Axion properties in Grand Unified Theories

Carlos Tamarit, Technische Universität München

JHEP 1802 (2018) 103
Phys.Rev. D98 (2018) no.9, 095011

In collaboration with...

Anne Ernst
DESY

Luca Di Luzio
Università di Pisa

Andreas Ringwald
DESY

The aim:

Study the properties of the axion in SU(5)/SO(10) GUTS, extended with a global U(1) symmetry so as to solve the strong CP problem. Account for constraints from unification, proton decay, star cooling, black hole superradiance, fermion mass fits.

The novelty:

Properties of GUT axions had not been studied systematically

Our formalism bridges the gap between the simple UV symmetries and the low-energy description, and clarifies subtleties about fermion field redefinitions

Minimal SU(5) axion model can be ruled out by upcoming experiments

The plan:

Motivation of GUTs and the axion solution to the strong CP problem

The guts of the axion solution to the strong CP problem

The GUTs of the axion solution to the strong CP problem

Motivation

Why unification?

Group structure

Matter content in each generation

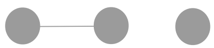
Anomaly cancellation, charge quantization

Hierarchies of masses and mixing angles

Can the SM model be a low energy effective description of a more predictive theory with a simple gauge group, and fewer representations?

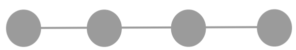
B and L are accidental symmetries in the SM: expect B violation, proton decay!

Bird's eye view of GUT models



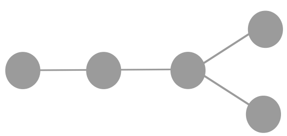
U(1)

SM group is of rank 4: Embed into simple groups of rank 4 or more.



SU(5) Each generation a 10 and $\bar{5}$ (without RH neutrino)

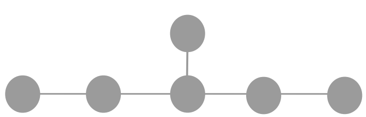
Minimal non-SUSY model ruled out by $\sin^2\theta_W$



SO(10) Each generation a single 16 irrep! (with RH neutrino)

Anomaly cancellation automatic

Larger rank allows for multi-step symmetry breakig with different chains



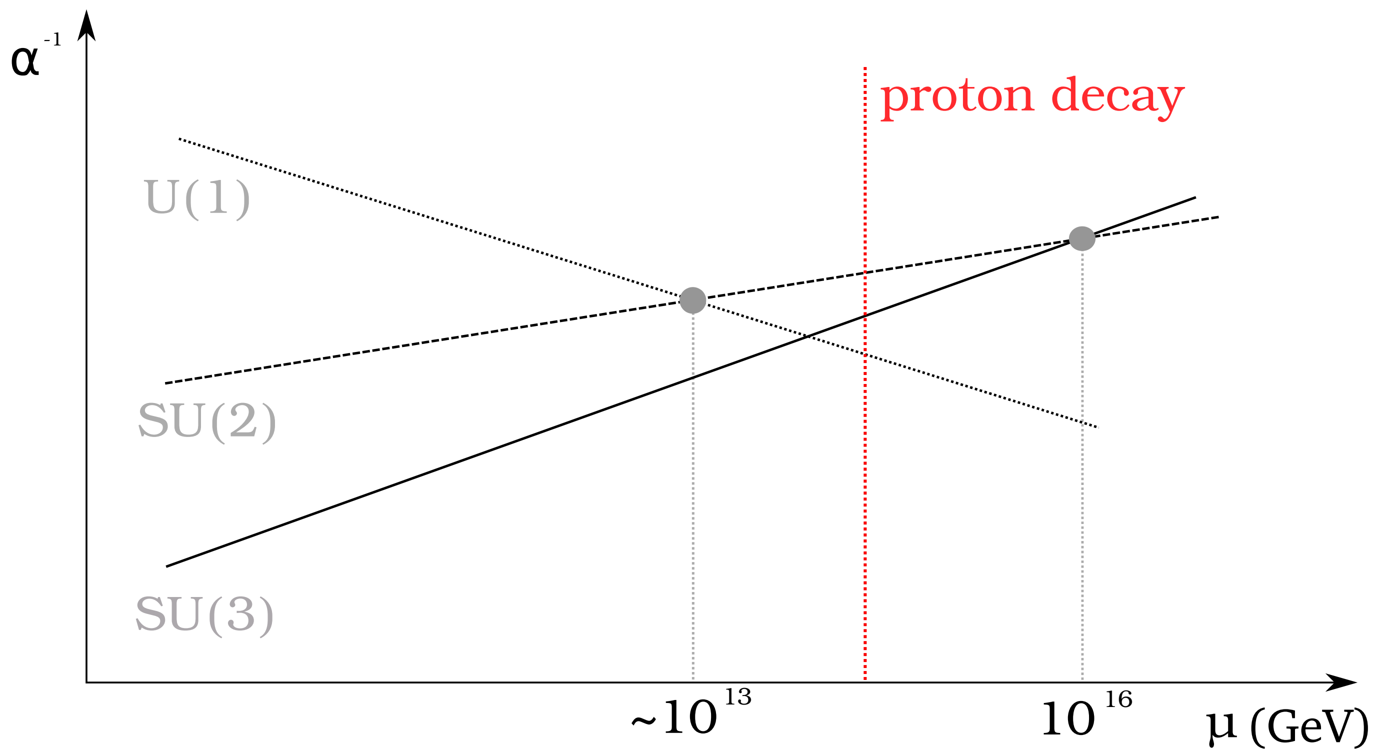
E6 A single generation plus Higgses fits in 27 irrep

Anomaly cancellation automatic

Multi-step breaking, multiple chains

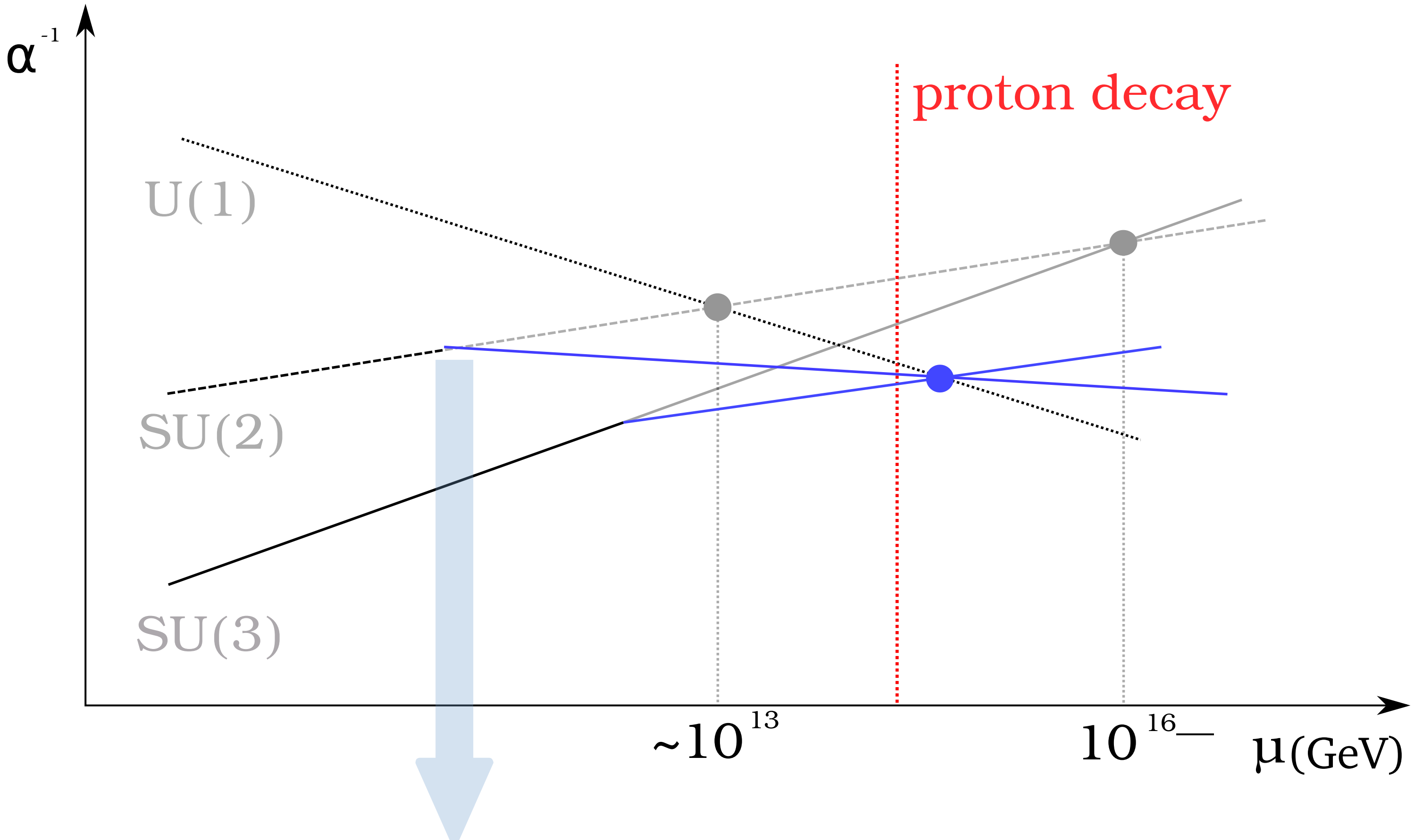
Minimal, non-SUSY SU(5)

Minimal SU(5) [Georgi, Glasgow] ruled out by neutrino masses and unification



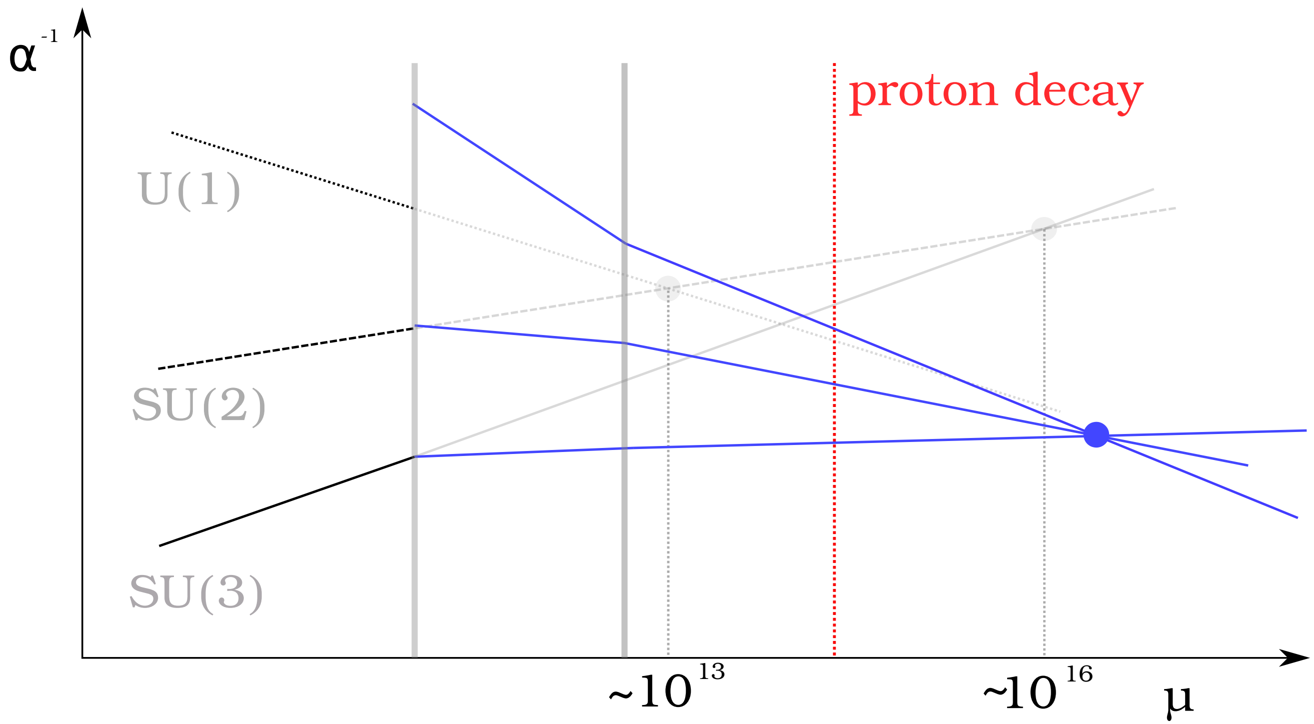
Rescueing SU(5)

SU(5) unification and nu masses can be solved by adding a 24_F [Bajc, Senjanovic]



Proton decay and unification constraints require a triplet scalar at energies accessible for LHC [Di Luzio, Mihaila]

SO(10) models



Minimality and anomaly freedom of matter representations

RH neutrinos automatically included, allowing for light neutrino masses and leptogenesis

Rich Higgs sector can accommodate axion, inflaton

Multi-step breaking suggests intermediate mass scales which could be tied to leptogenesis and the axion, while playing a role in unification.

The axion

Pseudo-Goldstone of U(1) "Peccei-Quinn" symmetry broken by a QCD anomaly, which makes the axion behave like a dynamical θ term with a potential stabilizing it at zero, solving the strong CP problem.

The axion is a combination of scalar phases, with shift symmetry broken by anomaly into a discrete subset.

Modding by trivial 2π rotations of scalars and center of gauge group gives a finite group whose dimension is the domain wall number (number of physically inequivalent vacua). This implies domain walls in the early universe!

Axion models classified by

Coupling to gauge bosons (nonderivative, from anomalies) $\propto \frac{1}{f_{A,k}}$

Axion mass (from QCD effects, fixed by coupling to gluons) $\propto \frac{1}{f_{A,SU(3)}}$

Coupling to fermions, nucleons (derivative)

Domain wall number

Axion mass and axion dark matter

Axion mass: Axion enters QCD partition function through the combination

$$\theta_{\rm phys} = \theta + \frac{A(x)}{f_A}$$

$Z_{QCD}[\theta_{phys}]$generates an axion mass!

$$m_A = 57.0(7)\left(\frac{10^{11}\rm GeV}{f_A}\right)\mu\rm eV.$$

[Borsanyi et al, di Cortona et al]

Axion dark matter: Axion field oscillating around its minimum behaves as dark matter [Preskill et all, Abbott and Sikivie, Dine and Fischler].

Post-inflationary PQ restoration: [Borsanyi et al, Gorghetto et al] For N_{DW}=1

$$\Omega_A h^2 \approx 0.12 \Rightarrow 3\times 10^{10}\,\rm GeV \lesssim f_A \lesssim 1.2\times 10^{11}\,\rm GeV \quad ; \quad (25)50\,\mu eV \lesssim m_A \lesssim 200(?)\,\mu eV$$

Pre-inflationary PQ breaking: $$\Omega_A h^2 \approx 0.35\left(\frac{\theta_I}{0.001}\right)^2\left(\frac{f_A}{3\times 10^{17}\,\rm GeV}\right)^{1.54}$$

A very light axion can only be DM in pre-inflationary scenario

See Hardy's talk!

Putting it together

Both GUT theories and the axion solution to the strong CP problem require intermediate thresholds on the way to the Planck mass

Can GUT and axion scales be correlated?

Aside from solving the strong CP problem, the axion provides a dark matter candidate which is otherwise usually absent in non-SUSY GUT theories

Non-SUSY GUTs can then solve the strong CP problem and explain neutrino masses, inflation, dark matter, baryogenesis

What has been done for the axion in GUTs

$U(1)_{PQ}$ extensions of SU(5) and SO(10) GUTs have been proposed a long time ago [Wise et al, Lazarides, Kim, Bacj et al, Babu et al, Altarelli et al].

A global U(1) motivated in SO(10) so as to make the Yukawa sector more predictive. It is anomalous, so can be used as PQ symmetry to implement axion solution

Axion identified in a few cases, models have been proposed with $N_{DW}=1$, arguing in terms of the UV symmetries.

… and what was missing

A systematic identification of axion field and axion decay constant/mass in relation to thresholds/VEVs in the theory

A systematic calculation of couplings to gauge bosons, fermions/nucleons, including low energy effects

An identification of the global symmetry corresponding to the physical axion, which is orthogonal to the massive gauge bosons

A direct calculation of domain wall number for the above symmetry

Studies of constraints from unification, proton decay, fermion mass fits, stelar cooling, superradiance.

The guts of the strong CP problem

Ingredients: a global U(1), anomalous, broken

Weyl fermions ψ_a

Complex scalars ϕ_j

Global U(1)

$$\psi_a \to e^{iq_a\alpha}\psi_a$$
$$\phi_j \to e^{iq_j\alpha}\phi_j$$

Symmetry breaking

$$\phi_j = \frac{1}{\sqrt{2}}(v_j + \rho_j)e^{iA_j/v_j}.$$

The axion/Goldstone:

$$A_i = \frac{q_i v_i}{f_{\rm PQ}} A + \text{ orthogonal excitations}, \;\; f_{\rm PQ} = \sqrt{\sum_j q_j^2 v_j^2}$$

$$A = \frac{1}{f_{\rm PQ}} \sum_i q_i v_i A_i$$

Ingredients: a global U(1), anomalous, broken

Physical PQ symmetry is different from naive one!

Axion must be orthogonal to unphysical modes (massive gauge bosons)

The axion hunting flow

$$A = \sum_i c_i A_i$$

Orthogonality
Masslessness
Canonical normalization

scalar PQ_{phys} charges

$$\frac{q_i}{f_{PQ}} = \frac{c_i}{v_i}$$

N_{DW}, f_A, $f_{A,EM}$,
Fermion/nucleon
couplings in axial basis

Axial basis can hide
symmetries of the theory

Special basis in which axion couples to axial currents
(fermion PQ charges related to scalar ones)

Solve for q_i in terms
of symmetries of the
theory

fermion PQ_{phys} charges
in "symmetry basis"

$$\frac{q_a}{f_{PQ}}$$

f_A, $f_{A,EM}$ same as in axial basis
$f_{A,SU(2)}$ changes

What to expect from GUT symmetry

One fundamental θ angle, which is inherited by the low energy groups

The GUT axion must solve the CP problem in all subgroups!

With GUT symmetry explicit, all θ_k and all f_{Ak} will be equal for all subgroups, but this can change after performing field redefinitions that explicitly break the GUT symmetry.

E.g.: Field redefinitions to go to axial basis. $f_{A,SU(2)} \neq f_{A,SU(3)}$

PQ_{phys} should be a combination of symmetries of GUT theory!

Minimal viable SU(5)

Relevant SU(5) representations

Each generation has 10 and $\bar{5}$

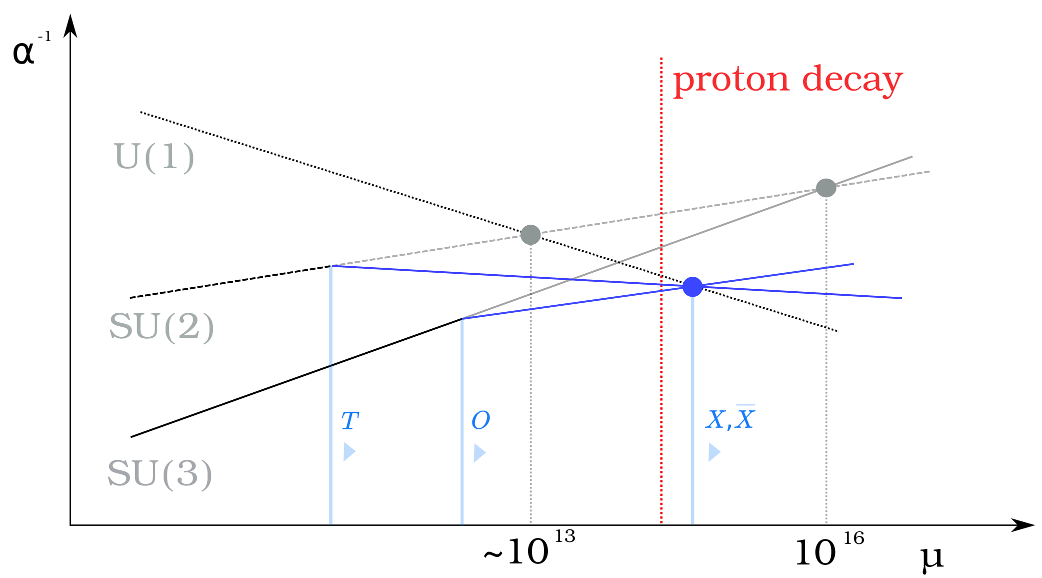
$SU(5)$	$3_C 2_L 1_Y$
$\bar{5}_F$	$(1,2,-\frac{1}{2}) := l$
	$(\bar{3},1,\frac{1}{3}) := d$
10_F	$(3,2,\frac{1}{6}) := q$
	$(1,1,1) := e$
	$(\bar{3},1,-\frac{2}{3}) := u$

Extra fermions in 24

$SU(5)$	$3_C 2_L 1_Y$
24_F	$(8,1,0) := O$
	$(1,3,0) := T$
	$(3,2,-\frac{5}{6}) := X$
	$(\bar{3},2,\frac{5}{6}) := \bar{X}$
	$(1,1,0) := S$

SU(5) breaking by scalar 24_H, ordinary Higgses in $5_H\ \bar{5}_H$

To fix unification (delay meeting of SU(2) and U(1))



Axion mass meets proton decay

PQ charge assignments (fixed by Yukawas) give axion scale $f_A \sim$GUT scale

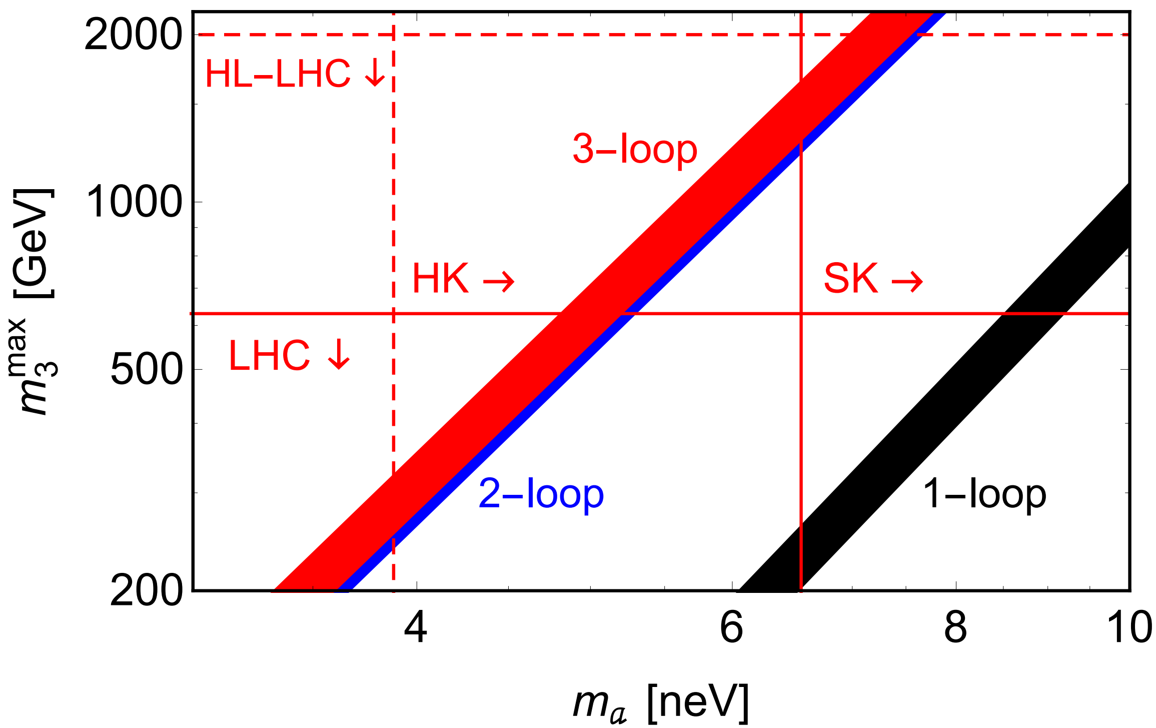
$\overline{5}_F$	10_F	24_F	5_H	$5'_H$	24_H
1	1	1	-2	2	2

Proton decay rate from heavy gauge boson exchange goes as $1/M_{\rm GUT}^4 \sim 1/f_A^4 \sim m_a^4$

$$\Gamma_{p\to\pi^0 e^+} \simeq \left(1.6\times 10^{34}\ {\rm yr}\right)^{-1} \left(\frac{m_a}{3.7\ {\rm neV}}\right)^4 \left(\frac{6}{11}\right)^4 \times \left[0.83\left(\frac{A_{SL}}{2.4}\right)^2 + 0.17\left(\frac{A_{SR}}{2.2}\right)^2\right]$$

Proton decay experiments can constrain axion mass
Decay rate prediction relaxed by an order of magnitude if flavour structure is tuned
[Dorsner, Fileviez-Perez]

SU(5) unification and proton decay constraints



$$m_3 \equiv (m_{T_F}^4 m_{T_H})^{1/5}$$

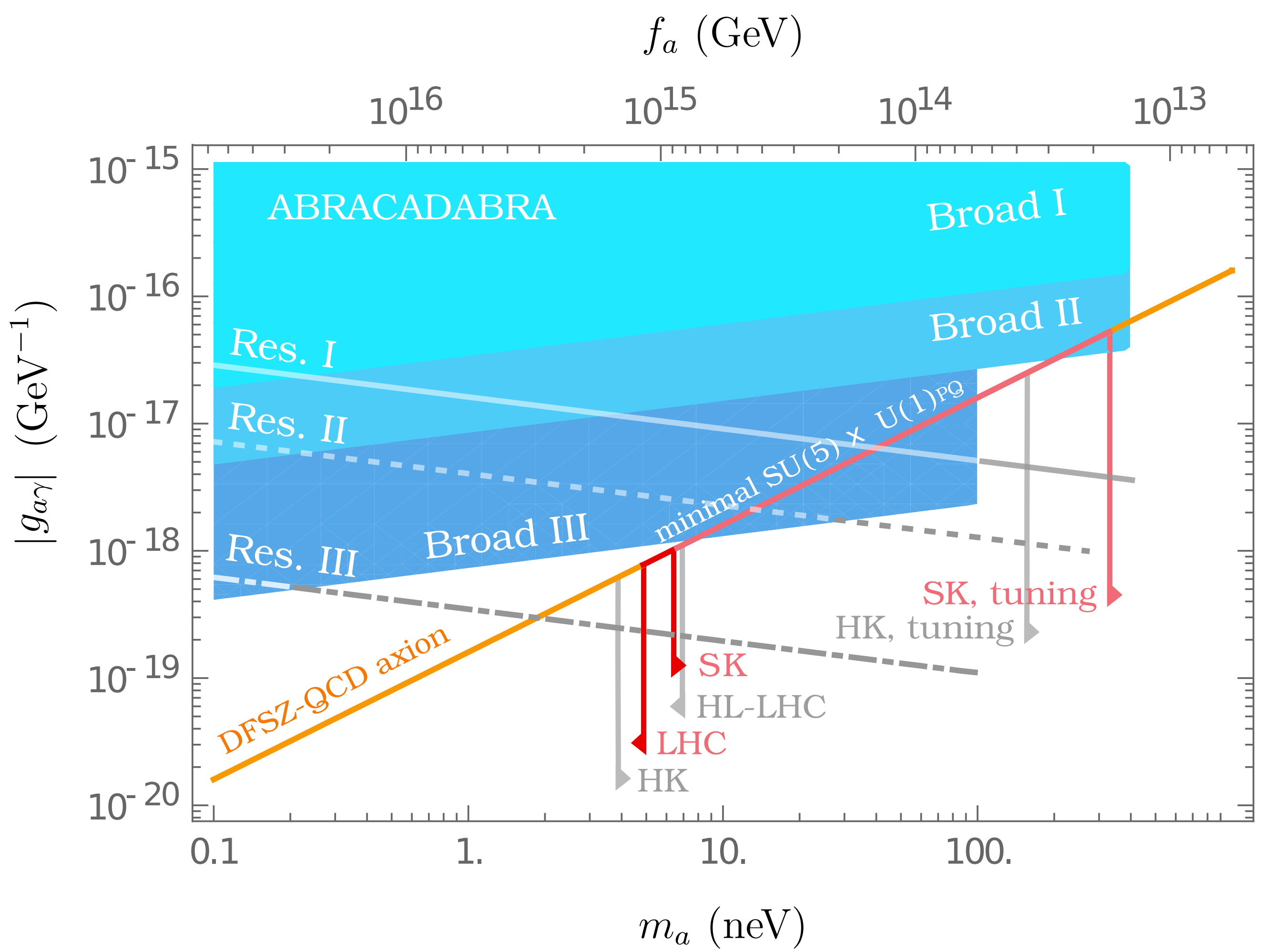
$$m_a \in \{4.8, 6.6\} \text{ neV}$$

3 loop RG+two-loop threshold corrections [Di Luzio, Mihaila]
Threshold structure much more constrained than in SO(10)!

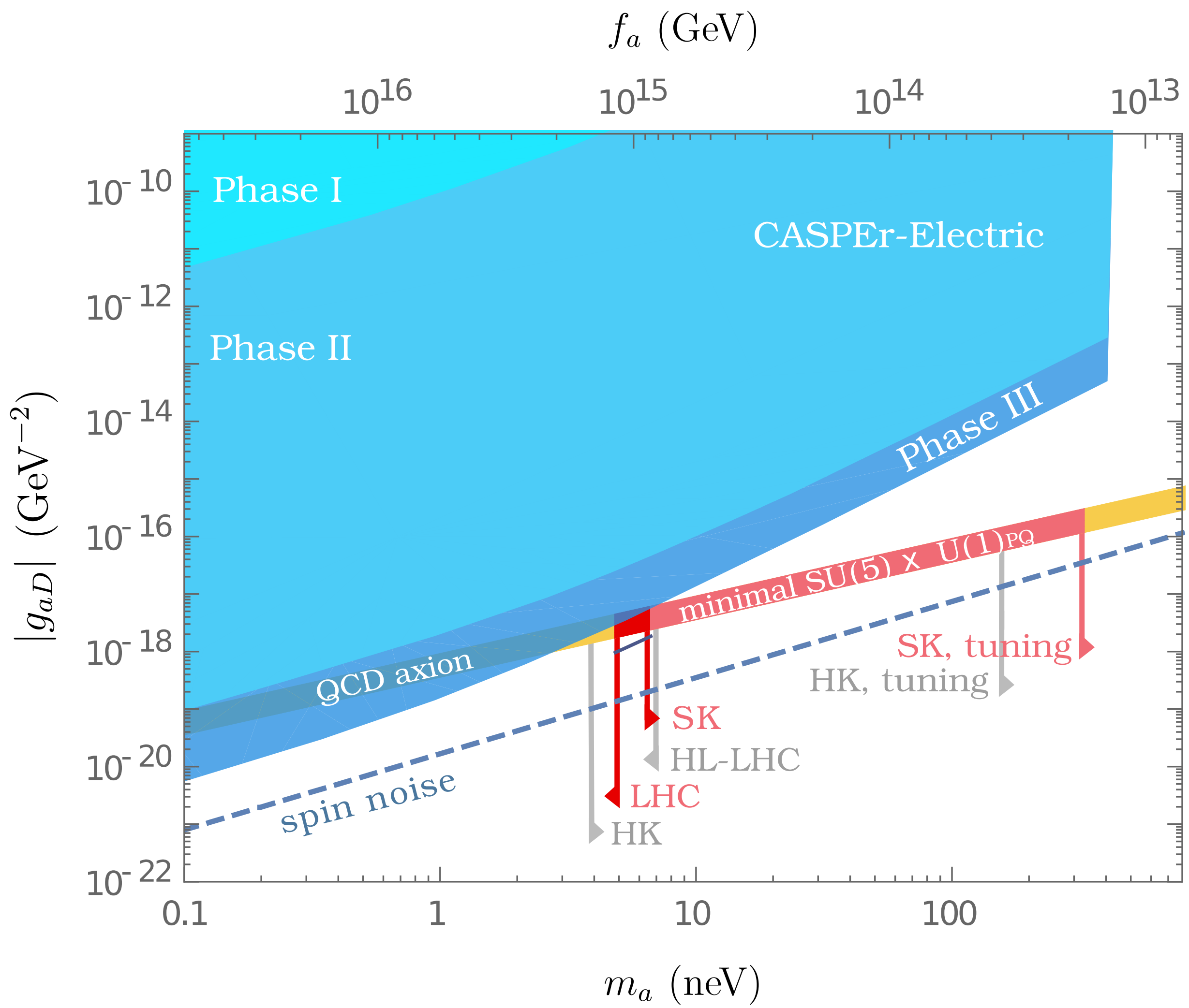
Proton decay bounds from Super-Kamiokande, projected Hyper-Kamiokande

LHC bounds from [CMS], HL-LHC bounds from [R.E.F. Ruiz et al]

Coupling of SU(5) axion to photons



Coupling of SU(5) axion to nucleon EDM



Axion-induced nucleon dipole moment from [Pospelov, Ritz]

SO(10)

Relevant SO(10) representations

Each generation comes come in spinorial 16 representation

$SO(10)$	$3_C 2_L 1_Y$
16_F	$(3,2,\frac{1}{6}) := q$
	$(1,2,-\frac{1}{2}) := l$
	$(\bar{3},1,\frac{1}{3}) := d$
	$(1,1,1) := e$
	$(\bar{3},1,-\frac{2}{3}) := u$
	$(1,1,0) := n$

We consider the following 2 and 3-step breaking chains

Two step

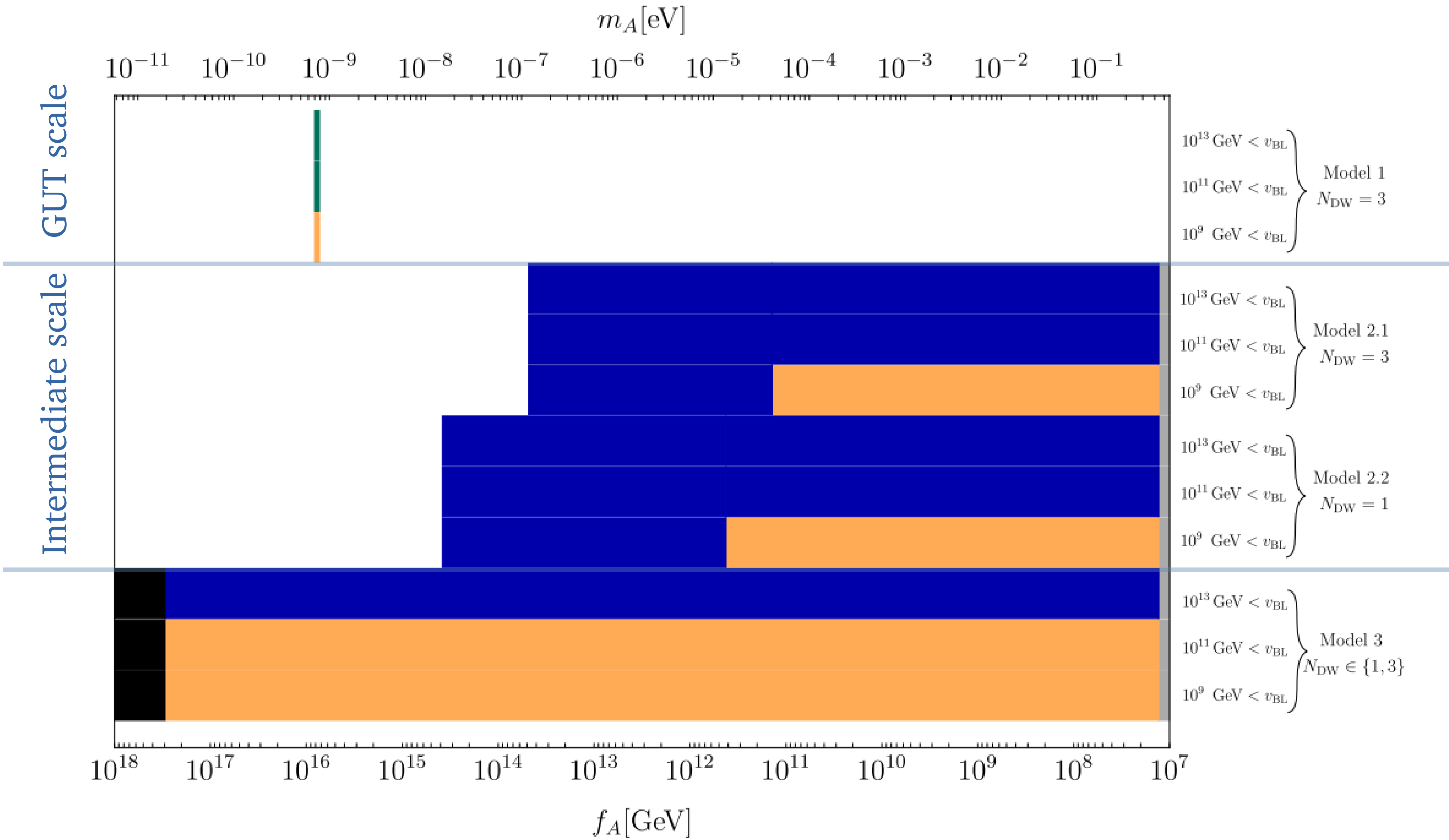
$$SO(10) \xrightarrow{v^{210}-210_H} 4_C\, 2_L\, 2_R \xrightarrow{v_{BL}-\overline{126}_H} 3_C\, 2_L\, 1_Y \xrightarrow{v_{u,d}^{10,126}-\overline{126}_H-10_H} 3_C\, 1_{\rm em}$$

Three step

$$SO(10) \xrightarrow{v^{210}-210_H} 4_C\, 2_L\, 2_R \xrightarrow{v_{\rm PQ}-45_H} 4_C\, 2_L\, 1_R \xrightarrow{v_{\rm BL}-\overline{126}_H} 3_C\, 2_L\, 1_Y \xrightarrow{v_{u,d}^{10,126}-\overline{126}_H-10_H} 3_C\, 1_{\rm em}.$$

GUT scale or intermediate axion depending on whether 210_H has nonzero PQ charge or not.

Unification constraints across models

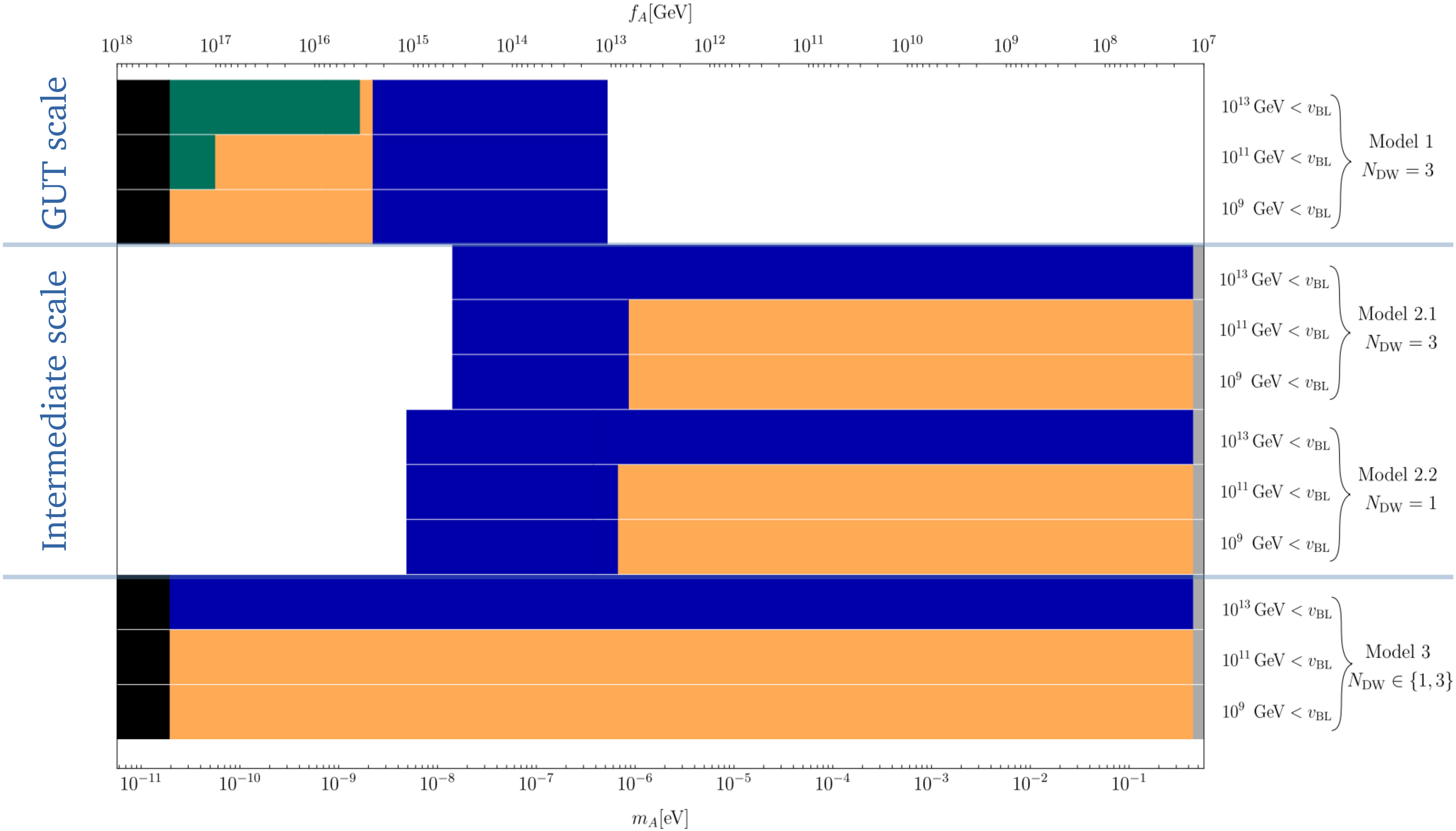


Orange: Allowed
Blue: Discarded by proton decay

Black: Discarded by superradiance
Green: Discarded, fermion mass fits
Gray: Stellar cooling constraint

No one-loop thresholds

Unification constraints across models

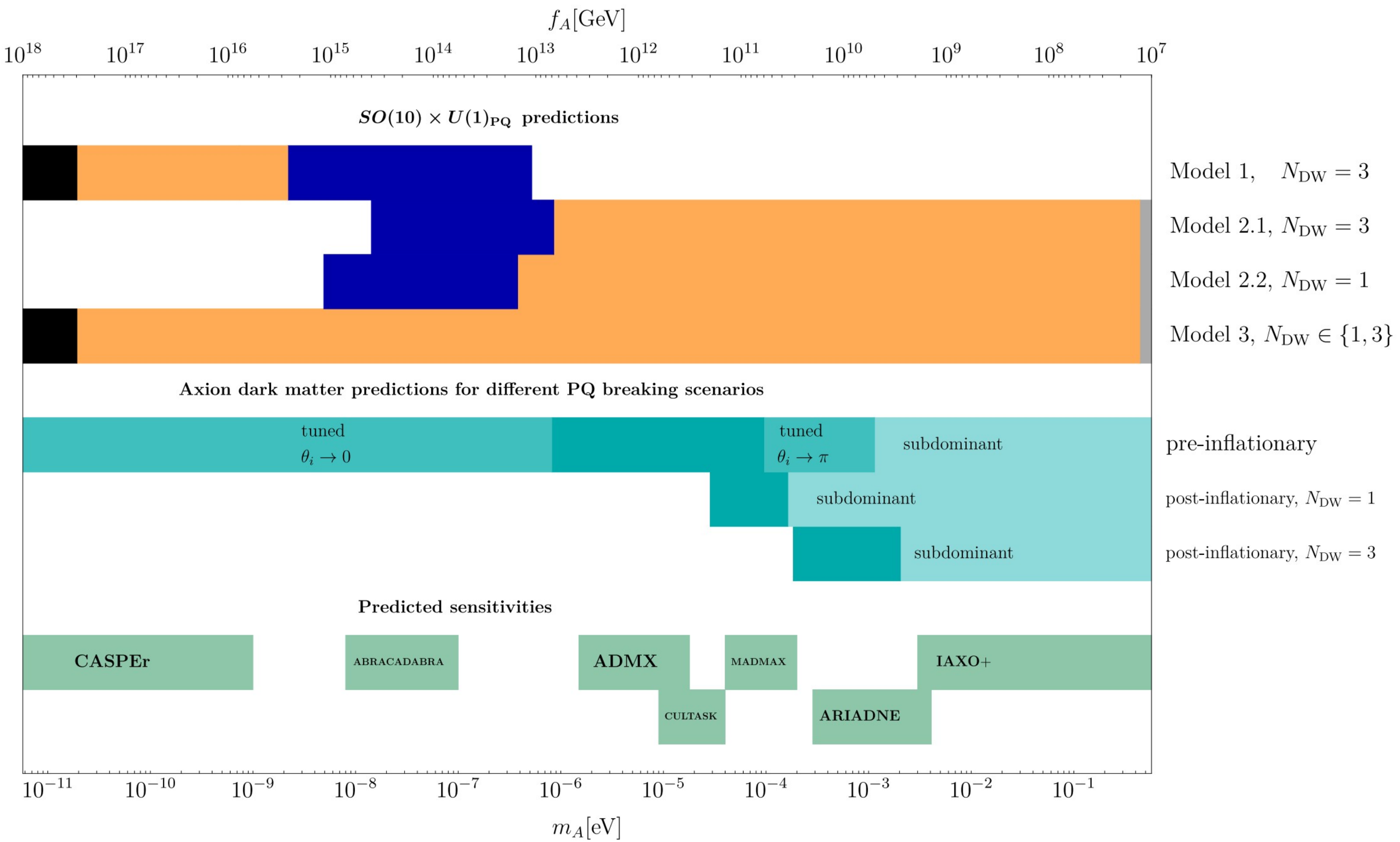


Orange: Allowed
Blue: Discarded by proton decay

Black: Discarded by superradiance
Green: Discarded, fermion mass fits
Gray: Stellar cooling constraint

Random 1-loop thresholds

Summary: Unification constraints

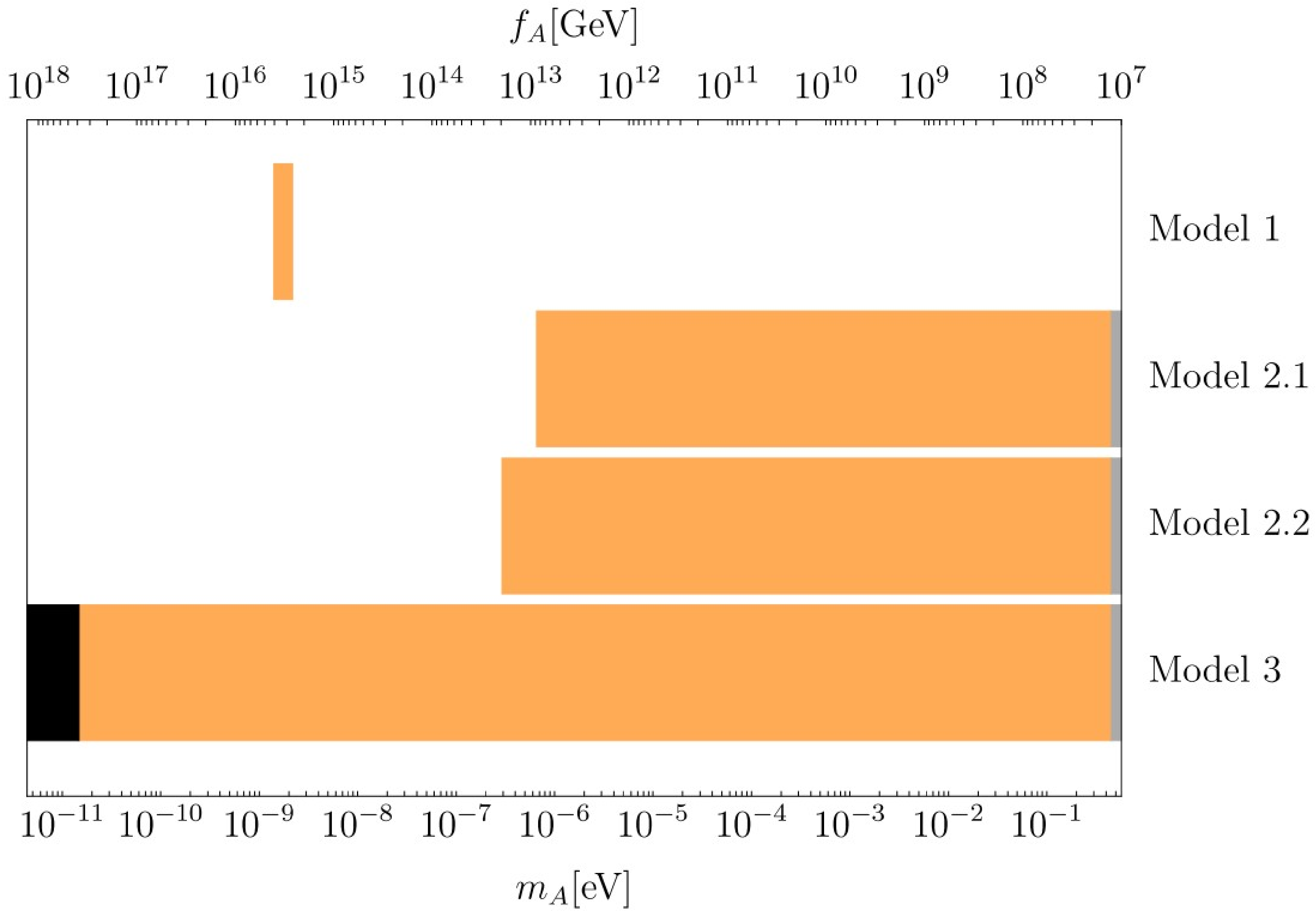


Orange: Allowed
Blue: Discarded by proton decay

Black: Discarded by superradiance
Gray: Stellar cooling constraint

Random 1-loop thresholds

..If Hyperkamiokande saw proton decay in first 10 years



Orange: Allowed

Black: Discarded by superradiance
Gray: Stellar cooling constraint

Random 1-loop thresholds

Summary of axion couplings

$$\mathcal{L} = \frac{1}{2}\partial_\mu A \partial^\mu A - \frac{1}{2} m_A^2 A^2 + \frac{\alpha}{8\pi}\frac{C_{A\gamma}}{f_A} A\, F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{2}\frac{C_{Af}}{f_A}\partial_\mu A\, \overline{\Psi}_f \gamma^\mu \gamma_5 \Psi_f \,,$$

SU(5)

$$N_{DW} = 11$$

$$C_{A\gamma} = \frac{8}{3} - 1.92(4)\,, \qquad C_{Ae} = \frac{2}{11}\sin^2\beta\,,$$

$$C_{Ap} = -0.47(3) + \frac{6}{11}[0.29\cos^2\beta - 0.15\sin^2\beta \pm 0.02]\,,$$

$$C_{An} = -0.02(3) + \frac{6}{11}[-0.14\cos^2\beta + 0.28\sin^2\beta \pm 0.02]\,,$$

$$\tan\beta = \frac{\langle 5_H \rangle}{\langle 5'_H \rangle}$$

SO(10)

$$N_{DW} = 1, 3$$

$$C_{A\gamma} = \frac{8}{3} - 1.92(4)\,, \qquad C_{Ae} = \frac{1}{N_{\mathrm{DW}}}\sin^2\beta\,,$$

$$C_{Ap} = -0.47(3) + \frac{3}{N_{\mathrm{DW}}}[0.29\cos^2\beta - 0.15\sin^2\beta \pm 0.02]\,,$$

$$C_{An} = -0.02(3) + \frac{3}{N_{\mathrm{DW}}}[-0.14\cos^2\beta + 0.28\sin^2\beta \pm 0.02]\,,$$

$$\tan^2\beta = ((v_u^{10})^2 + (v_u^{126})^2)/((v_d^{10})^2 + (v_d^{126})^2)$$

Conclusions

We identified the axion field, obtained its couplings to SM fields, and computed N_{DW} for the physical PQ symmetry, in a minimal SU(5) model and several SO(10) models.

We studied constraints from unification, superradiance, star cooling, fermion masses.

Our formalism bridges the gap between UV and IR symmetries. We identified the physical PQ symmetry as a combination of UV symmetries.

We clarified issues pertaining to fermion field redefinitions.

Minimal viable SU(5) model highly constrained, predicting axion mass in window of 4.8-6.6 neV that can be probed at future axion and proton decay searches.

SO(10) models can have axion in wide range of masses due to the possibility of multi-state breaking and intermediate scale VEVs.

Backup slides

From the anomaly to the effective Lagrangian

$$\partial_\mu J^\mu = \sum_{k=1}^{N_g} \frac{g_k^2 \hat{N}_k}{16\pi^2} \bar{\mathrm{Tr}}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu} = f_{\mathrm{PQ}} \Box A + \sum_a q_a \partial_\mu (\psi_a^\dagger \bar{\sigma}^\mu \psi_a)$$

Equivalent to Euler-Lagrange equations from the following effective action [Srednicki]

$$\mathcal{L}[A]_{\mathrm{eff}} = \frac{1}{2}\partial_\mu A \partial^\mu A + \partial_\mu A \sum_a \frac{q_a}{f_{\mathrm{PQ}}} (\psi_a^\dagger \bar{\sigma}^\mu \psi_a) + A \sum_{k=1}^{N_g} \frac{1}{f_{A,k}} \frac{g_k^2}{16\pi^2} \bar{\mathrm{Tr}}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu}$$

$$f_{A,k} = \frac{f_{\mathrm{PQ}}}{\hat{N}_k},$$

Alternatively, one can obtain Lagrangian by redefining fermion fields [Kim, Dias et al]

$$\mathcal{L} \supset y^i_{ab} \phi_i \psi_a \psi_b + c.c. \supset \frac{y^i_{ab} v_i}{\sqrt{2}} e^{i q_i A / f_{\mathrm{PQ}}} \psi_a \psi_b + c.c. = \frac{y^i_{ab} v_i}{\sqrt{2}} e^{-i(q_a + q_b) A / f_{\mathrm{PQ}}} \psi_a \psi_b + c.c.,$$

$\psi_a \to e^{i q_a A / f_{\mathrm{PQ}}} \psi_a$ eliminates axion field in phases and gives $\mathcal{L}[-A]_{\mathrm{eff}}$

(Not the only way to eliminate phase. One can define physically equivalent Lagrangians that differ by fermion field redefinitions.)

Hadronic/QCD effects

Axion coupling to nucleons:

Axial basis: Weyl fermions grouped into Dirac spinors:

$$\partial_\mu A \left[\frac{q_1}{f_{\rm PQ}} \psi^\dagger \bar{\sigma}^\mu \psi + \frac{q_2}{f_{\rm PQ}} \tilde{\psi}^\dagger \bar{\sigma}^\mu \tilde{\psi} \right] \to -\partial_\mu A \, \frac{q_1 + q_2}{2 f_{\rm PQ}} \, \overline{\Psi} \gamma^\mu \gamma_5 \Psi .$$

PQ invariance: $q_1 + q_2$ is equal to a scalar PQ charge. From chiral perturbation theory at NLO and lattice results [Villadoro et al]

$$\delta\mathcal{L}_{\rm eff} = -\,\partial_\mu A \, \frac{C_{AN}}{2 f_A} \, \overline{N} \gamma^\mu \gamma_5 N - \partial_\mu A \, \frac{C_{AP}}{2 f_A} \, \overline{P} \gamma^\mu \gamma_5 P,$$

$$C_{AN} = -\,0.02(3) + 0.41(2) \frac{q_{H_u} f_A}{f_{\rm PQ}} - 0.83(3) \frac{q_{H_d} f_A}{f_{\rm PQ}},$$

$$C_{AP} = -\,0.47(3) - 0.86(3) \frac{q_{H_u} f_A}{f_{\rm PQ}} + 0.44(2) \frac{q_{H_d} f_A}{f_{\rm PQ}}.$$

Fermion couplings in axial basis only depend on scalar PQ charges q_i / f_{PQ}.

Axion coupling to photons:

Axion field can mix with other pseudo-Goldstones, like the neutral pion. An appropriate field redefiniton removes the mixing and gives for the physical axion

$$\delta\mathcal{L}_{\rm eff} = \frac{\alpha}{8\pi f_A} \, \delta C_{A\gamma} A \tilde{F}_{\mu\nu} F^{\mu\nu}, \; \delta C_{A\gamma} = -\frac{2}{3} \left(\frac{4 m_u + m_d}{m_u + m_d} \right) + \text{higher order} = -1.92(4).$$

Wherefore art thou a physical Goldstone?

Axion must be orthogonal to massive gauge bosons: physical PQ symmetry $\mathrm{PQ}_{\mathrm{phys}}$ is generated by a combination of original PQ and other symmetries S_j:

$$PQ_{\mathrm{phys}} = PQ + \sum_j \lambda_j S_j \to A = \sum_i c_i A_i, \quad c_i = \frac{q_i v_i}{\sqrt{q_i^2 v_i^2}}$$

Unknown c_i giving charges of $\mathrm{PQ}_{\mathrm{phys}}$ can be obtained by solving masslessness and orthogonality conditions

Art thou massless? $$\mathcal{L} \supset m \left(\sum_m d_m A_m \right)^2 \Rightarrow \sum_m c_m d_m = 0$$

Art thou orthogonal to massive gauge bosons?

Avoidance of kinetic mixing with gauge bosons implies $\sum_{mn} v_m T^a_{mn} c_n = 0.$

For a massive U(1) boson with associated scalar charges $\tilde{q}_m$: $\sum_m v_m \tilde{q}_m c_m = 0.$

Need at least 2 scalars charged under massive U(1) for them to contain the axion.
f_A of the order of the smallest VEV of fields charged under massive U(1).

Domain wall number

$$\mathcal{L}_{\rm eff}[A] \supset A \sum_{k=1}^{N_g} \left(\theta_k + \frac{A \hat{N}_k}{f_{\rm PQ}} \frac{g_k^2}{16\pi^2} \right) \bar{\rm Tr}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu} + \text{derivatives}$$

Axion-Goldstone shifts under $\mathrm{PQ}_{\mathrm{phys}}$, $A \to A + \alpha f_{\rm PQ}$. The anomalies break this translation symmetry in the SU(3) sector to a discrete subset (equivalent to translations $\theta_3 \to \theta_3 + 2\pi$).

$$S_{\rm phys}(n) : A \to A + \frac{2\pi n}{\hat{N}} f_{\rm PQ}, \quad n \in \mathbb{Z}.$$

Some of this translations correspond to unphysical rotations of phases A_i by 2π

$$P_{\rm phys}(n_i) : A \to A + \sum_i \frac{2\pi n_i q_i v_i^2}{f_{\rm PQ}}, \quad n_i \in \mathbb{Z}.$$

$$N_{\rm DW} = \dim \left[\frac{S_{\rm phys}}{P_{\rm phys}} \right] = \text{ min. integer} \left\{ \frac{1}{f_A} \sum_i n_i c_i v_i,\, n_i \in \mathbb{Z} \right\} \left(\to \frac{\hat{N}}{\mathrm{MCD}[q_i]},\ \text{rational } q_i \right)$$

Relation to naive UV PQ symmetry (without imposing orthogonality conditions): Have to mod out by the discrete transformations in the center Z of the gauge group!

$$N_{\rm DW} = \dim \left[\frac{S_{\rm phys}}{P_{\rm phys}} \right] = \dim \left[\frac{S}{ZP} \right]$$

Example case: axion and couplings in model 2.2

Scalar components getting VEVS, in terms of decompositions under PS=SU(4)xSU(2)$_L$xSU(2)$_R$. SM arises from PS as follows:

$$SU(4) \supset SU(3) \times U(1)_{B-L} \qquad Y = U(1)_R + \frac{1}{2} U(1)_{B-L}$$
$$SU(2)_R \supset U(1)_R$$

$SO(10)$	$4_C 2_L 2_R$	$4_C 2_L 1_R$	$3_C 2_L 1_R 1_{B-L}$	$3_C 2_L 1_Y$	$3_C 1_{\mathrm{em}}$	scale	VEV
10_H	$(1,2,2)$	$(1,2,\frac{1}{2})$	$(1,2,\frac{1}{2},0)$	$(1,2,\frac{1}{2})$	$(1,0) =: H_u$	M_Z	v_u^{10}
		$(1,2,-\frac{1}{2})$	$(1,2,-\frac{1}{2},0)$	$(1,2,-\frac{1}{2})$	$(1,0) =: H_d$	M_Z	v_d^{10}
45_H	$(1,1,3)$	$(1,1,0)$	$(1,1,0,0)$	$(1,1,0)$	$(1,0) := \sigma$	M_{PQ}	v_{PQ}
$\overline{126}_H$	$(10,1,3)$	$(10,1,1)$	$(1,1,1,-2)$	$(1,1,0)$	$(1,0) := \Delta_R$	M_{BL}	v_{BL}
	$(15,2,2)$	$(15,2,\frac{1}{2})$	$(1,2,\frac{1}{2},0)$	$(1,2,\frac{1}{2})$	$(1,0) := \Sigma_u$	M_Z	v_u^{126}
		$(15,2,-\frac{1}{2})$	$(1,2,-\frac{1}{2},0)$	$(1,2,-\frac{1}{2})$	$(1,0) := \Sigma_d$	M_Z	v_d^{126}
210_H	$(1,1,1)$	$(1,1,0)$	$(1,1,0,0)$	$(1,1,0)$	$(1,0) := \phi$	M_{U}	v_{U}

Axion is a combination of phases of the first 6 scalars (because 210 has no PQ charge):

$$\phi_1 \equiv \Sigma_u, \phi_2 \equiv \Sigma_d, \phi_3 \equiv H_u, \phi_4 \equiv H_d, \phi_5 \equiv \Delta_R, \phi_6 \equiv \sigma, \qquad \phi_j = \frac{1}{\sqrt{2}}(v_j + \rho_j) e^{iA_j/v_j}.$$

$$A = \sum_i c_i A_i$$

Example case: axion and couplings in 2.2

Orthogonality with respect to $U(1)_R$, $U(1)_{B-L}$ and T_{L3}

$$\sum_m v_m (B-L)_m c_m = 0 \leftrightarrow c_v f_5 = 0$$
$$\sum_m v_m R_m c_m = 0 \leftrightarrow \frac{c_1 v_1}{2} + \frac{c_3 v_3}{2} + c_5 v_5 - \frac{c_2 v_2}{2} - \frac{c_4 v_4}{2}$$
$$\sum_m v_m T_{L3} c_m = 0 \leftrightarrow -\frac{c_1 v_1}{2} + \frac{c_2 v_2}{2} + \frac{c_4 v_4}{2} - \frac{c_3 v_3}{2} = 0$$

Orthogonality wrt massive nonabelian bosons: satisfied automatically, follows from analysis of SO(10) roots and weights

Masslessness conditions

$$\begin{aligned}
10_H \times \overline{126}_H \times \overline{126}_H \times 45_H &\quad\rightarrow\quad \phi\sigma\Sigma_u\Sigma_d + h.c. \propto \left(A_6 v_1 v_2 + v_6\left(A_2 v_1 + A_1 v_2\right)\right)^2 \\
210_H \times 10_H \times \overline{126}_H \times 45_H &\quad\rightarrow\quad \phi\sigma H_u\Sigma_d + h.c. \propto \left(A_6 v_2 v_3 + v_6\left(A_3 v_2 + A_2 v_3\right)\right)^2 \\
&\quad\rightarrow\quad \phi\sigma\Sigma_u H_d + h.c. \propto \left(A_6 v_1 v_4 + v_6\left(A_4 v_1 + A_1 v_4\right)\right)^2
\end{aligned}$$

$\cdots$

$$c_6 v_1 v_2 + v_6\left(c_2 v_1 + c_1 v_2\right) = 0$$
$$c_6 v_2 v_3 + v_6\left(c_3 v_2 + c_2 v_3\right) = 0$$
$$c_6 v_1 v_4 + v_6\left(c_4 v_1 + c_1 v_4\right) = 0$$

All equations compatible! (Only need 5 eqs. and normalization condition to find axion)

$$A = -\frac{(A_4 v_4 + A_2 v_2)(v_3^2 + v_1^2) + (A_3 v_3 + A_1 v_1)(v_4^2 + v_2^2) - A_6 v_6 v^2}{\sqrt{v^2((v_4^2+v_2^2)(v_3^2+v_1^2) + v_6^2 v^2)}}, \quad v^2 \equiv \sum_{i=1}^{4} v_i^2 .$$

Example case: axion and couplings in 2.2

From c_i to scalar $\mathrm{PQ_{phys}}$ charges: $$\frac{q_i}{f_{\mathrm{PQ}}} = \frac{c_i}{v_i}$$

As expected, $\mathrm{PQ_{phys}}$ is a combination of original PQ and symmetries in the Cartan of SO(10)!!

$$\mathrm{PQ_{phys}} = s_1\,\mathrm{PQ} + s_2\,U(1)_R + s_3\,U(1)_{B-L}.$$

$$\frac{s_1}{f_{\mathrm{PQ}}} = \frac{v}{4\sqrt{(v_1^2+v_3^2)(v_2^2+v_4^2)+v^2v_6^2}}, \quad \frac{s_2}{f_{\mathrm{PQ}}} = \frac{v_1^2-v_2^2+v_3^2-v_4^2}{v\sqrt{(v_1^2+v_3^2)(v_2^2+v_4^2)+v^2v_6^2}},$$
$$\frac{s_3}{f_{\mathrm{PQ}}} = \frac{v_1^2-3v_2^2+v_3^2-3v_4^2}{4v\sqrt{(v_1^2+v_3^2)(v_2^2+v_4^2)+v^2v_6^2}}$$

From this we can get $\mathrm{PQ_{phys}}$ of Weyl fermions and

$$f_{A,k} = \frac{f_{\mathrm{PQ}}}{\hat{N}_k} = 2\sum_a \left(\frac{q_a}{f_{\mathrm{PQ}}}\right) T_k(\rho_a), \quad f_{\mathrm{PQ}} = \sqrt{\sum_i q_i^2 v_i^2}, \quad \hat{N}_k = 2\sum_a q_a T_k(\rho_a)$$

Indeed all f_{Ak} are equal modulo hypercharge normalization, as follows from GUT symmetry!

$$f_{A,3c} = f_{A,2L} = \frac{5}{3} f_{A,Y} = \sqrt{\frac{v_6^2v^2 + (v_4^2+v_2^2)(v_3^2+v_1^2)}{v^2}}.$$

Example case: axion and couplings in 2.2

Domain wall number: from UV arguments we expect 1.

$$N_{\rm DW} = \text{ min. integer}\left\{\frac{1}{f_A}\sum_i n_i c_i v_i,\ n_i \in \mathbb{Z}\right\}$$

Calling this integer $n_{\rm min}$ one gets a system of equations (from demanding numerator proportional to denominator and collecting coefficients of powers of the v_i)

$$\begin{aligned} n_{\rm min} + (n_1+n_2) &= 0, \quad n_{\rm min} + (n_2+n_3) = 0, \\ n_{\rm min} + (n_1+n_4) &= 0, \quad n_{\rm min} + (n_3+n_4) = 0, \\ n_{\rm min} - n_6 &= 0. \end{aligned} \quad \Longrightarrow \quad n_{\rm min} = N_{\rm DW} = 1$$