Quantum vortex in neutron star's crust at finite temperatures

D. Pęcak[1, 2,], N. Chamel[3], P. Magierski[1, 4], and G. Wlazłowski[1, 4]

[1]Warsaw University of Technology, PL; [2]Institute of Physics, Polish Academy of Sciences, PL; [3]Université Libre de Bruxelles, BE; [4]University of Washington, USA

daniel.pecak@pw.edu.pl   danielpecak.github.io

Funded by the European Union

Abstract

MORE DETAILS

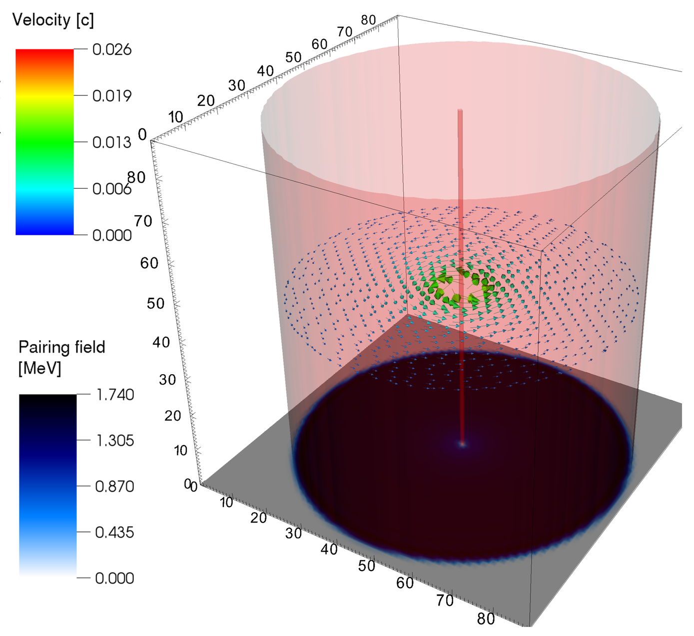
PRC 104, 055801 (2021)

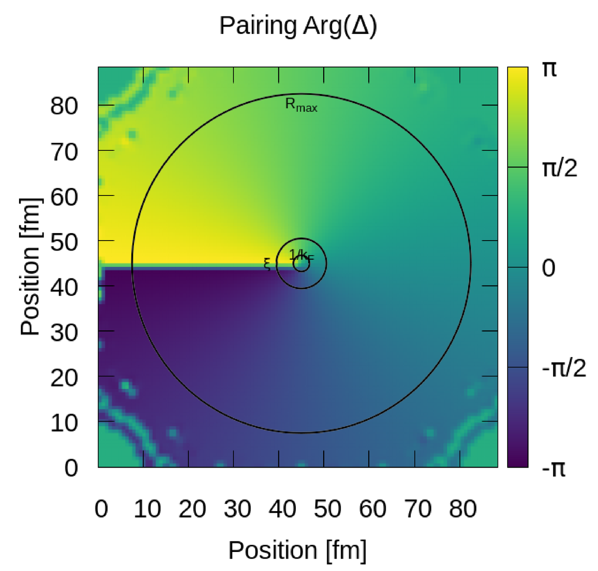
Bardeen–Cooper–Schrieffer theory explains how the heat capacity of the superfluid vanishes when the temperature approaches zero. Various mechanisms may suppress the pairing gap in the superfluid, leading to an increased heat capacity. In turn, this may translate to changing the cooling rate and the thermal evolution of neutron stars. The presence of a vortex in a superfluid neutron matter will add extra degrees of freedom in which the energy is stored, hence contributing to the heat capacity. From fully microscopic simulations, employing Superfluid Local Density Approximation (SLDA), it is possible to calculate the finite-temperature energy of the system. We use Brussels-Montreal type energy density functional, a very accurate nuclear functional designed to agree with existing astrophysical constraints. Using this state-of-the-art functional, we estimate the change in the heat capacity that results from the mere existence of a vortex in the system.

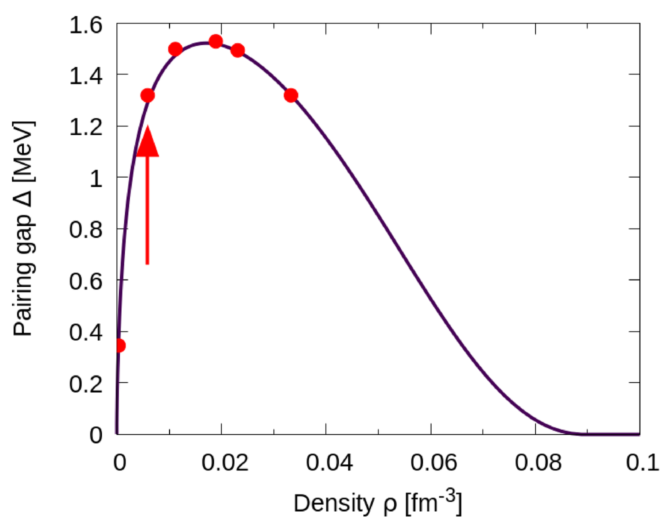
System

A typical setup with arrows representing currents, and the red contour a place where the pairing field vanishes.

- grid: 60x60x60 (120x120x24)
- lattice spacing: 1.5 fm
- periodic boundary conditions
- external potential: a tube
- imprinting phase of Δ for a vortex
- $N = 216 - 24000$
- $\rho = (0.000284 - 0.0316)\mathrm{fm}^{-3}$







Brussels-Montreal family of density functional

Experimental data

- atomic masses
- nuclear charge radii
- symmetry energy
- incompressibility

N-body calculations

- EoS of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter
- stability against spin and spin-isospin fluctuations

Numerical method

Quality of results highly depends on the quality of density functional:

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu),$$

which is used to calculate mean-field potentials and densities in a self-consisted way **without** any geometrical constraints.

Densities

$$\rho(r) = \sum_k |v_k(r)|^2$$
$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$
$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

Mean fields

$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla\frac{\delta\varepsilon}{\delta\tau}\nabla - \frac{i}{2}\left\{\frac{\delta\varepsilon}{\delta\boldsymbol{j}}, \nabla\right\}$$
$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

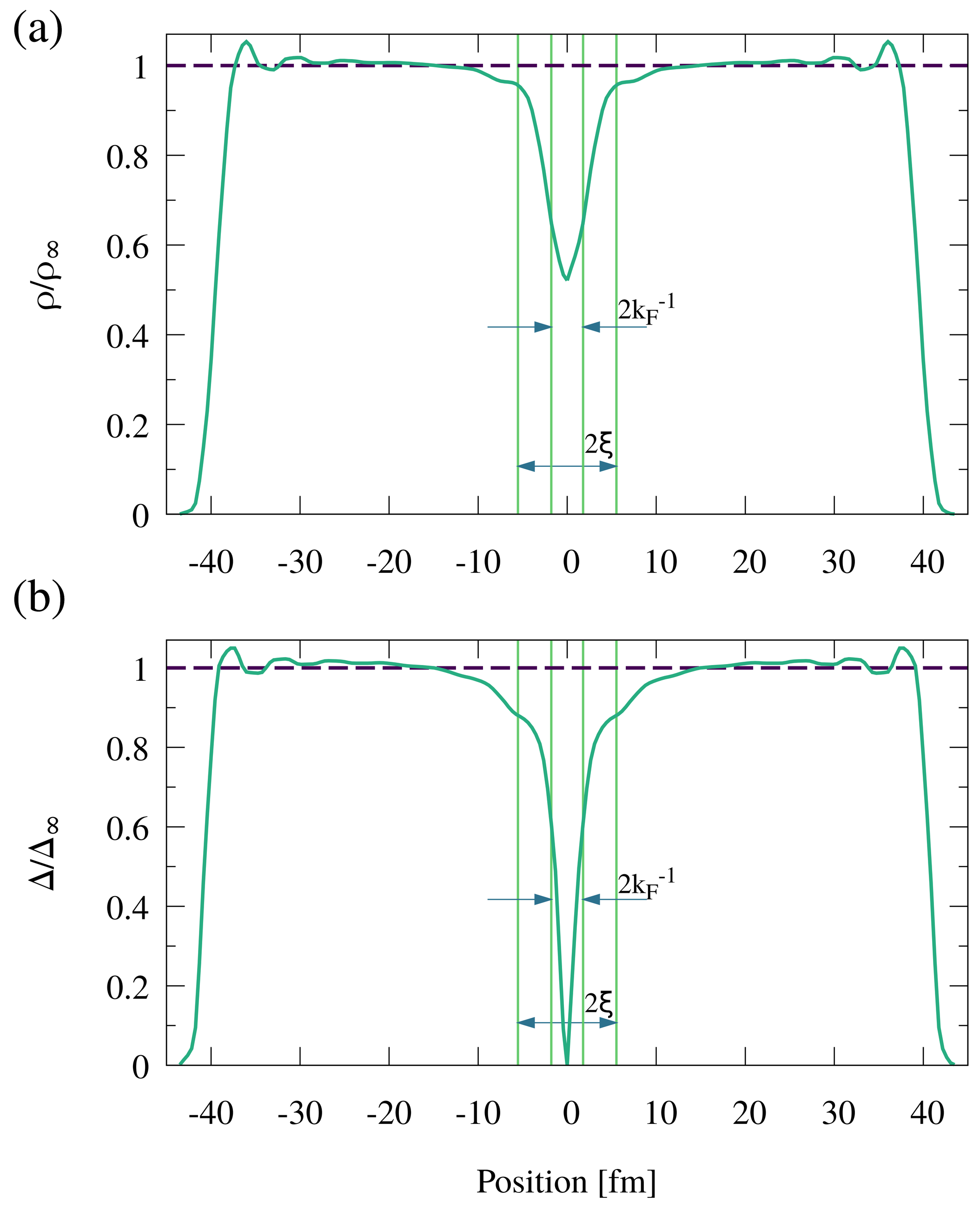
Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix}\begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

Superfluid **L**ocal **D**ensity **A**pproximation
A. Bulgac, Physical Review A **76**, 040502 (2007)

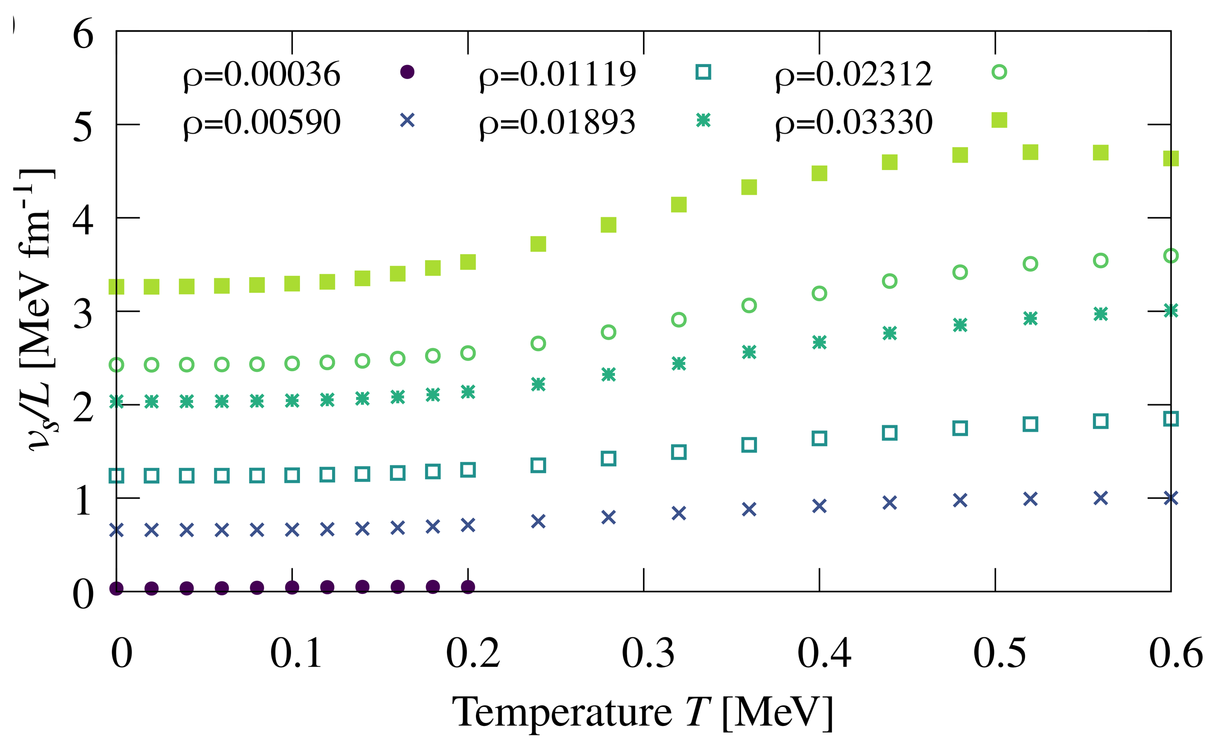
Length scales

Length scales in the system are set by the coherence length and inverse Fermi wave vector. Due to the fermionic nature of the superfluidity, the vorex core is in a normal state and surrounded with superfluid.



Tension

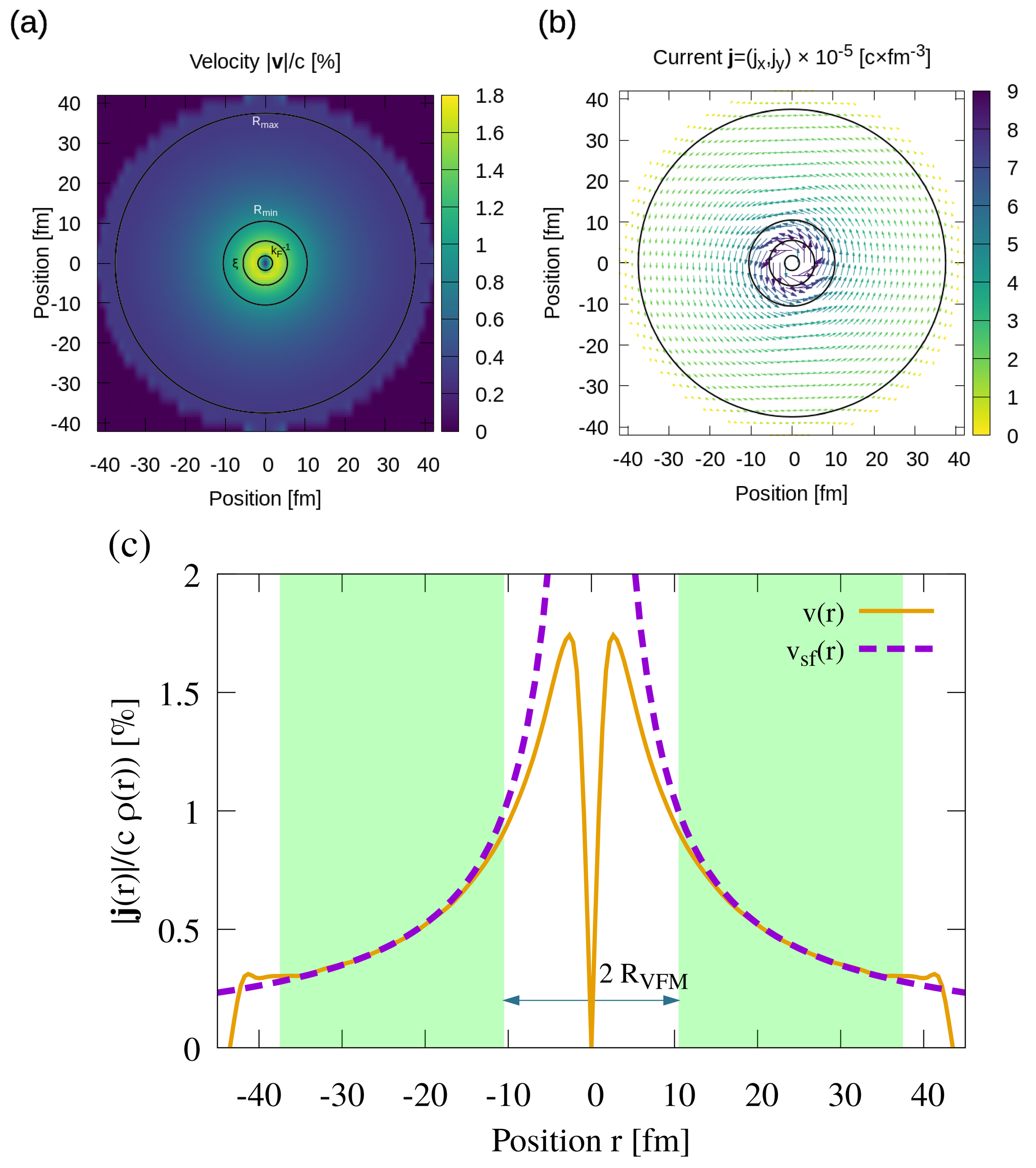
The vortex tension depends on temperature and is one of the parameters important for the **mesoscopic** models that be extracted from **microscopic** simulation.



Velocity

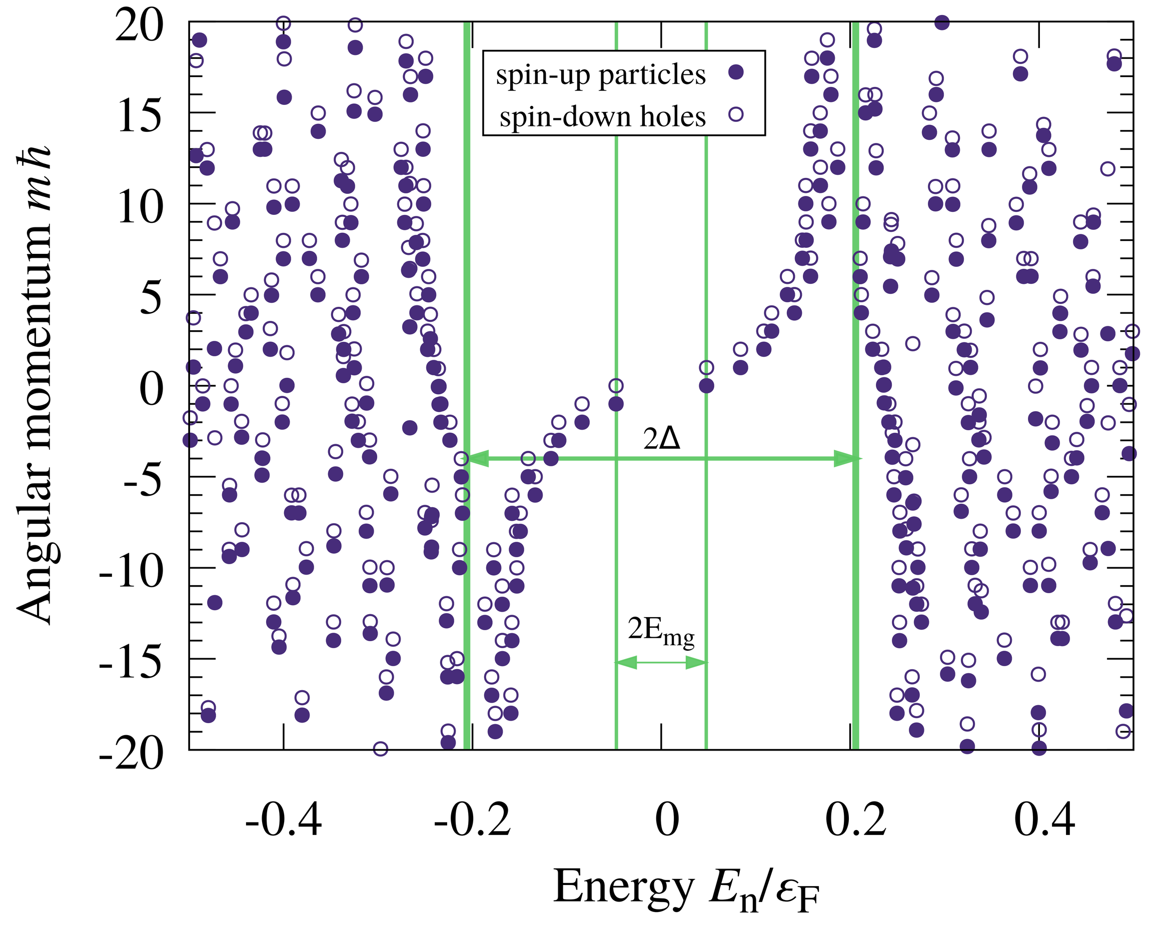
Effective size of the vortex core is one of the parameters of the Vortex Filament Model, for which there is no experimental data for nuclear matter. This length is associated with the distance from the core at which the finite-size effects does not play any role. We find this distance by comparing velocities of an ideal vortex $v_{\mathrm{sf}}(r)$ with a realistic one $v(r)$:

$$v_{\mathrm{sf}}(r) = \frac{\hbar c}{2m_n c^2}\frac{1}{r} \qquad v(r) = \hbar\frac{j(r)}{\rho(r)}$$

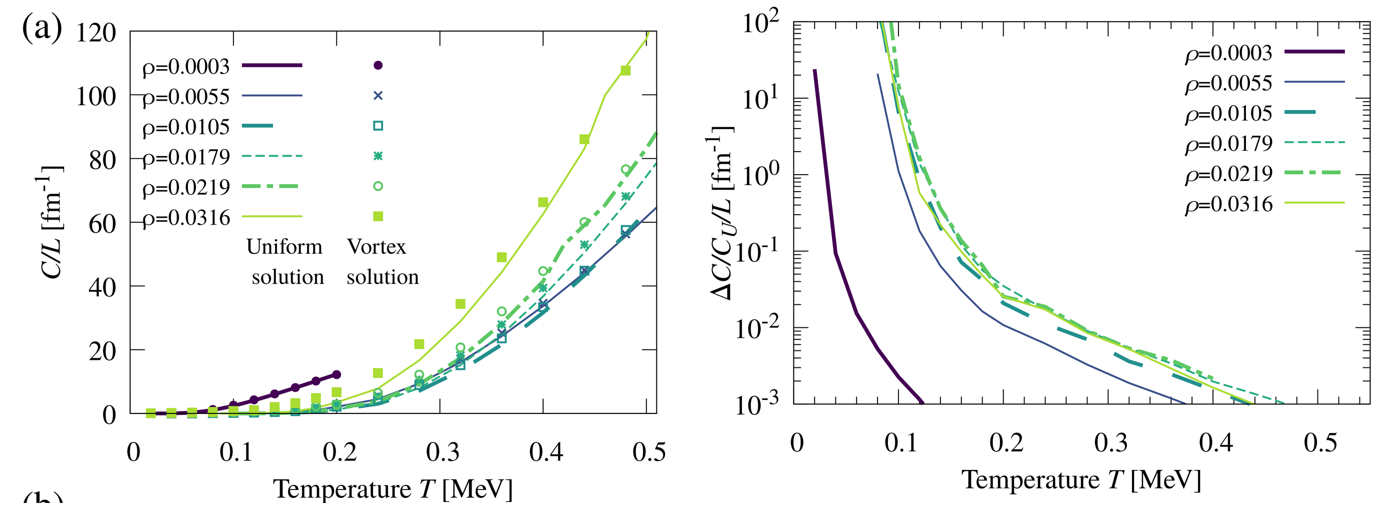


Heat capacity

The contribution to the heat capacity from the vortex core is connected to the localized so-called Caroli–de Gennes–Matricon in-gap states.



By finding the energy of a system with and without vortex as a function of temperature, one can find the heat capacity per unit of length. Note that the physical significance comes from comparizon to the heat capacity of a unifom system.



The heat capacity ratio of the considered system, compared to the uniform system shows that the increase of the heat capacity might be very large for high vortex areal densities. While the areal density of vortices in neutron stars are rather low, this contribution to the heat capacity might be important in other systems like liquid helium or ultracold atoms.

The approximate contribution of the heat capacity from the flow around the vortex as a function of temperature T, chemical potential μ and pairing Δ

$$\frac{\Delta C_V^{flow}}{L} = \frac{\hbar^2}{24M\Delta}\pi \ln\left(\frac{R_{out}}{R_{in}}\right)\frac{\mu}{\Delta}\left(\left(\frac{\Delta}{T}\right)^2 - 4\left(\frac{\Delta}{T}\right) + 2\right) C_V^{uniform}$$

Summary

- fully self-consistent 3D HFB calculations
- BSk31 Energy Density Functional adopted
- effective radius relevant for Vortex Filament Model
- superfluid fraction drops in the vortex core
- heat capacity contributions
- specific heat does not grow linearly as expected
- starting point for dynamics