Update on the estimate of the mt scheme and scale uncertainties

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Current Status

- Virtual & real (N)NLO QCD corrections in large top mass limit (HTL): ~100%

  Dawson,Dittmaier,Spira
  de Florian,Mazzitelli
  Grigo,Melnikov,Steinhauser

- Large top mass expansion: ~ ±10%

  Grigo, Hoff, Melnikov, Steinhauser

- NLO mass effects of the real NLO correction alone ~ -10 %

  Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

- NLO QCD corrections including the full top mass dependence: - 15 % NLO mass effects

  Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke

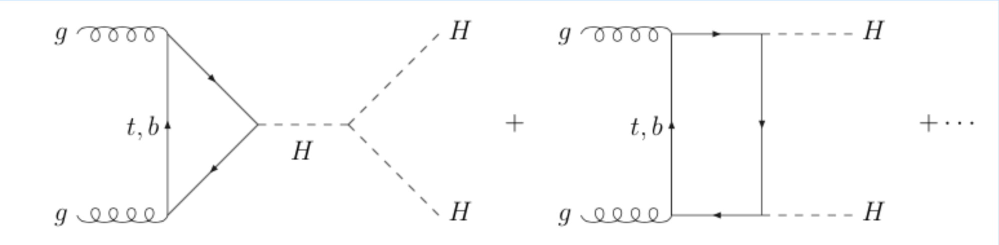
  Baglio,Campanario,SG,Mühlleitner, Spira,Streicher

- New expansion/extrapolation methods:
  - $1/m_t^2$ expansion & conformal mapping & Padé approximants — Gröber,Maier,Rauh
  - p_T^2 expansion — Bonciani,Degrassi,Giardino,Gröber
  - high-energy — Davies,Mishima,Steinhauser,Wellmann

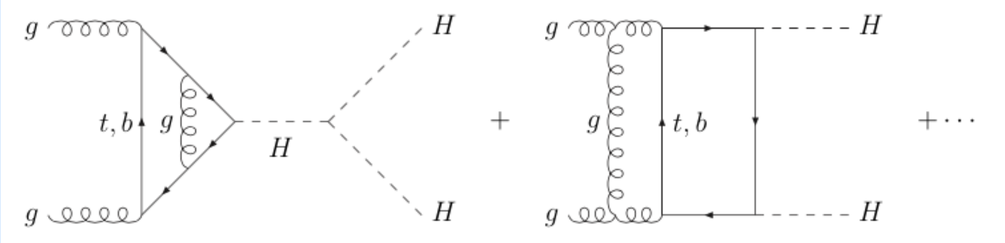
NLO Corrections

$$\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta\sigma_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

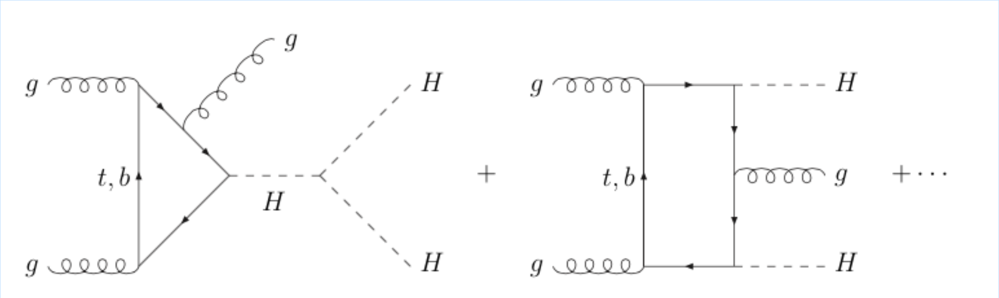
σ_{LO}:



$\Delta\sigma_{virt}$:

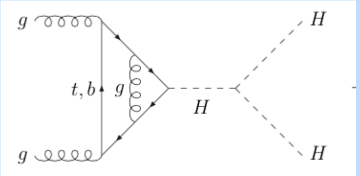


$\Delta\sigma_{ij}$:



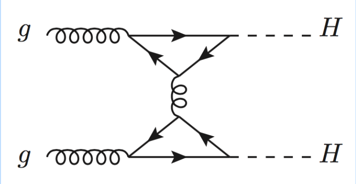
Virtual Corrections

Triangular diagrams



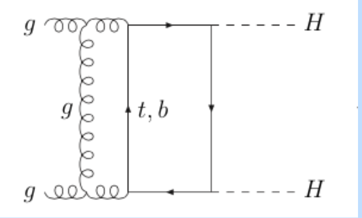
- Use existing results of single Higgs calculation

One-particle reducible diagrams



- Use existing results of $H \to Z\gamma$

Box diagrams



- Treat every diagram individually (no reduction to master integrals)
- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions
- Extract the infrared and collinear divergences using a 'proper' subtraction of the integrand
- Integration by parts due to numerical instabilities above the thresholds where $m_{hh}^2 > 0$, $m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1 - i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

Virtual Corrections

Total virtual corrections

- Numerical evaluation using Vegas (P. Lepage)

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$

$(Q^2 = m_{HH}^2)$

$$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$$

$$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$$

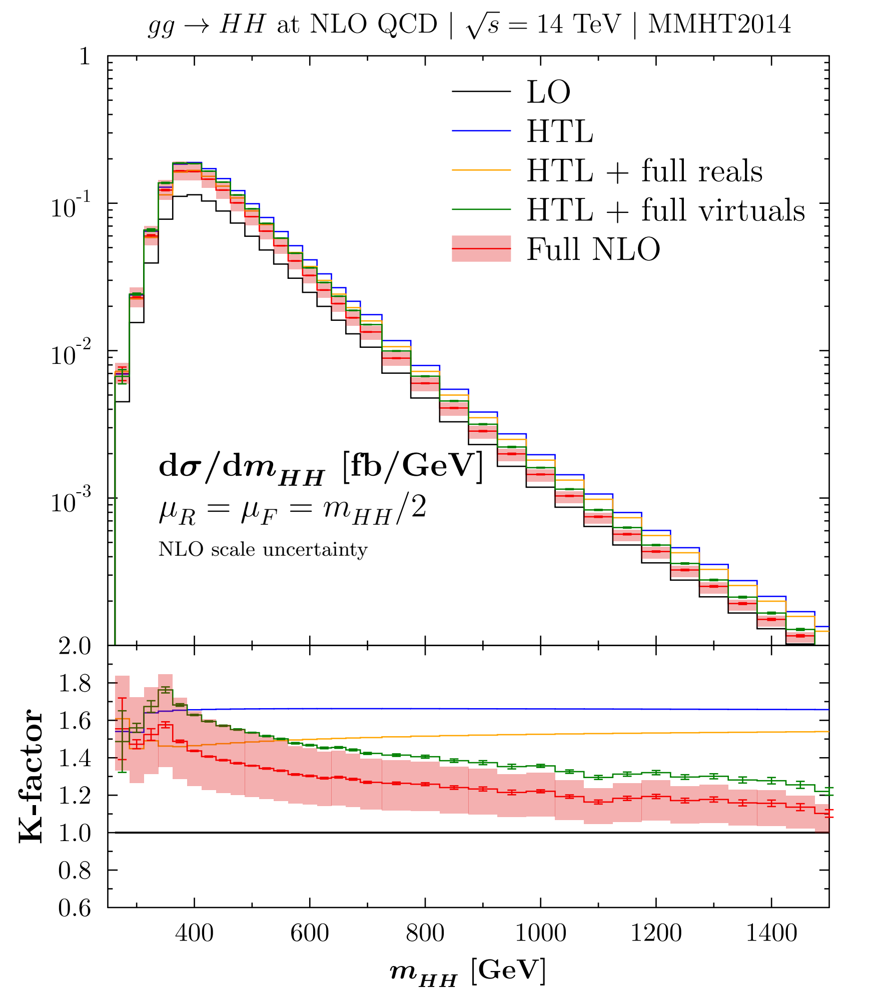
- Renormalization: α_s in $\overline{MS}$ with $N_F = 5$ and m_t on shell (central value)
- Subtraction of HTL $\longrightarrow$ IR finite top mass effects
- Numerical instabilities due to the small imaginary parts of the top mass above the thresholds: *Richardson extrapolation*

Real corrections

- Full matrix elements generated with FeynArts and FormCalc
- Matrix element in the HTL (massive LO) subtracted $\longrightarrow$ IR finite top mass effects

Results

Differential cross section



K-factor

$$K = \frac{\sigma_{\rm NLO}}{\sigma_{\rm LO}}$$

Results

Total hadronic cross section ($\sqrt{s}$ =14 TeV, m_t=172.5 GeV)

	PDF4LHC15	MMHT2014
σ_{LO}	19.80 fb	23.75 fb
σ_{NLO}^{HTL}	38.66 fb	39.34 fb
σ_{NLO}	32.78(7) fb	33.33(7) fb

(only numerical error!)

→ - 15 % NLO mass effects compared to HTL result

Borowka et al.: $\sigma_{NLO} = (32.91^{+13.6\%}_{-12.6\%})$ fb using PDF4LHC15 PDFs (m_t=173 GeV)

Factorisation / renormalisation scale dependence

varying both scales by a factor of two around central value of $\mu_F = \mu_R = m_{hh}/2$

Differential cross section:

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(8)^{+15.3\%}_{-13.0\%}\,\mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+14.4\%}_{-12.8\%}\,\mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=600\,\mathrm{GeV}} = 0.03204(9)^{+10.9\%}_{-11.5\%}\,\mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg} \to \mathrm{HH})}{\mathrm{dQ}}\right|_{Q=1200\,\mathrm{GeV}} = 0.000435(4)^{+7.1\%}_{-10.6\%}\,\mathrm{fb/GeV}$$

Total cross section:

$$\sigma(gg \to HH) = 32.78(7)^{+13.5\%}_{-12.5\%} \quad (\mathrm{PDF4LHC15})$$

$(\sqrt{s} = 14\,\mathrm{TeV})$

Uncertainty due to m_t: differential cross section

- uncertainty related to the scheme and scale choice of the top mass
- calculated the total NLO results for the differential cross section for the $\overline{MS}$ top mass at different scale choices
- $\overline{MS}$ top mass in the range [Q/4,Q], m_t

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=600\,\mathrm{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=1200\,\mathrm{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \mathrm{fb/GeV}$$

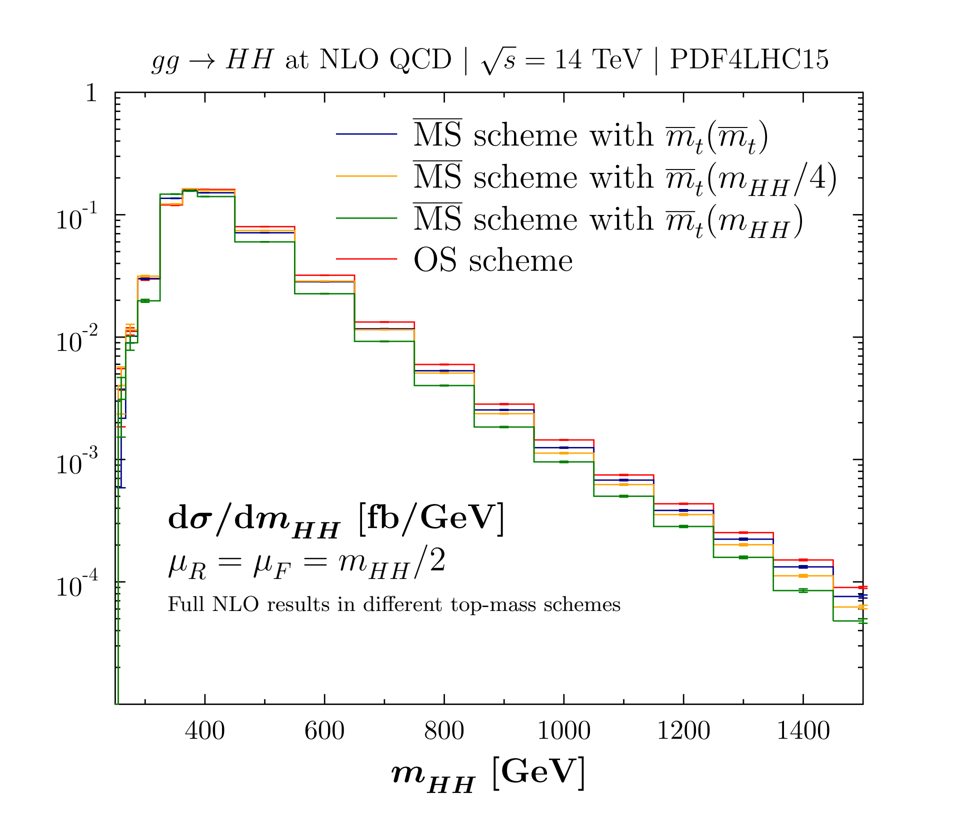
$(\sqrt{s} = 14\,\mathrm{TeV})$

Uncertainty due to m_t: total hadronic cross section

Take for individual Q values the maximum / minimum differential cross section and integrate

$$\sigma(gg \to HH) = 32.78(7)^{+4.0\%}_{-17\%}$$

with PDF4LHC15



Results

Uncertainty due to m_t for single Higgs

→ $\overline{MS}$ top mass in the range [Q/4,Q]

$$\sigma(gg \to H)\big|_{m_H=125\,\mathrm{GeV}} = 42.17^{+0.4\%}_{-0.5\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\big|_{m_H=300\,\mathrm{GeV}} = 9.85^{+7.5\%}_{-0.3\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\big|_{m_H=400\,\mathrm{GeV}} = 9.43^{+0.1\%}_{-0.9\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\big|_{m_H=600\,\mathrm{GeV}} = 1.97^{+0.0\%}_{-15.9\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\big|_{m_H=900\,\mathrm{GeV}} = 0.230^{+0.0\%}_{-22.3\%}\ \mathrm{pb}$$

$$\sigma(gg \to H)\big|_{m_H=1200\,\mathrm{GeV}} = 0.402^{+0.0\%}_{-26.0\%}\ \mathrm{pb}$$

Conclusions

- Higgs pair production via gluon fusion at NLO including the top mass
- NLO top mass effects of ~ -15% compared to HTL result
- Factorisation / renormalisation scale dependence: ~ 15% uncertainties
- Top mass scheme and scale uncertainties: $\lesssim$ 30%

total cross section at 14 TeV: $\sigma(gg \to HH) = 32.78(7)^{+4.0\%}_{-17\%}$

BACK-UP

NLO Corrections

$$\sigma_{\text{NLO}}(pp \to HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

$$\begin{aligned}
\sigma_{\text{LO}} &= \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \\
\Delta\sigma_{\text{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \; C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s (1-z)^2} \right. \\
&\qquad \left. + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \; d_{q\bar{q}}(z)
\end{aligned}$$

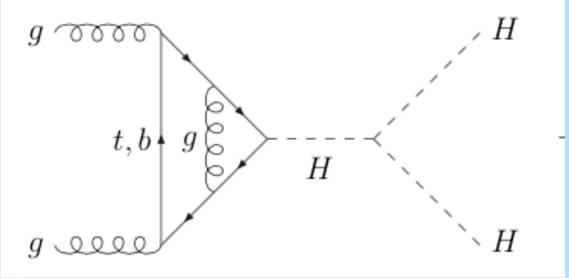
$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \qquad d_{gg} \to -\frac{11}{2}(1-z)^3, \qquad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \qquad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

Virtual Corrections

47 two-loop box diagrams + 8 triangular diagrams + 2 one-particle reducible diagrams

Triangular Diagrams

$\longleftarrow$ single Higgs case

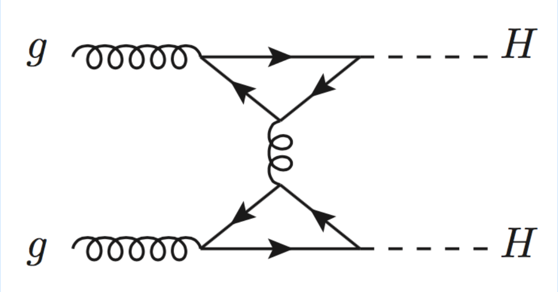


One-particle reducible diagrams

$\longrightarrow$ analytical results for $C_{\triangle\triangle}$

$(H \to Z\gamma)$

see e.g. Degrassi, Giardino, Gröber



Virtual Corrections

Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand → *Reduce, Mathematica, Form*)
- Perform Feynman parametrisation → additional 6-dimensional integrals
- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx\,\frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx\,\frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx\,\frac{f(x)-f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx\,\frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extract the infrared and collinear divergences using a ‘proper’ subtraction of the integrand (based on HTL calculation)
- Integration by parts due to numerical instabilities at the thresholds

$m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1-i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

$$\int_0^1 \mathrm{dx}\,\frac{\mathrm{f(x)}}{(\mathrm{a}+\mathrm{bx})^3} = \frac{\mathrm{f}(0)}{2\mathrm{a}^2\mathrm{b}} - \frac{\mathrm{f}(1)}{2\mathrm{b}(\mathrm{a}+\mathrm{b})^2} + \int_0^1 \mathrm{dx}\,\frac{\mathrm{f}'(\mathrm{x})}{2\mathrm{b}(\mathrm{a}+\mathrm{bx})^2}$$

Virtual Corrections

Differential cross section

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}} \qquad (Q^2 = m^2_{HH})$$

$$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$$

$$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$$

$\longrightarrow$ 7 dimensional integrals (6 Feynman and one phase space integration)

$\longrightarrow$ use Vegas for numerical integration (P. Lepage)

$\longrightarrow$ numerical instabilities due to the small imaginary parts of the top mass above the thresholds: Richardson extrapolation

Virtual Corrections

Renormalisation

α_s and m_t need to be renormalised

→ α_s in $\overline{MS}$ with $N_F = 5$

→ m_t on shell (→ central prediction)

$$\delta\sigma = \delta\alpha_s \frac{\delta\sigma_{LO}}{\delta\alpha_s} + \delta m_t \frac{\delta\sigma_{LO}}{\delta m_t}$$

Subtraction of the heavy-top limit → virtual mass effects only (infrared finite)

$$\Delta C_{mass} = C^0 - C^0_{HTL}$$

Adding back the results in the heavy-top limit (HPAIR)

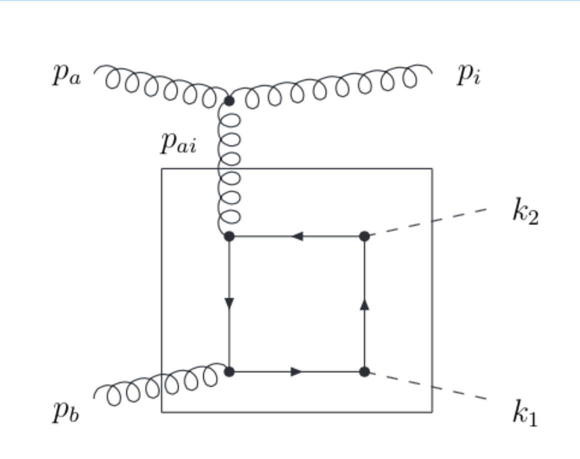
$$C = \underset{\substack{\uparrow \\ \text{HPAIR}}}{C_{HTL}} + \Delta C_{mass}$$

Real Corrections

Processes: $gg, q\bar{q} \to HHg;\ gq \to HHq$

- Full matrix elements generated with FeynArts and FormCalc
- Matrix elements in the heavy-top limit rescaled locally by massive LO matrix elements $\longrightarrow$ subtracted $\longrightarrow$ free of divergences

  (appropriate mapping on LO kinematics $\longleftarrow$ Catani Seymour)
- Adding back the results in the heavy-top limit (HPAIR)



Virtual Corrections

Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand → *Reduce, Mathematica*)

- Use dimensional regularisation: $D = 4 - 2\epsilon$

- Perform Feynman parametrisation → additional 6-dimensional integrals

$$\frac{1}{A_1^{\alpha_1}\cdots A_n^{\alpha_n}} = \frac{\Gamma(\alpha_1+\ldots+\alpha_n)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_n)}\int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-\ldots-u_{n-2}} du_{n-1} \frac{u_1^{\alpha_1-1}\cdots u_{n-1}^{\alpha_{n-1}-1}(1-u_1-\ldots-u_{n-1})^{\alpha_n-1}}{[u_1A_1+\ldots+u_{n-1}A_{n-1}+(1-u_1-\ldots-u_{n-1})A_n]^{\alpha_1+\ldots+\alpha_n}},$$

- Substitution to obtain integrals from 0 to 1

- Evaluating momentum integrals using:

$$\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-M^2+i\epsilon)^N} = i\frac{(-1)^N}{(4\pi)^{D/2}}\frac{\Gamma(N-\frac{D}{2})}{\Gamma(N)}\frac{1}{(M^2-i\epsilon)^{N-\frac{D}{2}}}$$

$$\int \frac{d^Dk}{(2\pi)^D}\frac{k_\mu k_\nu}{(k^2-M^2+i\epsilon)^N} = \frac{i}{2}\frac{(-1)^{N-1}}{(4\pi)^{D/2}}\frac{\Gamma(N-1-\frac{D}{2})}{\Gamma(N)}\frac{g_{\mu\nu}}{(M^2-i\epsilon)^{N-1-\frac{D}{2}}}$$

etc.

Virtual Corrections

Divergences

- Integration by parts due to numerical instabilities at the thresholds

$m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1 - i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

$$\int_0^1 \mathrm{dx}\, \frac{\mathrm{f(x)}}{(\mathrm{a+bx})^3} = \frac{\mathrm{f(0)}}{2\mathrm{a}^2\mathrm{b}} - \frac{\mathrm{f(1)}}{2\mathrm{b(a+b)}^2} + \int_0^1 \mathrm{dx}\, \frac{\mathrm{f'(x)}}{2\mathrm{b(a+bx)}^2}$$

(more involved for second order polynomials)

further integration by parts not successful since new divergences are created (investigated further)

Virtual Corrections

Divergences

- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}}$$

$$= \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extract the infrared and collinear divergences using a proper subtraction of the integrand

denominator: $N = ar^2 + br + c$ $\qquad a = \mathcal{O}(\rho)$ $\qquad \rho_s = \frac{\hat{s}}{m_Q^2}$

$N_0 = br + c$ $\qquad b = 1 + \mathcal{O}(\rho)$

$c = -\rho_s x(1-x)(1-s)t$

$$\int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \frac{rH(\vec{x},r)}{N^{3+2\epsilon}} = \int_0^1 \mathrm{d}\vec{x}\mathrm{d}r \left\{ \left(\frac{rH(\vec{x},r)}{N^{3+2\epsilon}} - \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \right) + \frac{rH(\vec{x},0)}{N_0^{3+2\epsilon}} \right\}$$

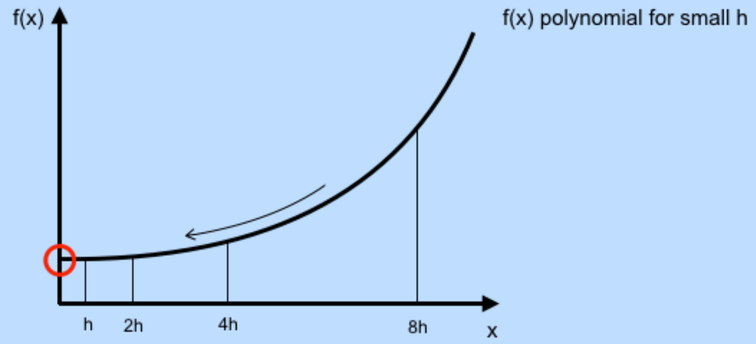
Taylor expansion in ϵ $\qquad$ analytical r-integration

Richardson extrapolation

⟶ sequence acceleration method to obtain a better convergence behaviour

Approximation polynomial

$$M_{i+1}(h) = \frac{t^{k_i} M_i(\frac{h}{t}) - M_i(h)}{t^{k_i} - 1}$$



h and *h/t* the two step sizes and k_i the truncation error

$$M_2[f(h), f(2h)] = 2f(h) - f(2h) \qquad = f(0) + \mathcal{O}(h^2)$$

$$M_4[f(h), f(2h), f(4h)] = (8f(h) - 6f(2h) + f(4h))/3 \qquad = f(0) + \mathcal{O}(h^3)$$

$$M_8[f(h), f(2h), f(4h), f(8h)] = (64f(h) - 56f(2h) + 14f(4h) - f(8h))/21 \qquad = f(0) + \mathcal{O}(h^4)$$

In our case $h = \bar{\epsilon}$ and $\bar{\epsilon}_n = 0.05 \times 2^n \quad n = 0 \ldots 9$

Theoretical error from Richardson extrapolation estimated by the difference of the fifth and the

Numerical Instabilities

1. due to phase-space integration over Mandelstam variable *t*
   → cut-off at $t = 10^{-8}$ for individual diagrams (total sum finite)
   → logarithmic substitution with $y = \log \frac{t - t_-}{m_t^2}$

2. due to the small imaginary parts $\bar{\epsilon}$ of the top mass above the thresholds, where
   $m_t^2 \to m_t^2(1 - i\bar{\epsilon})$
   need value in narrow width approximation where $\bar{\epsilon} \to 0$
   calculate partonic cross section for different $\bar{\epsilon}$ → Richardson extrapolation

Real Corrections

Processes: $gg, q\bar{q} \to HHg;\ gq \to HHq$

- Full matrix elements generated with FeynArts and FormCalc
- Matrix elements in the heavy-top limit rescaled locally by massive LO matrix elements ⟶ subtracted ⟶ free of divergences

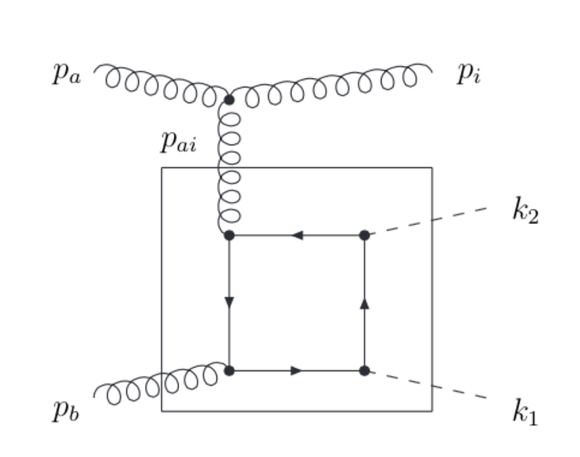
$$\frac{|\mathcal{M}^R(p_i,k_j)|^2}{|\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)|^2} - \frac{|\mathcal{M}^R_{HTL}(p_i,k_j)|^2}{|\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)|^2} = \frac{\Delta^{RM}(p_i,k_j)}{|\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)|^2}$$

$\mathcal{M}^R(p_i,k_j)$ = real NLO matrix element

$\mathcal{M}^R_{HTL}(p_i,k_j)$ = real NLO matrix element in the HTL

$\mathcal{M}_{LO}(\tilde{p}_i,\tilde{k}_j)$ = LO matrix element

$\Delta^{RM}(p_i,k_j)$ = remaining finite part of the NLO matrix element



- Adding back the results in the heavy-top limit (HPAIR)

Total hadronic cross section

Range 275-300 GeV:

- extension of Boole's rule (*h* step size of 5 GeV)

$$\int_{x_0}^{x_5} f(x)\mathrm{d}x \approx \frac{5h}{288}[19f(x_0) + 75f(x_1) + 50f(x_2) + 50f(x_3) + 75f(x_4) + 19f(x_5)]$$

error: $-\frac{275}{12096}h^7 f(x_6) \qquad (x_0 < x_6 < x_5)$

Range 300-1500 GeV:

- Richardson extrapolation for differential cross sections with bin sizes of 50, 100, 200 and 400 GeV
- Trapezoidal rule

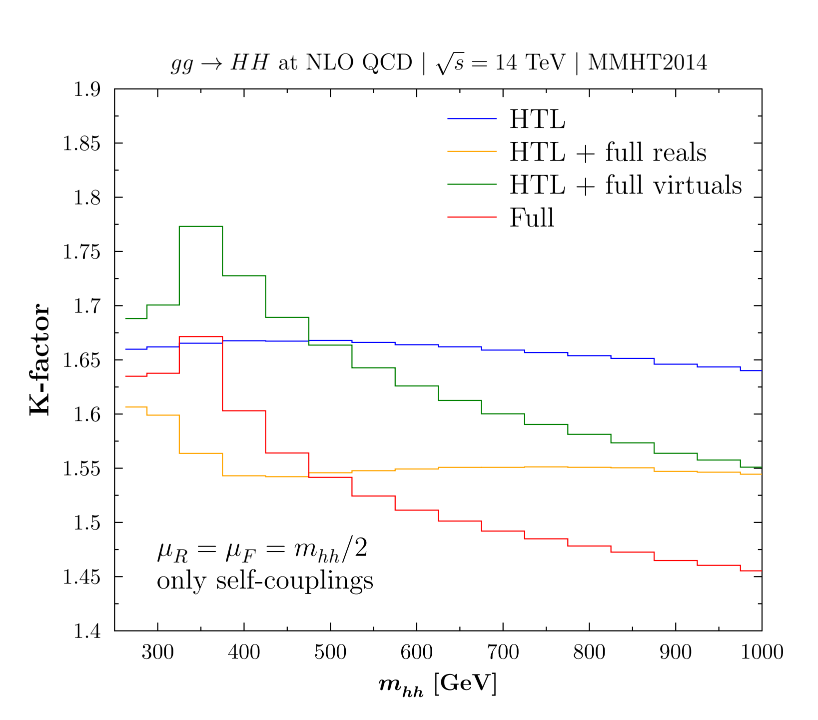
$$\int_a^b f(x)\mathrm{d}x \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

error: $-\frac{nh^3}{12} f''(\zeta) \qquad (a < \zeta < b)$

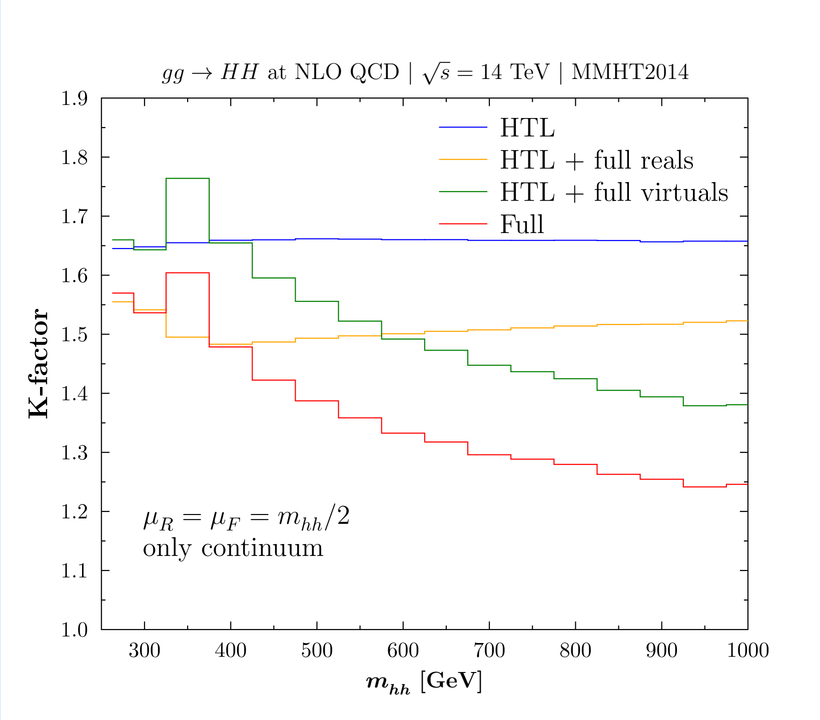
K-factor distribution

$$K = \frac{\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$$

Triangular contributions

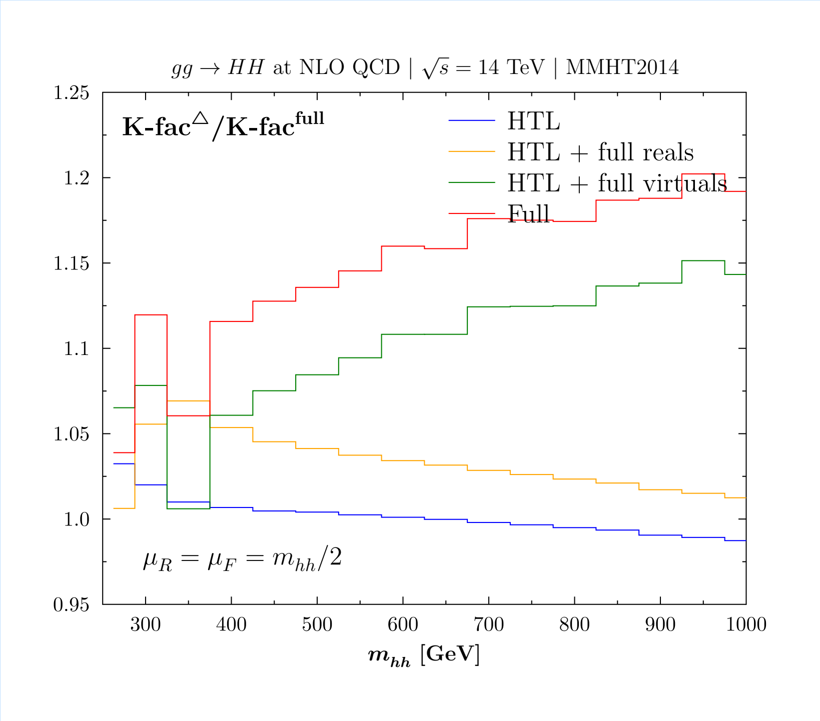


Box contributions



Results

Triangular contributions



Box contributions