Beam Optics in the Cooling Channel

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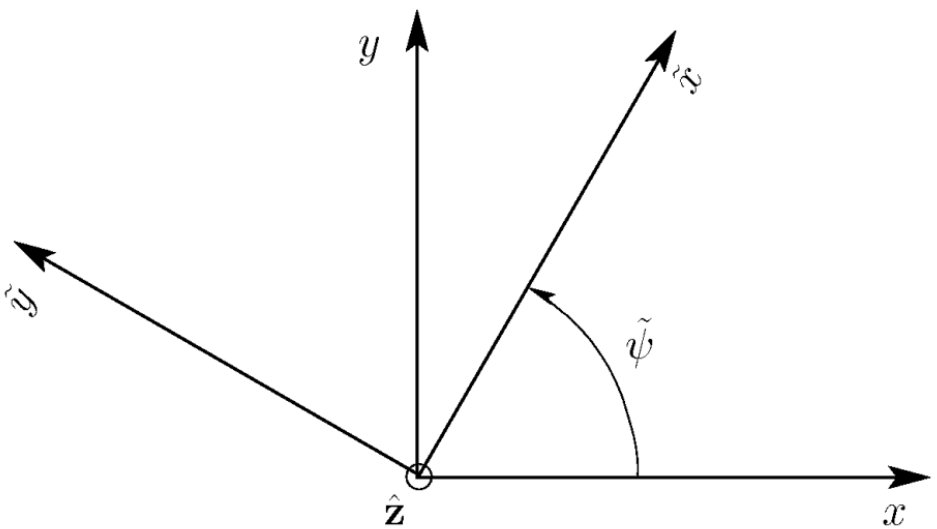
April 17, 2019

Overview

- Spent some time looking at the beam phase-space in the cooling channel
- Transformed beam into the rotating Larmor frame to decouple the x and y dynamics
- Further transformed xpx and ypy phase space evolution into normalised coordinates
- Used MC truth originating from data run 10448, 3 mm, 140 MeV/c

Larmor Angle



$\tilde{...}$ used to denote rotating frame variables

$$\tilde{x} = x\cos\tilde{\psi}(s) + y\sin\tilde{\psi}(s)$$
$$\tilde{y} = -x\sin\tilde{\psi}(s) + y\cos\tilde{\psi}(s)$$

$$\tilde{\psi}(s) = -\int_{s_i}^{s} d\bar{s}\; k_L(\bar{s})$$

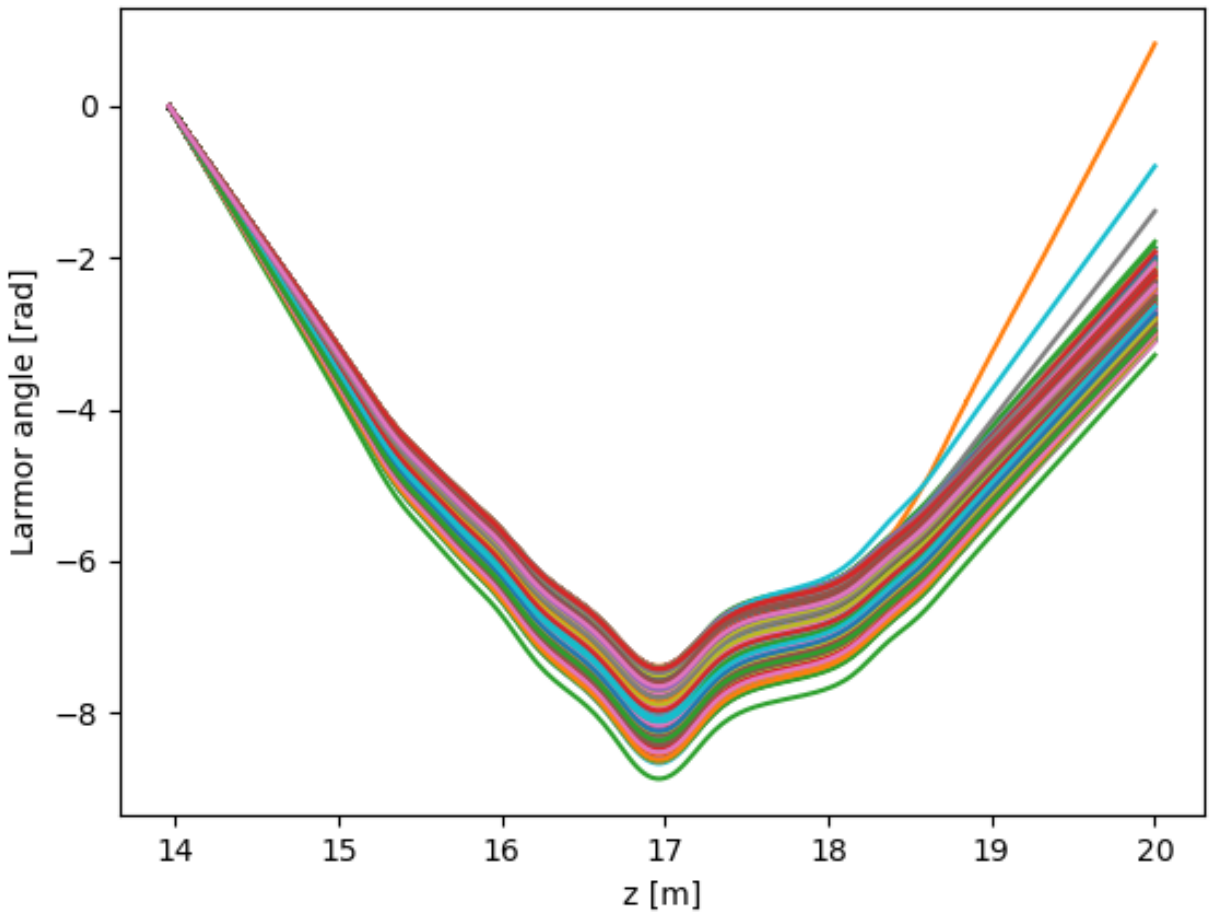
$$k_L(s) \equiv \frac{B_{z0}(s)}{2[B\rho]} = \frac{\omega_c(s)}{2\gamma_b\beta_b c}$$

$= \text{Larmor wave number}$

$s = s_i$ defines initial condition

Larmor Angle

- Computed the Larmor angle for all the particles in the sample, starting integration at TKU5

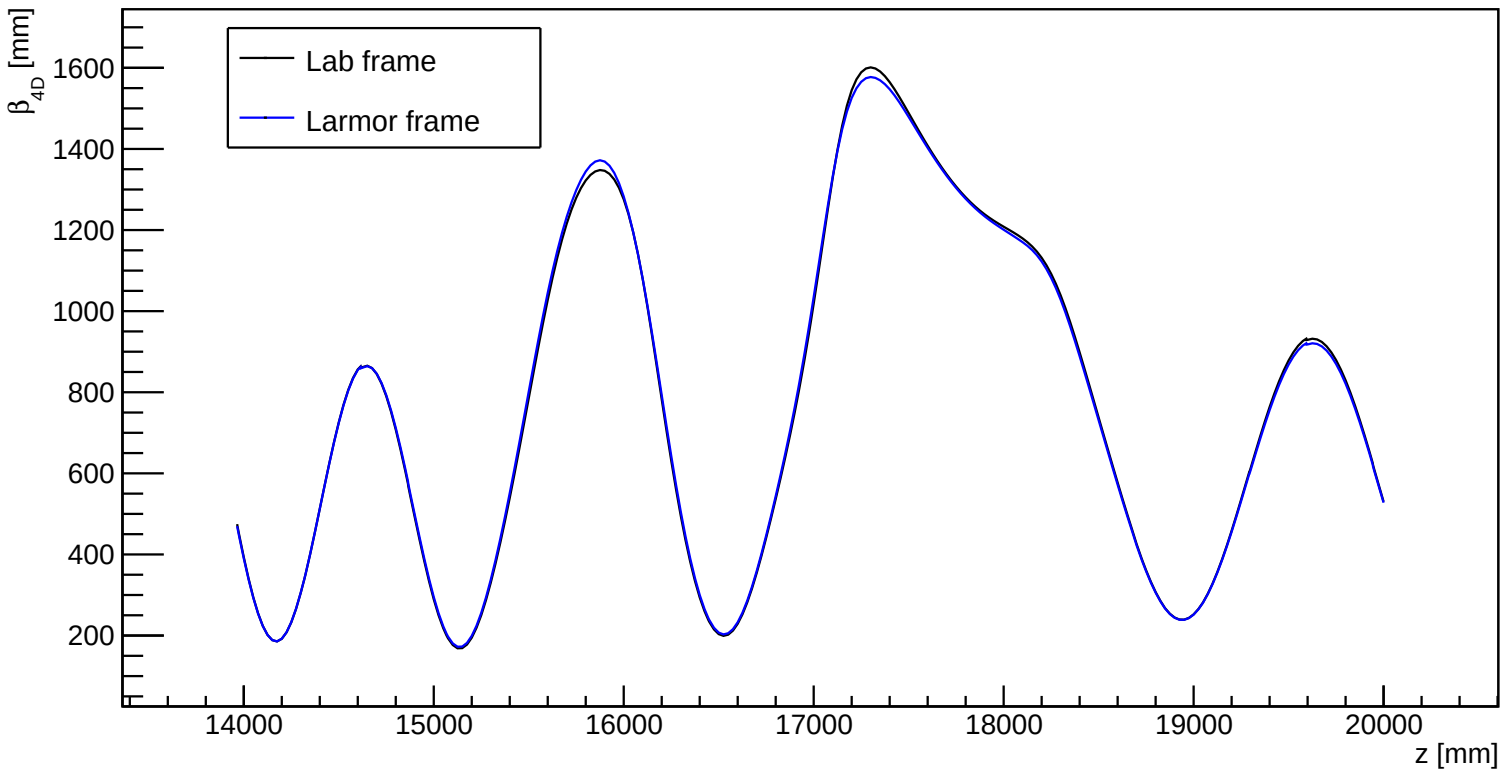


Transformation to Larmor Frame

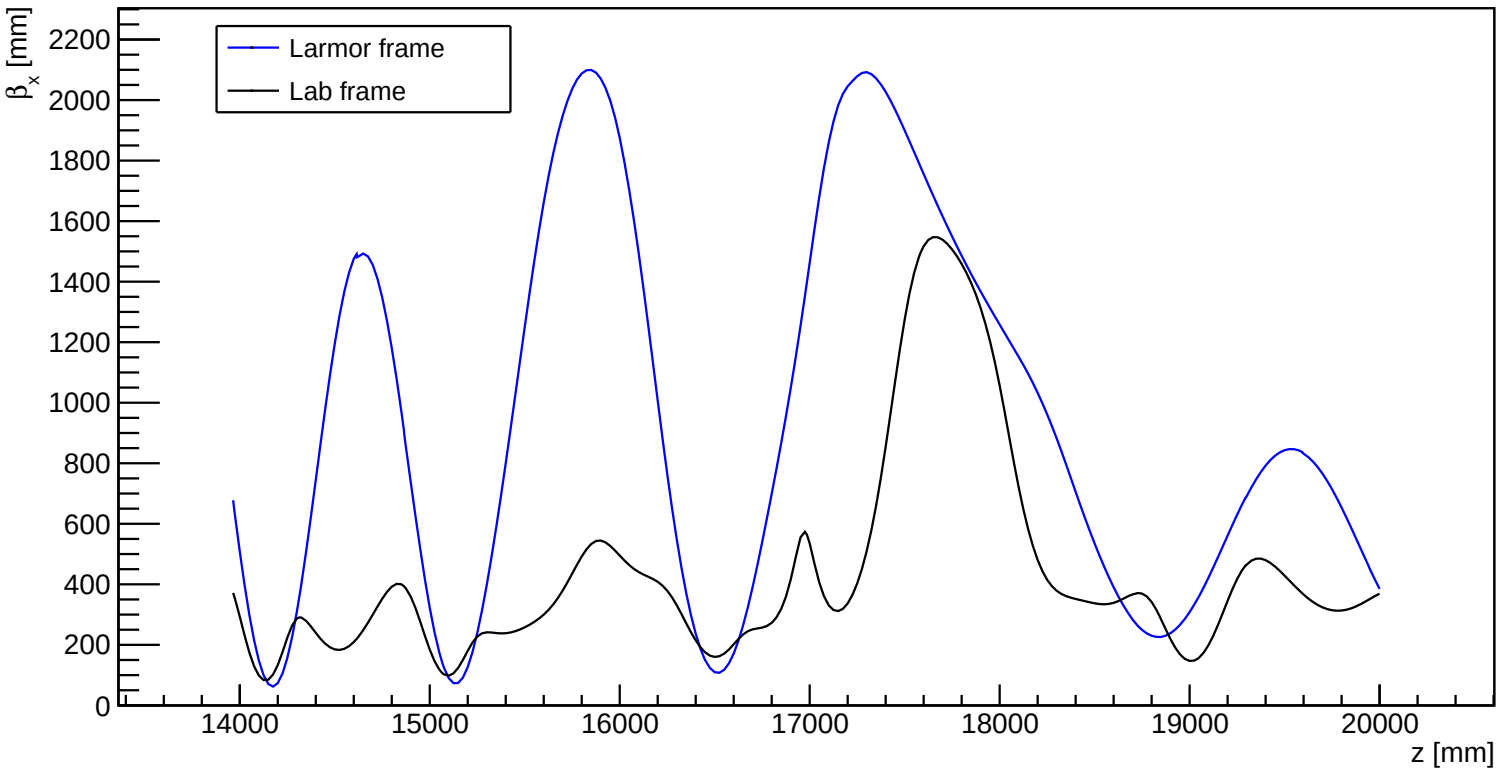
$$\begin{bmatrix} \tilde{x} \\ \tilde{x}' \\ \tilde{y} \\ \tilde{y}' \end{bmatrix} = \tilde{\mathbf{M}}_r^{-1}(s|s_i) \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}$$

$$\tilde{\mathbf{M}}_r^{-1}(s|s_i) = \begin{bmatrix} \cos\tilde{\psi} & 0 & \sin\tilde{\psi} & 0 \\ k_L \sin\tilde{\psi} & \cos\tilde{\psi} & -k_L \cos\tilde{\psi} & \sin\tilde{\psi} \\ -\sin\tilde{\psi} & 0 & \cos\tilde{\psi} & 0 \\ k_L \cos\tilde{\psi} & -\sin\tilde{\psi} & k_L \sin\tilde{\psi} & \cos\tilde{\psi} \end{bmatrix}$$

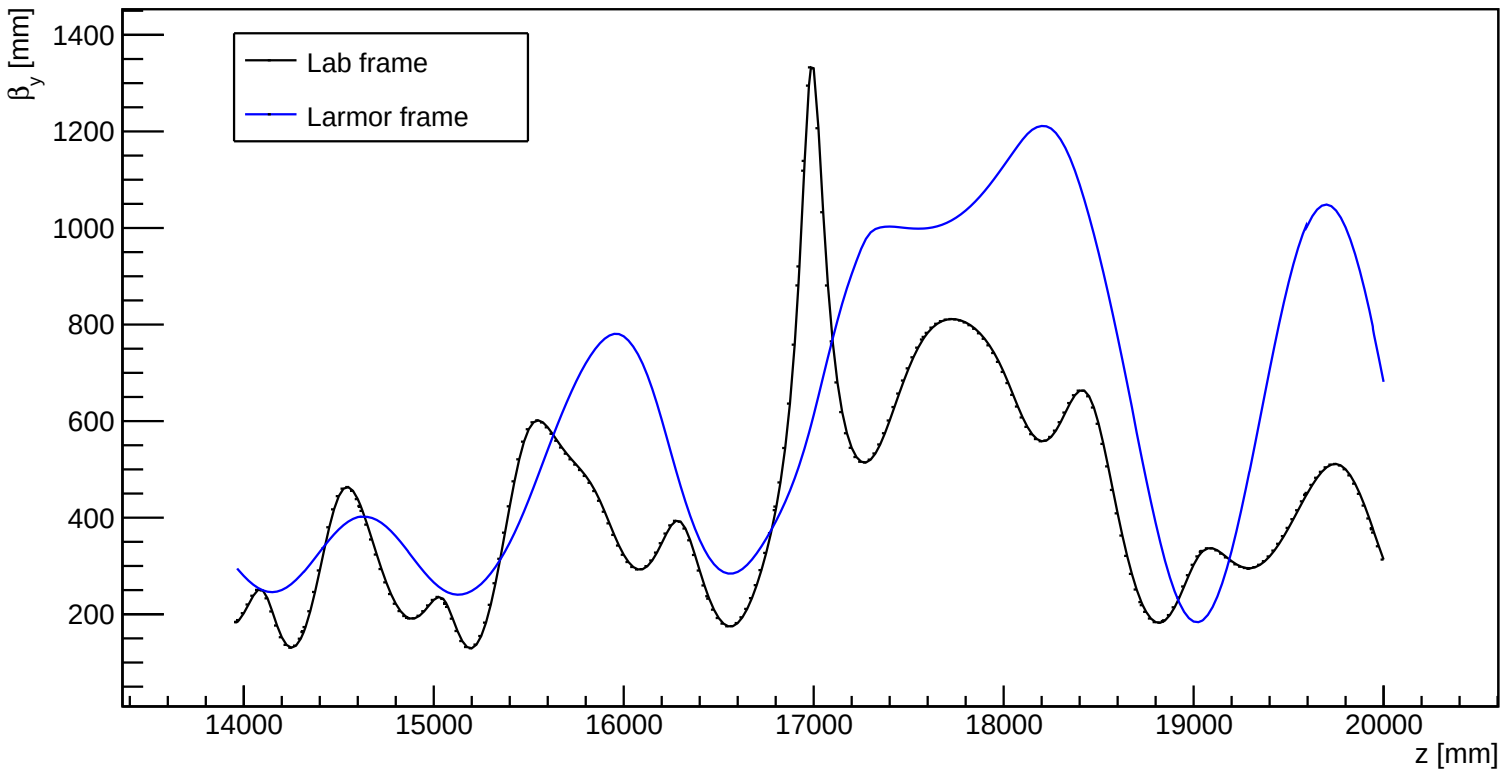
Twiss parameters: β_{4D}



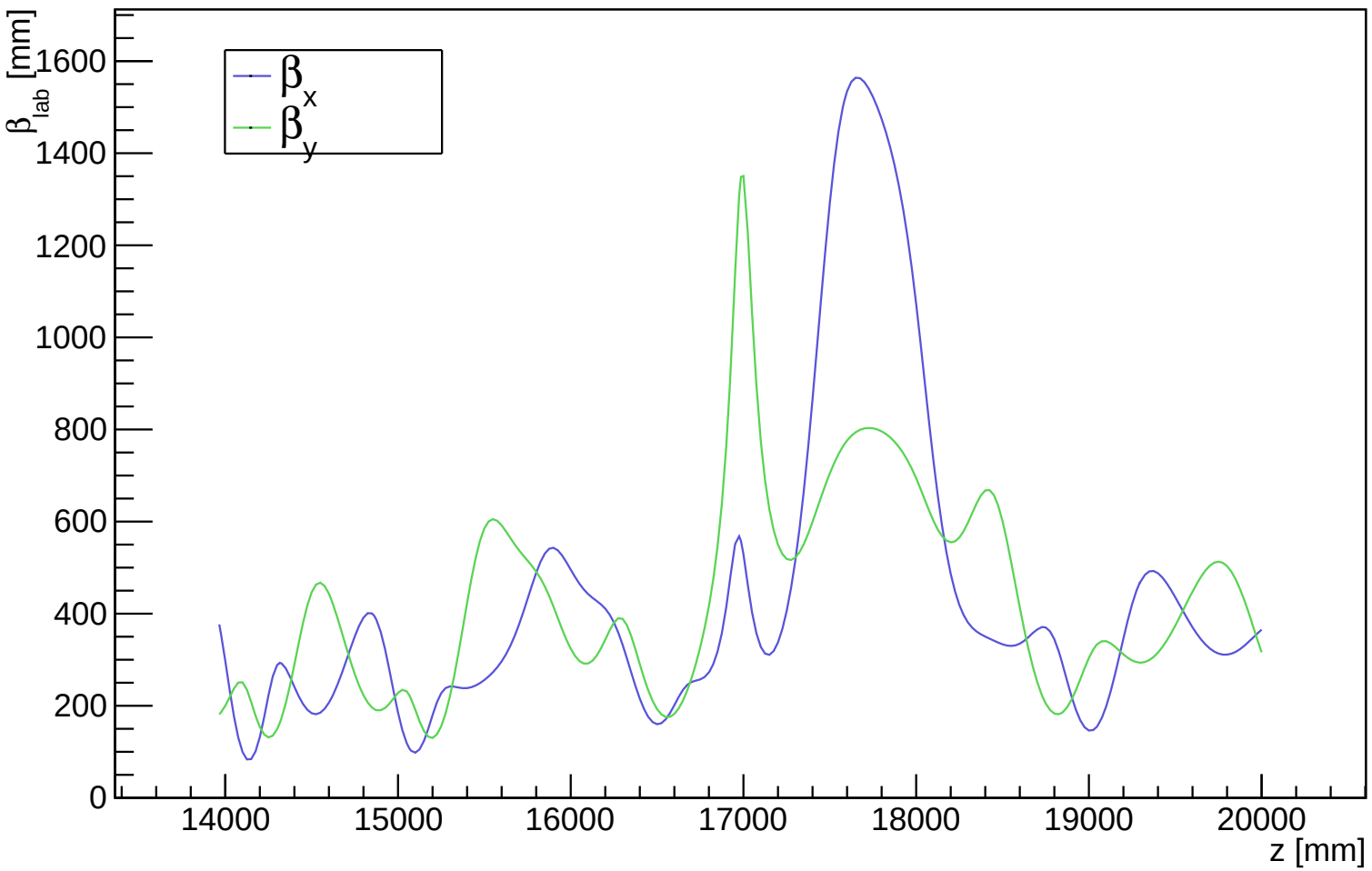
Twiss parameters: β_x



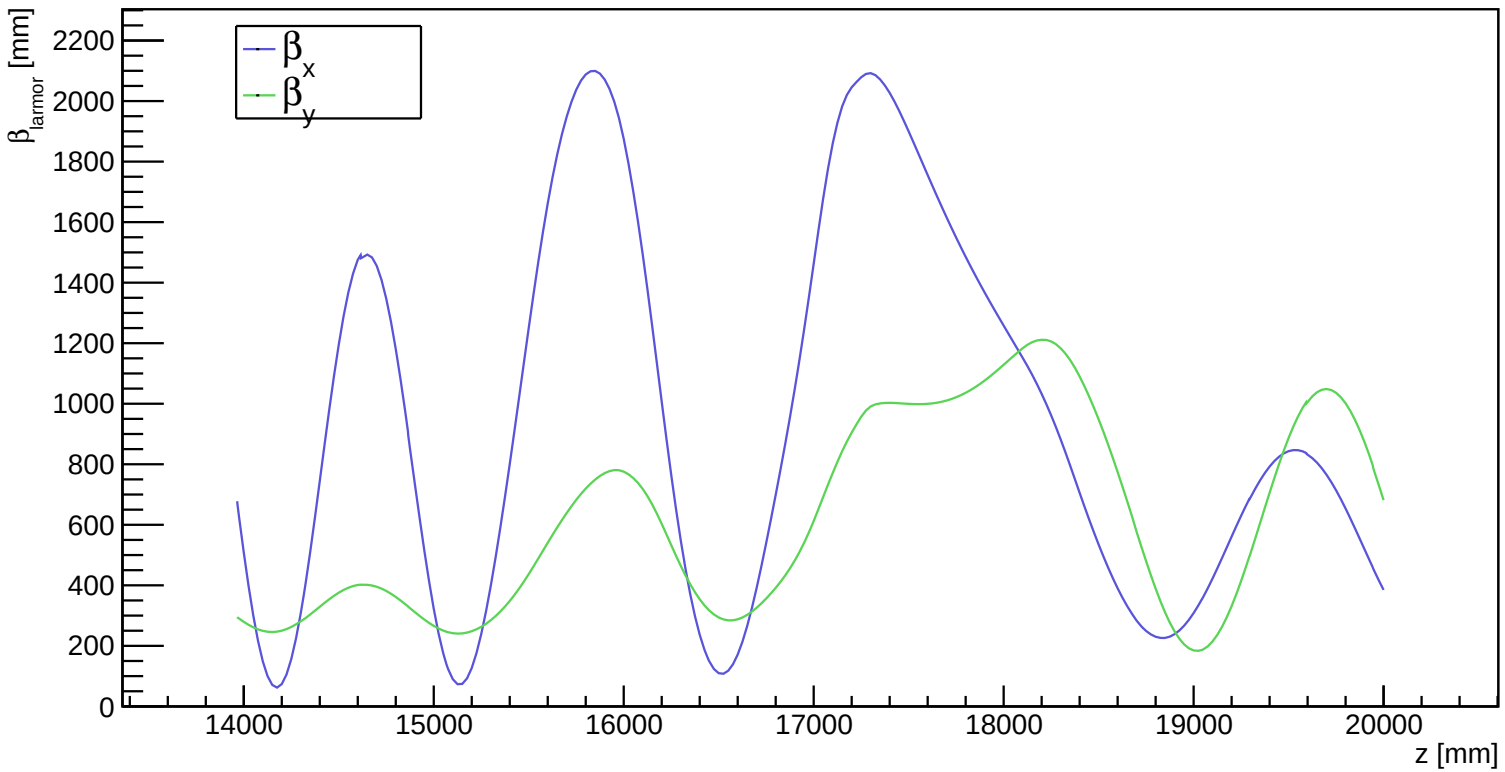
Twiss parameters: β_y



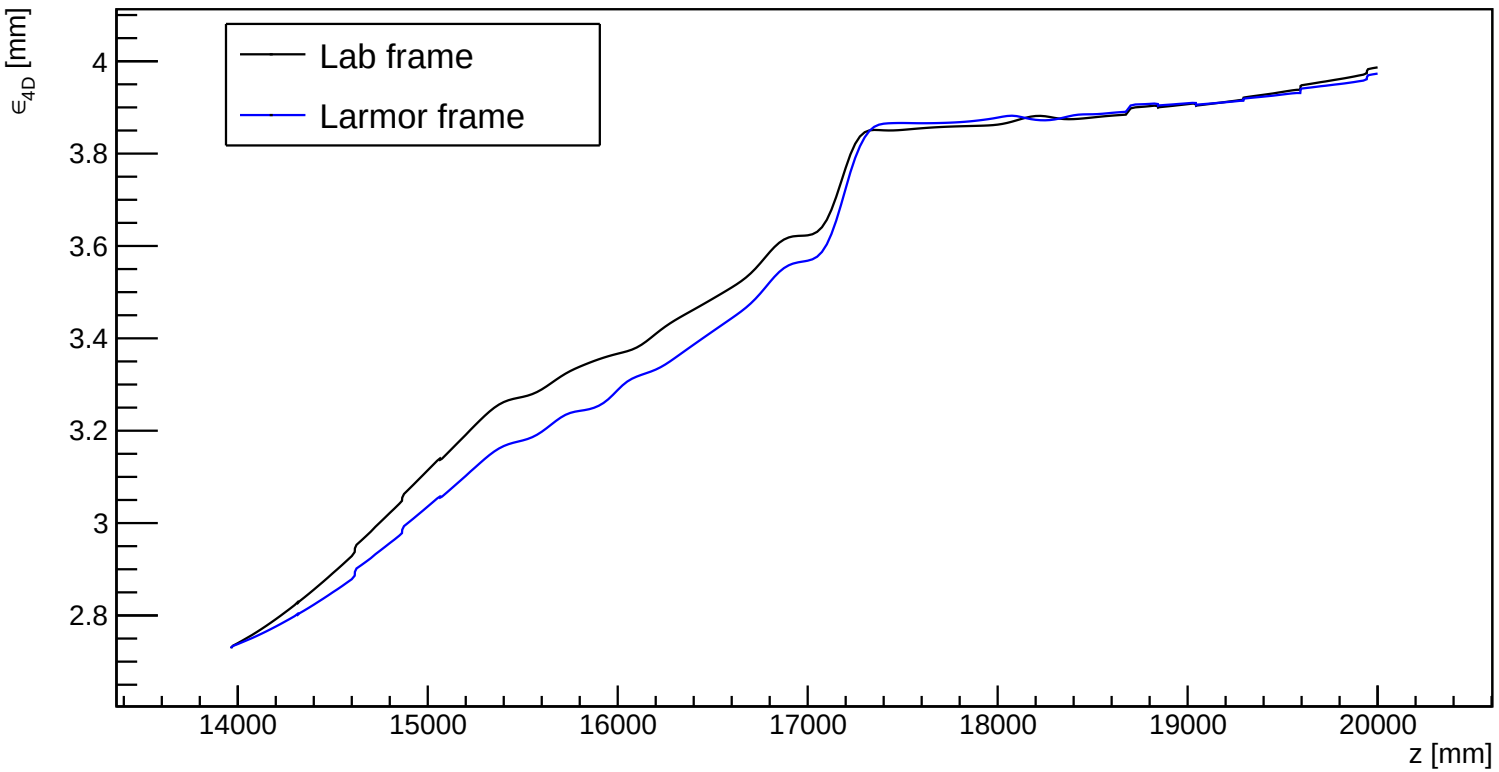
Twiss parameters: β_x vs β_y lab frame



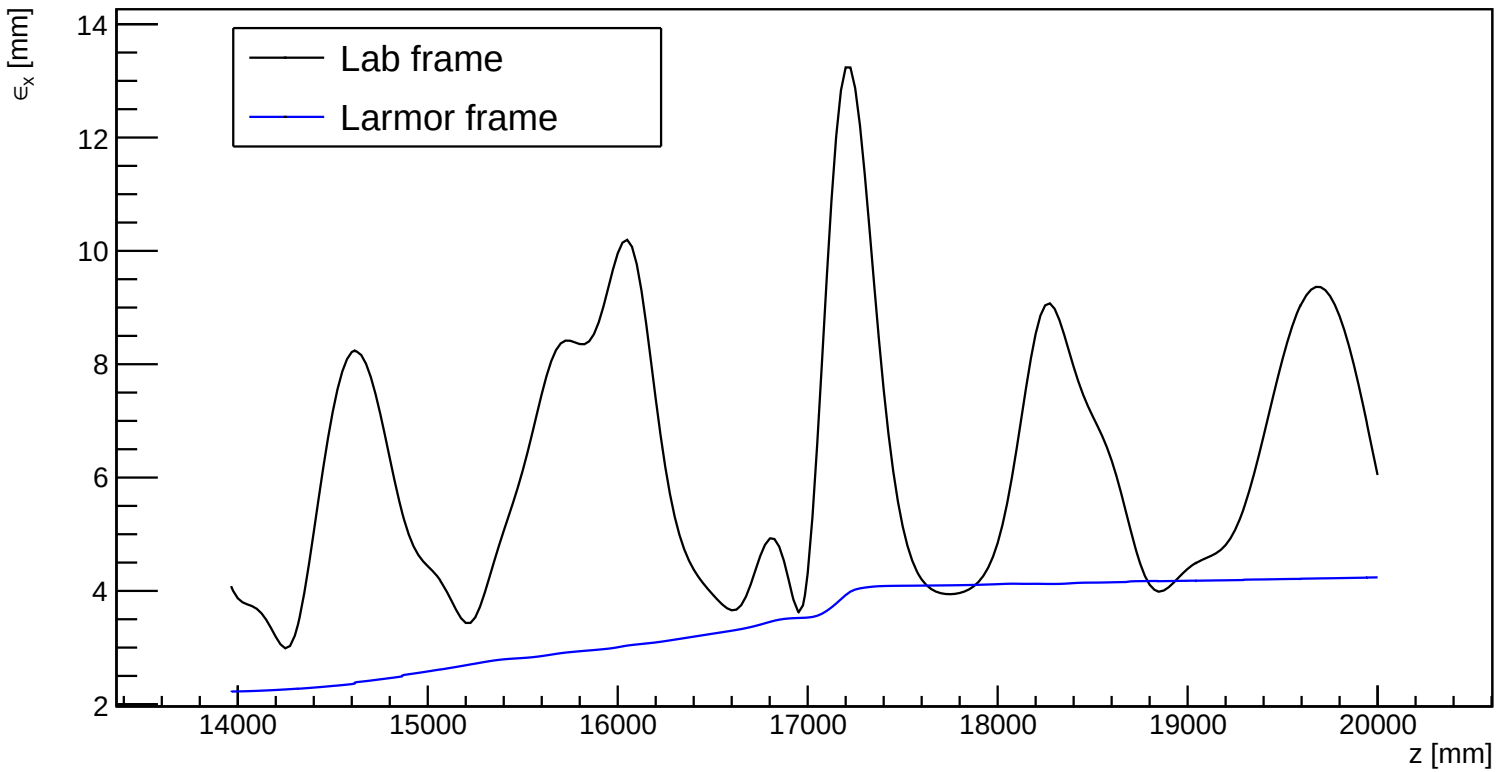
Twiss parameters: β_x vs β_y Larmor frame



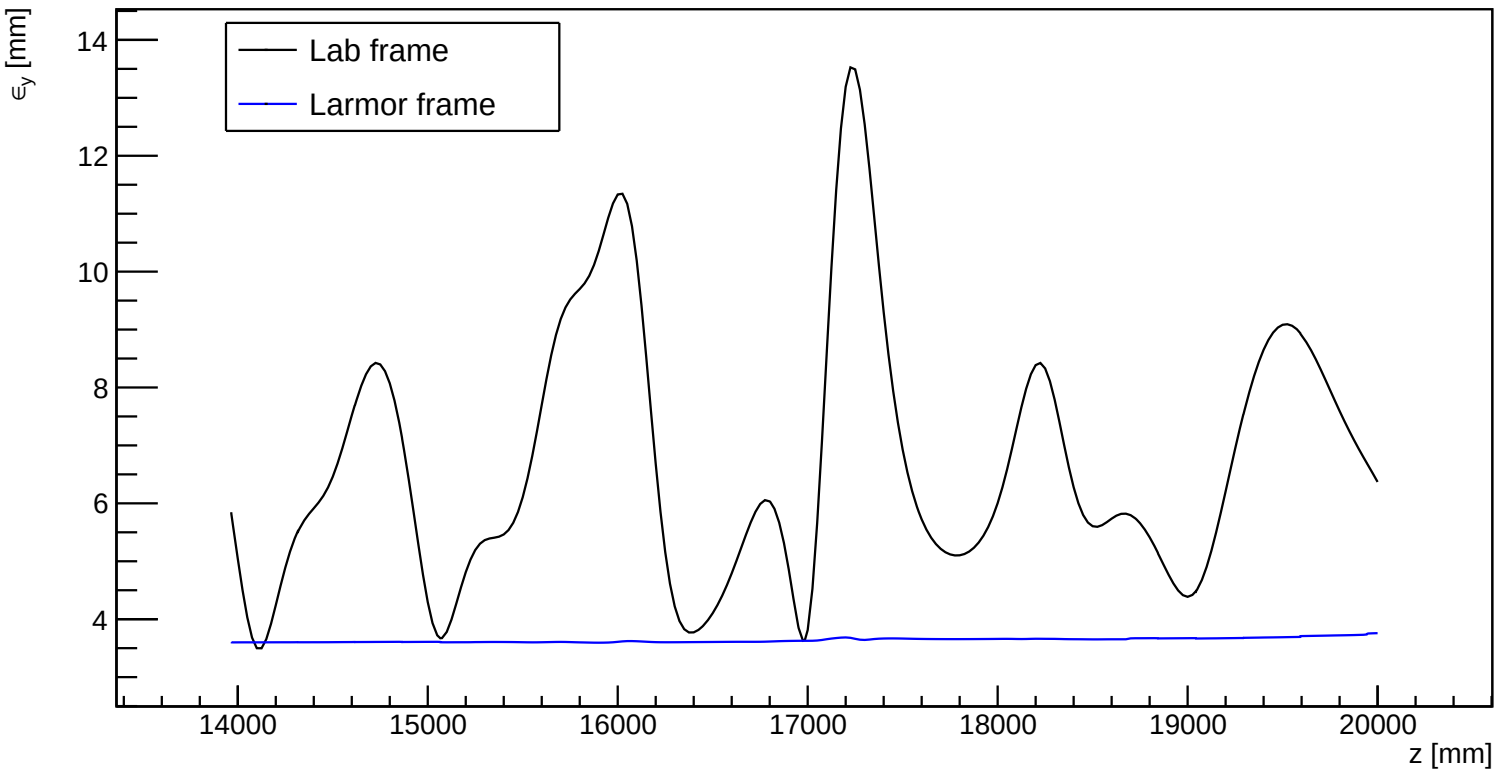
Emittance: ϵ_{4D}



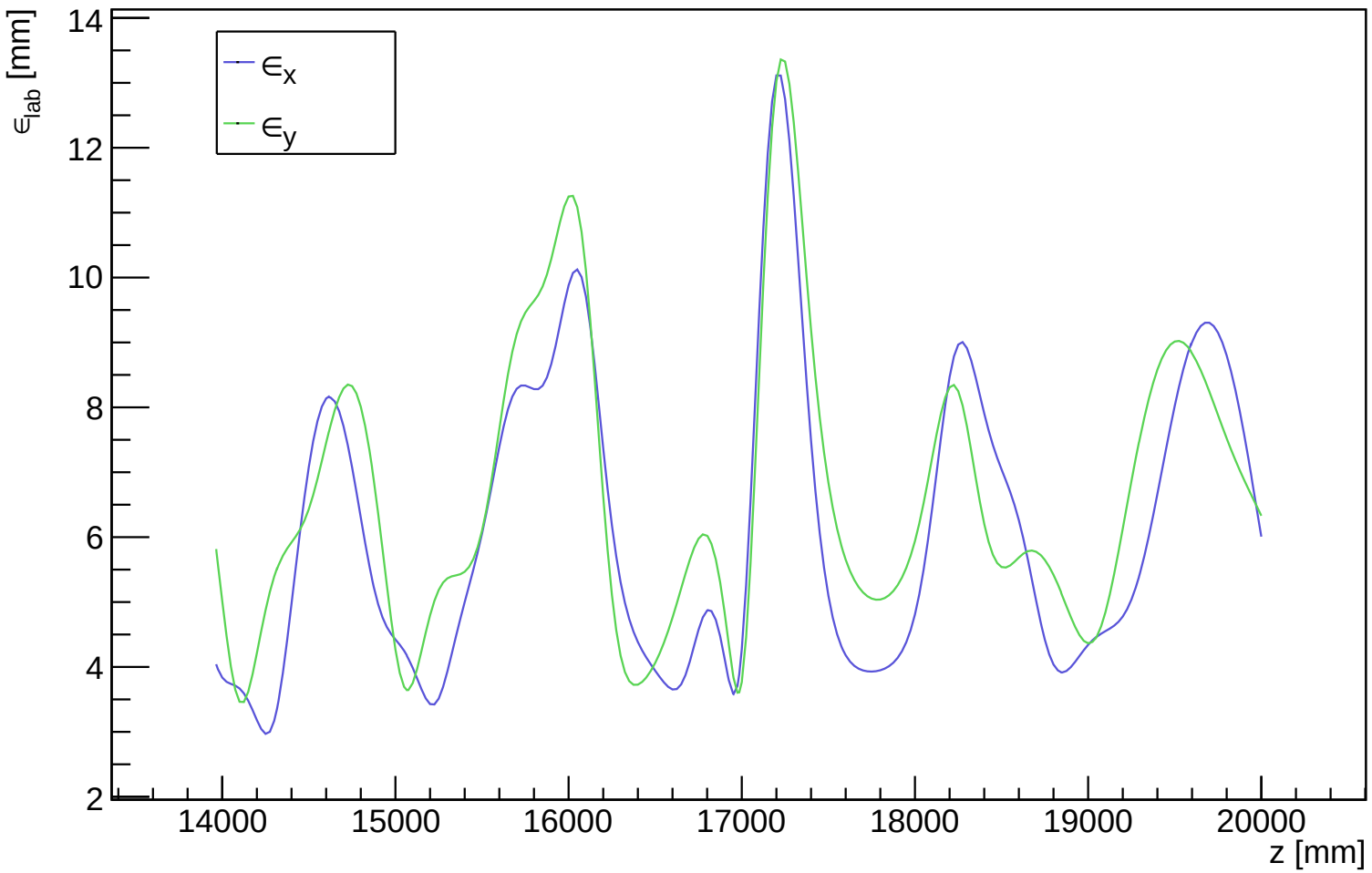
Emittance: ϵ_x



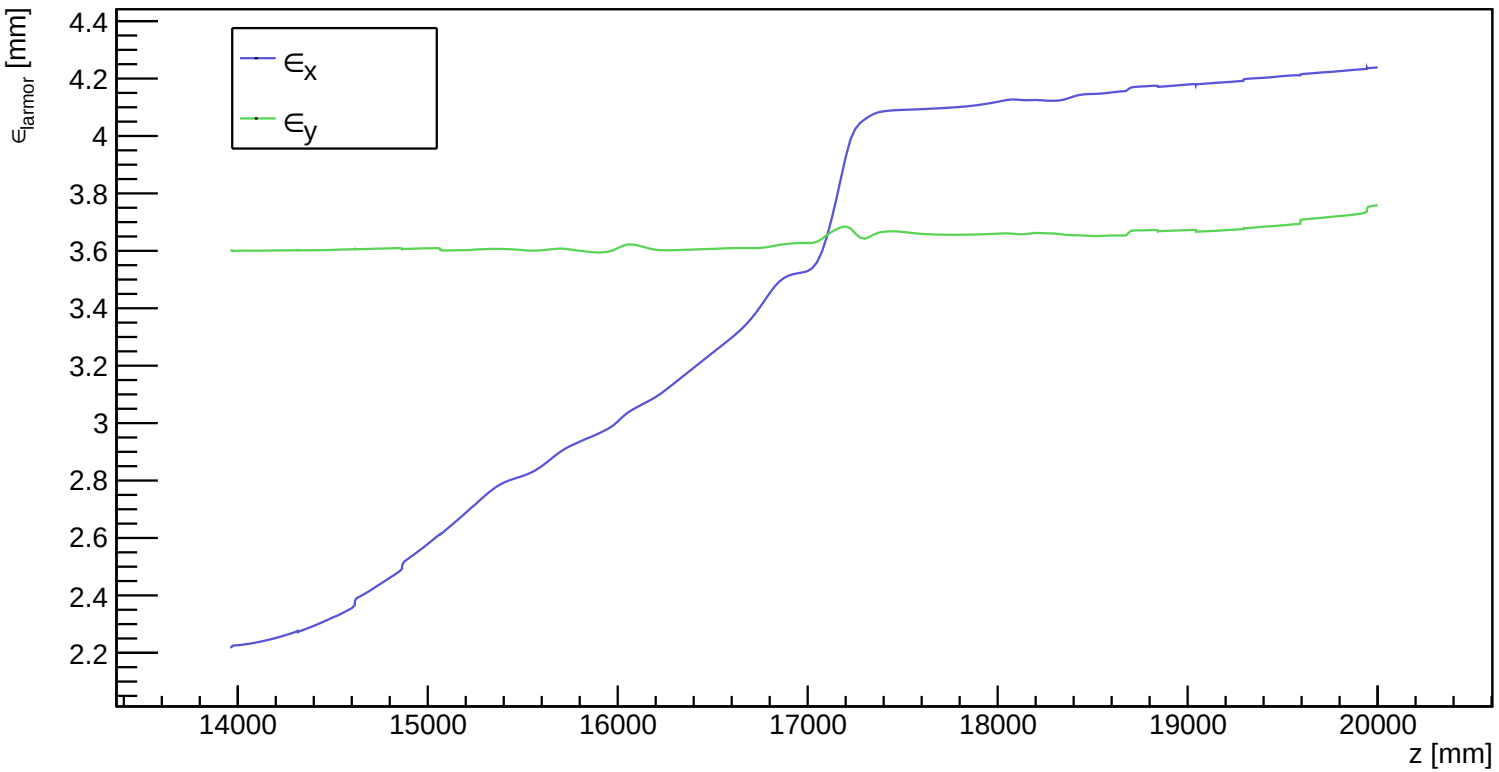
Emittance: ϵ_y



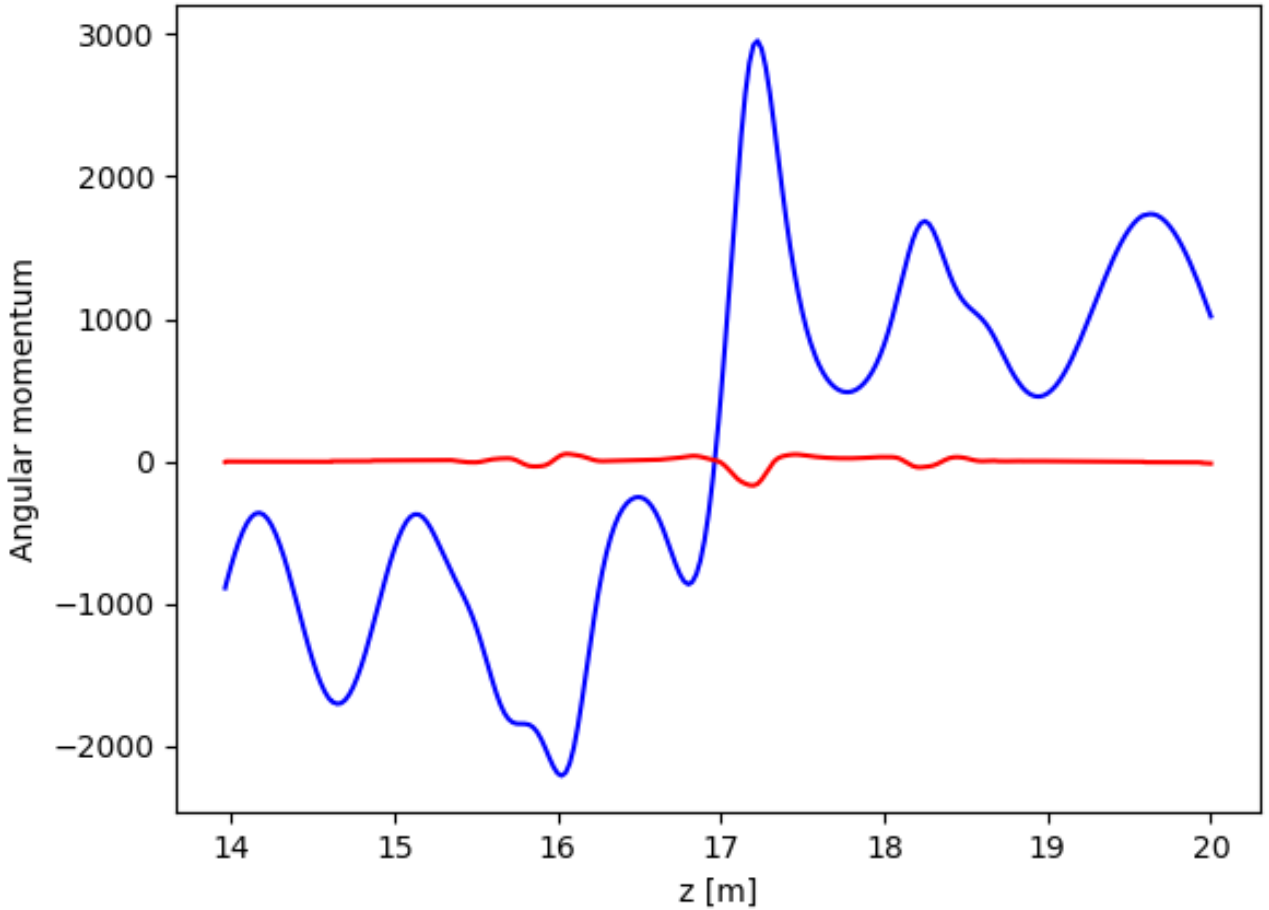
Emittance: ϵ_x vs ϵ_y lab frame



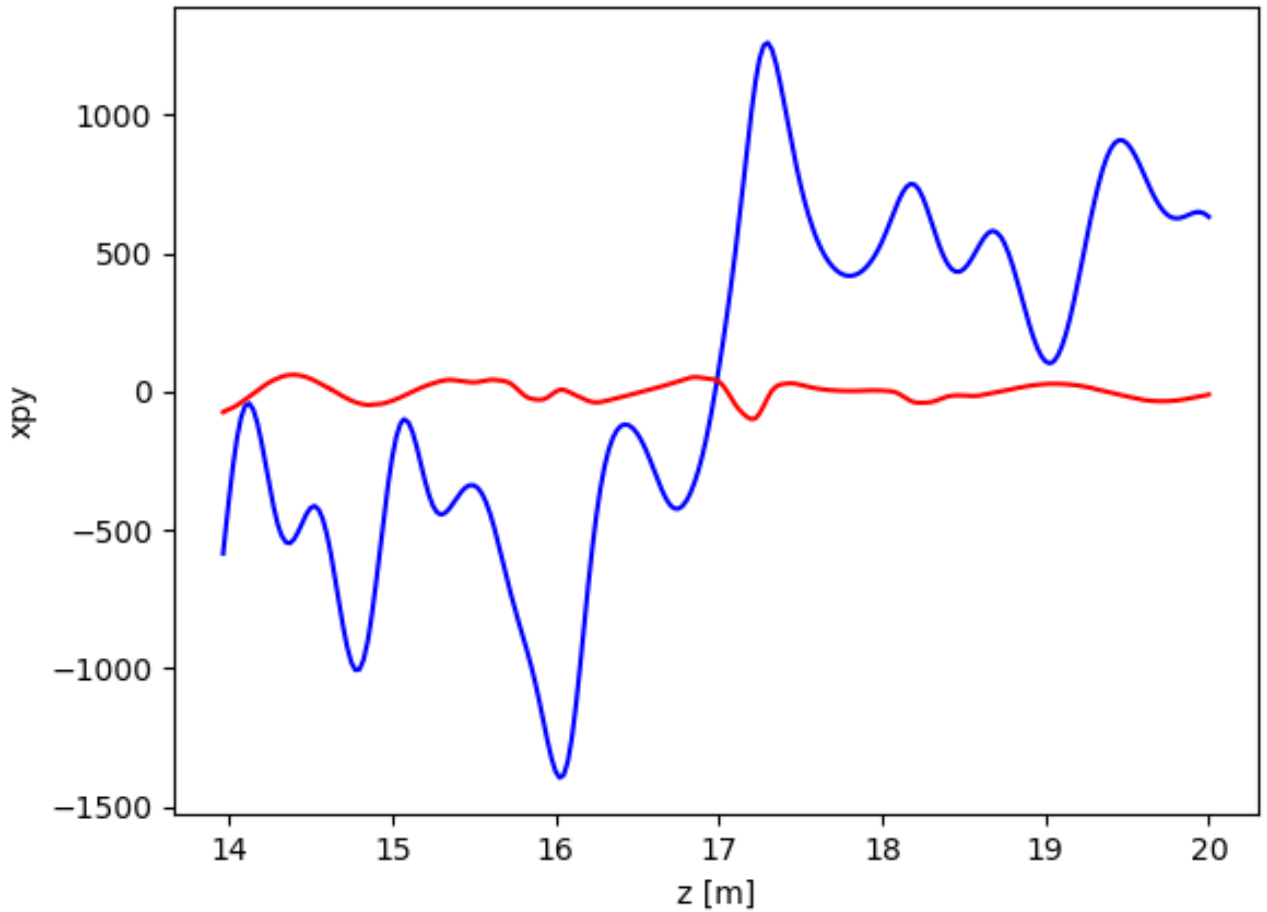
Emittance: ϵ_x vs ϵ_y Larmor frame



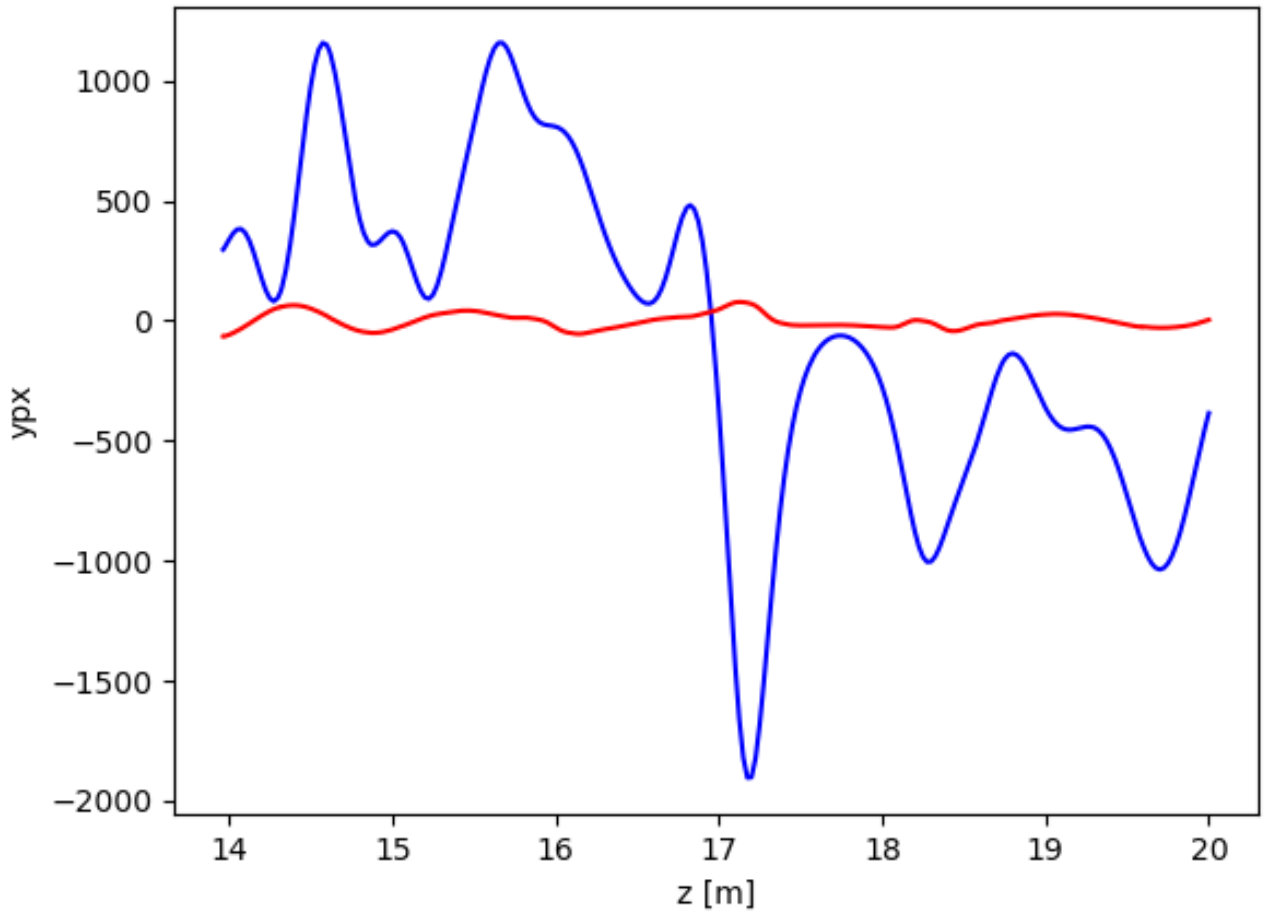
Angular momentum



Covariance matrix: *xpy* term

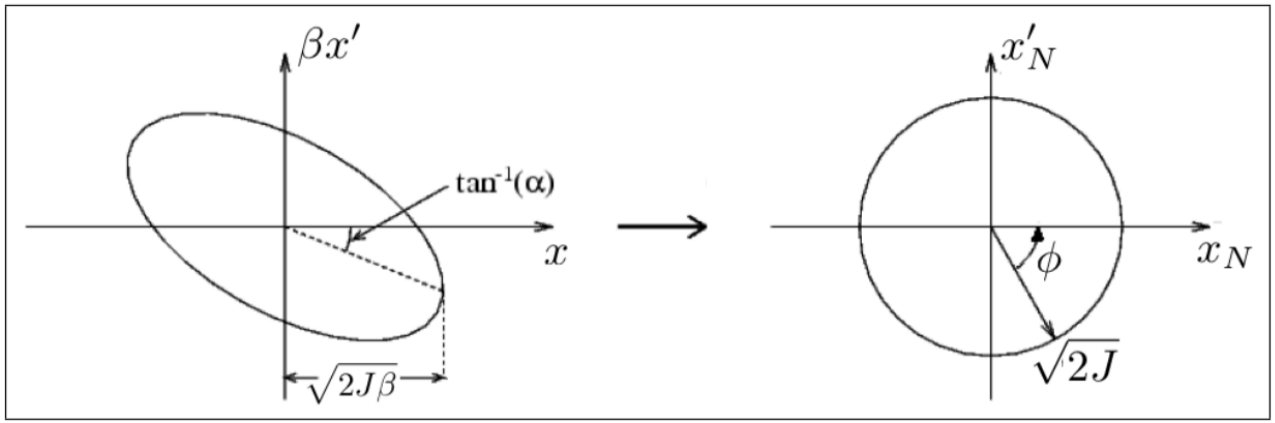


Covariance matrix: *ypx* term



Normalised Coordinates

$$\begin{pmatrix} x_N \\ x'_N \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$



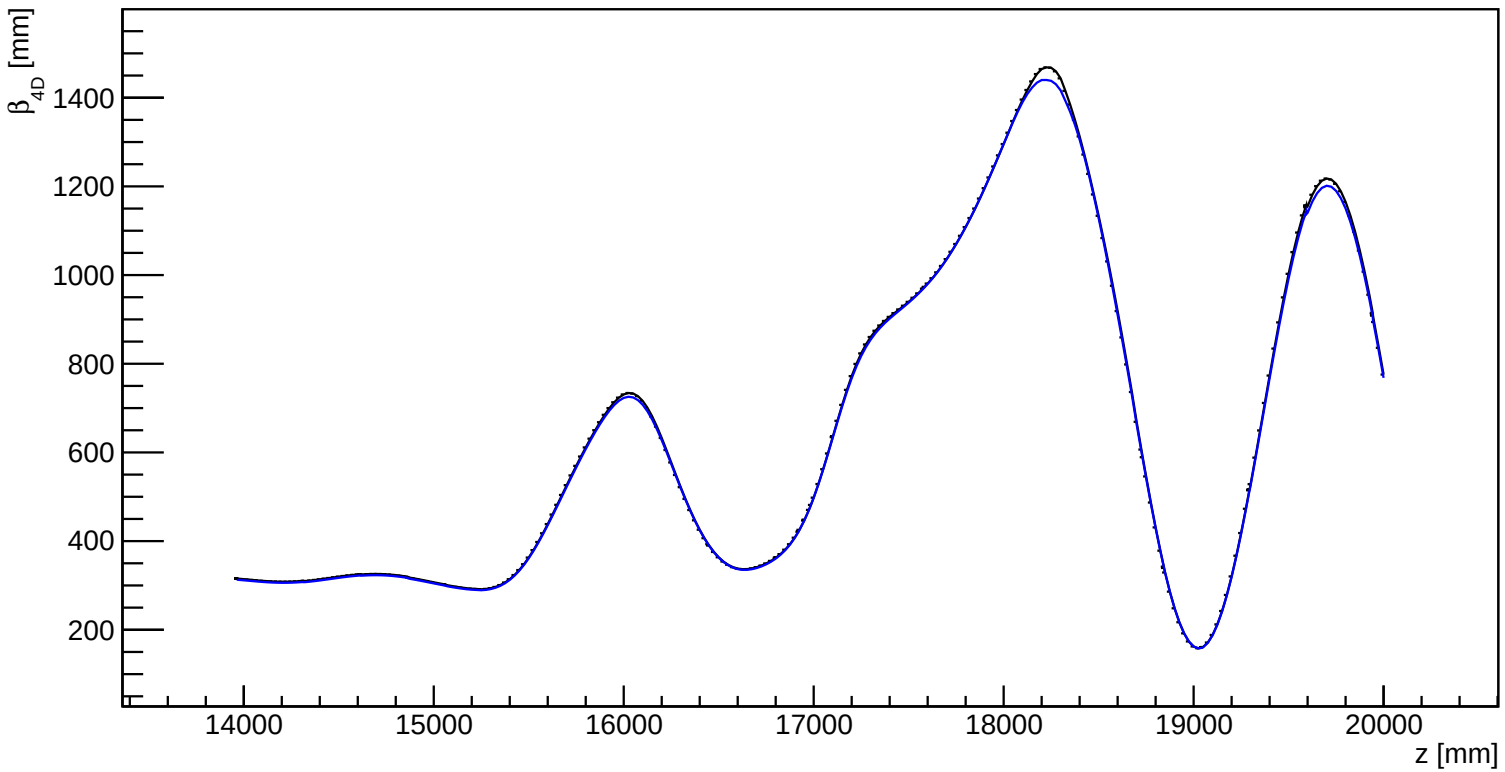
- Nonlinearities are expected to deform the circles in the normalised coordinates phase space

Videos with the phase space evolution in the cooling channel.

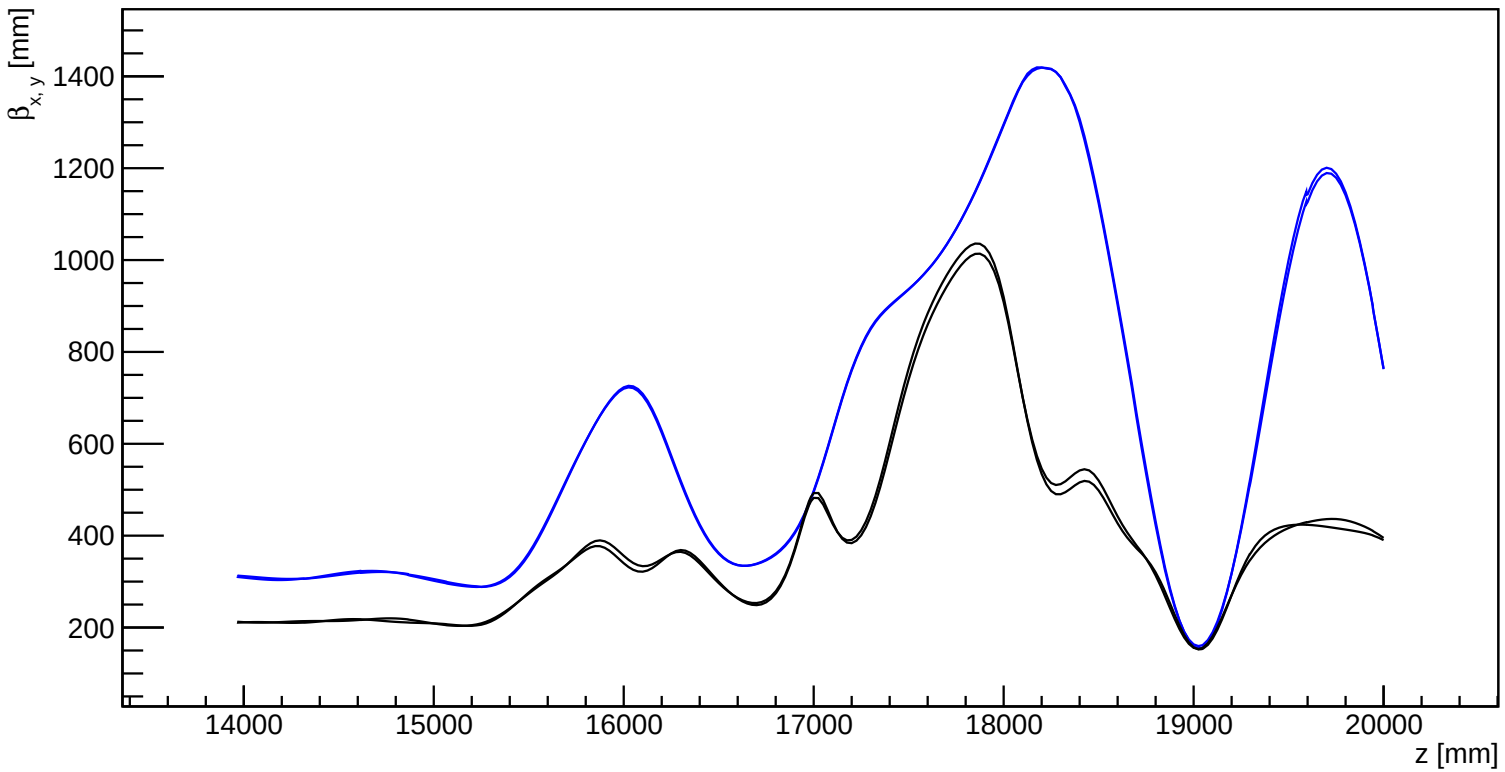
Backup: Toy Model

- Matched cylindrically symmetric beam
- $\beta_0 = 311$ mm
- $\alpha_0 = 0.0$
- $L_0 = 1.1$
- $\epsilon_0 = 3$ mm

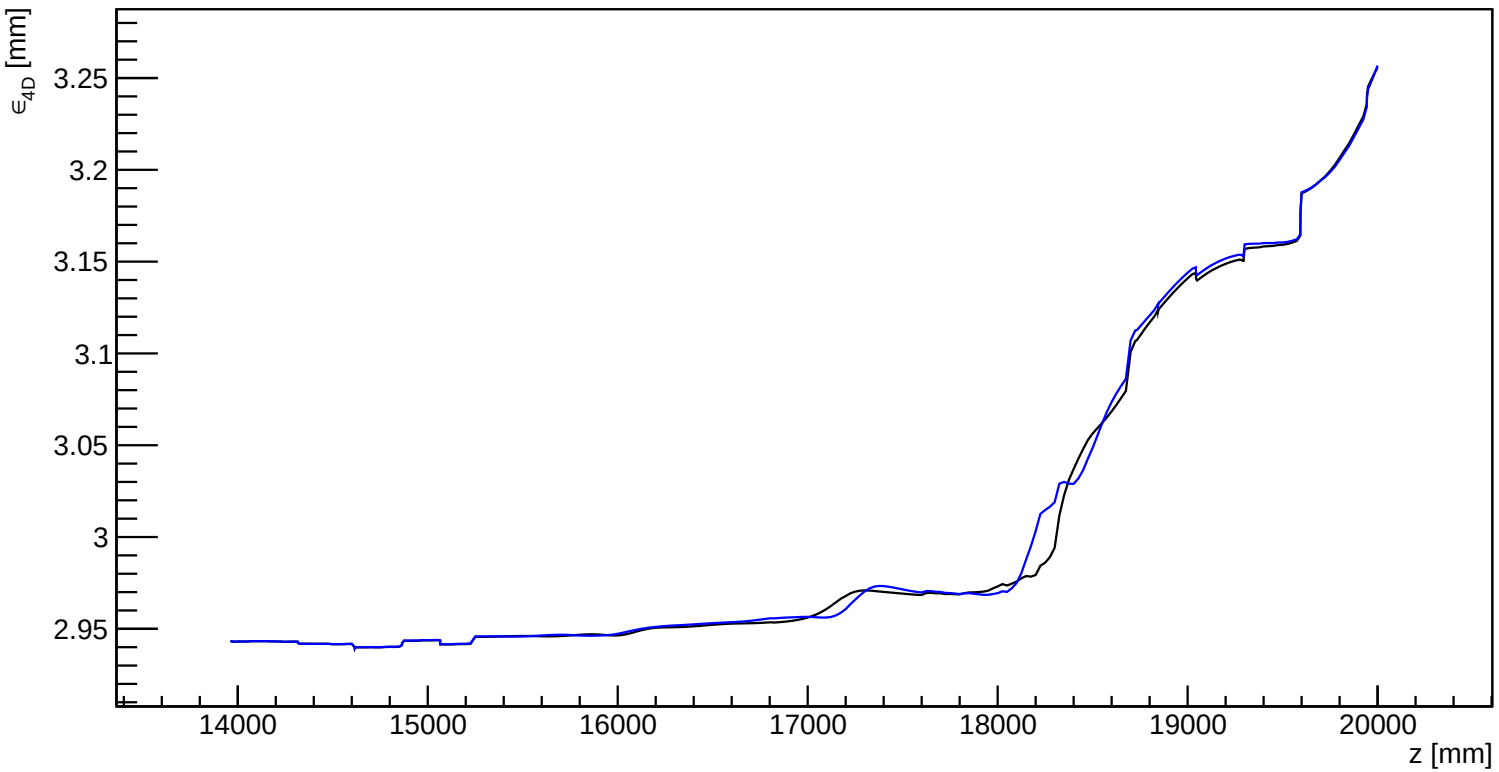
Twiss parameters: β_{4D}



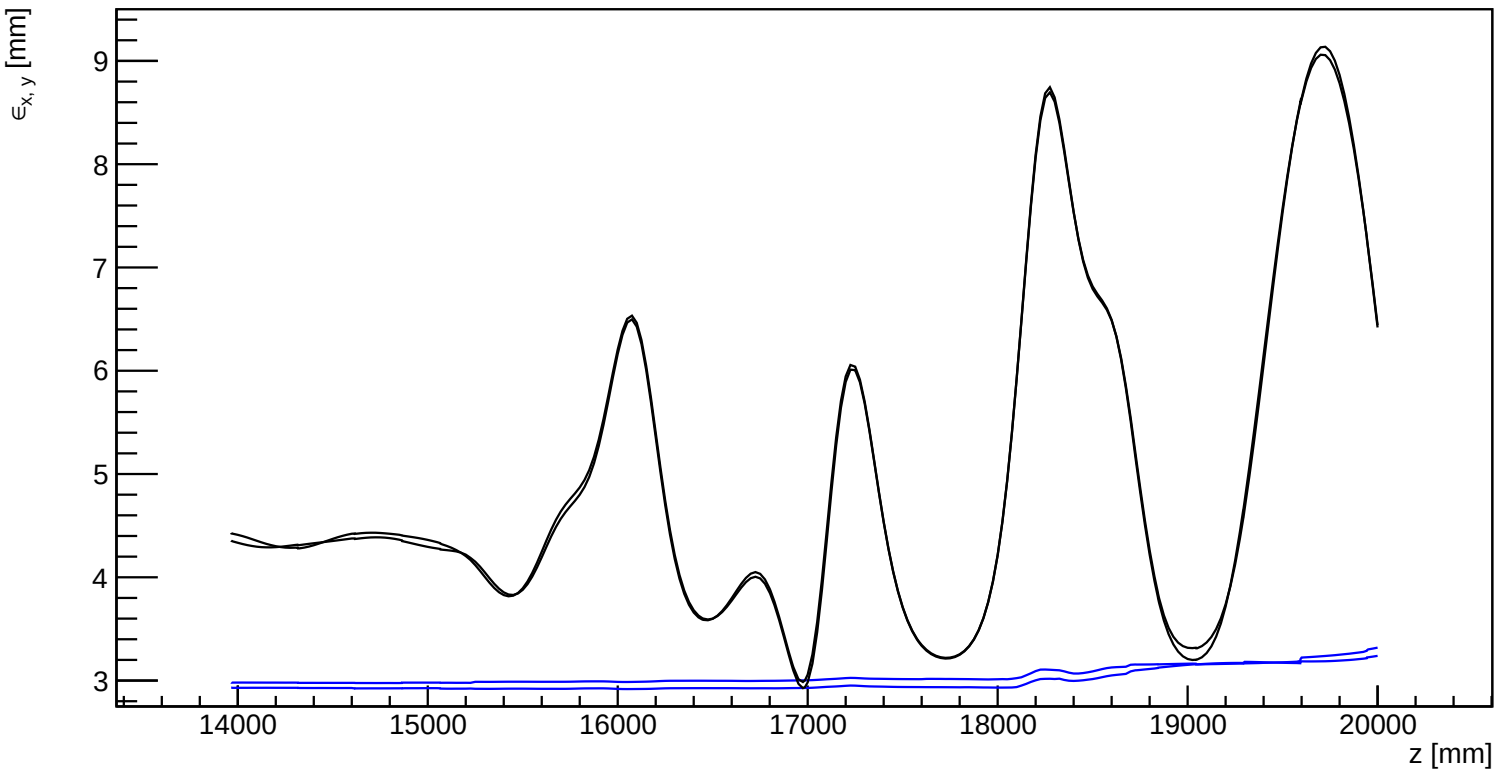
Twiss parameters: β_x vs β_y



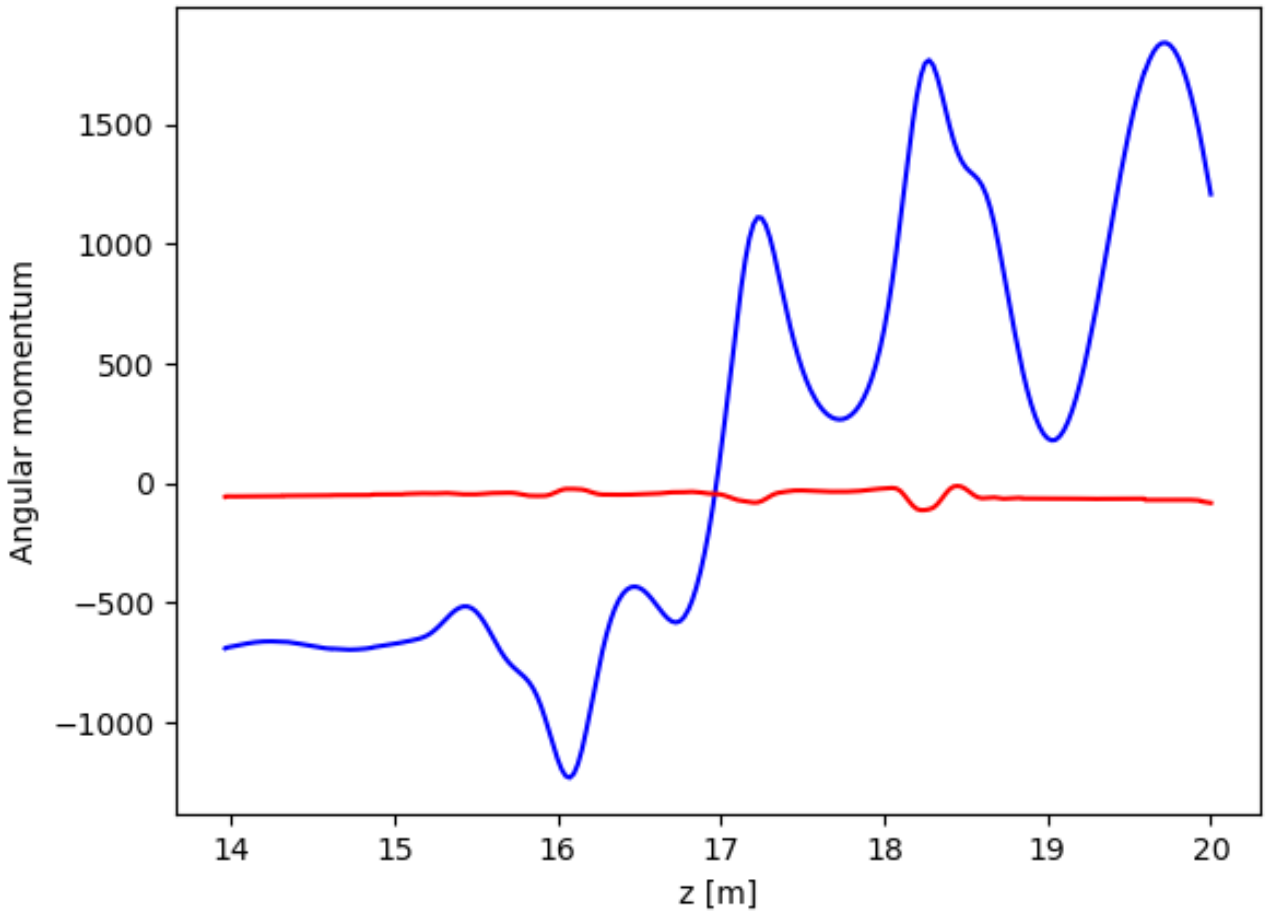
Emittance: ϵ_{4D}



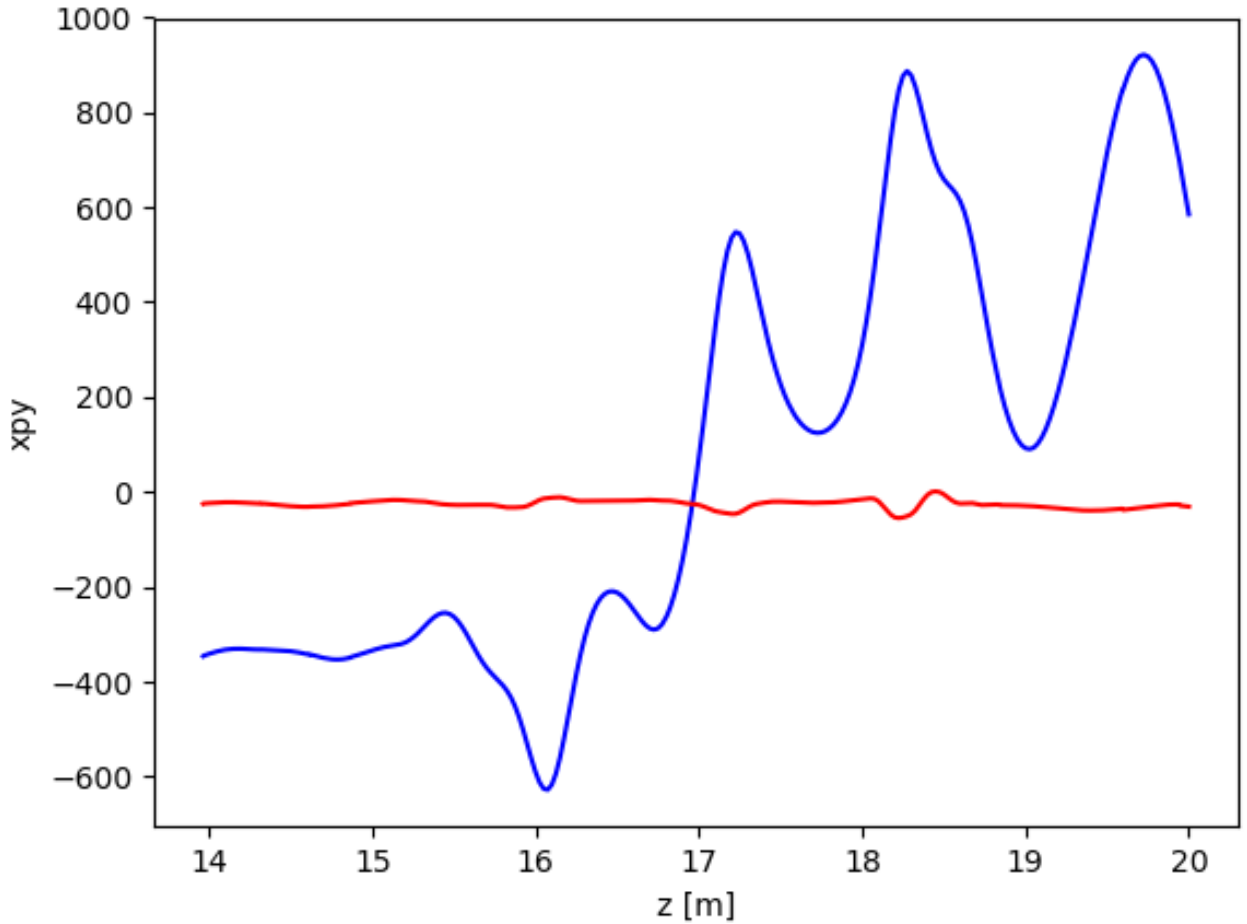
Emittance: ϵ_x vs ϵ_y



Angular momentum



Covariance matrix: *xpy* term



Covariance matrix: *ypx* term