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Variance-based sensitivity analysis in the r-process studies and a scalable extension

Yukiya Saito
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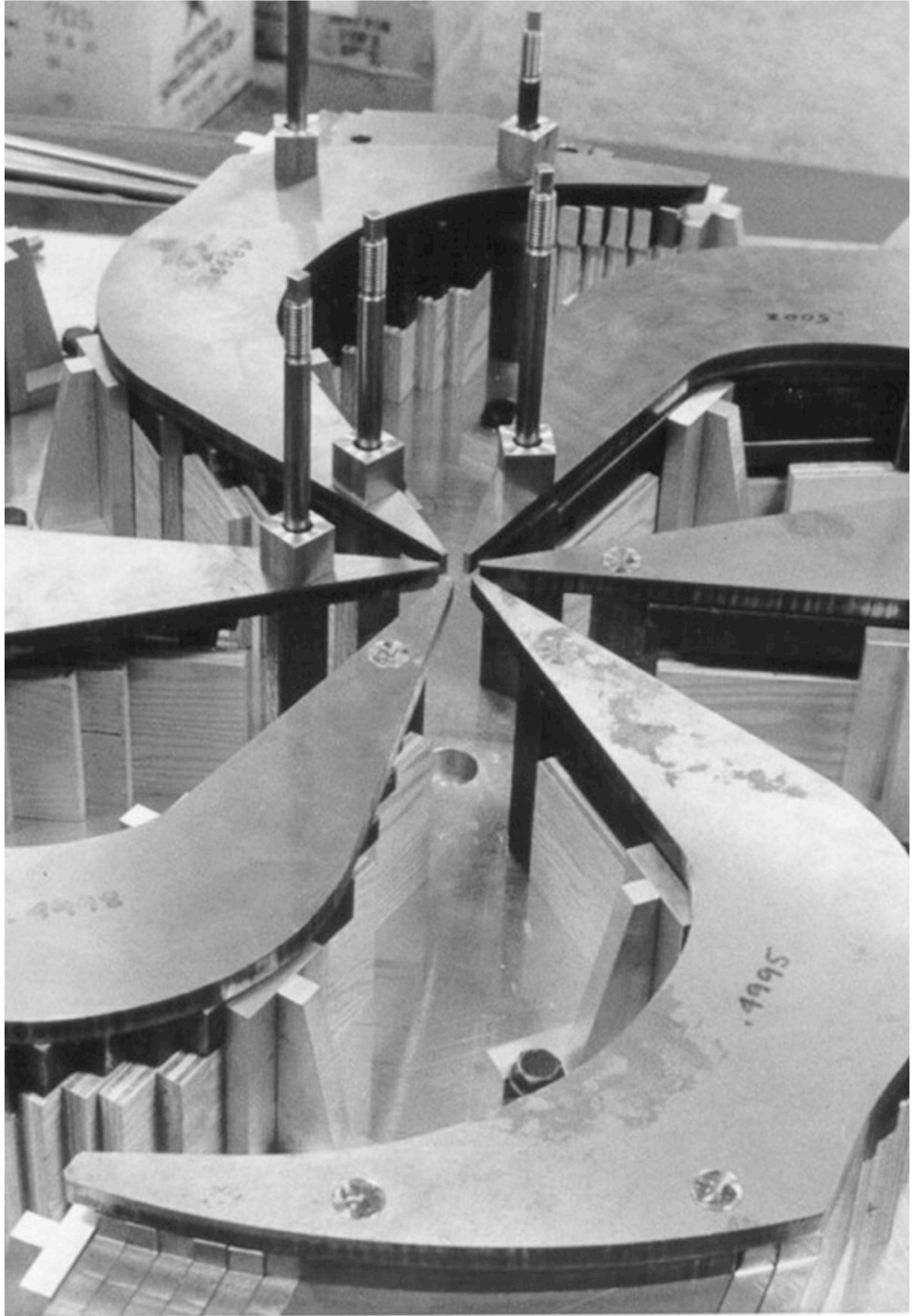
The University of British Columbia

NPA-X
2022-09-07



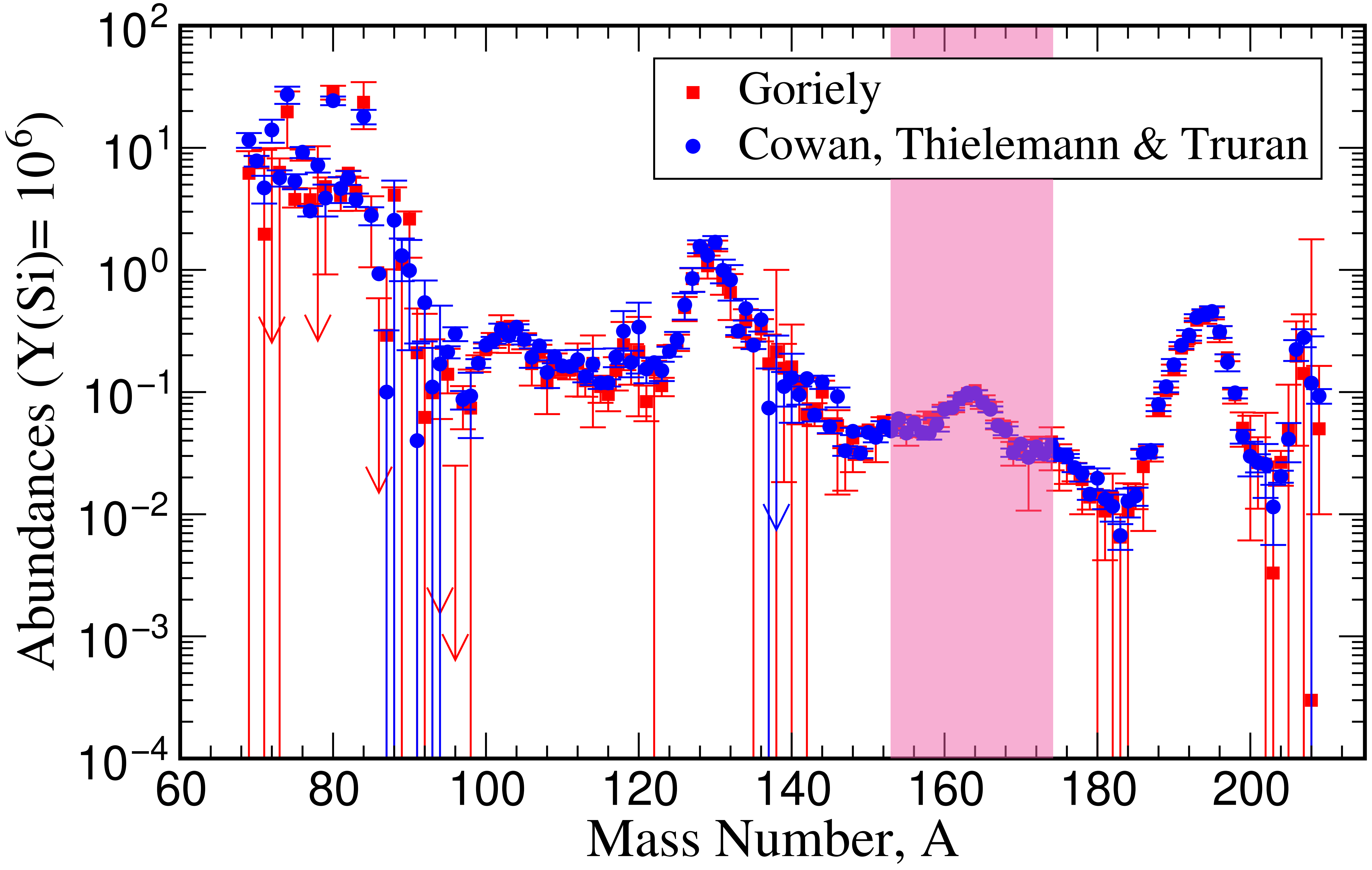
Discovery, accelerated

Rare-Earth Peak in the *r*-process

- Rare-earth peak (REP) forms during the “freeze-out” of the r-pcoess

- Details of nuclear physics inputs affect the features of the peak
  - Competition between β-decay and n-capture
  - β-delayed n-emission provide extra neutrons during freeze-out
  - Fission

- Recent β-decay experiment by the BRIKEN collaboration

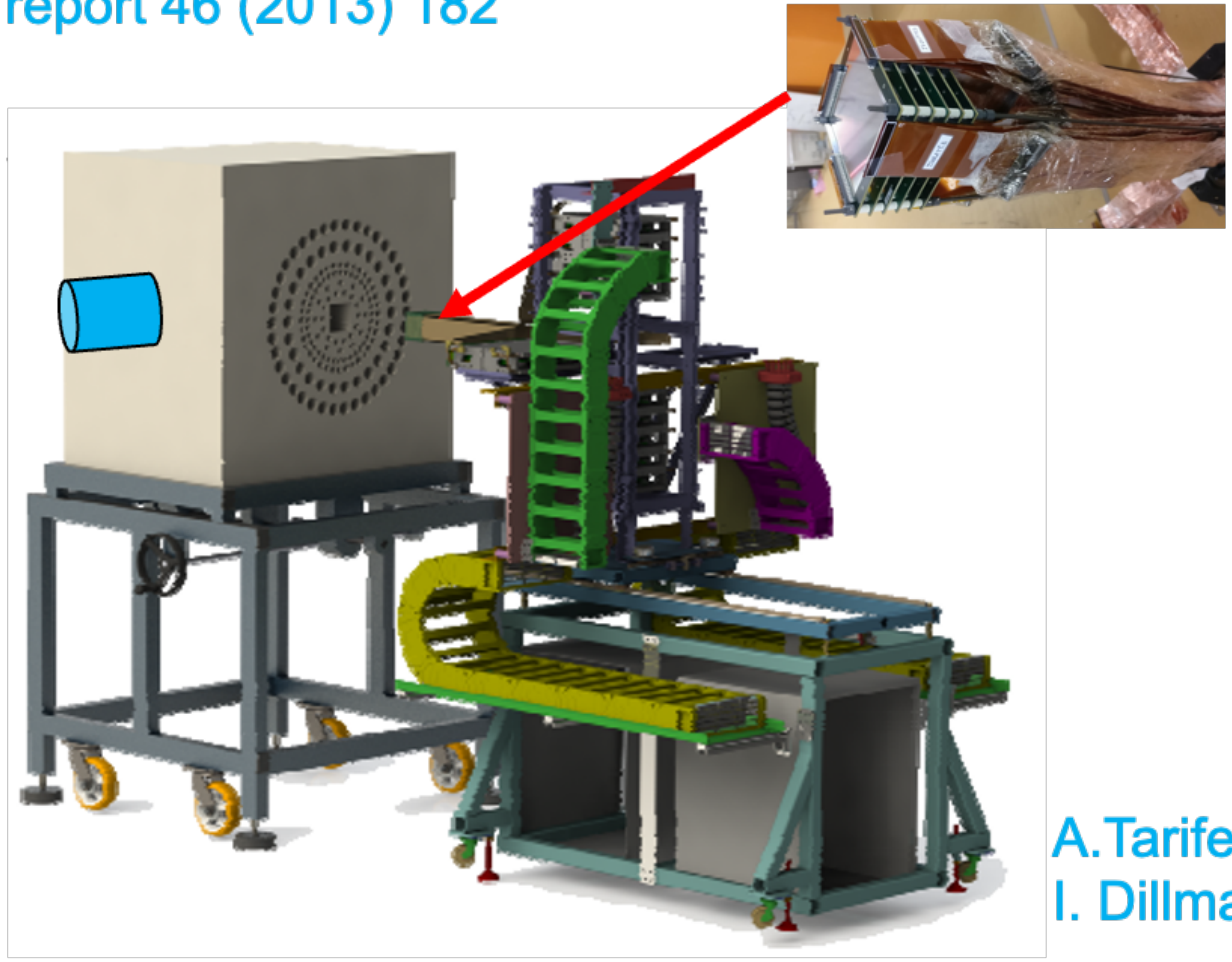


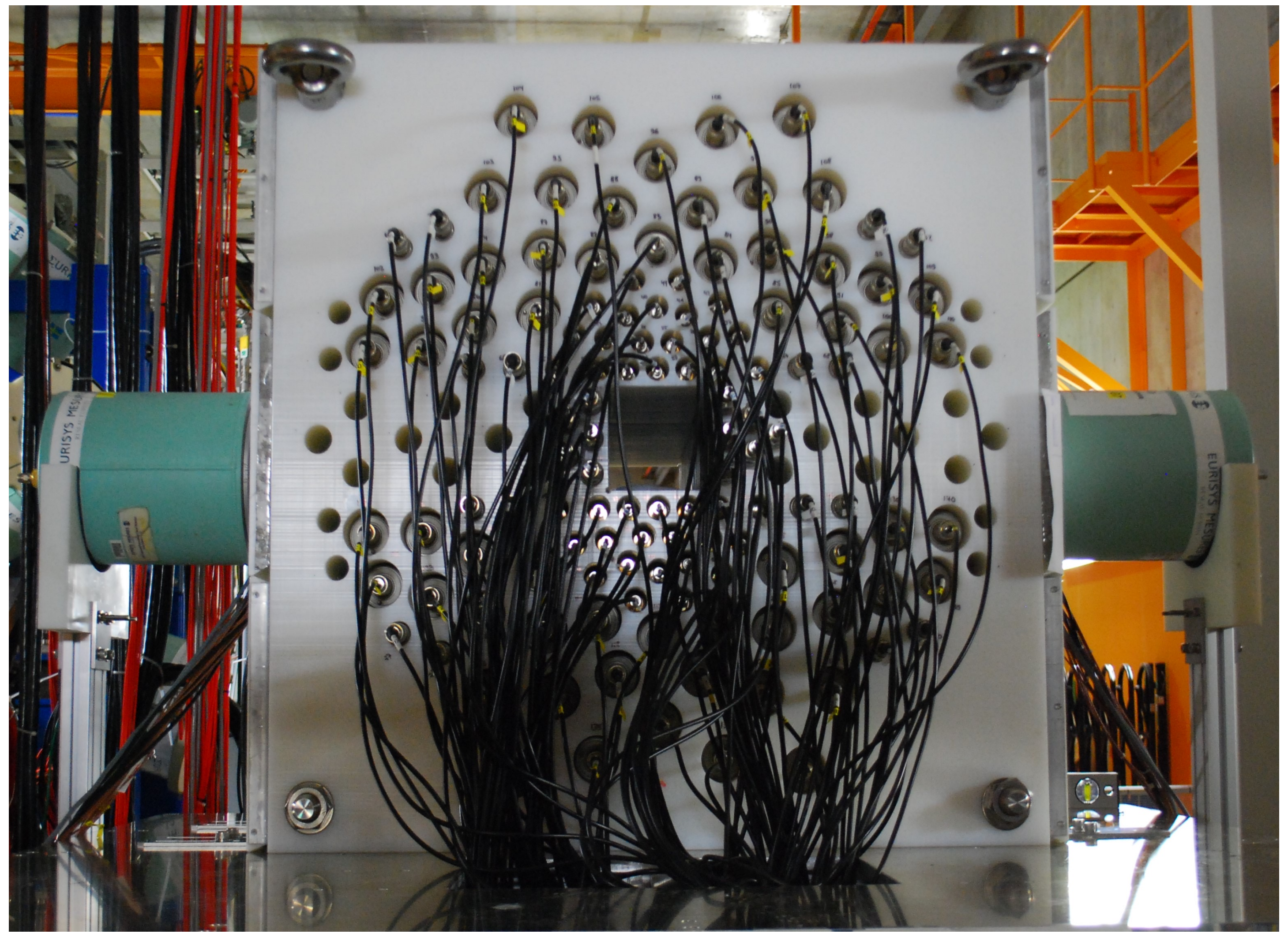
From Cowan et al. (2021), Rev. Mod. Phys. **93**, 015002

BRIKEN Experimental Setup

Advanced Implantation Detector Array (AIDA) – Univ. of Edinburgh, UK.
https://www2.ph.ed.ac.uk/~td/AIDA/

WASABI detector (RIKEN)
S.Nishimura et al. RIKEN Accel. progress report 46 (2013) 182





140 ^{3}He counters in 7 rings.
BRIKEN (2016…)

$$\varepsilon_{1n} \approx 68.6\%$$

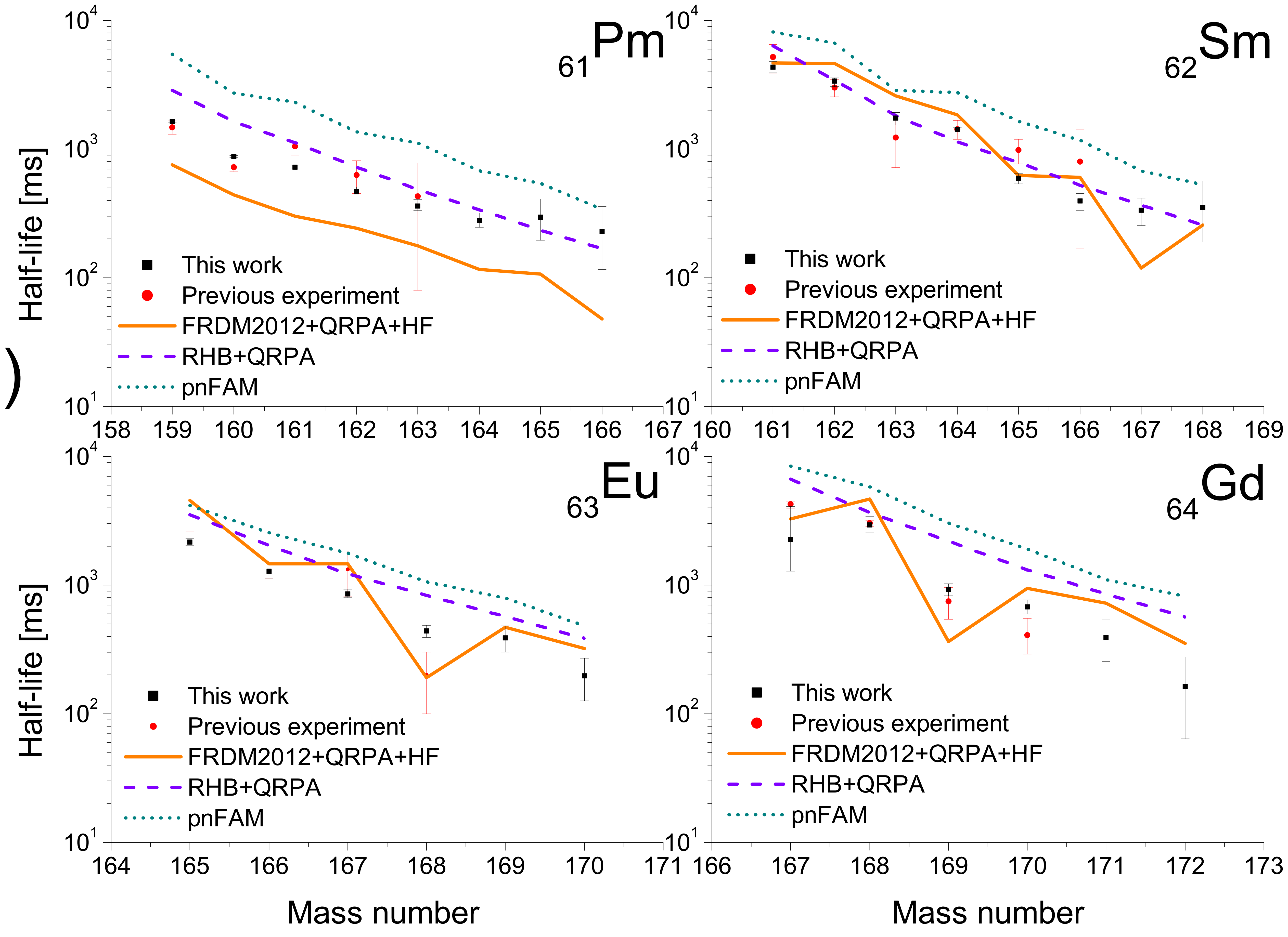
A.Tarifeño-Saldivia, et al, Journal of Instr. 12, P04006 (2017).
I. Dillmann and A. Tarifeño-Saldivia, Nucl. Phys. News 28,28 (2018).

Slide courtesy of
R. Caballero-Folch (TRIUMF)
& C. J. Griffin (TRIUMF)

For more details about BRIKEN, see talk by Alvaro Tolosa Delgado and poster by Max Pallas I Solis

Experiment and analysis overview

- P_{1n} and $T_{1/2}$ of $^{159\text{–}166}$Pm, $^{161\text{–}168}$Sm, $^{165\text{–}170}$Eu, and $^{167\text{–}172}$Gd (28 isotopes, **9 new** $T_{1/2}$ and **all new** P_{1n})
- Data analysis led by G. Kiss and A. Vitéz-Sveiczer (ATOMKI)



G. Kiss, A. Vitéz-Sveiczer, YS, et al., APJ (2022)

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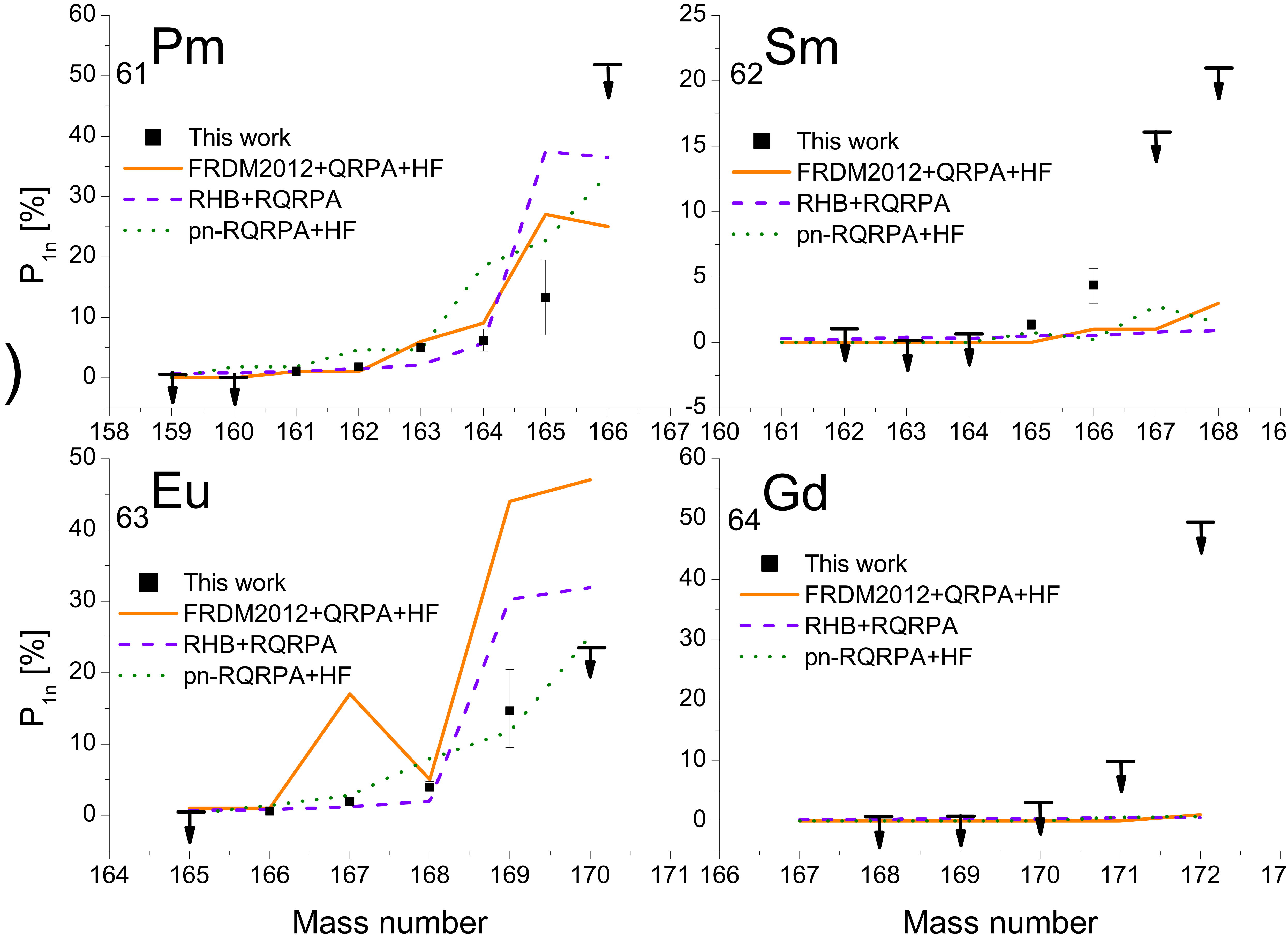

G. Kiss, A. Vitéz-Sveiczer, YS, et al., APJ (2022)

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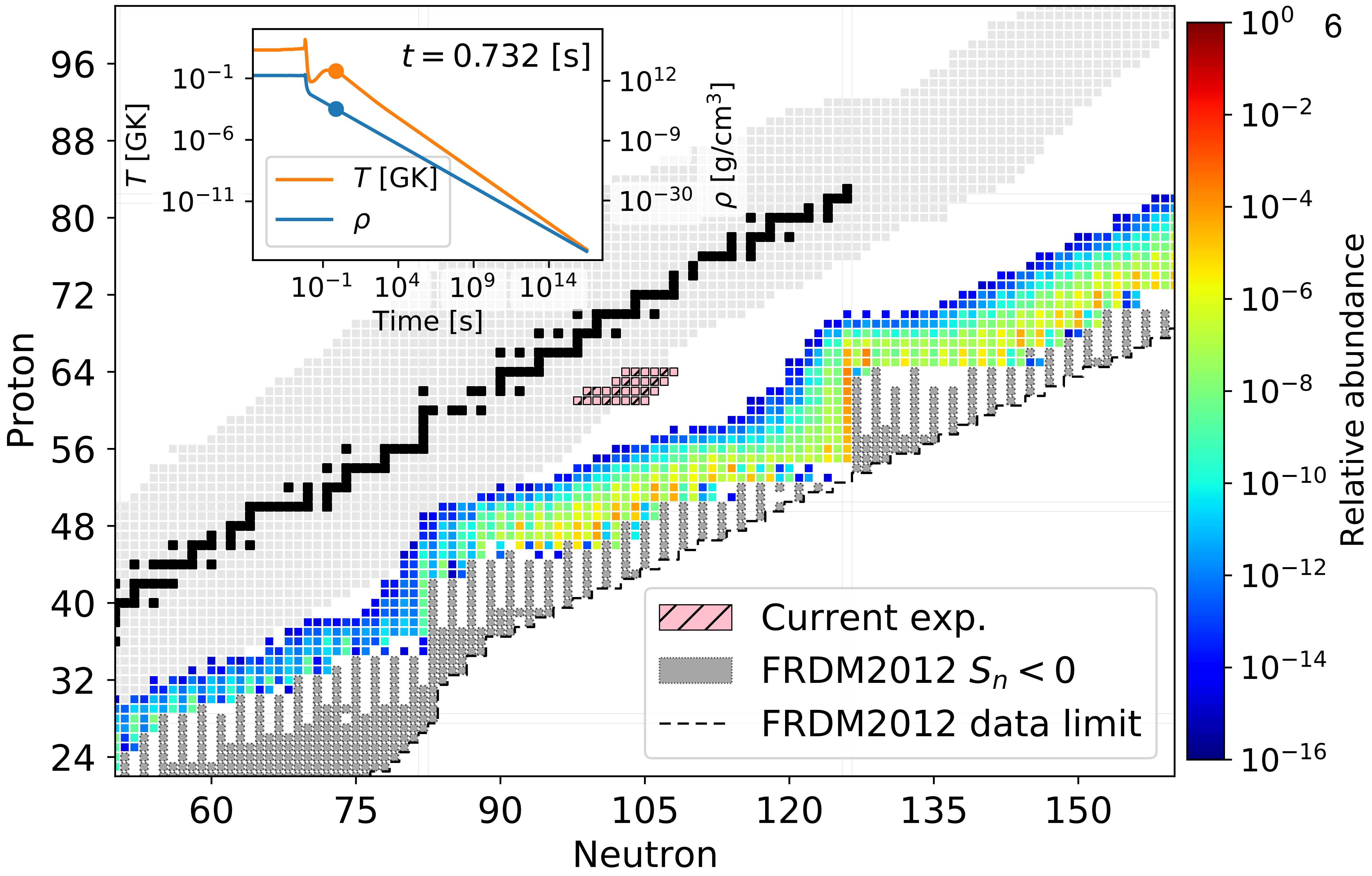
- Data analysis led by G. Kiss and A. Vitéz-Sveiczer (ATOMKI)

- Astrophysical analysis (YS)

➡ **How do the β-decay properties of the 28 nuclei affect the formation of the *r*-process rare earth peak (REP)?**

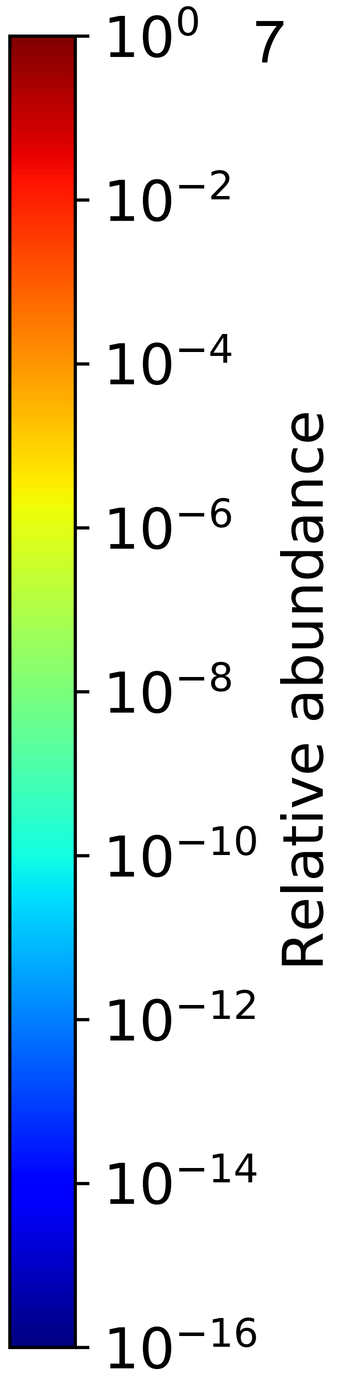
Snapshot of nucleosynthesis in NS merger



- Published yesterday on ApJ: **G. Kiss, A. Vitéz-Sveiczer, YS, et al. (2022), APJ, 936:107**

Experiment and analysis overview

Snapshot of nucleosynthesis in NS merger



- P_1

159

165

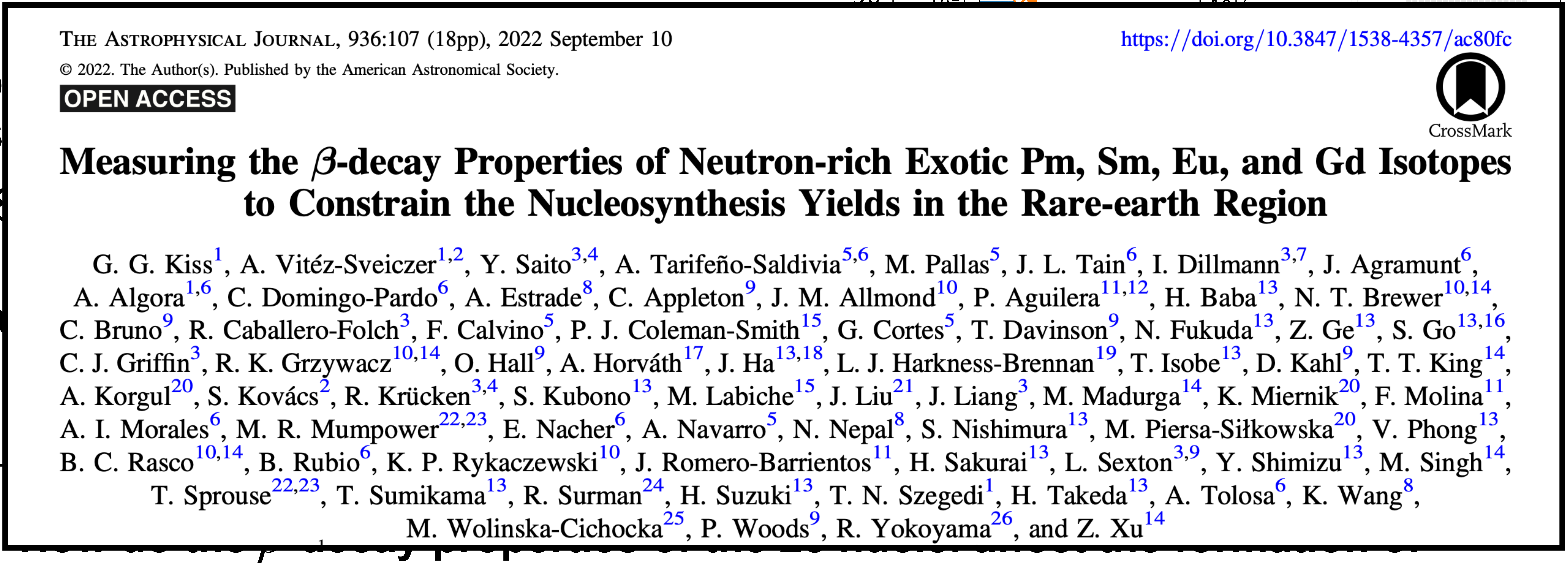
(28

- Da

G.

- As

➡ the *r*-process rare earth peak (REP)?



The Astrophysical Journal, 936:107 (18pp), 2022 September 10 https://doi.org/10.3847/1538-4357/ac80fc

CrossMark

Measuring the β-decay Properties of Neutron-rich Exotic Pm, Sm, Eu, and Gd Isotopes to Constrain the Nucleosynthesis Yields in the Rare-earth Region

G. G. Kiss[1], A. Vitéz-Sveiczer[1,2], Y. Saito[3,4], A. Tarifeño-Saldivia[5,6], M. Pallas[5], J. L. Tain[6], I. Dillmann[3,7], J. Agramunt[6], A. Algora[1,6], C. Domingo-Pardo[6], A. Estrade[8], C. Appleton[9], J. M. Allmond[10], P. Aguilera[11,12], H. Baba[13], N. T. Brewer[10,14], C. Bruno[9], R. Caballero-Folch[3], F. Calvino[5], P. J. Coleman-Smith[15], G. Cortes[5], T. Davinson[9], N. Fukuda[13], Z. Ge[13], S. Go[13,16], C. J. Griffin[3], R. K. Grzywacz[10,14], O. Hall[9], A. Horváth[17], J. Ha[13,18], L. J. Harkness-Brennan[19], T. Isobe[13], D. Kahl[9], T. T. King[14], A. Korgul[20], S. Kovács[2], R. Krücken[3,4], S. Kubono[13], M. Labiche[15], J. Liu[21], J. Liang[3], M. Madurga[14], K. Miernik[20], F. Molina[11], A. I. Morales[6], M. R. Mumpower[22,23], E. Nacher[6], A. Navarro[5], N. Nepal[8], S. Nishimura[13], M. Piersa-Siłkowska[20], V. Phong[13], B. C. Rasco[10,14], B. Rubio[6], K. P. Rykaczewski[10], J. Romero-Barrientos[11], H. Sakurai[13], L. Sexton[3,9], Y. Shimizu[13], M. Singh[14], T. Sprouse[22,23], T. Sumikama[13], R. Surman[24], H. Suzuki[13], T. N. Szegedi[1], H. Takeda[13], A. Tolosa[6], K. Wang[8], M. Wolinska-Cichocka[25], P. Woods[9], R. Yokoyama[26], and Z. Xu[14]

- Published yesterday on ApJ: **G. Kiss, A. Vitéz-Sveiczer, YS, et al. (2022), APJ, 936:107**

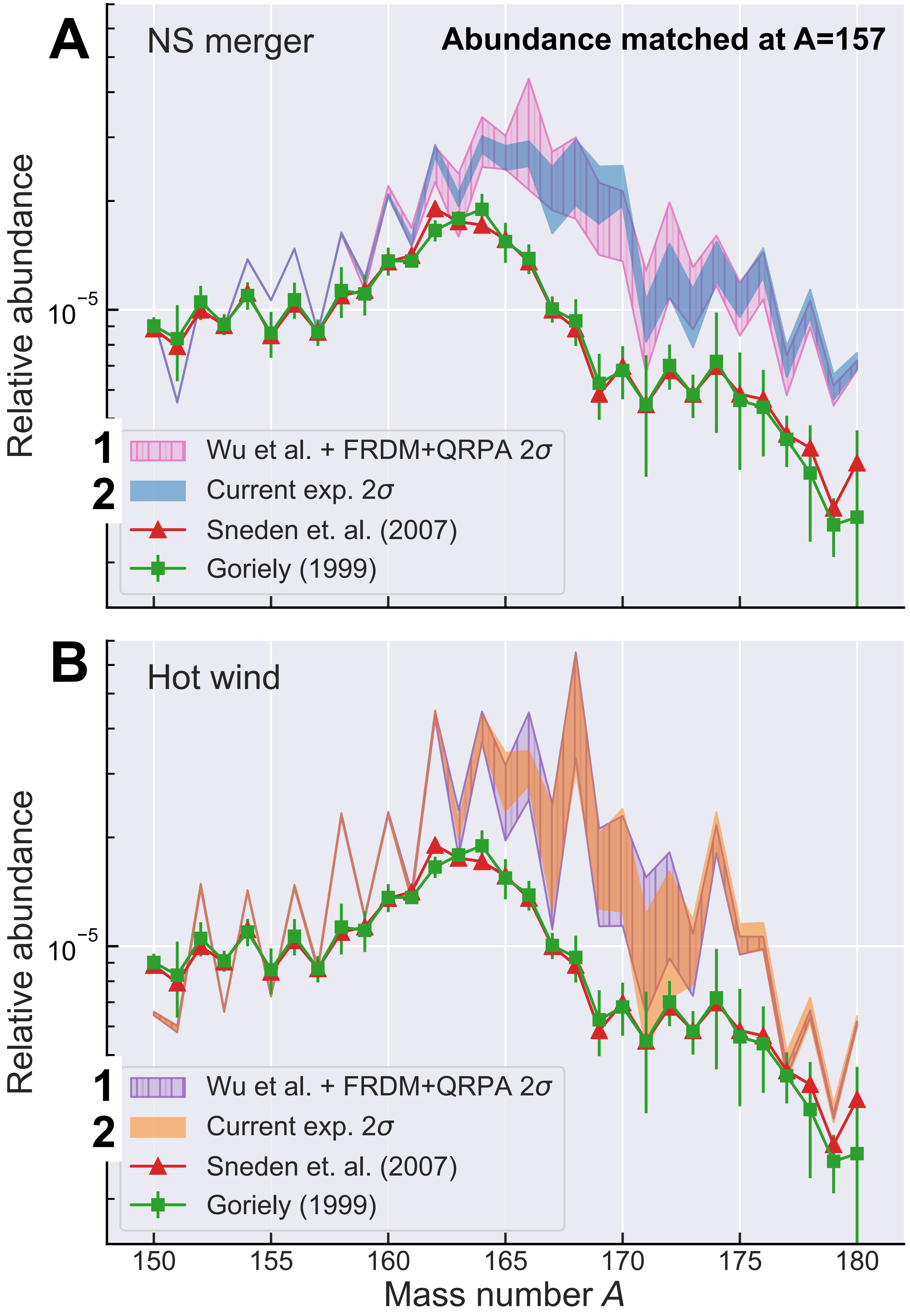
Propagated uncertainty

- Half-life uncertainty:
  1. Wu et al. PRL (2017) +
     “factor of 10” uncertainty for FRDM+QRPA
     (as in Mumpower et al. PrPNP (2016))
  2. Our results (G. Kiss et al. APJ (2022))

- Astrophysical conditions:
  A. Neutron star merger dynamical ejecta
     (Vassh et al., JPhG (2019))
  B. Hot neutrino driven wind
     (Mumpower et al. PrPNP (2016))

In all the calculations, **our new P_{1n} values** are used.

- **Where does the uncertainty come from?**



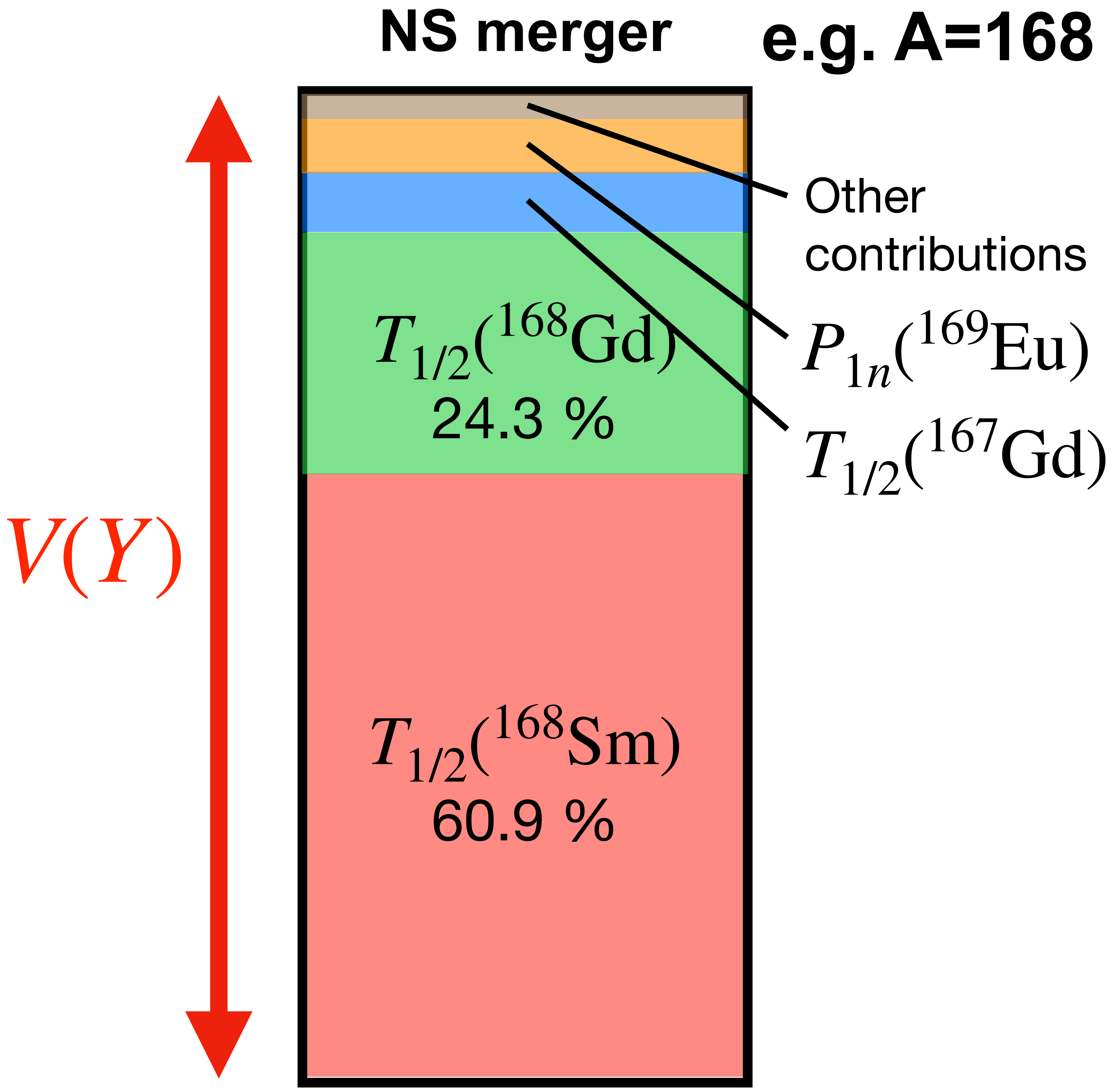
Variance-based sensitivity analysis

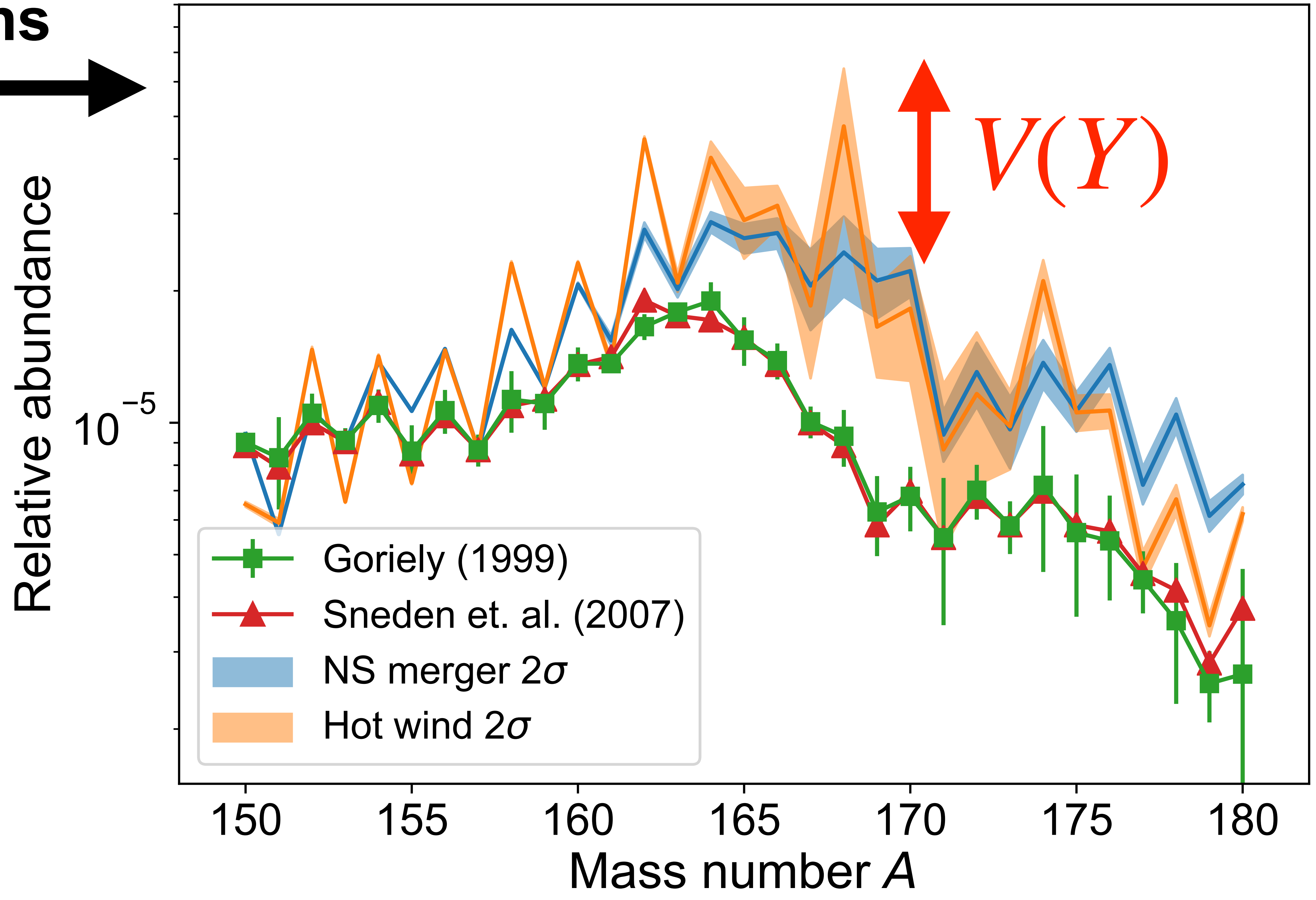
Exp. uncertainty of
$T_{1/2}$ and P_{1n}

Monte Carlo sampling and abundance calculations

$$T_{1/2}(^{168}\mathrm{Sm}) = 353^{+210}_{-164}\ [\mathrm{ms}]$$

e.g. A=168



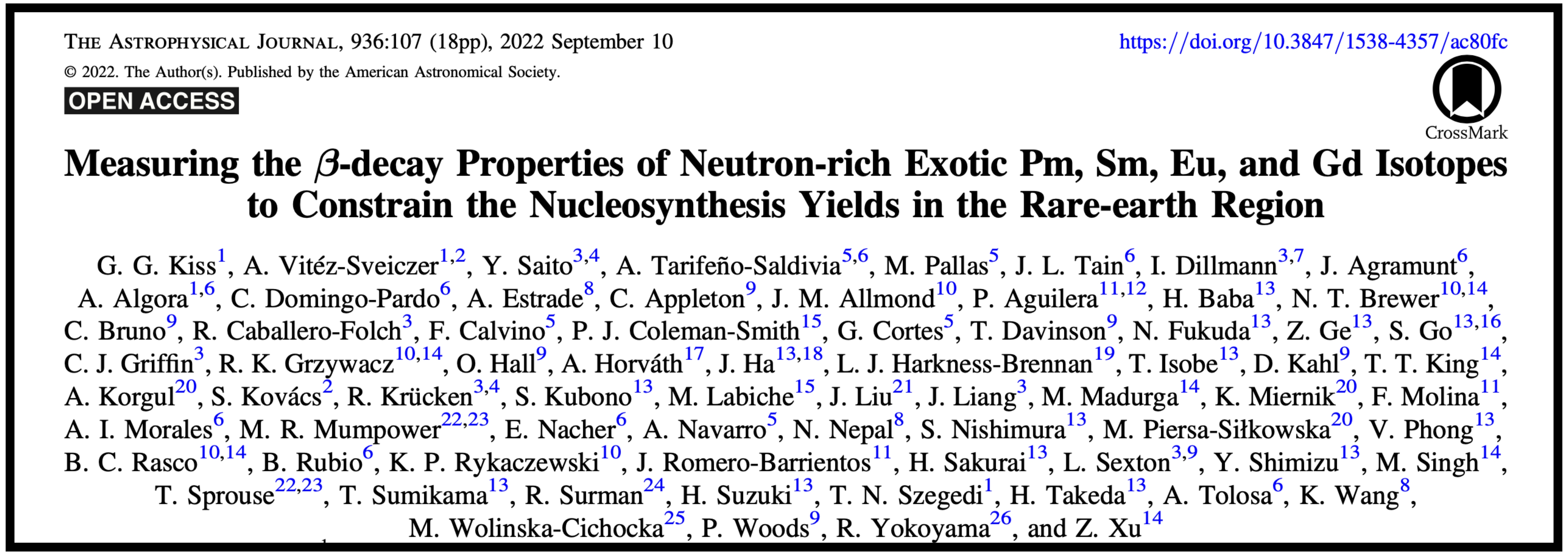


Variance decomposition

A. Saltelli, et al. 2010, Computer Physics Communications, 181, 259

Variance-based sensitivity analysis results

- Dominant contribution from the half-lives in both astrophysical scenarios (~90%)
- Strong contribution from $T_{1/2}$(^{168}Sm) to mass numbers *A*=168–173
- β-decay $T_{1/2}$ of nuclei synthesized at the beginning of **freeze-out** may strongly affect the flow of *n* capture
- For full results, ask me or see G. Kiss, A. Vitéz-Sveiczer, YS, et al. (2022), APJ, 936:107



THE ASTROPHYSICAL JOURNAL, 936:107 (18pp), 2022 September 10
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Measuring the β-decay Properties of Neutron-rich Exotic Pm, Sm, Eu, and Gd Isotopes to Constrain the Nucleosynthesis Yields in the Rare-earth Region

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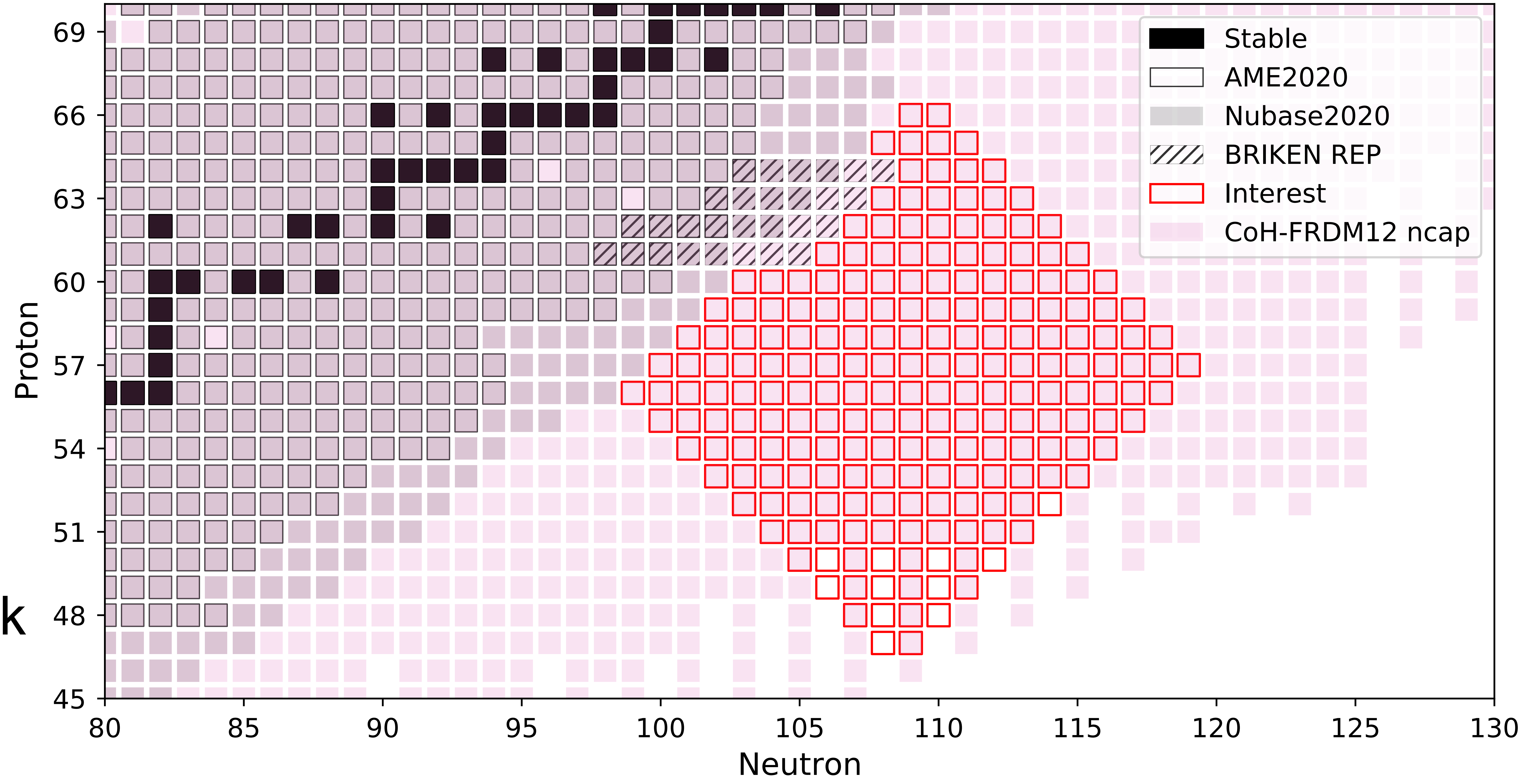
Towards scalable statistical analyses

- To draw more general conclusions, more isotopes and more types of nuclear physics inputs should be included
  - ➡ Cost may become prohibitive (millions of calculations required)
- Challenging to accelerate nuclear reaction network calculations with GPUs (Work by Paul Virally (U. Waterloo))
- An ***emulator*** would significantly reduce the computational cost
  - ➡ Model the map between
    input (nuclear physics) and output (abundance pattern)
  - ➡ One run is significantly cheaper than a full abundance calculation

Towards scalable statistical analyses

- 212 isotopes
- Variables: $T_{1/2}$ and S_n (propagated to n-capture rate) = 424 variable inputs
- Emulate abundance calculations with artificial neural network (ANN)



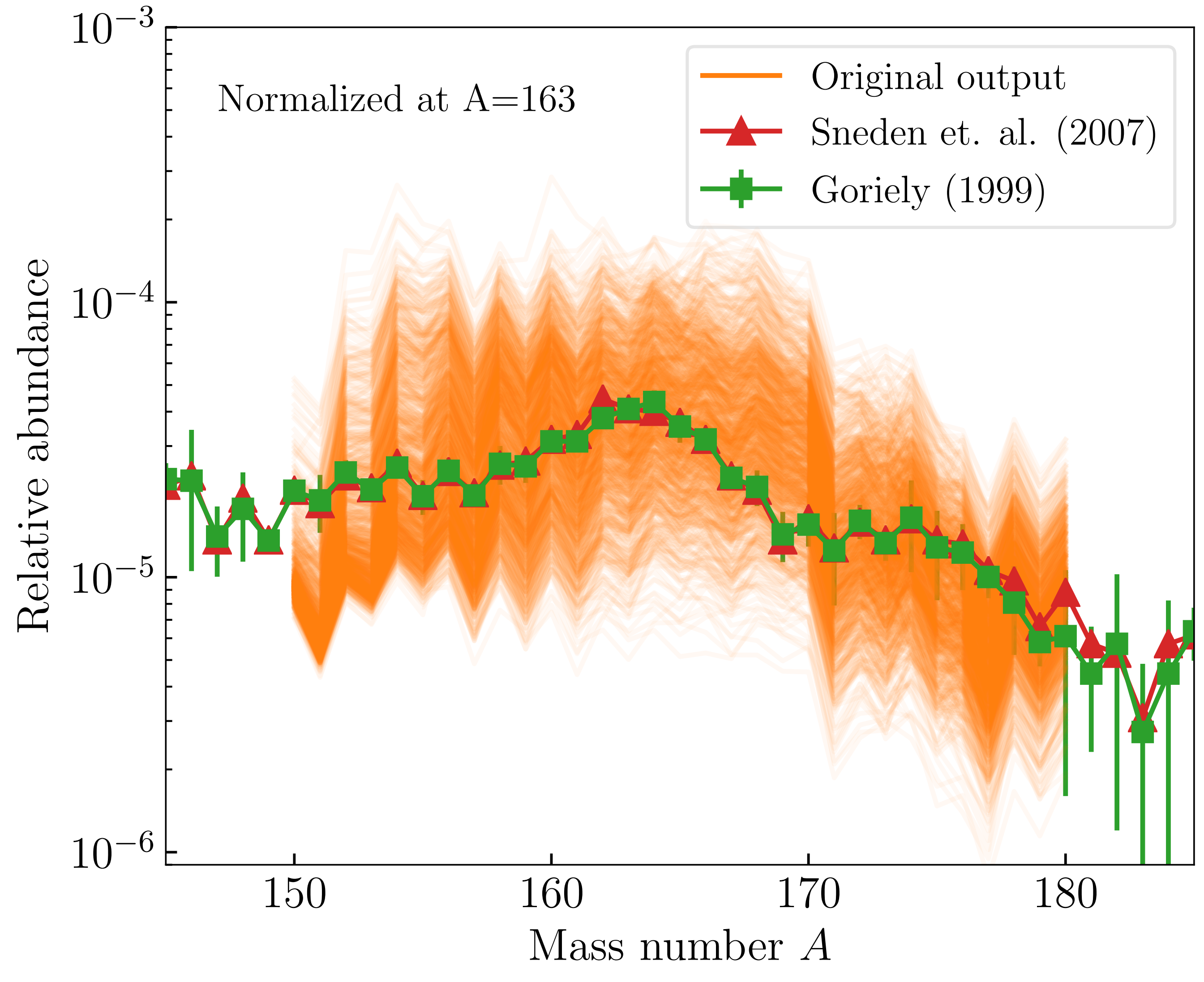
Artificial Neural Network Emulator

Simultaneously vary

- β-decay half-lives
- S_n (propagated to n capture rate)

Solve nuclear reaction network ODE:

$$\frac{dY_i}{dt} = \sum_j P^i_j \; \lambda_j Y_j + \sum_{j,k} P^i_{j,k} \; \frac{\rho}{m_u} \langle j,k \rangle Y_j Y_k + \sum_{j,k,l} P^i_{j,k,l} \; \frac{\rho^2}{m_u^2} \langle j,k,l \rangle Y_j Y_k Y_l.,$$



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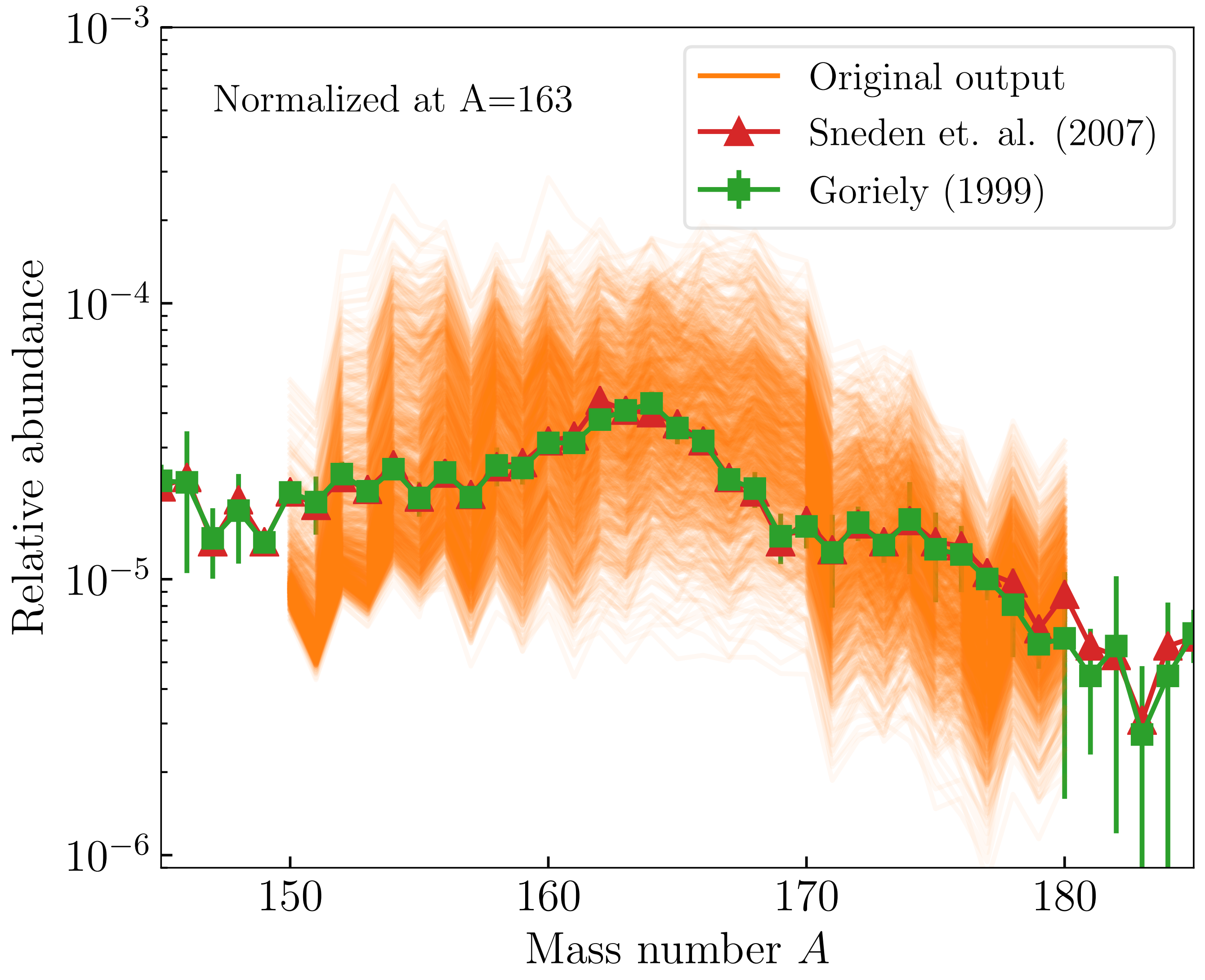
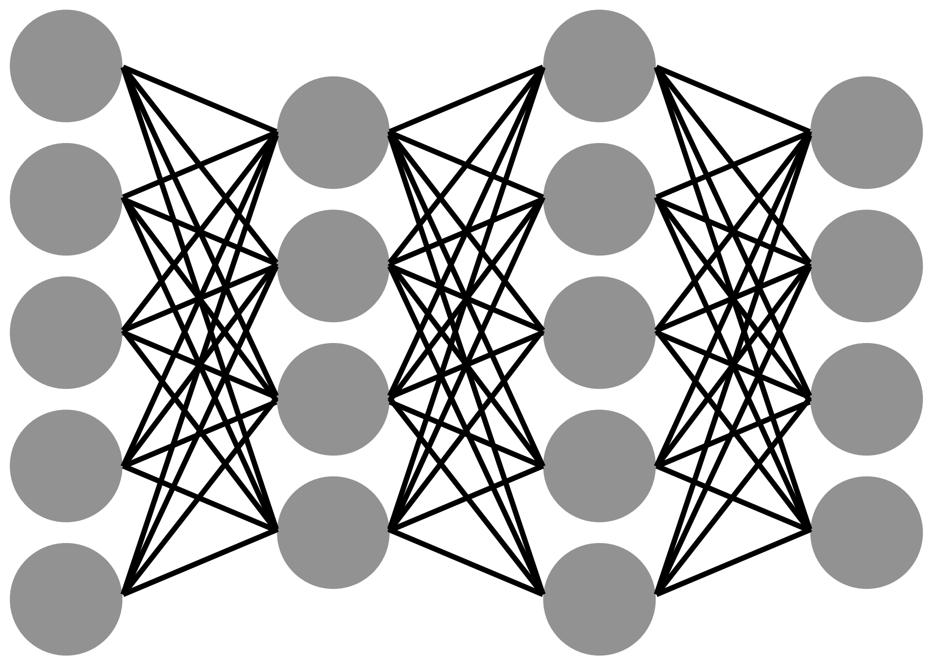
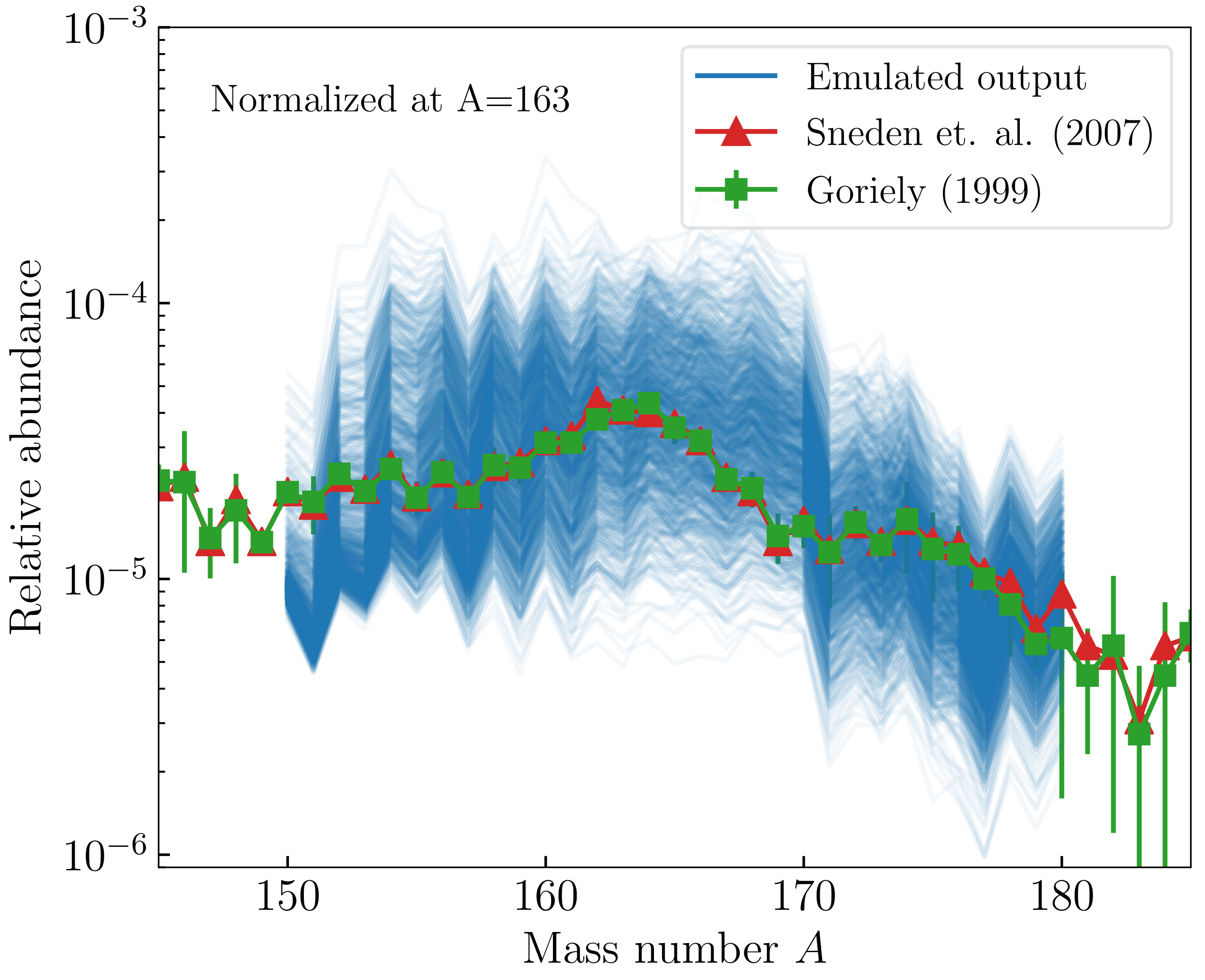


Train

ANN Emulator

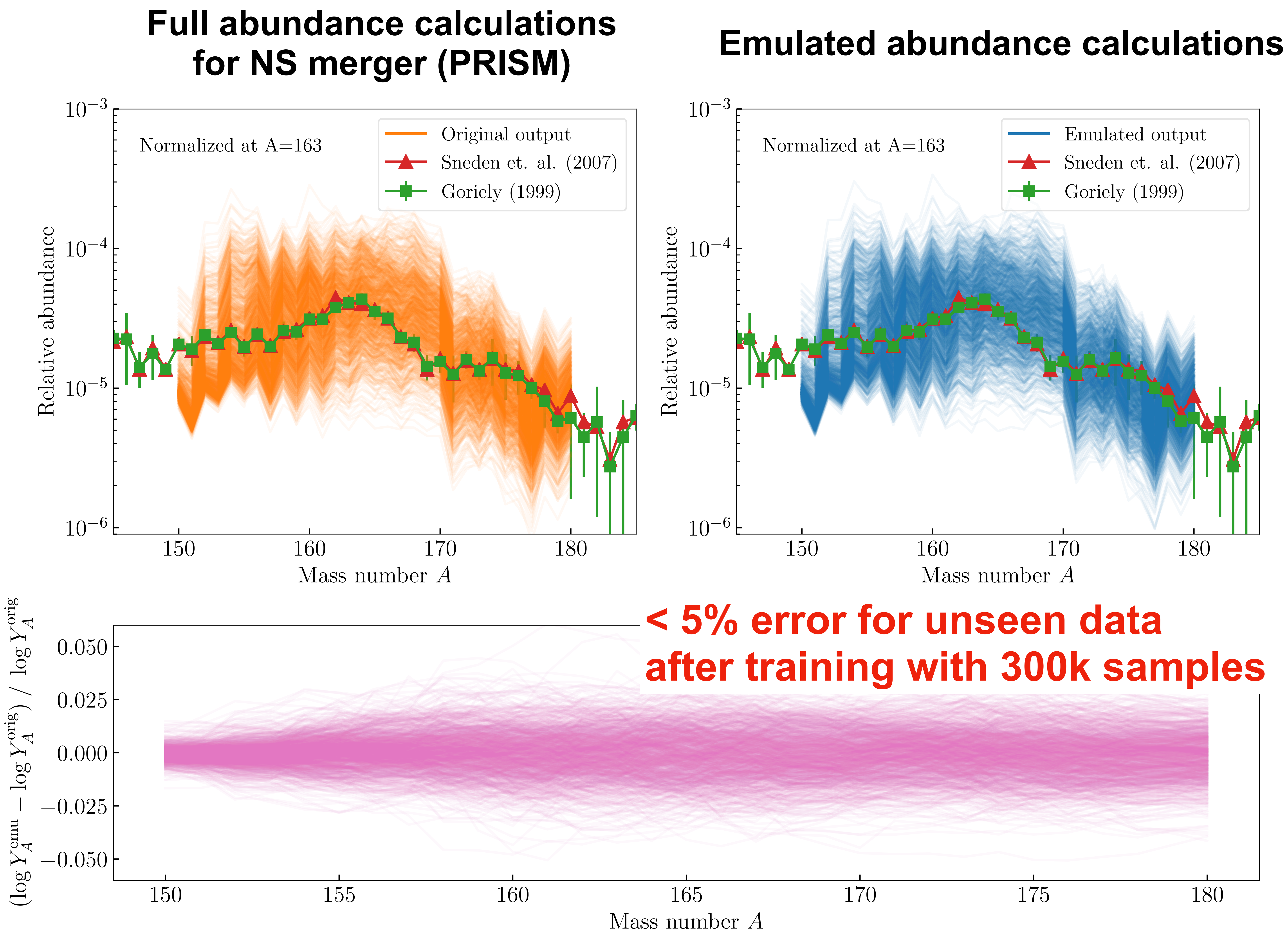


Predict



ANN Emulator

- $T_{1/2}$ and S_n of 212 isotopes (propagated to n-capture rate) = 424 variable inputs
- ~300 ms per prediction (cp. One full network calculation ~300 s)
- With uncertainty quantification of each prediction



< 5% error for unseen data after training with 300k samples

YS, et al. in preparation

PRISM: M. Mumpoer (LANL) and T. Sprouse (LANL)

Summary and outlook

- Astrophysical implication of new $T_{1/2}$ and P_{1n} data in the REP region
  - ➡ $T_{1/2}$ of influential isotopes affect the flow of n-capture
  - ➡ Nuclei synthesized right before neutrons are exhausted may be important
- Variance-based sensitivity analysis
  - ➡ Identify inputs where abundance uncertainty comes from

G. Kiss, A. Vitéz-Sveiczer, YS, et al., (2022), APJ for more detail

- ANN-based abundance calculation emulator
  - ➡ Enables further sensitivity analysis and various statistical inference tasks

Manuscripts in preparation

Collaborators: I. Dillmann (TRIUMF), R. Kruecken (UBC/TRIUMF/LBNL)
M. Mumpower (LANL), R. Surman (U. Notre Dame)
A. Ravlic (U. Zagreb), N. Paar (U. Zagreb), F. Minato (JAEA)

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Natural Sciences and Engineering Research Council of Canada

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Thank you!
Merci!

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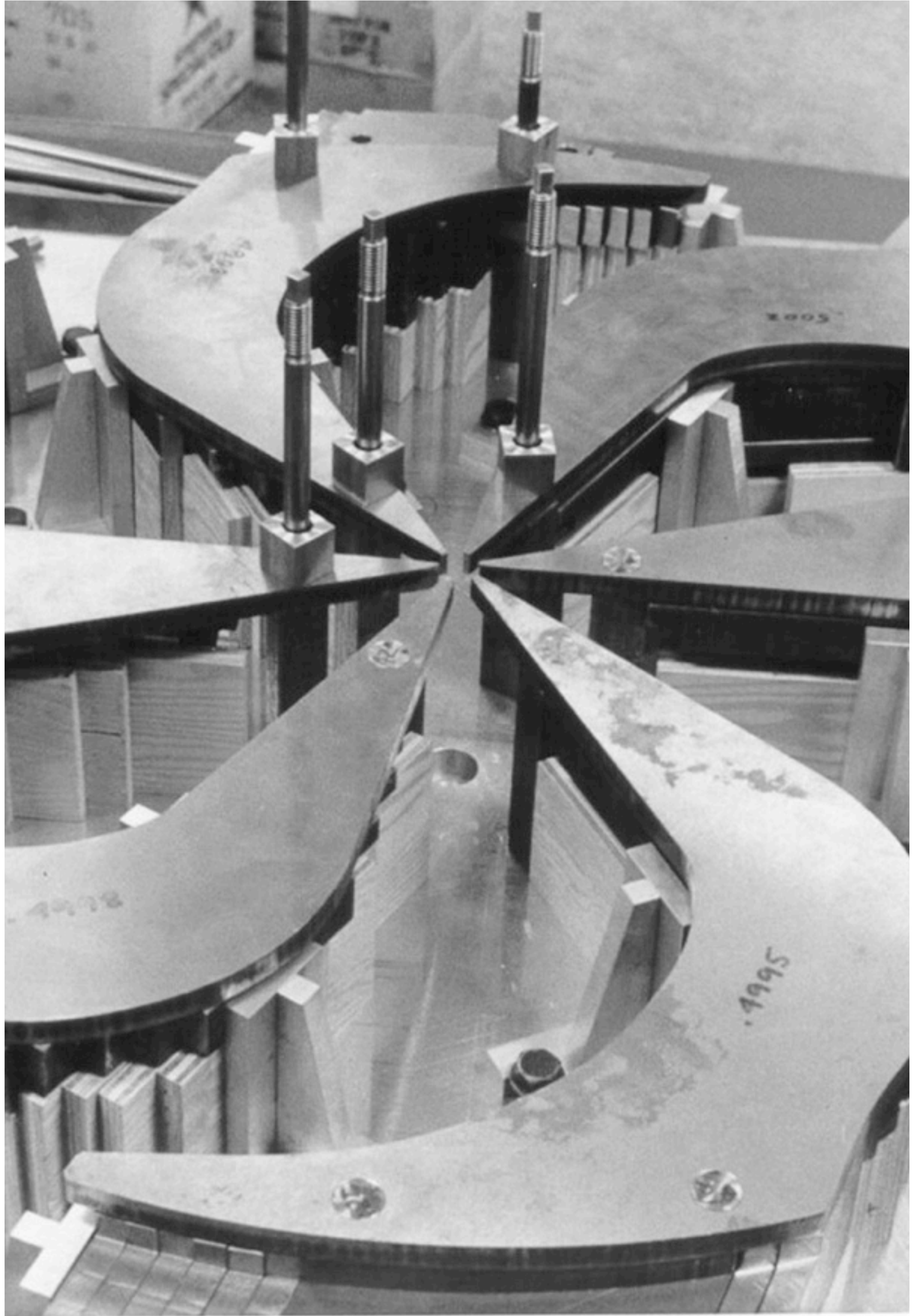


Discovery, accelerated

Variance-based sensitivity analysis results

- Dominant contribution from the uncertainty of half-lives in both astrophysical scenarios

Contribution of experimental uncertainty to the abundance uncertainty [%]
NS merger

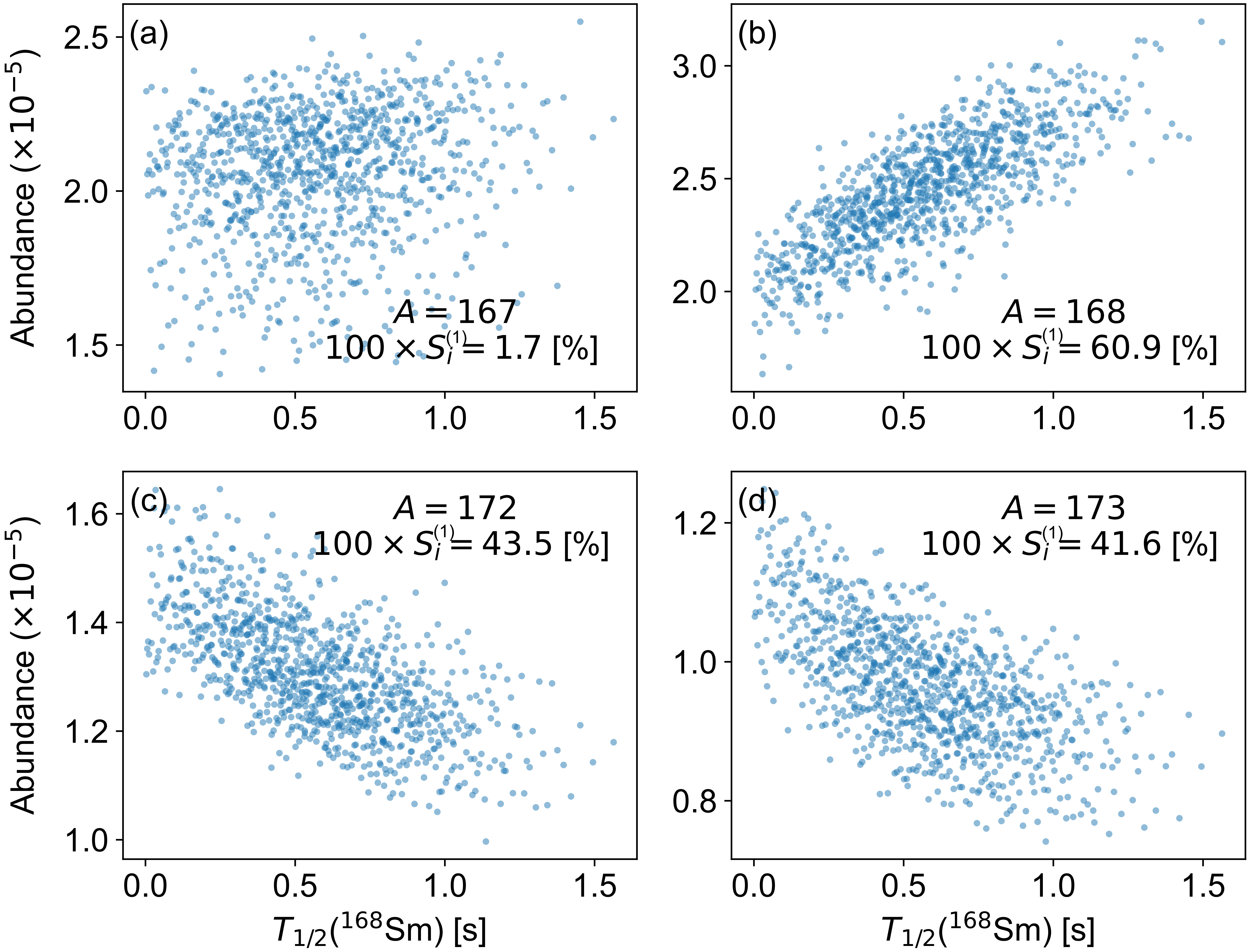
	$100{\times}S^{(1)}$ (95% C.I.) [%]					
	$A = 168$	169	170	171	172	173
$S^{(1)}(T_{1/2})$ total:	94.9 (± 8.6)	100.1 (± 9.2)	93.9 (± 9.9)	84.0 (± 8.5)	95.1 (± 8.3)	99.7 (± 8.6)
$S^{(1)}(P_{1n})$ total:	5.9 (± 2.3)	1.1 (± 1.1)	5.6 (± 2.0)	11.0 (± 2.9)	5.7 (± 2.0)	2.0 (± 1.1)
$S^{(1)}$ total:	100.9 (± 8.9)	101.3 (± 9.2)	99.5 (± 10.1)	95.0 (± 9.0)	100.7 (± 8.6)	101.6 (± 8.6)

Hot wind

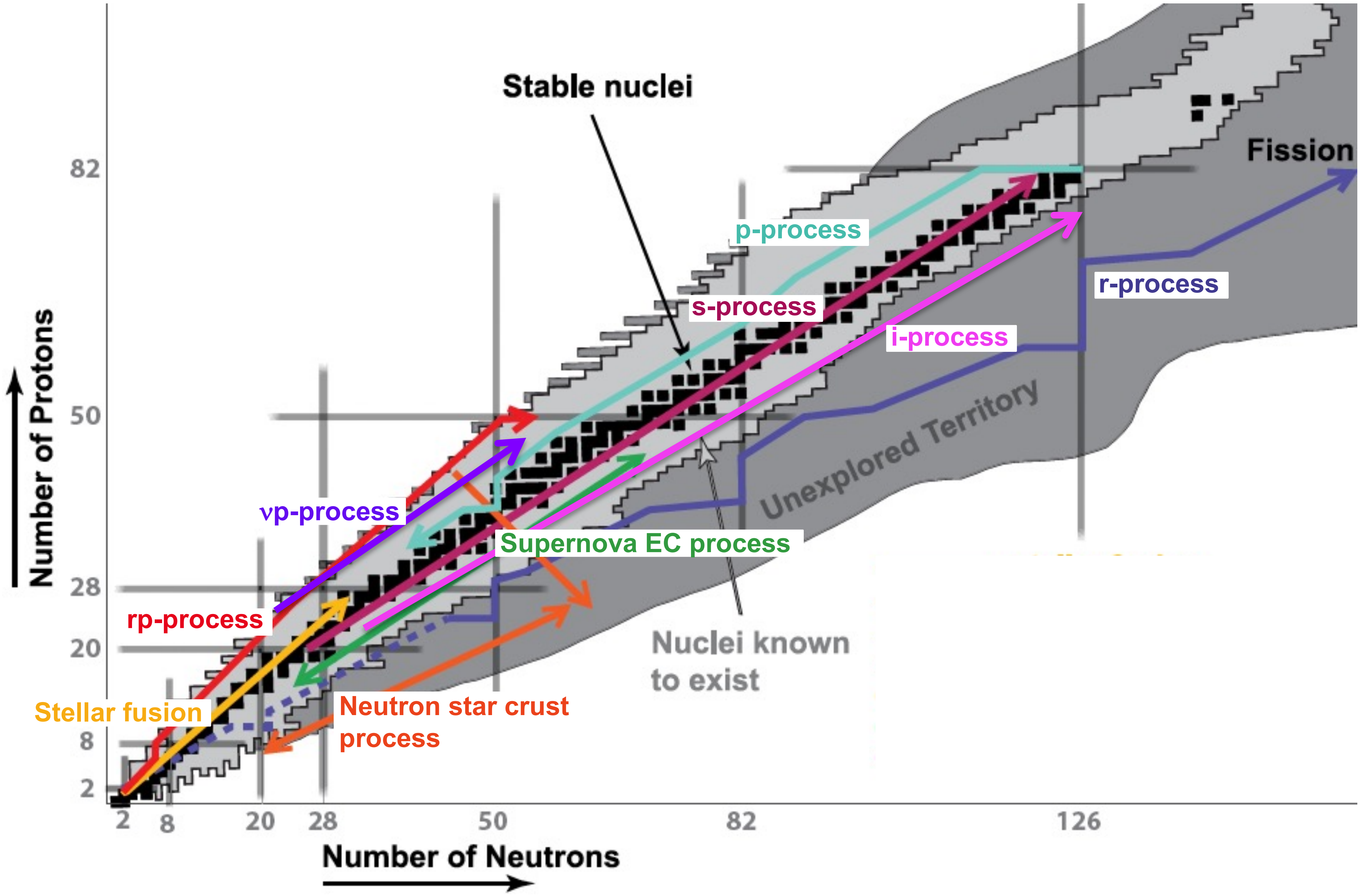
$S^{(1)}(T_{1/2})$ total:	97.0 (± 14.1)	78.9 (± 7.4)	97.4 (± 8.3)	74.6 (± 8.2)	98.6 (± 10.5)	93.8 (± 13.7)
$S^{(1)}(P_{1n})$ total:	3.0 (± 1.8)	21.5 (± 4.1)	3.7 (± 1.6)	25.9 (± 4.7)	2.8 (± 1.7)	5.6 (± 2.1)
$S^{(1)}$ total:	100.0 (± 14.3)	100.5 (± 8.5)	101.1 (± 8.4)	100.5 (± 9.5)	101.3 (± 10.7)	99.4 (± 13.9)

Variance-based sensitivity analysis

Effect of $T_{1/2}(^{168}\mathrm{Sm}) = 353^{+210}_{-164}$ [ms] in NS merger



Rapid neutron capture process (*r*-process)



From Schatz et al. (2022), arXiv:2205.07996

- Responsible for ~50% of heavy elements

- High neutron density (>10^{26} cm^{-3})
- Temperature can reach a few GK
  - ➡ Compact binary mergers
  - ➡ Some types of core-collapse SNe

- Interplay of nuclear physics processes
  - Neutron capture & photodissociation
  - β-decay & β-delayed neutron emission
  - Fission
  - etc…

Analysis of propagated uncertainty

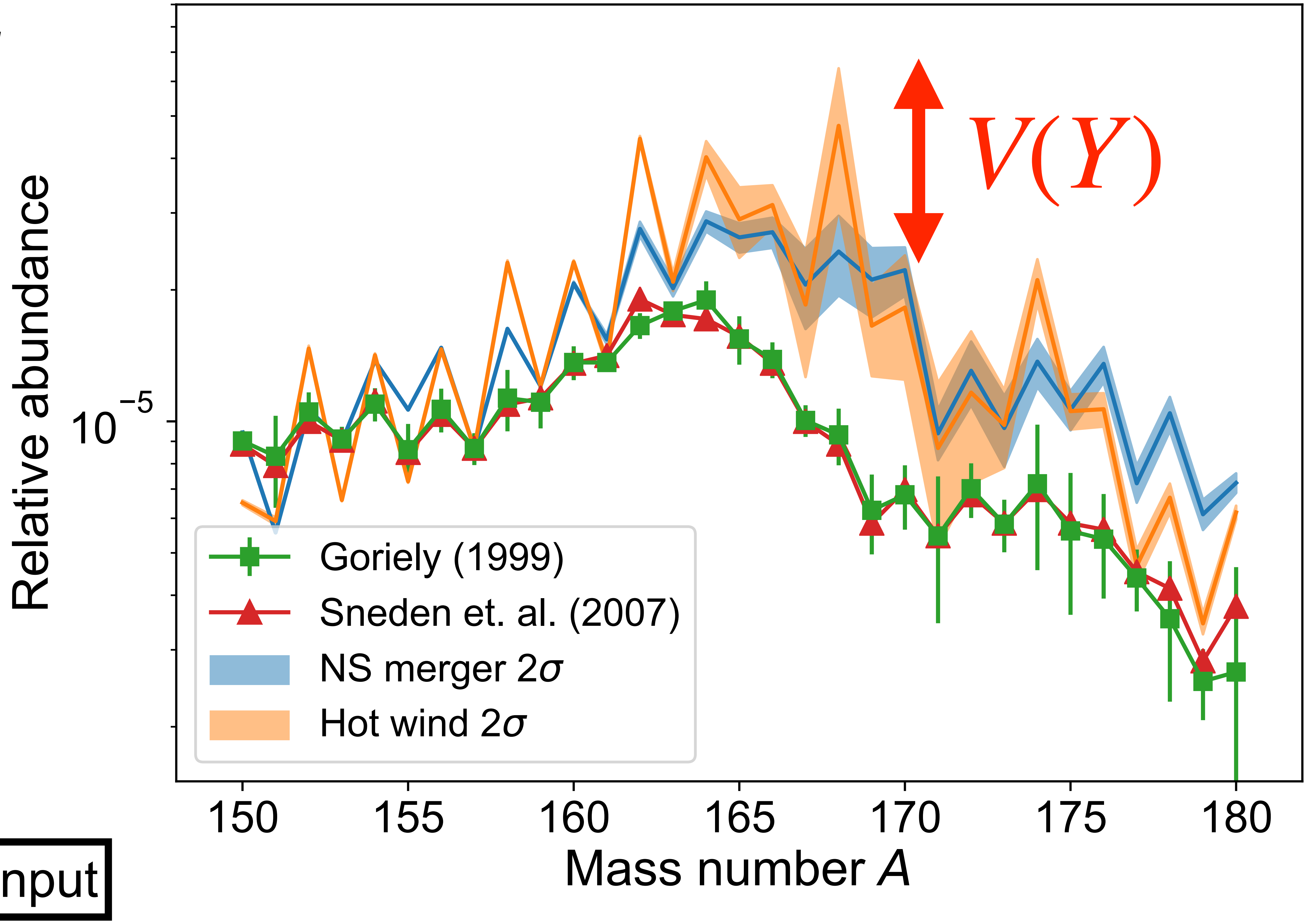
- Is it possible to quantify the contribution of each $T_{1/2}$ and P_{1n} (input) uncertainty to the abundance (output) uncertainty?

➡ Variance-based sensitivity analysis

- Decompose variance $V(Y)$

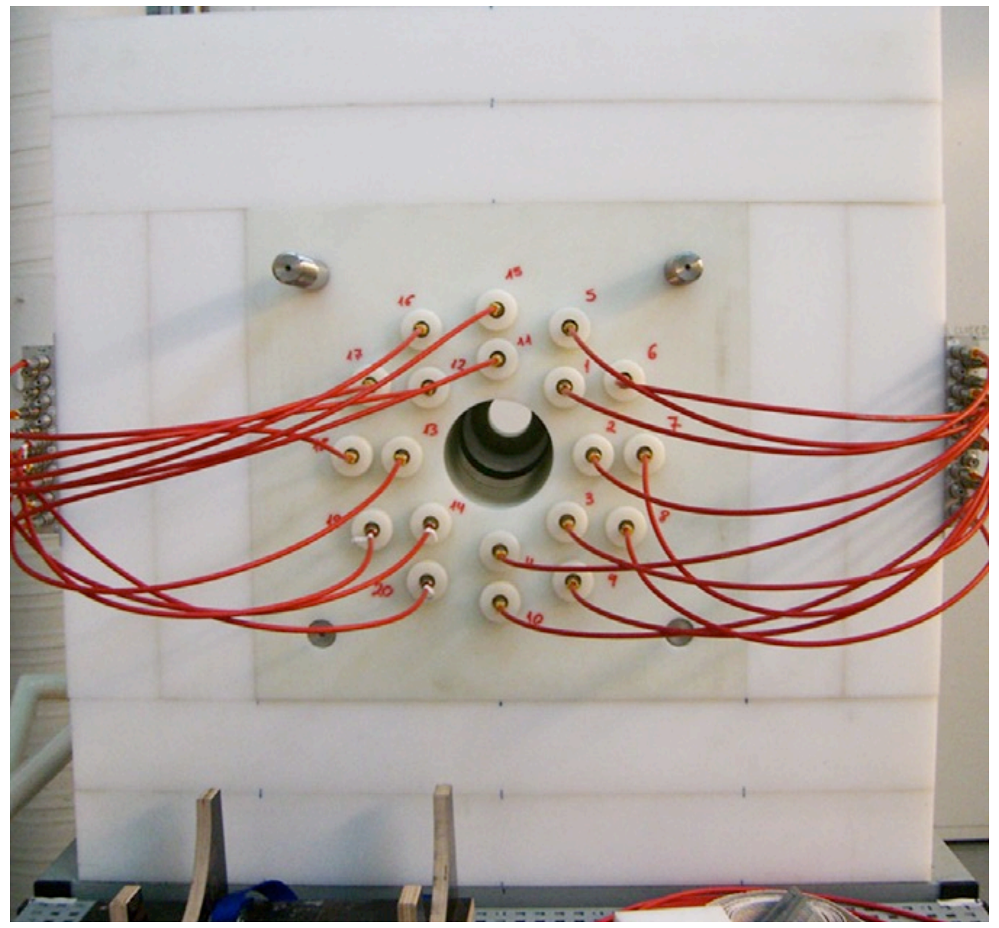
$$V(Y) = \sum_{i} V_i^{(1)} + \sum_{i} \sum_{j>i} V_{ij}^{(2)} + \cdots + V_{12\ldots k}^{(k)},$$

Variance (propagated uncertainty) from each uncertain input

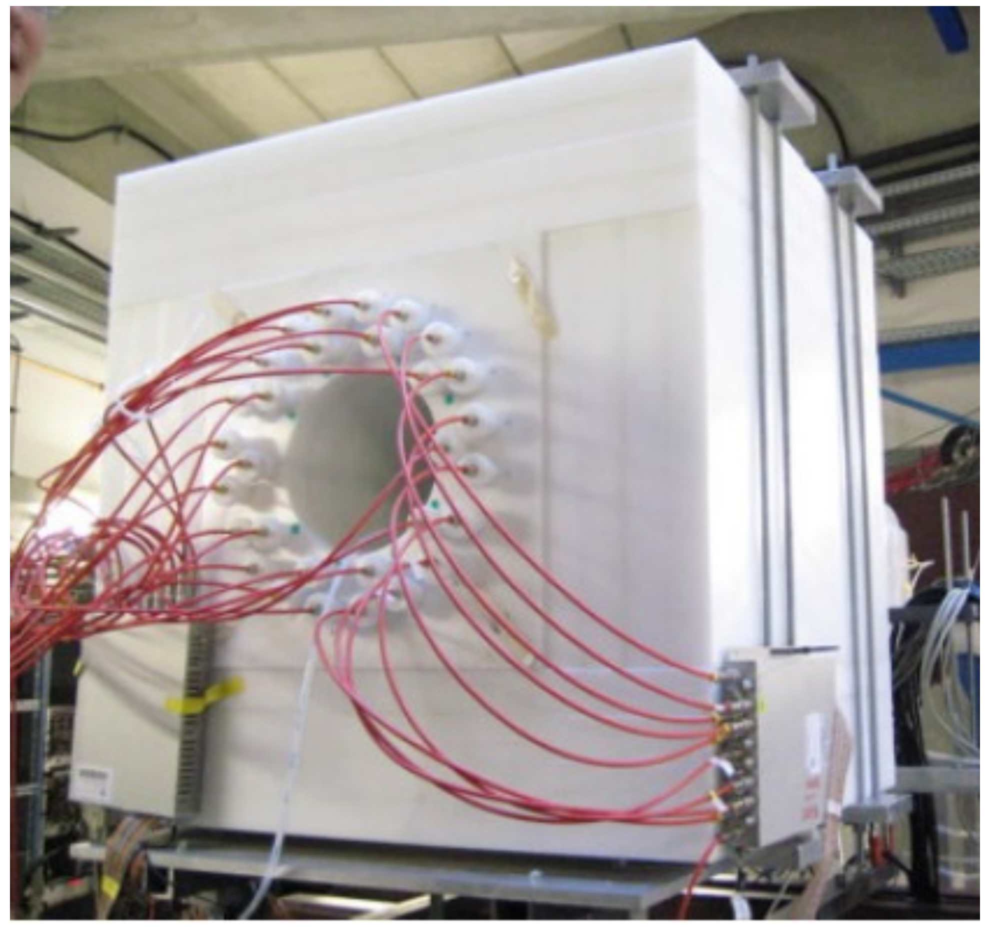


and compute Monte Carlo estimate of first-order sensitivity: $S_i^{(1)} = \dfrac{V_i^{(1)}}{V(Y)}$

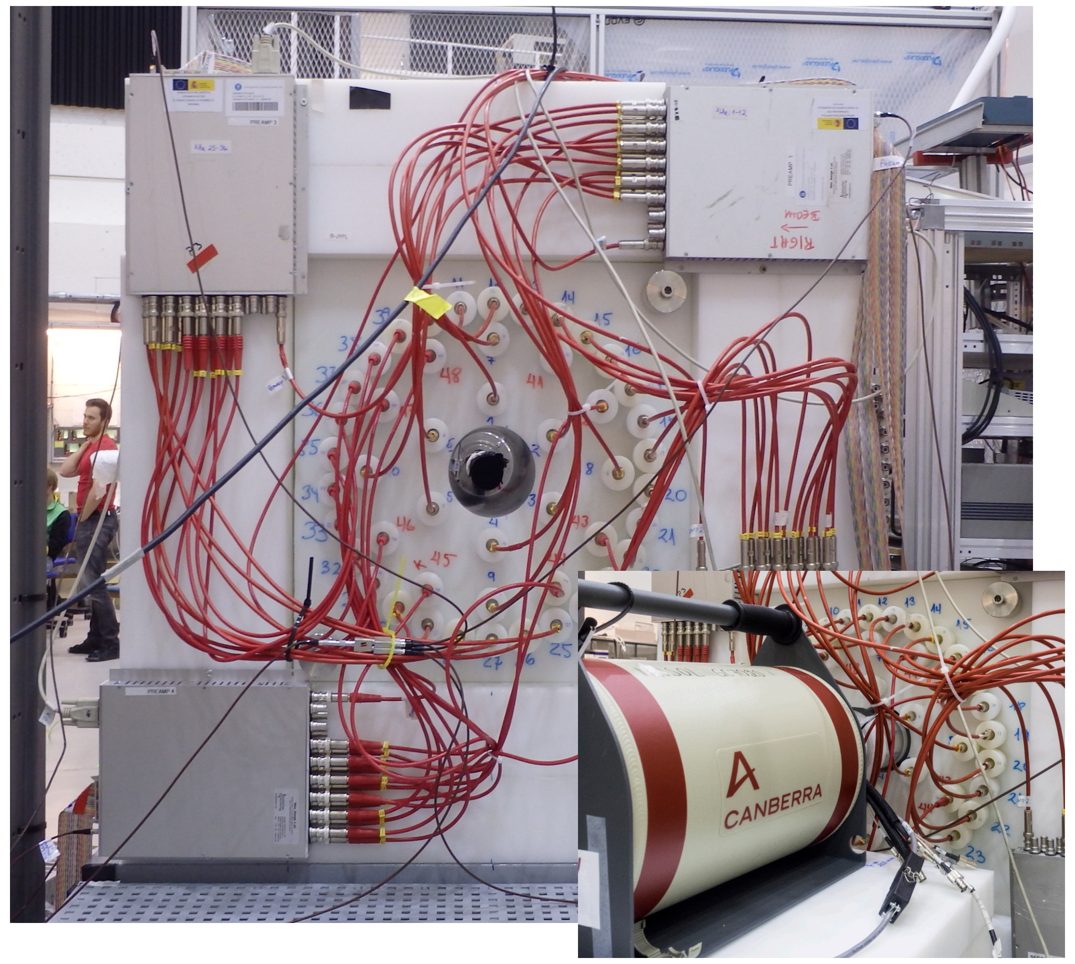
BRIKEN project background: BELEN detector (2009-2014) – BRIKEN 2016-2021



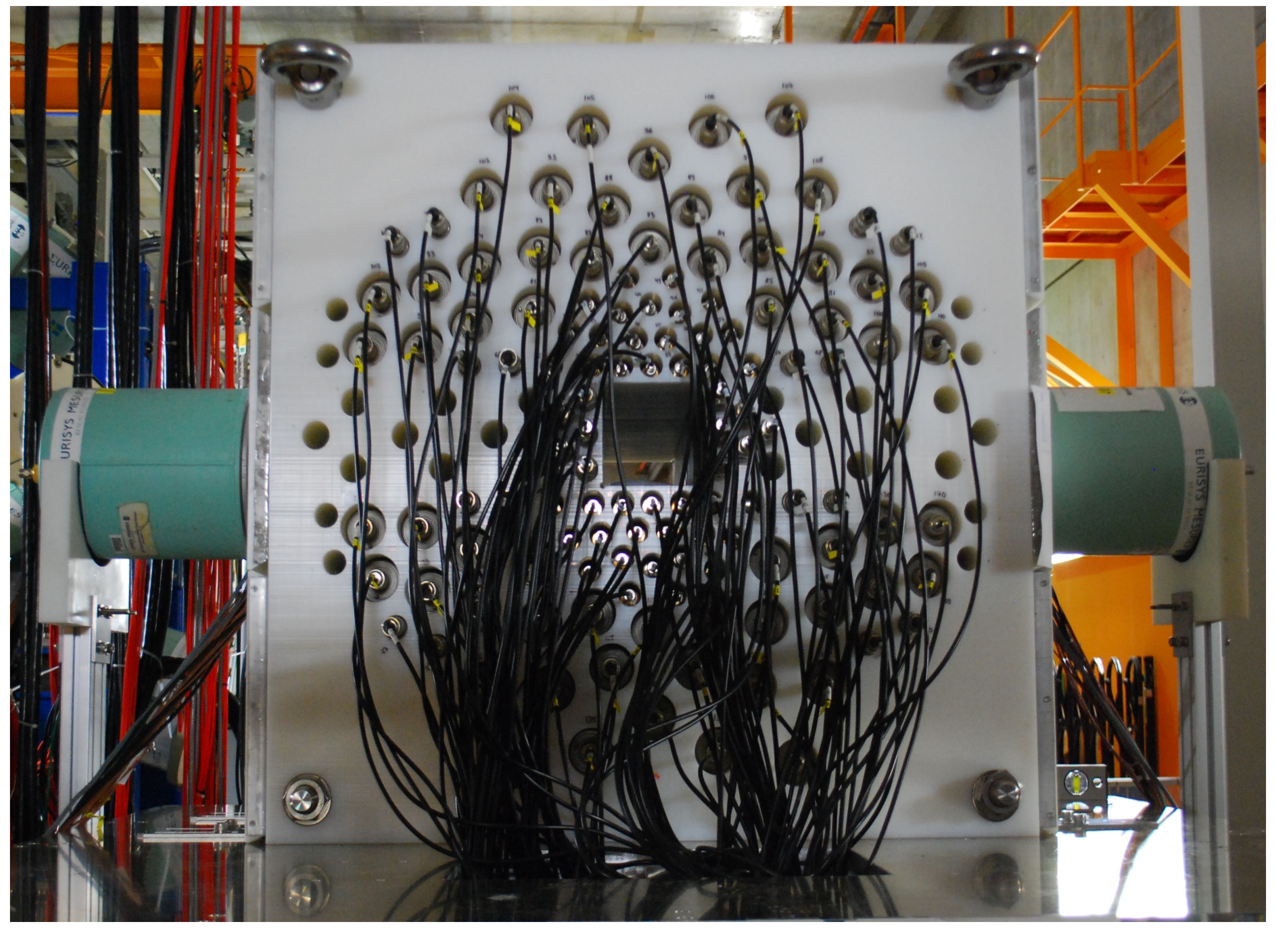
20 ^{3}He counters in 2 rings. IGISOL Jyväskylä (2010).
$\varepsilon_{1n} \approx 47\%$



30 ^{3}He counters in 2 rings. GSI-FRS (2011).
$\varepsilon_{1n} \approx 38\%$

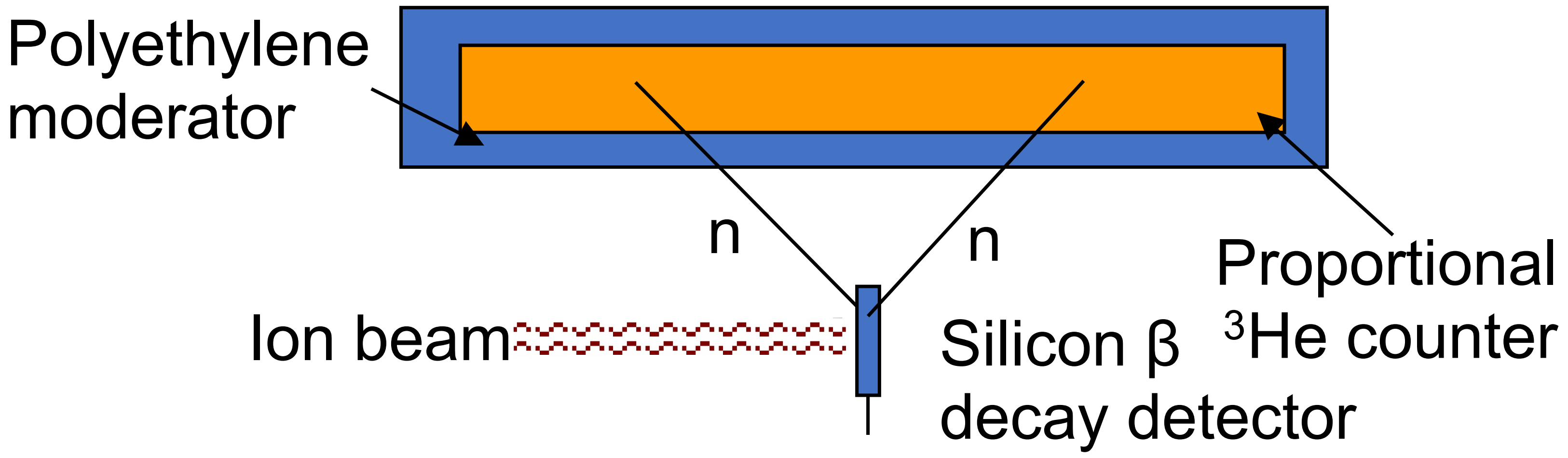


48 ^{3}He counters in 3 rings. IGISOL Jyväskylä (2014).
$\varepsilon_{1n} \approx 40\%$ (HPGe)
$\varepsilon_{1n} \approx 60\%$



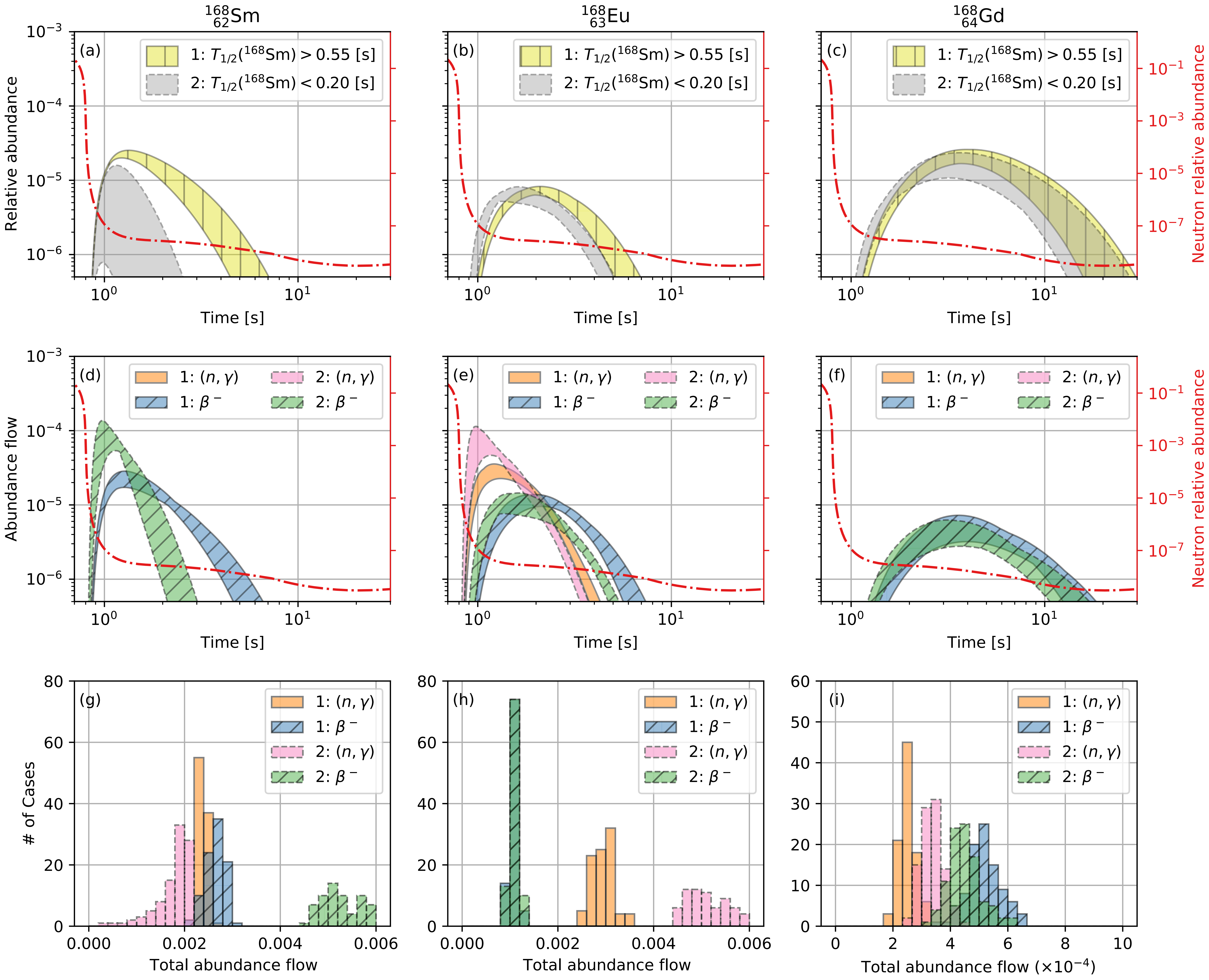
140 ^{3}He counters in 7 rings. BRIKEN (2016…)
$\varepsilon_{1n} \approx 68.6\%$ (HPGe)

$$^{3}He + n \rightarrow {}^{3}H + p + 764keV$$



All efficiencies are up to 1MeV, beam hole and counter location dependant.

Slide courtesy of R. Caballero-Folch



$^{168}_{62}$Sm
$^{168}_{63}$Eu
$^{168}_{64}$Gd
(a)
(b)
(c)
1: $T_{1/2}(^{168}\text{Sm}) > 0.55$ [s]
2: $T_{1/2}(^{168}\text{Sm}) < 0.20$ [s]
Relative abundance
Neutron relative abundance
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{0}
10^{1}
Time [s]
(d)
(e)
(f)
1: (n,γ)
1: β^-
2: (n,γ)
2: β^-
Abundance flow
(g)
(h)
(i)
# of Cases
0
20
40
60
80
10
30
50
0.000
0.002
0.004
0.006
2
4
6
8
10
Total abundance flow
Total abundance flow ($\times 10^{-4}$)

Variance decomposition and Sensitivity indices

$$V(Y) = \sum_i V_i^{(1)} + \sum_i \sum_{j>i} V_{ij}^{(2)} + \cdots + V_{12\ldots k}^{(k)},$$

$$1 = \sum_i S_i^{(1)} + \sum_i \sum_{j>i} S_{ij}^{(2)} + \cdots + S_{12\ldots k}^{(k)},$$

Where $S_i^{(1)} = \dfrac{V_i}{V(Y)}$

Monte Carlo estimator: $\hat{S}_i^{(1)} = \dfrac{\frac{1}{N}\sum_{j=1}^{N} f(\boldsymbol{A})_j \left(f(\boldsymbol{B}_{\boldsymbol{A}}^{[i]})_j - f(\boldsymbol{B})_j \right)}{\hat{V}(Y)},$