PDFs from first principles

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QCDSF Collaboration

Theoretical Framework

Starting point is the Compton Amplitude, to be evaluated at large values of q^2, i.e. small values of x:

$$T_{\mu\nu}(p,q) = \int \mathrm{d}^4x\, \mathrm{e}^{iqx} \langle N(p,s)|T J_\mu(x) J_\nu(0)|N(p,s)\rangle$$

$$= \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \mathcal{F}_1(\omega, q^2) + \left(p_\mu - \frac{pq}{q^2} q_\mu\right) \left(p_\nu - \frac{pq}{q^2} q_\nu\right) \frac{1}{pq} \mathcal{F}_2(\omega, q^2)$$

$$+ \; \epsilon_{\mu\nu\lambda\sigma}\, q_\lambda\, s_\sigma \frac{1}{pq}\, \mathcal{G}_1(\omega, q^2) + \; \epsilon_{\mu\nu\lambda\sigma}\, q_\lambda\, (pq\, s_\sigma - sq\, p_\sigma)\, \frac{1}{(pq)^2}\, \mathcal{G}_2(\omega, q^2)$$

In the physical region $1 \leq |\omega| \leq \infty$

$$\mathrm{Im}\, \mathcal{F}_{1,2}(\omega, q^2) = 2\pi F_{1,2}(\omega, q^2)\,, \; \mathrm{Im}\, \mathcal{G}_{1,2}(\omega, q^2) = 2\pi g_{1,2}(\omega, q^2) \qquad \omega = \frac{1}{x} = \frac{2pq}{q^2}$$

For $p_3 = q_3 = q_0 = 0$, substituting $\bar{\omega}$ by $1/x$

$$T_{33}(p,q) = \mathcal{F}_1(\omega, q^2) = 4\omega \int_0^1 dx \, \frac{\omega x}{1-(\omega x)^2} F_1(x, q^2) + \mathcal{F}_1(0, q^2)$$

$$= \sum_{n=2,4,\cdots}^{\infty} 4\omega^n \int_0^1 dx \, x^{n-1} F_1(x, q^2) + \mathcal{F}_1(0, q^2) \qquad \text{OPE}$$

$$T_{03}(p,q) \overset{\vec{s}\|\vec{p}}{=} \frac{(\vec{q}\times\vec{s})_3}{pq} \mathcal{G}_1(\omega, q^2) = \frac{(\vec{q}\times\vec{s})_3}{pq} 4\omega \int_0^1 dx \, \frac{1}{1-(\omega x)^2} g_1(x, q^2)$$

$$= \frac{(\vec{q}\times\vec{s})_3}{pq} \sum_{n=1,3,\cdots}^{\infty} 4\omega^n \int_0^1 dx \, x^{n-1} g_1(x, q^2)$$

$$T_{03}(p,q) \overset{\vec{s}\|\vec{q}}{=} -\frac{(\vec{p}\times\vec{q})_3 \, \vec{s}\vec{q}}{(pq)^2} \mathcal{G}_2(\omega, q^2) = -\frac{(\vec{p}\times\vec{q})_3 \, \vec{s}\vec{q}}{(pq)^2} 4\omega \int_0^1 dx \, \frac{1}{1-(\omega x)^2} g_2(x, q^2)$$

$$= -\frac{(\vec{p}\times\vec{q})_3 \, \vec{s}\vec{q}}{(pq)^2} \sum_{n=1,3,\cdots}^{\infty} 4\omega^n \int_0^1 dx \, x^{n-1} g_2(x, q^2)$$

OPE

Moments

$$\mu_n(q^2) = f\int_0^1 dx\, x^n\, F_1(x,q^2) = \boxed{\langle x^n\rangle}$$

PDFs are obtained by Mellin transform

$$F_1(x,q^2) = \frac{1}{2\pi i f}\int_{\mathcal{C}} ds\, x^{-s-1}\mu_s(q^2)$$

For example

$$\mu_1(q^2) = c_2(q^2a^2)\,\mathcal{V}_2(a) + \frac{c_4(q^2a^2)}{q^2}\,\mathcal{V}_4(a) + \cdots$$

Lattice: $q^2 \sim \dfrac{1}{a^2}$

Twist-2

$$\langle N(p)|\mathcal{O}_{\mu\nu}|N(p)\rangle = \mathcal{V}_2\,[p_\mu p_\nu - \text{traces}]$$

Twist-4

$$\langle N(p)|\mathcal{O}_{\mu\nu\lambda\lambda}|N(p)\rangle = \mathcal{V}_4\,[p_\mu p_\nu - \text{traces}]$$

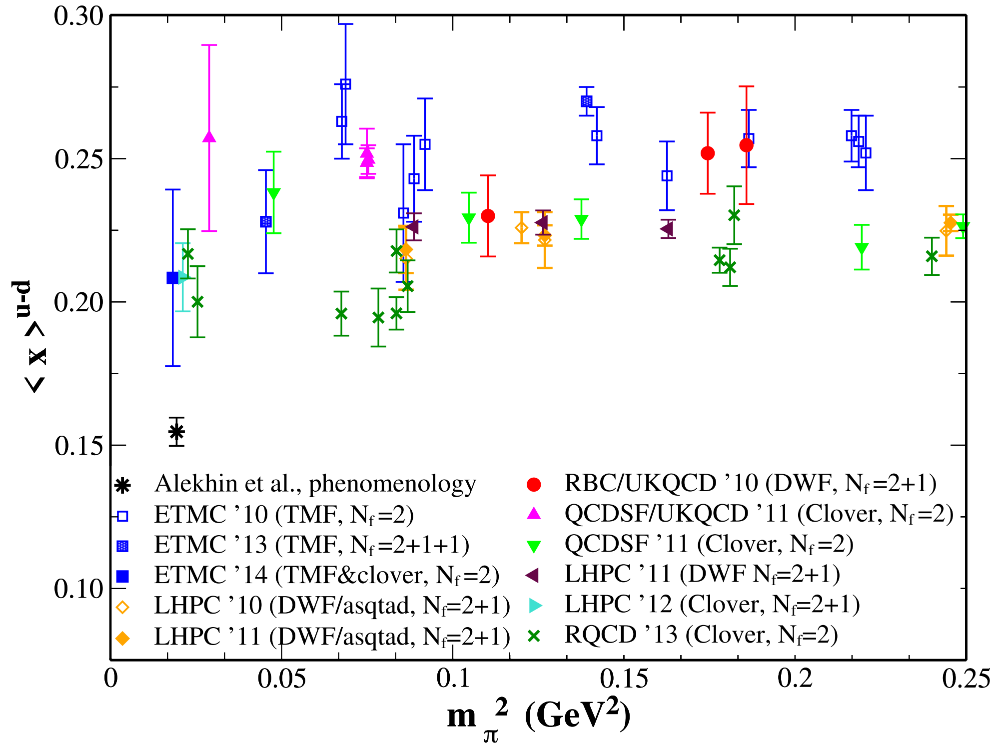
$$\mathcal{O}_{\mu_1\mu_2\cdots\mu_n} = \bar\psi\,\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}\,\psi$$

It happens that matrix elements of twist-2 mix with matrix elements of twist-4 and higher under renormalization due to power divergences

Martinelli & Sachrajda, Rossi & Testa

Twist-2 only

Exp. →



For comparison: Quasi-PDFs

Constantinou

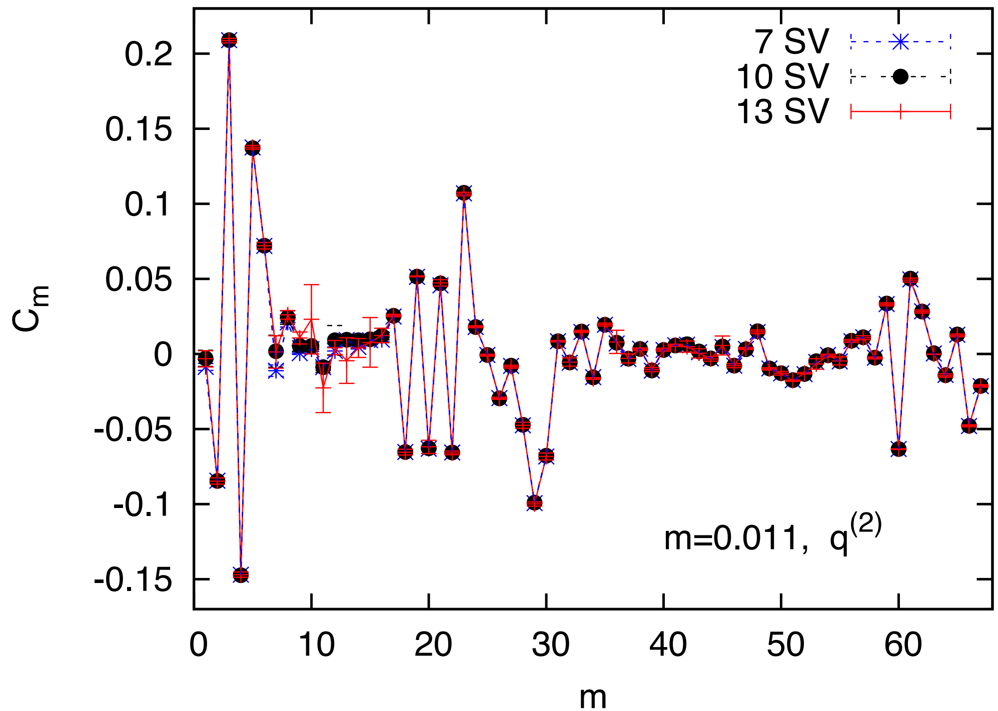
$$q(x) = \int \frac{dz}{4\pi}\, e^{i\,x\,p_z z} \langle N(p)|\mathcal{P}\,\bar{\psi}(z)\,\Gamma\, e^{-ig\int_0^z d\zeta\, A_z(\zeta)}\psi(0)|N(p)\rangle$$

Twist-2

Significance of higher twist?

Wilson coefficients

$$\ell_{\mu\nu} T_{\mu\nu} = \sum c^i_{\mu_1\mu_2\cdots\mu_n} \langle p | \mathcal{O}^i_{\mu_1\mu_2\cdots\mu_n} | p \rangle$$



QCDSF

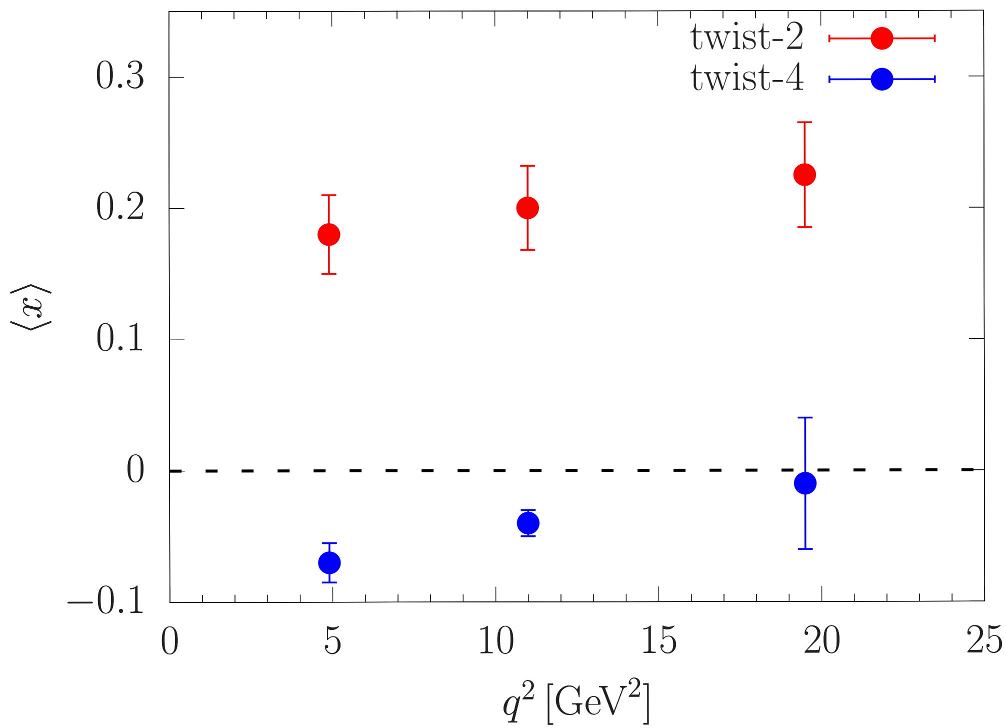
Operators taken into account

1	1	1	2	$i\gamma_\mu\gamma_i$	3						
3	$\gamma_\mu\mathcal{D}_\mu$	1	4	$\gamma_i\mathcal{D}_\mu$	3	5	$\gamma_\mu\mathcal{D}_i$	3	6	$\gamma_i\mathcal{D}_i$	3
7	$\gamma_i\mathcal{D}_j$	6									
8	$1\mathcal{D}_\mu\mathcal{D}_\mu$	1	9	$i\gamma_\mu\gamma_i\mathcal{D}_\mu\mathcal{D}_\mu$	3	10	$1\mathcal{D}_\mu\mathcal{D}_i$	3			
11	$i\gamma_\mu\gamma_i\mathcal{D}_\mu\mathcal{D}_i$	3	12	$i\gamma_\mu\gamma_i\mathcal{D}_\mu\mathcal{D}_j$	6	13	$1\mathcal{D}_i\mathcal{D}_\mu$	3	14	$i\gamma_\mu\gamma_i\mathcal{D}_i\mathcal{D}_i$	3
15	$i\gamma_\mu\gamma_i\mathcal{D}_j\mathcal{D}_\mu$	6	16	$1\mathcal{D}_i\mathcal{D}_i$	3	17	$i\gamma_\mu\gamma_i\mathcal{D}_i\mathcal{D}_i$	3	18	$i\gamma_\mu\gamma_i\mathcal{D}_j\mathcal{D}_j$	6
19	$1\mathcal{D}_i\mathcal{D}_j$	6	20	$i\gamma_\mu\gamma_i\mathcal{D}_i\mathcal{D}_j$	6	21	$i\gamma_\mu\gamma_i\mathcal{D}_j\mathcal{D}_i$	6	22	$i\gamma_\mu\gamma_i\mathcal{D}_j\mathcal{D}_k$	6
23	$\gamma_\mu\mathcal{D}_\mu\mathcal{D}_\mu\mathcal{D}_\mu$	1	24	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_\mu\mathcal{D}_\mu$	3	25	$\gamma_\mu\mathcal{D}_\mu\mathcal{D}_\mu\mathcal{D}_i$	3	26	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_\mu\mathcal{D}_\alpha$	3
27	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_\mu\mathcal{D}_i$	6	28	$\gamma_\mu\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_\mu$	3	29	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_\alpha\mathcal{D}_\mu$	3	30	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_\mu$	6
31	$\gamma_\mu\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_i$	3	32	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_\alpha\mathcal{D}_\alpha$	3	33	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_i$	6	34	$\gamma_\mu\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_j$	6
35	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_\alpha\mathcal{D}_i$	6	36	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_\alpha$	6	37	$\gamma_\alpha\mathcal{D}_\mu\mathcal{D}_i\mathcal{D}_j$	6	38	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{D}_\mu$	3
39	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{D}_\mu$	3	40	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\mu\mathcal{D}_\mu$	6	41	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{D}_\alpha$	3	42	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{D}_\alpha$	3
43	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\mu\mathcal{D}_\beta$	6	44	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{D}_i$	6	45	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{D}_i$	6	46	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\mu\mathcal{D}_\alpha$	6
47	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\mu\mathcal{D}_i$	6	48	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_\mu$	3	49	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_\mu$	3	50	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\beta\mathcal{D}_\mu$	6
51	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_\alpha$	3	52	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_\alpha$	3	53	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\beta\mathcal{D}_\beta$	6	54	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_i$	6
55	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_i$	6	56	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\beta\mathcal{D}_\alpha$	6	57	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\beta\mathcal{D}_i$	6	58	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_\mu$	6
59	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_\mu$	6	60	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\alpha\mathcal{D}_\mu$	6	61	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_i\mathcal{D}_\mu$	6	62	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_\alpha$	6
63	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_\alpha$	6	64	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\alpha\mathcal{D}_\beta$	6	65	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_i\mathcal{D}_\beta$	6	66	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_i$	6
67	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_i$	6	68	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\alpha\mathcal{D}_\alpha$	6	69	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_i\mathcal{D}_i$	6	70	$\gamma_\mu\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_j$	6
71	$\gamma_\alpha\mathcal{D}_\alpha\mathcal{D}_i\mathcal{D}_j$	6	72	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_\alpha\mathcal{D}_i$	6	73	$\gamma_\alpha\mathcal{D}_\beta\mathcal{D}_i\mathcal{D}_\alpha$	6			

Ordering not consistent with figure

Effect of twist-4 (for illustration)

$\bar{\psi}\gamma_\mu D_\nu \psi \,, \quad \bar{\psi}\gamma_\mu D_\nu D_\lambda D_\lambda \psi$

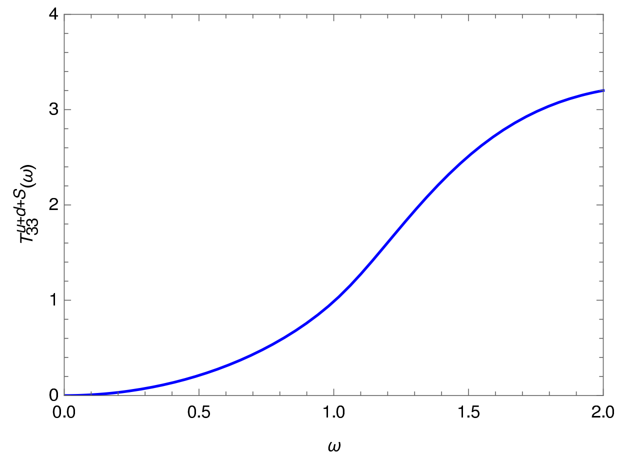


Quenched overlap

QCDSF

OPE Caveats

$$T_{33}^{u+d+S}(\omega) = 4\omega\, \mathrm{P}\int_0^1 dx\, \frac{\omega x}{1-(\omega x)^2} F_1^{u+d+S}(x)$$



Large ω / small x region incompatible with OPE Regge

Extension of OPE to Regge asymptotics:

R.A. Brandt, Nucl. Phys. B72 (1974) 125
H.-J. Thun, Nuovo Cim. A26 (1975) 329

Further reading

A.H. Mueller, Phys. Lett. B396 (1997) 251

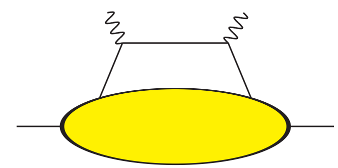
QCDSF Approach ComInv

The Compton amplitude can be computed most efficiently, including singlet (disconnected) matrix elements, by the Feynman-Hellmann technique. By introducing the perturbation to the Lagrangian

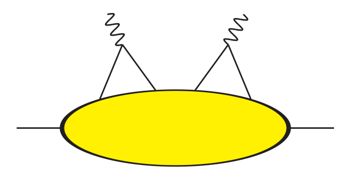
$$\mathcal{L}_{\rm QCD}(x) \to \mathcal{L}_{\rm QCD}(x) + \lambda \mathcal{J}_3(x)\,, \quad \mathcal{J}_3(x) = Z_V \cos(\vec{q}\vec{x})\; e_q\, \bar{q}(x)\gamma_3 q(x)$$

and taking the second derivative of $\langle N(\vec{p},t)\bar{N}(\vec{p},0)\rangle_\lambda \simeq C_\lambda\, {\rm e}^{-E_\lambda(p,q)\,t}$ with respect to λ on both sides, we obtain

$$-2E_\lambda(p,q)\,\frac{\partial^2}{\partial\lambda^2}E_\lambda(p,q)\,\Big|_{\lambda=0} = T_{33}(p,q)$$



The amplitude encompasses the dominating 'handbag' diagram as well as the power-suppressed 'cats ears' diagram. Varying q^2 will allow to test the twist expansion. No further renormalization is needed

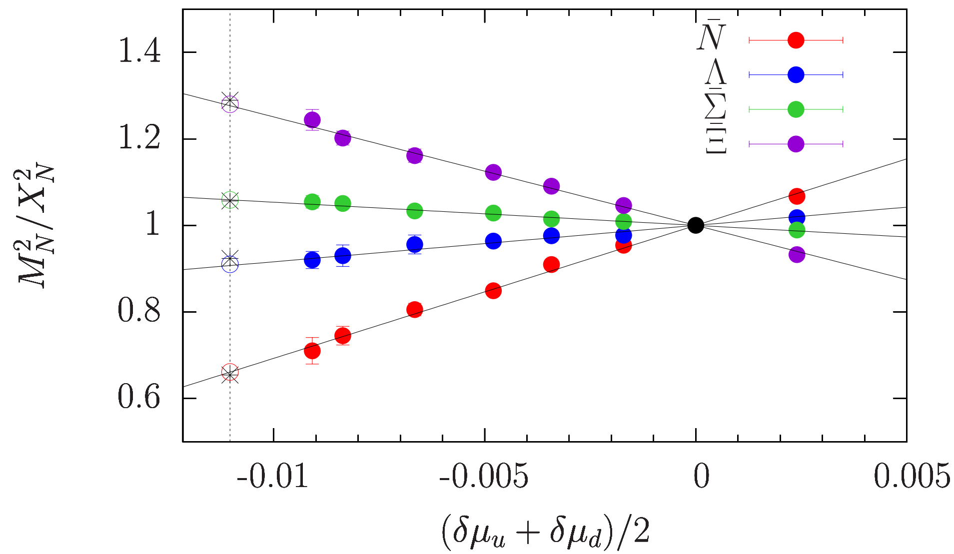


Lattices: SU(3) symmetric point

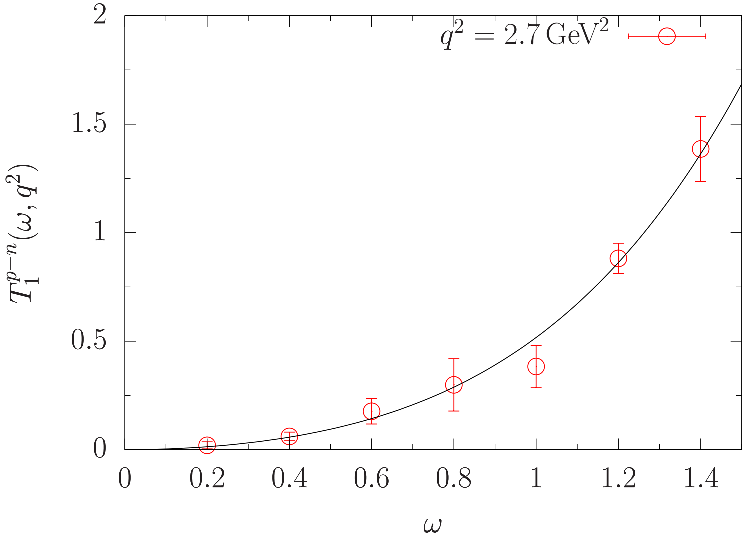
$M_\pi = M_K \approx 410\,\text{MeV}$

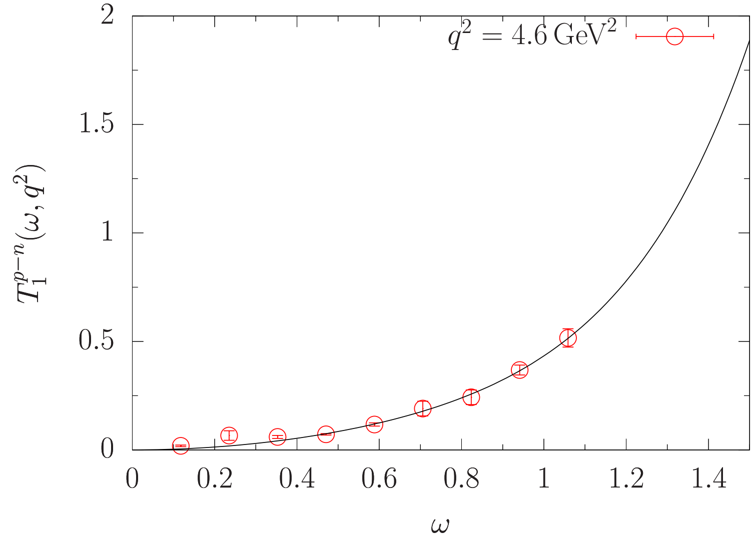
V	a [fm]	q^2 [GeV^2]
$32^3 \times 64$	0.074	1.37
$32^3 \times 64$	0.074	2.73 *
$32^3 \times 64$	0.074	3.55
$32^3 \times 64$	0.074	4.64 *
$32^3 \times 64$	0.074	5.48
$32^3 \times 64$	0.074	6.83
$32^3 \times 64$	0.074	7.10
$32^3 \times 64$	0.074	9.29
$32^3 \times 64$	0.074	12.29
$32^3 \times 64$	0.074	15.85
$48^3 \times 96$	0.068	1.44

* High statistics



Compton amplitude

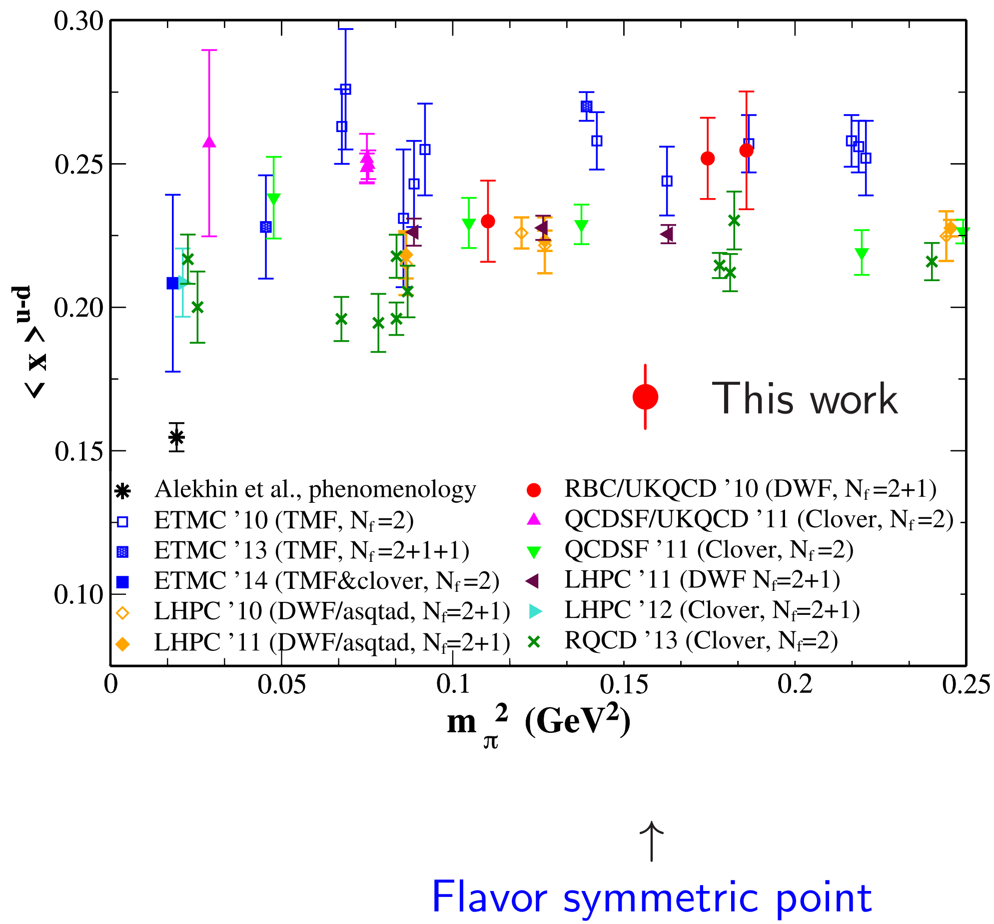




$$\langle x \rangle^{u-d} = 0.166(33)$$

$$\langle x \rangle^{u-d} = 0.167(45)$$

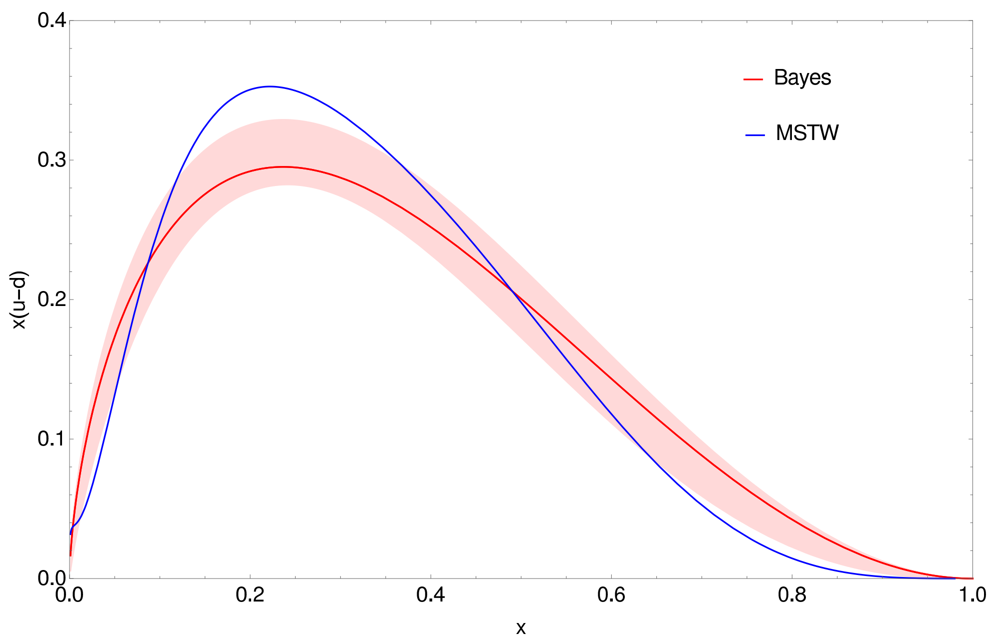
$$T^{p-n}(\omega, q) = \frac{3}{2} \langle x \rangle^{u-d} \omega^2 + O(\omega^4)$$


This work
Alekhin et al., phenomenology
ETMC '10 (TMF, Nf=2)
ETMC '13 (TMF, Nf=2+1+1)
ETMC '14 (TMF&clover, Nf=2)
LHPC '10 (DWF/asqtad, Nf=2+1)
LHPC '11 (DWF/asqtad, Nf=2+1)
RBC/UKQCD '10 (DWF, Nf=2+1)
QCDSF/UKQCD '11 (Clover, Nf=2)
QCDSF '11 (Clover, Nf=2)
LHPC '11 (DWF Nf=2+1)
LHPC '12 (Clover, Nf=2+1)
RQCD '13 (Clover, Nf=2)
0.30
0.25
0.20
0.15
0.10
0
0.05
0.1
0.15
0.2
0.25
< x >u-d
m_π^2 (GeV^2)
↑
Flavor symmetric point

PDF nonsinglet

Bayesian fit

Prior: $A\, x^{B}\, (1-x)^{C}$



$q^2 = 4.6$ GeV^2

Conclusions

ComInv appears to be a promising method to compute PDFs from first principles. Mixing of operators of leading twist with operators of higher dimension and twist under renormalization is taken into account automatically. Issues of the OPE are evaded

A generalization of ComInv to the computation of GPDs is straightforward. Consider, for example, the unpolarized case. In the Breit frame

$$T_{33}(\vec{p},-\vec{p},\vec{q}) = \int \mathrm{d}^4x\, \mathrm{e}^{iqx} \langle N(\vec{p},s)|T J_3(x) J_3(0)|N(-\vec{p},s)\rangle$$

$$= \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) 4\omega \int_0^1 dx\, \frac{\omega x}{1-(\omega x)^2} H(x,\Delta^2,q^2) \qquad \vec{\Delta} = 2\,\vec{p}$$

with

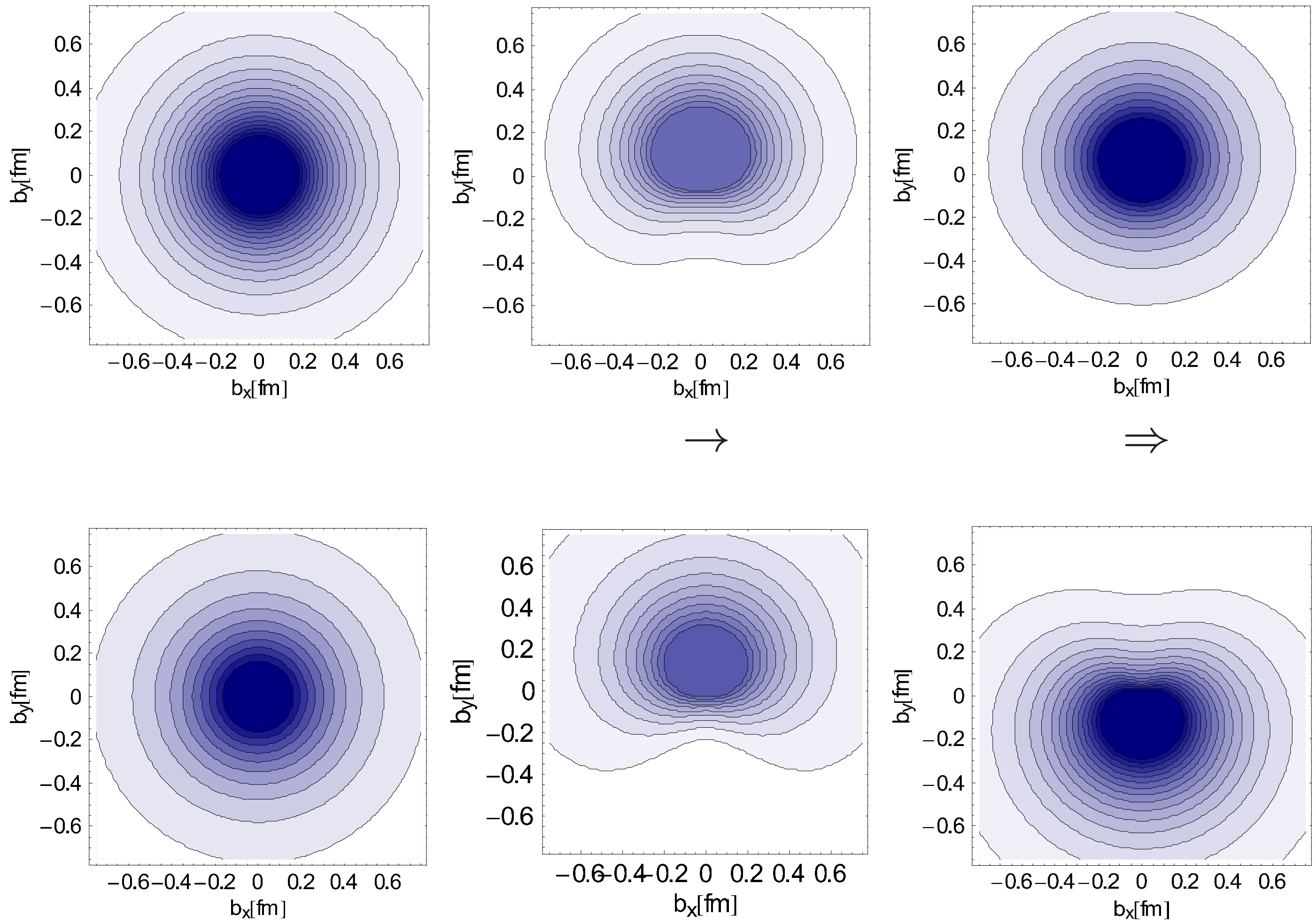
$$\mathcal{J}_3(x) = Z_V \cos(\vec{p}\vec{x})\; e_q\, \bar{q}(x)\gamma_3 q(x)$$

No extra propagators and background field configurations need to be generated

Transversity – Can we do better? $\langle x \rangle$

up

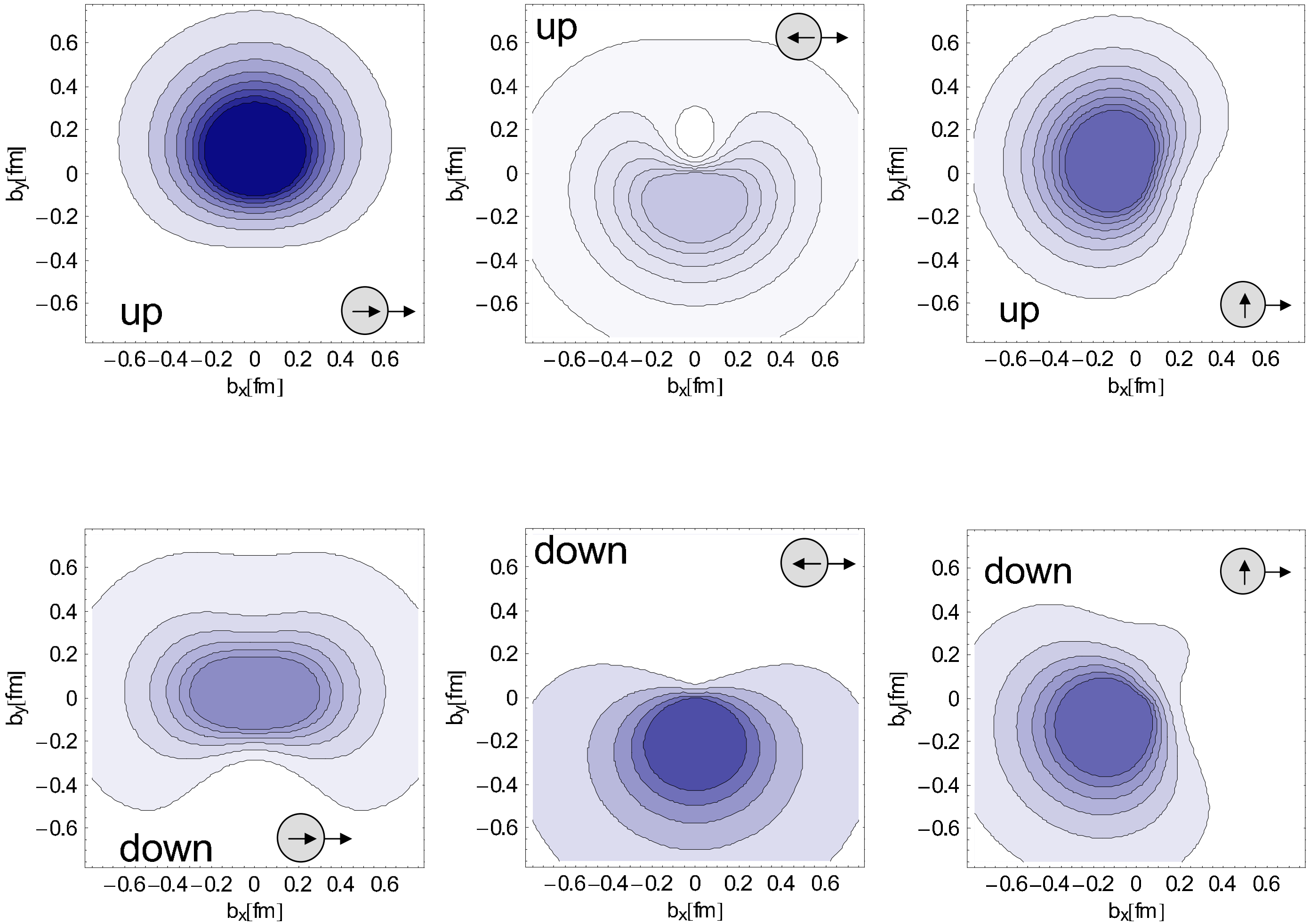
down



$\longrightarrow$ $\Longrightarrow$

$\longrightarrow$ $\Longrightarrow$

Boer–Mulders effect   Sivers effect



Displacement due to orbital angular momentum

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