Nucleon transversity distribution from lattice QCD

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PDFLattice 2019, 25 September, 2019,

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Introduction

- To leading-twist accuracy, the quark structure of hadrons is characterized by three distribution functions:

- Unpolarized distribution

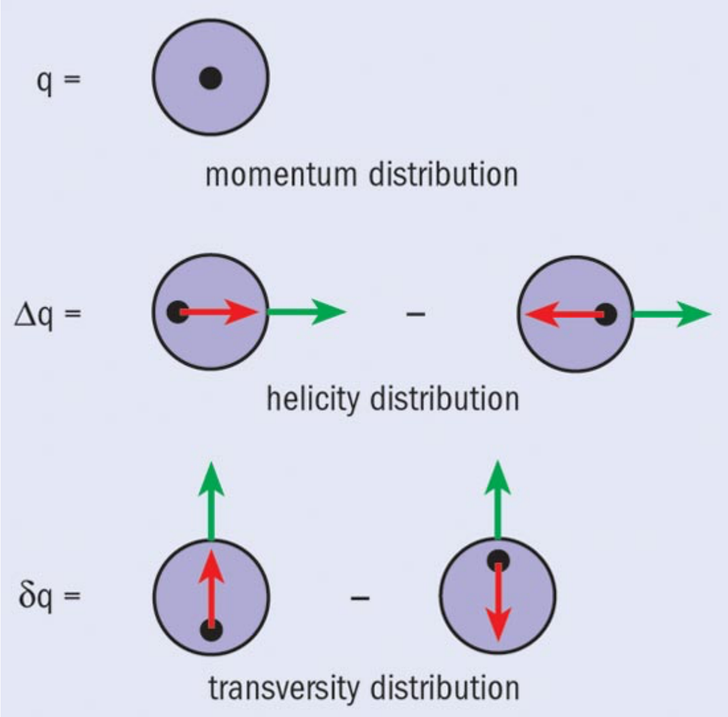
$$q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

- Helicity distribution

$$\Delta q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

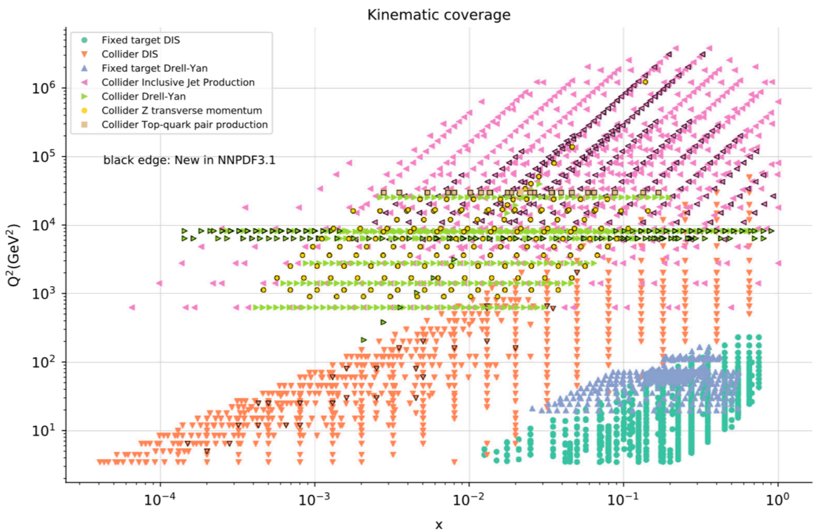
- Transversity distribution

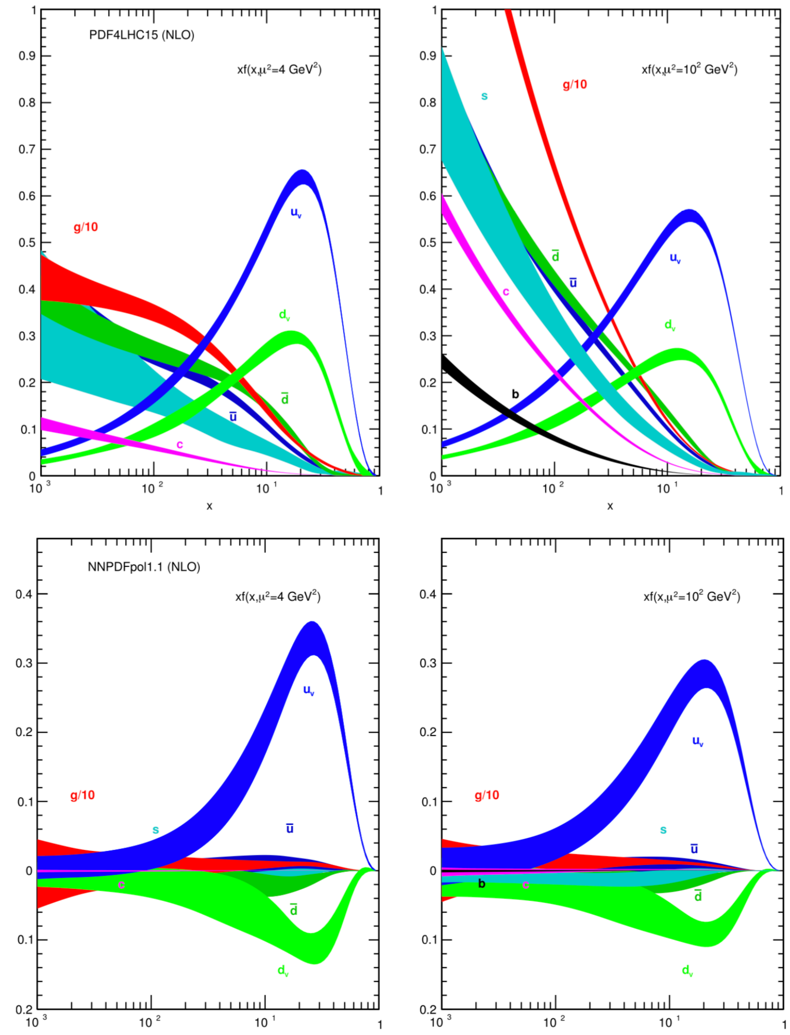
$$\delta q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma^\perp\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

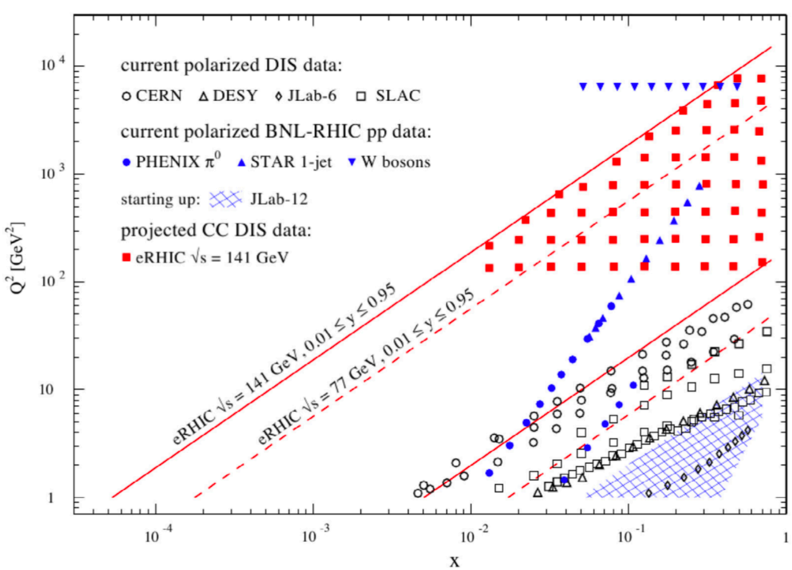


Introduction

- For the unpolarized and helicity distribution, considerable information has been accumulated





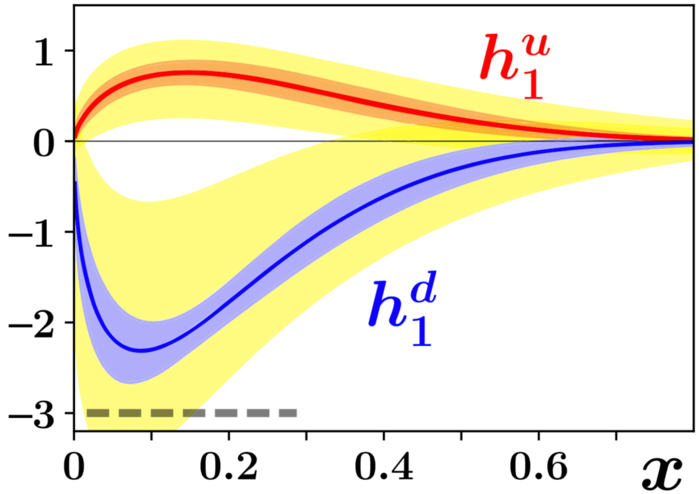


Lin et al, Prog.Part.Nucl.Phys. 18'

Introduction

- For the unpolarized and helicity distribution, considerable information has been accumulated

- In contrast, much less is known about the transversity distribution
  - Chiral-odd function. To measure it, chirality needs to be flipped twice
  - Semi-inclusive DIS, hadronic collisions



Lin, Melnitchouk, Prokudin, Sato, Shows III, PRL 18'

Lattice QCD can help QCD global analysis

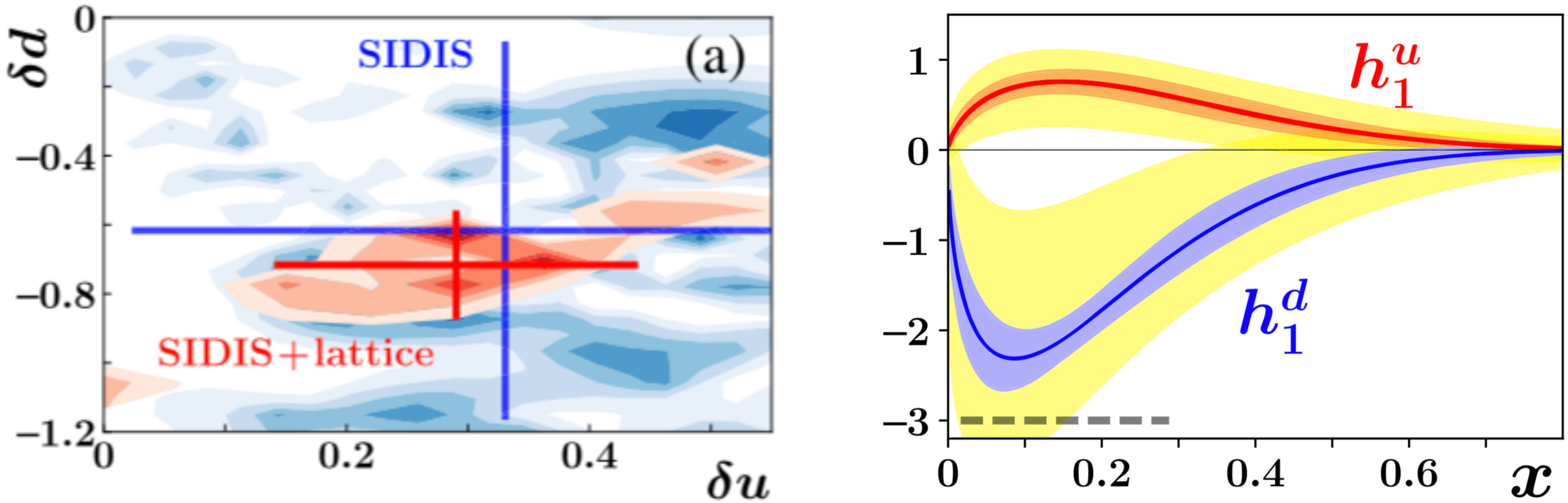
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O(\bar{\psi},\psi,A)$$

  - in Euclidean spacetime

$$t = -i\tau, \qquad e^{iS_M} = e^{-S_E}$$

  - Traditionally, the information on PDFs is extracted from lattice calculations of their moments
  - Transversity fit improved by lattice tensor charge



Lin, Melnitchouk, Prokudin, Sato, Shows III, PRL 18'

Lattice QCD can help QCD global analysis

- Accessing the full PDF also becomes possible on the lattice
  - [Liu and Dong, PRL 94']
  - [Detmold and Lin, PRD 06']
  - [Braun and Müller, EPJC 08']
  - [Davoudi and Savage, PRD 12']
  - [Ji, PRL 13' & Sci. China Phys. Mech. Astron. 14']
  - [Ma and Qiu, 14' & PRL 17']
  - [Chambers et al., PRL 17']
  - [Radyushkin, PRD 17']

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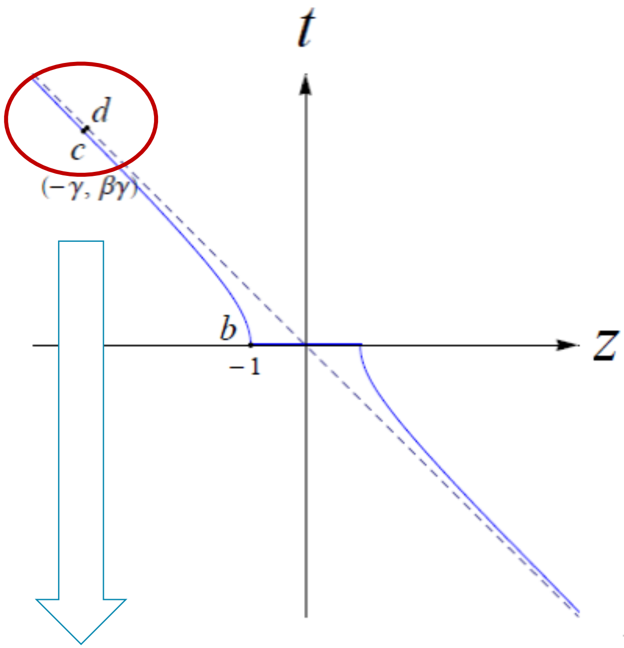
Large momentum effective theory



- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
  - Appropriately chosen quasi-PDF can be calculated on the lattice, e.g.

$$\tilde{q}(x,\tilde{\mu},P_z) = \int \frac{dz}{4\pi} e^{ixzP_z} \langle P|\bar{\psi}(z)\gamma^t L(z,0)\psi(0)|P\rangle, \qquad L(z,0) = Pe^{-ig\int_0^z dz' A_z(z')}$$

  - A finite but large already offers a good approximation, where (leading) frame-dependence can be removed through a factorization formula

$$\tilde{q}(x,\tilde{\mu},P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\tilde{\mu}}{yP_z}, \frac{\tilde{\mu}}{\mu}\right) q(y,\mu) + h.t.$$

PDFs from lattice

Bare lattice matrix element

Non-pert. Renorm.

↓

renormalized matrix element

Ji, JHZ, Zhao, PRL 18'
Ishikawa et al, PRD 17'
Green et al, PRL 18'
Stewart, Zhao, PRD 18'
Chen, JHZ et al, PRD 18'
Alexandrou et al, NPB 17'
Monahan, Orginos, JHEP 17'
Radyushkin PRD 17' & Orginos et al, PRD 17'
JHZ et al, PRL 19' & Wang, JHZ et al, 19'
Li et al, PRL 19'

$$\tilde{h}(z, Pz, a^{-1}) = \langle PS | O_\Gamma(z) | PS \rangle, \qquad O_\Gamma(z) = \bar{\psi}(z)\gamma^x \gamma^t \gamma_5 L(z,0)\psi(0)$$

$$\tilde{h}_R(z, P_z, p_z^R, \mu_R) = Z^{-1}(z, p_z^R, a^{-1}, \mu_R)\tilde{h}(z, P_z, a^{-1})\Big|_{a\to 0},$$

$$Z(z, p_z^R, a^{-1}, \mu_R) = \frac{\langle ps | O_\Gamma(z) | ps \rangle}{\langle ps | O_\Gamma(z) | ps \rangle_{\rm tree}}\Bigg|_{p^2=-\mu_R^2,\, p_z=p_z^R}$$

PDFs from lattice

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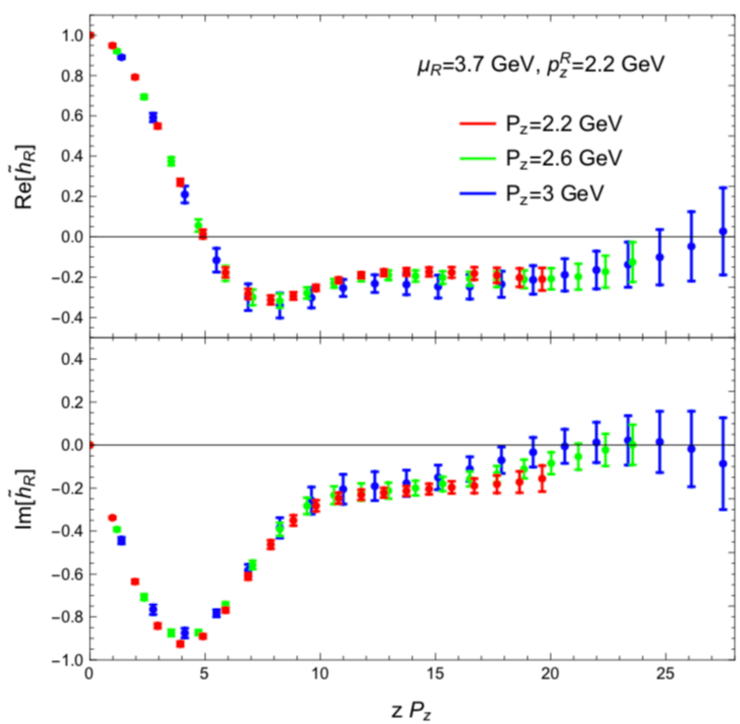
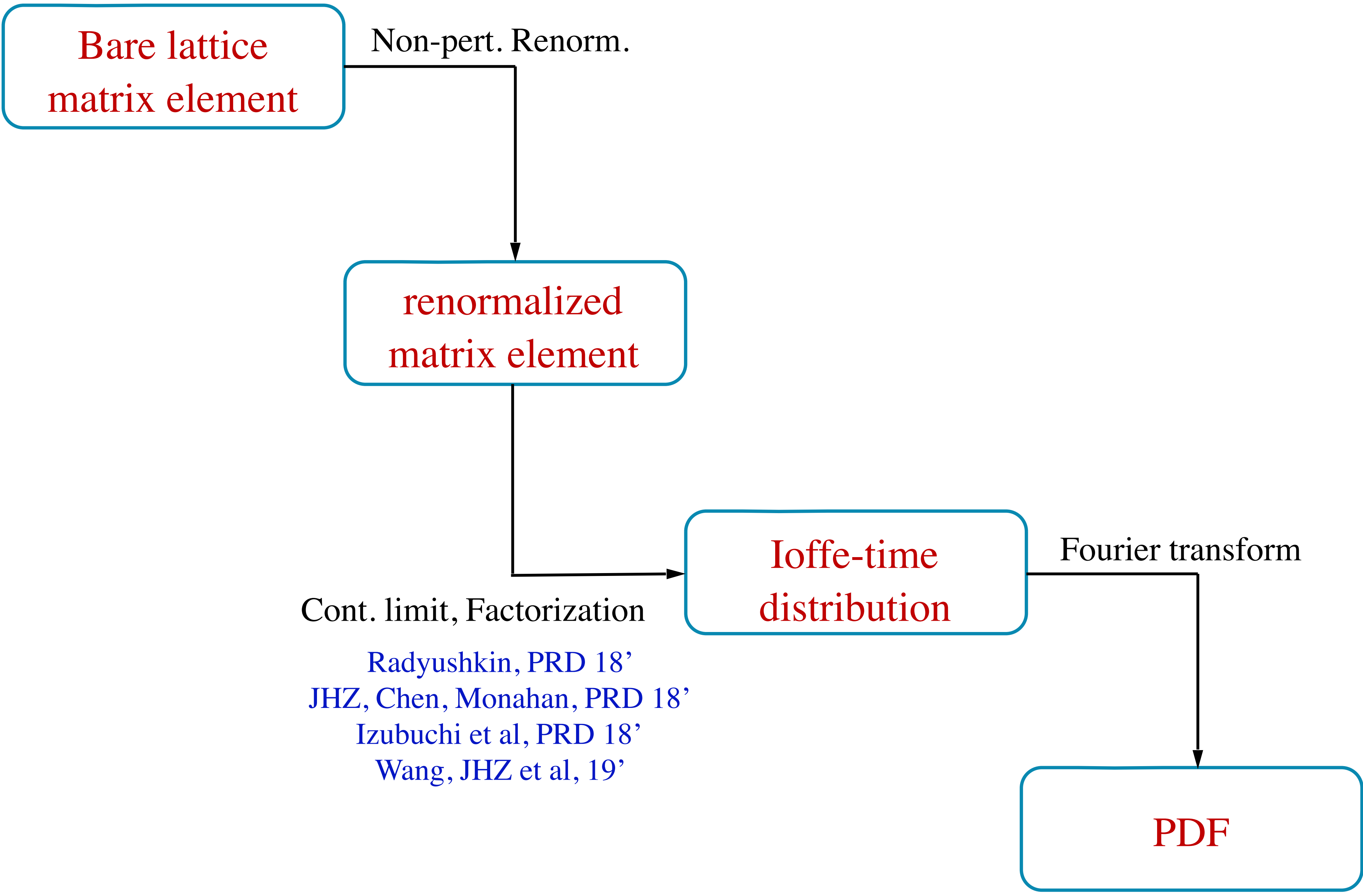


PDFs from lattice



PDFs from lattice

Bare lattice matrix element

Non-pert. Renorm.

renormalized matrix element

Cont. limit, Fourier transform

$$\delta\tilde{q}_R(x, P_z, p_z^R, \mu_R) = \int \frac{dz}{4\pi} e^{ixzP_z} \tilde{h}_R(z, P_z, p_z^R, \mu_R)$$

Quasi-PDF

Factorization

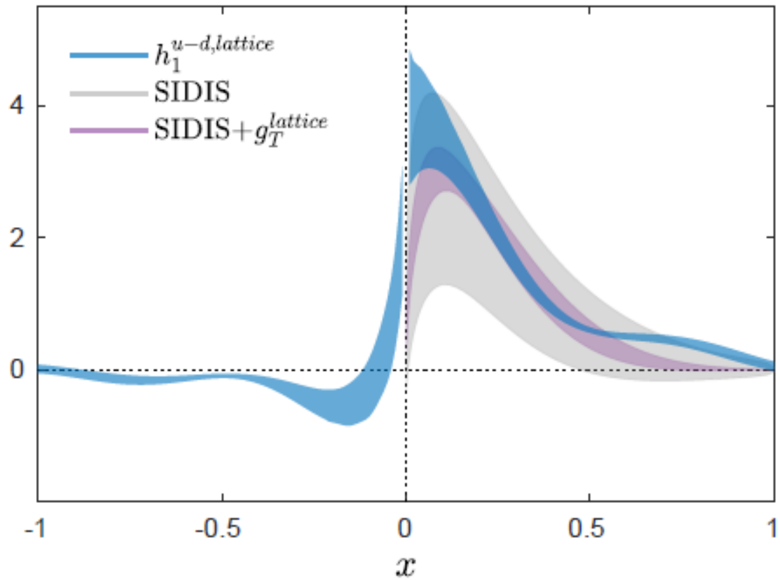
$$\delta\tilde{q}_R(x, P_z, p_z^R, \mu_R)$$

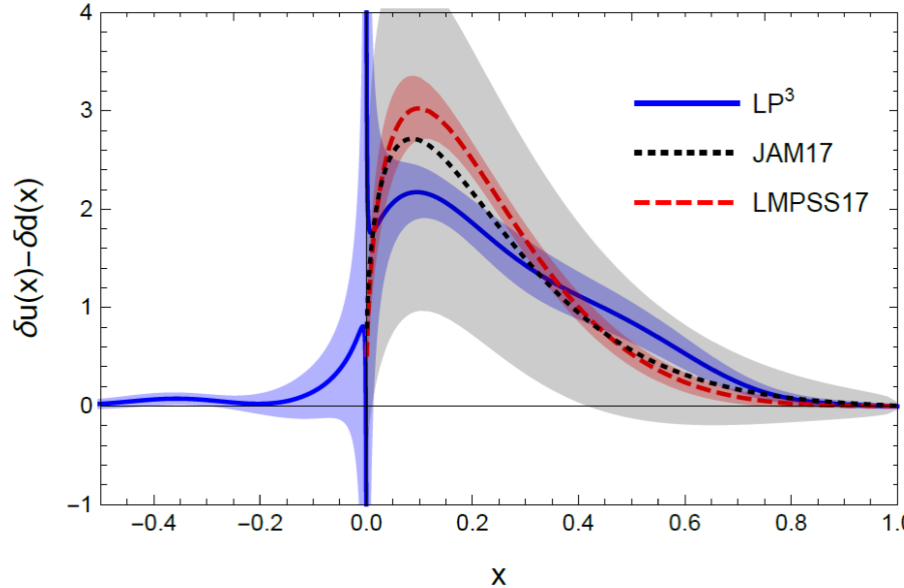
$$= \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu_R^2}{(p_z^R)^2}, \frac{\mu}{yP_z}, \frac{p_z^R}{yP_z}\right) \delta q(y, \mu) + h.t.$$

Ji, PRL 13'
Xiong, Ji, JHZ, Zhao, PRD 14'
Chen, JHZ et al, PRD 18'
Stewart, Zhao, PRD 18'
Izubuchi et al, PRD 18'
Wang, JHZ et al, 19'

PDF

Transversity distribution from lattice QCD



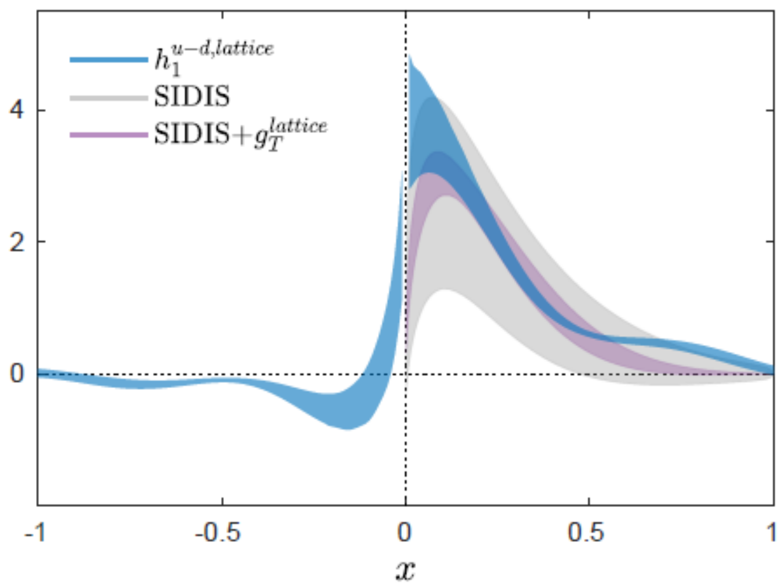


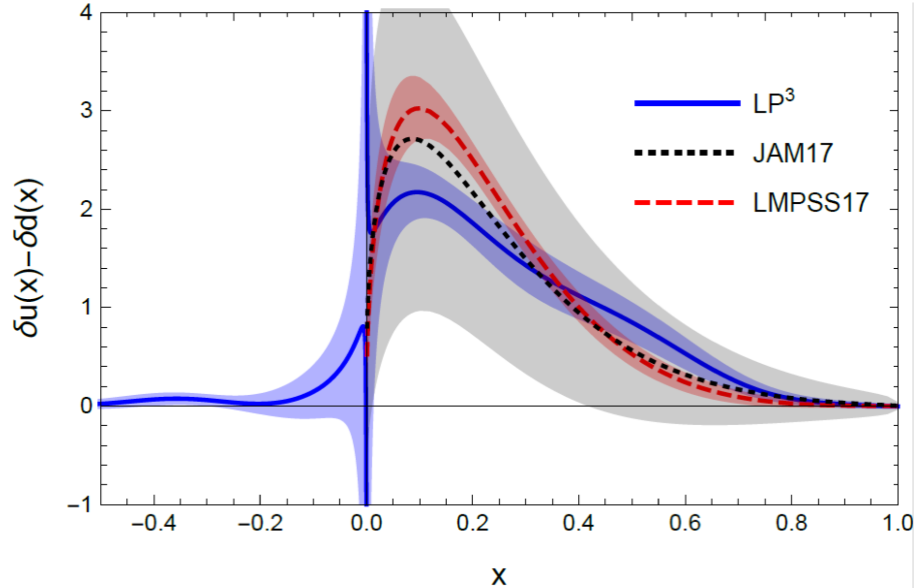
Alexandrou et al, PRD 18$'$, $m_\pi \approx 130\,MeV$, $a = 0.094\,fm, L \approx 4.5\,fm$

LP^3, 18$'$, $m_\pi \approx 135\,MeV$, $a = 0.09\,fm, L \approx 5.8\,fm$

- Qualitative agreement with improved transversity fit
- Error budget incomplete
- Sea flavor asymmetry?

Transversity distribution from lattice QCD





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- Reduce scale dependence: higher-order perturbative corrections
- Long range correlations: calculating/parametrizing higher-twist contributions

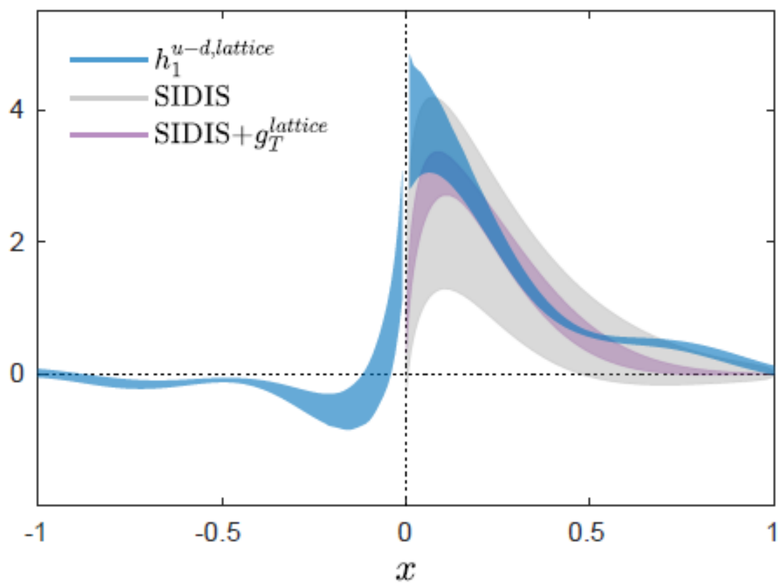
$$\tilde{q}(x, P_z) = q(x)\left\{1 + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{x^2(1-x)P_z^2}\right)\right\}$$

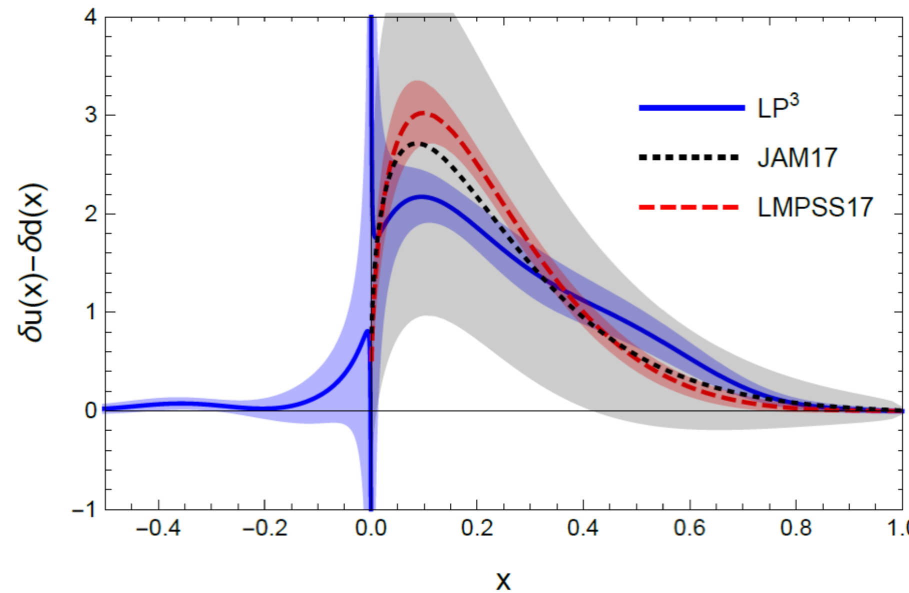
- Useful ansatz for fitting such contributions

Braun, Vladimirov, JHZ, PRD 19'

- Eventually combine all lattice data and fit?

Transversity distribution from lattice QCD





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- Useful ansatz for fitting such contributions — Braun, Vladimirov, JHZ, PRD 19'
- Lattice systematics