Parton distributions from lattice data: the nonsinglet case

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based on arxiv:1907.06037 in collaboration with L. Del Debbio and K. Cichy

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PDFs from global QCD analysis

1. Factorization theorem relating any sufficiently inclusive observable to the collinear PDFs

$$\mathcal{O}_I\left(x, Q^2\right) = \sum_i C_i^I\left(\alpha_s\left(Q^2\right)\right) \otimes f_i\left(Q^2\right) + \text{power corrections}$$

with

$$C_{I,i} \otimes f_i \equiv \int_x^1 \frac{dy}{y} C_i^I\left(\frac{x}{y}\right) f_i\left(y\right)$$

2. Perturbative input: coefficient functions and DGLAP evolution

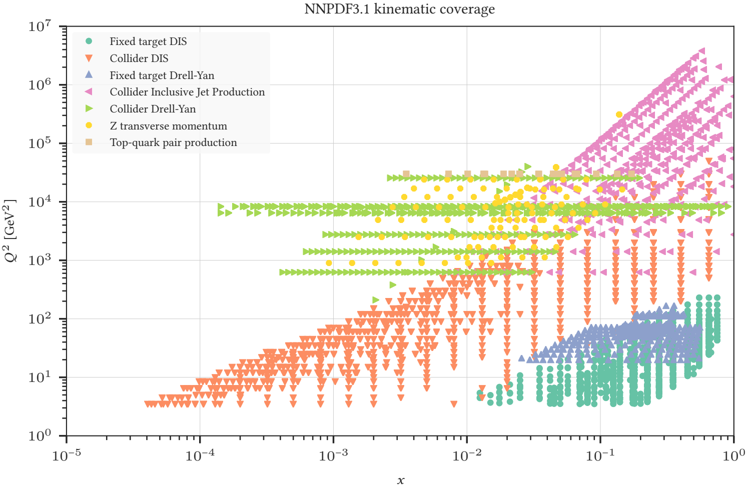
$$C_{I,i} = \sum_k C_{I,i}^{(k)} \alpha_s^k$$

$$f_i\left(x, Q^2\right) = \sum_j \Gamma_{ij}\left(Q^2, Q_0^2, \alpha_s\right) \otimes f_j\left(y, Q_0^2\right)$$

with

$$\Gamma_{ij} = \sum_k \Gamma_{ij}^{(k)} \alpha_s^k$$

3. Experimental data


NNPDF3.1 kinematic coverage
Fixed target DIS
Collider DIS
Fixed target Drell-Yan
Collider Inclusive Jet Production
Collider Drell-Yan
Z transverse momentum
Top-quark pair production
Q^2 [GeV^2]
x
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{0}
10^{1}
10^{2}
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10^{4}
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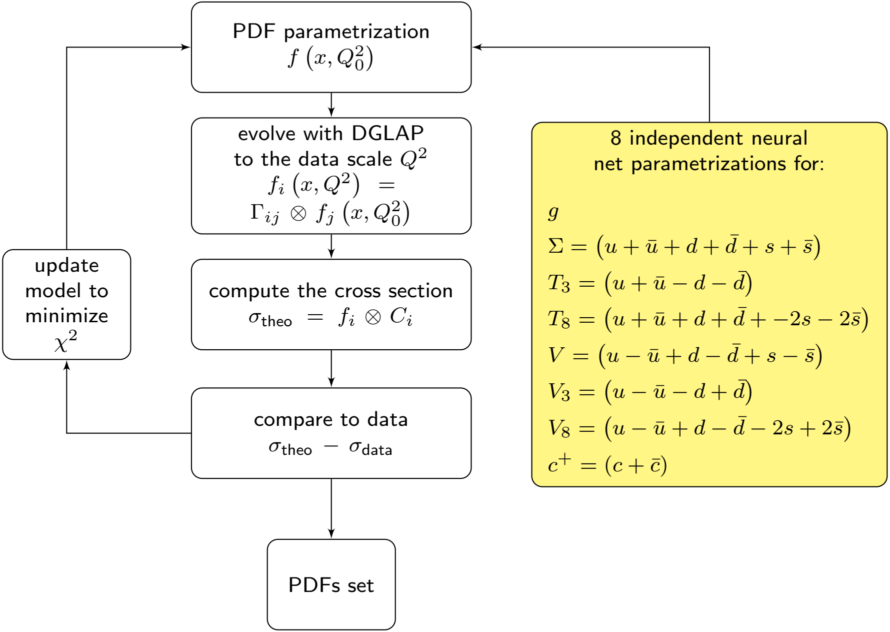
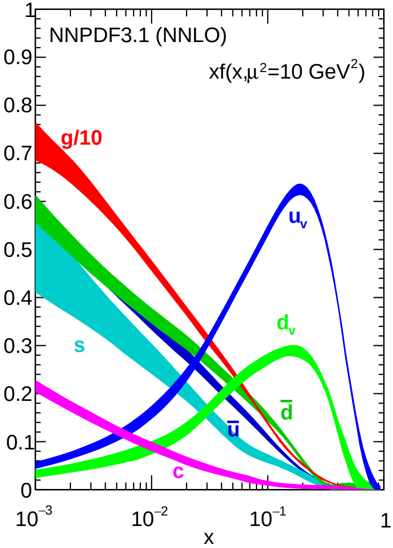
10^{6}
10^{7}


PDF parametrization
$f\left(x, Q_0^2\right)$
evolve with DGLAP
to the data scale Q^2
$f_i\left(x, Q^2\right) = \Gamma_{ij} \otimes f_j\left(x, Q_0^2\right)$
compute the cross section
$\sigma_{\text{theo}} = f_i \otimes C_i$
compare to data
$\sigma_{\text{theo}} - \sigma_{\text{data}}$
update model to minimize χ^2
PDFs set
8 independent neural net parametrizations for:
g
$\Sigma = (u + \bar{u} + d + \bar{d} + s + \bar{s})$
$T_3 = (u + \bar{u} - d - \bar{d})$
$T_8 = (u + \bar{u} + d + \bar{d} + -2s - 2\bar{s})$
$V = (u - \bar{u} + d - \bar{d} + s - \bar{s})$
$V_3 = (u - \bar{u} - d + \bar{d})$
$V_8 = (u - \bar{u} + d - \bar{d} - 2s + 2\bar{s})$
$c^+ = (c + \bar{c})$



- excellent description of all the experimental data included in the analysis
- some PDFs are constrained better than others and some x-regions have bigger error bands

What about lattice data?

As before we need 3 things:

1. Factorization theorem relating some lattice observable to the collinear PDFs

$$\mathcal{O}_I\left(z, Q^2\right) = \sum_i \mathcal{C}_i^I\left(\alpha_s\left(Q^2\right)\right) \circledast f_i\left(Q^2\right) + \text{power corrections}$$

   where the specific expression of $\mathcal{C}_i^I$ and the definition of the operation $\circledast$ will depend on the specific observable we are interested in.

2. Perturbative input: coefficient functions and DGLAP evolution

$$\mathcal{C}_{I,i} = \sum_k \mathcal{C}_{I,i}^{(k)}\,\alpha_s^k$$

$$f_i\left(x, Q^2\right) = \sum_j \Gamma_{ij}\left(Q^2, Q_0^2\right) \otimes f_j\left(Q_0^2\right) \quad \text{with} \quad \Gamma_{ij} = \sum_k \Gamma_{ij}^{(k)}\,\alpha_s^k$$

3. Lattice data

- we can use a generic fitting framework to include different lattice observables in the same fit [Yan-Qing Ma, Jian-Wei Qiu 2017]
- we can include both lattice and experimental data points in the same analysis

An example: quasi-PDF as an observable

Starting from the Ioffe-time distribution

$$\mathcal{M}_\mu^{(0)}(n,P) = \langle P|\bar{\psi}_{\rm q}^{(0)}(n)\,\Gamma_\mu\,U(n,0)\,\psi_{\rm q}^{(0)}(0)\,|P\rangle$$

$$= 2P_\mu\,h_\Gamma\left(n\cdot P, n^2\right) + n_\mu\,\tilde{h}_\Gamma\left(n\cdot P, n^2\right)$$

and taking
- gamma structure: γ^0
- pure-spatial separation: $n=(0,0,0,z)\,,\quad n^2=-z^2$

we can define

$$\boxed{\tilde{f}_{\sf q}^{(0)}(x) = \int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{i\,x\,P_z\,z}h_{\gamma^0}\left(-zP_z,-z^2\right)}$$

quasi-PDF [X.Ji, 2013]

The relation to the light-cone PDFs is given by

$$\boxed{\tilde{f}_A\left(x,P_z,\mu^2\right) = \int_{-1}^{+1}\frac{dy}{|y|}\,C_A\left(\frac{x}{y},\frac{\mu}{|y|P_z}\right)\,f_A\left(y,\mu^2\right) + \mathcal{O}\left(\frac{M^2}{P_z^2},\frac{\Lambda^2_{\sf QCD}}{P_z^2}\right)}$$

Matching relations

Factorization theorem

Unpolarized isovector parton distribution

$$f_3\left(x,\mu^2\right) = \begin{cases} u\left(x,\mu^2\right) - d\left(x,\mu^2\right)\,, & \text{if} \quad x > 0 \\ -\bar{u}\left(-x,\mu^2\right) + \bar{d}\left(-x,\mu^2\right)\,, & \text{if} \quad x < 0 \end{cases}$$

$$\mathsf{h}_{\gamma^0}\left(z,\mu\right) = \int_{-\infty}^{\infty} dx\, e^{-i(xP_z)z} \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f_3\left(y,\mu^2\right)$$

$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(z,\mu\right) \equiv \int_{-\infty}^{\infty} dx\, \cos\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f_3\left(y,\mu^2\right)$$
$$= \int_0^1 dx\, \mathcal{C}^{\mathsf{Re}}_3\left(x, z, \frac{\mu}{P_z}\right) V_3\left(x,\mu\right)$$

$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}\left(z,\mu\right) \equiv \int_{-\infty}^{\infty} dx\, \sin\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f_3\left(y,\mu^2\right)$$
$$= \int_0^1 dx\, \mathcal{C}^{\mathsf{Im}}_3\left(x, z, \frac{\mu}{P_z}\right) T_3\left(x,\mu\right)$$

In the NNPDF framework, all the observables are implemented in terms of FastKernel tables

$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}(z,\mu) = \int_0^1 dx\, \mathcal{C}^{\mathsf{Im}}_3\left(x, z, \frac{\mu}{P_z}\right) T_3\left(x,\mu^2\right)$$

$$T_3\left(x,\mu^2\right) = \int_x^1 \frac{dy}{y}\, \mathsf{K}^{(+)}_3\left(\frac{x}{y}, \alpha_s, \alpha_s^0\right) T_3\left(y,\mu_0^2\right)$$

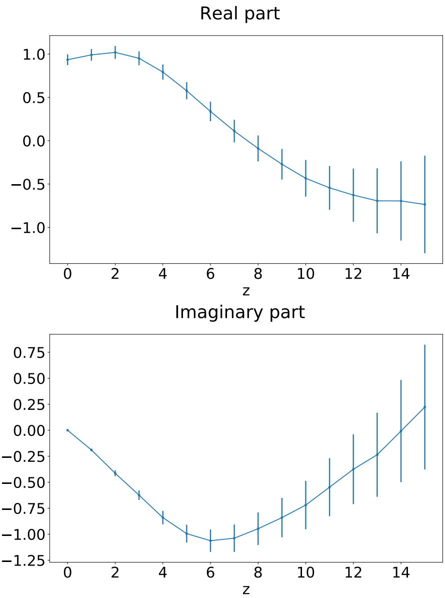
Interpolation of the PDF at a chosen scale (μ and μ_0)

$$T_3\left(x,\mu^2\right) = \sum_\beta T_3\left(x_\beta,\mu^2\right) \mathcal{I}^{(\beta)}(x) + \mathcal{O}\left[\left(x_{\beta+1} - x_\beta\right)^p\right]$$

$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}(z,\mu) = \sum_\alpha T_3\left(x_\alpha,\mu^2\right) \int_0^1 dx\, \mathcal{C}^{\mathsf{Im}}_3\left(x, z, \frac{\mu}{P_z}\right) \mathcal{I}^{(\alpha)}(x)$$

$$= \sum_{\alpha,\beta} T_3\left(x_\beta,\mu_0^2\right) \underbrace{\boxed{\int_{x_\alpha}^1 \frac{dy}{y}\, \mathsf{K}^{(+)}_3\left(\frac{x_\alpha}{y}, \alpha_s, \alpha_s^0\right) \mathcal{I}^{(\beta)}(y)}}_{K_{\alpha\beta}} \underbrace{\boxed{\int_0^1 dx\, \mathcal{C}^{\mathsf{Im}}_3\left(x, z, \frac{\mu}{P_z}\right) \mathcal{I}^{(\alpha)}(x)}}_{\mathcal{C}_{z\alpha}}$$

$$\rightarrow \underbrace{\boxed{\mathcal{O}^{\mathrm{Im}}_{\gamma^0}(z,\mu) = \sum_\beta \mathcal{H}^{\mathrm{Im}}_{z\beta}\, T_3\left(x_\beta,\mu_0^2\right)}}_{\text{Observable in terms of FK table}}$$



- data from ETMC collaboration
- γ_0 Dirac structure
- non-perturbative renormalization procedure
- simulation at the physical pion mass
- data available for unpolarized, polarized and transversity cases
- $P_z = 10\pi/L$ (1.38GeV)

Lattice data systematics

Different sources of systematics from the lattice simulation

- cut-off effects (finite value of lattice spacing a, UV regulator)
- finite volume effects (finite size of the box L, IR reguator)
- excited states contaminations
- truncation effects (coefficients to go from RI'-MOM to minimal subtraction)

We came up with different possible scenarios

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

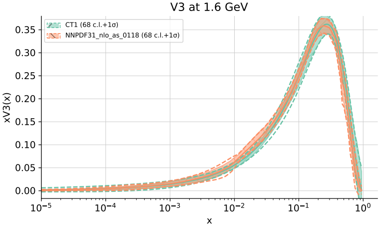
Fit settings (NNPDF framework)

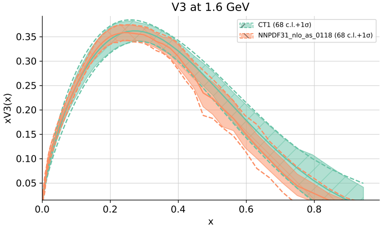
- **covariance matrix:**
  built using the available information about the lattice systematics, to be used for both the computation of the χ^2 and for replicas generation
- **Monte Carlo fit:**
  for each lattice point we generate 100 Gaussian pseudo-data (100 replicas), according to the covarinace matrix constructed using the available information about the systematics. 100 independents fits are run, the 68% CL on the ensemble of the fits gives the PDF error.
- **minimization and stopping:**
  we minimize numerically the χ^2 (CMA algorithm) over half of the available data (8 lattice points). The remaining points are used as a validation set: the final fit correposnds to the minimum of the validation error.

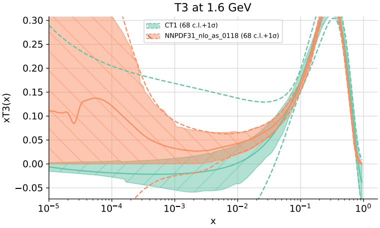
Results

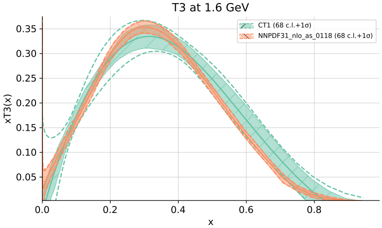
1. **closure tests**:
   we generate fake data using an input PDFs set, and we run a fit over these artificial data. By comparing our results with the input PDFs we can assess
   - how good the convolution to get the lattice observables is in constraining PDFs
   - what we should expect given different systematics scenarios
2. **fit**:
   - fit results with different systematics scenarios
   - how lattice PDFs look like in comparison to pheno ones

- small fake statistical uncertainty
- no systematics



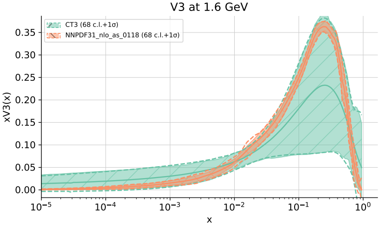


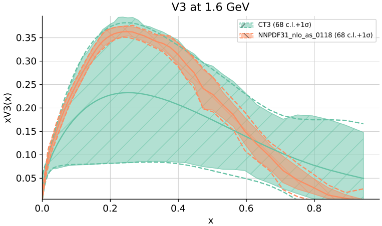


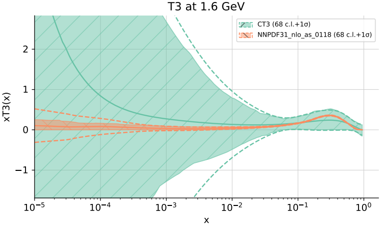


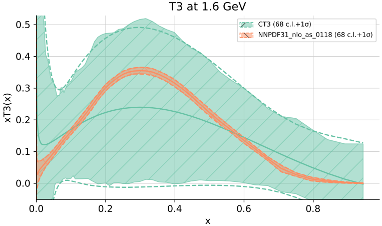
Closure test 2: real world

- real statistical uncertainty
- real systematics scenario

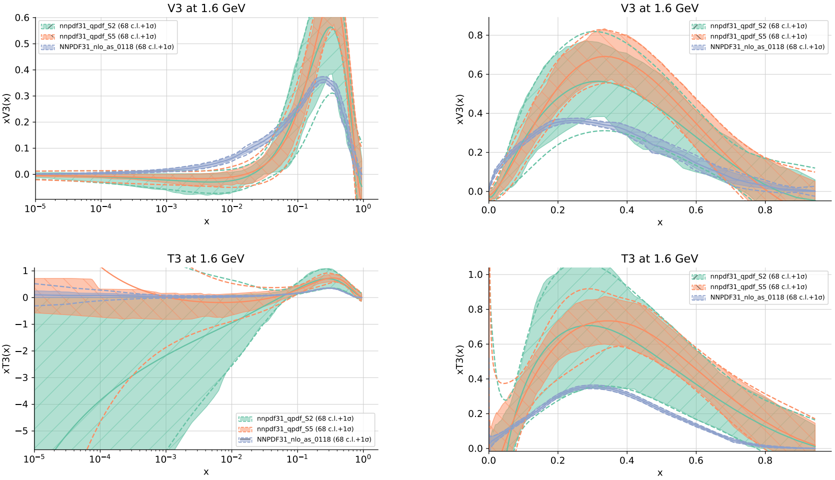








S2 and S5 scenarios (more realistic ones)



- we can treat lattice data on the same footing as experimental data
- in this framework it would be easy to run a global fit over different lattice data: any data available..?
- polarized and transversity cases: less experimental data available $\rightarrow$ could be more interesting from the phenomenological point of view

Thanks!