Perturbative simplicity in lower dimensions

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Integrability in 2d

Conventions on momenta variables

$$p = (p_0, p_1) \quad , \quad p_0^2 - p_1^2 = m^2$$

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Suppose to have a theory with $n-1$ couplings:

$$V_{int} \sim C^{(3)}_{b_1 b_2 b_3} \phi_{b_1} \phi_{b_2} \phi_{b_3} + \ldots + C^{(n-1)}_{b_1 b_2 \ldots b_{n-1}} \phi_{b_1} \phi_{b_2} \ldots \phi_{b_{n-1}}$$

Scattering amplitudes are rational functions of a_{j_i}:

$$A_n(a_{j_1}, a_{j_2}, \ldots, a_{j_n}) = \frac{N(a_{j_1}, a_{j_2}, \ldots, a_{j_n})}{D(a_{j_1}, a_{j_2}, \ldots, a_{j_n})}$$

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We can set the next coupling $C^{(n)}_{a_{j_1} a_{j_2} \ldots a_{j_n}}$ in such a way to cancel this constant and obtain

$$A_n(a_{j_1}, a_{j_2}, \ldots, a_{j_n}) = 0$$

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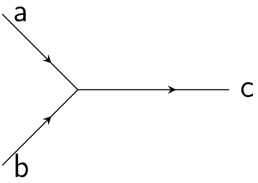

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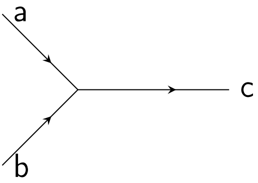

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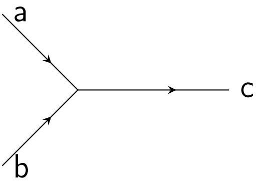

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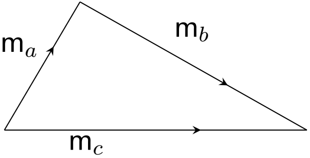
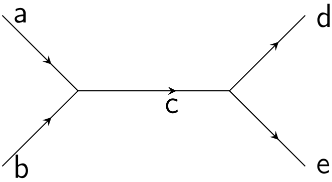
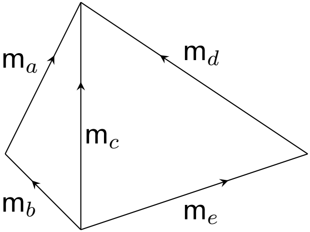
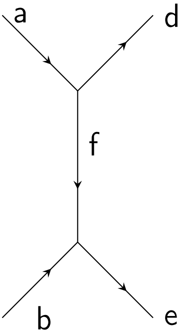
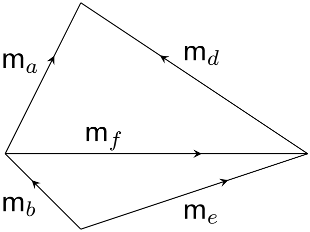


No production of different kind of particles $(a, b) \neq (d, e)$









Thank you

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Scattering Amplitudes:
from Geometry to Experiment