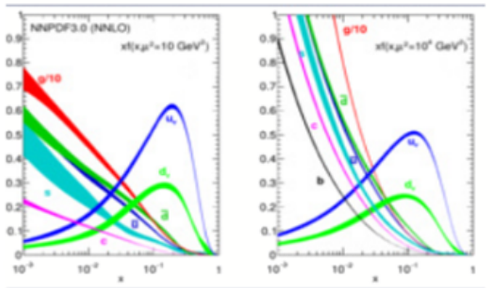
W. K . Kellogg Biological Station
MICHIGAN STATE UNIVERSITY



Parton Distributions and Lattice Calculations (PDFLattice 2019)
25-27 Sept. 2019

Transversity Overview

Marco Radici
INFN - Pavia

In collaboration with A. Bacchetta (Univ. Pavia)

a phase transition

quark polarization

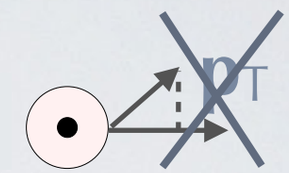
nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

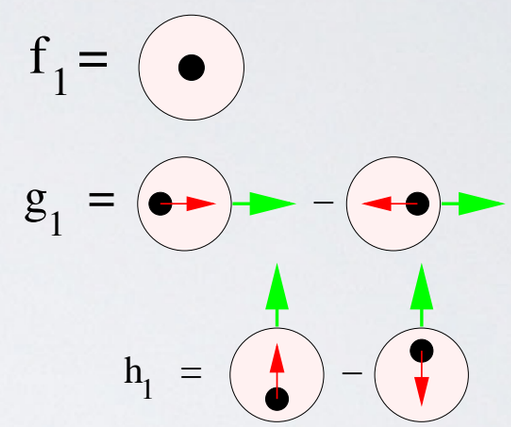
thousands data

hundreds data

tens data



collinear PDFs



chiral-odd

a phase transition

quark polarization

nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
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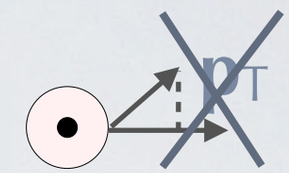
thousands data **hundreds** data **tens** data



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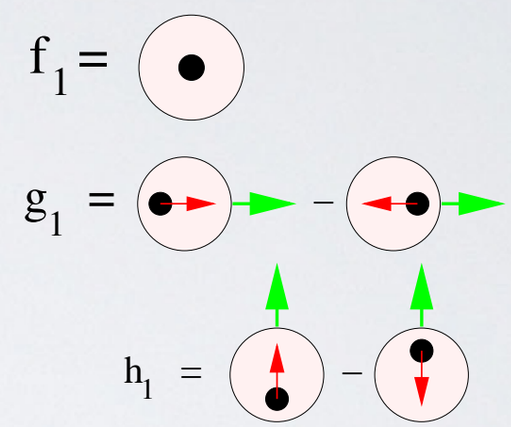


chiral-odd

first global fit
(= lepton-hadron scatt. and hadron collisions) of **PDF h_1**

Radici and Bacchetta, P.R.L. ***120*** *(18) 192001*

1 Dim (PDFs)

Explorations

Global fits

Parton model

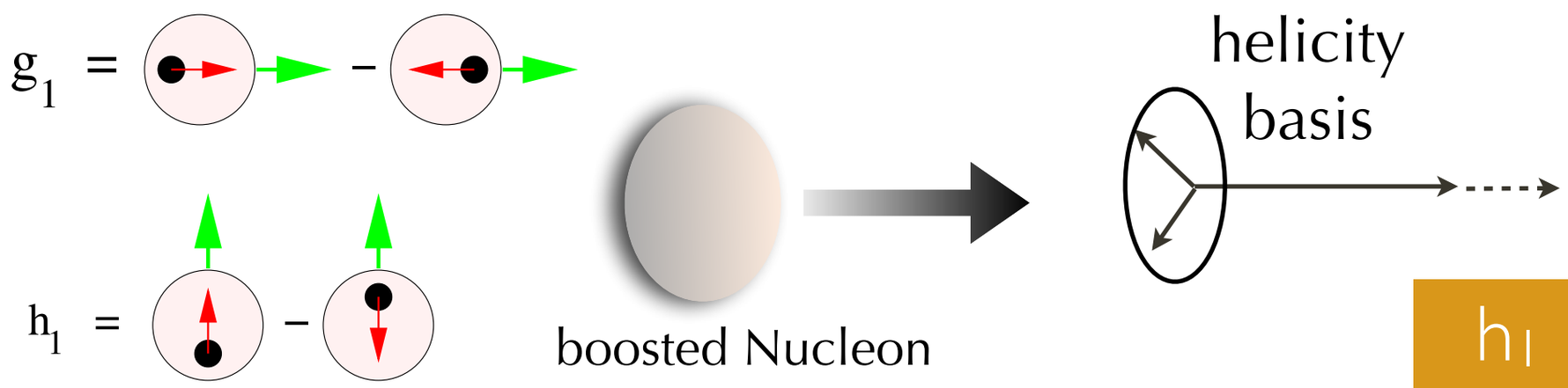
QCD analysis + data

Phase 1

Phase 2

Transversity : Why

- transversity is very different from helicity

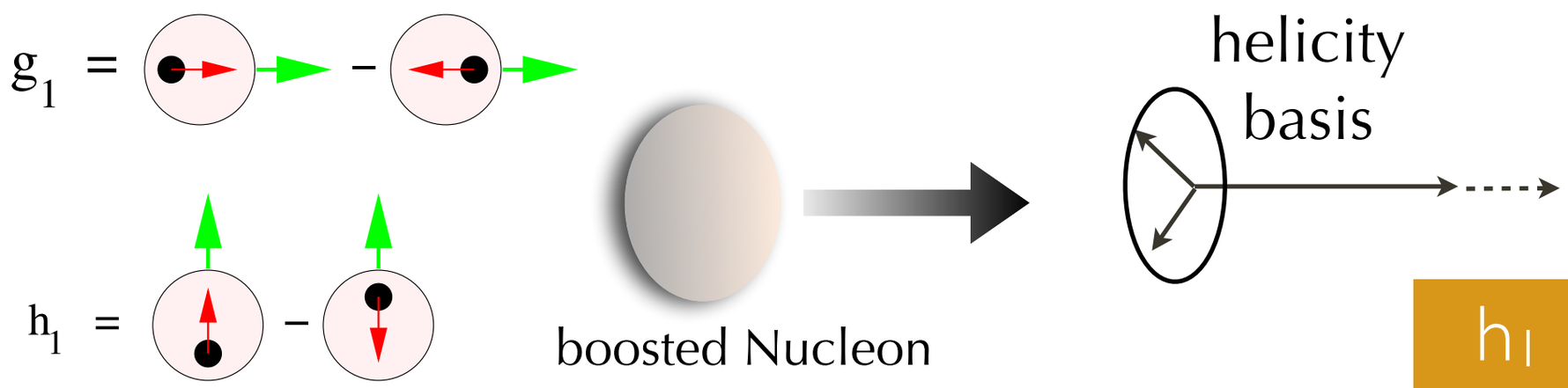


g_1 diagonal

h_1 non diagonal
("chiral-odd")

Transversity : Why

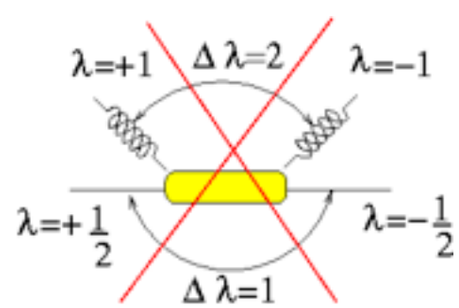
- transversity is very different from helicity



g_1 diagonal

h_1 non diagonal
("chiral-odd")

- no h_1 for gluons
  (in Nucleon)



→ pure non-singlet evolution

playground for tests of perturbative and nonperturbative QCD

Tensor Charge

- 1st Mellin moment of transversity ⇒ tensor "charge"

$$\delta q \equiv g_T^q = \int_0^1 dx \; \left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right]$$

no associated conserved current in $\mathcal{L}_{\rm QCD}$

↓

tensor "charge" g_T	scales with Q^2	C-odd
axial charge g_A	conserved	C-even

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no associated conserved current in $\mathcal{L}_{\mathrm{QCD}}$

↓

tensor "charge" g_T	scales with Q^2	C-odd
axial charge g_A	conserved	C-even

tensor charge not directly accessible in $\mathcal{L}_{\mathrm{SM}}$
low-energy footprint of new physics at higher scales ?

potential for BSM discovery ?

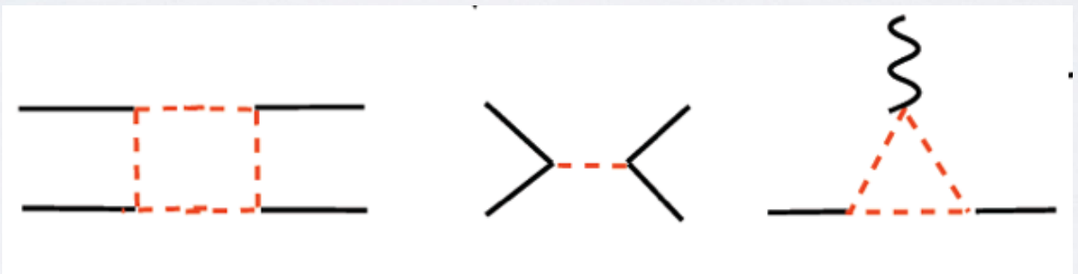
E

search for new physics **B**eyond **S**tandard **M**odel

M_{BSM}
high energy

direct access:
new particles

$E_{exp} \ll M_{BSM}$
low energy
high precision



indirect access:
virtual effects

potential for BSM discovery ?

E

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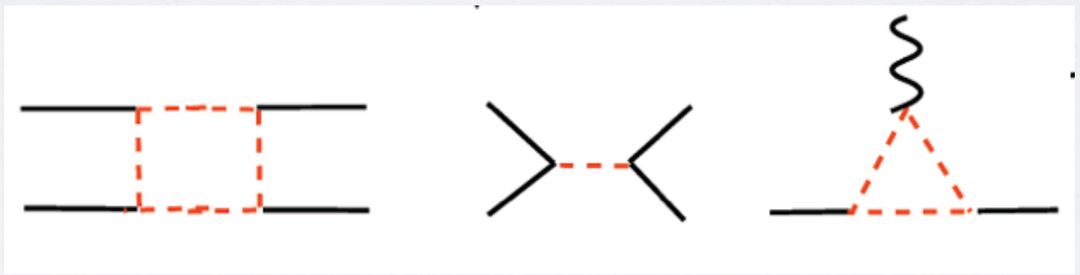
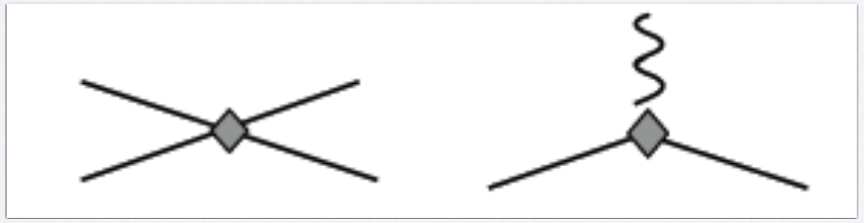
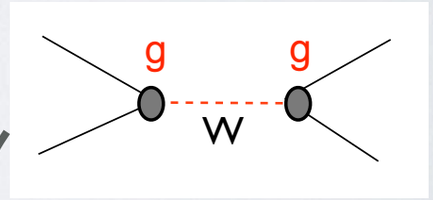


indirect access:
virtual effects

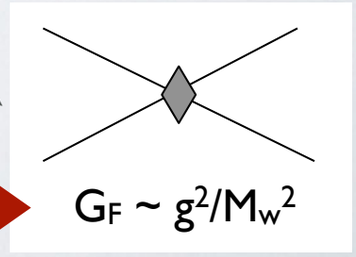
footprint:
new local
operators



Example:
weak CC
interaction



$q^2 \ll M_W^2$



Examples of indirect access

- **nuclear β-decay**: effective field theory including operators not in SM Lagrangian; for example, tensor operator

<u>hadron level</u> : n→p e^- $\bar{\nu}_e$

<u>quark level</u> : d→u e^- $\bar{\nu}_e$

$C_T \; \bar{p}\sigma^{\mu\nu} n \;\; \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e$

$\langle p|\, \bar{u}\, \sigma^{\mu\nu}\, d\, |n\rangle \;\; \epsilon_T \; \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e$

$$C_T \;\leftrightarrow\; g_T \epsilon_T$$

exp. data

?

$$\sim \frac{M_W^2}{M_{\rm BSM}^2}$$

$$g_T = \delta u - \delta d$$

isovector tensor charge

precision of 0.1% ⇒ BSM scale > [3-5] TeV

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isovector tensor charge

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- **neutron EDM**: estimate CPV induced by quark chromo-EDM d_q

$$\mathcal{L}_{\rm CPV} \supset ie \sum_{f=u,d,s,e} d_f \, \bar{f}\, \sigma_{\mu\nu}\gamma_5 f \; F^{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$d_n = \delta u\, d_u + \delta d\, d_d + \delta s\, d_s$$

exp. bounds + **tensor charge**

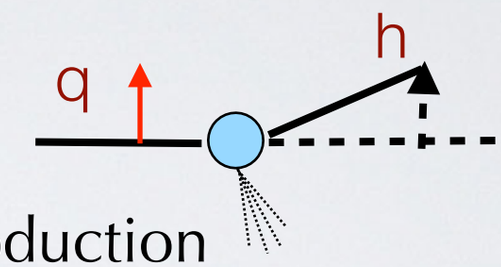
constraints on CP violation encoded in q EDM

extraction of transversity

transversity is chiral-odd → need a chiral-odd partner

- itself : fully polarized Drell-Yan ✘

- Collins function : the Collins effect

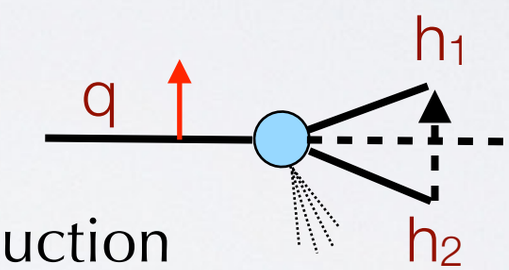


1-hadron semi-inclusive production

TMD framework

h_1 as TMD

- IFF : the di-hadron mechanism



2-hadrons semi-inclusive production

collinear framework

h_1 as PDF

- hadron-in-jet mechanism : mixed framework **h_1 as PDF**

- lattice "quasi-h1" : using Ji's LaMET *Chen et al., N.P. **B911** (16) 246* talk by Zhang

- collinear limit of chiral-odd GPD's GPD session, tomorrow morning

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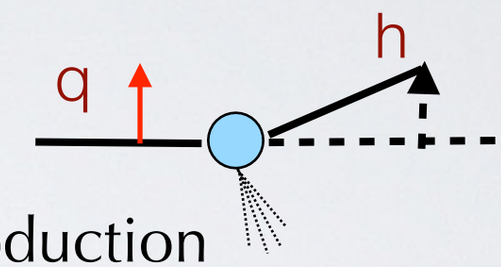

TMD framework
h_1 as TMD

1-hadron semi-inclusive production

TMD session, tomorrow afternoon

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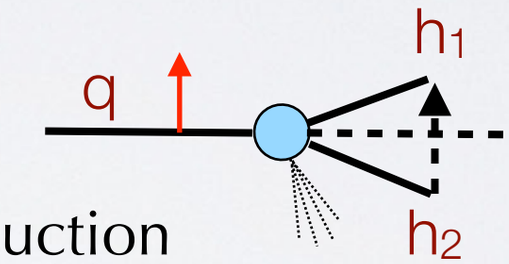

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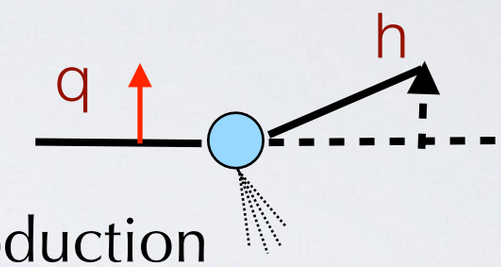

1-hadron semi-inclusive production

TMD framework

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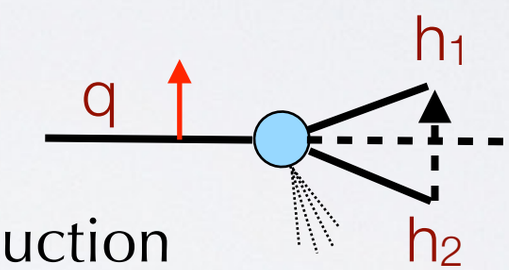


2-hadrons semi-inclusive production

collinear framework

h_1 as PDF

this & next talks

- hadron-in-jet mechanism : mixed framework **h_1 as PDF**

- lattice "quasi-h1" : using Ji's LaMET *Chen et al., N.P. **B911** (16) 246*

- collinear limit of chiral-odd GPD's

why di-hadron mechanism ?

collinear framework

- simple product of PDF and IFF

chiral-odd
di-hadron FF

Ex.: SIDIS $$A_{\rm SIDIS}^{\sin(\phi_R+\phi_S)}(x,z,M_h^2) \sim -\frac{\sum_q e_q^2\ h_1^q(x)\ \frac{|\boldsymbol{R}_T|}{M_h}\ H_{1,q}^{\sphericalangle}(z,M_h^2)}{\sum_q e_q^2\ f_1^q(x)\ D_{1,q}(z,M_h^2)}$$

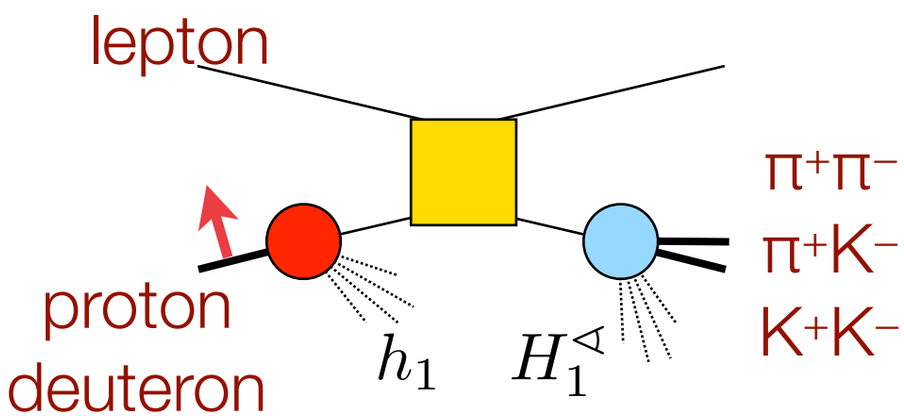
x-dependence of $A_{\rm SIDIS}$ all in PDF

- factorization theorems for all hard processes
  → universality of $h_1\ H_1^{\sphericalangle}$ mechanism

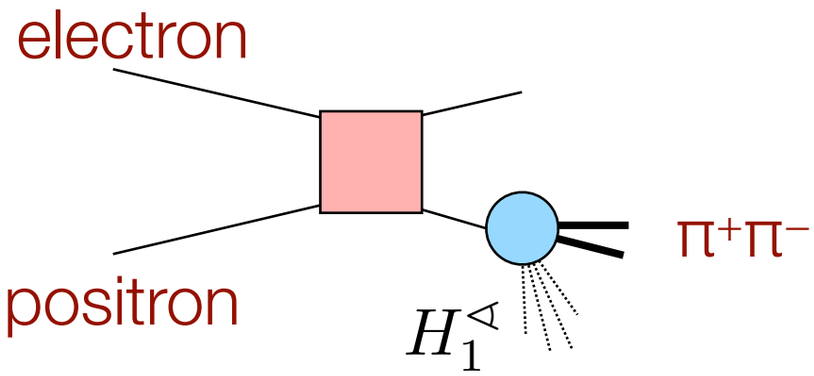
advantages of di-hadron mechanism

factorization theorems for all hard processes

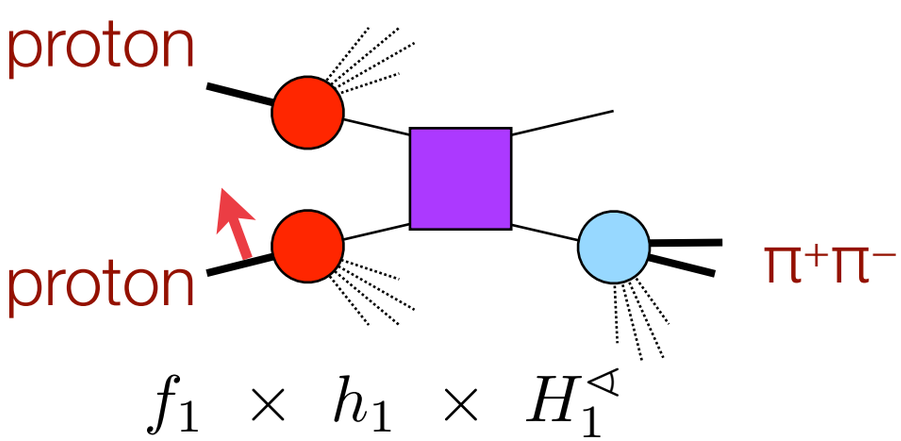
SIDIS



e+e−



p p↑



data used in the global fit

hermes

*Airapetian et al., JHEP **0806** (08) 017*

COMPASS

*Adolph et al., P.L. **B713** (12)*
*Braun et al., E.P.J. Web Conf. **85** (15)*

BELLE

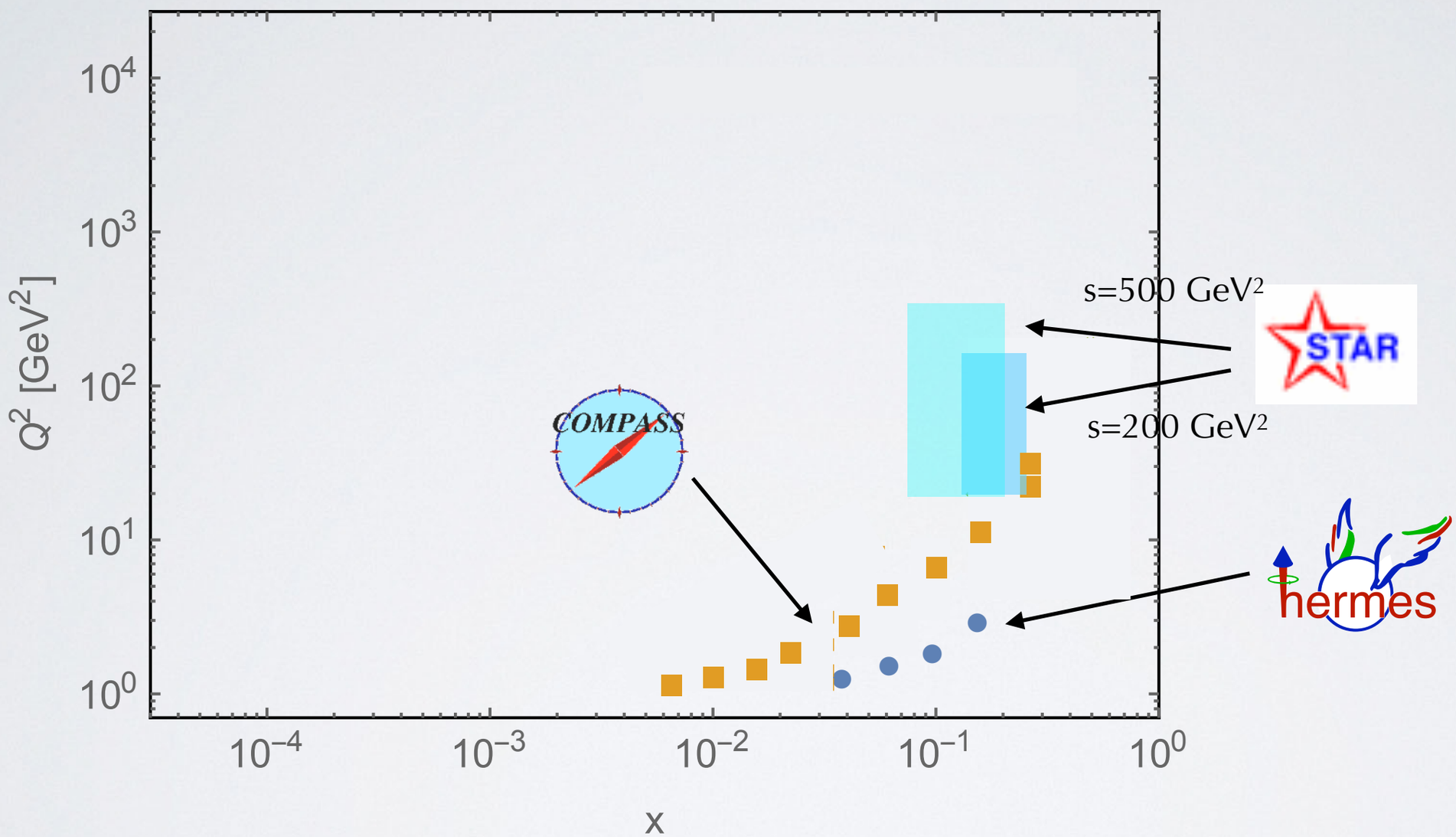
*Vossen et al., P.R.L. **107** (11) 072004*

STAR

run 2006 (s=200 GeV²) — *Adamczyk et al. (STAR), P.R.L. **115** (15) 242501*

run 2011 (s=500 GeV²) — *Adamczyk et al. (STAR), P.L. **B780** (18) 332*

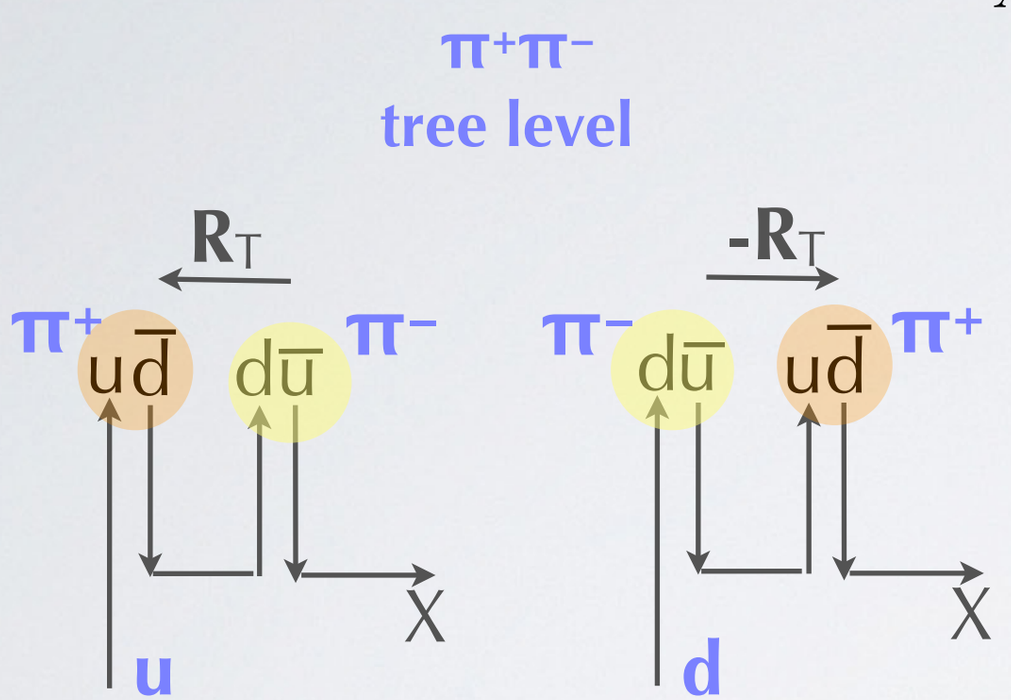
the phase space



- mostly medium/high x
- guess low-x behavior (relevant for calculation of tensor charge - see later)

currently, only LO analysis

$$A_{UT}^{\sin(\phi_R+\phi_S)}(x,z,M_h^2) \propto -\frac{|\mathbf{R}|}{M_h}\,\frac{\sum_q e_q^2\, h_1^q(x)\; H_1^{\sphericalangle q}(z,M_h^2)}{\sum_q e_q^2\, f_1^q(x)\; D_1^q(z,M_h^2)}$$



$$H_1^{\sphericalangle u} = -H_1^{\sphericalangle d} \qquad \text{isospin symmetry}$$

$$\left.\begin{aligned} H_1^{\sphericalangle q} &= -H_1^{\sphericalangle \bar{q}} \\ D_1^q &= D_1^{\bar{q}} \end{aligned}\right\} \text{charge conjugation}$$

access only $q\text{-}\bar{q} = q_v$, q=u,d
valence flavors in SIDIS A_{UT}

theoretical uncertainties

unpolarized Di-hadron Fragmentation Function D_1

- **quark** D_1^q is **well** constrained by $e^+e^- \to (\pi^+\pi^-)\,X$ (Montecarlo)
- **gluon** D_1^g is **not** constrained by $e^+e^- \to (\pi^+\pi^-)\,X$ (currently, LO analysis)
- **no data** available yet for $p\,p \to (\pi^+\pi^-)\,X$

we don't know anything about the gluon D_1^g

our choice: set $D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u(Q_0)/4 \\ D_1^u(Q_0) \end{cases}$ ← ~ 1-hadron $D_1^g(Q_0)$

deteriorates our e^+e^- fit as $\chi^2/\mathrm{dof} = \begin{cases} 1.69 & 1.28 \\ 1.81 & 1.37 \\ 2.96 & 2.01 \end{cases}$

background ρ channels

choice of functional form

functional form whose Mellin transform can be computed analytically and complying with Soffer Bound at any x and scale Q^2

$$h_1^{q_v}(x;Q_0^2) = F^{q_v}(x)\ \left[\mathrm{SB}^q(x) + \overline{\mathrm{SB}}^{\bar{q}}(x)\right]$$

Soffer Bound

$$2|h_1^q(x,Q^2)| \leq 2\ \mathrm{SB}^q(x,Q^2) = |f_1^q(x,Q^2) + g_1^q(x,Q^2)|$$

MSTW08 DSSV

$$F^{q_v}(x) = \frac{N_{q_v}}{\max_x[|F^{q_v}(x)|]}\, x^{A_{q_v}}\ [1 + B_{q_v}\,\mathrm{Ceb}_1(x) + C_{q_v}\,\mathrm{Ceb}_2(x) + D_{q_v}\,\mathrm{Ceb}_3(x)]$$

$\mathrm{Ceb}_n(x)$ Cebyshev polynomial

10 fitting parameters

constrain parameters

$|N_{q_v}| \leq 1 \Rightarrow |F^{q_v}(x)| \leq 1$ Soffer Bound ok at any Q^2

constrain parameters : low-x trend

$$\left.\begin{array}{l}\lim\limits_{x\to 0} x\mathrm{SB}^q(x) \propto x^{a_q} \\ \\ \lim\limits_{x\to 0} F^{q_v}(x) \propto x^{A_q}\end{array}\right\} \quad h_1^q(x) \overset{x\to 0}{\approx} x^{A_q + a_q - 1}$$

tensor charge $\delta q(Q^2) = \int_{x_{\min}}^{1} dx\, h_1^{q-\bar{q}}(x, Q^2)$

low-x behavior important

constrain parameters

δq finite => $A_q + a_q > 0$

our choice

$A_q + a_q > \frac{1}{3}$

$$\left|\int_0^{x_{\min}} dx\right| \sim 1\% \text{ of } \left|\int_{x_{\min}}^{1} dx\right|$$

for $x_{min}=10^{-6}$ from MSTW08

Other choices

"massive" jet in DIS → h_1 at twist 3
violation of Burkardt-Cottingham s.r.
Accardi and Bacchetta, P.L. ***B773*** *(17) 632*

$$\int_0^1 dx\, g_2(x) \propto \int_0^1 dx\, \frac{h_1(x)}{x} \longrightarrow A_q + a_q > 1$$

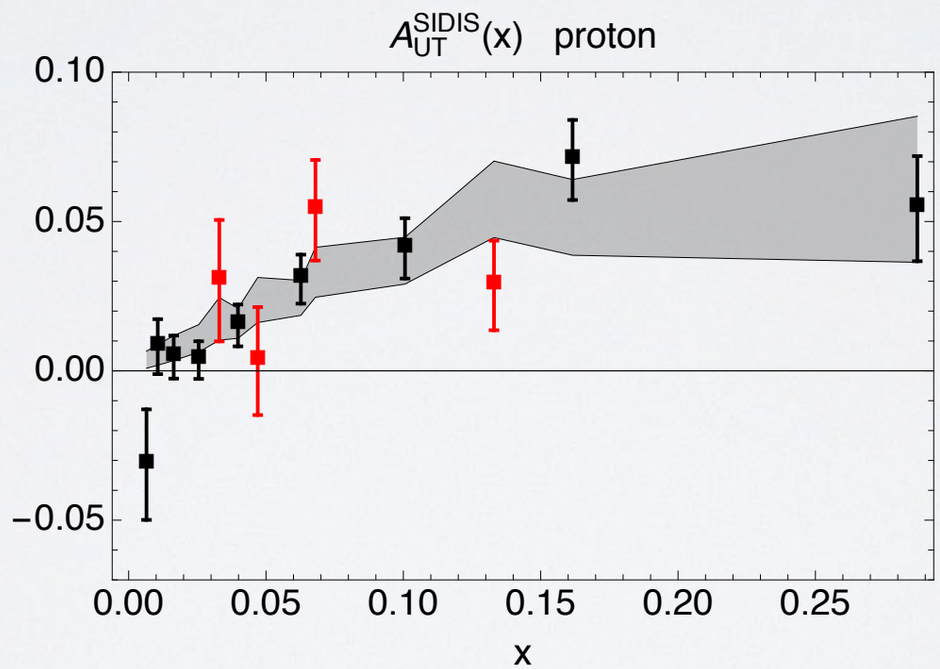
small-x dipole picture $h_1^{q_v}(x) \overset{x\to 0}{\approx} x^{1-2\sqrt{\frac{\alpha_s(Q^2)N_c}{2\pi}}}$ $\longrightarrow$ at Q_0 $A_q + a_q \sim 1$

Kovchegov & Sievert, arXiv:1808.10354

statistical uncertainty

the bootstrap method

- shift each exp. point by Gaussian noise within exp. variance
- create sets of virtual points to be fitted: 50, 100, 200 sets… until average and standard deviation reproduce original exp. variance (**here, 200x3=600**)
- exclude largest and smallest 5% => **90% band**



automatically accounts for correlations

Results

Our first global fit

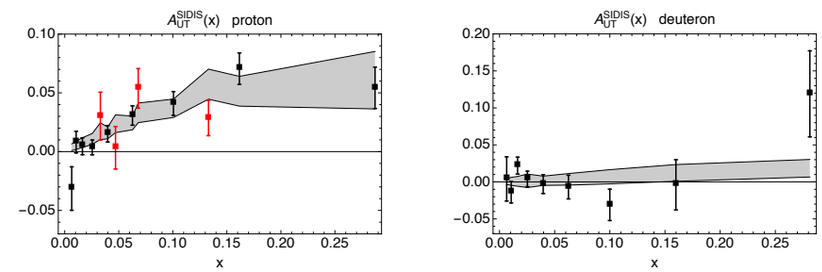
first ever extraction of transversity from data of SIDIS and proton-proton collisions

Radici and Bacchetta, P.R.L. ***120*** *(18) 192001*

SIDIS

18 data points

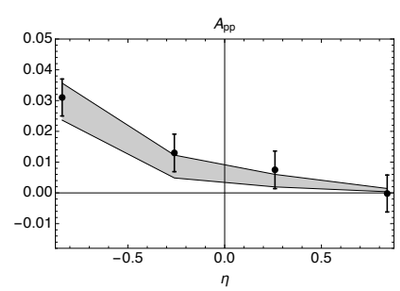
4 data points

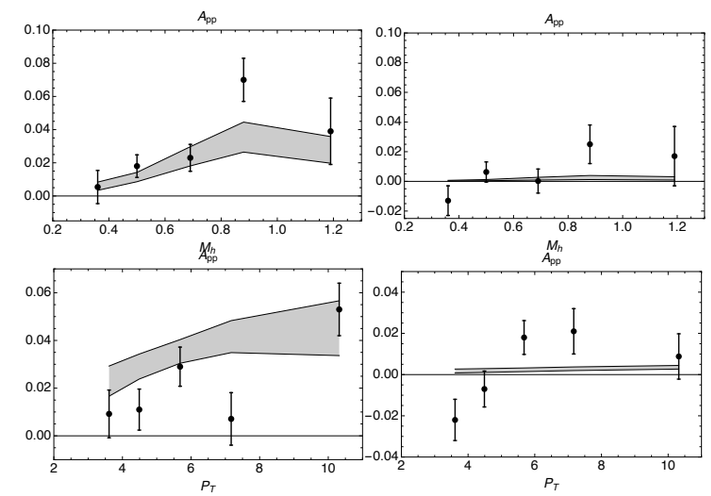


pp collisions

run 2006
(s=200 GeV²)

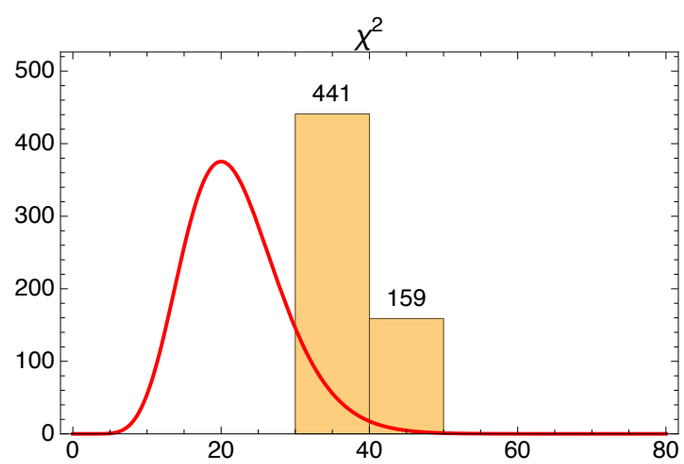
10 independent data points





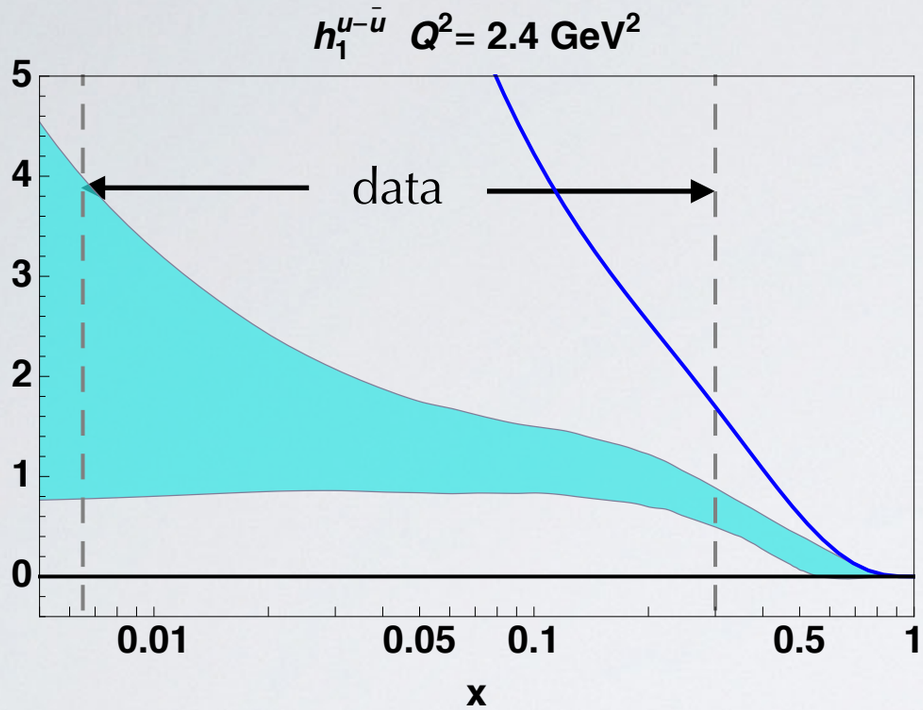
probability density function of
χ^2 distribution for 22 d.o.f.

(for χ^2/dof = 1 perfect overlap)



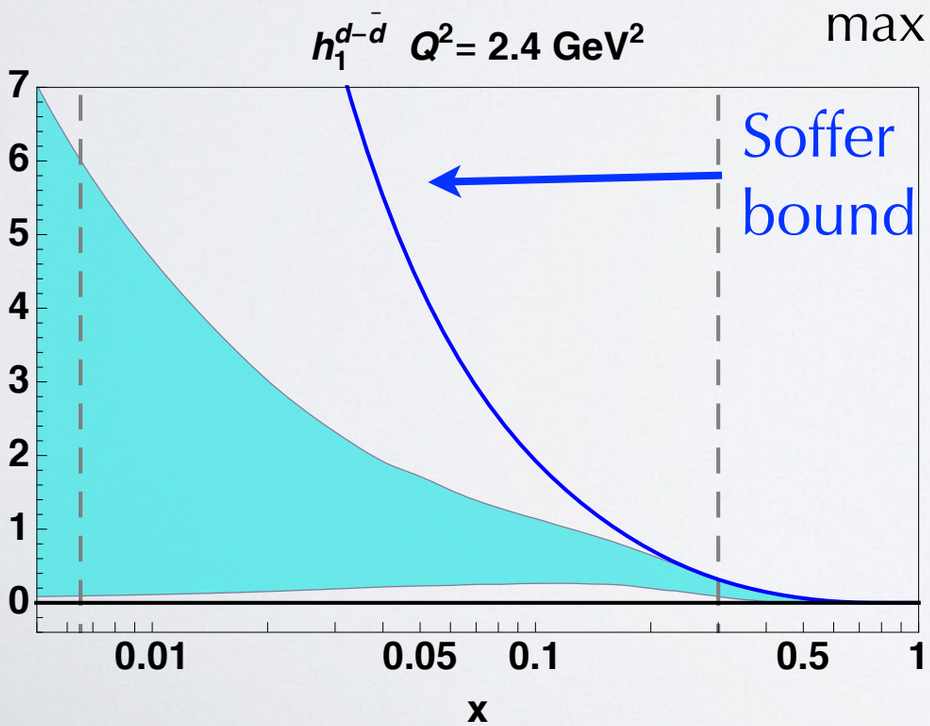
$\chi^2/\text{dof} = 1.76 \pm 0.11$

the extracted transversity



up

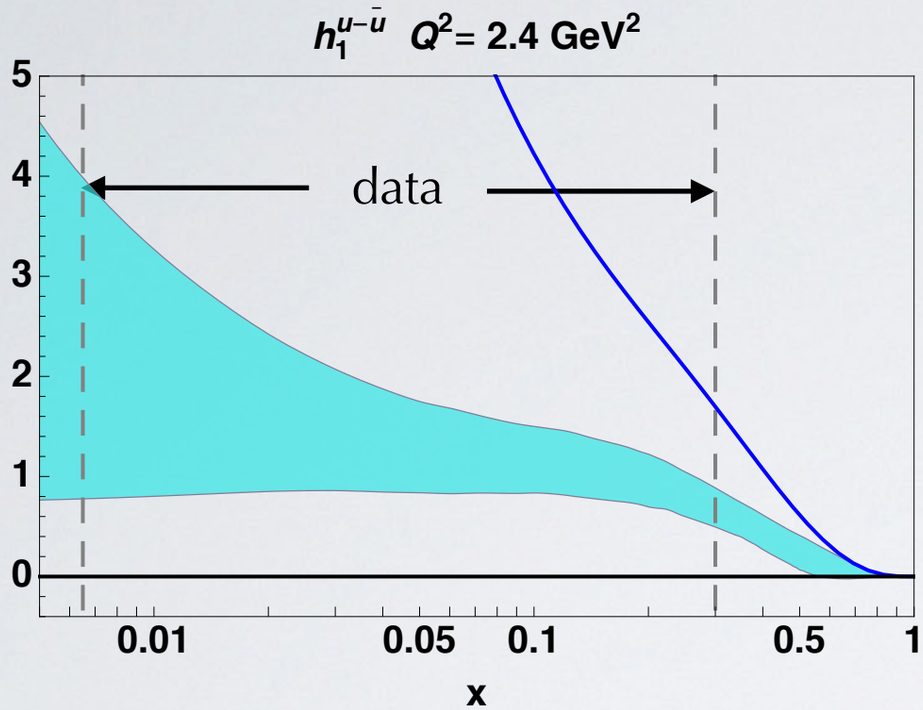
uncertainty band from
90% of 600 replicas =
max uncertainty on $D_1^g(Q_0)$

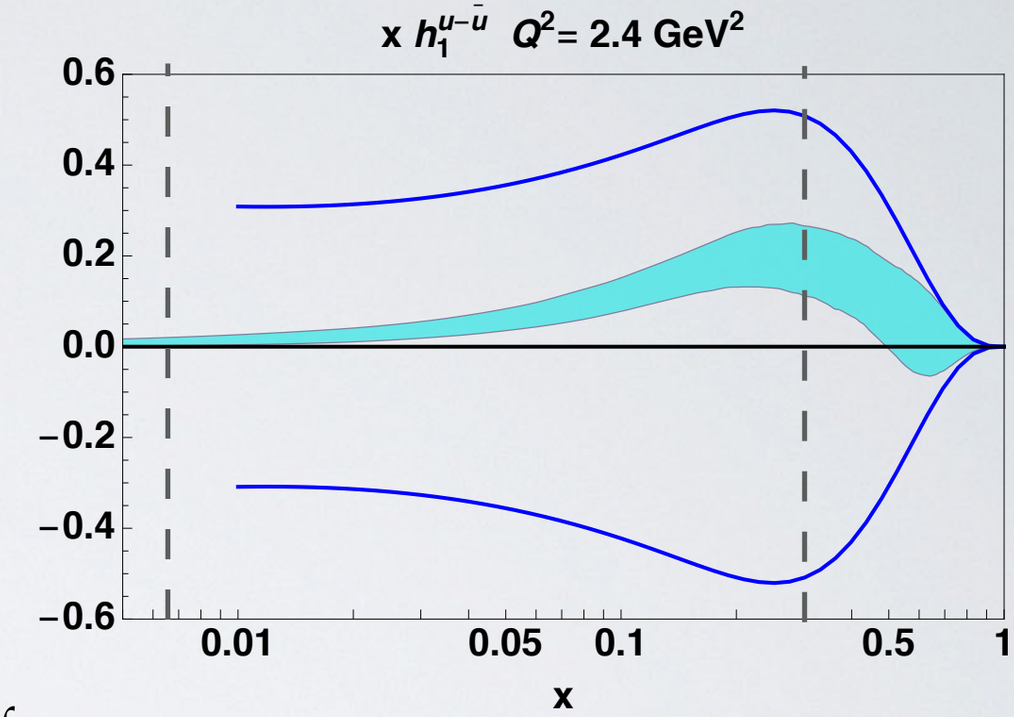


down

the extracted transversity

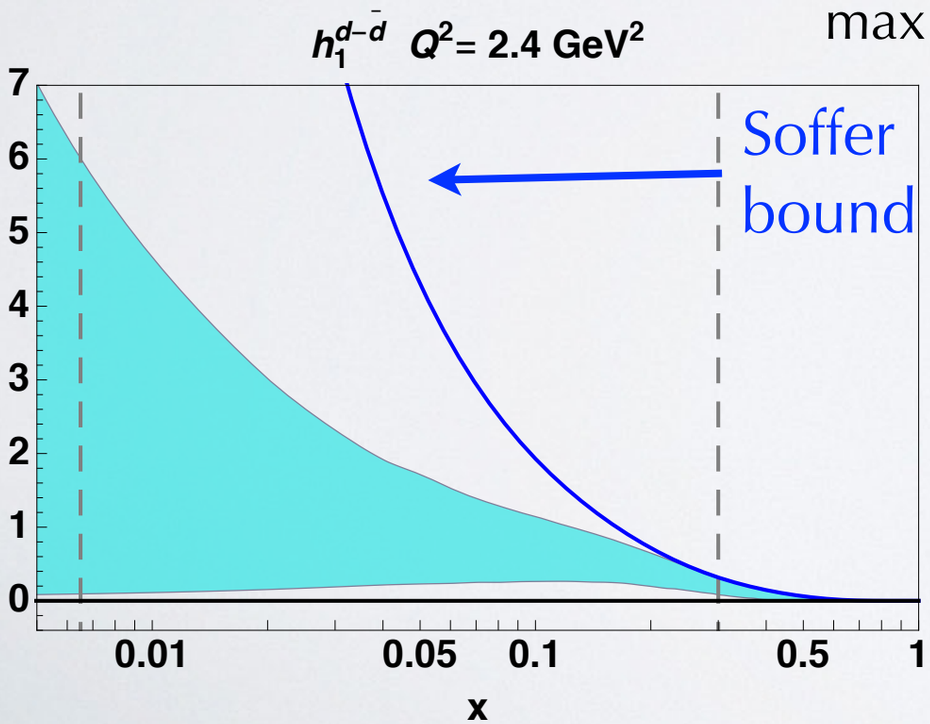
up



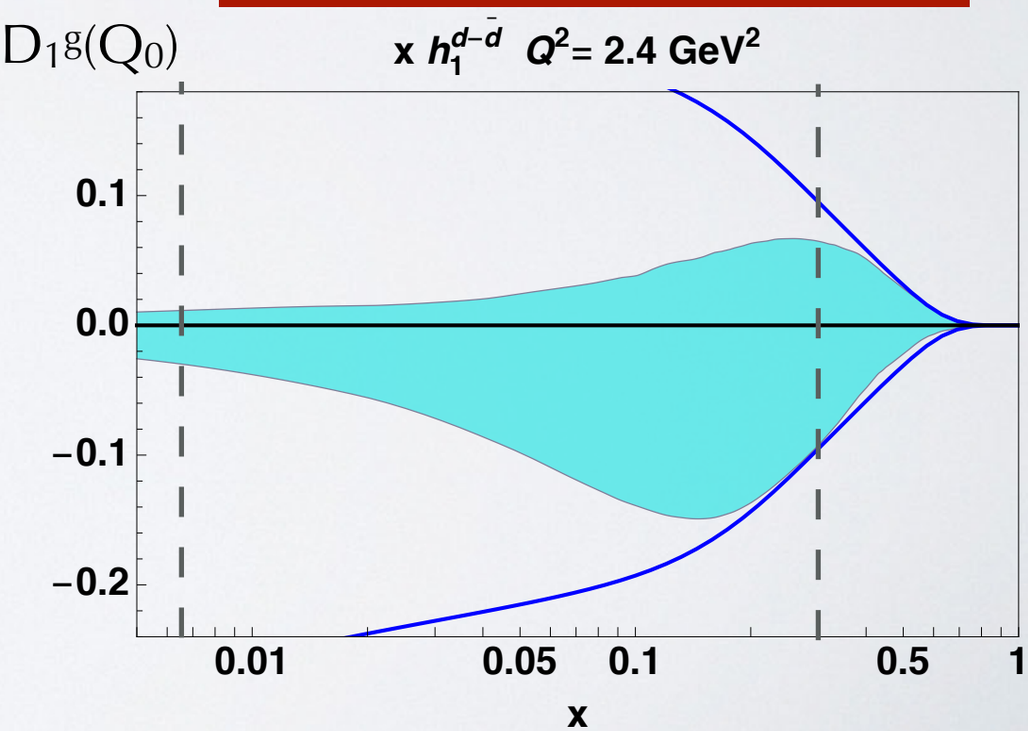


uncertainty band from
90% of 600 replicas =
max uncertainty on $D_1^g(Q_0)$

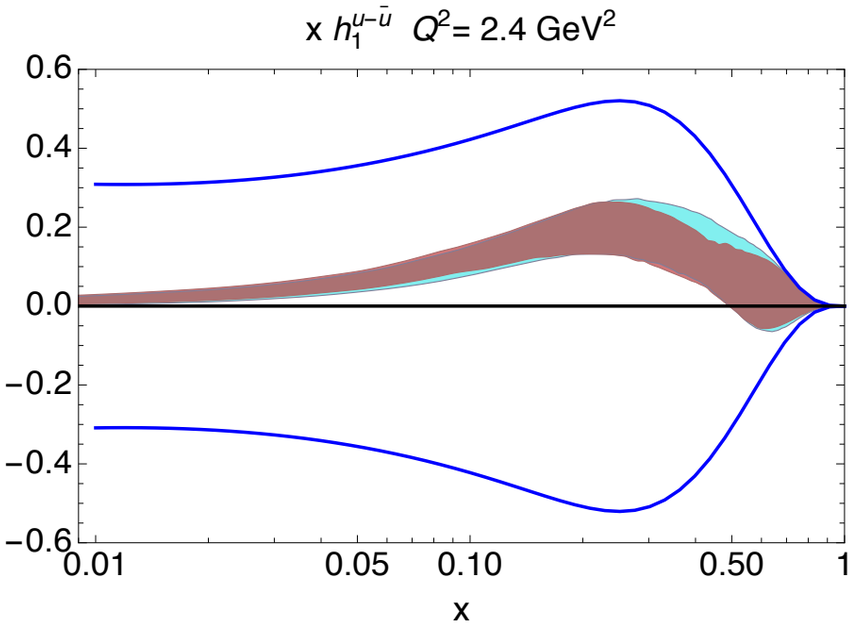
diverges less than 1/x



down



sensitivity to th. uncertainty

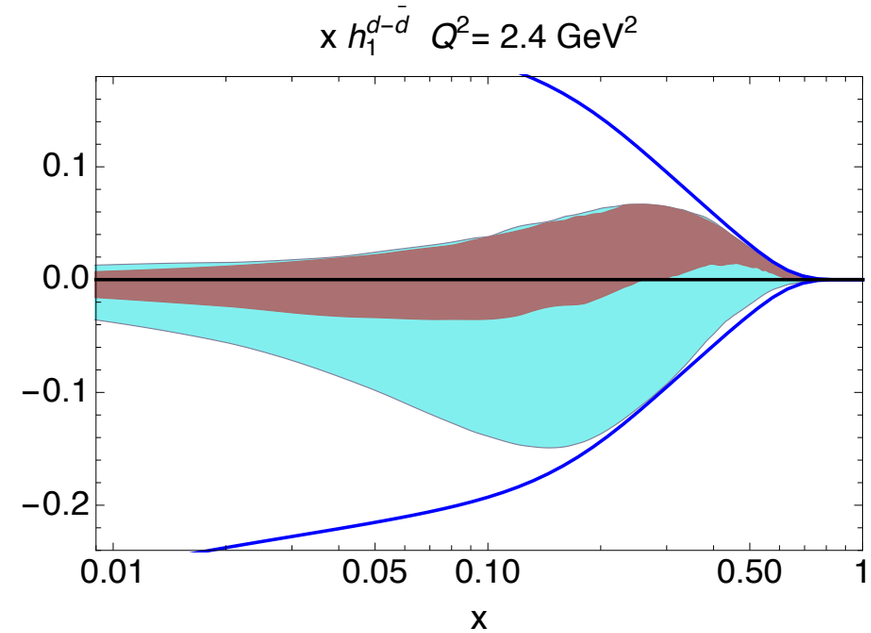


$D_1^g(Q_0) = 0$

$$D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u/4 \\ D_1^u \end{cases}$$

global fit

Radici & Bacchetta, P.R.L. ***120*** *(18) 192001*



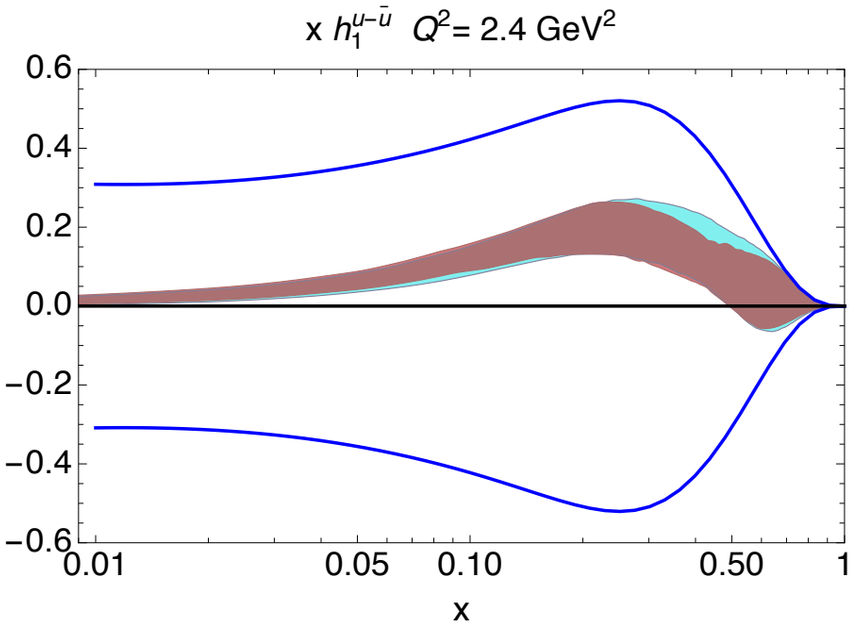
up

insensitive to uncertainty on gluon D_1

down

sensitive to uncertainty on gluon D_1

sensitivity to th. uncertainty



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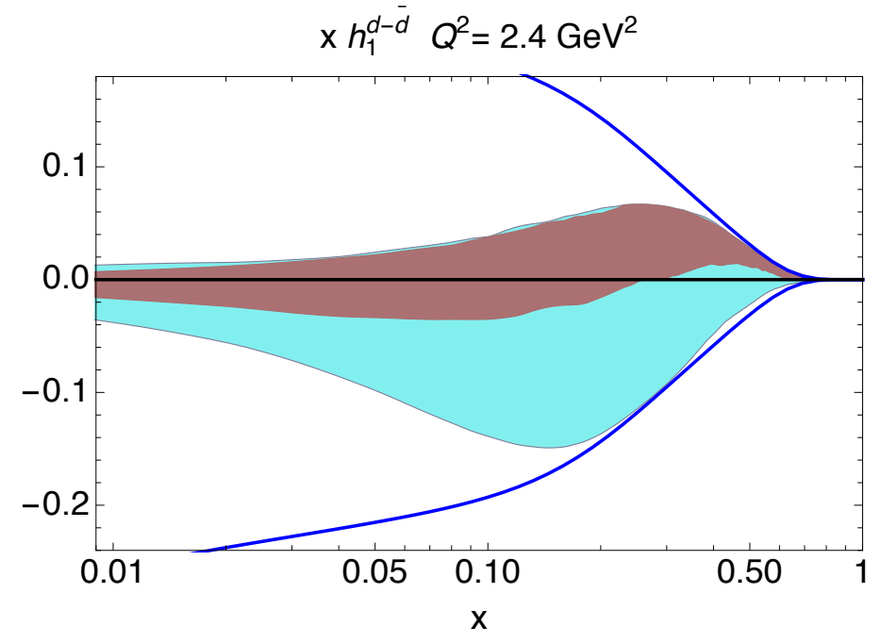
global fit

Radici & Bacchetta, P.R.L. ***120*** *(18) 192001*



up

insensitive to uncertainty on gluon D_1

down

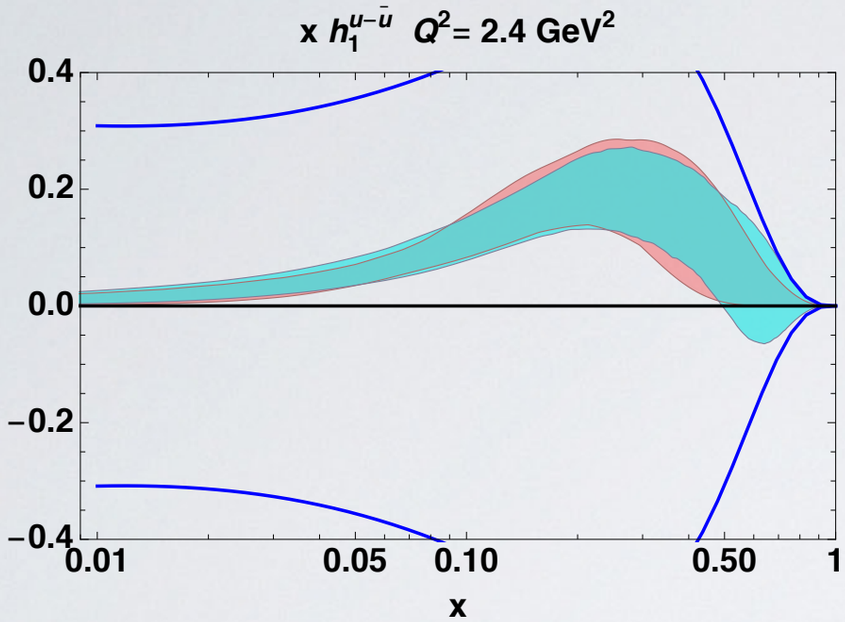
sensitive to uncertainty on gluon D_1

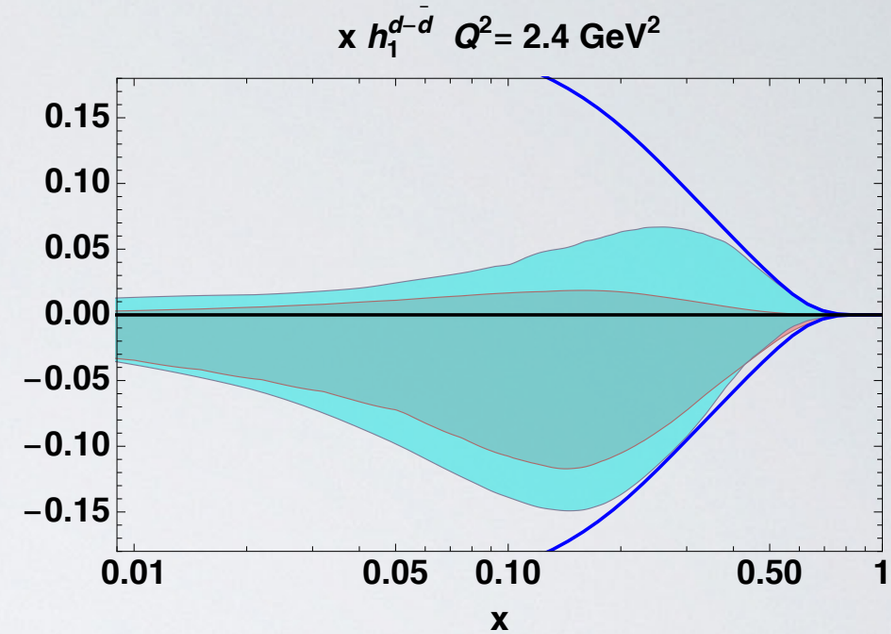
p-p : u~d , gluon @LO but **SIDIS :** u~(8x)d , gluon @NLO

need data from target more sensitive to down (deuteron, ^{3}He) and need data from multiplicities in p+p → (ππ)+X

Comparison with other extractions

Collins effect, only SIDIS data





*Anselmino et al., P.R. D**87** (13) 094019*

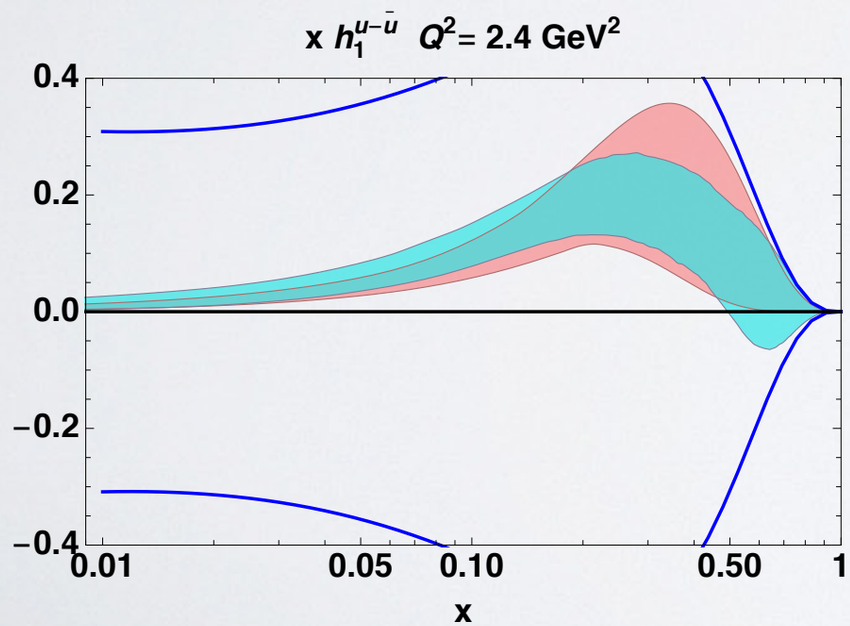
Torino

global fit

*Radici and Bacchetta, P.R.L. **120** (18) 192001*

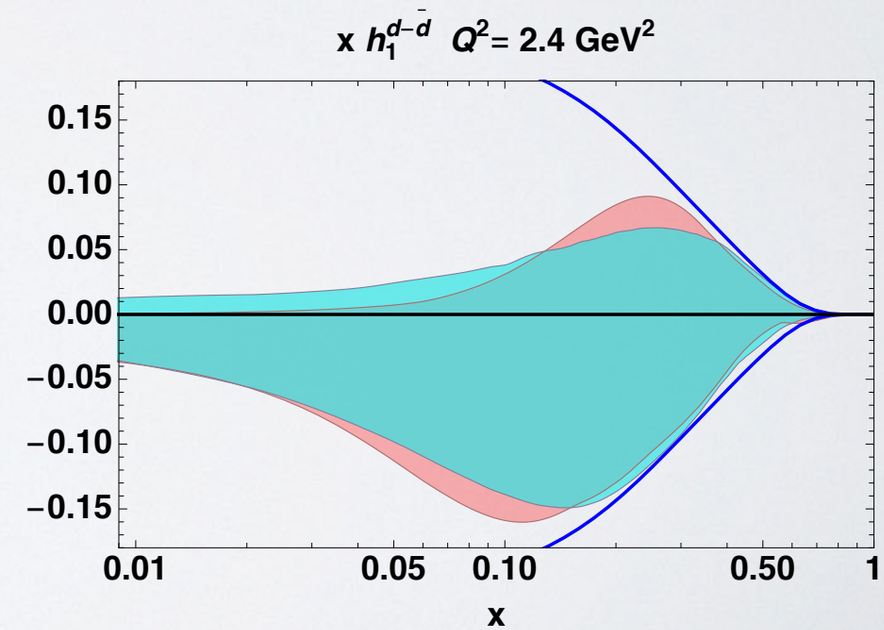
up

down

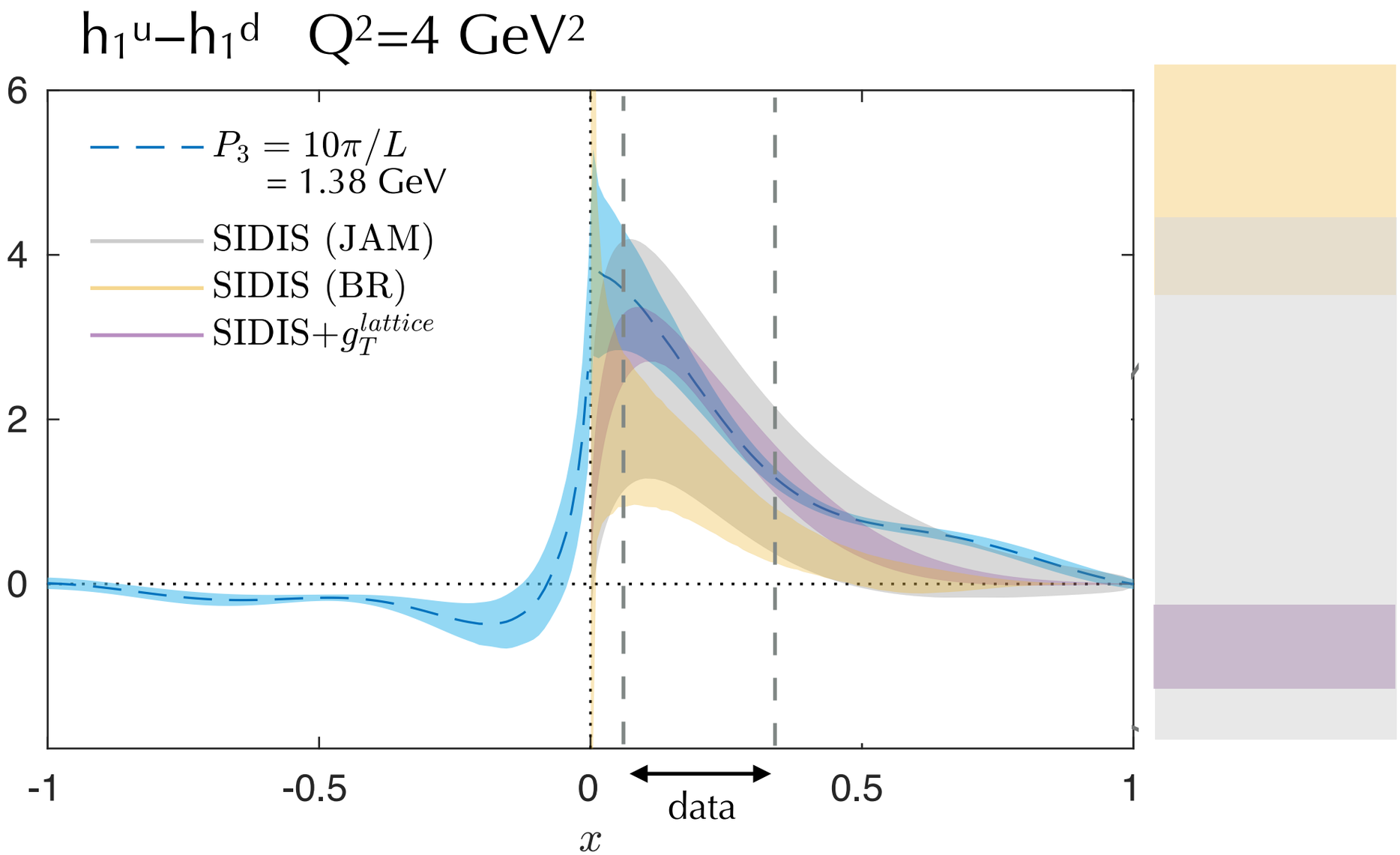


TMD

*Kang et al., P.R. D**93** (16) 014009*



Comparison with lattice



our global fit

Radici and Bacchetta, P.R.L. ***120*** *(18) 192001*

JAM Collab.

Lin et al., P.R.L. ***120*** *(18) 152502*

Collins effect only SIDIS data

constrained to g_T^{u-d} from lattice

ETMC quasi-h_1

*Alexandrou et al., P.R. D****99*** *(19) 114504*

courtesy of F. Steffens

The tensor "charge" of the proton

1st Mellin moment of transversity PDF ⇒ tensor "charge"

$$\delta q \equiv g_T^q = \int_0^1 dx \; \left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right]$$

tensor charge connected to tensor operator

$$\langle P, S_p | \, \bar{q}\sigma^{\mu\nu} q \, | P, S_p \rangle = (P^\mu S_p^\nu - P^\nu S_p^\mu)\, \delta q$$

$$= (P^\mu S_p^\nu - P^\nu S_p^\mu) \int dx \, h_1^{q-\bar{q}}(x,Q^2)$$

compute on lattice

lattice δq

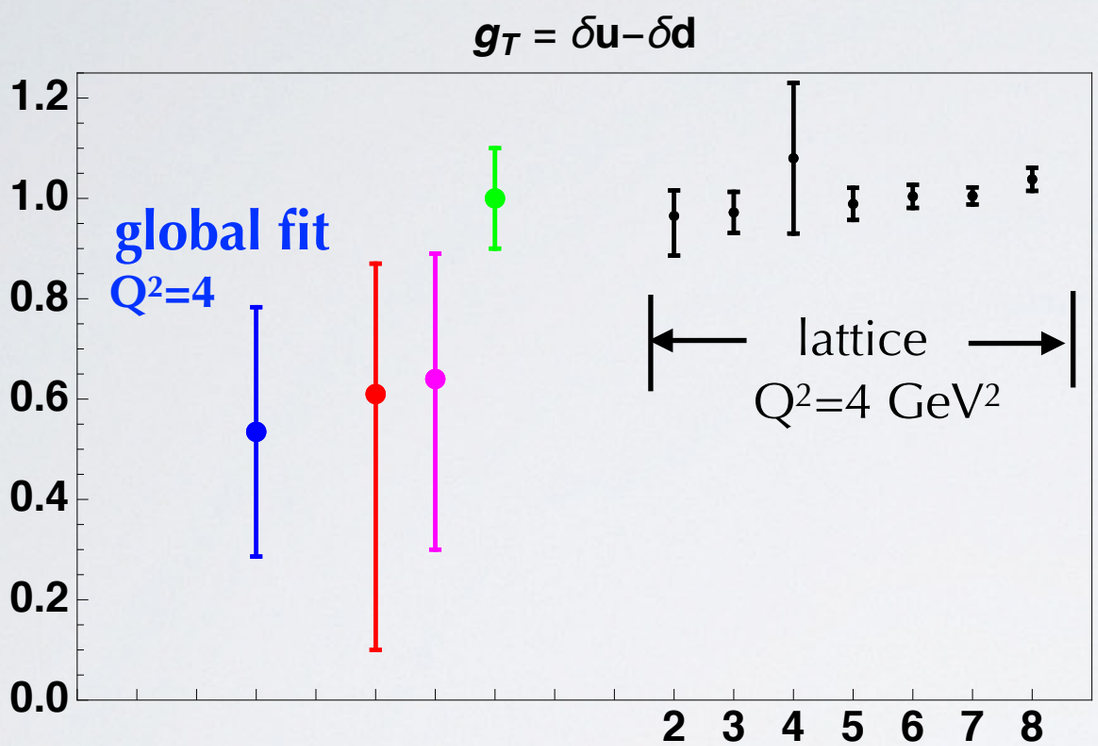
preferably the isovector $g_T = \delta u - \delta d$
(cancellation of "disconnected" diagrams)

extract transversity from data with transversely polarized protons

pheno δq

Comparison on tensor charge

isovector tensor charge



Collins effect
no p-p data

JAM (Q^2=2)
Torino (Q^2=1)
TMD (Q^2=10)

updated August 2019

only pheno extractions
and recent lattice calc.

2) **Mainz '19** *Harris et al., P.R. D**100** (19) 034513*

3) **LHPC '19** *Hasan et al., P.R. D**99** (19) 114505*

4) **JLQCD '18** *Yamanaka et al., P.R. D**98** (18) 054516*

5) **PNDME '18** *Gupta et al., P.R. D**98** (18) 034503*

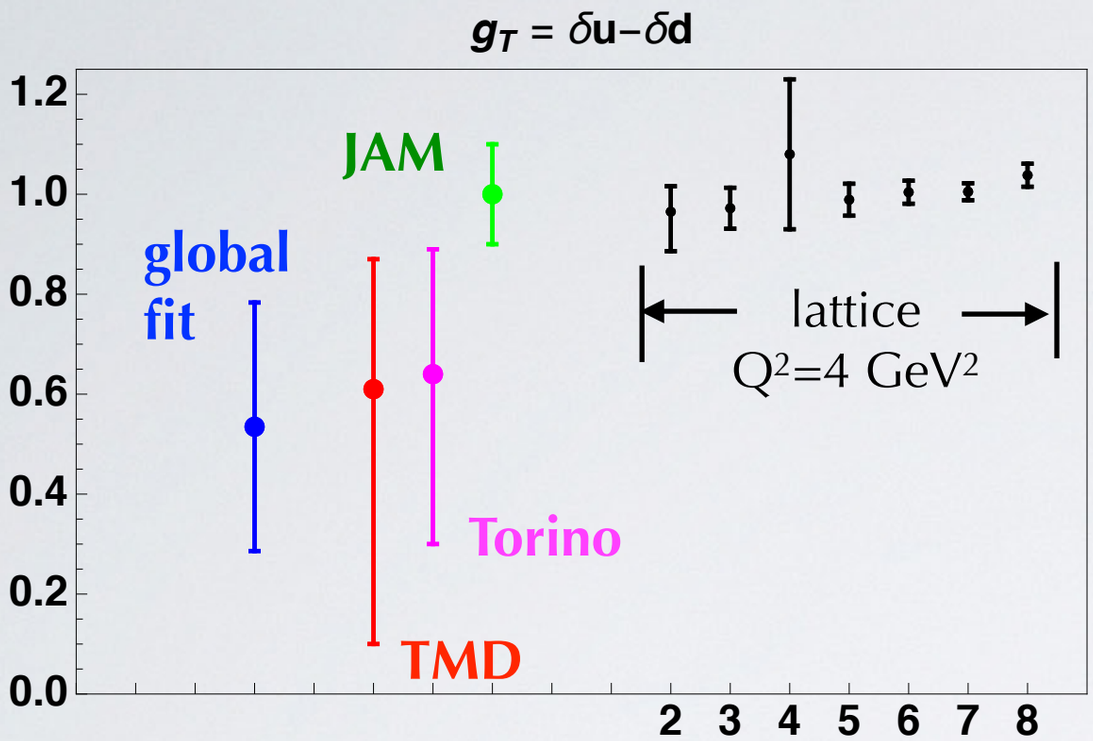
6) **ETMC '17** *Alexandrou et al., P.R. D**95** (17) 114514; E P.R. D**96** (17) 099906*

7) **RQCD '14** *Bali et al., P.R. D**91** (15) 054501*

8) **LHPC '12** *Green et al., P.R. D**86** (12) 114509*

JAM includes constraint from "lattice g_T"
but uses Collins effect only from SIDIS data (no e+e−)

Comparison on tensor charge



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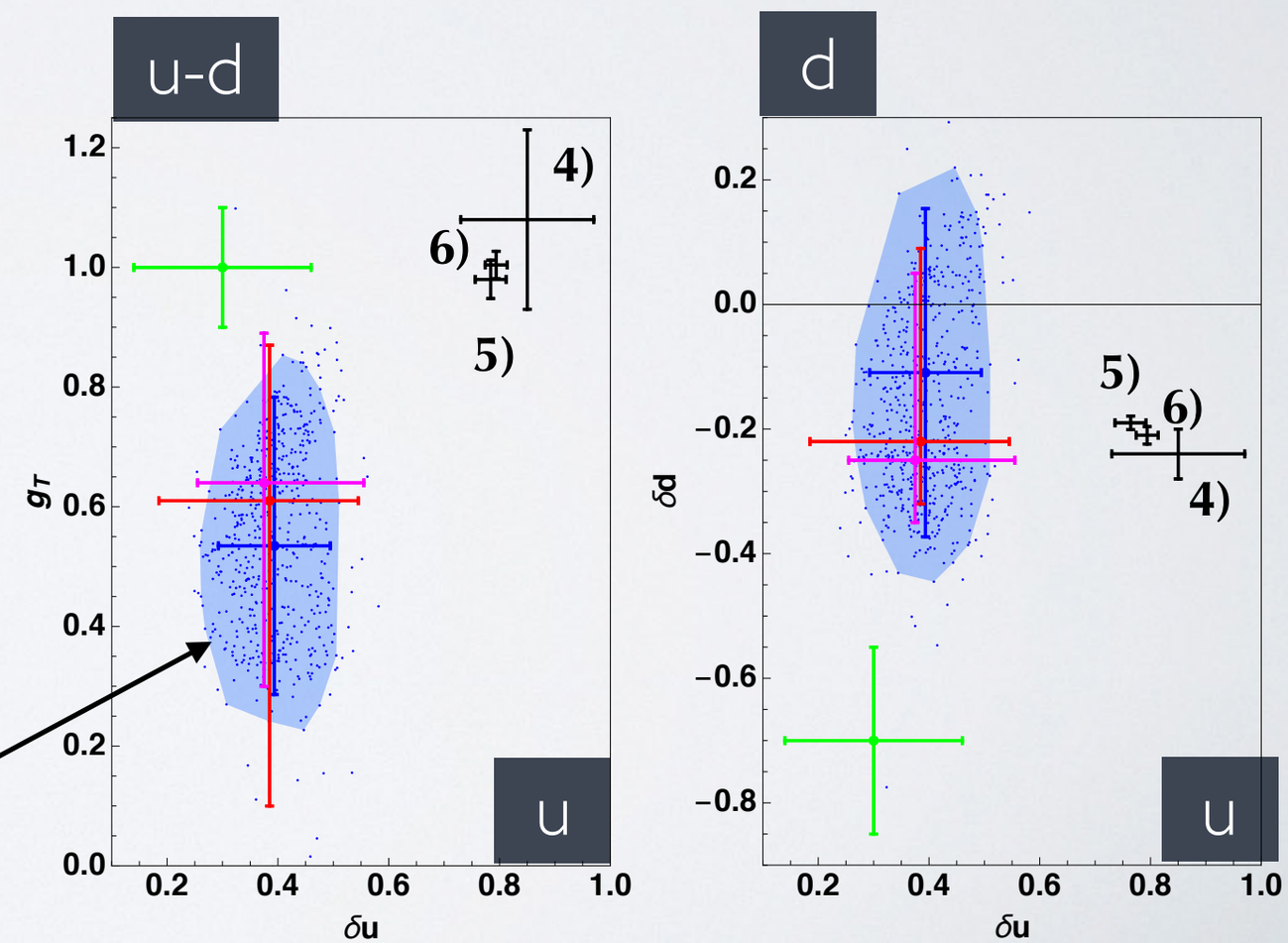
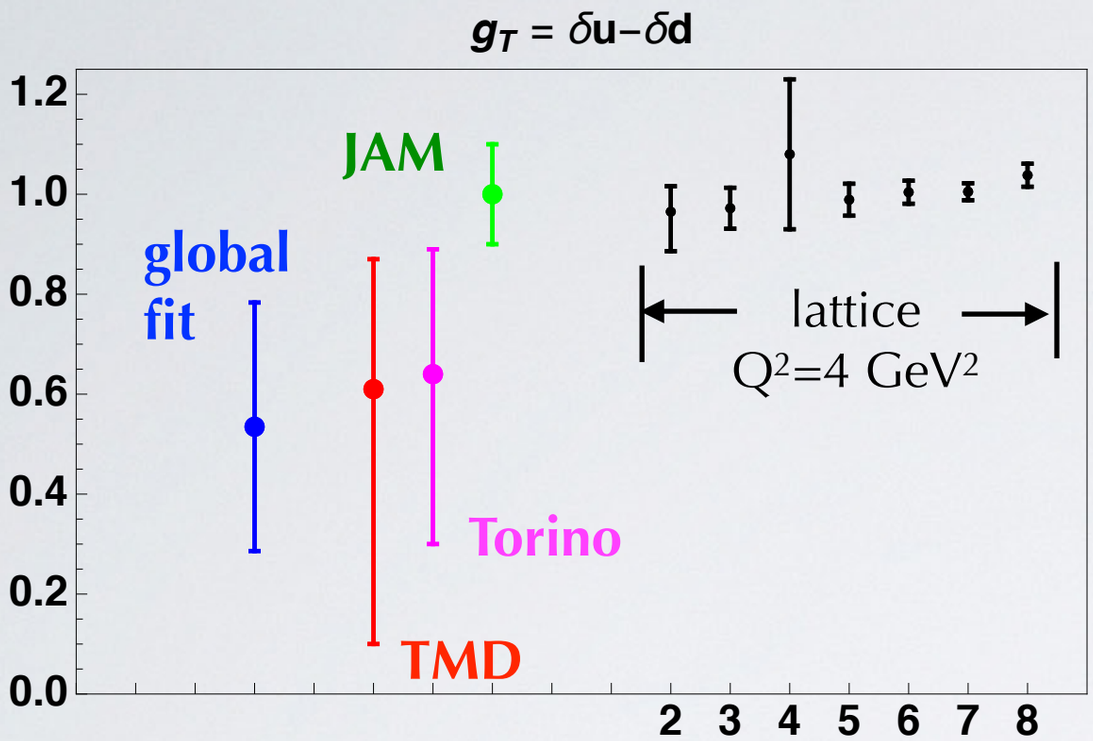
8) **LHPC '12** *Green et al., P.R. D**86** (12) 114509*

But if we look also at δu and δd …

shaded area = 90% C.L.



Comparison on tensor charge



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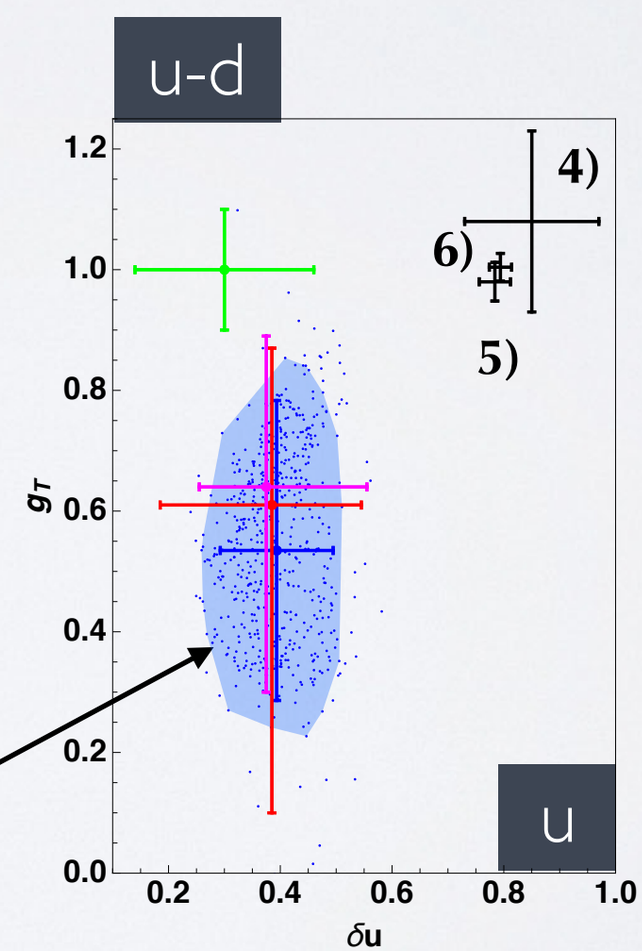
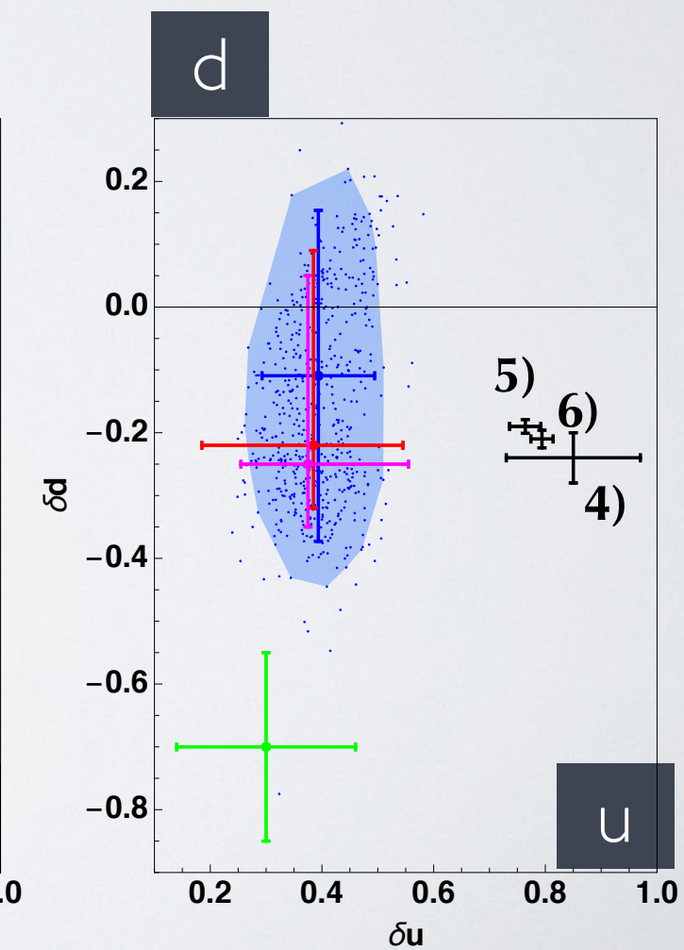




no simultaneous compatibility between "pheno δq" and "lattice δq"

shaded area = 90% C.L.

pheno vs. lattice tensor charge

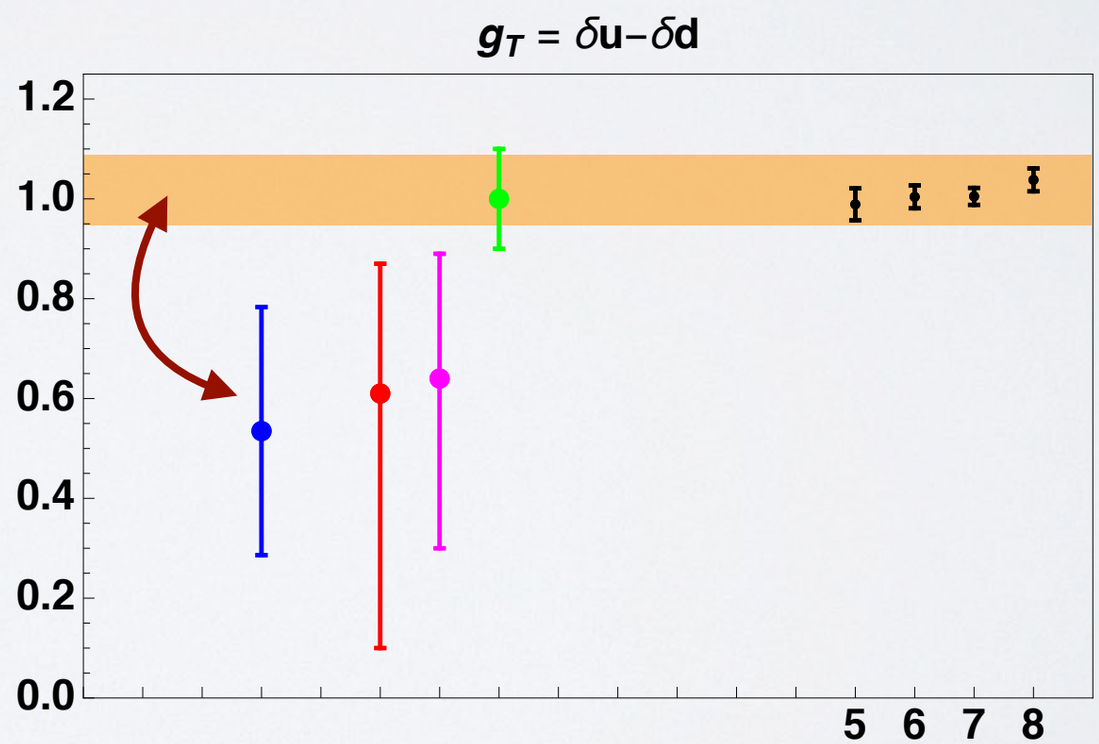
main problem of "pheno δq" is extrapolating outside data..

$$\delta q = \int_0^{x_{\min}} dx\, h_1^{q-\bar{q}} + \int_{x_{\min}}^{x_{\max}} dx\, h_1^{q-\bar{q}} + \int_{x_{\max}}^{1} dx\, h_1^{q-\bar{q}}$$

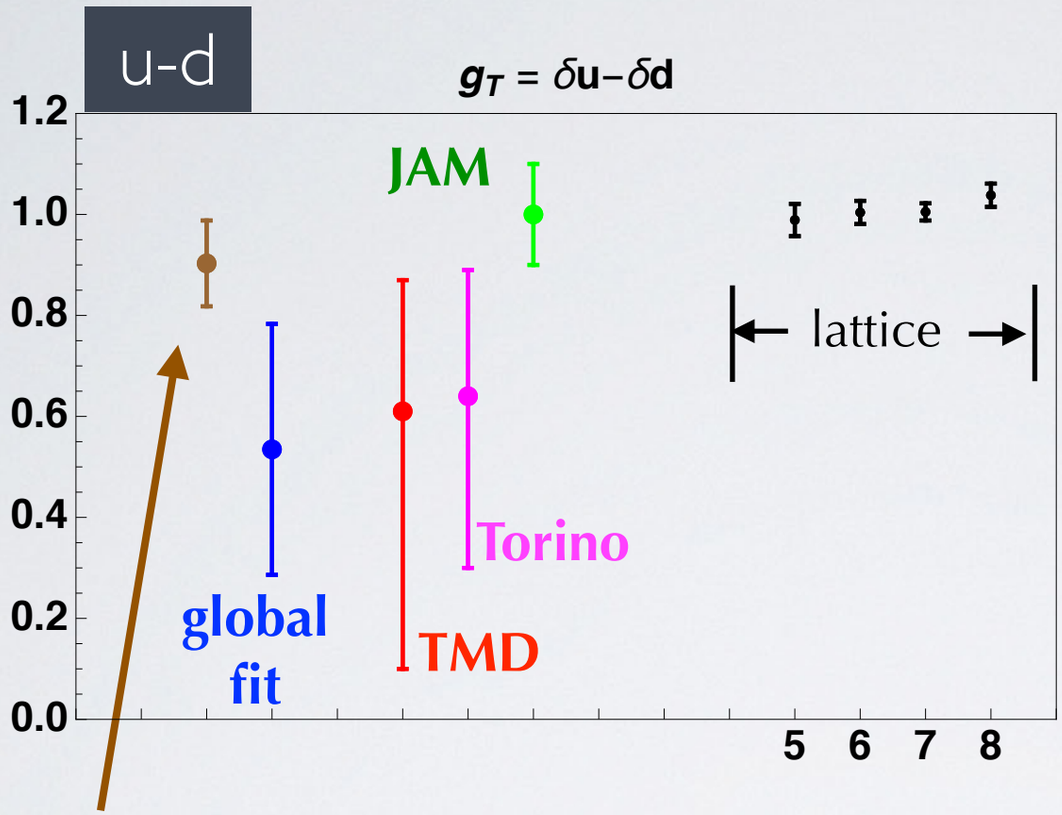
constraining "pheno g_T" with "lattice g_T" similarly to **JAM** ?

$\overline{g_T}^{\text{latt}} = 1.004 \pm 0.057$

are they compatible?

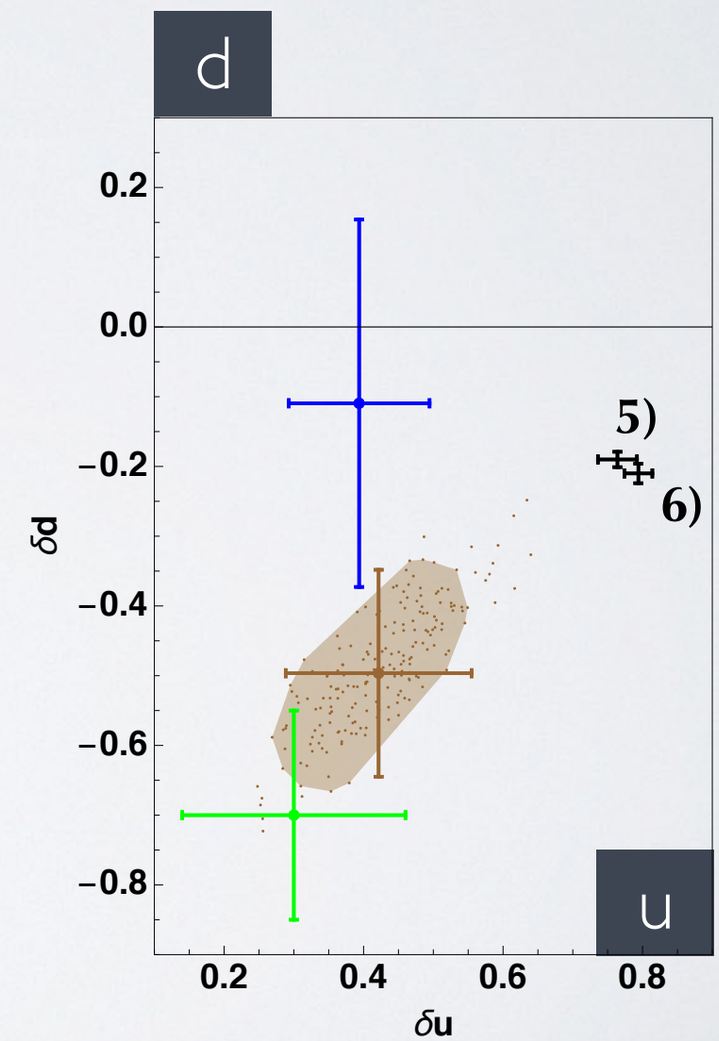


Constraining our global fit with lattice g_T



constraining **global fit** with **lattice g_T**

confirm **JAM** results:
constraining "pheno g_T" with "lattice g_T"
at the price of
incompatibility for **δu and δd**



Tension "pheno" - "lattice"

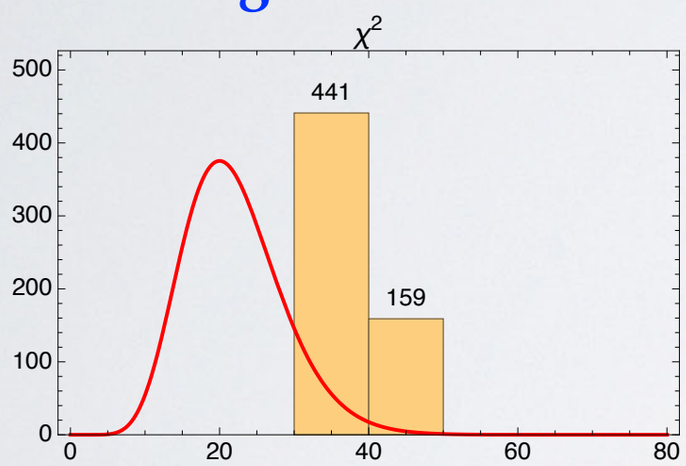
if we constrain our **global fit** with lattice results for all components of tensor charge (up, down, isovector) the χ^2 clearly deteriorate

$\overline{g_T}^{latt} = 1.004 \pm 0.057$

$\overline{\delta u}^{latt} = 0.782 \pm 0.031$

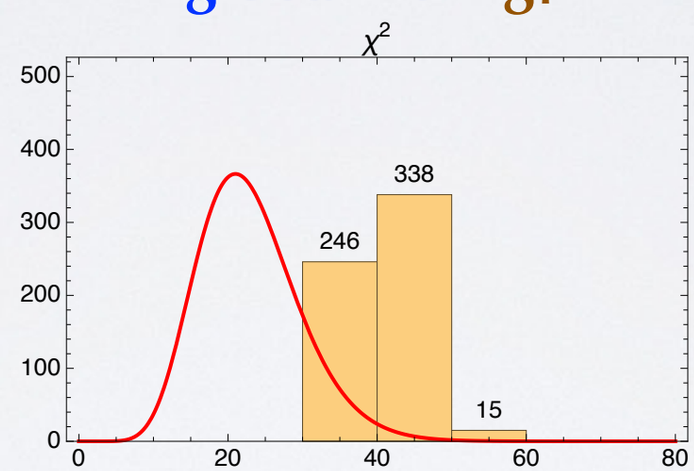
$\overline{\delta d}^{latt} = -0.218 \pm 0.026$

global fit → **global fit + g_T** → **global fit + g_T + δu + δd**



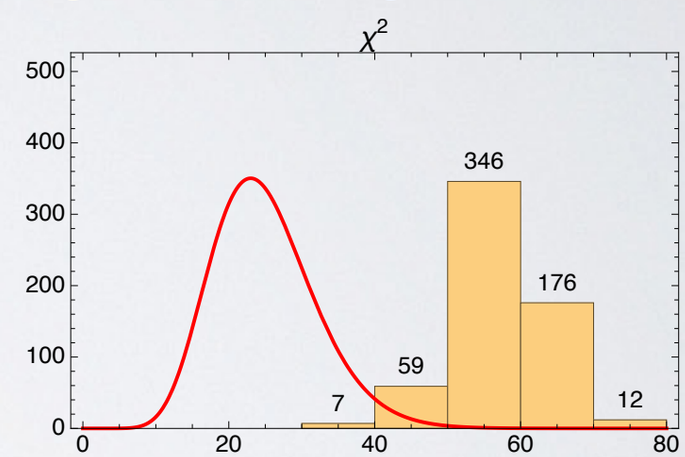
χ^2/dof = 1.76 ± 0.11

22 d.o.f.



χ^2/dof = 1.82 ± 0.25

23 d.o.f.



χ^2/dof = 2.29 ± 0.25

25 d.o.f.

probability density function of χ^2 distribution for … d.o.f.

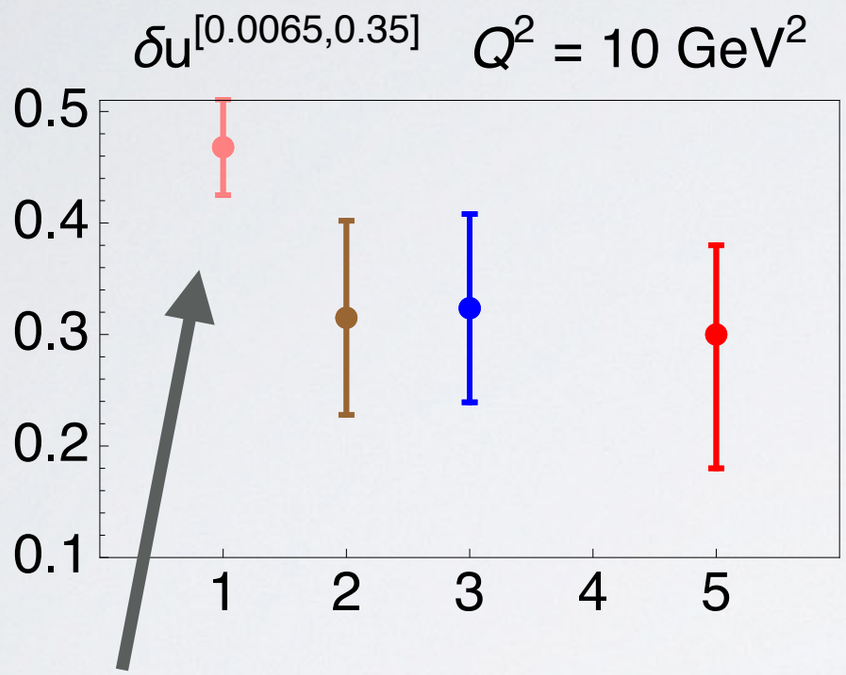
statistically very unlikely ….

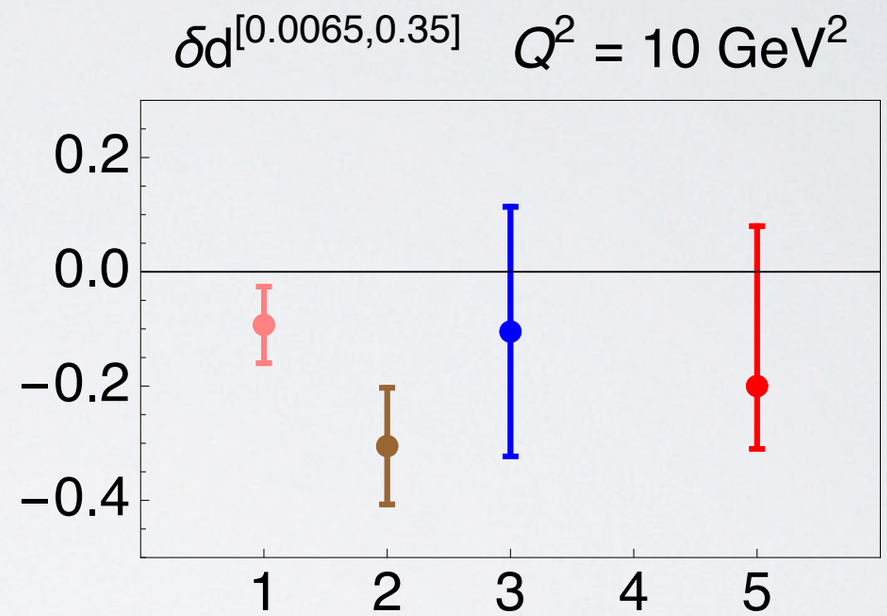
truncated tensor charge

truncated

$\delta q^{[0.0065,0.35]}$ $Q^2 = 10$

up





down

expect stability when integrating on x-range of exp. data…

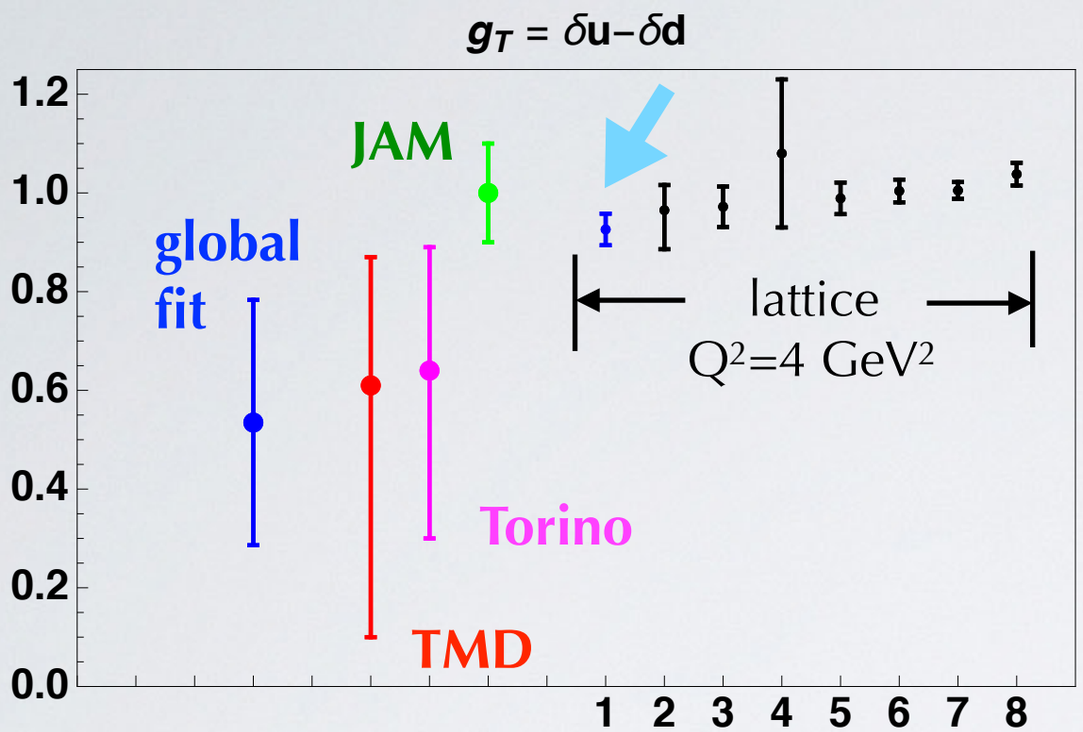
1) global fit + constrain g_T , δu , δd

2) global fit + constrain g_T

3) global fit '17 *Radici & Bacchetta, P.R.L.* ***120*** *(18) 192001*

5) "TMD fit" *Kang et al., P.R. D****93*** *(16) 014009*

Comparison on tensor charge (latest update)



1) ETMC '19 — *Alexandrou et al., arXiv:1909.00485*
2) Mainz '19 — *Harris et al., P.R. D**100** (19) 034513*
3) LHPC '19 — *Hasan et al., P.R. D**99** (19) 114505*
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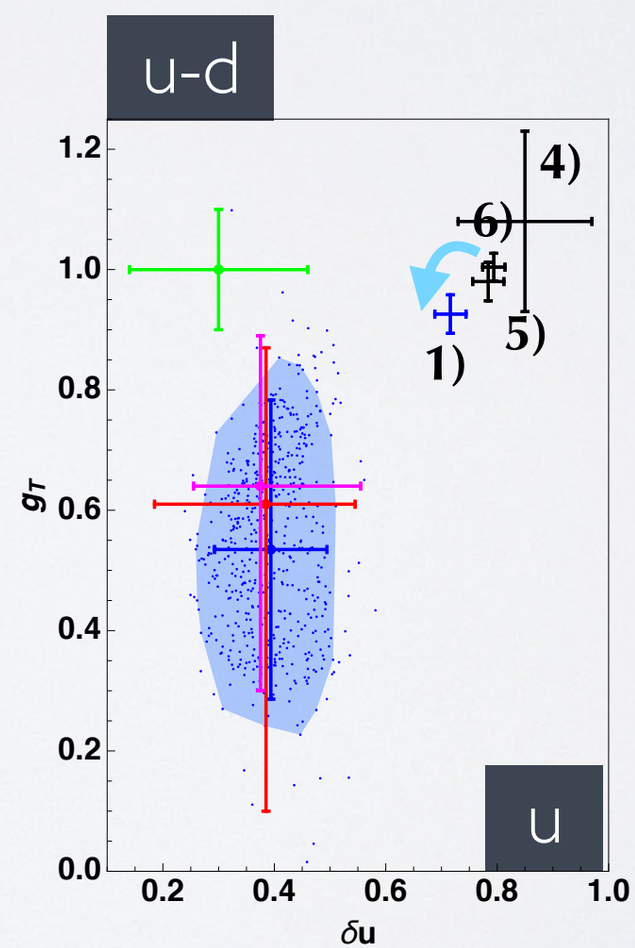
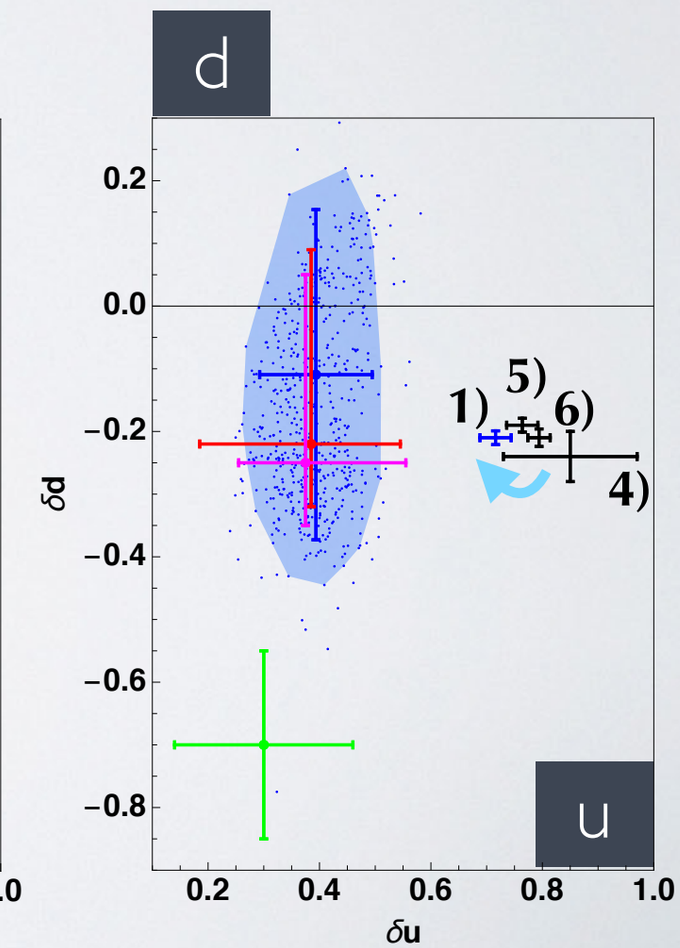
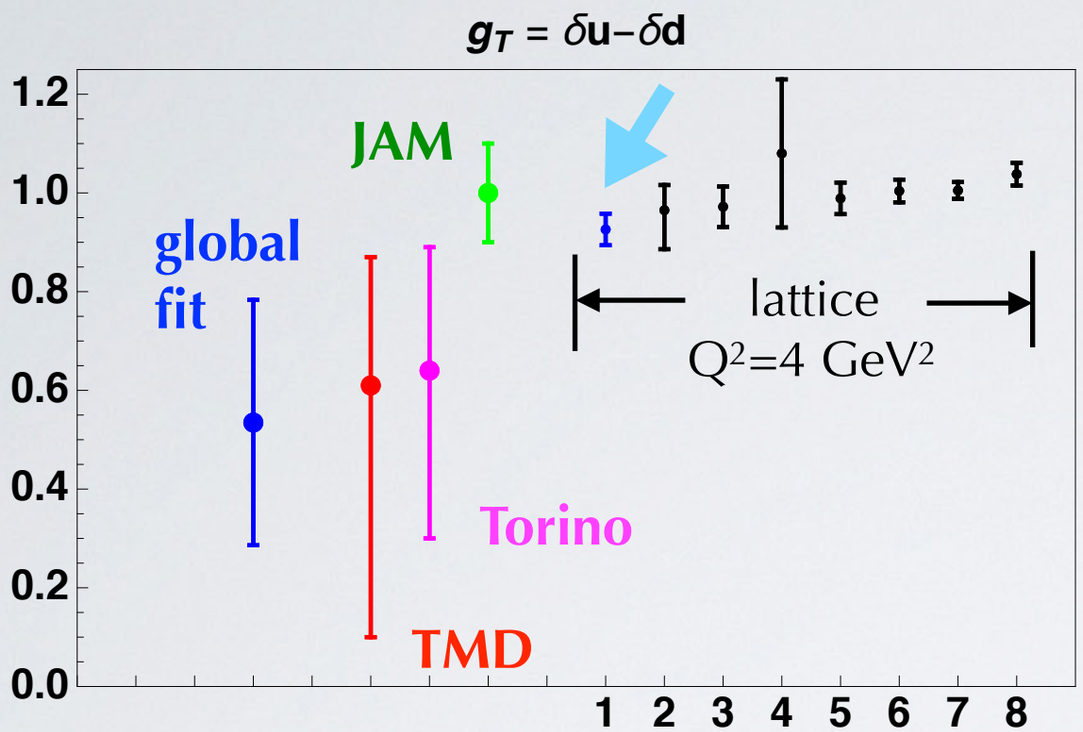
6) ETMC '17 — *Alexandrou et al., P.R. D**95** (17) 114514; E P.R. D**96** (17) 099906*
7) RQCD '14 — *Bali et al., P.R. D**91** (15) 054501*
8) LHPC '12 — *Green et al., P.R. D**86** (12) 114509*





Comparison on tensor charge (latest update)



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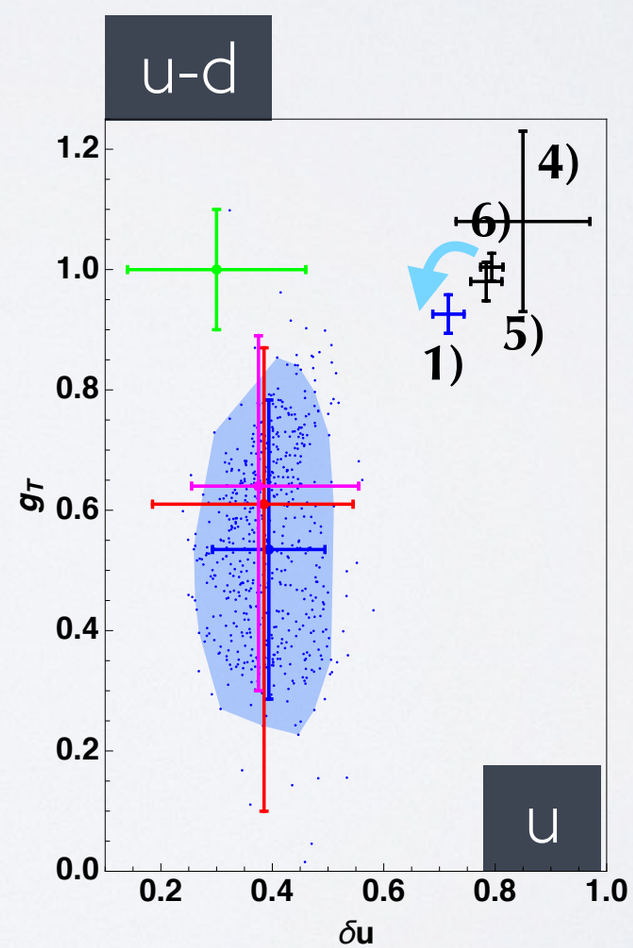
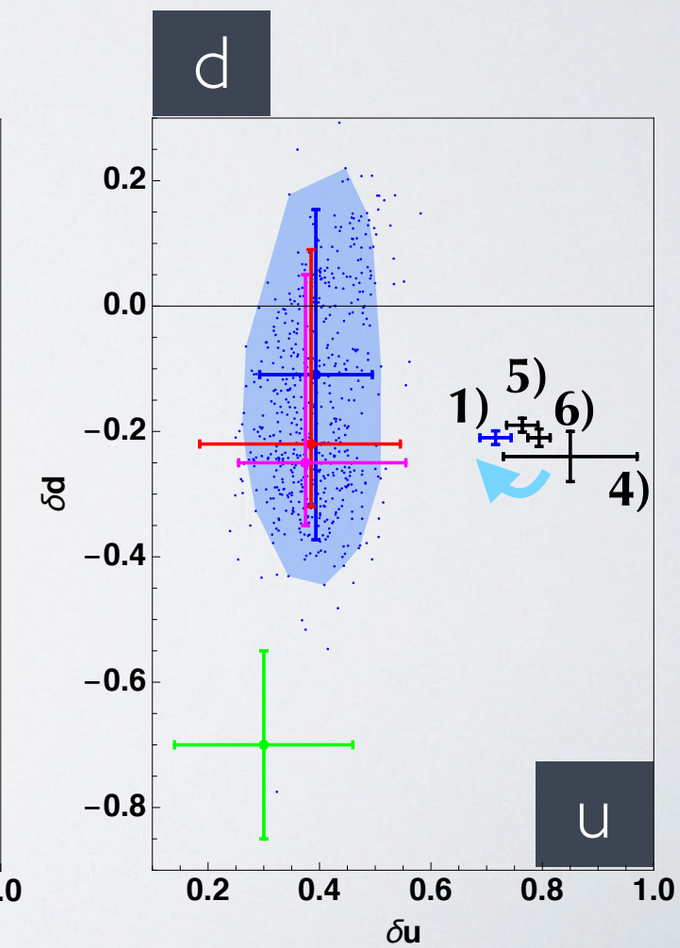




ETMC '19:
- physical m_π, no chiral extrapolation
- 1 → 3 gauge configurations
- new method for disconnected loops

→ lower δu → lower g_T
(and ~40% larger errors…)

Projections

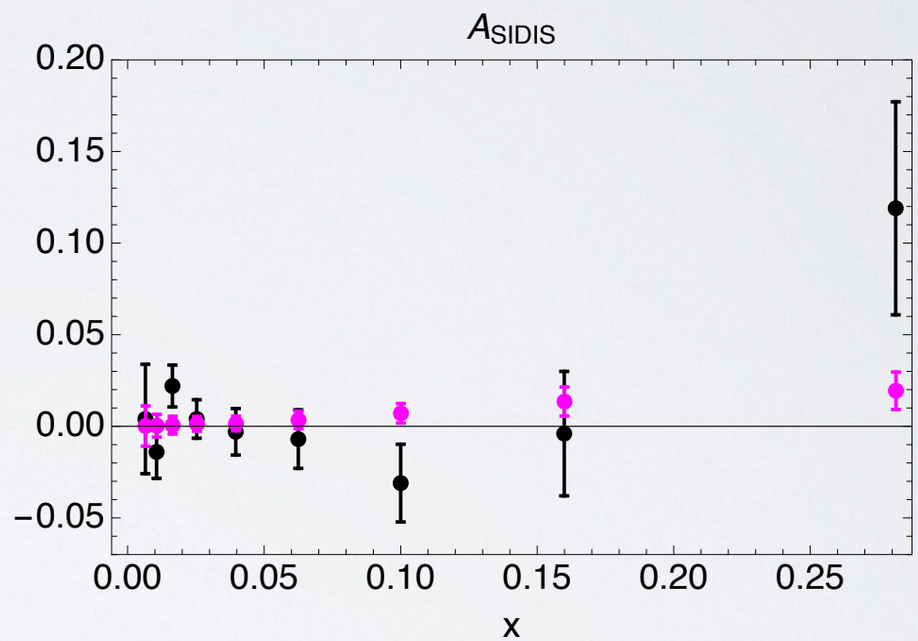
Compass pseudo-data

add to data of our global fit
a new set of SIDIS pseudo-data for **deuteron** target

Adolph et al., P.L. ***B713*** *(12)*

pseudodata

arXiv:1812.07281



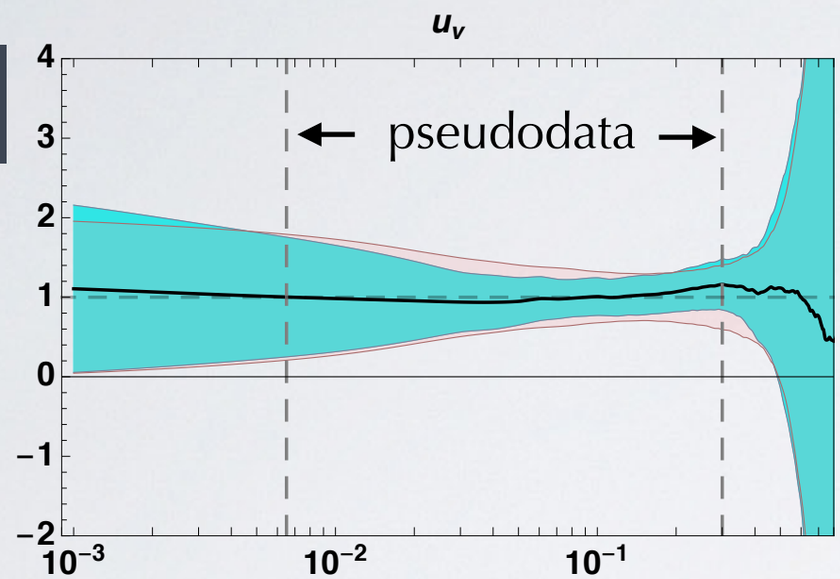
statistical error ~ 0.6 x [error in 2010 proton data]
<A> = average value of replicas in previous global fit

study impact on precision of previous global fit

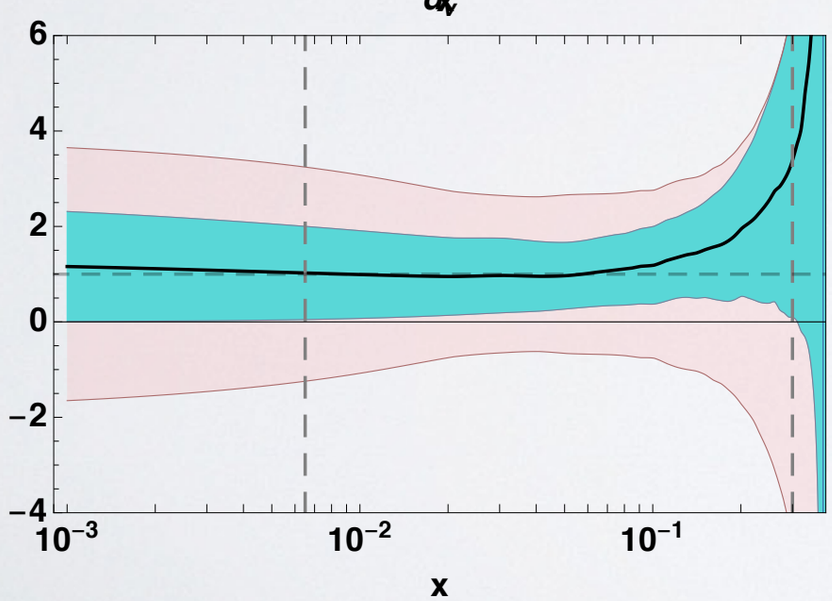
Adding Compass pseudodata

range [0.0065, x , 0.28]

u



d



uncertainty band $\left[\frac{h_{1min}}{\langle h_1 \rangle_{\substack{global \\ fit}}} \quad \frac{\langle h_1 \rangle}{\langle h_1 \rangle_{\substack{global \\ fit}}} \quad \frac{h_{1max}}{\langle h_1 \rangle_{\substack{global \\ fit}}} \right]$

deuteron target
→ better precision on down

global fit

global fit + pseudodata

CLAS12 pseudo-data

add to data of our global fit
a new set of SIDIS pseudo-data for **proton** target

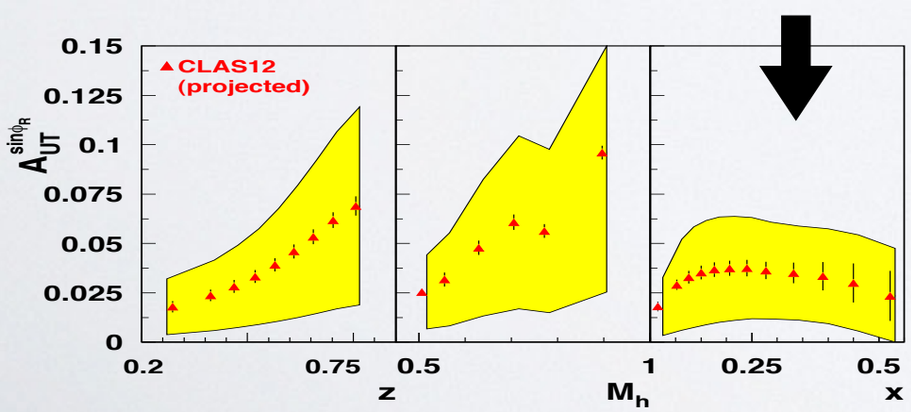
Adolph et al., P.L. ***B713*** *(12)*

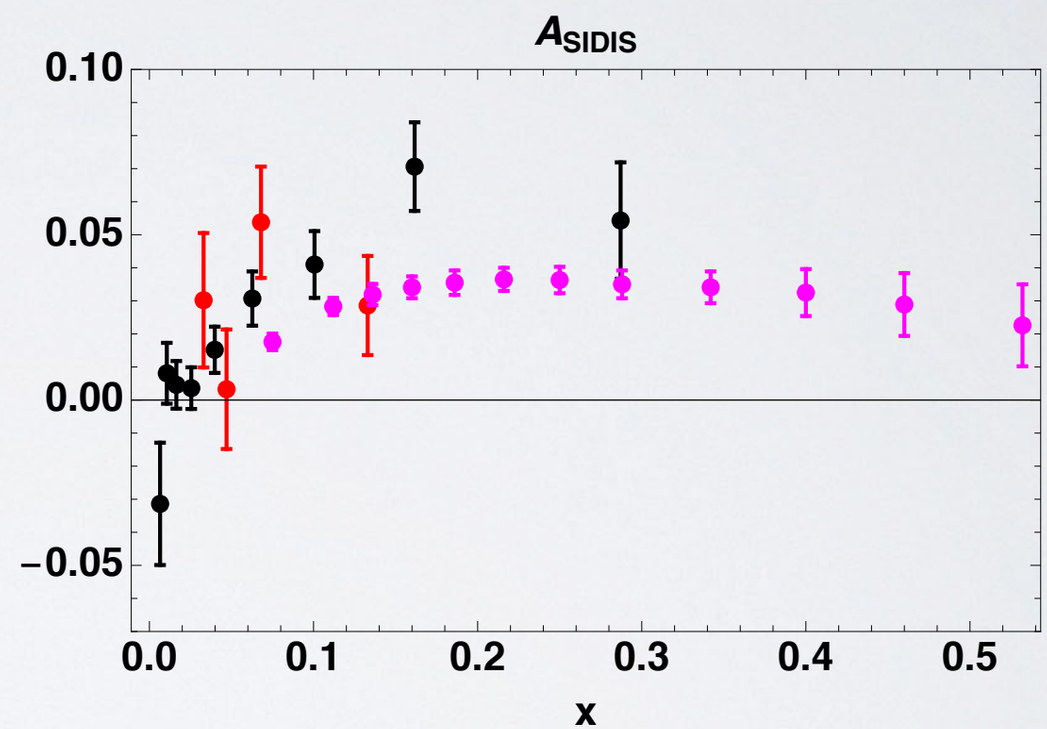
Airapetian et al.,
JHEP ***0806*** *(08) 017*

pseudodata C12-12-009

A 12 GeV Research Proposal to Jefferson Lab (PAC 39)

Measurement of transversity with dihadron production
in SIDIS with transversely polarized target





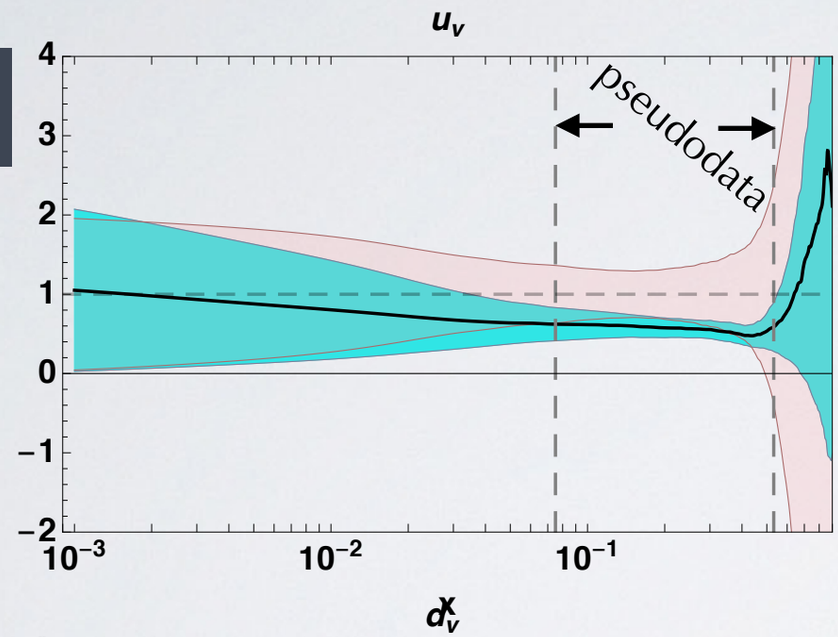
study impact on precision
of published global fit

Adding CLAS12 pseudodata

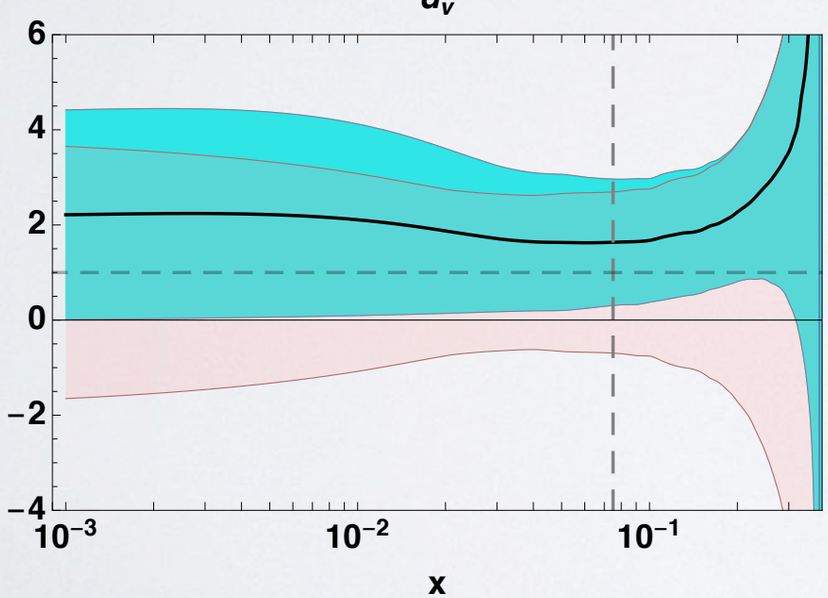
range [0.075, x , 0.53]

proposal C12-12-009

u



d



uncertainty band $\left[\frac{h_{1min}}{\langle h_1 \rangle_{\substack{global \\ fit}}} \quad \frac{\langle h_1 \rangle}{\langle h_1 \rangle_{\substack{global \\ fit}}} \quad \frac{h_{1max}}{\langle h_1 \rangle_{\substack{global \\ fit}}} \right]$

proton target
→ better precision on up

global fit

global fit + pseudodata

New PRELIMINARY results

PRELIMINARY

add to data of our global fit
the set of **STAR data at s=500 GeV²**

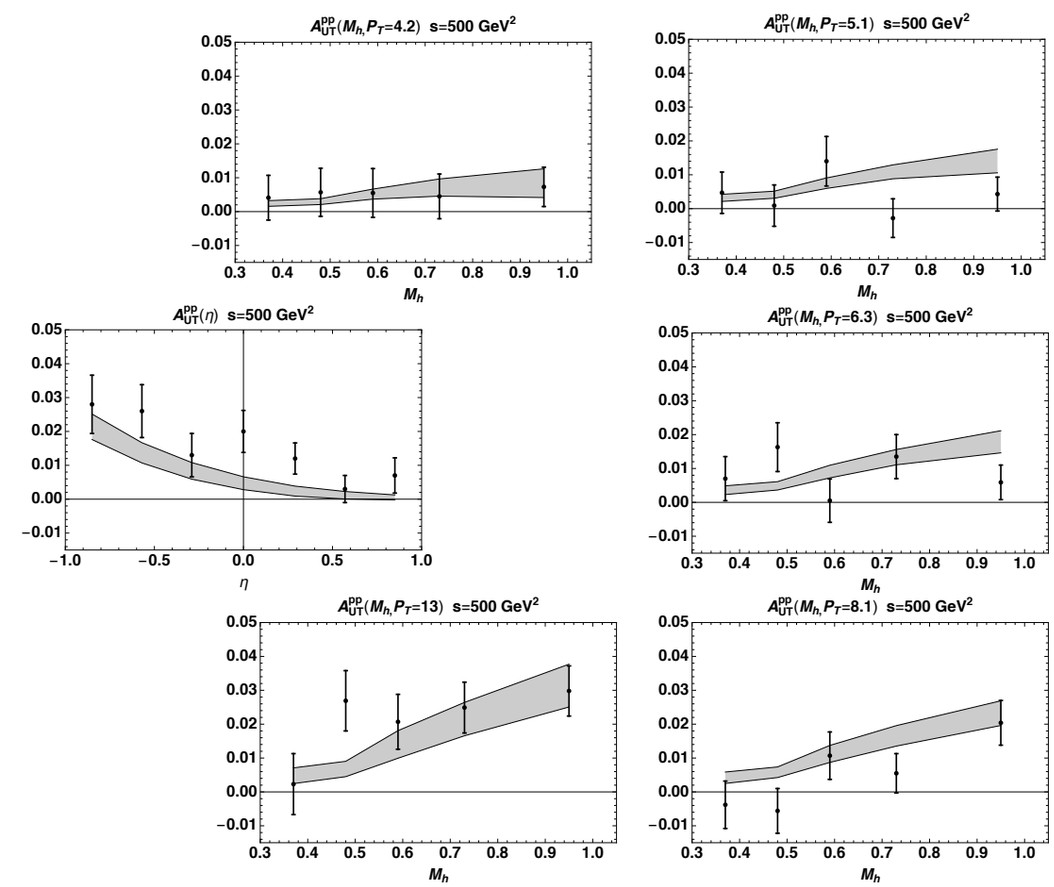
*Adamczyk et al. (STAR), P.L. **B780** (18) 332*

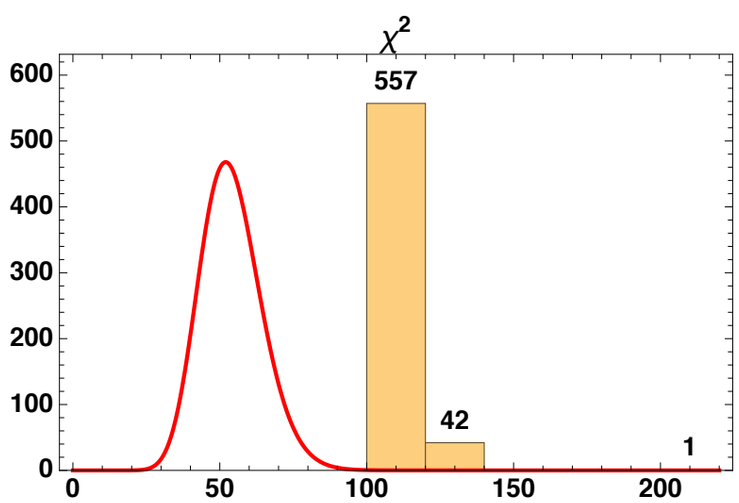
SIDIS **22** data points

pp collisions

run 2006 (s=200 GeV²) **10** indep. data points

run 2011 (s=500 GeV²) **32** indep. data points





(for χ^2/dof = 1 perfect overlap)

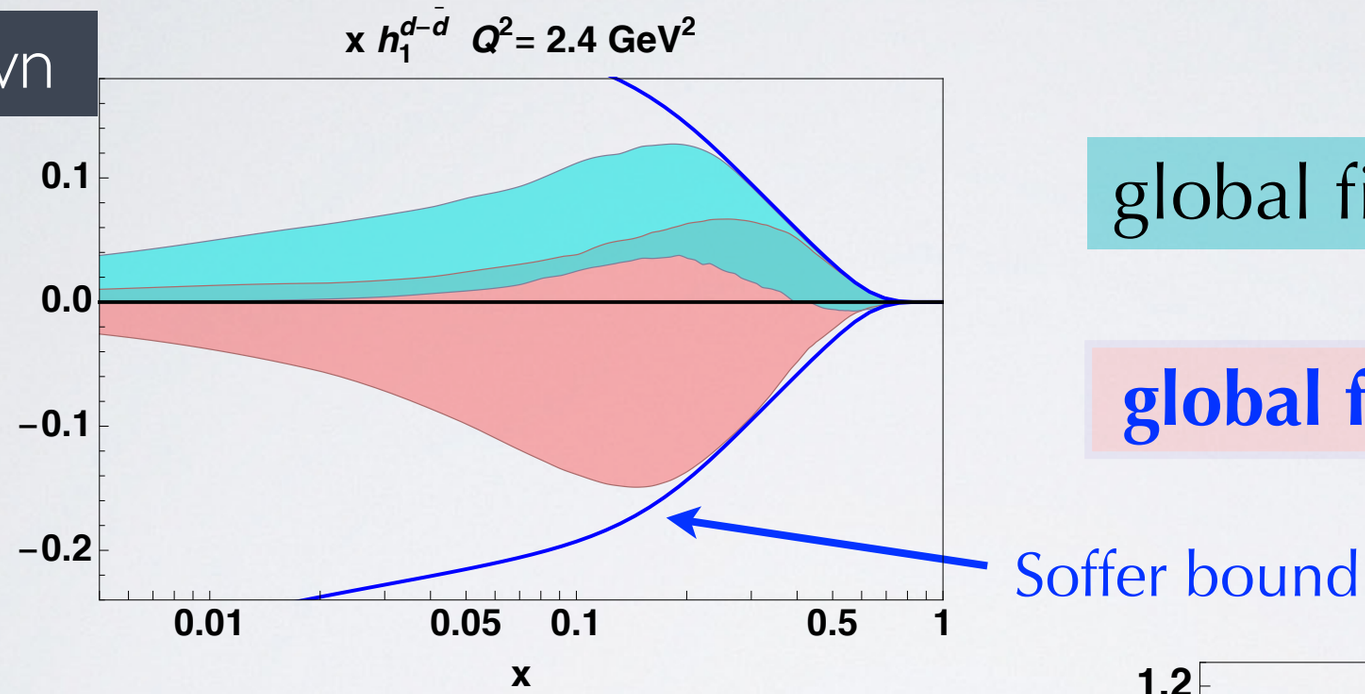
probability density function of χ^2 distribution for 54 d.o.f.

$\chi^2/\text{dof} = 2.12 \pm 0.09$

PRELIMINARY

up basically not modified

down



global fit + STAR s=500 data

global fit

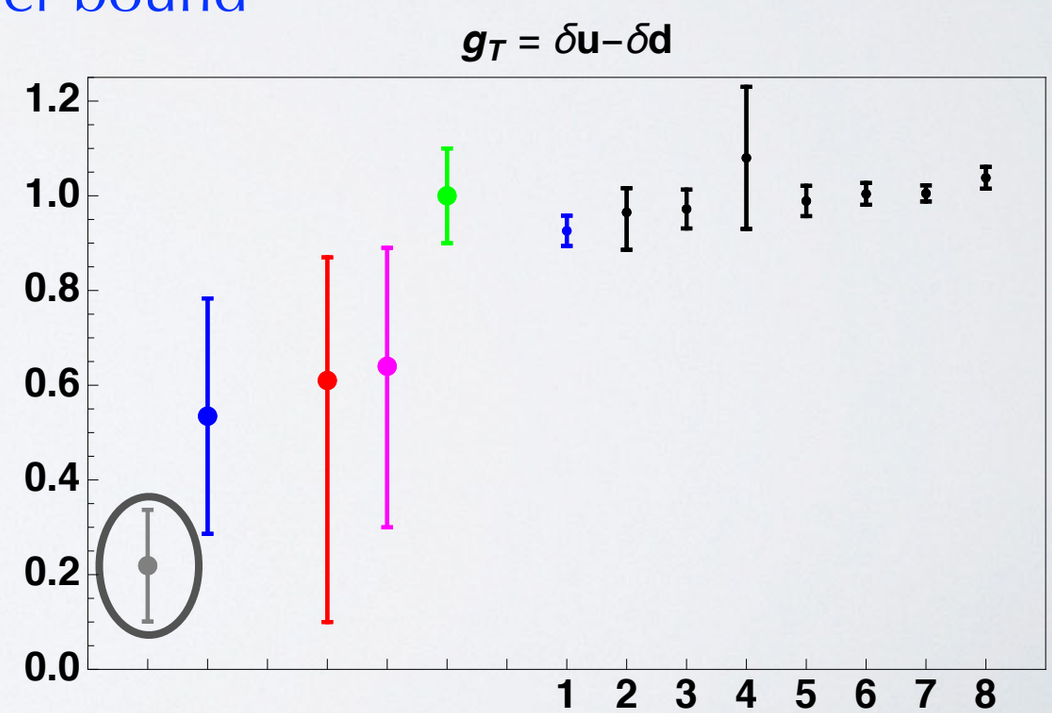
Radici and Bacchetta, P.R.L. ***120*** *(18) 192001*

more precise and fully positive

→ positive δd

→ smaller g_T = δu-δd !

unexpected opposite trend : need h_1 for sea quarks ?



Conclusions

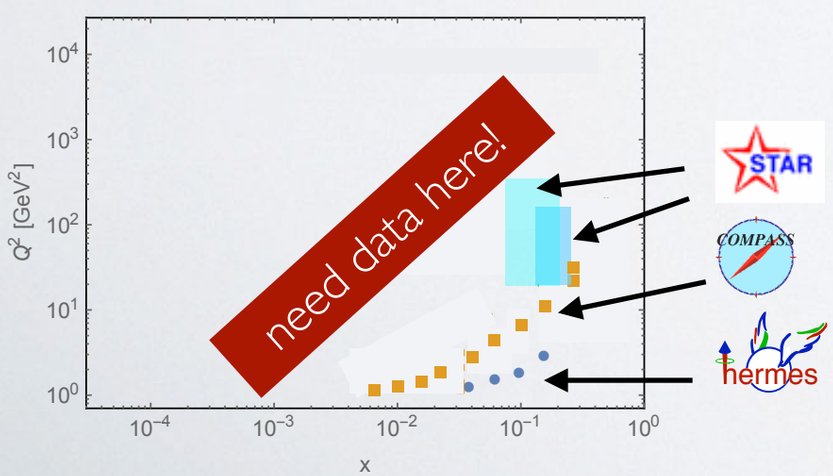
- **global fit** for h_1 is now possible as for f_1 & g_1

- uncertainty on **gluon channel in pp** collisions drives uncertainty on h_1^{down} , **need data** on **deuteron/^{3}He** and on ($\pi\pi$) **multiplicities**

- **NO** simultaneous **compatibility with lattice** for tensor charge in up, down, and isovector channels; new lattice results seem to slowly tend towards pheno points (within large errors)

- it is possible to **force compatibility** but it is **statistically very unlikely**

- adding **STAR** s=500 data gives puzzling results: need sea quarks?



- adding **Compass** and **CLAS12** SIDIS **pseudodata** increases precision of down and up, respectively

- ultimate resource: **EIC**

Backup

"X-ray" of pheno extractions of g_T^{u-d}

	mechanism	data set (independ. pts)	error analysis	quality of fit	result
our global fit *Radici & Bacchetta, P.R.L. **120** (18) 192001*	di-hadron production collinear framework DGLAP evolution LO analysis automatic Soffer Bound 10 fitting parameters	SIDIS data (22 pts) p-p data (42 pts) e+e- data (46 pts)	bootstrap (600 replicas) syst. error from unknown gluon channel in p-p	χ^2/dof = 1.76 (11) 90% confid. level (~ 2σ)	g_T = 0.53 (25) at Q^2 = 4 GeV^2
TMD *Kang et al., P.D. D**93** (16) 014009*	Collins effect TMD framework TMD evolution LO+NLL analysis enforced Soffer Bound 13 fitting parameters	SIDIS data (140 pts) **49** e+e- data (122 pts) **104**	Hessian + tolerance $\Delta\chi^2$ $\chi^2 \leq \chi^2_{min} + \Delta\chi^2$	χ^2/dof = 0.88 probab. P=92% $\Delta\chi^2$ = 22.2	g_T = 0.61 (-51 +26) at Q^2 = 10 GeV^2
Torino *Anselmino et al., P.R. D**87** (13) 094019*	Collins effect extended parton model (only DGLAP evolution of collinear part) LO analysis enforced Soffer Bound 9 fitting parameters	SIDIS data (140 pts) **49** e+e- data (32 pts)	Hessian + tolerance $\Delta\chi^2$ $\chi^2 \leq \chi^2_{min} + \Delta\chi^2$	χ^2/dof = 0.80 probab. P=95.4% $\Delta\chi^2$ = 17.21	g_T = 0.64 (-34 +25) at Q^2 = 0.8 GeV^2
JAM *Lin et al., P.R.L. **120** (18) 152502*	Collins effect TMD framework extended parton model (no evolution) LO analysis 19 fitting parameters	SIDIS data (106 pts) **45** +constraint on lattice g_T from only multiple $\{a, m_\pi, L\}$	Bayesian statistics + Nested Monte Carlo	χ^2/dof = 0.79	g_T = 1.0 (1) at Q^2 = 2 GeV^2

the FLAG standard

*Aoki et al. (the **F**lavor **L**attice **A**verage **G**roup), arXiv:1902.08191*

"What is the best lattice result for a given observable ?" *

or, better,

How do we compare two different lattice computations of the same observable ?

* critique on averaging method:

$$Q_i = x_i + \sigma_i \ , \quad i = 1, .., \#\mathrm{exp}$$

$$\sigma_i = \sqrt{[\sigma_i^{\mathrm{stat}}]^2 + \sum_{j=1}^{\mathrm{syst}} [\sigma_i^{(j)}]^2}$$

$$\omega_i = \frac{\sigma_i^{-2}}{\sum_{j=1}^{\#\mathrm{exp}} \sigma_j^{-2}} \qquad x_{\mathrm{av}} = \sum_{i=1}^{\#\mathrm{exp}} x_i \, \omega_i$$

emphasizes measurements with smaller errors σ_i

Definition of FLAG standard

*Aoki et al. (the **F**lavor **L**attice **A**verage **G**roup), arXiv:1902.08191*

rate / error	★	○	■
chiral extrapolation	$m_{\pi,\min} < 200$ MeV and $m_{\pi,i}$ $i \geq 3$ **or** $125 < m_{\pi,\min} < 145$ and $m_{\pi,2} < 200$	$200 < m_{\pi,\min} < 400$ and $m_{\pi,i}$ $i \geq 3$ **or** $i=2$ and $m_{\pi,\min} < 200$ **or** $125 < m_{\pi} < 145$	otherwise
continuum extrapolation	$\{a_1, ..., a_i, ...\}$ $i \geq 3$ $a_l, a_m < 0.1$ fm $\left(\frac{a_{\max}}{a_{\min}}\right)^2 \geq 2$	$\{a_1, ..., a_i, ...\}$ $i \geq 2$ $a_l < 0.1$ fm $\left(\frac{a_{\max}}{a_{\min}}\right)^2 \geq 1.4$	otherwise
finite-volume effects	$\left(\frac{m_{\pi,\min}}{200}\right)^2 e^{4-m_{\pi,\min} L^{\max}(m_{\pi,\min})} < 1$ or $L_1 \neq L_2 \neq L_3 > 2\,\mathrm{fm}$	$\ldots e^{3-\cdots} < 1$ or $L_1 \neq L_2 > 2\,\mathrm{fm}$	otherwise
renormalization	nonperturbative (RI-MOM)	perturbative (1-loop or higher) + trunc. error estimate	otherwise
excited states	$(\text{source-sink})_i$, $i \geq 3$ and two of them >1 fm	$(\text{source-sink})_i$, $i \geq 2$ and one of them >1 fm	otherwise

"X-ray" of lattice calculations of g_T^{u-d}

FLAG rates

	main features	extrapolations: chiral	extrapolations: continuum	extrapolations: volume	renorm.	exct. states	errors
ETMC 19 *Alexandrou et al., arXiv:1909.00485*	2+1+1 dynam. flavors m_π = 139 MeV 2 a : 0.08, 0.094 fm 3 own config's	○	○	★	★	★	g_T = 0.926 (32) syst. renorm. and excited state cont. & vol. effects not seen, but no study of extrapol. error
LHPC 19 *Hasan et al., P.D. D99 (19) 114505*	2+1 dynam. flavors m_π = 133 MeV 2 a : 0.093, 0.116 fm hundreds config.'s	○	○	★	★	★	g_T = 0.972 (41) syst. renorm. and excited states no cont. & vol. extrapol. errors discretiz. error (underestimated?)
Mainz 19 *Harris et al., P.R. D100 (19) 034513*	2+1 dynam. flavors 353 > m_π > 203 MeV 4 a : 0.050, … , 0.086 fm 11 config.'s (CLS)	○	★	★	★	★	g_T = 0.965 (38) (+13 -41) (38) = stat. (+13 -41) = syst. from renorm., excited states and combined
JLQCD 18 *Yamanaka et al., P.R. D98 (18) 054516*	2+1 dynam. flavors 540 > m_π > 290 MeV 1 a : 0.11 fm	○	■	○	★	★	g_T = 1.08 (3) (3) (9) (3) = stat. (3) = syst. chiral extrapol. (9) = syst. discretiz. effects
PNDME 18 *Gupta et al., P.R. D98 (18) 034503*	2+1+1 dynam. flavors 3 m_π : 315, 225, 135 MeV 4 a : 0.06, … , 0.15 fm 11 config's (MILC+HISQ)	★	★	★	★	★	g_T = 0.989 (32) (10) (10) = extrapolations (simult. fit) (32) = stat. + all other syst.
ETMC 17 *Alexandrou et al., P.R. D95 (17) 114514; E P.R. D96 (17) 099906*	2 dynam. flavors m_π = 135 MeV only 1 a : 0.0938 only 1 config.	○	■	○	★	★	g_T = 1.004 (21)(02)(19) (21) = stat. (02) = syst. renorm. (19) = syst. excited states no cont. & vol. extrapol. errors
RQCD 14 *Bali et al., P.R. D91 (15) 054501*	2 dynam. flavors 490 > m_π > 150 MeV 3 a : 0.06, …, 0.081 fm several config's (RQCD+QCDSF)	★	○	★	★	■	g_T = 1.005 (17) (29) (17) = stat. + syst. (chiral fit, ..) (29) = syst. continuum no volume effects visibile
LHPC 12 *Green et al., P.R. D86 (12) 114509*	2+1 dynam. flavors 597 > m_π > 149 MeV 4 a : 0.084, … , 0.124 fm various config's (BMW, RBC, …)	★	■	★	★	★	g_T = 1.038 (11) (12) (11) = stat. (12) = syst. excited states no cont. & vol. extrapol. errors