

Gauge Field & Fermion Production via Chern-Simons Coupling

Kyohei Mukaida

DESY, HAMBURG

Based on [1806.08769](#), [1812.08021](#), [19xx.xxxxxx](#)

In collaboration with Y. Ema, V. Domcke, B. von Harling, E. Morgante, R. Sato



1.

Introduction

Introduction

Light Scalar w/ CS-coupling

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha \phi}{4\pi f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}$$

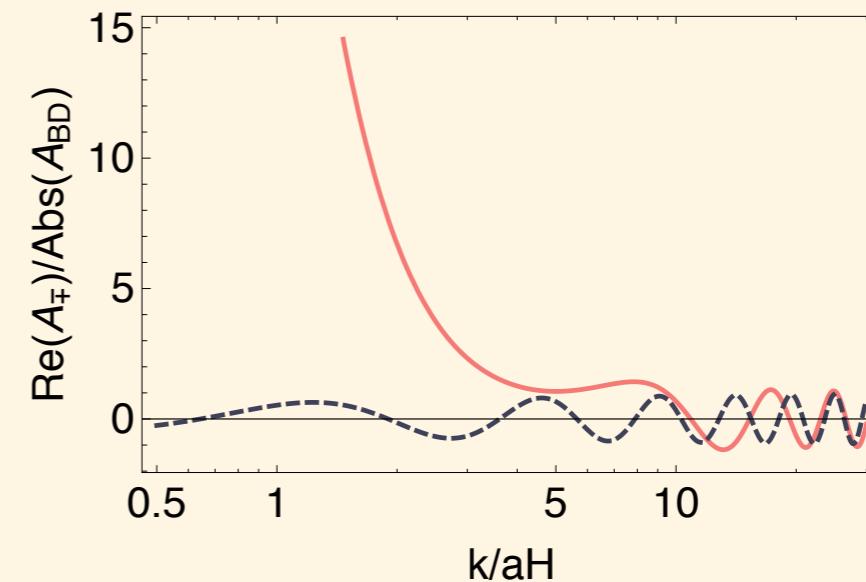
- “Lightness” protected by the shift symmetry: $\phi \mapsto \phi + c$

- Efficient **helical-gauge field** production by $\dot{\phi} \neq 0$.

$$0 = [\partial_\eta^2 + k(k \pm 2\xi aH)] A_\pm(\eta, k)$$

where $\xi \equiv \frac{\alpha \dot{\phi}}{2\pi f_a H}$

0 $\neq \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle = -4 \langle E \cdot B \rangle$



- axion inflation, relaxion, chiral GWs, baryogenesis, magnetogenesis,...

Introduction

Light Scalar w/ CS-coupling

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} F_{a\mu\nu} F^{a\mu\nu} + \boxed{\frac{\alpha\phi}{4\pi f_a} F_{a\mu\nu} \tilde{F}^{a\mu\nu}} \right\}$$

non-Abelian

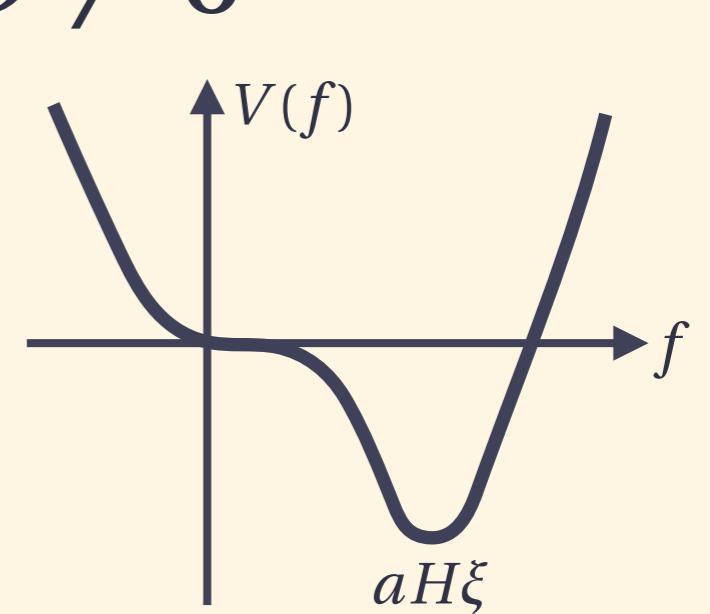
- “Lightness” protected by the shift symmetry (classically): $\phi \mapsto \phi + c$

- Efficient **helical-gauge field** production by $\dot{\phi} \neq 0$.
+ **Homogeneous & isotropic** gauge field may develop.

$$f''(\eta) + 2f^3(\eta) - 2aH\xi f^2(\eta) = 0$$

where $A_0^a = 0$, $A_i^a = -g^{-1} f(\eta) \delta_i^a$, $\xi \equiv \frac{\alpha\dot{\phi}}{2\pi f_a H}$

$$0 \neq \langle F_{a\mu\nu} \tilde{F}^{a\mu\nu} \rangle = -4 \langle E^a \cdot B^a \rangle$$



- **axion inflation, relaxion, chiral GWs, baryogenesis, magnetogenesis,...**

Introduction

Coupling to the SM Gauge Group ?

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + \frac{\alpha_Y \phi}{4\pi f_a} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right\}$$

Matters charged under $\mathbf{U(1)_Y}$

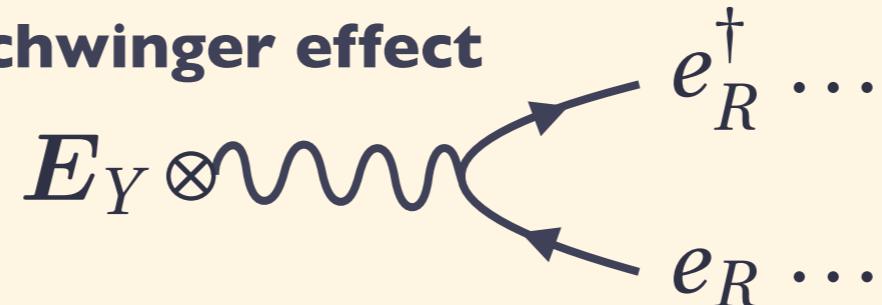
$$+ \sum_{\alpha} \psi_{\alpha}^\dagger \sigma \cdot (i\partial - g_Y Q_{\alpha} A_Y) \psi_{\alpha} + \dots \}$$

- ▶ Production of **matters** charged under the **SM gauge group**.
 - Asymmetry generation via the **SM chiral anomaly**

$$\partial_\mu J_{B+L}^\mu = \frac{3}{16\pi^2} \left(-g_2^2 W_{\mu\nu}^a \tilde{W}^{a\mu\nu} + g_Y^2 Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right) \neq 0$$

Helical gauge
 \Leftrightarrow **B+L asym.**

- Pair production via the **Schwinger effect**



- I. **Unified** understanding of these processes, 2. **Implications**

2.

Fermion Production

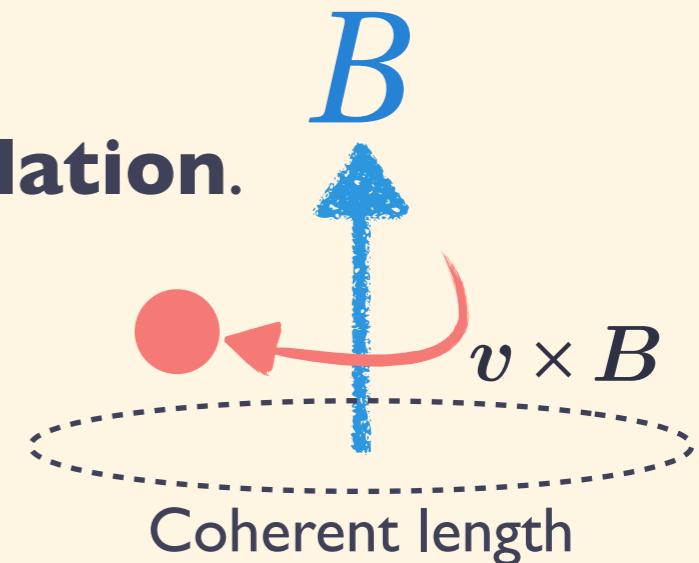
Fermion Production

Landau Levels

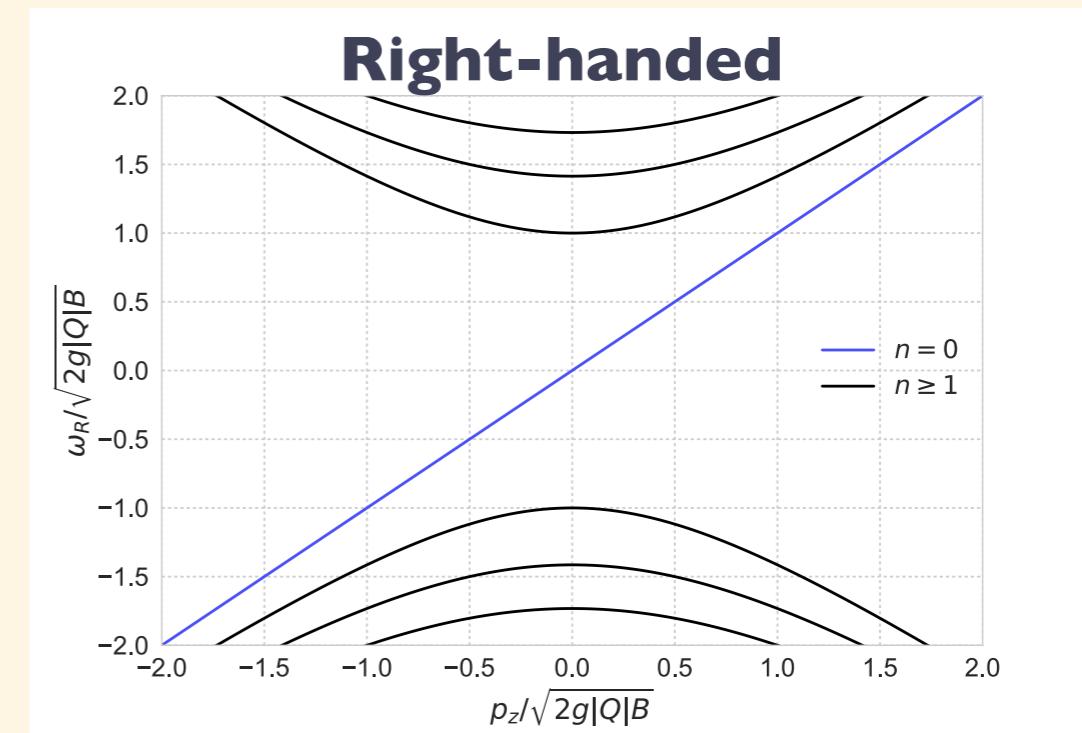
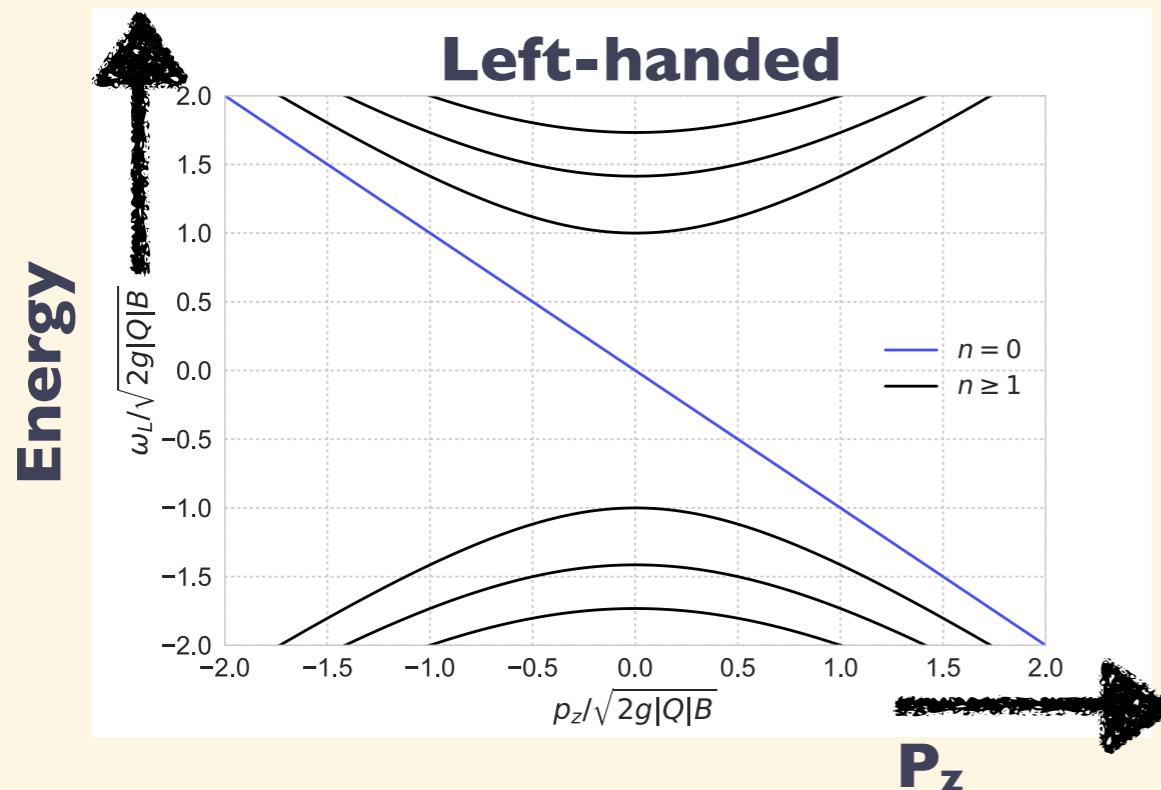
- Turn off E; B field modifies the **dispersion relation**.

$$0 = (i\partial_\eta \pm i\nabla \cdot \sigma - gQA_0 \pm gQA \cdot \sigma) \psi_{R/L}$$

where $(A_\mu) = (0, 0, -Bx, 0)$



- Landau Level n:** transverse motion; \mathbf{p}_z : parallel motion

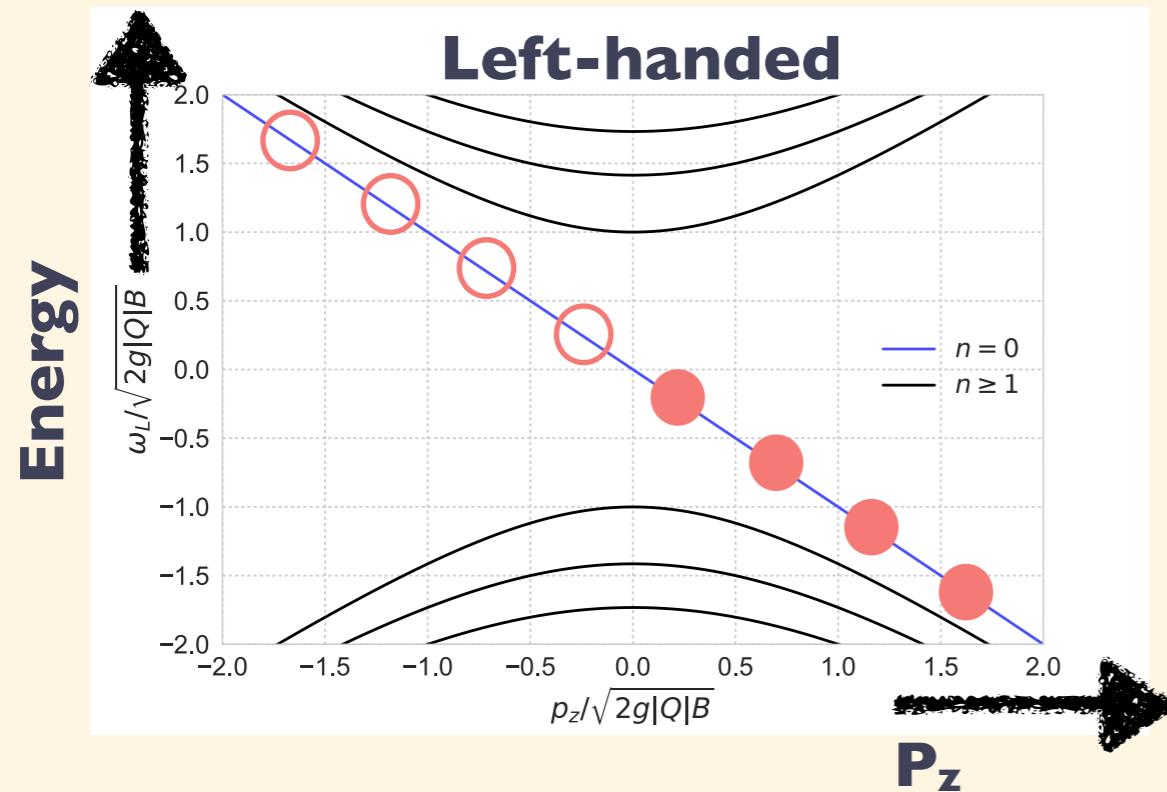


Fermion Production

Lowest Landau Level ($n=0$) & Chiral Anomaly

- ▶ Turn on E and see what happens.

Nielsen, Ninomiya, Phys.Lett. **I 30B** (1983)

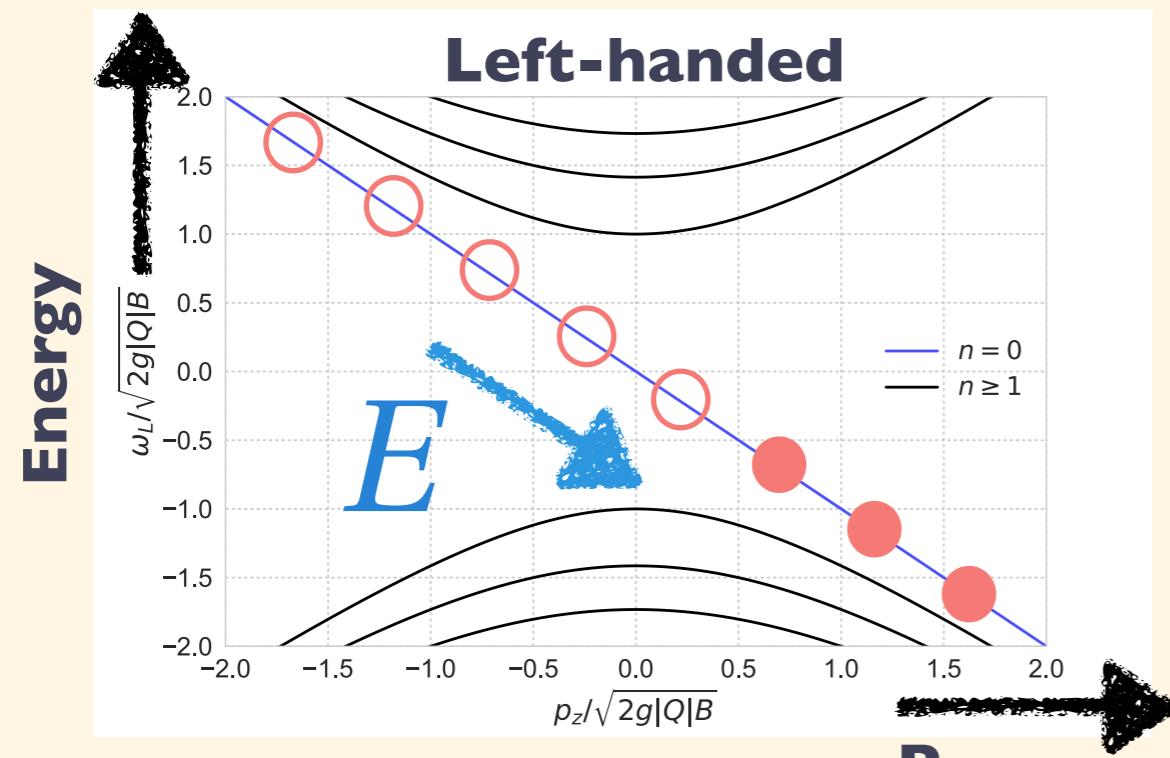


Fermion Production

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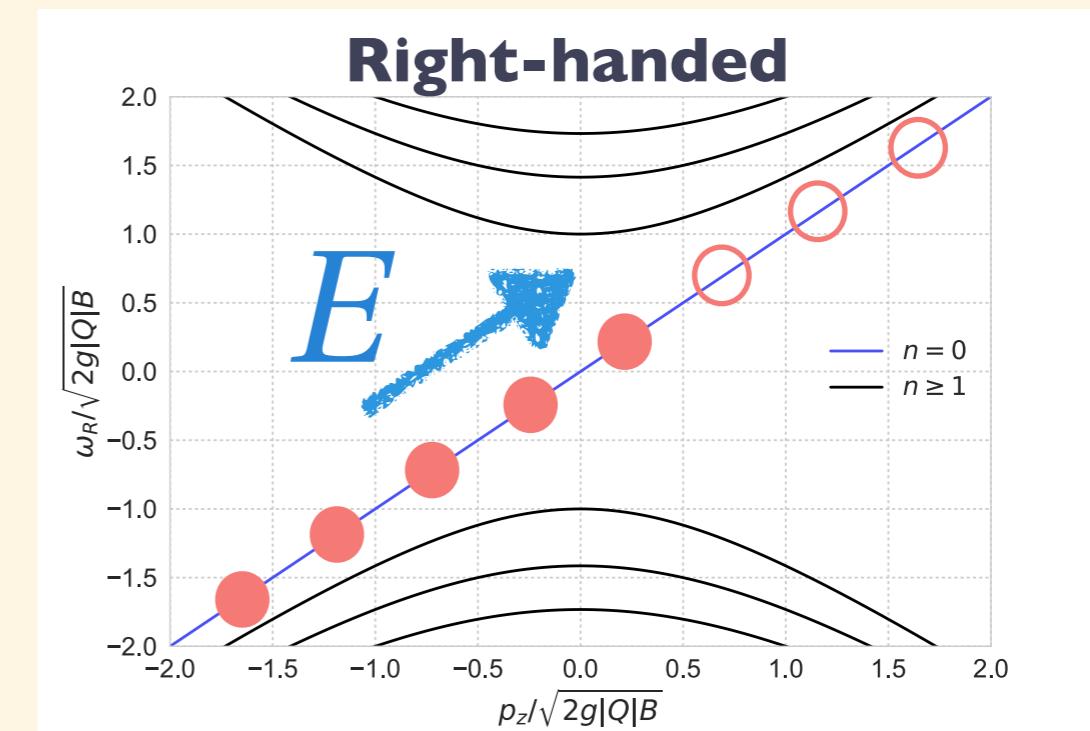
- ▶ Turn on **E** and see what happens.

Nielsen, Ninomiya, Phys.Lett. **I 30B** (1983)



Anti-particle prod:

$$\dot{q}_L = \dot{n}_L - \dot{\bar{n}}_L = -\frac{g^2 Q^2}{4\pi^2} E B$$



Particle prod:

$$\dot{q}_R = \dot{n}_R - \dot{\bar{n}}_R = +\frac{g^2 Q^2}{4\pi^2} E B$$

→ Reproduce **ABJ anomaly**:

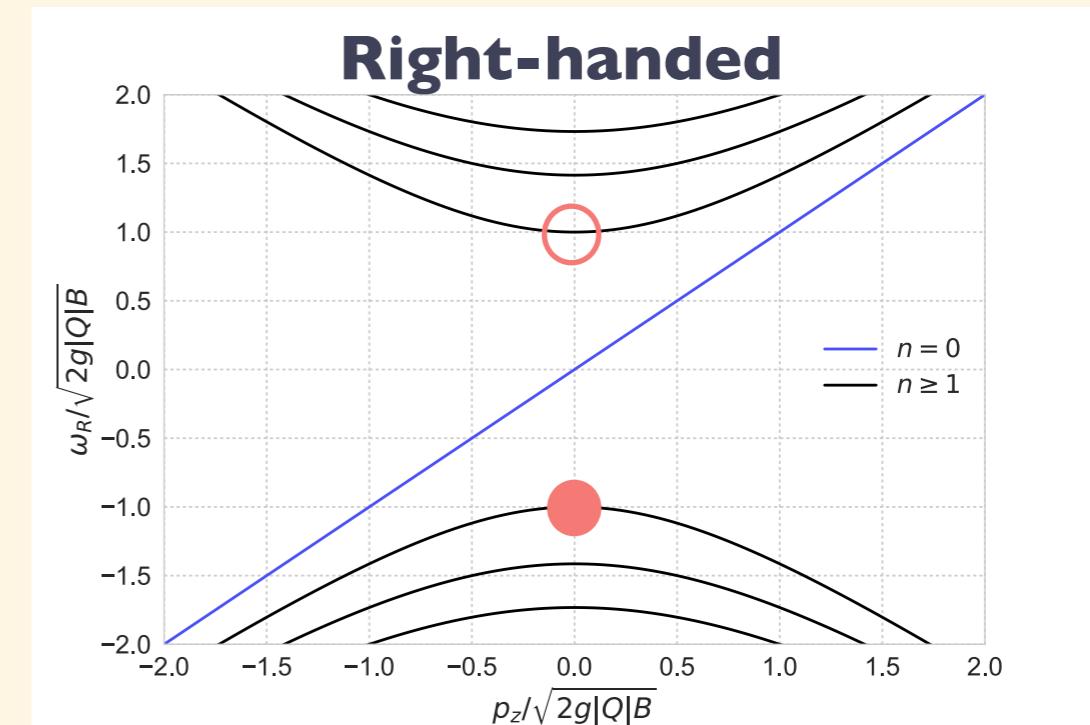
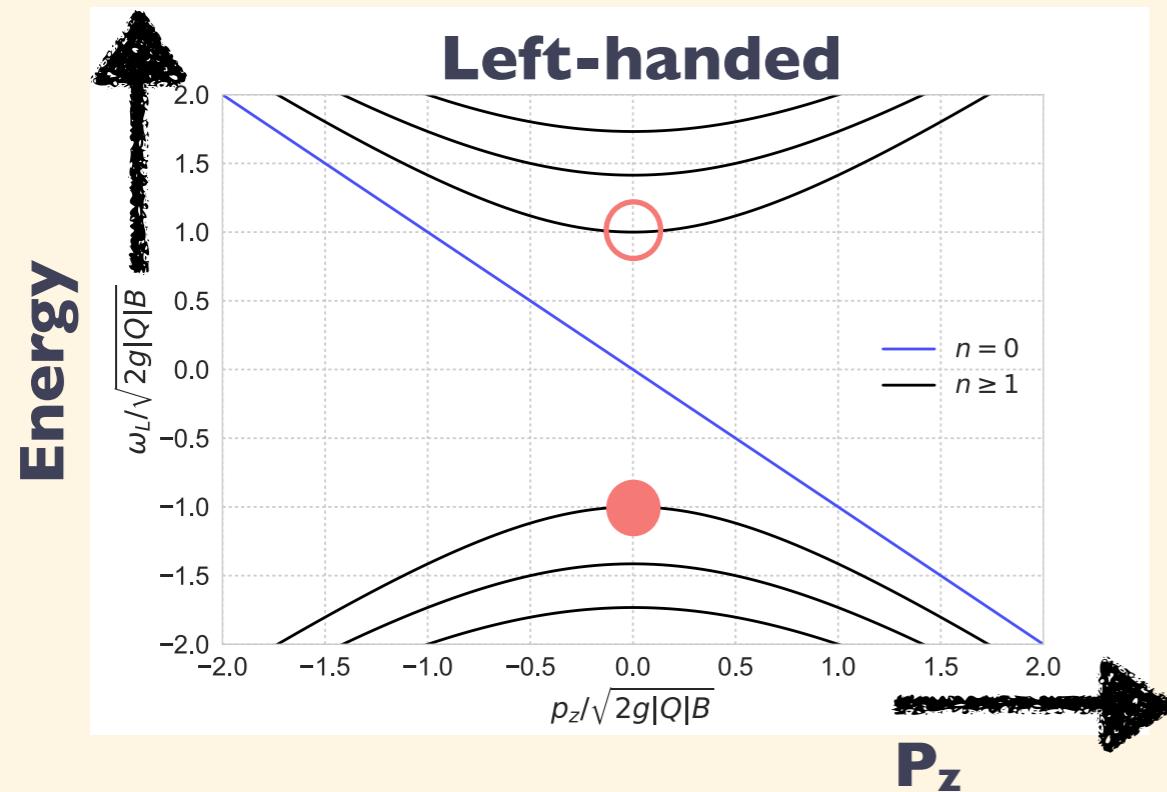
$$\dot{q}_5 = \dot{q}_R - \dot{q}_L = \frac{g^2 Q^2}{2\pi^2} E B = -\frac{g^2 Q^2}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

Fermion Production

Higher Landau Levels ($n \geq 1$) & Pair Production

- ▶ Turn on E and see what happens.

V.Domcke and KM 1806.08769

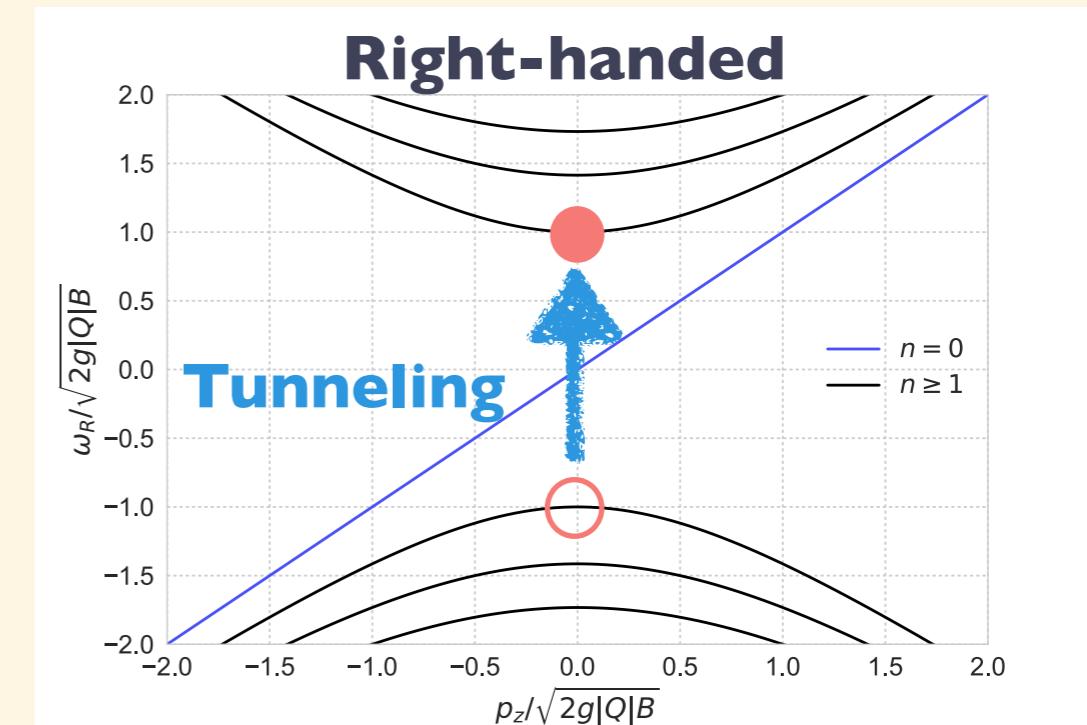
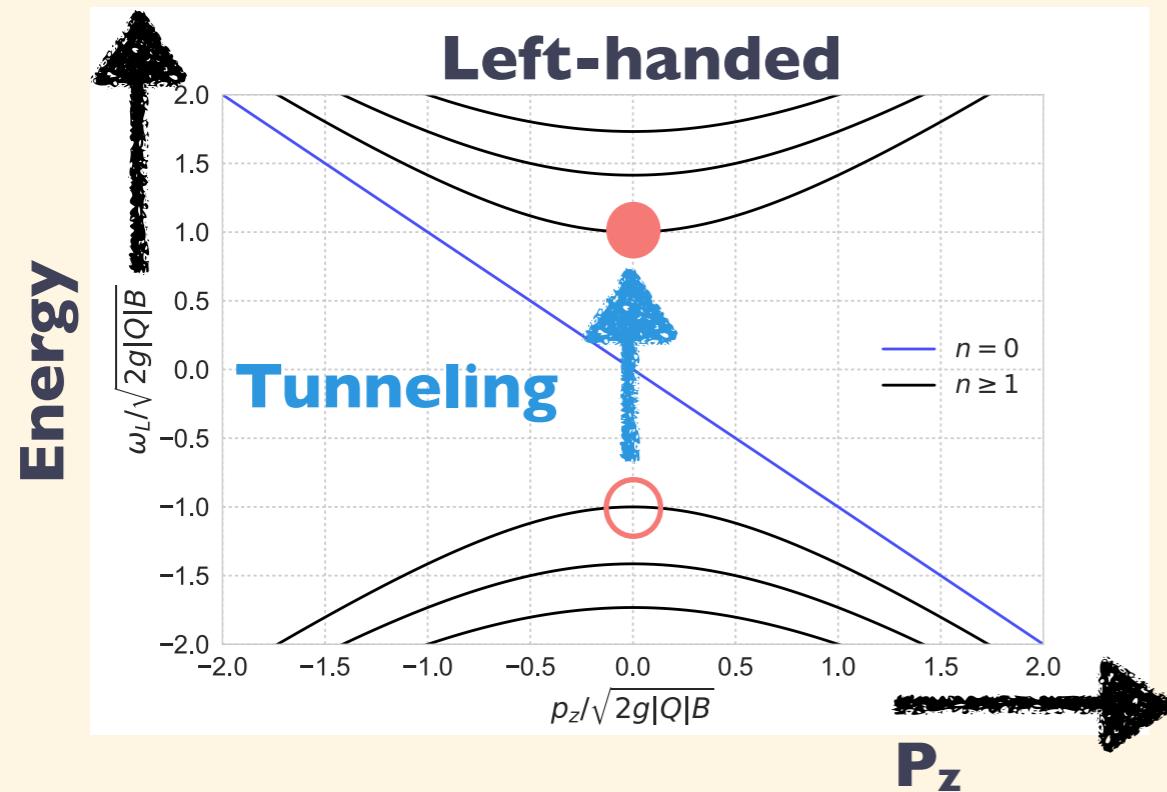


Fermion Production

Higher Landau Levels ($n \geq 1$) & Pair Production

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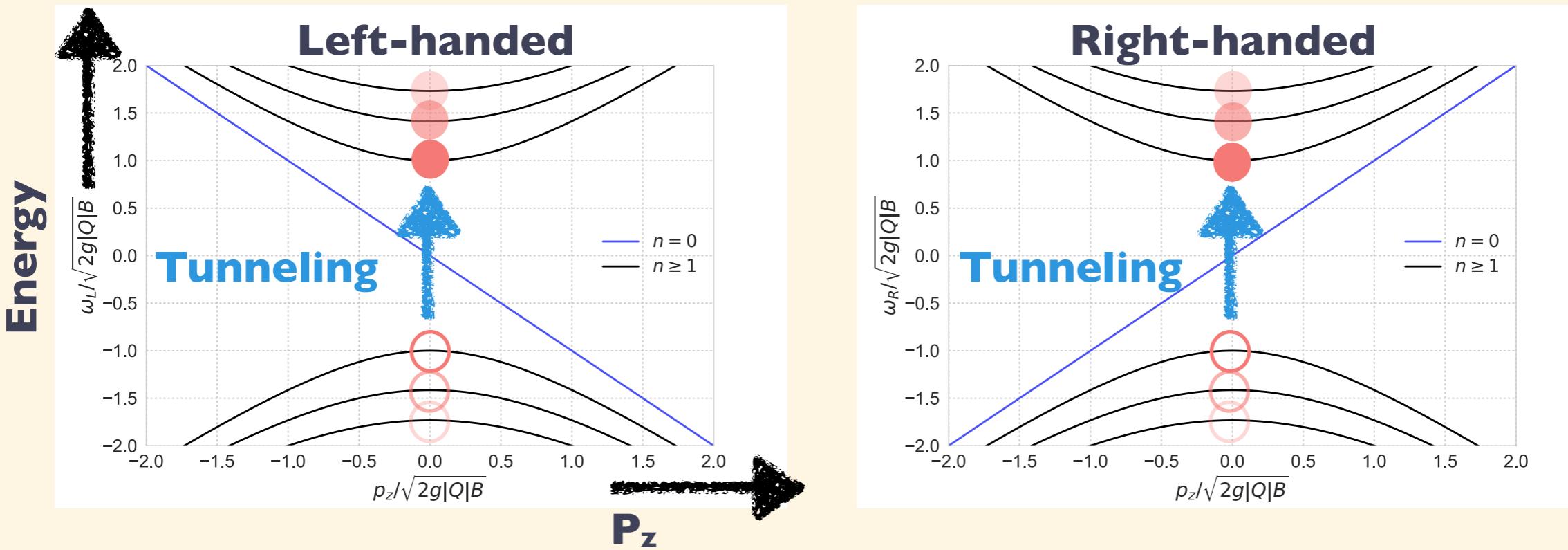
- Pair-production via Schwinger effect

Fermion Production

Higher Landau Levels ($n \geq 1$) & Pair Production

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V.Domcke and **KM** 1806.08769



- Pair-production via Schwinger effect

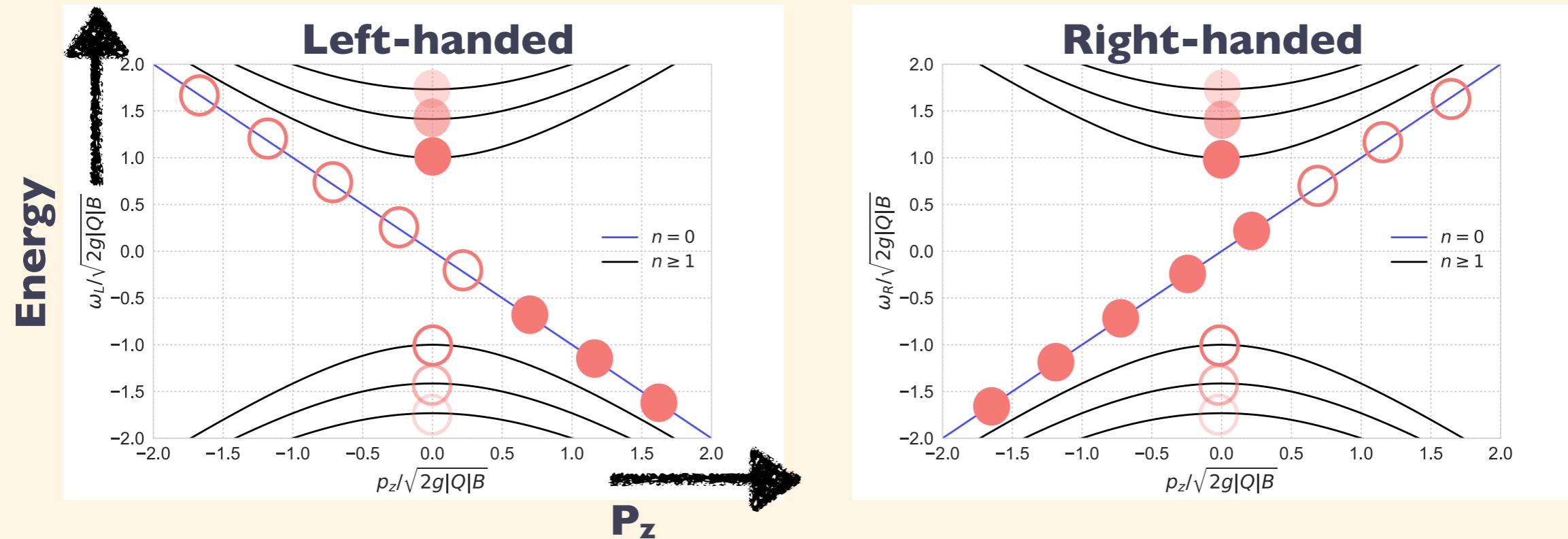
$$\dot{n}_R|_{n \geq 1} = \dot{\bar{n}}_R|_{n \geq 1} = \dot{n}_L|_{n \geq 1} = \dot{\bar{n}}_L|_{n \geq 1} = \sum_{n=1} \frac{g^2 Q^2}{4\pi^2} E B e^{-\frac{2\pi n B}{E}} = \frac{g^2 Q^2}{4\pi^2} E B \frac{1}{e^{\frac{2\pi B}{E}} - 1}$$

❖ Never contribute to the asymmetry! $\dot{q}_L|_{n \geq 1} = (\dot{n}_L - \dot{\bar{n}}_L)|_{n \geq 1} = 0$, $\dot{q}_R|_{n \geq 1} = (\dot{n}_R - \dot{\bar{n}}_R)|_{n \geq 1} = 0$

Fermion Production

Fermion Production in $B \parallel E$

V.Domcke and KM 1806.08769



► ABJ anomaly from LLL

$$\dot{q}_5 = \dot{q}_R - \dot{q}_L = \frac{g^2 Q^2}{2\pi^2} EB = -\frac{g^2 Q^2}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} \rightarrow 0 \text{ for } B \rightarrow 0$$

► Schwinger pair-production from HLLs

$$\dot{n}_R|_{n \geq 1} = \dot{\bar{n}}_R|_{n \geq 1} = \dot{n}_L|_{n \geq 1} = \dot{\bar{n}}_L|_{n \geq 1} = \frac{g^2 Q^2}{4\pi^2} EB \frac{1}{e^{\frac{2\pi B}{E}} - 1} \rightarrow \frac{g^2 Q^2}{8\pi^3} E^2 \text{ for } B \rightarrow 0$$

3.

Implications

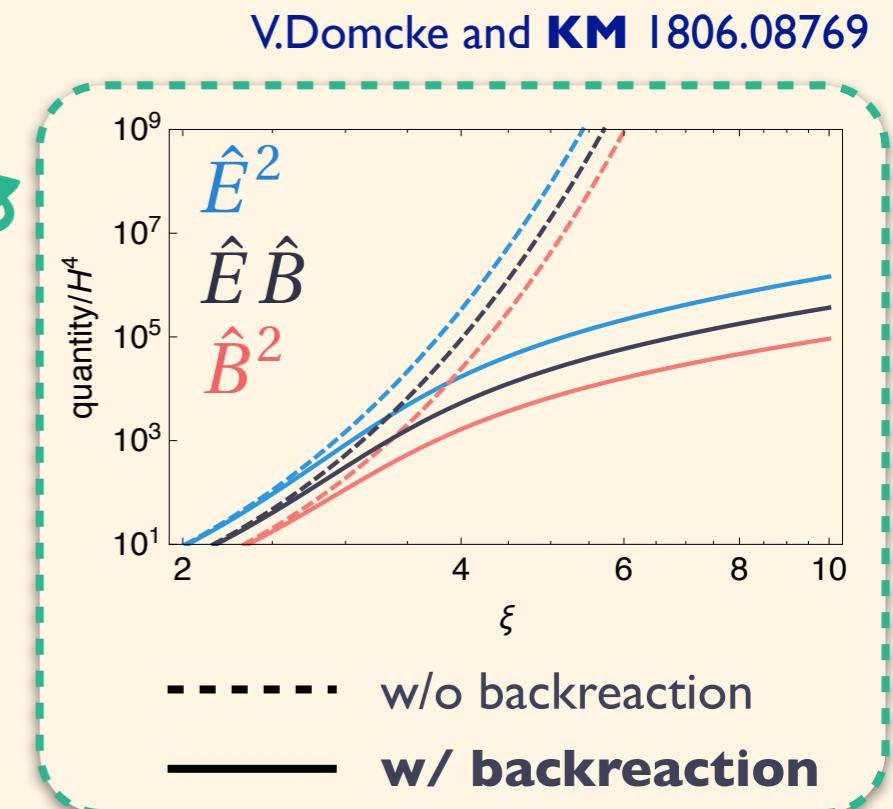
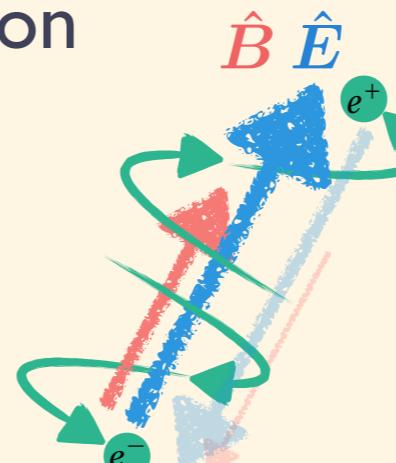
Backreaction

Backreaction suppresses gauge field.

► Induced current as backreaction

$$0 = \square A + a \frac{g^2}{4\pi^2} \frac{\dot{\phi}}{f_a} \nabla \times A - g J$$

$$g J = -a \left[\sum_{\alpha} \frac{g^3 |Q_{\alpha}|^3}{12\pi^2} \coth \left(\frac{\pi \hat{B}}{\hat{E}} \right) \hat{B} \right] \frac{\partial}{\partial \eta} A$$



► Implications on axion inflation

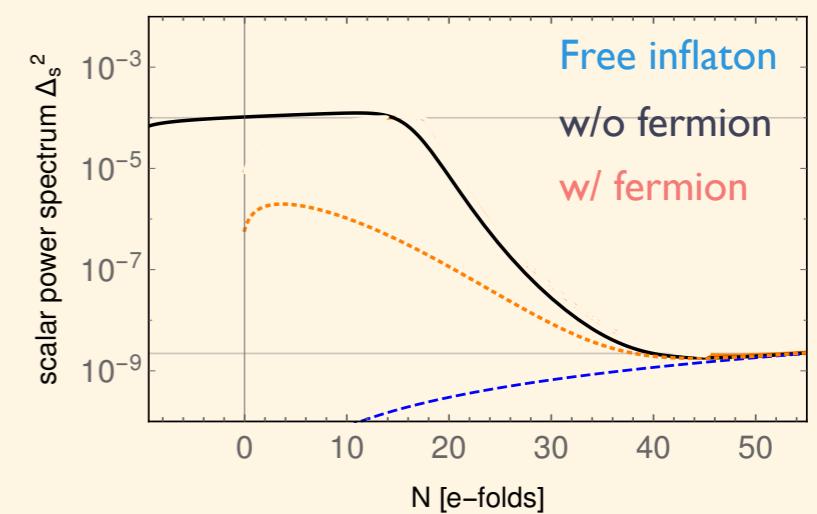
- Reduction of Δ_s sourced by gauge field

$$\Delta_s^2 \simeq \left(\frac{H^2}{2\pi\dot{\phi}} \right)^2 + \left(\frac{\alpha \langle \hat{E} \cdot \hat{B} \rangle}{3\pi\beta H\dot{\phi} f_a} \right)^2 \text{ w/ } \beta = 1 + \frac{2\xi\alpha \langle \hat{E} \cdot \hat{B} \rangle}{3\pi H\dot{\phi} f_a}$$

A.Linde, S.Mooij, E.Pajer 1212.1693

- Reduction of **GWs** sourced by gauge field (?)

Fermion contributions to GWs? Need further investigation...



Baryogenesis?

Evolution after Axion Inflation

B.von Harling, V.Domcke, E.Morgante, **KM**
19xx.xxxxxx

► Primordial generation of B+L asymmetry \Leftrightarrow helical gauge field

- **Asymmetry in e_R** survives from wash-out for $\hat{T} > 10^5 \text{ GeV}$ Electron Yukawa enters equilibrium

$$\frac{\hat{\mu}_{e_1}}{\hat{T}} \sim -\frac{\alpha_Y}{\pi} \frac{\langle \hat{E}_Y \cdot \hat{B}_Y \rangle_{\text{inf}}}{H_{\text{inf}}^4} \left(\frac{\hat{T}_R}{M_{\text{Pl}}} \right)^3 \quad \text{where} \quad \frac{\hat{\mu}_{B+L}}{\hat{T}} = \frac{396}{481} \frac{\hat{\mu}_{e_1}}{\hat{T}}$$

► Chiral Plasma Instability (CPI)

- **Annihilation** between the **asymmetry in e_R** and the **helical gauge field**

w/o electron Yukawa

$$\partial_\eta Q_{e_1} = -\frac{\alpha_Y}{4\pi} \int_x Y_{\mu\nu} \tilde{Y}^{\mu\nu} =: -\partial_\eta Q_{\text{CS}}$$

Initially: $Q_{e_1} = 0, Q_{\text{CS}} = 0$

During inflation: $Q_{e_1} \nearrow \& Q_{\text{CS}} \searrow \because (\partial_\eta \phi / f_a) Q_{e_1}$

After inflation: $Q_{e_1} \rightarrow 0 \& Q_{\text{CS}} \rightarrow 0$

Chiral Plasma Instability: $\hat{T}_{\text{CPI}} \sim 10^5 \text{ GeV} \left(\frac{\hat{\mu}_{e_1}^{\text{rh}} / \hat{T}_{\text{rh}}}{10^{-4}} \right)^2$

- $\hat{T}_{\text{CPI}} > 10^5 \text{ GeV}$; the asymmetry and helical gauge field annihilate. *Need Chiral MHD simulation to draw a quantitative conclusion
- $\hat{T}_{\text{CPI}} < 10^5 \text{ GeV}$; helical gauge field survives and sources B+L asym. @ EWPT

K.Kamada, A.Long 1610.03074; D.Jimenez+ 1707.07943

Summary

Scalar w/ CS-coupling + Charged Matters

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + \frac{\alpha_Y \phi}{4\pi f_a} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right\}$$

Matters charged under $\mathbf{U(1)_Y}$

$$+ \sum_{\alpha} \psi_{\alpha}^\dagger \sigma \cdot (i\partial - g_Y Q_{\alpha} A_Y) \psi_{\alpha} + \dots \right\}$$

► Production of **charged** matters & **helical** gauge field in $\dot{\phi} \neq 0$.

- **Lowest** Landau Level: **asymmetry** (e.g. B+L) \leftrightarrow **helical** gauge field

- **Higher** Landau Levels: **pair**-production via Schwinger effect

(For the homogeneous & isotropic gauge field, see Domcke, Ema, **KM**, Sato **1812.08021**.)

→ **Backreaction** from matters suppresses the gauge field.

- Axion inflation: Suppress Δ_s sourced by gauge field; how about GWs(?)

→ Primordial generation of **B+L asymmetry** & **helical** gauge field.

- Could be useful for **Baryogenesis/Magnetogenesis**

Back up

Induced Current

How does Induced Current look like?

- **Weak** electromagnetic field in thermalized plasma

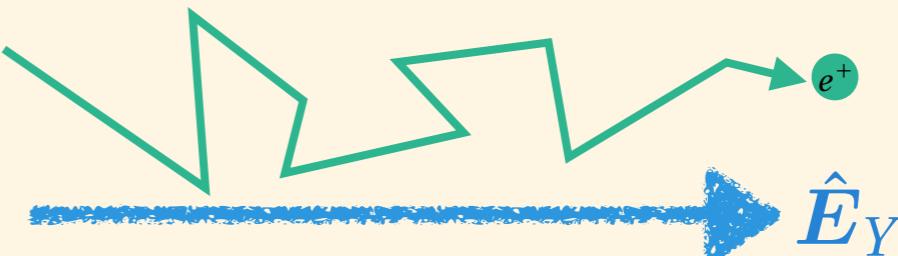
$$\hat{J}_Y = \hat{\sigma}_Y \hat{E}_Y + \frac{2\alpha_Y}{\pi} \hat{\mu}_{Y,5} \hat{B}_Y \quad \text{i.e., Generalized Ohm's law (neglecting velocity field)}$$

- **No magnetic mass** for transverse mode of Abelian gauge field.

E. Fradkin, Proc. of the Lebedev Institute 29, 6 (1965).

- This expression holds if **energy by acceleration** \ll **temperature**

$$g_Y |Q| \hat{E}_Y \hat{\tau}_{\text{int}} \ll \hat{T}$$



- **Strong** electromagnetic field ?

- Estimate the current operator for the **accelerated energy** \gg **temperature**

$$\hat{J}_Y^z = \sum_{\alpha} \frac{(g|Q_{\alpha}|)^3}{12\pi^2} \coth\left(\frac{\pi \hat{B}_Y}{\hat{E}_Y}\right) \hat{E}_Y \hat{B}_Y \frac{1}{H} \quad \rightarrow \quad \hat{J}_Y^z = \sum_{\alpha} \frac{(g|Q_{\alpha}|)^3}{6\pi^3} \frac{\hat{E}_Y^2}{H} \quad \text{for } B_Y \rightarrow 0$$

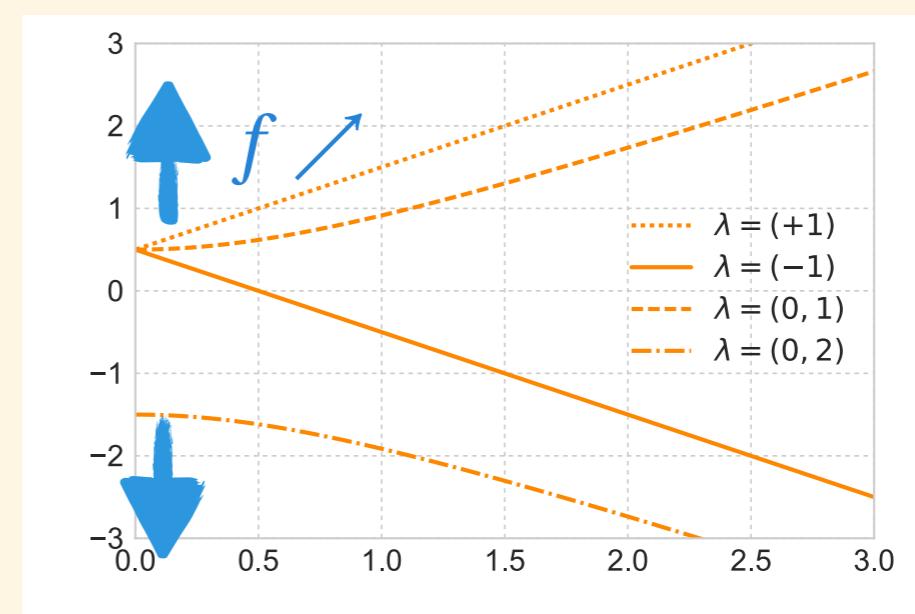
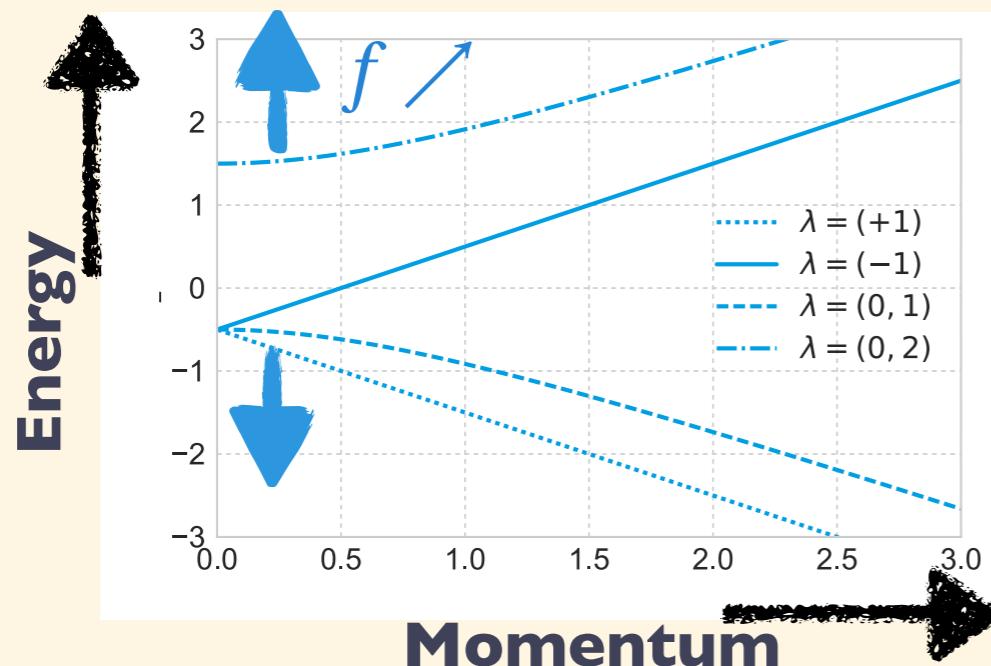
Reproduce e.g., Kobayashi, Afshordi 1408.4141

Fermion Production

Subtleties for $A_0^a = 0$, $A_i^a = -g^{-1} f(\eta) \delta_i^a$

V.Domcke, Y.Ema, **KM**, R.Sato
1812.08021

- **No Landau levels**, but we can still play the same game...



- The spectrum is different between the **initial** and **final** states.
- Need to include the contributions from **vacuum** not only **particles**.

$$Q_\alpha = \boxed{Q_\alpha} + \boxed{Q_\alpha^{(\text{vac})}} \quad \text{w/ } Q_\alpha^{(\text{vac})} \equiv \lim_{\hat{\Lambda} \rightarrow \infty} \text{vol}(\mathbb{R}^3) \int \frac{d^3 k}{(2\pi)^3} \left[-\frac{1}{2} \sum_\lambda \text{sgn}(\omega_\alpha^{(\lambda)}) R \left(\frac{|\omega_\alpha^{(\lambda)}|}{a\hat{\Lambda}} \right) \right]$$

Fundamental repr: $\frac{1}{6}$ (Anomaly) $\frac{5}{6}$ (Anomaly)

The eta-invariant; see Atiyah and Singer