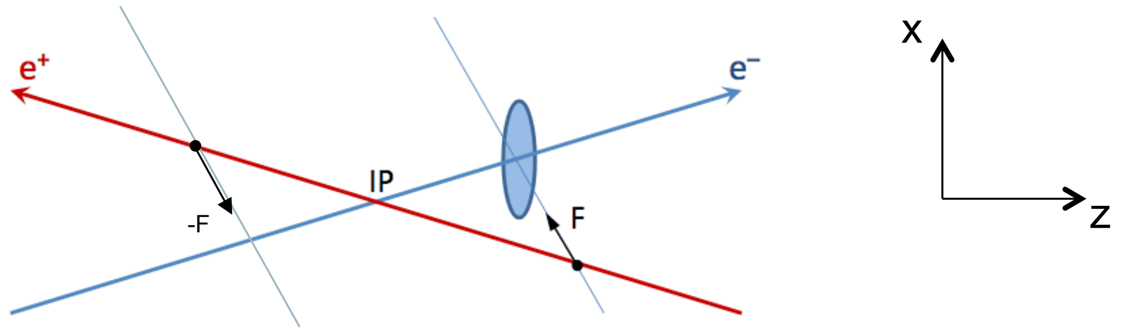
Measurement of the “Energy Kick”

- **Reminder : Energy change due to crossing angle**
  - **See talks from Dmitry Shatilov and Emmanuel Perez**



- Transverse kick from a charged “slice” of the opposite bunch is perpendicular to its trajectory (in ultra-relativistic case). To first order, the kick is proportional to the opposite bunch population.
- Due to the crossing angle (actually, large Piwinski angle), transverse kicks have longitudinal components for the particles, and therefore affect their energy.
- The signs of energy change are different “before” and “after” IP.
- The whole energy change depends on the particle’s Z-coordinate.
- Integrated effect is 0, except when the particle experiences a collision with a particle of the opposite bunch
  The truncated integral is equivalent to an average shift of the beam energy
- The transverse kick has also a component along the x axis,
  The truncated integral is equivalent to an average shift of the collision crossing angle

- **Average beam energy shift +70 keV [+60 keV (Dmitry) to +80 keV (Emmanuel)]**

Why does it matter ?

- The beam energy is measured with resonant depolarization
  - With a few transversally polarized single bunches

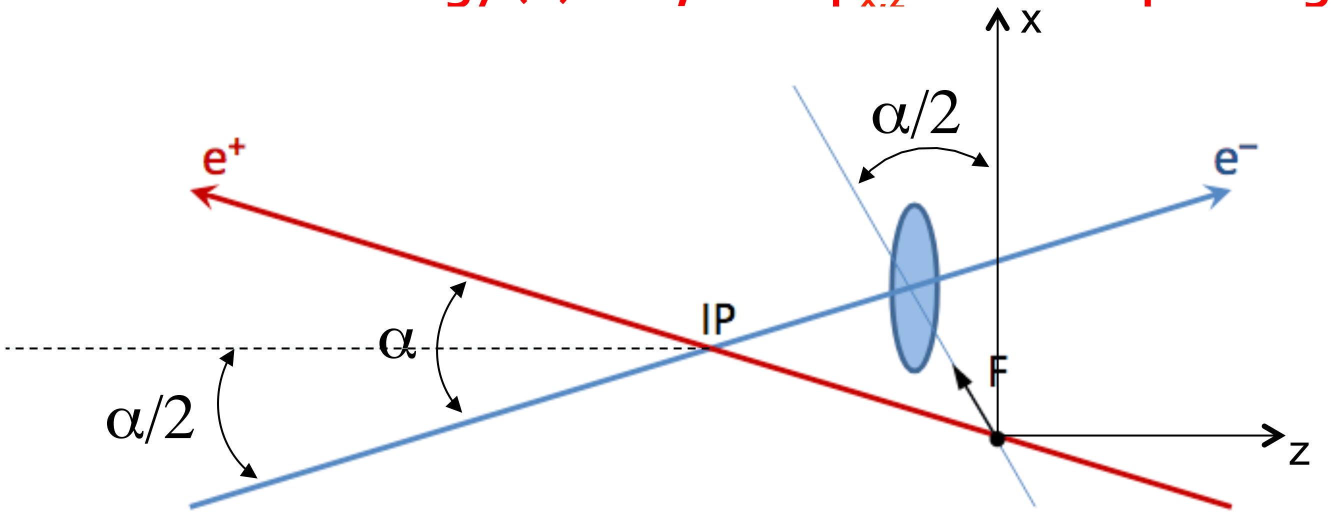
However

- "Single" means that these bunches do not experience collisions
  - And therefore, they do not experience this energy shift

- The measured beam energy is smaller than the collision beam energy
  - By (70±10) keV on average
    - i.e., of the same order as the precision of the measurement
      - ➨ This shift needs therefore to be measured with a reasonable accuracy (<10%)

A few useful relations

- Let δE be the beam energy (E) kick, and $\delta p_{x,z}$ the corresponding $p_{x,z}$ kicks



- $p_x = E \sin\alpha/2$; $p_z = E \cos\alpha/2$.
  - For E = 45.6 GeV and α = 30 mrad, p_x = 684 MeV and p_z = 45.594 GeV
- $\delta p_x = k \cos\alpha/2$; $\delta p_z = k \sin\alpha/2$ (the force is perpendicular to the opposite bunch trajectory)
- $E^2 = p_x^2 + p_y^2 + p_z^2 \Rightarrow E\delta E = p_x\delta p_x + p_z\delta p_z = 2kE \cos\alpha/2 \sin\alpha/2 = 2\, p_x\delta p_x = 2\, p_z\delta p_z$.

$$\delta p_x = E\delta E/2p_x \quad ; \delta p_z = E\delta E/2p_z$$

  - For E = 45.6 GeV, α = 30 mrad, and δE = 70 keV : δp_x = 2.3 MeV and δp_z = 35 keV
- Note: $\delta p_x/p_x$ = 0.34% , while $\delta p_z/p_z < 10^{-6} \Rightarrow$ Increase of crossing angle

A few useful relations (cont'd)

- **Let $\delta\alpha$ the increase of the crossing angle α**
  - Reminder : $p_x = E \sin\alpha/2$
    - $\delta p_x/p_x \sim \delta E/E + \delta\alpha/\alpha \Rightarrow \delta\alpha/\alpha = \delta p_x/p_x - \delta E/E = 0.34\% - 0.00015\% = 0.34\%$
      - ➨ For α = 30 mrad, $\delta\alpha$ = 0.102 mrad

- **Centre-of-mass energy $\sqrt{s} = 2E\cos(\alpha/2) = 2p_z$**
  - Centre-of-mass energy increase $\delta\sqrt{s} = 2\delta p_z = E/p_z \times \delta E$
    - $\delta\sqrt{s} \simeq \delta E = 70$ keV
      - ➨ Warning ! $\delta\sqrt{s} \neq 2\delta E$: the boost along x does not change $\sqrt{s}$

- **Relation between $\delta\sqrt{s}$ and $\delta\alpha$**
  - $\delta\sqrt{s} = 2\delta p_z = 2\delta p_x \times p_x/p_z = 2\delta p_x \times \tan(\alpha/2) \simeq \delta p_x\, \alpha = p_x\, \delta\alpha = E \sin\alpha/2\, \delta\alpha$
    - Let ΔE, $\Delta\alpha$ and $\Delta\delta\alpha$, the precisions with which E, α, and $\delta\alpha$ are measured
      - ➨ The uncertainty on the centre-of-mass energy shift amounts to:

$$\Delta\delta\sqrt{s} / \delta\sqrt{s} = \Delta E/E \oplus \Delta\alpha/\alpha \oplus \Delta\delta\alpha/\delta\alpha \cong \Delta\delta\alpha/\delta\alpha$$

  - Measuring the C.M. energy shift amounts to measure the crossing angle increase with the same relative precision, typically < 10%.

Measurement of the crossing angle (reminder)

- **Make use of $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events**
  - Assuming one photon emitted along one of the two beams + E,p conservation

$$
\begin{aligned}
E^+ \sin\theta^+ \cos\varphi^+ + E^- \sin\theta^- \cos\varphi^- + |p_z^\gamma| \tan\alpha/2 &= \sqrt{s}\tan\alpha/2, \\
E^+ \sin\theta^+ \sin\varphi^+ + E^- \sin\theta^- \sin\varphi^- &= 0, \\
E^+ \cos\theta^+ + E^- \cos\theta^- + p_z^\gamma &= 0, \\
E^+ + E^- + |p_z^\gamma| / \cos\alpha/2 &= \sqrt{s}/\cos\alpha/2,
\end{aligned}
$$

- Where $E^\pm$ are the measured energies of the $\mu^\pm$
- Where α is the beam crossing angle (nominal : 30 mrad),
- Where the z axis is the bisector of the two beam axes,
- Where the two beam axes form the (x,z) plane,
- Where $\theta^\pm$ are measured with respect to the z axis in the FCC-ee frame,
- Where $\phi^\pm$ are measured with to the x axis in the plane transverse to the z axis,
- Where $\sqrt{s}$ is the centre-of-mass energy of the collision

Solve E,p conservation for α and $x_\gamma = p_z(\gamma)/\sqrt{s}$

- As a function of muon angles only (assumed resolution: 0.1 mrad)

$$\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

$$x_\gamma = -\frac{x_+\cos\theta^+ + x_-\cos\theta^-}{\cos(\alpha/2) + |x_+\cos\theta^+ + x_-\cos\theta^-|}$$

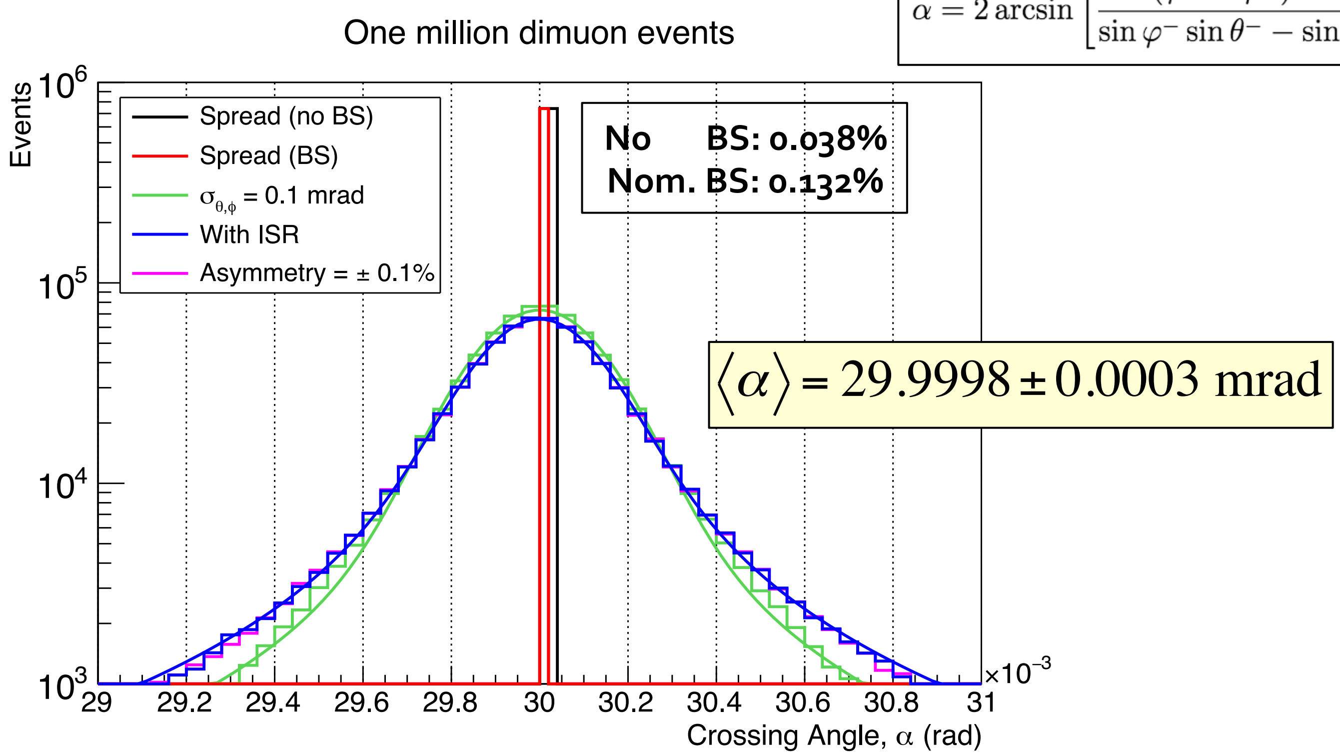
With $$x_\pm = \frac{\mp\sin\theta^\mp\sin\varphi^\mp}{\sin\theta^+\sin\varphi^+ - \sin\theta^-\sin\varphi^-}$$

- Absolute angles defined with respect to the (x,y,z) axes
  - z axis = bissector of the two beam axes
  - (x,z) plane = plane that contains the two beam axes
  - y = axis going upwards perpendicularly to that plane
- Tracker can be aligned perfectly with respect to these axes
  - By minimizing the RMS of the α distribution with respect to its Euler angles
    - ➨ See backup slides (and energy calibration paper)

Beam crossing angle determination

- With 10^6 dimuon events (every 5 minutes at the Z pole, at full luminosity)

$$\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$



- **Precision = 0.3 mrad / $\sqrt{N_{\mu\mu}}$, e.g., 0.01 mrad with 1000 dimuon events**
  - **Spread sensitive to anything happening in the transverse plane**
    - **ϕ resolution, p_T of emitted photons, and (x,y,z) axes knowledge**

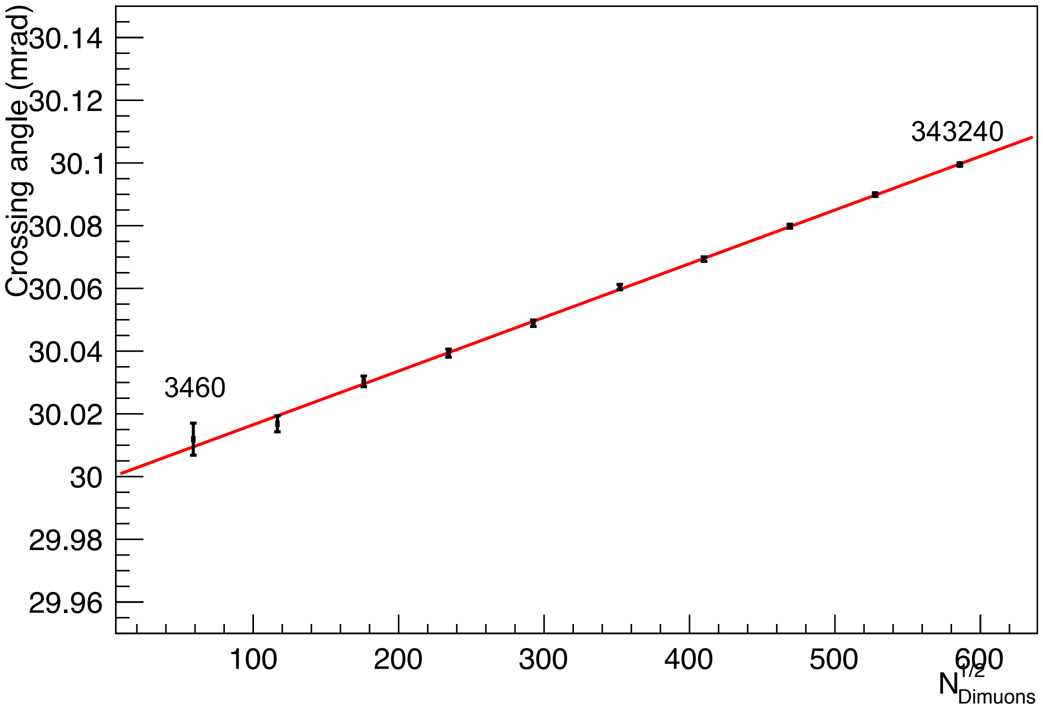
Beam crossing angle increase determination

- Need bunches with increasing population (from 0 to nominal)
  - The filling period with the "bootstrapping" method is ideal for this purpose
    - Every 104 seconds, inject 10% of the bunch intensity at once for 1/8 of the bunches
      - ➨ Until the next 10% (104 seconds after), these bunches collide with nominal β^*
        - With bunch populations of 10%, 20%, …, 100% of the nominal value
        - i.e., with a luminosity corresponding to 1%, 4%, 9%, 16%, 25%, …, 100% of the nominal value
      - ➨ The crossing angle increases by 10%, 20%, 30%, …, 100% of the total increase
        - Proportionally to the bunch intensity
        - And can be measured with a precision of 0.3 mrad / $\sqrt{N_{\mu\mu}}$
    - Then repeat with the other 7/8 of the collider

  - The following figure shows the measured crossing angle for each of 10 steps
    - Averaged over the 8 filling sequences, with its uncertainty 0.3 mrad / $\sqrt{N_{\mu\mu}}$
  - As a function of the squareroot of the total number of recorded dimuon events $N_{\mu\mu}$
    - Summed over the 8 filling sequences, randomized according to statistics
      - ➨ ($N_{\mu\mu}$ is proportional to the luminosity, and $\sqrt{N_{\mu\mu}}$ is proportional to the bunch intensity)

Results

- **Measurement of the beam crossing angle increase at the Z pole**



- Nominal crossing angle α : 29.9994 ± 0.0011 mrad (precision 3×10^{-5})
- Average crossing angle increase $\delta\alpha$: 0.1023 ± 0.0013 mrad (precision 1.3%)
  - $\Delta E/E$ and $\Delta\alpha/\alpha \ll \Delta\delta\alpha/\delta\alpha \Rightarrow \Delta\delta\sqrt{s} / \delta\sqrt{s} = \Delta\delta\alpha/\delta\alpha = 1.3\%$
    - Average $\sqrt{s}$ increase : $\delta\sqrt{s} = E\ \alpha/2\ \delta\alpha$ = 69.9 ± 0.9 keV
    - Average p_x kick: $\delta p_x = p_x\ \delta\alpha / \alpha$ = 2.33 ± 0.03 MeV

Precisions are directly proportional to the muon azimuthal resolution

Results (cont'd)

- **Off-peak points**
  - Dimuon rate smaller by a factor 7.8 (3.2) at √s = 87.9 (93.8) GeV
    - Precision on δ√s during the filling period:
      - ➨ 70 ± 2.7 keV at 87.9 GeV
      - ➨ 70 ± 1.7 keV at 93.8 GeV

        Still very small with respect to the absolute √s calib. uncertainty (~100 keV)

        Very much acceptable contribution to the point-to-point uncertainty

- **WW threshold**
  - Dimuon rate smaller by a factor 3000 (than at the Z pole)
  - Filling time (266 s instead of 1035 s) smaller by a factor 4
  - δE, δ√s (90-120 keV) and δα (0.15 mrad) larger by a factor 1.5
    - Standalone precision on δ√s during the filling period:
      - ➨ 105 ± 45 keV

        Still acceptable w.r.t. the absolute √s calib. uncertainty (~300 keV)
    - Can use nominal α from regular Z calibration runs (and larger statistics at full lumi)
      - ➨ Precision on δ√s reduced to ±15 keV (and to ±5 keV with 5 add'l minutes)

Results (cont'd)

- **The method does not work at 240, 350, and 365 GeV**
  - Dimuon rate too small (less than one event expected every 14 or 11 seconds)
  - Also: No transverse polarization, hence no calibration with resonant depolarization

- **Need to rely on other methods (or not)**
  - Calibration with Zγ and WW events for the <$\sqrt{s}$> calibration
    - ±1.7 MeV at 240 GeV, ±5 MeV at 350 GeV, ±2 MeV at 365 GeV
  - Calibration of the theoretical prediction for $\delta\sqrt{s}$ at the Z pole and the WW threshold
    - And extrapolation to 240, 350, and 365 GeV.
  - Effect anyway much smaller than 2 MeV, typically 100 keV
    - This bias contributes the systematic uncertainty in a negligible manner

Conclusion & outlook

- **The beam energy and crossing angle kicks can be measured**
  - At the Z pole and at the WW threshold
  - During the filling period
  - With an adequate precision

- **There is no need to measure these kicks at higher energy**
  - Effects can be predicted with an adequate precision with calibration at lower energies

- **Data taking may prove difficult during transfer from booster to main ring**
  - Once every 104 (27 seconds) seconds at the Z pole (WW thresold)
    - Tracker designs must accommodate this delicate step with HV On

- **Alternative methods at full luminosity should be explored if too difficult**
  - e.g., with a few bunches with lower intensity ?
  - Or using the natural bunch intensity spread ? (I have no idea of what it could be…)

Backup slides

- Prepared for FCC week in Amsterdam
  - Written up in the energy calibration paper
    - See draft at https://www.overleaf.com/11630130cmkmfpvyhhgb

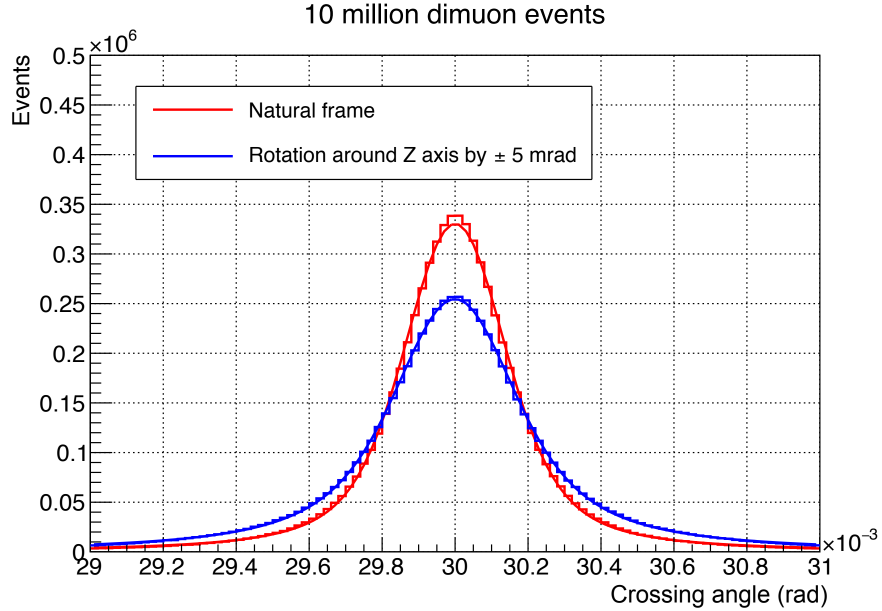
Control the angular resolution to 0.01 mrad ?

- **Q: How to measure the angular resolution to 10% or better**
  - For any value of θ and ϕ ?

- **A: Take a muon track in dimuon events**
  - Refit it with the odd hits, on the one hand, and with the even hits, on the other
    - And compare the angles
  - Need only 100 tracks in each (θ , ϕ) bin for a 10% precision
    - 10^6 dimuon events = 5 minutes at the Z pole = bins of 3×3 $(\text{mrad})^2$
  - Expected to be stable in time
    - Precision (or bin size) improves with dimuon statistics

Absolute tracker alignment

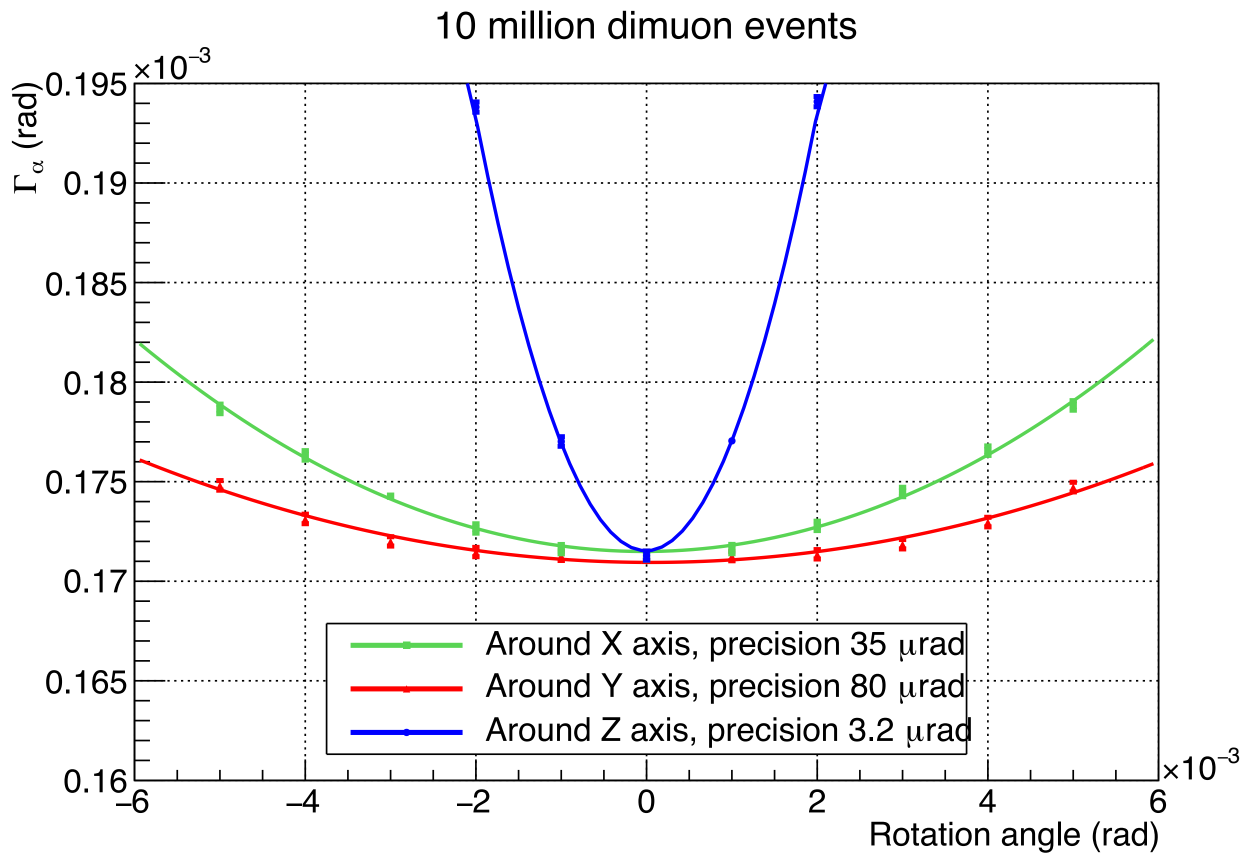
- Absolute angle determination is (usually) not an easy task
  - Requires alignment of the local (detector) frame with the natural (FCC-ee) frame
    - Z axis = solenoid axis vs bissector of the two beam axes
    - (X,Z) plane = horizontal plane vs plane containing the two beam axes
- Spread of α increases with anything happening in the transverse plane
  - E.g., rotation around the Z axis changes both X and Y directions



    - Similarly, rotation around the X (Y) axis changes Y (X) direction

Detector alignment

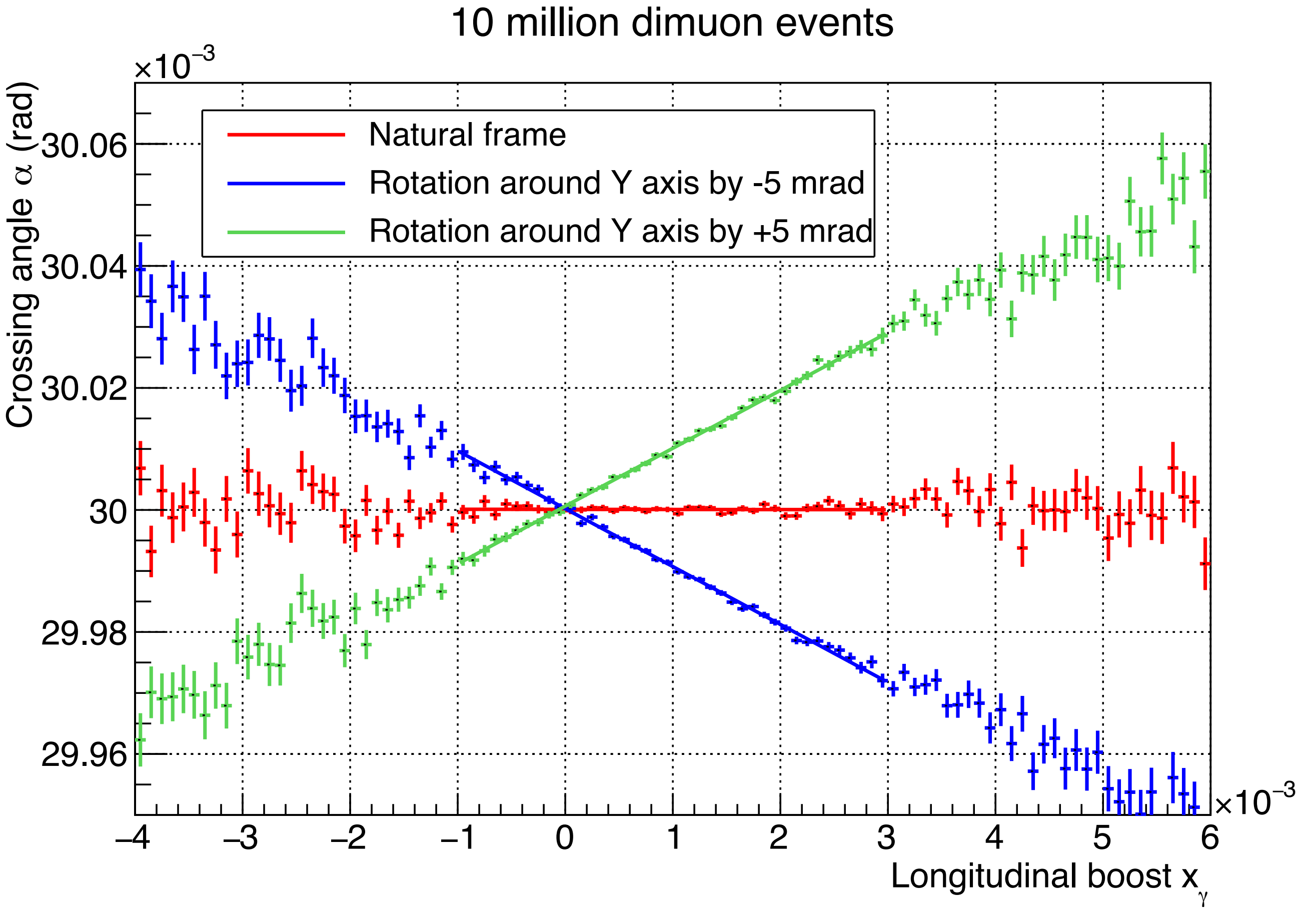
- Minimize the spread of the α distribution to find the three Euler angles



- Note: α spread dominated by the ϕ resolution (here 0.1 mrad)
  - Precisions quadratically improves with the resolution in ϕ (here 0.1 mrad)

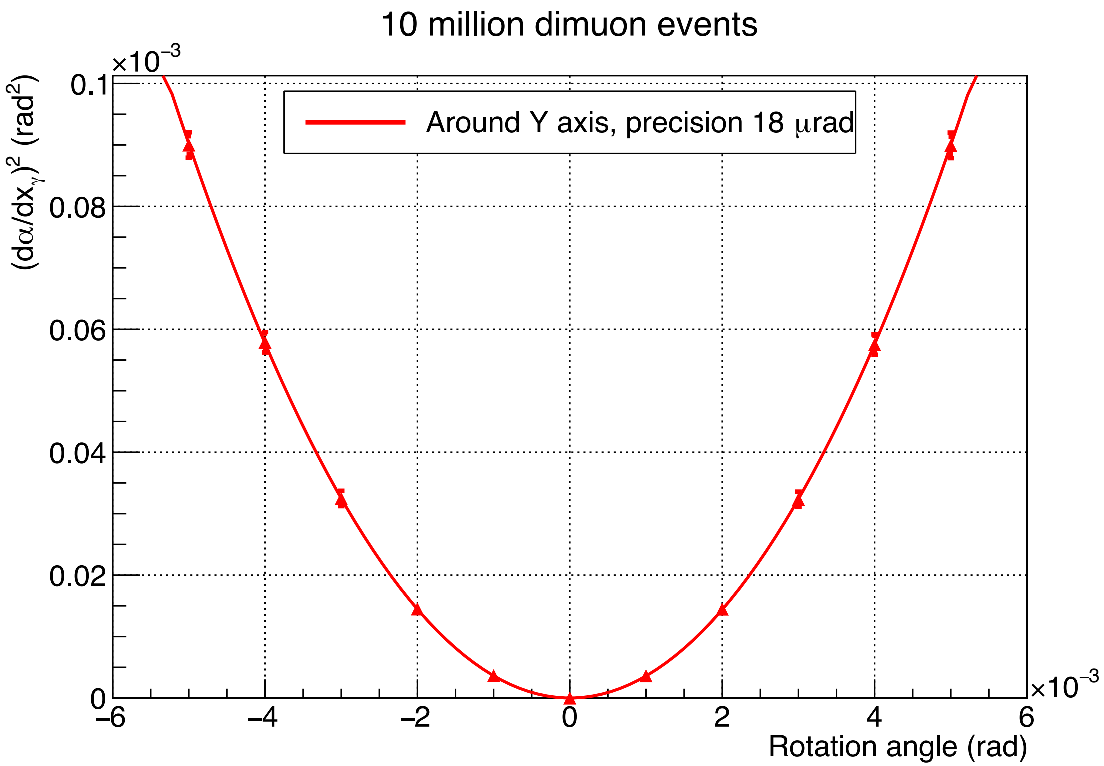
Detector alignment

- **Improve the angle corresponding to a rotation around the Y axis**
  - **X and Z information get mixed by such a rotation**
    - **Resuting in a strong (linear) correlation between x_γ and α:**



Detector alignment

- Minimize the correlation between x_γ and α:



- Improves the precision on that angle by a factor of five.
  - Reach a precision of 0.1 μrad on α and of 10^{-7} on x_γ
  - Variation of the x_γ spread already insignificant with 100 times less events