

# WP 5.4: Exotic Undulators

## Microwave Undulator

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Coherent wavelength is given by

$$\lambda = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$

$\lambda_u$  is the period of the undulator

$n$  is the harmonic number

$K$  is rms undulator strength parameter

$\gamma$  is the Lorentz-Fitzgerald relativistic factor

Liang Zhang, Strathclyde, Cockcroft Institute  
PDRA, 1/04/2017 to 31/6/2019



State of the art values are (e.g. PSI Swiss FEL)

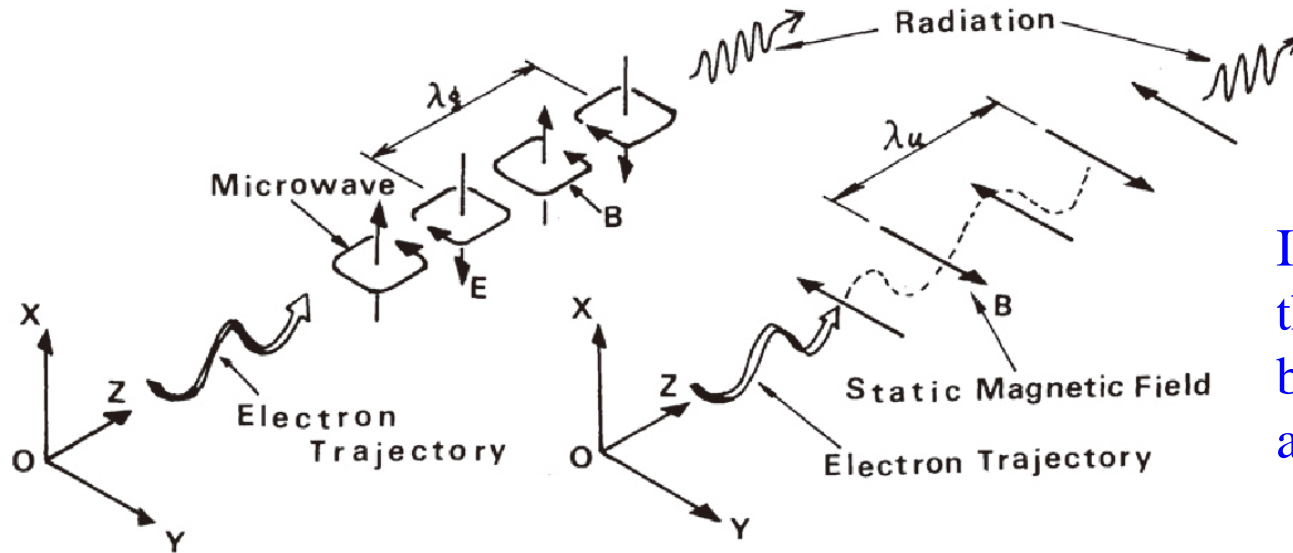
$$\lambda_u = 15\text{mm}$$

$$K = \frac{|e|\lambda_u B_0}{2\pi mc}$$

PSI undulator gap range can vary between 3mm and 6mm  
giving a tuning range for  $K$  of 1.8 to 1.0

Consequently for  $\lambda = 0.1\text{nm}$ ,  $\sim 10\text{keV}$        $E \sim 6\text{GeV}$

# Microwave undulator



(a) Microwave Undulator

(b) Magnetic Undulator

In microwave undulator, the electron bunch sees both the electric field and magnetic field.

$$E_x = E_0 \sin(2\pi z / \lambda_g) \cdot \sin(\omega t)$$

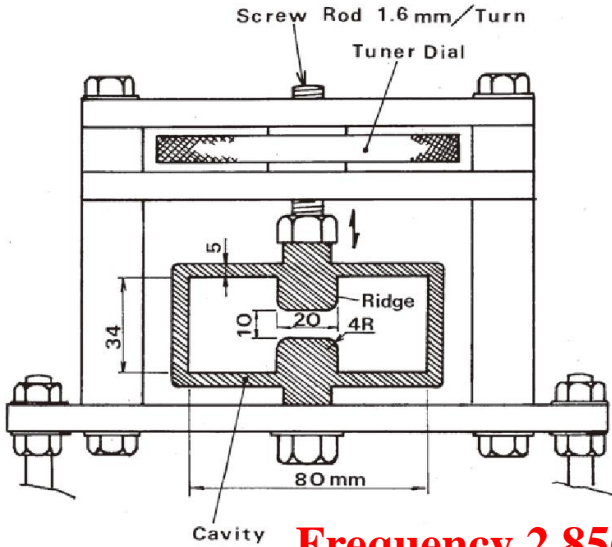
$$B_y = B_0 \cos(2\pi z / \lambda_g) \cdot \cos(\omega t) \quad B_z = B_0 \sin(2\pi z / \lambda_u) = B_0 \sin(k_u z)$$

$$F_x = -e(E_x - v_z B_y) \quad F_x = e v_z B_y$$

**Advantages:** (1) Fast dynamic control of polarization; (2) Easy to control the field strength by adjusting the input power; (3) Short wavelength; (4) Large aperture (cm vs mm); (5) Resilient to damage by radiation



# Previous experiments



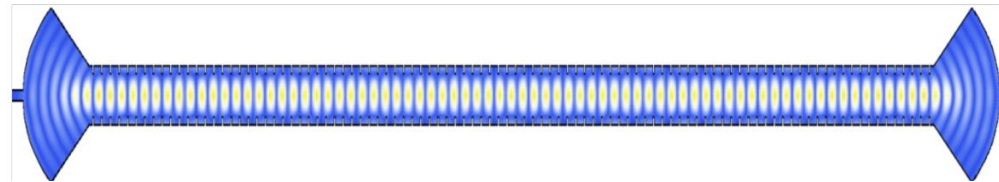
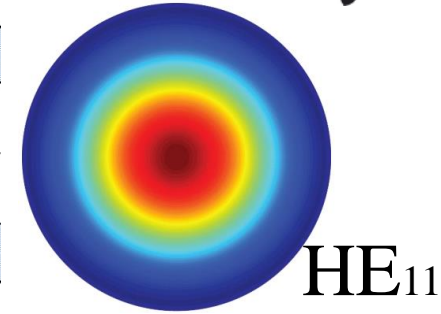
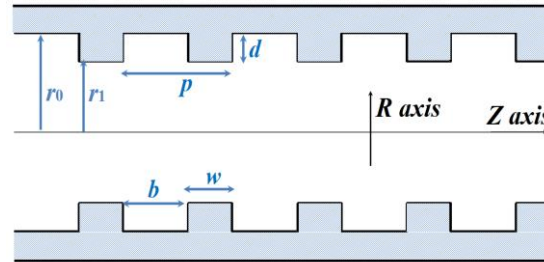
**Frequency 2.856 GHz (1983)**

Table II. Measured cavity parameters.

Quality factor	$Q_1$	7100
Transverse shunt impedance	$R_1/Q_1$	$4.34 \times 10^4 \Omega/m$
	$R_1$	308 M $\Omega/m$
Guide wavelength	$\lambda_g$	$115.56 \pm 0.78$ mm
Undulator period	$\lambda_u$	$55.01 \pm 0.19$ mm

Table IV. Microwave and undulator parameters.

Microwave power	300 kW
Pulse duration	4 $\mu$ sec
Repetition rate	10 pps
Peak electric field	12.8 MV/m
Equivalent magnetic field	430 Gauss
Undulator period	5.5 cm
K-parameter	0.24



Freq. = 11.424 GHz  
 $K = 1$  for 50 MW  
 $B_u = 0.77$  T  
 $\lambda_u = 1.39$  cm  
 $Q = 91000$  (meas.)  
 $Q = 94000$  (simu.)  
 Length = 1 meter  
**(PRL, 2014)**

Demonstration experiment at NLCTA, SLAC

# Cavity-type MU

- (1) Aiming at Ka-band (**36 GHz**), to achieve short undulator (**~ 4.4 mm**) period
- (2) Low loss HE mode; High field at the cavity center.

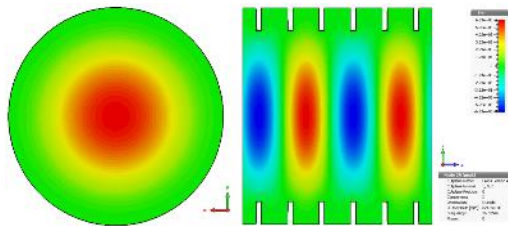
	State-of-the-art $\mu$ -wave undulator	Record breaking $\mu$ -wave undulator	Dream $\mu$ -wave Undulator
Period (mm)	13.9	13.9	4.4
Beam Aperture (mm)	5.0	5.0	5.0
Peak B Field (T)	0.92	1.62	2.0
K Parameter	1.2	2.1	0.82
Length (m)	4.0	1.0 - 4.0	1.0 - 4.0
Operating frequency (GHz)	11.424	11.424	36
Required microwave power (MW)	152	185 - 464	108 - 272
Required pulse length (us)	5.8	1.4 - 5.7	0.8 - 3.2

$$P \propto L^{2/3}$$

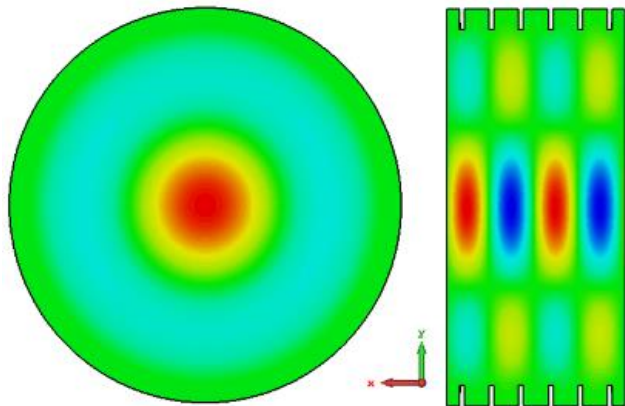
$$T \propto L$$

University of Strathclyde HPM source produced 65MW of power at 36GHz, I.V. Konoplev, A.W. Cross, et al, Appl. Phys. Letts., **92**, 211501, 2008





HE11 mode



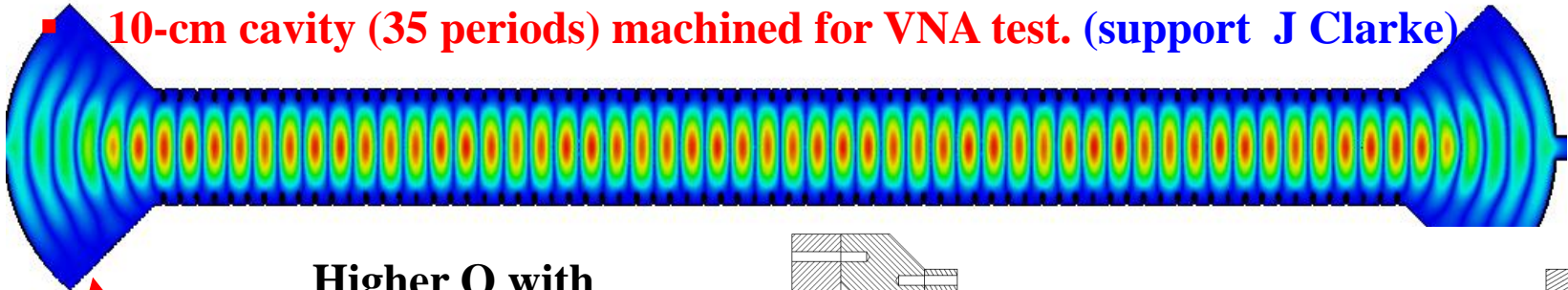
HE12 mode

Dimensions calculated  
using theoretical equations

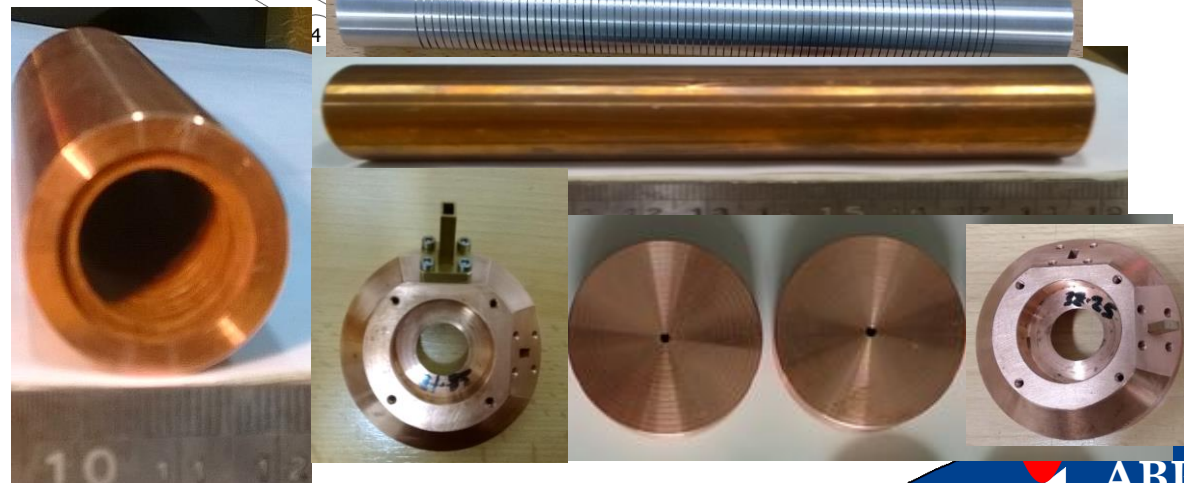
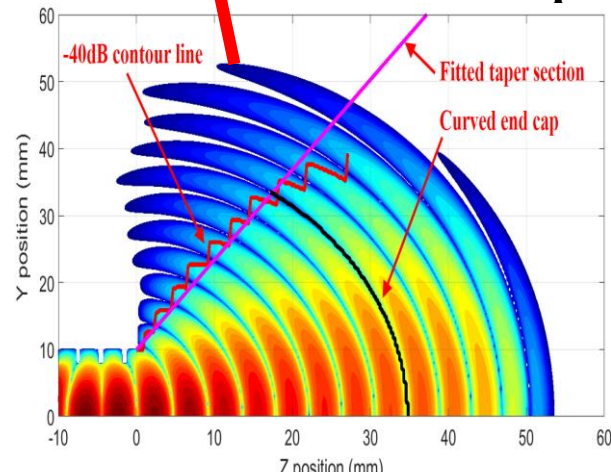
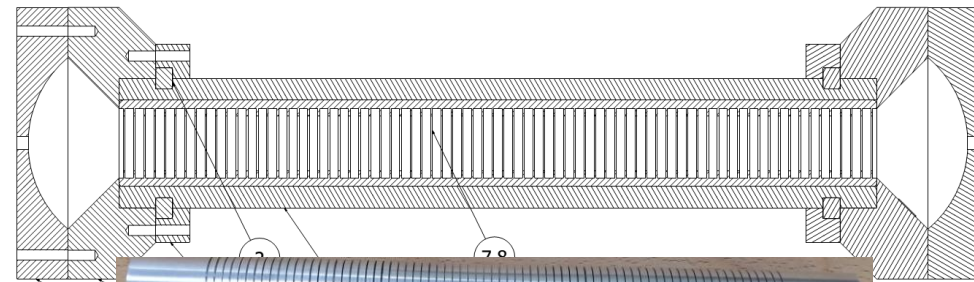
Operating mode	$HE_{11}$	$HE_{12}$
Operating frequency (GHz)	36	36
$\lambda_0$ (mm)	8.33	8.33
$R_b$ (mm)	2.0	2.0
$r_1$ (mm)	$4R_b=8.0$	$9R_b=18.0$
depth = $\lambda_0/4$ (mm)	2.1	2.1
$\lambda_g$ (mm)	9.06	9.12
$p = \lambda_g/3$ (mm)	3.00	3.02
$s$ (mm)	0.5	0.5
$b = p - s$ (mm)	2.50	2.52
Q factor	94,344	187,073
<b>Input power (MW)</b>	<b>50</b>	<b>50</b>
Peak Ex on axis (V/m)	3.8E8	3.7E8
Peak E on wall (V/m)	7.3E6	9.5E6
$B_u$ (T)	<b>1.27</b>	<b>1.23</b>
$\lambda_u$ (mm)	<b>4.34</b>	<b>4.35</b>
$K_u$	<b>0.52</b>	<b>0.50</b>

# 36 GHz corrugated cavity

- **Taper / Coupler design based on near field radiation pattern for 22-cm cavity (74 periods) (Maintain  $Q = 89648$ )**
- **An empirical formula was derived. (Allow scalable design)**
- **10-cm cavity (35 periods) machined for VNA test. (support J Clarke)**



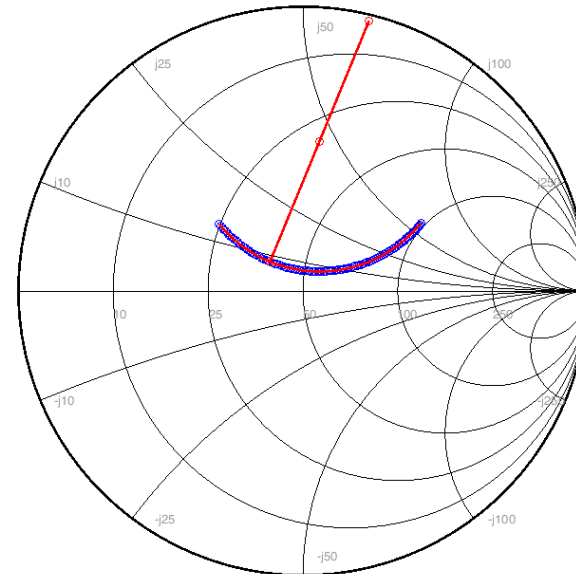
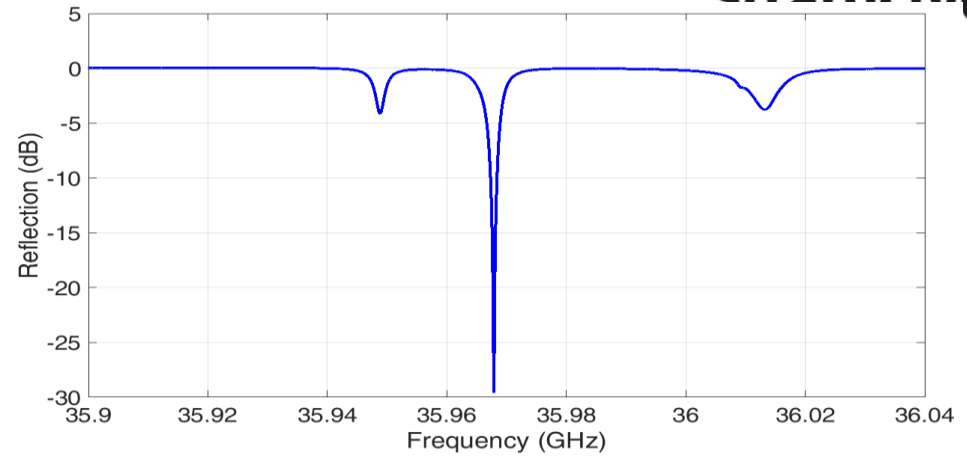
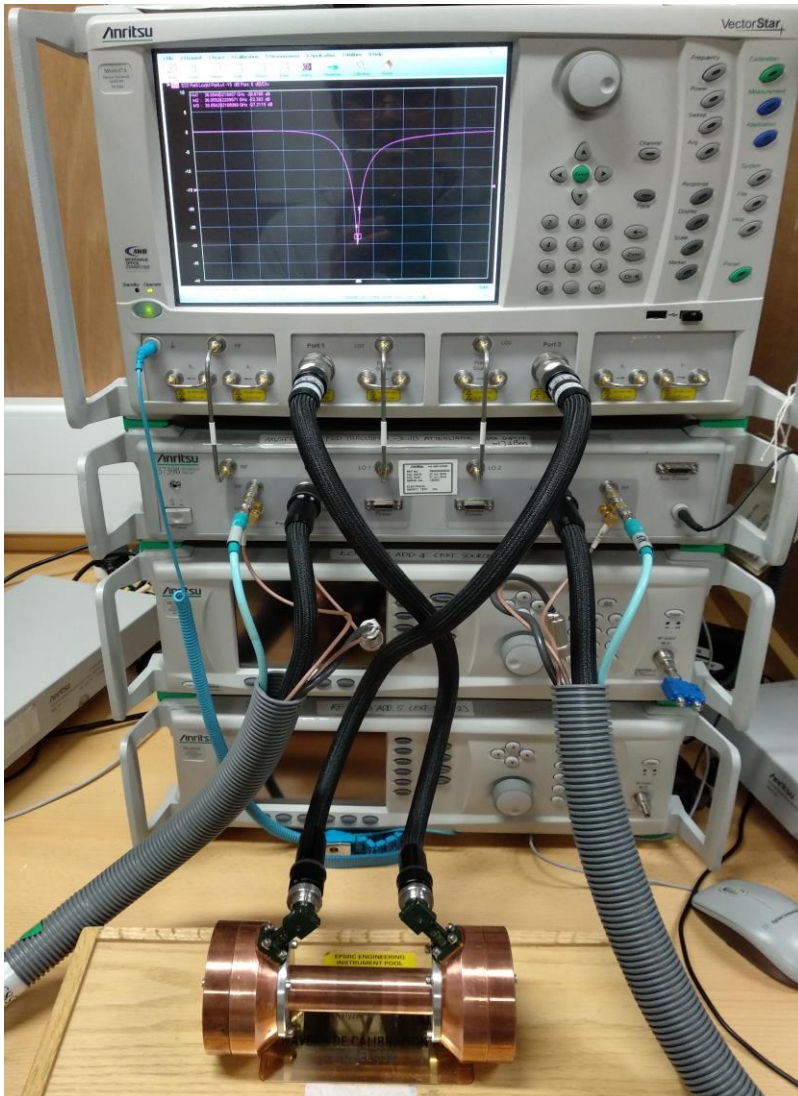
**Higher Q with more periods**



$$L(N) = \frac{\lambda}{2} N \left[ 1 + F(N) \frac{\lambda_g}{\lambda} \right]$$

$$F(N) = 0.123e^{-0.213N} + 0.015$$

# Measurement results



File name:  
Microwve Undulator.txt  
Freq= 35.968 GHz  
 $Q_0 = 3.406e+04$   
 $Q_L = 1.046e+04$   
 $\kappa = 2.256$   
 $r_s = 0.51$

- ~40% of the  $Q_0$  compared with simulation due to:
- Surface roughness (need bright dip etching)
  - Small gaps at the junctions (brazing)



Rewrite the motion of the electrons in a cavity-type microwave undulator as

$$\frac{dp_x}{dt} = \frac{eE_0}{2} \left( \frac{\zeta}{Z_w} + 1 \right) \cos \left( \omega t + \frac{2\pi z}{\lambda_g} \right) + \frac{eE_0}{2} \left( \frac{\zeta}{Z_w} - 1 \right) \cos \left( \omega t - \frac{2\pi z}{\lambda_g} \right)$$

## Backward wave

$$B_{ub} = \frac{E_0}{2c} \left( \frac{\zeta}{Z_w} + 1 \right)$$

$$\frac{1}{\lambda_{ub}} = \frac{1}{\lambda_0} + \frac{1}{\lambda_g}$$

short wavelength.

## Forward wave

$$B_{uf} = \frac{E_0}{2c} \left( \frac{\zeta}{Z_w} - 1 \right)$$

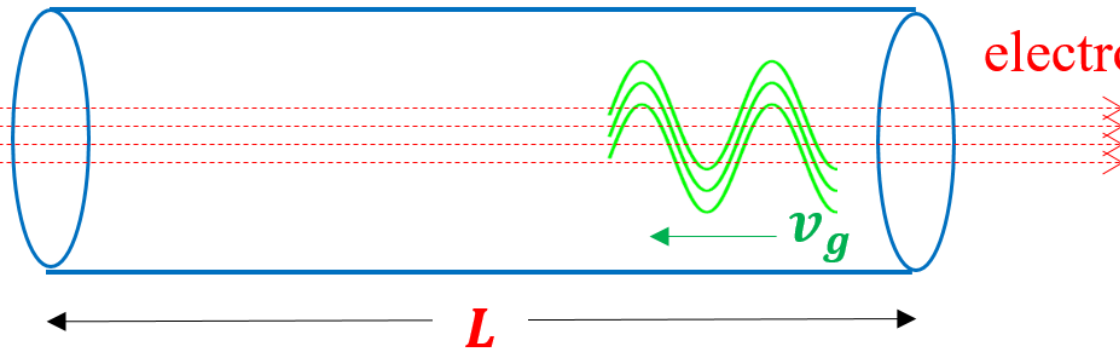
$$\frac{1}{\lambda_{uf}} = \frac{1}{\lambda_0} - \frac{1}{\lambda_g}$$

long wavelength.

Usually  $\lambda_0$  and  $\lambda_g$  are close values, therefore the backward wave is the dominant component. The impact of the forward wave can be minimized to operate the microwave undulator far away from the cutoff frequency.

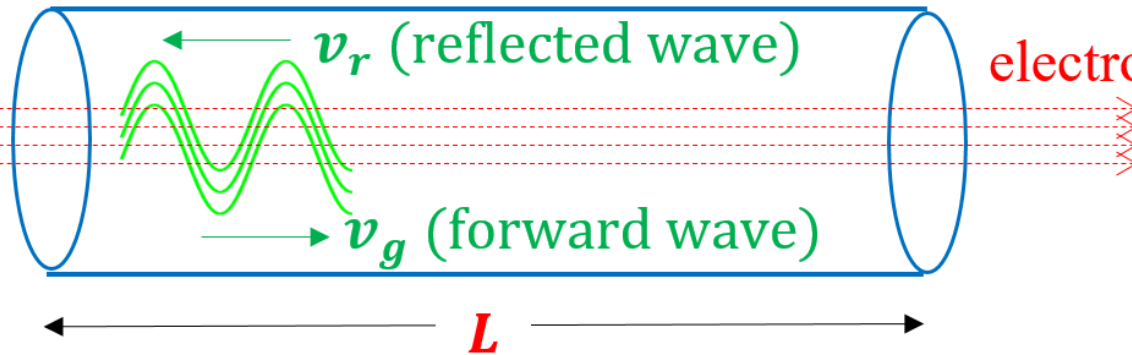
# “Flying” undulator

A waveguide can be used as an undulator instead of a cavity structure. The wave can co-propagate or counter-propagate with the electron beam.



electron beam  $v_e$

$$L_{eff} = \frac{v_{gr}\tau}{1 + v_{gr}/c}$$



electron beam  $v_e$

$$L_{eff} = \frac{v_{gr}\tau}{1 - v_{gr}/c}$$

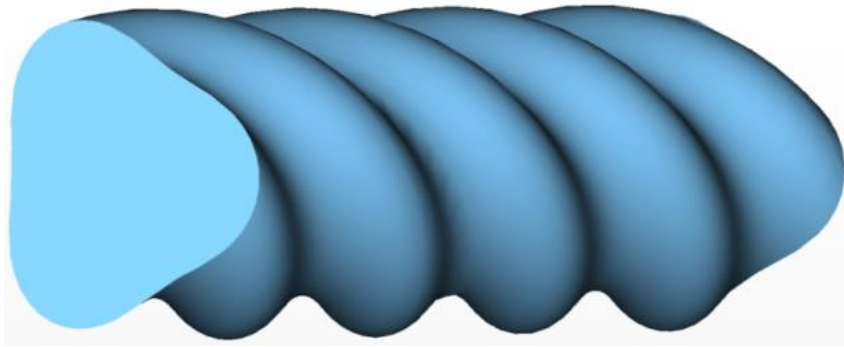
**A co-propagating wave with backward group velocity has longer effective interaction length.**

[1] S. V. Kuzikov, et al., "Flying radio frequency undulator," Appl. Phys. Lett., vol. 105, no. 3, p. 033504, 2014.

[2] S. V. Kuzikov, et al., "Configurations for short period rf undulators," Phys. Rev. S.T., vol. 16, no. 7, p. 070701, 2013.

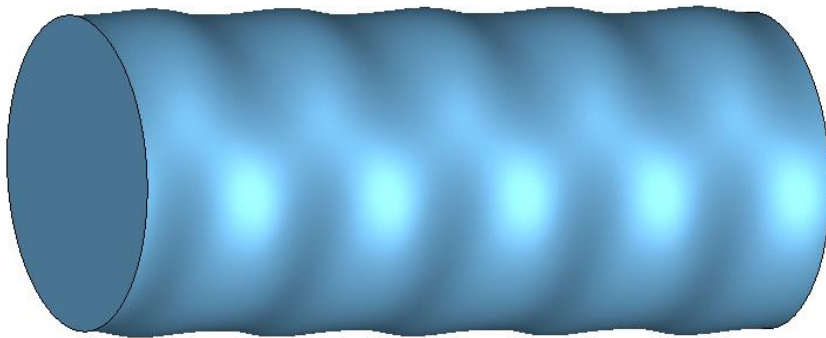
# “Flying” undulator

“flying” undulator by helically corrugated waveguide



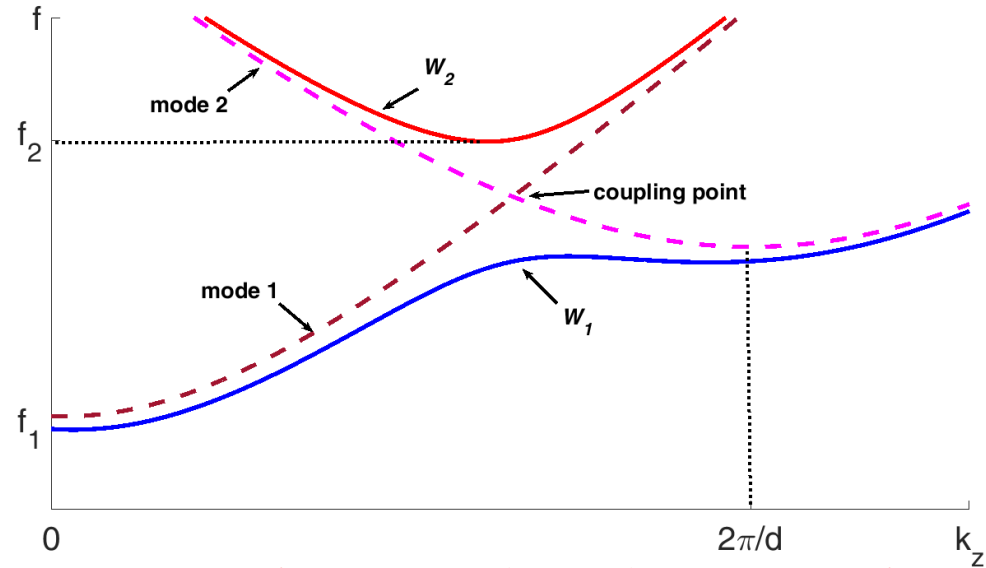
$$r(\theta, z) = R_0 + R_1 \cos(m_B \theta + 2\pi z/d)$$

**Circular polarization**



$$r(\theta, z) = R_0 + R_2 \cos(m_B \theta) \cos\left(\frac{2\pi z}{d}\right)$$

**Linear polarization**



**The coupling modes 1 and 2 have opposite directions of group velocities.**

Cavity-type	Flying-type
energy per pulse (~200J)	energy per pulse (~1J)
High power, long pulse microwave source (50 MW, 2 us), PRF (~50Hz)	Ultra high power Short pulse microwave source (1 GW, 1 ns), high PRF 1kHz



# Design of “Flying” undulator

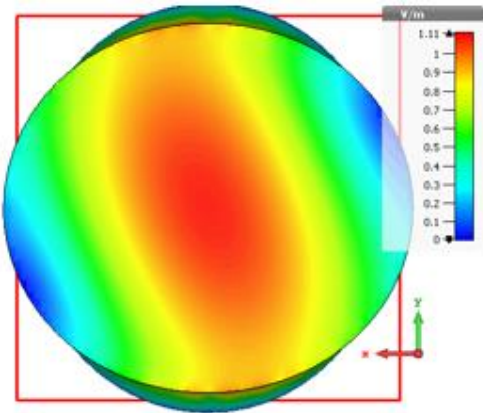


The Cockcroft Institute  
of Accelerator Science and Technology

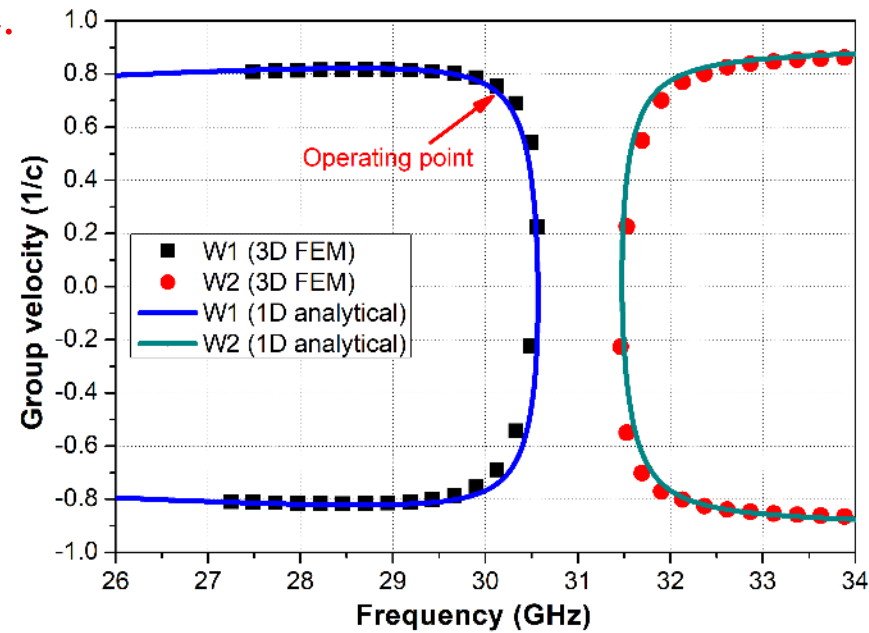
Operating mode: TE<sub>11</sub> coupled with TE<sub>11</sub> mode.

Operating frequency: 30.3 GHz

The dimensions are chosen from the dispersion relation of the operating mode.



$$R_0 = 5.8 \text{ mm},$$
$$R_2 = 0.3 \text{ mm},$$
$$d = 5.6 \text{ mm}$$



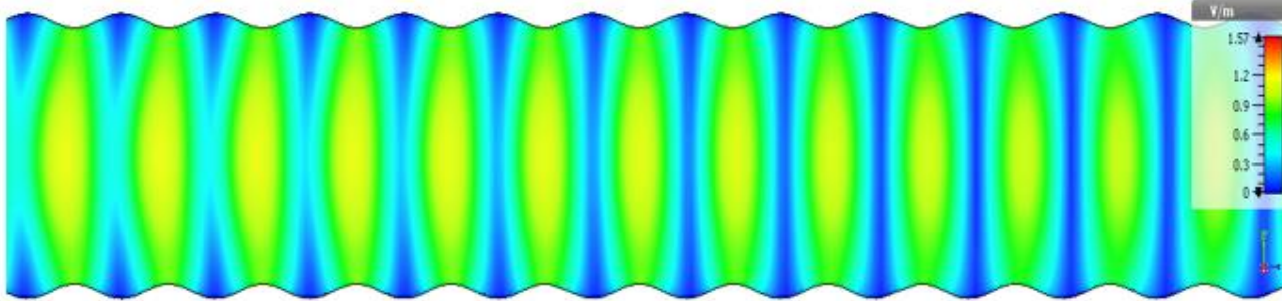
$$v_g \approx 0.6c$$

**1 GW input power:**

$$B_u = 0.3 \text{ T}$$

$$\lambda_u = 4.95 \text{ mm}$$

$$K = 0.14$$

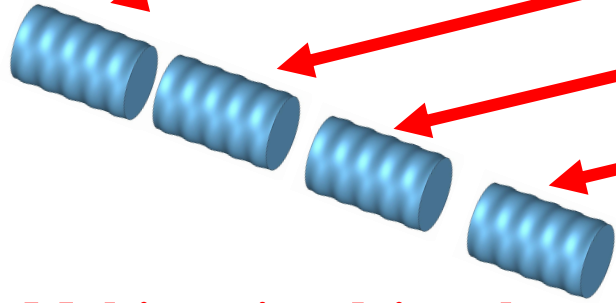
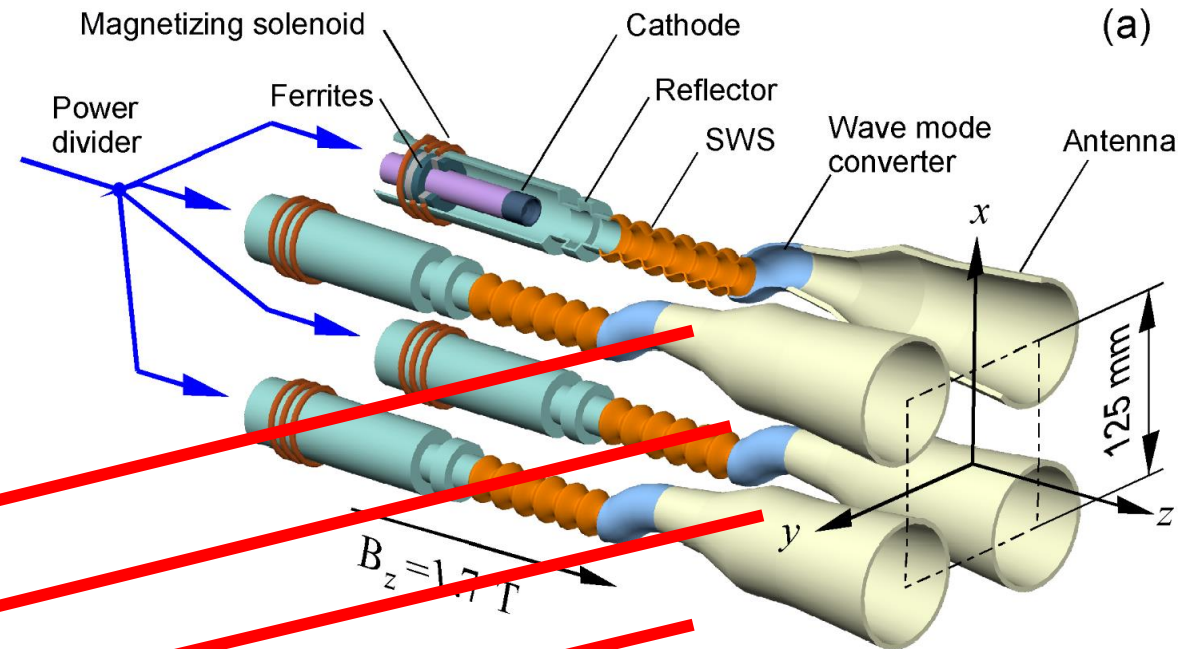


Possible to be driven by Short pulse BWO (0.6 GW, Ka band)

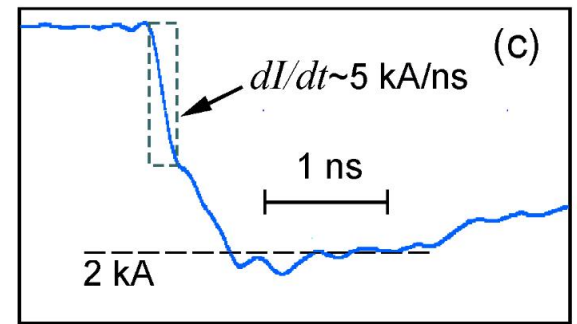
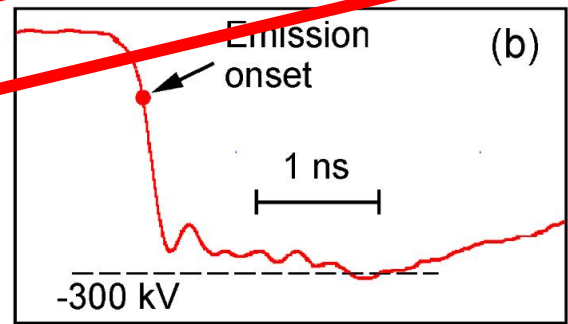
L. Zhang, W. He, J. Clarke, K. Ronald, A. D. R. Phelps, and A. W. Cross, “Microwave undulator using a helically corrugated waveguide” *IEEE Trans. Electron Device*, vol. 65, no. 12, 2018.

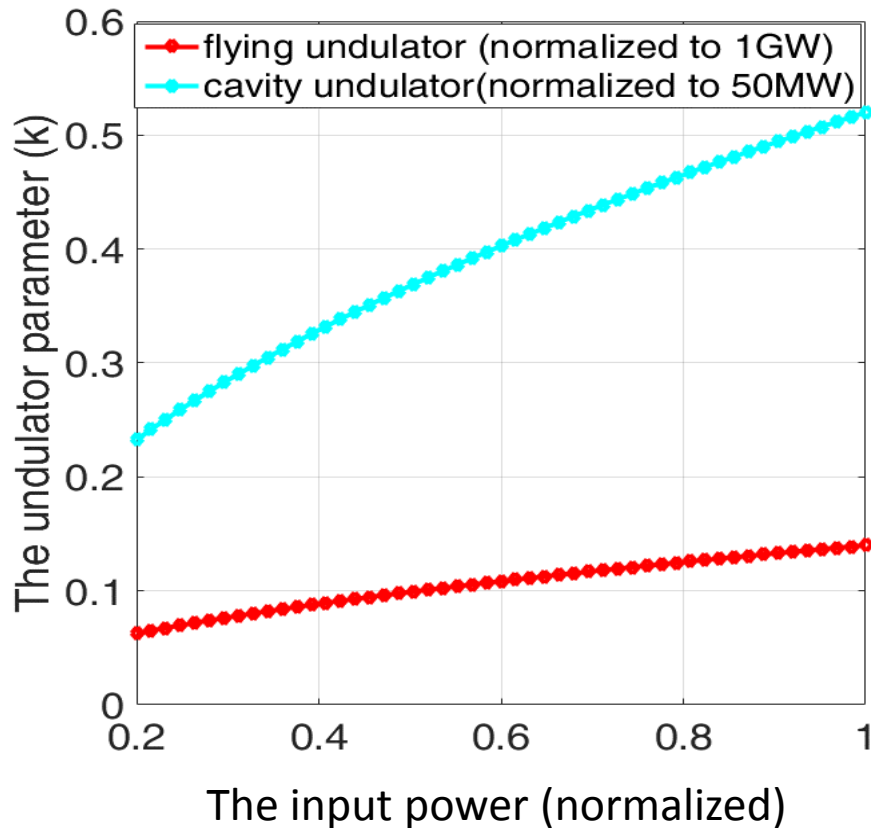
- Principal scheme of four-channel Ka-band backward wave oscillator. (b) Accelerating voltage pulse applied to each cathode in the experiment. (c) Electron beam current recorded at the entrance of interaction space, corresponding to the cross-section  $z=3.5$  cm

(a) Layout of the four-channel Ka-band generator;  
(b) accelerating voltage pulse applied to each cathode;  
(c) e-beam current recorded at the entrance of SWS.



**Multi-section driven by multiple SR sources!**





## Microwave undulator

### Advantage:

- Small undulator period ( **$\sim 4.4\text{mm}$** ) enables high energy X-ray radiation
  - $K$  peak value 0.5 for 50MW, 36GHz
  - $K$  rms value 0.7 for 100MW, 36GHz

### Challenges:

- Cavity undulator has a good undulator strength parameter  $K=0.5$  for 50MW undulator of period  **$\sim 4.4\text{mm}$**
- Flying undulator has a lower undulator strength parameter  $K=0.14$

	Cavity-type undulator	Flying-type undulator
Power supply	High energy per pulse ( $\sim 200\text{J}$ )	Low energy per pulse ( $\sim 1\text{J}$ )
Microwave source	High power, long pulse microwave source Co-axial Gyro-klystron or Magnicon (50 MW, 2 $\mu\text{s}$ ), PRF ( $\sim 50\text{Hz}$ )	Ultra high power Short pulse microwave source (1 GW, 1 ns), high PRF 1kHz

- Two types microwave undulators have been investigated and designed.
- A cavity type microwave undulator has been machined and cold tested using Vector Network Analyser
  - Bead push-pull experiment, Louise Cowie, ASTeC, Daresbury
- Electron beam dynamics are being simulated
  - D. Zhu, Astra for beam dynamics and Spectra for photon output
  - N. Thompson & D. Dunning, ASTRA, Python, Genesis2
- Microwave undulators are at an R&D stage
  - a proof-of-concept experiment is proposed for Cockcroft core grant renewal (2021) using FEBE (0.2GeV or 1GeV) at Daresbury labs

## Acknowledgements

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# Thank you!

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