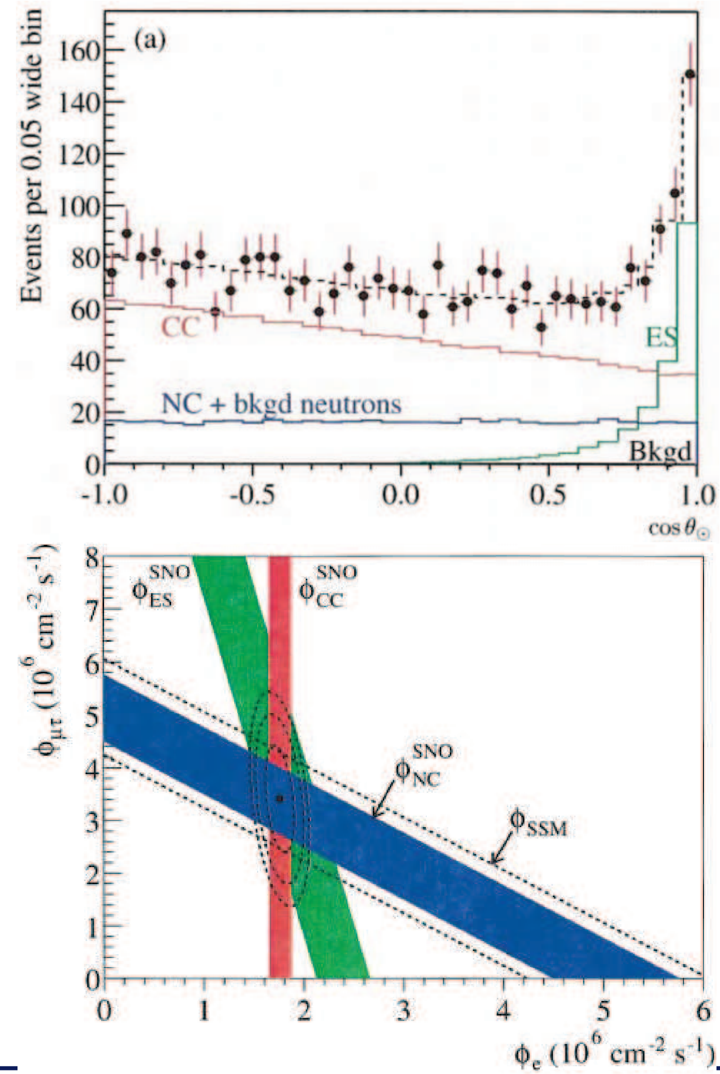
Cherenkov

		sensitive to
ES	$\nu_x + e^- \rightarrow \nu_x + e^-$	$\nu_e\ (\nu_\mu, \nu_\tau)$
CC	$\nu_e + d \rightarrow p + p + e^-$	ν_e
NC	$\nu_x + d \rightarrow p + n + \nu_x$	ν_e, ν_μ, ν_τ

Clear finding: NC > ES > CC

⇒ excistence of ν_μ, ν_τ

⇒ ν oscillation & $m(\nu_i) \neq 0$



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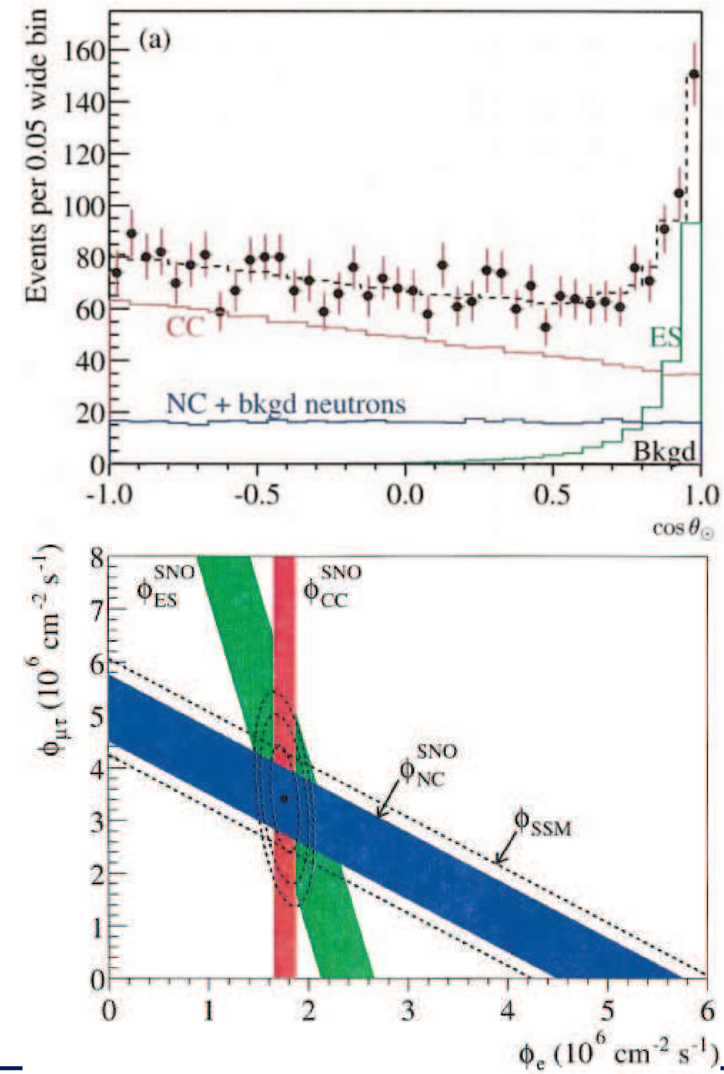

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GT $1^+ \rightarrow 0^+$ transition $(a = -1/3)$

Lab frame and angular averaging

Implications for relic ν capture: lab frame

Standard cosmology predicts isotropic neutrino velocity distributions in CνB rest frame. As neutrinos are in helicity eigenstates in that frame – neutrino spin distributions are also isotropic. That is, angular averaging gives

$$\langle \vec{v}_j \rangle = 0\,, \qquad \langle \vec{s}_j \rangle = 0\,.$$

The Sun moves with a velocity $-\vec{u}$ ($u \ll 1$) with respect to the CνB rest frame $\Rightarrow$ neutrino variables in the lab frame obtained by a Lorentz boost. To $\mathcal{O}(u)$:

$$E_j' = E_j + \vec{u}\cdot\vec{q}\,, \qquad \vec{q}\,' = \vec{q} + \vec{u}E_j\,, \qquad \vec{v}_j' = \vec{v}_j(1 - \vec{u}\cdot\vec{v}_j) + \vec{u}\,.$$

In the regime $u \ll v_j$ (in the standard 3-flavour neutrino picture corresponds to hierarchical neutrino mass spectrum) in this limit

$$\frac{\vec{v}_j'}{v_j'} = \frac{1}{v_j}\Big\{\vec{v}_j + \Big[\vec{u} - \frac{(\vec{u}\cdot\vec{v}_j)}{v_j^2}\vec{v}_j\Big]\Big\} \equiv \frac{1}{v_j}\big(\vec{v}_j + \vec{u}_{\perp j}\big)\,,$$

($\vec{u}_{\perp j}$ is the component of $\vec{u}$ orthogonal to $\vec{v}_j$).

$$\vec{s}_j^{\,\prime} = \vec{s}_j + \frac{(\vec{q}\cdot\vec{s}_j)\vec{u} - (\vec{u}\cdot\vec{s}_j)\vec{q}}{E_j + m_j}\,.$$

If in the CνB rest frame neutrinos are in the exact helicity eigenstates,

$$\vec{s}_j = \lambda_j \frac{\vec{v}_j}{v_j}\,, \qquad\qquad (\lambda_j = \pm 1)$$

$$\Rightarrow \qquad \vec{s}_j^{\,\prime} = \lambda_j \Big\{ \frac{\vec{v}_j}{v_j} + v_j \frac{E_j}{E_j + m_j} \vec{u}_{\perp j} \Big\}.$$

N.B.: Although helicity is not a Lorentz-invariant quantity, in the regime $u \ll v_j$ (valid for hierarchical ν masses and no ν_s) and to first order in u neutrinos are also helical in the lab frame with the same λ_j (corrections are $\mathcal{O}(u^2)$).

[This would not be in general correct for $u \gtrsim v_j$ $(m_j \gtrsim 0.5$ eV$)$ even if $u \ll 1$. E.g. for $v_j \ll u \ll 1$ we would have $\vec{v}_j^{\,\prime} \simeq \vec{u}$, $\vec{s}_j^{\,\prime} \simeq \vec{s}_j$ and $\vec{s}_j^{\,\prime} \cdot \vec{v}_j^{\,\prime}/v_j^{\prime} \simeq \vec{s}_j \cdot \vec{u}/u$ ($\neq \lambda_j$ in general)].

From $\langle \vec{v}_j \rangle = 0$, $\langle \vec{s}_j \rangle = 0$:

$$\langle E'_j \rangle = E_j \,, \qquad \langle \vec{q}\,' \rangle = \vec{u} E_j \,, \qquad \langle \vec{v}\,'_j \rangle = \Big(1 - \frac{v_j^2}{3}\Big) \vec{u} \,.$$

$$\langle \vec{s}\,'_j \rangle = \lambda_j v_j \, \frac{2}{3} \frac{E_j}{E_j + m_j} \vec{u} \,,$$

$$\big\langle \vec{v}\,'_j \cdot \vec{s}\,'_j \big\rangle = \lambda_j v_j = \big\langle \vec{v}_j \cdot \vec{s}_j \big\rangle \,.$$

Polarized targets

Upon averaging over the angular distributions of the CνB neutrinos:

$$\diamondsuit \quad \left\langle \frac{\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu}}{4E_e E_j'} \right\rangle = \frac{1}{2E_e}\big(A^0 - \vec{A}\cdot\vec{s}_N\big)\mathcal{F}_j(\lambda_j)\,,$$

$$\boxed{\mathcal{F}_j(\lambda_j) \equiv \left\{1 - \lambda_j v_j + \left(1 - \frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{s}_N\right\}.}$$

For L-helical νs:

$$\mathcal{F}_j(\lambda_j = -1) = 1 + v_j + \left(1 + \frac{2}{3}v_j - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{s}_N\,.$$

For R-helical νs: need to distinguish between Dirac and Majorana neutrinos.
$\mathcal{F}_j^{\rm Dir}(\lambda_j = 1) = 0$;

$$\mathcal{F}_j^{\rm Maj}(\lambda_j = 1) = 1 - v_j + \left(1 - \frac{2}{3}v_j - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{s}_N\,.$$

Vanishes in the limit $v_j \to 1$ (no difference between Dir. and Maj. νs).

Standard cosmology predicts CνB to contain equal numbers of L-helical and R-helical neutrinos. $\Rightarrow$ Summing over the helicities:

$$\mathcal{F}_j \equiv \sum_{\lambda_j=\pm 1} \mathcal{F}_j(\lambda_j) = \begin{cases} 1 + v_j + \left(1 + \frac{2}{3}v_j - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{s}_N\,, & \text{Dirac neutrinos}\,, \\ 2\left[1 + \left(1 - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{s}_N\right], & \text{Majorana neutrinos}\,. \end{cases}$$

In the limit of non-relativistic neutrinos:

$$\mathcal{F}_j \simeq \begin{cases} 1 + v_j + (1 + \frac{2}{3}v_j)\vec{u}\cdot\vec{s}_N\,, & \text{Dirac neutrinos}\,, \\ 2\left(1 + \vec{u}\cdot\vec{s}_N\right), & \text{Majorana neutrinos}\,. \end{cases}$$

For highly relativistic neutrinos:

$$\mathcal{F}_j^{\rm Dir} \simeq \mathcal{F}_j^{\rm Maj} \simeq 2\left(1 + \frac{2}{3}\vec{u}\cdot\vec{s}_N\right).$$

Non-relativistic case:

The results for Dirac and Majorana neutrinos differ. the total detection cross section for Majorana νs is about twice as large as that for Dirac νs (Long, Lunardini & Sabancilar, 2014). $\Rightarrow$ Can in principle be used to discriminate between Dir. amd Maj. neutrinos.

<u>But:</u> complicated by the fact that the local CνB density at the Earth may differ from $n_{\nu 0}$ due to gravitational clustering effects, not precisely known.

The degeneracy between unknown local n_ν relic and Dirac/Majorana ν nature can in principle be lifted by measuring time-dependent angular correlations in relic ν capture.

In practice, however, this will be very difficult because the difference between the angular correlatiopn coeffs. contains an extra factor v_j compared to the main $\vec{u}\cdot\vec{s}_N$ term.

Unpolarized targets

Averaging over the polarizations of the parent nuclei we find

$$\left\langle \frac{1}{3}\sum_{\lambda}\frac{\varepsilon_\mu(\lambda)\varepsilon_\nu^*(\lambda)X^{\mu\nu}}{4E_eE_j'}\right\rangle = \frac{1}{2E_e}\left\{A^0(1-\lambda_j v_j) - \frac{1}{3}\Big(1-\frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\Big)\vec{u}\cdot\vec{A}\right\}.$$

If the spin state of the produced electron is not measured – sum over s_e:

$$\tilde{\mathcal{F}}_j(\lambda_j) \equiv \left\langle \frac{1}{3}\sum_{\lambda,s_e}\frac{\varepsilon_\mu(\lambda)\varepsilon_\nu^*(\lambda)X^{\mu\nu}}{4E_eE_j'}\right\rangle = 1-\lambda_j v_j - \frac{1}{3}\Big(1-\frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\Big)\vec{u}\cdot\vec{v}_e\,.$$

Summing over helicities:

$$\tilde{\mathcal{F}}_j \equiv \sum_{\lambda_j=\pm 1}\tilde{\mathcal{F}}_j(\lambda_j) = \begin{cases} 1+v_j-\frac{1}{3}\Big(1+\frac{2}{3}v_j-\frac{v_j^2}{3}\Big)\vec{u}\cdot\vec{v}_e\,, & \text{Dirac neutrinos}\,, \\ 2\Big[1-\frac{1}{3}\Big(1-\frac{v_j^2}{3}\Big)\vec{u}\cdot\vec{v}_e\Big]\,, & \text{Majorana neutrinos}\,. \end{cases}$$

Unpolarized targets

For highly relativistic neutrinos:

$$\tilde{\mathcal{F}}_j^{\rm Dir} \simeq \tilde{\mathcal{F}}_j^{\rm Maj} \simeq 2\Big[1 - \frac{2}{9}\vec{u}\cdot\vec{v}_e\Big].$$

In the limit of non-relativistic neutrinos:

$$\tilde{\mathcal{F}}_j \simeq \begin{cases} 1 + v_j - \frac{1}{3}\big(1 + \frac{2}{3}v_j\big)\vec{u}\cdot\vec{v}_e\,, & \text{Dirac neutrinos}\,, \\ 2\Big[1 - \frac{1}{3}\vec{u}\cdot\vec{v}_e\Big]\,, & \text{Majorana neutrinos}\,. \end{cases}$$

On top of the overall factor $\simeq 2$ for Dirac neutrinos, different angular correlation coeffs. for non-relat. Dirac and Majorana νs:

$$\begin{array}{ll} 1 - \frac{1}{3}\Big(1 - \frac{v_j}{3}\Big)\vec{u}\cdot\vec{v}_e\,, & \text{Dirac neutrinos}\,, \\ 1 - \frac{1}{3}\vec{u}\cdot\vec{v}_e\,, & \text{Majorana neutrinos}\,. \end{array}$$

Difference $\sim v_j\vec{u}\cdot\vec{v}_e$ – extra small factor v_j.

Comparison with ν capture on polarized ^{3}H

Compare our results for pure Gamow-Teller $1^\pi \to 0^\pi$ transitions with those for ν capture on polarized tritium (mixed Fermi–Gamow-Teller $1/2^+ \to 1/2^+$ transition) in the case when spins of the electron and daughter ^{3}He are not observed (Lisanti et al., 2014):

$$1 - \vec{v}_j\cdot\vec{s}_j + A(1-\vec{v}_j\cdot\vec{s}_j)\vec{v}_e\cdot\vec{s}_N + BK_j\vec{v}_j\cdot\vec{s}_N - B\frac{m_j}{E_j}\vec{s}_j\cdot\vec{s}_N + aK_j\vec{v}_e\cdot\vec{v}_j - a\frac{m_j}{E_j}\vec{v}_e\cdot\vec{s}_j\,.$$

Correlation coeffs. A, B and a depend on the ratio of the F. and G.-T. nuclear matrix elements. Numerically

$$A \simeq -0.095\,, \qquad B \simeq 0.99\,, \qquad a \simeq -0.087\,.$$

To compare with our results, sum our squared amplitude over s_e $\Rightarrow$

$$1 - \vec{v}_j\cdot\vec{s}_j - (1-\vec{v}_j\cdot\vec{s}_j)\vec{v}_e\cdot\vec{s}_N + K_j\vec{v}_j\cdot\vec{s}_N - \frac{m_j}{E_j}\vec{s}_j\cdot\vec{s}_N - K_j(\vec{v}_j\cdot\vec{s}_N)(\vec{v}_e\cdot\vec{s}_N) + \frac{m_j}{E_j}(\vec{v}_e\cdot\vec{s}_N)(\vec{s}_j\cdot\vec{s}_N)\,.$$

The first five terms have the same structure; the two sets coincide if one takes $A=-1$ and $B=1$ in the tritium result. Terms bilinear in $\vec{s}_N$ are absent in the tritium case. Reason: the special nature of the $1/2^+ \to 1/2^+$ transition, for which the tensor alignment $\propto [J(J+1) - 3\langle(\vec{J}\cdot\vec{s}_N)^2\rangle]$ vanishes (Jackson et al., 1957).

Upon averaging over the directions of $\vec{s}_N$ the last two terms in G.-T. case would reproduce those in the tritium case but with $a=-1/3$.

To $\mathcal{O}(u)$ in the lab frame:

$$E'_j = E_j + \vec{u}\cdot\vec{q}, \qquad \vec{q}\,' = \vec{q} + \vec{u}E_j\,, \qquad \vec{v}\,'_j = \vec{v}_j(1-\vec{u}\cdot\vec{v}_j) + \vec{u}\,.$$

$$\vec{s}\,'_j = \vec{s}_j + \frac{(\vec{q}\cdot\vec{s}_j)\vec{u} - (\vec{u}\cdot\vec{s}_j)\vec{q}}{E_j + m_j}\,.$$

$$\Downarrow$$

$$\vec{v}\,'_j\cdot\vec{s}\,'_j = \lambda_j v_j\left\{1 + (\vec{u}\cdot\vec{v}_j)\frac{1-v_j^2}{v_j^2}\right\}.$$

$$K'_j = 1 - \frac{E'_j}{E'_j + m_j}(\vec{v}\,'_j\cdot\vec{s}\,'_j) = 1 - \frac{E_j}{E_j + m_j}\lambda_j v_j\left(1 + \frac{\vec{u}\cdot\vec{v}_j}{v_j^2}\frac{m_j}{E_j}\right).$$

This gives

$$\frac{1}{E'_j}\vec{B}' \;=\; K'_j\vec{v}\,'_j - \frac{m_j}{E'_j}\vec{s}\,'_j \;=\; (1-\lambda_j v_j)\left\{\vec{u} - (1-\vec{u}\cdot\vec{v}_j)\lambda_j\frac{\vec{v}_j}{v_j}\right\}.$$

Angular averaging

Taking into account that $\langle \vec{v}_j \rangle = 0$ and $\langle \vec{s}_j \rangle = 0$:

$$\langle E_j' \rangle = E_j\,, \qquad \langle \vec{q}\,' \rangle = \vec{u} E_j\,, \qquad \langle \vec{v}_j' \rangle = \Big(1 - \frac{v_j^2}{3}\Big)\vec{u}\,,$$

$$\Big\langle \frac{\vec{v}_j'}{v_j'} \Big\rangle = \frac{2}{3}\frac{\vec{u}}{v_j}\,,$$

$$\langle \vec{s}_j' \rangle = \lambda_j v_j \,\frac{2}{3}\frac{E_j}{E_j + m_j}\vec{u}\,, \qquad \langle \vec{v}_j'\!\cdot\!\vec{s}_j' \rangle = \lambda_j v_j = \langle \vec{v}_j\!\cdot\!\vec{s}_j \rangle\,.$$

This yields

$$\Big\langle \frac{1}{E_j'} B^{0\prime} \Big\rangle = 1 - \langle \vec{v}_j'\!\cdot\!\vec{s}_j' \rangle = 1 - \lambda_j v_j\,,$$

$$\Big\langle \frac{1}{E_j'} \vec{B}' \Big\rangle = \Big\langle K_j' \vec{v}_j' - \frac{m_j}{E_j'}\vec{s}_j' \Big\rangle = \Big(1 - \frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\Big)\vec{u}\,.$$

With these relations, one can find the expressions for $\mathcal{F}_j(\lambda_j)$.

Unpolarized targets

Averaging over the polarizations of the parent nuclei we find

$$\left\langle \frac{1}{3}\sum_{\lambda} \frac{\varepsilon_\mu(\lambda)\varepsilon_\nu^*(\lambda)X^{\mu\nu}}{4E_eE_j'}\right\rangle = \frac{1}{2E_e}\left\{A^0(1-\lambda_j v_j) - \frac{1}{3}\left(1-\frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{A}\right\}.$$

If the spin state of the produced electron is not measured – sum over s_e:

$$\left\langle \frac{1}{3}\sum_{\lambda,s_e} \frac{\varepsilon_\mu(\lambda)\varepsilon_\nu^*(\lambda)X^{\mu\nu}}{4E_eE_j'}\right\rangle = 1-\lambda_j v_j - \frac{1}{3}\left(1-\frac{2}{3}\lambda_j v_j - \frac{v_j^2}{3}\right)\vec{u}\cdot\vec{v}_e\,.$$

If the electron direction is not observed but its spin state is:

$$\int \frac{d\Omega_e}{4\pi}\left\langle \frac{1}{3}\sum_{\lambda} \frac{\varepsilon_\mu(\lambda)\varepsilon_\nu^*(\lambda)X^{\mu\nu}}{4E_eE_j'}\right\rangle = \frac{1}{2}\left\{1-\lambda_j v_j+\frac{1}{9}\left(1+2\frac{m_e}{E_e}\right)\left(1-\frac{2}{3}\lambda_j v_j-\frac{v_j^2}{3}\right)\vec{u}\vec{s}_e\right\}.$$